Quark mass and isospin dependence of the deconfining critical temperature

E. S. Fraga,[1] L. F. Palhares,[1] and C. Villavicencio[2]

[1]*Instituto de Física, Universidade Federal do Rio de Janeiro, C.P. 68528, Rio de Janeiro, RJ 21941-972, Brazil*
[2]*Departamento de Física, Comisión Nacional de Energía Atómica, (1429) Buenos Aires, Argentina*
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We describe the deconfining critical temperature dependence on the pion mass and on the isospin chemical potential in remarkably good agreement with lattice data. Our framework incorporates explicit dependence on quark masses, isospin and baryonic chemical potentials for the case of two flavors through ingredients from well-known high- and low-energy theories. In the low-energy sector, the system is described by a minimal chiral perturbation theory effective action, corresponding to a hot gas of pion quasiparticles and heavy nucleons. For the high-temperature sector we adopt a simple extension of the fuzzy bag model. We also briefly discuss the effects of mass asymmetry and baryon chemical potential.

I. INTRODUCTION

The phase diagram of quark matter has been the object of intense investigation during the last years, and yet several open questions within the thermodynamics of strong interactions still remain unsolved [1,2]. In this quest, lattice QCD represents the main nonperturbative approach within the full theory [3], always complemented by effective models [4].

In this article we investigate the effects of finite quark masses and isospin number on the equation of state of hot and dense strongly interacting matter and on the deconfining phase transition within a framework inspired by chiral perturbation theory (χPT) and lattice results for the pressure and the trace anomaly. The setting we propose is simple and *completely* fixed by vacuum QCD properties (measured or simulated on the lattice) and lattice simulations of finite temperature QCD. More explicitly, there is no fitting of mass or isospin chemical potential dependence at all. Nevertheless, our findings for the behavior of the critical temperature as a function of *both* the pion mass and the isospin chemical potential are in remarkably good agreement with lattice data. It is crucial to note that several detailed studies of chiral models failed to describe $T_c(m_\pi)$ [5–7], while Polyakov-loop models, whose predictions can be fitted to the lattice points for $T_c(m_\pi)$ [5], cannot at the present form address isospin effects. In this approach, we can also investigate in a straightforward manner the effects of quark-mass asymmetry and nonzero baryon chemical potential, physical cases in which the sign problem develops, constraining systematic lattice studies. The predictions within our framework for these regimes, which are not yet fully probed by lattice QCD, are left for a longer publication (for preliminary results, see [8]). Throughout the present paper, the baryon chemical potential is fixed to zero. It is not our aim in this short paper to provide a complete description of the full richness of the QCD phase diagram. Still, we present a description of the dependence of the critical deconfining temperature on the quark-average mass and on the isospin chemical potential *simultaneously.*

Small quark masses and a nonzero baryon chemical potential have always represented a major challenge for lattice simulations. Presently, although viable lattice sizes still prevent extensive and precise studies with realistic quark masses, and the sign problem considerably restricts the applicability of Monte Carlo simulations to the description of chemically asymmetric media, lattice QCD is starting to provide results with smaller quark masses, and probing a larger domain at finite chemical potential [9–11]. For the latter, it is believed that much can be learned from simulations of realizations of QCD that avoid the sign problem, such as those with vanishing baryon chemical potential and finite isospin density, which has a positive fermionic determinant.

QCD at finite isospin density is certainly, but not only, a playground to test numerical approaches to the case of finite baryon density on the lattice [12,13]. It is also part of the physical phase diagram for strong interactions, and exhibits a very rich phenomenology [14]. It has been under careful study during the last years, both theoretically and experimentally, with a clear identification of certain phenomena that depend directly on the isospin asymmetry of nuclear matter at intermediate-energy heavy ion experiments (see Ref. [15] and references therein).

Nevertheless, theoretical and phenomenological studies often focus on the chiral limit of QCD, putting aside effects from finite quark masses, and on isospin symmetric hot matter, mainly stimulated by the physical scenario found in current high-energy heavy ion collision experiments [16] and the quark-gluon plasma [17]. Some exceptions are, however, chiral model analyses of pion-mass dependence of the finite temperature transition (e.g., [5–7]), which fail completely to reproduce the lattice behavior, and Polyakov-loop models predictions as mentioned above. At finite isospin chemical potential, though there are phase-diagram investigations in Polyakov–Nambu–Jona-Lasinio models [18], none of them has addressed the

critical temperature dependence on the isospin chemical potential, in particular.

To investigate the effects of nonzero quark masses and isospin asymmetry on the deconfining transition, we build an effective theory that combines ingredients from χPT in the low-energy sector with the phenomenological fuzzy bag model at high energy. The high-energy regime is described perturbatively by two-flavor QCD with massive quarks and explicit isospin symmetry breaking. Nonperturbative (confinement) effects at this scale are incorporated through a fuzzy bag description [19] with coefficients extracted from lattice data. For the low-energy sector, we adopt an effective action inspired by χPT that exhibits the same structure of symmetries contained in the high-energy theory [20]. The quasinucleon degrees of freedom described in this regime seem to be crucial for understanding the pion-mass dependence of the deconfining temperature. The definition of parameters, as well as masses, is such that variations in the deconfined sector are totally consistent with variations in the confined one, which guarantees the bookkeeping in the different degrees of freedom that are present in the description.

By matching the two branches of the equations of state, corresponding to the high- and low-temperature regimes, we of course obtain a first-order transition. Recently improved Lattice QCD calculations, with almost realistic quark masses, seem to indicate a crossover instead [9]. On the other hand, from the experimental standpoint a weakly first-order transition is not ruled out, and in fact corresponds to the scenario adopted in very successful hydrodynamic calculations [16]. Although this is a crucial question for the understanding of the phase structure of QCD, it is essentially of no consequence to the analysis we undertake. The value of the critical temperature we obtain is $\sim$5–10% different from current values extracted from the lattice [9,21], which is always the case in effective field theory approaches and of no harm to our analysis, either. Our concern is with providing a good description of the behavior of the critical temperature for increasing values of quark masses and the isospin chemical potential.

II. LOW-ENERGY SECTOR

The physical setting for the low-energy regime of strong interactions is that of a system of heavy nucleons in the presence of a hot gas of pions whose masses are already dressed by corrections from temperature, isospin chemical potential, and quark masses. The effective Lagrangian reads $\mathcal{L}_{\text{eff}} = \mathcal{L}_N + \mathcal{L}_\pi + \mathcal{L}_{\pi N}$, where

$$\mathcal{L}_N = \overline{N}\Big(i\slashed{\partial} - M_N + \tfrac{1}{2}\mu_I\gamma_0\tau^3 + \tfrac{3}{2}\mu_B\gamma_0\Big)N, \quad (1)$$

$$\mathcal{L}_\pi = (\partial_\mu - ih\delta_{\mu 0})\pi^+(\partial_\mu + ih\delta_{\mu 0})\pi^- - \bar{m}_\pm^2\pi^+\pi^-$$
$$+ \tfrac{1}{2}[(\partial\pi^0)^2 - \overline{m}_0^2(\pi^0)^2], \quad (2)$$

$$\mathcal{L}_{\pi N} = \frac{g_A}{f_\pi}\overline{N}i\gamma_5\Big(\slashed{\partial}\pi - \frac{1}{2}\mu_I\gamma_0[\tau^3, \pi]\Big)N, \quad (3)$$

where g_A is the axial vector current coefficient of the nucleon, which accounts for renormalization in the weak decay rate of the neutron, f_π is the pion decay constant, and h is a function of temperature and isospin chemical potential. Nucleons are represented by $N = (p, n)$ with p, n being the proton and neutron spinors, respectively, and have a mass matrix $M_N = \text{diag}(M_p, M_n) = \text{diag}(M - \delta M, M + \delta M)$. This corresponds to the $\mathcal{O}(P)$ nucleon chiral Lagrangian [22], but considering mass corrections at zero temperature and chemical potential, and the coupling to dressed pions.

The effective (dressed) masses of the pions $\pi^0 = \pi^3$ and $\pi^\pm = \frac{1}{\sqrt{2}}(\pi^1 \mp i\pi^2)$, which depend on temperature T, isospin chemical potential μ_I and mass asymmetry $\delta m = (m_d - m_u)/2$, are denoted, respectively, by $\overline{m}_0 = \overline{m}_0(T, \mu_I, \delta m)$ and $\bar{m}_\pm = \bar{m}_\pm(T, \mu_I)$ [23] and their explicit expressions were calculated in Ref. [20]. In the so-called first phase, a regime in which $|\mu_I| < m_\pi$, they have the form

$$m_{\pi^0} = \overline{m}_0 = m_\pi[1 + \tfrac{1}{2}\alpha\sigma_{00}], \quad (4)$$

$$m_{\pi^\pm} = \overline{m}_\pm \mp h = m_\pi\left[1 + \frac{1}{2}\alpha\sigma_1 \pm \frac{1}{2}\alpha\frac{\sigma_0}{m_\pi}\right] \mp \mu_I, \quad (5)$$

up to first order in $\alpha = (m_\pi/4\pi f_\pi)^2$. Here, σ_{00}, σ_0, σ_1, and h are functions of temperature, isospin chemical potential, and quark masses [20].

In the second phase ($|\mu_I| > m_\pi$), a condensation of pions occurs, and a superfluid phase sets in [24]. In this new phase, in order to reestablish the vacuum structure, a chiral rotation is produced due to the isospin symmetry breaking. All this produces a pion mixing, and the nucleons also couple in a different way. The degrees of freedom do not correspond anymore to pions, but we can still call them quasipions since their masses in the two phases match at the transition point. The tree-level masses do not have the shape as in the equations above. Instead, $m_{\pi^0} = |\mu_I|$, $m_{\pi^-} = 0$, and $m_{\pi^+} = \mu_I\sqrt{1 + 3(m_\pi/\mu_I)^4}$ [14]. However, it is possible to treat this phase with a simple approximation near the superfluid phase transition. In the regime in which $|\mu_I| \gtrsim m_\pi$, the natural expansion parameter is given by $s^2 = 1 - m_\pi^4/\mu_I^4$ [20], after scaling all the parameters by $|\mu_I|$. The result of the first terms in this expansion ($s^2 = 0$) provides the same equations as in the normal phase, just replacing m_π by $|\mu_I|$. Strictly speaking, this is valid only for values of $|\mu_I|$ very close to m_π, i.e. $|\mu_I| \lesssim \sqrt{8/7}m_\pi$ (for a more detailed discussion cf. [20]). For simplicity, we also apply this results for slightly higher values of the isospin chemical potential [25]. These results in χPT are confirmed by a Nambu–Jona-Lasinio analysis [26].

The direct effect of the baryonic chemical potential in the pure pion quasiparticle gas is omitted since, without considering gluonic corrections, it appears as an anomalous term in the $\mathcal{O}(P^4)$ chiral Lagrangian [27], and will be present only in two-loop corrections according to power counting. For very large values of μ_B, one has in principle to incorporate effects from the color superconductivity gap in the calculation of meson masses in an effective theory near the Fermi surface [28,29]. In the present analysis, we treat the case $\mu_B = 0$.

The nucleon masses, $M_p = M - \delta M$ and $M_n = M + \delta M$, are dressed by leading-order contributions in zero-temperature baryon χPT. Using the results from Ref. [30], for the isospin symmetric case with explicit chiral symmetry breaking, and Ref. [31], which includes explicit isospin breaking effects, we have (neglecting terms $\sim m_q^2$, $\sim m_q^2 \log(m_q)$, and of higher order in m_q):

$$M(m) = M_0 + 2\gamma_1 m + 2^{3/2}\gamma_{3/2}m^{3/2}, \qquad (6)$$

$$\delta M(\delta m) = 2\gamma_1^{\text{asymm}}\delta m, \qquad (7)$$

M_0 being the nucleon mass in the chiral limit, $m = (m_u + m_d)/2$ the average quark mass, and [32]

$$\gamma_1 = \frac{-4c_1}{f_\pi^2}(-\langle\overline{q}q\rangle), \qquad (8)$$

$$\gamma_{3/2} = -\frac{3g_A^2}{32\pi f_\pi^5}(-\langle\overline{q}q\rangle)^{3/2}, \qquad (9)$$

$$\gamma_1^{\text{asymm}} = \frac{2\overline{\alpha} - \overline{\beta}}{3}\frac{(-\langle\overline{q}q\rangle)}{f_\pi^2}. \qquad (10)$$

Here, all parameters and coefficients are fixed to reproduce properties of the QCD vacuum either measured or extracted from recent lattice simulations. Explicitly, $\langle\overline{q}q\rangle = -(225 \text{ MeV})^3$ is the (one-flavor) chiral condensate in the chiral limit [33] and, from Table I in Ref. [30], $M_0 = (0.882 \pm 0.003)$ GeV, $c_1 = (-0.93 \pm 0.04)$ GeV^{-1}, $g_A = 1.267$, and $f_\pi = 92.4$ MeV, so that $\gamma_1 = 4.9630 \pm 0.2135$, and $\gamma_{3/2} = -0.273424$ MeV$^{-1/2}$. Finally, from Table 3 in Ref. [31], one can extract $(2\overline{\alpha} - \overline{\beta})/3$. Converting from lattice units to GeV ($1(\text{lattice units}) = b = 0.125$ fm), in the case of the $O(m_q)$ fit, we arrive at $\gamma_1^{\text{asymm}} = 0.16734 \pm 0.07858$. This fixes the dispersion relation satisfied by the proton and neutron as

$$E_{\text{p/n}}(\mathbf{p}) = \sqrt{\mathbf{p}^2 + (M \mp \delta M)^2} + \frac{3}{2}\mu_B \pm \frac{1}{2}\mu_I, \qquad (11)$$

where $\mathbf{p}$ is the three momentum and the antiparticle dispersion relations are obtained from the ones above by the substitution $\mu_i \mapsto -\mu_i$.

III. HIGH-ENERGY SECTOR

The fuzzy bag model has been proposed by Pisarski [19] as a phenomenological parameterization of the equation of state to account for the plateau in the trace anomaly normalized by T^2, $(\epsilon - 3p)/T^2$, observed in lattice results above the critical temperature. Besides the usual MIT-type bag constant, the total pressure for QCD in this model has also a nonperturbative contribution $\sim T^2$ [34]: $p_{\text{deconf}}(T) \simeq p_{\text{pQCD}}(T) - B_{\text{fuzzy}}T^2 - B_{\text{MIT}}$. The trace anomaly associated with this equation of state, assuming that $p_{\text{pQCD}} \sim T^4$, is then $\epsilon - 3p = 2B_{\text{fuzzy}}T^2 + 4B_{\text{MIT}}$.

Recently, a similar parameterization has been used to fit lattice results for the trace anomaly at high temperatures [9], $T > 1.5T_c \approx 300$ MeV, yielding

$$\left(\frac{\epsilon - 3p}{T^4}\right)_{\text{highT}} = \frac{3}{4}b_0g^4 + \frac{b}{T^2} + \frac{c}{T^4}, \qquad (12)$$

with coefficients b and c given in Table VIII of Ref. [9]. Notice that the first term in Eq. (12) comes from a $O(\alpha_s^2)$ perturbative contribution to the pressure and is important for the fit only at very high temperatures. Hence, we neglect this term in what follows, obtaining the following values for the bag coefficients: $B_{\text{fuzzy}} = 0.05$ GeV^2 and $B_{\text{MIT}} = 0.006$ GeV^5.

In our effective theory, we adopt, phenomenologically, a simple extension of the fuzzy bag equation of state, which includes the influence of finite chemical potentials and masses in the perturbative pressure, neglecting for simplicity nonperturbative contributions due to the finite quark chemical potentials μ_f, so that $p_{\text{deconf}} \simeq p_{\text{pQCD}}(T, \mu_f, m_f) - B_{\text{fuzzy}}T^2 - B_{\text{MIT}}$.

IV. RESULTS FOR THE CRITICAL TEMPERATURE

As detailed above, we constructed a model using results of well-studied theories and choosing carefully the relevant ingredients to study mass and isospin number effects. Both low- and high-energy sectors are completely fixed, and now we turn to the determination of the prediction of this model for the behavior of the deconfining critical temperature. From our results for the massive free gas contribution of the perturbative QCD pressure in the fuzzy bag model at finite temperature, isospin, and baryon number, and the free gas pressure of quasipions and nucleons in the low-energy regime, the critical temperature and chemical potential for the deconfining phase transition are extracted by maximizing the total pressure. The validity of our approach is, of course, restricted by the scale of χPT: e.g. for $m_\pi \approx m_\rho \approx 770$ MeV, the expansion parameter in χPT is $\alpha \approx 0.45$, so that the extension of the predictions to $m_\pi \lesssim 1$ GeV is justified [35].

The pion mass dependence, or equivalently the quark average mass dependence, of the critical temperature is displayed in Fig. 1 for vanishing chemical potentials with the temperature and the pion mass normalized by the
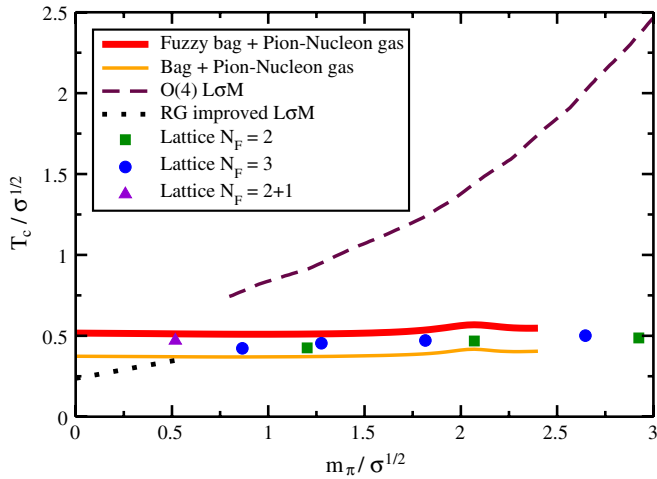
FIG. 1 (color online). Critical temperature as a function of m_π normalized by the string tension $\sqrt{\sigma} = 425$ MeV. Our results are compared to lattice data [9,36] and other approaches [5,6].

FIG. 2 (color online). Critical temperature as a function of μ_I for two different values of the pion mass. Our results are compared to lattice data [13] and other approaches [37].

square root of the string tension. Our curves stop at a point from which our χPT approach clearly breaks down. The results of our framework are compared to lattice data from Ref. [36] ($N_F = 2, 3$) and [9] ($N_F = 2 + 1$) and two other phenomenological treatments: the $O(4)$ linear sigma model [5] and a renormalization group improved computation [6] (cf. also Ref. [7] that discusses the quark-meson model using the proper-time renormalization group approach). The approximate mass independence observed in the lattice data is very well reproduced within our framework, while the other descriptions tend to generate a qualitatively different behavior. This feature is yet another indication that the functional dependence of $T_c(m_\pi)$ requires confinement ingredients to be reproduced, being incompatible with a phase transition dictated by pure chiral dynamics. This argument goes in the same direction of Ref. [5], the main difference here being the fact that we construct the mass dependence from the $m_{\text{quark}} = 0$ limit, with the heavy quasinucleons as the key new element at low energies. Moreover, our results are not strongly sensitive to the choice between the fuzzy bag model and the usual MIT bag model. The critical values for the MIT bag model are systematically lowered, but the qualitative behavior is not altered, as illustrated in Fig. 1. This indicates that a consistent treatment of the quark mass dependence connecting both perturbative regimes of energy is probably the essential ingredient to describe this observable.

In Fig. 2, the critical temperature is plotted as a function of the isospin chemical potential. The critical temperature is normalized by its value in the absence of chemical asymmetry, whereas the isospin chemical potential is normalized by the critical temperature itself. The curves obtained within our framework are represented by solid lines. Once again our results are in good agreement with lattice computations [13], even though the curve that is closer to the lattice points corresponds to a small vacuum pion mass,
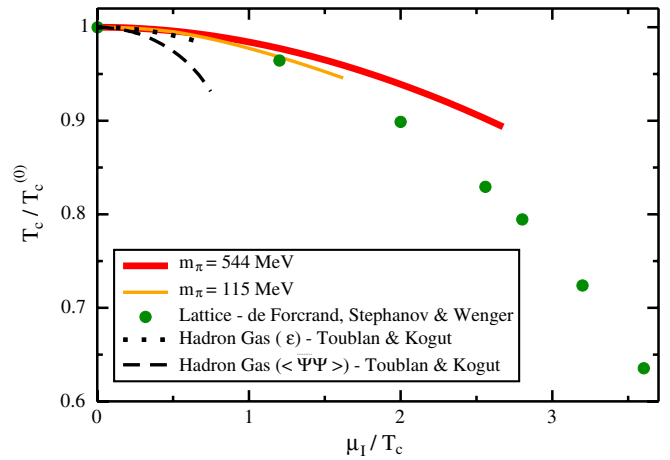
which is not the situation simulated on the lattice [13]. Our curves for different values of m_π start to disagree more appreciably for $\mu_I > 2T_c$. Recall that our treatment is valid up to isospin chemical potentials that are larger but not much larger than the pion mass, and contains the effects from pion condensation for $\mu_I > m_\pi$. Figure 2 also exhibits, for comparison, results using the hadron resonance gas model [37] that appear to depart from the lattice data at a much lower value of the isospin chemical potential. The two curves are produced by two different methods to determine the critical temperature: the dotted curve is obtained from the observation that the deconfined phase emerges at a constant energy density, whereas the dashed one uses the fact that the quark-antiquark condensate for the light quarks almost disappears at the quark-hadron transition [37]. Similar to what we observe for the quark-mass dependence of T_c, it is clear from Fig. 2 that plain chiral considerations render the largest discrepancies for the behavior of $T_c(\mu_I)$ as compared to lattice data.

An up-down quark-mass imbalance, characterized by the relative difference in quark mass, which is in principle not much smaller than 1, tends to increase the critical temperature, though by a quantitatively small amount, as expected. The value of the critical baryonic chemical potential, beyond which matter is deconfined, also seems to increase with the vacuum pion mass. Detailed results in these and other thermodynamic observables will be presented in a longer publication (see also [8]).

V. FINAL REMARKS

We constructed a frugal effective framework selecting features from established theories that are relevant in the determination of the quark-mass and chemical-asymmetry dependence of the deconfining transition. From a simple free gas calculation of the equation of state, we found

surprisingly good agreement with different lattice data for the behavior of the critical temperature for the deconfining transition with masses and the isospin chemical potential, indicating that the model captures the essential features brought about by the inclusion of those effects. It should be emphasized that the framework provides simultaneous predictions for these two functional dependences of the critical temperature, $T_c(m_\pi)$ and $T_c(\mu_I)$, with only one set of coefficients completely determined by observed QCD vacuum properties and lattice simulations at finite temperature and zero chemical potentials and fixed quark masses. As far as we are aware of, this is the first framework to well reproduce lattice data. Our description of $T_c(m_\pi)$ reinforces, through a completely different approach, the discussion in Ref. [5] in which the quark-mass dependence of the finite temperature transition on the lattice is shown to be compatible with results from slightly perturbed Polyakov-loop models, contrasting with the failure of different chiral models. Confinement properties seem to influence strongly the mass dependence of the QCD transitions observed on the lattice. Moreover, the approximate framework built in the present paper, constituted of selected ingredients from well-known high- and low-energy theories, is able to access physical settings including quark-mass asymmetry and finite baryon chemical potential, regimes in which lattice simulations encounter severe difficulties due to non-positive definite fermion determinants. Therefore, this model provides a pragmatic tool to investigate the role played by nonzero quark masses and chemical potentials, also going beyond the free gas approximation, complementing in a healthy direction results from other model approaches and lattice simulations. Since the (mass-symmetric) isospin chemical potential regime does not suffer from the sign problem, the road is open for detailed Monte Carlo studies and further comparisons. On the experimental side, intermediate-energy experiments in nuclear physics are providing data and observables that are sensitive to chemical asymmetry, whereas high-energy heavy ion collisions to start soon at RHIC-BES and FAIR-GSI will probe a region of the phase diagram of QCD where effects from μ_B become important. Here, we have focused on the influence of finite quark masses and isospin chemical potential. Predictions of this framework for the finite μ_B regime will be reported in the near future.

[1] J. B. Kogut and M. A. Stephanov, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol. **21**, 1 (2004).

[2] K. Yagi, T. Hatsuda, and Y. Miake, Cambridge Monogr. Part. Phys., Nucl. Phys., Cosmol.**23**, 1 (2005).

[3] E. Laermann and O. Philipsen, Annu. Rev. Nucl. Part. Sci. **53**, 163 (2003).

[4] M. A. Stephanov, Proc. Sci., LAT2006 (2006) 024.

[5] A. Dumitru, D. Roder, and J. Ruppert, Phys. Rev. D **70**, 074001 (2004).

[6] J. Berges, D. U. Jungnickel, and C. Wetterich, Phys. Rev. D **59**, 034010 (1999).

[7] J. Braun, B. Klein, H. J. Pirner, and A. H. Rezaeian, Phys. Rev. D **73**, 074010 (2006).

[8] L. F. Palhares, E. S. Fraga, and C. Villavicencio, arXiv:0810.2742.

[9] M. Cheng *et al.*, Phys. Rev. D **77**, 014511 (2008).

[10] O. Philipsen, arXiv:0808.0672.

[11] Y. Aoki *et al.*, Phys. Lett. B **643**, 46 (2006); Z. Fodor, Proc. Sci., LAT2007 (2007) 011.

[12] J. B. Kogut and D. K. Sinclair, Phys. Rev. D **70**, 094501 (2004); D. K. Sinclair and J. B. Kogut, Proc. Sci., LAT2006 (2006) 147; LAT2007 (2007) 225; J. B. Kogut and D. K. Sinclair, Phys. Rev. D **77**, 114503 (2008).

[13] P. de Forcrand, M. A. Stephanov, and U. Wenger, Proc. Sci., LAT2007 (2007) 237.

[14] D. T. Son and M. A. Stephanov, Phys. Rev. Lett. **86**, 592 (2001); K. Splittorff, D. T. Son, and M. A. Stephanov, Phys. Rev. D **64**, 016003 (2001).

[15] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. **464**, 113 (2008).

[16] Procs. of Quark Matter 2006 Collaboration, J. Phys. G **34**, S173 (2007).

[17] D. H. Rischke, Prog. Part. Nucl. Phys. **52**, 197 (2004).

[18] S. Mukherjee, M. G. Mustafa, and R. Ray, Phys. Rev. D **75**, 094015 (2007).

[19] R. D. Pisarski, Phys. Rev. D **74**, 121703 (2006); Prog. Theor. Phys. Suppl. **168**, 276 (2007).

[20] M. Loewe and C. Villavicencio, Phys. Rev. D **67**, 074034 (2003); **70**, 074005 (2004); **71**, 094001 (2005).

[21] Currently, there is still no agreement between different groups regarding the value of the critical temperature (cf. Ref. [11]). This important issue will hopefully be settled in the near future. By convenience, we choose to compare with the set of results that was used in previous tests of other effective models to which we also compare our results.

[22] J. Gasser, M. E. Sainio, and A. Svarc, Nucl. Phys. **B307**, 779 (1988); A. Krause, Helv. Phys. Acta **63**, 3 (1990).

[23] μ_I and μ_B can be written in terms of the chemical potentials for the up and down quarks. Here, we adopt the following convenient definition: $\mu_I = \mu_u - \mu_d$ and $\mu_B = \mu_u + \mu_d$. (μ_B is also frequently defined with an overall factor 2/3.) We also use the customary notation $\pi \equiv \pi^a \tau^a$, with τ^a being the Pauli matrices.

[24] Here, we consider the π^- condensation taking $\mu_I = -|\mu_I|$.

[25] For higher values of $|\mu_I|$ one needs to consider more terms, not only in the expansion of the pion Lagrangian but also in the coupling with the nucleons, due to the appearance of the pion condensate. These terms will also be present at the perturbative QCD level via the chiral rotation. The case in which $|\mu_I| \gg m_\pi$ can be explored, nevertheless, using m_π^2/μ_I^2 as an expansion parameter, even though the validity of the whole treatment is restricted to $|\mu_I|$ smaller than the η or ρ masses [14].

[26] L. y. He, M. Jin, and P. f. Zhuang, Phys. Rev. D **71**, 116001 (2005).

[27] R. F. Alvarez-Estrada and A. Gomez Nicola, Phys. Lett. B **355**, 288 (1995); **380**, 491(E) (1996).

[28] M. G. Alford, K. Rajagopal, T. Schaefer, and A. Schmitt, Rev. Mod. Phys. **80**, 1455 (2008).

[29] S. R. Beane, P. F. Bedaque, and M. J. Savage, Phys. Lett. B **483**, 131 (2000).

[30] M. Procura *et al.*, Phys. Rev. D **73**, 114510 (2006).

[31] S. R. Beane, K. Orginos, and M. J. Savage, Nucl. Phys. **B768**, 38 (2007).

[32] To relate the pion mass (in the isospin symmetric case) with the quark masses, we use the Gell-Mann-Oakes-Renner relation $m_\pi^2 f_\pi^2 = -(m_u + m_d)\langle \overline{q}q \rangle = 2m(-\langle \overline{q}q \rangle)$.

[33] T. W. Chiu and T. H. Hsieh, Nucl. Phys. **B673**, 217 (2003); Nucl. Phys. B, Proc. Suppl. **129**, 492 (2004).

[34] The underlying theoretical framework is that of an effective theory of Wilson lines and their electric field.

[35] A rough estimate of the error in the effective masses of the low-energy sector of our approximation for $m_\pi \sim 1$ GeV yields $\sim 30\%$.

[36] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. **B605**, 579 (2001); F. Karsch, K. Redlich, and A. Tawfik, Eur. Phys. J. C **29**, 549 (2003).

[37] D. Toublan and J. B. Kogut, Phys. Lett. B **605**, 129 (2005).