PHYSICAL REVIEW D **79,** 014007 (2009)

Order-α_s corrections to the quarkonium electromagnetic current at all orders in the heavy-quark velocity

Geoffrey T. Bodwin,[1] Hee Sok Chung,[2] Jungil Lee,[1,2] and Chaehyun Yu[2]

[1]*High Energy Physics Division, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439, USA*

[2]*Department of Physics, Korea University, Seoul 136-701, Korea*

(Received 29 August 2008; published 13 January 2009)

We compute in order α_s the nonrelativistic QCD (NRQCD) short-distance coefficients that match quark-antiquark operators of all orders in the heavy-quark velocity v to the electromagnetic current. We employ a new method to compute the one-loop NRQCD contribution to the matching condition. The new method uses full-QCD expressions as a starting point to obtain the NRQCD contribution, thus greatly streamlining the calculation. Our results show that, under a mild constraint on the NRQCD operator matrix elements, the NRQCD velocity expansion for the quark-antiquark-operator contributions to the electromagnetic current converges. The velocity expansion converges rapidly for approximate J/ψ operator matrix elements.

DOI: 10.1103/PhysRevD.79.014007 PACS numbers: 12.38.Bx, 12.39.St, 13.20.Gd, 14.40.Gx

I. INTRODUCTION

The electromagnetic decays of quarkonia through a single virtual photon have played an important role in the experimental and theoretical development of quarkonium physics. On the experimental side, the decays of charge-conjugation-odd quarkonium states to a lepton pair provide unique signals for the detection of those states. On the theoretical side, the decays of $^3S_1$ quarkonium states to a lepton pair allow one to determine one of the fundamental parameters of the heavy-quark–antiquark ($Q\bar{Q}$) bound state, namely, the square of the wave function at the origin. (See, for example, Ref. [1].) The square of the wave function at the origin enters into many calculations of quarkonium decay and production rates.

The expression for the $^3S_1$ quarkonium decay rate into a lepton pair at leading order in the QCD coupling α_s and at leading order in v, the Q or $\bar{Q}$ velocity in the quarkonium rest frame, has been known since the first discovery of quarkonium and is based on the Van Royen-Weisskopf formula [2] of quantum electrodynamics. The order-α_s corrections to this formula at leading order in v were calculated in Refs. [3,4]. The order-α_s^0 relativistic corrections at relative orders v^2 and v^4 were calculated in Refs. [5,6], respectively. Order-α_s^2 corrections to the decay rate were calculated in Refs. [7,8]. The correction to the electromagnetic current of a quarkonium at relative order $\alpha_s v^2$ was calculated in Ref. [9].

In this paper, we calculate relativistic corrections to the quarkonium electromagnetic current at order α_s. We carry out our calculation in the context of nonrelativistic QCD (NRQCD) [5]. We obtain closed-form expressions whose Taylor-series expansions in v give the short-distance coefficients for the NRQCD $Q\bar{Q}$ operators, of all orders in v, that match to the electromagnetic current. We do not consider $Q\bar{Q}$ operators that contain gauge fields. Therefore, our operators are not gauge invariant, and we evaluate their matrix elements in the Coulomb gauge. In the Coulomb gauge, $Q\bar{Q}$ operators involving gauge fields first contribute at relative order v^4. Our results confirm the calculation at relative order $\alpha_s v^2$ in Ref. [9]. Since the corrections at relative order $\alpha_s v^2$ are not very significant at the current level of precision of calculations of $^3S_1$ quarkonium electromagnetic decay rates, we do not expect the order-α_s corrections at still higher orders in v to be important numerically.

We present our calculation primarily as a demonstration of a new method for computing the one-loop NRQCD contribution that enters into the matching of NRQCD to full QCD. The direct computation of one-loop NRQCD expressions to all orders in v would be a formidable task, in that it would require knowledge of the NRQCD interactions and electromagnetic-current operators, their Born-level short-distance coefficients, and their Feynman rules to all orders in v. Instead of following the direct NRQCD approach, we note that NRQCD through infinite order in v is equivalent to QCD, but with the interactions rearranged in an expansion in powers of v. Therefore, we can obtain the one-loop NRQCD contribution by starting from full-QCD expressions and expanding integrands in powers of momenta divided by the heavy-quark mass m *before* we carry out the dimensional regularization. In dimensional regularization, this method is related to the method of regions [10]. We explain this relationship in Sec. II. The method of regions has been used previously at leading order in v to compute NRQCD short-distance coefficients from full-QCD expressions. (See, for example, Ref. [8].)

Our results show that, under a mild constraint on the NRQCD operator matrix elements, the NRQCD velocity expansion for the quark-antiquark-operator contributions to the electromagnetic current converges. The velocity expansion converges rapidly for approximate J/ψ operator matrix elements.

The remainder of this paper is organized as follows. In Sec. II we discuss the one-loop matching of NRQCD to QCD at all orders in v. We define the notation that we use to describe the kinematics of the calculation in Sec. III. Section IV contains detailed formulas for the NRQCD $Q\bar{Q}$ short-distance coefficients. In Sec. V we compute the one-loop QCD corrections to the electromagnetic current, while in Sec. VI we use our new method to compute the one-loop NRQCD corrections to the electromagnetic current. We give analytic and numerical results for the short-distance coefficients in Sec. VII, present a formula that resums a class of relativistic corrections to all orders in v, and discuss the convergence of the velocity expansion. Our conclusions are given in Sec. VIII. The Appendices contain compilations of integrals and identities that are useful in the calculation.

II. MATCHING TO ALL ORDERS IN v

We define the hadronic part of the quarkonium electromagnetic decay amplitude $\mathcal{A}_H^\mu$ as

$$(-iee_Q)i\mathcal{A}_H^\mu = \langle 0|J^\mu_{\rm EM}|H\rangle, \tag{1}$$

where H is the quarkonium, e is the electromagnetic charge, e_Q is the heavy-quark charge, and $J^\mu_{\rm EM}$ is the heavy-quark electromagnetic current:

$$J^\mu_{\rm EM} = (-iee_Q)\bar{\psi}\gamma^\mu\psi. \tag{2}$$

Here, ψ is the heavy-quark Dirac field, and γ^μ is a Dirac matrix.

In the quarkonium rest frame, $i\mathcal{A}_H^0 = 0$ because of conservation of the electromagnetic current. According to NRQCD factorization [5], we can write the spatial components $i\mathcal{A}_H^i$ as

$$i\mathcal{A}_H^i = \sqrt{2m_H}\sum_n c_n\langle 0|\mathcal{O}_n^i|H\rangle, \tag{3}$$

where the c_n are short-distance coefficients, the $\mathcal{O}_n^i$ are NRQCD operators, and m_H is the quarkonium mass. We regulate the operator matrix elements in Eq. (3) dimensionally in $d = 4 - 2\epsilon$ dimensions. The factor $\sqrt{2m_H}$ on the right side of Eq. (3) appears because the NRQCD operator matrix elements have nonrelativistic normalization, while we choose the amplitude on the left side of Eq. (3) to have relativistic normalization for the quarkonium H.

The aim of this paper is to calculate the short-distance coefficients c_n that correspond to $Q\bar{Q}$ color-singlet operators in order α_s^1. We can determine these c_n by making use of a matching equation that is the statement of NRQCD factorization for perturbative $Q\bar{Q}$ color-singlet states:

$$i\mathcal{A}^i_{Q\bar{Q}_1} = \sum_n c_n\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle, \tag{4}$$

where the subscript 1 indicates a color-singlet state. Throughout this paper, we suppress the factor $\sqrt{N_c}$ that comes from the implicit color trace in $i\mathcal{A}^i_{Q\bar{Q}_1}$, where $N_c = 3$ is the number of colors. Through order α_s^1, the matching equation is

$$i\mathcal{A}^{i(0)}_{Q\bar{Q}_1} + i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} = \sum_n (c_n^{(0)} + c_n^{(1)})\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)} + \sum_n c_n^{(0)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(1)}, \tag{5}$$

where the superscripts (0) and (1) indicate the order in α_s. In the first sum in Eq. (5), only color-singlet $Q\bar{Q}$ operators contribute, while in the last sum, additional operators can contribute if they mix into color-singlet $Q\bar{Q}$ operators under one-loop corrections.

We define the quantity

$$[i\mathcal{A}^{i(0)}_{Q\bar{Q}_1}]_{\rm NRQCD} = \sum_n c_n^{(0)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)}, \tag{6}$$

which is the expansion of $i\mathcal{A}^{i(0)}_{Q\bar{Q}_1}$ in powers of q/m, where q is half the relative momentum of the heavy quark and heavy antiquark. At order α_s^0, the matching equation (5) yields

$$i\mathcal{A}^{i(0)}_{Q\bar{Q}_1} = \sum_n c_n^{(0)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)}, \tag{7}$$

from which the $c_n^{(0)}$ can be determined. The $c_n^{(0)}$ have been computed previously in Ref. [1].

At order α_s^1, the matching equation (5) yields

$$i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} = \sum_n c_n^{(1)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)} + [i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD}, \tag{8}$$

from which the $c_n^{(1)}$ can be computed. We compute the quantities $i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}$ and $[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD}$ in Secs. V and VI, respectively.

The quantity

$$[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD} = \sum_n c_n^{(0)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(1)} \tag{9}$$

would be formidable to calculate directly in NRQCD because it involves operators and interactions of all orders in v. Rather than carry out such a direct calculation, we take a new approach. We note that, by construction, NRQCD reproduces all of the interactions in full QCD, but with those interactions reorganized in an expansion in powers of v. Therefore, we can obtain $[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD}$ from the expression for $i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}$ by expanding the integrand in powers of the momentum divided by m. Before making this expansion, we carry out the integration over the temporal component of the loop momentum, using contour integration. This procedure establishes the scale of the temporal component of the loop momentum, which varies from contribution to contribution, and it avoids the generation of ill-defined pinch singularities that can arise when one expands the Q and $\bar{Q}$ propagators prematurely in powers of the momentum. We expand the integrand in powers of both the external momenta divided by m and the loop momenta divided by m. We then regulate the integrals dimensionally, setting scaleless, power-divergent integrals equal to zero. Ultimately, we renormalize ultraviolet divergences according to the $\overline{\text{MS}}$ prescription.

The procedure of expanding both external and loop momenta in power series before regulating dimensionally was first utilized in the Appendix of Ref. [5]. The rationale for it was discussed in Refs. [9,11]. This procedure amounts to the prescription that infrared-finite contributions that arise from loop momenta in the vicinity of zero are kept in the short-distance coefficients [11].[1]

If one uses dimensional regularization, then the quantity $\sum_n c_n^{(1)}\langle 0|\mathcal{O}_n^i|Q\bar{Q}_1\rangle^{(0)}$ corresponds to the contribution from the hard region in the method of regions [10], while the quantity $[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\text{NRQCD}}$ corresponds to the sum of the contributions from the potential, soft, and ultrasoft regions, i.e., the contribution from the small-loop-momentum region. In the method of regions, it is assumed that there are no contributions from the region in which the temporal component of the gluon momentum is of order m, but the spatial component of the gluon momentum is of order mv. As we shall see explicitly in our calculation, this assumption is justified because the contribution from this region of integration vanishes in dimensional regularization. We note, however, that the contribution from this region does not vanish in the case of a hard-cutoff regulator. One potentially useful feature of the approach that we present here is that it can be applied in the case of a hard cutoff, such as lattice regularization, while the method of regions is applicable only in dimensional regularization. In the method of regions, one can compute the contribution from the hard region directly, rather than computing it, as we do, by subtracting the small-loop-momentum contribution from the full-QCD contribution. As we shall explain later, there may be advantages to our indirect procedure in calculating the hard contribution to all orders in v.

III. KINEMATICS

Before proceeding to write explicit formulas for the short-distance coefficients, let us define some notation for the kinematics of the heavy-quark electromagnetic vertex. We take p_1 and p_2 to be the momenta of the incoming heavy quark Q and heavy antiquark $\bar{Q}$, respectively. p_1 and p_2 can be expressed as linear combinations of their average p and half their difference q:

$$p_1 = p + q, \tag{10a}$$
$$p_2 = p - q. \tag{10b}$$

[1]In the case of hard-cutoff regularization, such as lattice regularization, the expansion in powers of loop momentum divided by m would be uniformly convergent and would yield the same result as the unexpanded expression.

In the $Q\bar{Q}$ rest frame, the momenta are given by

$$p_1 = (E, \boldsymbol{q}), \tag{11a}$$
$$p_2 = (E, -\boldsymbol{q}), \tag{11b}$$
$$p = (E, \boldsymbol{0}), \tag{11c}$$
$$q = (0, \boldsymbol{q}), \tag{11d}$$

where $E = \sqrt{m^2 + \boldsymbol{q}^2}$. The quark Q and antiquark $\bar{Q}$ are on their mass shells: $p_1^2 = p_2^2 = m^2$. For later use, it is convenient to define a parameter

$$\delta = \frac{|\boldsymbol{q}|}{E}, \tag{12}$$

which is related to the velocity

$$v = \frac{|\boldsymbol{q}|}{m}. \tag{13}$$

We can write δ in terms of v as

$$\delta = \frac{v}{\sqrt{1+v^2}}. \tag{14}$$

E^2 and $\boldsymbol{q}^2$ are expressed in terms of m and δ as

$$E^2 = \frac{m^2}{1-\delta^2}, \tag{15a}$$
$$\boldsymbol{q}^2 = \frac{m^2\delta^2}{1-\delta^2}. \tag{15b}$$

IV. FORMULAS FOR THE SHORT-DISTANCE COEFFICIENTS

Now let us make use of the matching conditions (7) and (8) to compute the short-distance coefficients for the specific color-singlet $Q\bar{Q}$ operators that we consider in this paper. These operators are

$$\mathcal{O}_{An}^i = \chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{\nabla}}\right)^{2n}\sigma^i\psi, \tag{16a}$$
$$\mathcal{O}_{Bn}^i = \chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{\nabla}}\right)^{2n-2}\left(-\frac{i}{2}\overleftrightarrow{\nabla}^i\right)\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{\nabla}}\right)\cdot\boldsymbol{\sigma}\psi, \tag{16b}$$

where ψ is the Pauli spinor field that annihilates a heavy quark, $\chi^\dagger$ is the Pauli spinor field that annihilates a heavy antiquark, and σ^i is a Pauli matrix. Our operators contain ordinary derivatives, rather than covariant derivatives. Therefore, our operators are not gauge invariant, and we evaluate their matrix elements in the Coulomb gauge. We do not consider $Q\bar{Q}$ operators involving the gauge fields, which first contribute at relative order v^4. We note that $\mathcal{O}_{Bn}^i$ can be decomposed into a linear combination of the S-wave operator $\mathcal{O}_{An}^i$ and the D-wave operator $\mathcal{O}_{Dn}^i$:

$$\mathcal{O}_{Bn}^i = \frac{1}{d-1}\mathcal{O}_{An}^i + \mathcal{O}_{Dn}^i, \tag{17}$$

where $\mathcal{O}_{Dn}^i$ is defined by

$$\mathcal{O}^i_{Dn} = \chi^\dagger\Big(-\frac{i}{2}\overleftrightarrow{\nabla}\Big)^{2n-2}\Big[\Big(-\frac{i}{2}\overleftrightarrow{\nabla}^i\Big)\Big(-\frac{i}{2}\overleftrightarrow{\nabla}\Big)\cdot\boldsymbol{\sigma} - \frac{1}{d-1}\Big(-\frac{i}{2}\overleftrightarrow{\nabla}\Big)^2\sigma^i\Big]\psi. \tag{18}$$

In the basis of operators $\mathcal{O}^i_{An}$ and $\mathcal{O}^i_{Bn}$, the matching conditions (7) and (8) become

$$i\mathcal{A}^{i(0)}_{Q\bar{Q}_1} = \sum_n a^{(0)}_n \langle 0|\mathcal{O}^i_{An}|Q\bar{Q}_1\rangle^{(0)} + \sum_n b^{(0)}_n \langle 0|\mathcal{O}^i_{Bn}|Q\bar{Q}_1\rangle^{(0)}, \tag{19a}$$

$$i\mathcal{A}^{i(1)}_{Q\bar{Q}_1} = \sum_n a^{(1)}_n \langle 0|\mathcal{O}^i_{An}|Q\bar{Q}_1\rangle^{(0)} + \sum_n b^{(1)}_n \langle 0|\mathcal{O}^i_{Bn}|Q\bar{Q}_1\rangle^{(0)} + [i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD}, \tag{19b}$$

where a_n and b_n are the corresponding short-distance coefficients. A similar equation holds in the basis $\mathcal{O}^i_{An}$ and $\mathcal{O}^i_{Dn}$, where the associated short-distance coefficients are

$$s_n = a_n + \frac{1}{d-1}b_n, \tag{20a}$$

$$d_n = b_n, \tag{20b}$$

respectively.[2]

The $Q\bar{Q}$ matrix elements in Eq. (19) are

$$\langle 0|\mathcal{O}^i_{An}|Q\bar{Q}_1\rangle^{(0)} = \boldsymbol{q}^{2n}\eta^\dagger\sigma^i\xi, \tag{21a}$$

$$\langle 0|\mathcal{O}^i_{Bn}|Q\bar{Q}_1\rangle^{(0)} = \boldsymbol{q}^{2n-2}q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi, \tag{21b}$$

where ξ and η are two-component spinors. In order to maintain consistency with our calculations in full QCD, we have taken the $Q\bar{Q}$ states to have nonrelativistic normalization and we have suppressed the factor $\sqrt{N_c}$ that comes from the color trace.

Because of current conservation, the most general form of $i\mathcal{A}^i_{Q\bar{Q}_1}$ is

$$i\mathcal{A}^i_{Q\bar{Q}_1} = \bar{v}(p_2)(G\gamma^i + Hq^i)u(p_1), \tag{22}$$

where

$$G = Z_Q(1+\Lambda). \tag{23}$$

Z_Q is the fermion wave-function renormalization, and Λ is the multiplicative correction to the fermion electromagnetic vertex. Similarly,

$$i[\mathcal{A}^i_{Q\bar{Q}_1}]_{\rm NRQCD} = \bar{v}(p_2)(G_{\rm NRQCD}\gamma^i + H_{\rm NRQCD}q^i)u(p_1), \tag{24}$$

where

$$G_{\rm NRQCD} = [Z_Q]_{\rm NRQCD}(1+\Lambda_{\rm NRQCD}). \tag{25}$$

Using nonrelativistic normalization for the spinors u and v, we obtain

$$\bar{v}(p_2)\gamma^i u(p_1) = \eta^\dagger\sigma^i\xi - \frac{q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi}{E(E+m)}, \tag{26a}$$

$$q^i\bar{v}(p_2)u(p_1) = -\frac{q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi}{E}. \tag{26b}$$

Then,

$$i\mathcal{A}^i_{Q\bar{Q}_1} = G\eta^\dagger\sigma^i\xi - \Big[\frac{G}{E(E+m)} + \frac{H}{E}\Big]q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi. \tag{27}$$

Similarly,

$$i[\mathcal{A}^i_{Q\bar{Q}_1}]_{\rm NRQCD} = G_{\rm NRQCD}\eta^\dagger\sigma^i\xi - \Big[\frac{G_{\rm NRQCD}}{E(E+m)} + \frac{H_{\rm NRQCD}}{E}\Big]q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi. \tag{28}$$

Using the matching condition (19a) and Eqs. (21) and (27), we obtain the short-distance coefficients at order α_s^0:

$$a^{(0)}_n = \frac{1}{n!}\Big(\frac{\partial}{\partial\boldsymbol{q}^2}\Big)^n G^{(0)}\Big|_{\boldsymbol{q}^2=0} = \delta_{n0}, \tag{29a}$$

$$b^{(0)}_n = d^{(0)}_n = -\frac{1}{(n-1)!}\Big(\frac{\partial}{\partial\boldsymbol{q}^2}\Big)^{n-1}\Big[\frac{G^{(0)}}{E(E+m)} + \frac{H^{(0)}}{E}\Big]\Big|_{\boldsymbol{q}^2=0} = -\frac{1}{(n-1)!}\Big(\frac{\partial}{\partial\boldsymbol{q}^2}\Big)^{n-1}\Big[\frac{1}{E(E+m)}\Big]\Big|_{\boldsymbol{q}^2=0}, \tag{29b}$$

$$s^{(0)}_n = a^{(0)}_n + \frac{1}{3}b^{(0)}_n. \tag{29c}$$

Using the matching condition (19b) and Eqs. (21), (27), and (28), we obtain the short-distance coefficients at order α_s^1:

$$a^{(1)}_n = \frac{1}{n!}\Big(\frac{\partial}{\partial\boldsymbol{q}^2}\Big)^n \Delta G^{(1)}\Big|_{\boldsymbol{q}^2=0}, \tag{30a}$$

$$b^{(1)}_n = d^{(1)}_n = -\frac{1}{(n-1)!}\times\Big(\frac{\partial}{\partial\boldsymbol{q}^2}\Big)^{n-1}\Big[\frac{\Delta G^{(1)}}{E(E+m)} + \frac{\Delta H^{(1)}}{E}\Big]\Big|_{\boldsymbol{q}^2=0}, \tag{30b}$$

$$s^{(1)}_n = a^{(1)}_n + \frac{1}{d-1}b^{(1)}_n, \tag{30c}$$

[2]If we replace the ordinary derivatives $\overleftrightarrow{\nabla}$ with covariant derivatives $\overleftrightarrow{\boldsymbol{D}}$ in an S-wave operator $\mathcal{O}^i_{An}$, then we obtain one of the conventional gauge-invariant S-wave NRQCD operators. Because the squared covariant derivatives $(\overleftrightarrow{\boldsymbol{D}})^2$ commute with themselves, the substitution of covariant derivatives for ordinary derivatives leads to a unique S-wave operator at each order n. Therefore, the S-wave short-distance coefficients s_n that we compute are also the short-distance coefficients of the S-wave operator in which ordinary derivatives have been replaced with covariant derivatives. In the case of the D-wave operators $\mathcal{O}^i_{Dn}$, the replacement of ordinary derivatives with covariant derivatives does not lead to a unique operator because $(\overleftrightarrow{D})^i$ and $(\overleftrightarrow{D})^j$ do not commute. Therefore, each of the D-wave short-distance coefficients d_n that we compute is the sum of the short-distance coefficients for the various operators at order n that can be constructed from covariant derivatives.

where

$$\Delta G^{(1)} = G^{(1)} - G^{(1)}_{\mathrm{NRQCD}}, \tag{31a}$$

$$\Delta H^{(1)} = H^{(1)} - H^{(1)}_{\mathrm{NRQCD}}. \tag{31b}$$

The infrared divergences in $G^{(1)}_{\mathrm{NRQCD}}$ and $H^{(1)}_{\mathrm{NRQCD}}$ cancel in $\Delta G^{(1)}$ and $\Delta H^{(1)}$ because NRQCD reproduces full QCD in the infrared region. The one-loop NRQCD matrix elements in $G^{(1)}_{\mathrm{NRQCD}}$ contain ultraviolet divergences, which we renormalize according to the $\overline{\mathrm{MS}}$ prescription. The quantity $H^{(1)}_{\mathrm{NRQCD}}$ is free of ultraviolet divergences. The quantities Λ and Z_Q also contain ultraviolet divergences. However, because of the usual cancellation between the vertex and fermion-wave-function renormalizations, $G^{(1)}$ is free of ultraviolet divergences. $H^{(1)}$ is also free of ultraviolet divergences. Carrying out the renormalization, we have

$$[a_n^{(1)}]_{\overline{\mathrm{MS}}} = \frac{1}{n!}\left(\frac{\partial}{\partial \boldsymbol{q}^2}\right)^n \Delta G^{(1)}_{\overline{\mathrm{MS}}}\bigg|_{\boldsymbol{q}^2=0}, \tag{32a}$$

$$[b_n^{(1)}]_{\overline{\mathrm{MS}}} = [d_n^{(1)}]_{\overline{\mathrm{MS}}} = -\frac{1}{(n-1)!} \times \left(\frac{\partial}{\partial \boldsymbol{q}^2}\right)^{n-1}\left[\frac{\Delta G^{(1)}_{\overline{\mathrm{MS}}}}{E(E+m)} + \frac{\Delta H^{(1)}}{E}\right]\bigg|_{\boldsymbol{q}^2=0}, \tag{32b}$$

$$[s_n^{(1)}]_{\overline{\mathrm{MS}}} = [a_n^{(1)}]_{\overline{\mathrm{MS}}} + \frac{1}{3}[b_n^{(1)}]_{\overline{\mathrm{MS}}}. \tag{32c}$$

In deriving the expression for $[s_n^{(1)}]_{\overline{\mathrm{MS}}}$, we have used the fact that, in minimal subtraction, one removes the $1/\epsilon$ pole times the order-α_s^0 d-dimensional matrix element. Hence, a term proportional to $(d-1)^{-1}\epsilon^{-1}$ is subtracted in Eq. (30c) in carrying out the renormalization.

V. QCD CORRECTIONS

In this section, we calculate the QCD corrections to the heavy-quark electromagnetic current. That is, we compute $i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}$.

A. Vertex correction

In the Feynman gauge, the vertex correction to the electromagnetic current is given by

$$\Lambda^\mu = -ig_s^2 C_F \times \int_k \frac{\bar{v}(p_2)\gamma_\alpha(-\not{p}_2 + \not{k} + m)\gamma^\mu(\not{p}_1 + \not{k} + m)\gamma^\alpha u(p_1)}{D_0 D_1 D_2}, \tag{33}$$

where $g_s^2 = 4\pi\alpha_s$ is the strong coupling, $C_F = (N_c^2 - 1)/(2N_c) = 4/3$, and

$$\int_k \equiv \mu^{2\epsilon}\int \frac{d^d k}{(2\pi)^d}, \tag{34a}$$

$$D_0 = k^2 + i\varepsilon, \tag{34b}$$

$$D_1 = k^2 + 2k\cdot p_1 + i\varepsilon, \tag{34c}$$

$$D_2 = k^2 - 2k\cdot p_2 + i\varepsilon. \tag{34d}$$

μ is the renormalization scale. The loop momentum k is chosen to be the gluon momentum.

By making use of Eq. (10) and applying the equations of motion,

$$\bar{v}(p_2)\not{p}u(p_1) = 0, \tag{35a}$$

$$\bar{v}(p_2)\not{q}u(p_1) = m\bar{v}(p_2)u(p_1), \tag{35b}$$

we find that Eq. (33) can be written as

$$\Lambda^\mu = -ig_s^2 C_F \int_k \frac{1}{D_0 D_1 D_2}\bar{v}(p_2)\{[(d-2)k^2 - 4(2p^2 - m^2) + 8k\cdot q]\gamma^\mu + 4mk^\mu - 8q^\mu\not{k} + 2(2-d)k^\mu\not{k}\}u(p_1). \tag{36}$$

Tensor reductions of the integrals in Eq. (36) are given in Appendix A. The result is

$$\Lambda^\mu = -ig_s^2 C_F \bar{v}(p_2)\bigg\{[(d-2)J_1 - 4(2p^2 - m^2)J_2 + 4J_3 + 2(2-d)J_4]\gamma^\mu + \frac{2mp^\mu}{p^2}J_5 - \frac{2mq^\mu}{q^2}J_3 + 2(2-d)m\left(\frac{q^\mu}{q^2}J_6 + \frac{p^\mu}{p^2 q^2}J_7\right)\bigg\}u(p_1), \tag{37}$$

where the integrals J_i are defined by

$$J_i = \int_k \frac{N_i}{D_0 D_1 D_2}, \tag{38}$$

and

$$N_1 = k^2, \tag{39a}$$

$$N_2 = 1, \tag{39b}$$

$$N_3 = 2k\cdot q, \tag{39c}$$

$$N_4 = \frac{1}{d-2}\left[k^2 - \frac{(k\cdot p)^2}{p^2} - \frac{(k\cdot q)^2}{q^2}\right], \tag{39d}$$

$$N_5 = 2k\cdot p, \tag{39e}$$

$$N_6 = \frac{1}{d-2}\left[-k^2 + \frac{(k\cdot p)^2}{p^2} + (d-1)\frac{(k\cdot q)^2}{q^2}\right], \tag{39f}$$

$$N_7 = k\cdot p k\cdot q. \tag{39g}$$

The integrals $J_1 - J_7$ are evaluated in Appendix B. The results are tabulated in Eq. (B12). We note that J_5 and J_7 vanish, as is required by conservation of electromagnetic current in Eq. (37).

Writing the vertex correction as $\Lambda^\mu = \bar{v}(p_2)\times(\Lambda\gamma^\mu + Hq^\mu)u(p_1)$, we have

$$\Lambda = -ig_s^2 C_F[(d-2)J_1 - 4(2p^2-m^2)J_2 + 4J_3 + 2(2-d)J_4]$$
$$= \frac{\alpha_s C_F}{4\pi}\left\{\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 2(1+\delta^2)L(\delta)\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) + (1+\delta^2)\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + \frac{3\delta^2}{1+\delta^2}\right)\right]\right\}, \tag{40a}$$

$$H = -ig_s^2 C_F\left[-\frac{2m}{q^2}J_3 + \frac{2(2-d)m}{q^2}J_6\right] = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left[2L(\delta) - \frac{i\pi}{\delta}\right], \tag{40b}$$

where the subscripts on $1/\epsilon$ denote the origins of the divergences and γ_E is the Euler-Mascheroni constant. The functions $L(\delta)$ and $K(\delta)$ are given by

$$L(\delta) = \frac{1}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right), \tag{41a}$$
$$K(\delta) = \frac{1}{4\delta}\left[\mathrm{Sp}\left(\frac{2\delta}{1+\delta}\right) - \mathrm{Sp}\left(-\frac{2\delta}{1-\delta}\right)\right], \tag{41b}$$

where Sp is the Spence function:

$$\mathrm{Sp}\,(x) = \int_x^0 \frac{\log(1-t)}{t}\,dt. \tag{42}$$

In Eq. (40), we have neglected terms of order ϵ^1 and higher. In the remainder of this paper, we drop such higher-order terms.

B. Wave-function renormalization

The heavy-quark wave-function renormalization Z_Q, evaluated in dimensional regularization, is given in Ref. [12]:

$$Z_Q = 1 + \frac{\alpha_s C_F}{4\pi}\left(-\frac{1}{\epsilon_{\rm UV}} - \frac{2}{\epsilon_{\rm IR}} - 3\log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} - 4\right). \tag{43}$$

C. Summary of QCD results

By making use of Eqs. (23), (40), and (43), we find that G and H are given by

$$G = 1 + \frac{\alpha_s C_F}{4\pi}\left\{2[(1+\delta^2)L(\delta) - 1]\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) - 4 + (1+\delta^2) \times\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + \frac{3\delta^2}{1+\delta^2}\right)\right]\right\}, \tag{44a}$$

$$H = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left[2L(\delta) - \frac{i\pi}{\delta}\right]. \tag{44b}$$

Expanding Eq. (27) through order v^2, using Eq. (44), we obtain

$$i\mathcal{A}^i_{Q\bar{Q}_1} = \eta^\dagger\sigma^i\xi\left[1 + \frac{\alpha_s C_F}{4\pi}\left\{\frac{8}{3}v^2\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) - 8 + \frac{2v^2}{9} + \left(1 + \frac{3v^2}{2}\right)\left[\frac{\pi^2}{v} - \frac{i\pi}{v}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right] - 3i\pi v\right\}\right] - \frac{q^i\eta^\dagger\boldsymbol{q}\cdot\boldsymbol{\sigma}\xi}{2m^2}\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[-4 + \frac{\pi^2}{v} - \frac{i\pi}{v}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + 2\right)\right]\right\} + O(v^3). \tag{45}$$

Equation (45) agrees with Eq. (4.16) of Ref. [9].

VI. NRQCD CORRECTIONS

In this section, we calculate the NRQCD corrections to the heavy-quark electromagnetic current. That is, we compute $[i\mathcal{A}^{i(1)}_{Q\bar{Q}_1}]_{\rm NRQCD}$. In order to demonstrate our method for calculating these corrections from full-QCD expressions, we present the calculation in some detail.

Divergent integrals are regulated using dimensional regularization, with $d = 4 - 2\epsilon$. We define the following notation for the loop integrals in $d-1$ dimensions:

$$\int_{\boldsymbol{k}} \equiv \mu^{2\epsilon}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}. \tag{46}$$

We also define $\not{\int}_k$ and $\not{\int}_{\boldsymbol{k}}$, which have the same meaning as $\int_k$ and $\int_{\boldsymbol{k}}$, except that it is understood for $\not{\int}_k$ that one carries out the k^0 integration first, and it is understood for both $\not{\int}_k$ and $\not{\int}_{\boldsymbol{k}}$ that one expands the integrand in powers of the momenta divided by m.

A. Vertex correction

Now, we calculate the NRQCD vertex correction to the electromagnetic current. From Eq. (36), we see that the vertex correction is given by

$$\Lambda^i_{\rm NRQCD} = -ig_s^2 C_F \oint_k \frac{1}{D_0 D_1 D_2} \bar{v}(p_2)\{[(d-2)k^2 - 4(2p^2 - m^2) + 8k\cdot q]\gamma^i + 4mk^i - 8q^i \not{k} + 2(2-d)k^i \not{k}\} u(p_1). \quad (47)$$

The vertex correction (47) can be written as

$$\Lambda^i_{\rm NRQCD} = -ig_s^2 C_F \bar{v}(p_2)\{[(d-2)S_1 - 4(2p^2 - m^2)S_2 + 8q_\mu S_3^\mu]\gamma^i + 2[2mS_3^i - 4\gamma_\mu S_3^\mu q^i + (2-d)\gamma_\mu S_4^{\mu i}]\}u(p_1), \quad (48)$$

where

$$S_1 = \oint_k \frac{1}{D_1 D_2}, \quad (49a)$$

$$S_2 = \oint_k \frac{1}{D_0 D_1 D_2}, \quad (49b)$$

$$S_3^\mu = \oint_k \frac{k^\mu}{D_0 D_1 D_2}, \quad (49c)$$

$$S_4^{\mu\nu} = \oint_k \frac{k^\mu k^\nu}{D_0 D_1 D_2}. \quad (49d)$$

The factors in the denominator of the integrands are defined in Eq. (34). In the $Q\bar{Q}$ rest frame, the factors D_i in Eq. (34) are

$$D_0 = (k^0)^2 - \boldsymbol{k}^2 + i\varepsilon = (k^0 - |\boldsymbol{k}| + i\varepsilon)(k^0 + |\boldsymbol{k}| - i\varepsilon), \quad (50a)$$

$$D_1 = (k^0 + E)^2 - \Delta^2 + i\varepsilon = (k^0 + \Delta + E - i\varepsilon)(k^0 - \Delta + E + i\varepsilon), \quad (50b)$$

$$D_2 = (k^0 - E)^2 - \Delta^2 + i\varepsilon = (k^0 + \Delta - E - i\varepsilon)(k^0 - \Delta - E + i\varepsilon), \quad (50c)$$

where Δ is defined by

$$\Delta = \sqrt{m^2 + (\boldsymbol{k} + \boldsymbol{q})^2}. \quad (51)$$

The following are identities that we use frequently:

$$\Delta - E = \frac{\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q}}{\Delta + E}, \quad (52a)$$

$$\Delta^2 - (E \pm |\boldsymbol{k}|)^2 = \mp 2|\boldsymbol{k}|(E \mp \boldsymbol{q}\cdot\hat{\boldsymbol{k}}), \quad (52b)$$

where $\hat{\boldsymbol{a}} = \boldsymbol{a}/|\boldsymbol{a}|$ for any spatial vector $\boldsymbol{a}$. We first evaluate the k^0 integral by contour integration, closing the contour in the upper half-plane in every case. The contributions from the poles in the gluon, quark, and antiquark propagators are defined as S_{ig}, S_{iQ}, and $S_{i\bar{Q}}$, respectively. Certain integrals that we use frequently are tabulated in Appendix C.

We note that the contributions $S_{i\bar{Q}}$ correspond to the potential region in the method of regions, and the contributions S_{ig} correspond to the soft and ultrasoft regions in the method of regions [10]. The contributions S_{iQ} correspond to a region of integration in which the temporal component of the gluon momentum is of order m, but the spatial component of the gluon momentum is of order mv. As we have mentioned, in the method of regions it is assumed that this region of integration does not contribute [10]. We shall see explicitly in the calculations that follow that this assumption is justified because the contributions from this region of integration consist of scaleless, power-divergent integrals, which vanish in dimensional regularization. In the case of a hard-cutoff regulator these contributions do not vanish, and they must be included in the calculation of the NRQCD corrections.

1. S_1

The integral S_1 is the sum of two contributions: $S_1 = S_{1Q} + S_{1\bar{Q}}$.

By making use of Eq. (50), we evaluate the k^0 integral. The contribution from the quark pole is

$$S_{1Q} = -\frac{i}{8E}\oint_{\boldsymbol{k}} \frac{1}{\Delta(\Delta + E)}. \quad (53)$$

Expanding $1/\Delta$ and $1/(\Delta + E)$ in Eq. (53) in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$, we find that all of the terms in the expansion are scaleless, power-divergent integrals. Hence,

$$S_{1Q} = 0. \quad (54)$$

The contribution from the antiquark pole is

$$S_{1\bar{Q}} = \frac{i}{8E}\oint_{\boldsymbol{k}} \frac{1}{\Delta(\Delta - E - i\varepsilon)}. \quad (55)$$

We use the identity (52a) to reduce the integrand in Eq. (55) to the following form:

$$S_{1\bar{Q}} = \frac{i}{8E}\oint_{\boldsymbol{k}}\left(1 + \frac{E}{\Delta}\right)\frac{1}{\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon}. \quad (56)$$

Expanding $1/\Delta$ in Eq. (56) in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$, we find that the expansion brings in additional factors of $(\boldsymbol{k}+\boldsymbol{q})^2$. In each additional factor, only the term $\boldsymbol{q}^2$ survives, as the terms $\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q}$ lead to scaleless, power-divergent integrals, which vanish. As a result, we can replace Δ with E in Eq. (56). Hence, $S_{1\bar{Q}}$ is proportional to an elementary integral n_1, which is defined in Eq. (C3a):

$$S_{1\bar{Q}} = \frac{i}{4E}n_1 = -\frac{|\boldsymbol{q}|}{16\pi E}. \quad (57)$$

Using Eqs. (12), (54), and (57), we obtain

$$S_1 = \frac{i}{(4\pi)^2} i\pi\delta. \quad (58)$$

2. S_2

The integral S_2 is the sum of three contributions: $S_2 = S_{2g} + S_{2Q} + S_{2\bar{Q}}$.

By making use of Eq. (50), we evaluate the k^0 integral. The gluon-pole contribution is

$$S_{2g} = -\frac{i}{2}\mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|[\Delta^2-(E+|\boldsymbol{k}|)^2-i\varepsilon][\Delta^2-(E-|\boldsymbol{k}|)^2-i\varepsilon]} = \frac{i}{8}\int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|^3[E^2-(\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{59}$$

where we have used the identity (52b). Making use of Eq. (C4b), we find that S_{2g} is proportional to n_0 in Eq. (C2):

$$S_{2g} = \frac{i}{16E|\boldsymbol{q}|} n_0 \log\left(\frac{E+|\boldsymbol{q}|}{E-|\boldsymbol{q}|}\right). \tag{60}$$

Using Eqs. (15) and (C2), we express S_{2g} in terms of m and δ as

$$S_{2g} = \frac{i}{32\pi^2 m^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)\frac{1-\delta^2}{2\delta}\log\left(\frac{1+\delta}{1-\delta}\right). \tag{61}$$

The contribution from the quark pole is

$$S_{2Q} = -\frac{i}{8E}\mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{\Delta(\Delta+E)[(\Delta+E)^2-\boldsymbol{k}^2+i\varepsilon]} = -\frac{i}{8E}\sum_{n=0}^{\infty}\mathcal{N}\!\!\int_{\boldsymbol{k}}\frac{\boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+3}}. \tag{62}$$

Now we expand $1/\Delta$ and $1/(\Delta+E)$ in Eq. (62) in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$. All of the terms in the expansion yield scaleless, power-divergent integrals, which vanish. Therefore, we have

$$S_{2Q} = 0. \tag{63}$$

The contribution from the antiquark pole is

$$S_{2\bar{Q}} = -\frac{i}{8E}\mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{\Delta(\Delta-E-i\varepsilon)[\boldsymbol{k}^2-(\Delta-E)^2-i\varepsilon]}. \tag{64}$$

If we use the relation (52a), we obtain

$$S_{2\bar{Q}} = -\frac{i}{8E}\mathcal{N}\!\!\int_{\boldsymbol{k}}\left(1+\frac{E}{\Delta}\right) \times \frac{1}{\boldsymbol{k}^2(\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q}-i\varepsilon)[1-\frac{1}{\boldsymbol{k}^2}(\frac{\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q}}{\Delta+E})^2]}. \tag{65}$$

The denominator factor in the brackets can be expanded to give

$$S_{2\bar{Q}} = -\frac{i}{8E}\mathcal{N}\!\!\int_{\boldsymbol{k}}\left(1+\frac{E}{\Delta}\right)\left[\frac{1}{\boldsymbol{k}^2(\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q}-i\varepsilon)} + \sum_{n=1}^{\infty}\frac{(\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q})^{2n-1}}{\boldsymbol{k}^{2n+2}(\Delta+E)^{2n}}\right]. \tag{66}$$

Now we expand E/Δ and $1/(\Delta+E)$ in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$. The expansion brings in additional factors of $(\boldsymbol{k}+\boldsymbol{q})^2$ in each term in the integrand of Eq. (66). In each additional factor $(\boldsymbol{k}+\boldsymbol{q})^2$, only the term $\boldsymbol{q}^2$ survives, as the terms $\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q}$ lead to scaleless, power-divergent integrals. Therefore, we can replace Δ in Eq. (66) with E. Furthermore, in the numerator of the second term in brackets in Eq. (66), only the term $(2\boldsymbol{k}\cdot\boldsymbol{q})^{2n-1}$ survives, as the other terms lead to scaleless, power-divergent integrals. Then, we have

$$S_{2\bar{Q}} = -\frac{i}{4E}\mathcal{N}\!\!\int_{\boldsymbol{k}}\left[\frac{1}{\boldsymbol{k}^2(\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q}-i\varepsilon)} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{(\boldsymbol{k}\cdot\boldsymbol{q})^{2n-1}}{\boldsymbol{k}^{2n+2}E^{2n}}\right]. \tag{67}$$

The term proportional to $(\boldsymbol{k}\cdot\boldsymbol{q})^{2n-1}$ yields a scaleless, logarithmically divergent integral. However, this integral vanishes because the integrand is an odd function of $\boldsymbol{k}$. Thus, only the first term in the brackets in Eq. (67) survives, and we find that

$$S_{2\bar{Q}} = -\frac{i}{4E}n_2 = -\frac{1}{64\pi E|\boldsymbol{q}|}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + i\pi\right), \tag{68}$$

where n_2 is defined in Eq. (C3b).

Making use of Eqs. (15), (61), (63), and (68), we obtain

$$S_2 = \frac{i}{(4\pi)^2}\frac{1-\delta^2}{4m^2}\left[2L(\delta)\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right) - \frac{\pi^2}{\delta} + \frac{i\pi}{\delta}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right], \tag{69}$$

where $L(\delta)$ is defined in Eq. (41a).

3. S_3^μ

The integral S_3^μ is the sum of three contributions: $S_3^\mu = S_{3g}^\mu + S_{3Q}^\mu + S_{3\bar{Q}}^\mu$.

We first evaluate S_3^0. The integral of S_3^0 over k^0 is identical to the integral of S_2 over k^0 except that, in S_3^0, the result contains an additional factor of k^0 evaluated at the gluon, quark, or antiquark pole. Thus, by making use of Eqs. (52a), (59), (62), and (66), we obtain

$$S_{3g}^0 = -\frac{i}{8}\int_{\boldsymbol{k}}\frac{1}{\boldsymbol{k}^2[E^2-(\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{70a}$$

$$S_{3Q}^0 = \frac{i}{8E}\sum_{n=0}^{\infty}\mathcal{N}\!\!\int_{\boldsymbol{k}}\frac{\boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+2}}, \tag{70b}$$

$$S_{3\bar{Q}}^0 = \frac{i}{8E}\sum_{n=0}^{\infty}\mathcal{N}\!\!\int_{\boldsymbol{k}}\frac{(\boldsymbol{k}^2+2\boldsymbol{k}\cdot\boldsymbol{q})^{2n}}{\boldsymbol{k}^{2n+2}\Delta(\Delta+E)^{2n}}. \tag{70c}$$

S_{3g}^0 is a scaleless, power-divergent integral, which vanishes. In S_{3Q}^0 and $S_{3\bar{Q}}^0$ we expand $1/\Delta$ and $1/(\Delta+E)$ in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$. We find that every term in the expansions leads to a scaleless, power-divergent integral, which vanishes. Hence,

$$S_3^0 = 0. \tag{71}$$

Next we compute the spatial components S_3^i. The integral of S_3^i over k^0 is identical to the integral of S_2 over k^0

except that, in S_3^i, the result contains an additional factor of k^i. Thus, by making use of Eqs. (59) and (62), we obtain

$$S^i_{3g} = \frac{i}{8}\int_{\boldsymbol{k}} \frac{k^i}{|\boldsymbol{k}|^3[E^2 - (\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{72a}$$

$$S^i_{3Q} = -\frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{k^i \boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+3}}. \tag{72b}$$

S^i_{3g} is a scaleless, power-divergent integral, which vanishes. Expanding $1/\Delta$ and $1/(\Delta+E)$ in S^i_{3Q} in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$, we also obtain only scaleless, power-divergent integrals, which vanish. If we multiply the second term in brackets in Eq. (67) by k^i, we obtain only scaleless, power-divergent integrals. Hence,

$$S^i_{3\bar{Q}} = -\frac{i}{4E}\int_{\boldsymbol{k}} \frac{k^i}{\boldsymbol{k}^2(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon)}. \tag{73}$$

After making a standard reduction of the tensor integral in Eq. (73) to a scalar integral, we obtain

$$S^i_{3\bar{Q}} = -\frac{iq^i}{8E\boldsymbol{q}^2}\int_{\boldsymbol{k}} \frac{(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q}) - \boldsymbol{k}^2}{\boldsymbol{k}^2(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon)} = \frac{iq^i}{8E\boldsymbol{q}^2}n_1 = -\frac{1}{32\pi}\frac{q^i}{E|\boldsymbol{q}|}, \tag{74}$$

where n_1 is defined in Eq. (C3a) and we have discarded scaleless, power-divergent integrals. Hence,

$$S^i_3 = -\frac{1}{32\pi}\frac{q^i}{E|\boldsymbol{q}|}. \tag{75}$$

Writing our results in Eqs. (71) and (75) in covariant form, we obtain

$$S^\mu_3 = \frac{i}{(4\pi)^2}\frac{1-\delta^2}{2m^2}\frac{i\pi}{\delta}q^\mu, \tag{76}$$

where we have made use of Eq. (15) to express E and $|\boldsymbol{q}|$ in terms of δ.

4. $S^{\mu\nu}_4$

The integral $S^{\mu\nu}_4$ is the sum of three contributions: $S^{\mu\nu}_4 = S^{\mu\nu}_{4g} + S^{\mu\nu}_{4Q} + S^{\mu\nu}_{4\bar{Q}}$.

We first evaluate S^{00}_4. The integral of S^{00}_4 over k^0 is identical to the integral of S^0_3 over k^0 except that, in S^{00}_4, the result contains an additional factor of k^0 evaluated at the gluon, quark, or antiquark pole. Thus, by making use of Eqs. (52a) and (70), we obtain

$$S^{00}_{4g} = \frac{i}{8}\int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|[E^2 - (\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{77a}$$

$$S^{00}_{4Q} = -\frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{\boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+1}}, \tag{77b}$$

$$S^{00}_{4\bar{Q}} = -\frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q})^{2n+1}}{\boldsymbol{k}^{2n+2}\Delta(\Delta+E)^{2n+1}}. \tag{77c}$$

Every integral in Eq. (77) is a scaleless, power-divergent integral. Hence,

$$S^{00}_4 = 0. \tag{78}$$

Next we compute S^{0i}_4. The integral of S^{0i}_4 over k^0 is identical to the integral of S^0_3 over k^0 except that, in S^{0i}_4, the result contains an additional factor of k^i. Thus, by making use of Eq. (70), we obtain

$$S^{0i}_{4g} = -\frac{i}{8}\int_{\boldsymbol{k}} \frac{k^i}{\boldsymbol{k}^2[E^2 - (\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{79a}$$

$$S^{0i}_{4Q} = \frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{k^i\boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+2}}, \tag{79b}$$

$$S^{0i}_{4\bar{Q}} = \frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{k^i(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q})^{2n}}{\boldsymbol{k}^{2n+2}\Delta(\Delta+E)^{2n}}. \tag{79c}$$

S^{0i}_{4g} is a scaleless, power-divergent integral, which vanishes. S^{0i}_{4Q} and $S^{0i}_{4\bar{Q}}$ also vanish, once we expand $1/\Delta$ and $1/(\Delta+E)$ in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$. Thus,

$$S^{0i}_4 = 0. \tag{80}$$

Finally, we evaluate the integrals S^{ij}_4. The integral of S^{ij}_4 over k^0 is identical to the integral of S^i_3 over k^0 except that, in S^{ij}_4, the result contains an additional factor of k^j. Thus, by making use of Eqs. (72) and (73), we obtain

$$S^{ij}_{4g} = \frac{i}{8}\int_{\boldsymbol{k}} \frac{k^ik^j}{|\boldsymbol{k}|^3[E^2 - (\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2]}, \tag{81a}$$

$$S^{ij}_{4Q} = -\frac{i}{8E}\sum_{n=0}^{\infty} \mathcal{H}\!\!\!\int_{\boldsymbol{k}} \frac{k^ik^j\boldsymbol{k}^{2n}}{\Delta(\Delta+E)^{2n+3}}, \tag{81b}$$

$$S^{ij}_{4\bar{Q}} = -\frac{i}{4E}\int_{\boldsymbol{k}} \frac{k^ik^j}{\boldsymbol{k}^2(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon)}. \tag{81c}$$

S^{ij}_{4g} is a scaleless, power-divergent integral, which vanishes. S^{ij}_{4Q} also vanishes, once we expand $1/\Delta$ and $1/(\Delta+E)$ in powers of $(\boldsymbol{k}+\boldsymbol{q})^2/m^2$. The tensor integral $S^{ij}_{4\bar{Q}}$ in Eq. (81c) must be a linear combination of the two symmetric tensors δ^{ij} and q^iq^j. By contracting these tensors into Eq. (81c), we determine the coefficients of the linear combination. The result is

$$\begin{aligned} S^{ij}_{4\bar{Q}} &= -\frac{i}{4E(d-2)}\bigg[\delta^{ij}\Big(n_1 - \frac{1}{4\boldsymbol{q}^2}n_3\Big) - \frac{q^iq^j}{\boldsymbol{q}^2}\Big(n_1 - \frac{d-1}{4\boldsymbol{q}^2}n_3\Big)\bigg] \\ &= \frac{|\boldsymbol{q}|}{32\pi(d-2)E}\bigg[\delta^{ij} + (d-3)\frac{q^iq^j}{\boldsymbol{q}^2}\bigg], \end{aligned} \tag{82}$$

where n_3 is defined in Eq. (C3c). Because S^{ij}_{4g} and $S^{ij}_{4\bar{Q}}$ vanish, we find that $S^{ij}_4 = S^{ij}_{4\bar{Q}}$. The integral in Eq. (82) is finite and, therefore, we may set $d = 4$.

The covariant form of the integral $S_4^{\mu\nu}$ at $d = 4$ is then

$$S_4^{\mu\nu} = \frac{i}{(4\pi)^2}\frac{i\pi\delta}{4}\left[g^{\mu\nu} - \frac{1-\delta^2}{m^2}\left(p^\mu p^\nu + \frac{q^\mu q^\nu}{\delta^2}\right)\right]. \quad (83)$$

5. Summary of the NRQCD vertex correction

Substituting $S_1 - S_4^{\mu\nu}$ in Eqs. (58), (69), (76), and (83) into Eq. (48) and using the equations of motion in Eq. (35), we obtain

$$\Lambda_{\rm NRQCD} = \frac{\alpha_s C_F}{4\pi}(1+\delta^2)\left[2L(\delta)\left(\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right) + \frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + \frac{3\delta^2}{1+\delta^2}\right)\right], \quad (84a)$$

$$H_{\rm NRQCD} = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left(-\frac{i\pi}{\delta}\right). \quad (84b)$$

B. Wave-function renormalization

In the Feynman gauge, the self-energy of the heavy quark, evaluated at four-momentum p_1, is

$$[\Sigma(p_1)]_{\rm NRQCD} = -ig_s^2 C_F \mathcal{N}\!\!\int_k \frac{\gamma_\mu(\not p_1 + \not k + m)\gamma^\mu}{(k^2 + i\varepsilon)[(p_1+k)^2 - m^2 + i\varepsilon]}, \quad (85)$$

where m is the mass of the heavy quark and k is the loop momentum, which has been chosen to be the momentum of the virtual gluon. In d dimensions, we find that the numerator factor reduces to

$$[\Sigma(p_1)]_{\rm NRQCD} = -ig_s^2 C_F \mathcal{N}\!\!\int_k \frac{(2-d)(\not p_1 + \not k) + dm}{(k^2 + i\varepsilon)[(p_1+k)^2 - m^2 + i\varepsilon]}. \quad (86)$$

The heavy-quark wave-function renormalization Z_Q is defined by

$$[Z_Q]_{\rm NRQCD} = \left[1 - \frac{p_1^\mu}{m}\frac{\partial[\Sigma(p_1)]_{\rm NRQCD}}{\partial p_1^\mu}\bigg|_{\not p_1 = m}\right]^{-1} = 1 + \frac{p_1^\mu}{m}\frac{\partial[\Sigma(p_1)]_{\rm NRQCD}}{\partial p_1^\mu}\bigg|_{\not p_1 = m} + O(\alpha_s^2). \quad (87)$$

Differentiating Eq. (86), we find that

$$\frac{p_1^\mu}{m}\frac{\partial[\Sigma(p_1)]_{\rm NRQCD}}{\partial p_1^\mu}\bigg|_{\not p_1 = m} = -ig_s^2 C_F \mathcal{N}\!\!\int_k \left\{\frac{2-d}{D_0 D_1} - \frac{2[(2-d)(\not k + m) + dm](p_1\cdot k + m^2)}{m D_0 D_1^2}\right\}$$
$$= -ig_s^2 C_F \mathcal{N}\!\!\int_k \left[\frac{2-d}{D_0 D_1} - \frac{(2-d)\not k + 2m}{m}\left(\frac{1}{D_0 D_1} - \frac{1}{D_1^2} + \frac{2m^2}{D_0 D_1^2}\right)\right], \quad (88)$$

where D_0 and D_1 are defined in Eq. (34). The expression in Eq. (88) can be written in terms of the integrals T_{02}, T_{11}, T_{12}, T_{02}^μ, T_{11}^μ, and T_{12}^μ, which are defined by

$$T_{ab} = \mathcal{N}\!\!\int_k \frac{1}{D_0^a D_1^b}, \quad (89a)$$

$$T_{ab}^\mu = \mathcal{N}\!\!\int_k \frac{k^\mu}{D_0^a D_1^b}. \quad (89b)$$

These integrals are evaluated in Appendix D, and the results are summarized in Eqs. (D7) and (D8). The only nonvanishing integral is T_{12}. Hence,

$$\frac{p_1^\mu}{m}\frac{\partial[\Sigma(p_1)]_{\rm NRQCD}}{\partial p_1^\mu}\bigg|_{\not p_1 = m} = 4ig_s^2 C_F m^2 T_{12}. \quad (90)$$

Making use of Eqs. (87), (90), and (D8), we obtain the heavy-quark wave-function renormalization in NRQCD:

$$[Z_Q]_{\rm NRQCD} = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right) + O(\alpha_s^2). \quad (91)$$

C. Summary of NRQCD results

Making use of Eqs. (84) and (91), we find that

$$G_{\rm NRQCD} = 1 + \frac{\alpha_s C_F}{4\pi}\left\{2[(1+\delta^2)L(\delta) - 1]\left(\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right) + (1+\delta^2)\left[\frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + \frac{3\delta^2}{1+\delta^2}\right)\right]\right\}, \quad (92a)$$

$$H_{\rm NRQCD} = \frac{\alpha_s C_F}{4\pi}\frac{1-\delta^2}{m}\left(-\frac{i\pi}{\delta}\right). \quad (92b)$$

Expanding Eq. (28) through order v^2, we obtain

$$i[\mathcal{A}^i_{Q\bar{Q}_1}]_{\rm NRQCD} = \eta^\dagger \sigma^i \xi\left[1 + \frac{\alpha_s C_F}{4\pi}\left\{\frac{8v^2}{3}\left(\frac{1}{\epsilon_{\rm IR}} - \frac{1}{\epsilon_{\rm UV}}\right) + \left(1 + \frac{3v^2}{2}\right)\left[\frac{\pi^2}{v} - \frac{i\pi}{v}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right)\right] - 3i\pi v\right\}\right]$$
$$- \frac{q^i \eta^\dagger \boldsymbol{q}\cdot\boldsymbol{\sigma}\xi}{2m^2}\left\{1 + \frac{\alpha_s C_F}{4\pi}\left[\frac{\pi^2}{v} - \frac{i\pi}{v}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2}\right) - \frac{2i\pi}{v}\right]\right\} + O(v^3). \quad (93)$$

Comparing Eq. (93) with Eqs. (4.28) and (4.29) of Ref. [9], we find agreement. We have also checked Eq. (93) by carrying out a conventional calculation in NRQCD.

VII. RESULTS FOR THE SHORT-DISTANCE COEFFICIENTS

Now we can collect the results of our calculations and obtain the short-distance coefficients. By making use of Eqs. (31), (44), and (92), we find that

$$\Delta G^{(1)} = \frac{\alpha_s C_F}{4\pi}\left\{2[(1+\delta^2)L(\delta) - 1]\left(\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right) + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) - 4\right\}, \quad (94a)$$

$$\Delta H^{(1)} = \frac{\alpha_s C_F}{4\pi}\frac{2(1-\delta^2)}{m}L(\delta). \quad (94b)$$

As expected, the infrared poles in $G^{(1)}$ and $G^{(1)}_{\rm NRQCD}$ have canceled in $\Delta G^{(1)}$. Note that $\Delta G^{(1)}$ and $\Delta H^{(1)}$ are real and contain only even powers of $v = |\boldsymbol{q}|/m$. Renormalizing the matrix elements in the $\overline{\rm MS}$ scheme, we obtain

$$\Delta G^{(1)}_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\left\{2[(1+\delta^2)L(\delta) - 1]\log\frac{\mu^2}{m^2} + 6\delta^2 L(\delta) - 4(1+\delta^2)K(\delta) - 4\right\}, \quad (95)$$

where now μ is the NRQCD factorization scale. Using Eq. (29), we obtain the short-distance coefficients $a_n^{(0)}$ and $b_n^{(0)}$:

$$a_n^{(0)} = \delta_{n0}, \quad (96a)$$
$$b_1^{(0)} = -\frac{1}{2m^2}, \quad (96b)$$
$$b_2^{(0)} = \frac{3}{8m^4}, \quad (96c)$$
$$b_3^{(0)} = -\frac{5}{16m^6}. \quad (96d)$$

The results in Eqs. (96a)–(96c) agree with those in Eq. (5.5) of Ref. [6] and those in Eqs. (3.13)–(3.20) of Ref. [13]. Using Eqs. (32), (94b), and (95), we obtain the short-distance coefficients $[a_n^{(1)}]_{\overline{\rm MS}}$ and $[b_n^{(1)}]_{\overline{\rm MS}}$:

$$[a_0^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}(-8), \quad (97a)$$
$$[a_1^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\frac{1}{m^2}\left(\frac{2}{9} + \frac{8}{3}\log\frac{\mu^2}{m^2}\right), \quad (97b)$$
$$[a_2^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\frac{1}{m^4}\left(-\frac{92}{75} - \frac{8}{5}\log\frac{\mu^2}{m^2}\right), \quad (97c)$$
$$[a_3^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\frac{1}{m^6}\left(\frac{13744}{11025} + \frac{128}{105}\log\frac{\mu^2}{m^2}\right), \quad (97d)$$
$$[b_1^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\frac{2}{m^2}, \quad (97e)$$
$$[b_2^{(1)}]_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\frac{1}{m^4}\left(\frac{7}{9} + \frac{4}{3}\log\frac{\mu^2}{m^2}\right), \quad (97f)$$
$$[b_3^{(1)}]_{\overline{\rm MS}} = \frac{\alpha_s C_F}{4\pi}\frac{1}{m^6}\left(\frac{107}{150} + \frac{9}{5}\log\frac{\mu^2}{m^2}\right). \quad (97g)$$

The operators $\mathcal{O}_{A0}$, $\mathcal{O}_{A1}$, and $\mathcal{O}_{B1}$ in Eq. (16) correspond to the operators that were considered in Ref. [9], provided that one neglects the gauge fields in the latter operators. Therefore, short-distance coefficients $[a_0]_{\overline{\rm MS}}$, $[a_1]_{\overline{\rm MS}}$, and $[b_1]_{\overline{\rm MS}}$ are related to the coefficients c_i in Eq. (4.29) of Ref. [9] as follows:

$$[a_0]_{\overline{\rm MS}} = c_1, \quad (98a)$$
$$[a_1]_{\overline{\rm MS}} = -\frac{1}{m^2}c_3, \quad (98b)$$
$$[b_1]_{\overline{\rm MS}} = -\frac{1}{2m^2}c_2. \quad (98c)$$

Our results for these short-distance coefficients agree with those in Eq. (4.29) of Ref. [9].

A. Resummation

Let us define ratios of the S-wave $Q\bar{Q}$ operator matrix elements to the S-wave $Q\bar{Q}$ operator matrix element of lowest order in v:

$$\langle \boldsymbol{q}^{2n}\rangle_{H(^3S_1)} = \frac{\langle 0|\mathcal{O}^i_{An}|H(^3S_1)\rangle}{\langle 0|\mathcal{O}^i_{A0}|H(^3S_1)\rangle}, \quad (99)$$

where $\mathcal{O}^i_{An}$ is defined in Eq. (16a), and we have used the property that the ratios are independent of the value of the index i. In Ref. [14], it was shown that these ratios of operator matrix elements are related according to a generalized Gremm-Kapustin relation [15]:

$$[\langle \boldsymbol{q}^{2n}\rangle_{H(^3S_1)}]_{\overline{\rm MS}} = [\langle \boldsymbol{q}^2\rangle_{H(^3S_1)}]^n_{\overline{\rm MS}}. \quad (100)$$

This relation holds for the matrix elements in spin-independent-potential models. Hence, for each value of n, it holds up to corrections of relative order v^2.

We can use the relation (100) to resum a class of relativistic corrections to the quarkonium electromagnetic current. From Eqs. (20a) and (32), we find that

$$\sum_{n=0}^{\infty}(s_n^{(0)} + [s_n^{(1)}]_{\overline{\rm MS}})\langle 0|\mathcal{O}^i_{An}|H(^3S_1)\rangle = \left\{\left[1 - \frac{\boldsymbol{q}^2}{E(E+m)(d-1)}\right]\left(1 + \Delta G^{(1)}_{\overline{\rm MS}}\right) - \frac{\boldsymbol{q}^2}{E(d-1)}\Delta H^{(1)}\right\}\Bigg|_{\boldsymbol{q}^2=\langle\boldsymbol{q}^2\rangle_{H(^3S_1)}}\langle 0|\mathcal{O}^i_{A0}|H(^3S_1)\rangle. \quad (101)$$

Because the relation (100) contains corrections of relative order v^2 at each order v^{2n}, the resummation in Eq. (101) does not improve the nominal accuracy beyond order v^4. The resummation might, however, improve the numerical accuracy beyond the accuracy that is obtained through order v^4 if the coefficients in the velocity expansion grow rapidly with the order in v. In any case, it is interesting to use the resummed result to examine the rate of convergence of the velocity expansion.

B. Numerical results and convergence of the velocity expansion

Let us evaluate the sums of products of S-wave short-distance coefficients and operator matrix elements, using the relation (100). For $\langle\boldsymbol{q}^2\rangle_{H(^3S_1)}$, we take the central value of the J/ψ matrix element from Ref. [1]: $\langle\boldsymbol{q}^2\rangle_{J/\psi} = 0.441\ \mathrm{GeV}^2$. Taking $m_c = 1.5$ GeV and setting $\mu = m_c$, we find that

$$\sum_{n=0}^{0}[s_n^{(1)}]_{\overline{\rm MS}}[\langle\boldsymbol{q}^2\rangle_{J/\psi}]^n_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\times 8, \quad (102a)$$

$$\sum_{n=0}^{1}[s_n^{(1)}]_{\overline{\rm MS}}[\langle\boldsymbol{q}^2\rangle_{J/\psi}]^n_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\times 7.826, \quad (102b)$$

$$\sum_{n=0}^{2}[s_n^{(1)}]_{\overline{\rm MS}}[\langle\boldsymbol{q}^2\rangle_{J/\psi}]^n_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\times 7.883, \quad (102c)$$

$$\sum_{n=0}^{3}[s_n^{(1)}]_{\overline{\rm MS}}[\langle\boldsymbol{q}^2\rangle_{J/\psi}]^n_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\times 7.872, \quad (102d)$$

$$\sum_{n=0}^{\infty}[s_n^{(1)}]_{\overline{\rm MS}}[\langle\boldsymbol{q}^2\rangle_{J/\psi}]^n_{\overline{\rm MS}} = -\frac{\alpha_s C_F}{4\pi}\times 7.873. \quad (102e)$$

In the last line of Eq. (102), we have used the resummed result in Eq. (101). Taking $\alpha_s = \alpha_s(2m_c) = 0.25$, we see that the corrections of order $\alpha_s v^2$ and $\alpha_s v^4$ are 0.5% and -0.2%, respectively. These are not very significant at the current level of precision of the theory of J/ψ decays to a lepton pair.

As can be seen from Eq. (102), the velocity expansion converges rapidly for approximate charmonium matrix elements. In fact, the expressions for $\Delta G^{(1)}_{\overline{\rm MS}}$ in Eq. (95) and $\Delta H^{(1)}$ in Eq. (94b), taken as functions of $v = |\boldsymbol{q}|/m$, have finite radii of convergence. The logarithms in $L(\delta)$ [Eq. (41a)] and the Spence functions in $K(\delta)$ [Eq. (41b)] have branch points at $\delta = \pm 1$, i.e., $v = \pm\infty$. The quantity $\delta = v/\sqrt{1+v^2}$ has branch points at $v = \pm i$. Therefore, the closest singularities to the origin in $\Delta G^{(1)}_{\overline{\rm MS}}$ or $\Delta H^{(1)}$ are at $v = \pm i$. Consequently, the radii of convergence of $\Delta G^{(1)}_{\overline{\rm MS}}$ and $\Delta H^{(1)}$ as functions of v are one. It follows that the velocity expansion for the $Q\bar{Q}$ operators is absolutely convergent, provided that the absolute values of the operator matrix elements are bounded by a geometric sequence in which the ratio between elements of the sequence is less than m^2.

VIII. CONCLUSIONS

We have presented a calculation in NRQCD of the order-α_s corrections to the quarkonium electromagnetic current. Our calculation gives expressions for the short-distance coefficients of all of the $Q\bar{Q}$ NRQCD operators that contain any number of derivatives but no gauge fields. Our operators are not gauge invariant, and we evaluate their matrix elements in the Coulomb gauge. Our principal results are given in Eqs. (94b) and (95). The NRQCD short-distance coefficients can be obtained, according to Eq. (32), from the Taylor-series expansions of $\Delta G^{(1)}_{\overline{\rm MS}}$ in Eq. (95) and $\Delta H^{(1)}$ in Eq. (94b). Our results at relative order v^2 agree with those in Ref. [9].

Our calculation makes use of a new method for computing, to all orders in v, the one-loop NRQCD corrections that enter into the matching of NRQCD to full QCD. In this new method, we begin with QCD expressions for the loop integrands. We obtain the NRQCD corrections from these QCD expressions by carrying out the integration over the temporal component of the loop momentum and then expanding the loop integrands in powers of the loop and external momenta divided by the heavy-quark mass m. We carry out this expansion *before* implementing the dimensional regularization. The new approach allows one to avoid the daunting task of obtaining NRQCD operators and interactions to all orders in v, along with their Born-level short-distance coefficients, and computing their contributions to the one-loop corrections. In terms of the total labor involved, the computation of the NRQCD corrections to all orders through the new approach is comparable to the calculation of the NRQCD corrections at relative order v^2 through conventional NRQCD methods. This new method should be applicable to matching calculations for a variety of effective field theories, including heavy-quark effective theory and soft-collinear effective theory.

As we have mentioned, our approach is related to the method of regions [10]. The NRQCD corrections in our approach correspond in the method of regions to the sum of the contributions from the potential, soft, and ultrasoft regions, i.e., the contribution from the small-loop-momentum region [10]. In our approach we have computed the quantities $\Delta G^{(1)}$ and $\Delta H^{(1)}$ by subtracting the NRQCD corrections from the full-QCD corrections. In the method of regions, $\Delta G^{(1)}$ and $\Delta H^{(1)}$ could, in principle, be computed directly from the contribution from the hard region. However, a straightforward computation of the contribution from the hard region, carried out by expanding the integrand in powers of the small momentum, would yield Taylor-series expansions of $\Delta G^{(1)}$ and $\Delta H^{(1)}$ in Eq. (94) in powers of δ. It would be nontrivial to sum those expansions to obtain the compact expressions in Eq. (94). In contrast, in our approach, expansions of the integrand occur only in the NRQCD expressions and lead to very simple series that can be summed at the integrand level. Hence, our method may be more efficient than the method of regions for computations of short-distance coefficients to all orders in v. Our method is also applicable in the case of a hard-cutoff regulator, such as lattice regularization, while the method of regions applies only in the case of dimensional regularization.

Because we have omitted operators that contain gauge fields, the operators that we consider are not the complete set of NRQCD operators that describe the quarkonium electromagnetic current. In the Coulomb gauge, the gauge-field operators first enter at relative order v^4, and so our results cannot be considered to be complete beyond order v^2. However, the operators that we consider account for all of the contributions that are contained in the Coulomb-gauge wave function of the quarkonium $Q\bar{Q}$ Fock state. The correction to the S-wave component of the electromagnetic current that we find in relative order $\alpha_s v^4$ is only about -0.2%, which is not significant at the current level of the precision of the theory of J/ψ decays to a lepton pair.

We have examined the convergence of the NRQCD velocity expansion for S-wave $Q\bar{Q}$ operators. In Eq. (102), we give the numerical values for the sums of the first few S-wave contributions to the electromagnetic current and for the sum of all of the S-wave contributions. In these computations, we have made use of the value of the relative-order-v^2 J/ψ matrix element that is given in Ref. [1] and the approximate relation between operator matrix elements in Eq. (100), which holds in spin-independent-potential models [14]. It can be seen from Eq. (102) that the velocity expansion converges rapidly in this case. In fact, the expressions for $\Delta G^{(1)}_{\overline{\rm MS}}$ in Eq. (95) and $\Delta H^{(1)}$ in Eq. (94b), taken as functions of $v = |\boldsymbol{q}|/m$, have radii of convergence one. Therefore, the velocity expansion for the $Q\bar{Q}$ operators is absolutely convergent, provided that the absolute values of the operator matrix elements are bounded by a geometric sequence in which the ratio between elements of the sequence is less than m^2.

ACKNOWLEDGMENTS

The work of G. T. B. was supported by the U.S. Department of Energy, Division of High Energy Physics, under Contract No. DE-AC02-06CH11357. The work of H. S. C. was supported by the BK21 program. The work of C. Y. was supported by the Korea Research Foundation under MOEHRD Basic Research Promotion Grant No. KRF-2006-311-C00020. The work of J. L. was supported by the Korea Science and Engineering Foundation (KOSEF) funded by the Korea government (MEST) under Grant No. R01-2008-000-10378-0.

APPENDIX A: TENSOR-INTEGRAL REDUCTION

In this Appendix, we describe the tensor-integral reduction that we use to simplify Eq. (36).

Tensor integrals of rank-1 and -2 that depend on p or on both p and q can be expressed in terms of scalar integrals as follows:

$$\int_k k^\mu f(k,p) = \frac{p^\mu}{p^2}\int_k p\cdot k f(k,p), \tag{A1a}$$

$$\int_k k^\mu k^\nu f(k,p) = \int_k [d_1(k,p) g^{\mu\nu} + d_2(k,p) p^\mu p^\nu] f(k,p), \tag{A1b}$$

$$\int_k k^\mu f(k,p,q) = p^\mu \int_k d_3(k,p,q) f(k,p,q) + q^\mu \int_k d_4(k,p,q) f(k,p,q), \tag{A1c}$$

$$\int_k k^\mu k^\nu f(k,p,q) = g^{\mu\nu}\int_k d_5(k,p,q) f(k,p,q) + p^\mu p^\nu \int_k d_6(k,p,q) f(k,p,q) + q^\mu q^\nu \int_k d_7(k,p,q) f(k,p,q) + (p^\mu q^\nu + p^\nu q^\mu)\int_k d_8(k,p,q) f(k,p,q), \tag{A1d}$$

where f is an arbitrary scalar function of the argument four-vectors. If $p\cdot q = 0$, then the functions d_i in Eq. (A1) are given by

$$d_1(k,p) = \frac{1}{d-1}\left[k^2 - \frac{(k\cdot p)^2}{p^2}\right], \tag{A2a}$$

$$d_2(k,p) = \frac{1}{(d-1)p^2}\left[-k^2 + d\frac{(k\cdot p)^2}{p^2}\right], \tag{A2b}$$

$$d_3(k,p,q) = \frac{k\cdot p}{p^2}, \tag{A2c}$$

$$d_4(k,p,q) = \frac{k\cdot q}{q^2}, \tag{A2d}$$

$$d_5(k,p,q) = \frac{1}{d-2}\left[k^2 - \frac{(k\cdot p)^2}{p^2} - \frac{(k\cdot q)^2}{q^2}\right], \tag{A2e}$$

$$d_6(k,p,q) = \frac{1}{(d-2)p^2}\left[-k^2 + (d-1)\frac{(k\cdot p)^2}{p^2} + \frac{(k\cdot q)^2}{q^2}\right], \tag{A2f}$$

$$d_7(k,p,q) = \frac{1}{(d-2)q^2}\left[-k^2 + \frac{(k\cdot p)^2}{p^2} + (d-1)\frac{(k\cdot q)^2}{q^2}\right], \tag{A2g}$$

$$d_8(k,p,q) = \frac{k\cdot p}{p^2}\frac{k\cdot q}{q^2}. \tag{A2h}$$

APPENDIX B: INTEGRALS FOR THE QCD CORRECTIONS

In this Appendix, we evaluate the integrals in Eq. (38). Throughout this Appendix, we neglect expressions of order ϵ or higher. The integrals in Eq. (38) can be expressed in terms of elementary integrals I_{010}, I_{110}, I_{011}, I_{-111}, and I_{111}:

$$J_1 = I_{011}, \tag{B1a}$$

$$J_2 = I_{111}, \tag{B1b}$$

$$J_3 = I_{110} - I_{011}, \tag{B1c}$$

$$J_4 = \frac{1}{d-2}\left(I_{011} - \frac{I_{-111} - I_{010}}{4q^2}\right), \tag{B1d}$$

$$J_5 = 0, \tag{B1e}$$

$$J_6 = -J_4 + \frac{1}{4q^2}I_{-111} + \frac{p^2 - 2m^2}{4q^2m^2}I_{010}, \tag{B1f}$$

$$J_7 = 0, \tag{B1g}$$

where the scalar integral I_{abc} is defined by

$$I_{abc} = \int_k \frac{1}{D_0^a D_1^b D_2^c}. \tag{B2}$$

In deriving Eq. (B1), we have used the fact that $I_{abc} = I_{acb}$, which follows from the symmetry of the integrals under $p_1 \leftrightarrow p_2$ and $k \to -k$. We have also discarded the scaleless, power-divergent integral I_{100}, which vanishes in dimensional regularization. In deriving the expressions for J_4 and J_6, we have made a further tensor reduction, using Eq. (A1a), which leads to

$$I_{11-1} = \frac{2p^2}{m^2}I_{010}. \tag{B3}$$

I_{010} and I_{110}, which depend only on m^2, are given by

$$I_{010} = \frac{i}{(4\pi)^2}m^2\left(\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 1\right), \tag{B4a}$$

$$I_{110} = \frac{i}{(4\pi)^2}\left(\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 2\right). \tag{B4b}$$

The scalar integrals I_{011} and I_{-111} can be evaluated by using Feynman parametrization. After integrating over k, we obtain

$$I_{011} = \frac{i}{(4\pi)^2}\left(\frac{4\pi\mu^2}{p^2}\right)^{\epsilon}\Gamma(\epsilon)\int_0^1 dz(z^2 - \delta^2 - i\varepsilon)^{-\epsilon}, \tag{B5a}$$

$$I_{-111} = \frac{i}{(4\pi)^2}\left(\frac{4\pi\mu^2}{p^2}\right)^{\epsilon}\Gamma(\epsilon)\left(\frac{3-2\epsilon}{1-\epsilon}\right)p^2 \times \int_0^1 dz(z^2 - \delta^2 - i\varepsilon)^{1-\epsilon}, \tag{B5b}$$

where $z = 2x - 1$ and x is the original Feynman parameter. Expanding the integrands of Eq. (B5) in powers of ϵ, integrating over z, and using Eq. (15), we find that

$$I_{011} = \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 2 - 2\delta^2 L(\delta) + i\pi\delta\right], \tag{B6a}$$

$$I_{-111} = \frac{i}{(4\pi)^2}\frac{m^2}{1-\delta^2}\left[(1-3\delta^2)\left(\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 1\right) - 2\delta^2 + 4\delta^4 L(\delta) - 2\pi i\delta^3\right], \tag{B6b}$$

where $L(\delta)$ is defined in Eq. (41a) and we have used the following results, which hold for $0 \le \delta < 1$:

$$\int_0^1 \log(z^2 - \delta^2 - i\varepsilon)dz = -2 + 2\delta^2 L(\delta) + \log(1-\delta^2) - i\pi\delta, \tag{B7a}$$

$$\int_0^1 (z^2 - \delta^2 - i\varepsilon)\log(z^2 - \delta^2 - i\varepsilon)dz = \frac{1}{3}\left[-\frac{2}{3} + 4\delta^2 - 4\delta^4 L(\delta) + (1-3\delta^2)\log(1-\delta^2) + 2\pi i\delta^3\right]. \tag{B7b}$$

I_{111} can be evaluated by using Feynman parametrization. After integrating over k, we obtain

$$I_{111} = -\frac{i}{(4\pi)^2}\left(\frac{4\pi\mu^2}{p^2}\right)^\epsilon \frac{\Gamma(1+\epsilon)}{p^2}\int_0^1 dy y^{-1-2\epsilon} \times \int_0^1 dz(z^2-\delta^2-i\varepsilon)^{-1-\epsilon}, \tag{B8}$$

where $z = 2x - 1$ and the original Feynman parameters are x and y. The infrared divergence is isolated in the integral over y:

$$\int_0^1 dy y^{-1-2\epsilon} = -\frac{1}{2\epsilon_{\rm IR}}. \tag{B9}$$

The integral over z can be evaluated by expanding the integrand in powers of ϵ. Then, we obtain

$$I_{111} = \frac{i}{(4\pi)^2}\frac{1-\delta^2}{4m^2}\left\{\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right)\left[-2L(\delta) + \frac{i\pi}{\delta}\right] + 4K(\delta) - \frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\log\frac{4\delta^2}{1-\delta^2}\right\}, \tag{B10}$$

where $K(\delta)$ is defined in Eq. (41b) and we have used the following results, which hold for $0 \le \delta < 1$:

$$\int_0^1 \frac{dz}{z^2-\delta^2-i\varepsilon} = \frac{i\pi}{2\delta} - L(\delta), \tag{B11a}$$

$$\int_0^1 \frac{\log(z^2-\delta^2-i\varepsilon)}{z^2-\delta^2-i\varepsilon}dz = -\log(1-\delta^2)L(\delta) - 2K(\delta) + \frac{\pi^2}{2\delta} + \frac{i\pi}{\delta}\log(2\delta). \tag{B11b}$$

By making use of Eqs. (B1), (B4), (B6), and (B10), we find that

$$J_1 = \frac{i}{(4\pi)^2}\left[\frac{1}{\epsilon_{\rm UV}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2} + 2 - 2\delta^2 L(\delta) + i\pi\delta\right], \tag{B12a}$$

$$J_2 = \frac{i}{(4\pi)^2}\frac{1-\delta^2}{4m^2}\left\{\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{4\pi\mu^2 e^{-\gamma_E}}{m^2}\right)\left[-2L(\delta) + \frac{i\pi}{\delta}\right] + 4K(\delta) - \frac{\pi^2}{\delta} - \frac{i\pi}{\delta}\log\frac{4\delta^2}{1-\delta^2}\right\}, \tag{B12b}$$

$$J_3 = \frac{i}{(4\pi)^2}[2\delta^2 L(\delta) - i\pi\delta], \tag{B12c}$$

$$J_4 = \frac{1}{4}\left[\frac{i}{(4\pi)^2} + J_1\right], \tag{B12d}$$

$$J_5 = 0, \tag{B12e}$$

$$J_6 = -\frac{1}{4}J_3, \tag{B12f}$$

$$J_7 = 0. \tag{B12g}$$

The results for $J_1 - J_4$ in Eq. (B12) agree with those in Ref. [16].

APPENDIX C: INTEGRALS FOR THE NRQCD CORRECTIONS

Here, we tabulate some integrals that are useful in computing the NRQCD corrections.

In dimensional regularization, scaleless, power-divergent integrals vanish:

$$\int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|^n} = 0 \tag{C1}$$

for $n \neq 3$. The only scaleless logarithmically divergent integral that we encounter is

$$n_0 \equiv \int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|^3} = \frac{1}{4\pi^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right). \tag{C2}$$

There are a few integrals that depend on $\boldsymbol{q}$ that appear in the evaluations of the S_i in Eq. (49):

$$n_1 \equiv \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon} = \frac{i}{4\pi}|\boldsymbol{q}|, \tag{C3a}$$

$$n_2 \equiv \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2(\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon)} = -\frac{i}{16\pi|\boldsymbol{q}|}\left(\frac{1}{\epsilon_{\rm IR}} + \log\frac{\pi\mu^2 e^{-\gamma_E}}{\boldsymbol{q}^2} + i\pi\right), \tag{C3b}$$

$$n_3 \equiv \int_{\boldsymbol{k}} \frac{\boldsymbol{k}^2}{\boldsymbol{k}^2 + 2\boldsymbol{k}\cdot\boldsymbol{q} - i\varepsilon} = \frac{i}{2\pi}|\boldsymbol{q}|^3. \tag{C3c}$$

We also make use of the angular averages:

$$\int_{\boldsymbol{k}} \frac{f(\boldsymbol{k}^2)}{E \pm \boldsymbol{q}\cdot\hat{\boldsymbol{k}}} = \frac{1}{2|\boldsymbol{q}|}\log\left(\frac{E+|\boldsymbol{q}|}{E-|\boldsymbol{q}|}\right)\int_{\boldsymbol{k}} f(\boldsymbol{k}^2), \tag{C4a}$$

$$\int_{\boldsymbol{k}} \frac{f(\boldsymbol{k}^2)}{E^2 - (\boldsymbol{q}\cdot\hat{\boldsymbol{k}})^2} = \frac{1}{2E}\int_{\boldsymbol{k}} f(\boldsymbol{k}^2)\left(\frac{1}{E+\boldsymbol{q}\cdot\hat{\boldsymbol{k}}} + \frac{1}{E-\boldsymbol{q}\cdot\hat{\boldsymbol{k}}}\right) = \frac{1}{2E|\boldsymbol{q}|}\log\left(\frac{E+|\boldsymbol{q}|}{E-|\boldsymbol{q}|}\right)\int_{\boldsymbol{k}} f(\boldsymbol{k}^2), \tag{C4b}$$

where $f(\boldsymbol{k}^2)$ is any function of $\boldsymbol{k}^2$.

APPENDIX D: EVALUATION OF THE INTEGRALS FOR $[Z_Q]_{\rm NRQCD}$

In this Appendix, we evaluate the integrals T_{02}, T_{11}, T_{12}, T_{02}^μ, T_{11}^μ, and T_{12}^μ, which enter into the calculation of $[Z_Q]_{\rm NRQCD}$ and are defined in Eq. (89). We make use of the same strategy that we used in evaluating the S_i integrals in Sec. VI, except that we carry out the evaluation in the rest frame of the heavy quark, $p_1 = (m, \mathbf{0})$, where the expressions become compact. The change of frame shifts momenta by an amount of order mv. Therefore, the NRQCD expansion in powers of the external momentum divided by m remains valid. In the heavy-quark rest frame, the gluon- and quark-propagator denominators are

$$[D_0]_{\text{rest}} = (k^0 + |\boldsymbol{k}| - i\varepsilon)(k^0 - |\boldsymbol{k}| + i\varepsilon), \tag{D1a}$$

$$[D_1]_{\text{rest}} = (k^0 + m + \sqrt{m^2 + \boldsymbol{k}^2} - i\varepsilon) \times (k^0 + m - \sqrt{m^2 + \boldsymbol{k}^2} + i\varepsilon). \tag{D1b}$$

We evaluate the k^0 integrals by using contour integration, closing the contour in the upper half-plane in every case. We denote the contributions of gluon and quark poles by subscripts g and Q, respectively.

The integral T_{02Q} is

$$T_{02Q} = \frac{i}{4} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{(m^2 + \boldsymbol{k}^2)^{3/2}}. \tag{D2}$$

The integral T_{11} yields

$$T_{11g} = \frac{i}{4m} \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2}, \tag{D3a}$$

$$T_{11Q} = -\frac{i}{4m} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2}\left(1 - \frac{m}{\sqrt{m^2 + \boldsymbol{k}^2}}\right). \tag{D3b}$$

The integral T_{12} yields

$$T_{12g} = -\frac{i}{8m^2} \int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|^3} = -\frac{i}{8m^2} n_0, \tag{D4a}$$

$$T_{12Q} = \frac{i}{8m^2} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{(m^2 + \boldsymbol{k}^2)^{3/2}}, \tag{D4b}$$

where n_0 is defined in Eq. (C2).

In the cases of the integrals T_{02}^{μ}, T_{11}^{μ}, and T_{12}^{μ}, the integrand of the temporal component T_{ab}^0 is identical to that of T_{ab}, except that the integrand in T_{ab}^0 contains an additional factor of k^0. Integrating over k^0, we obtain

$$T_{02Q}^0 = -\frac{im}{4} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{(m^2 + \boldsymbol{k}^2)^{3/2}}, \tag{D5a}$$

$$T_{11g}^0 = -\frac{i}{4m} \int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|}, \tag{D5b}$$

$$T_{11Q}^0 = \frac{i}{4m} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{\sqrt{m^2 + \boldsymbol{k}^2}}, \tag{D5c}$$

$$T_{12g}^0 = \frac{i}{8m^2} \int_{\boldsymbol{k}} \frac{1}{|\boldsymbol{k}|^2}, \tag{D5d}$$

$$T_{12Q}^0 = -\frac{i}{8m^2} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^2}\left[1 - \frac{m^3}{(m^2 + \boldsymbol{k}^2)^{3/2}}\right]. \tag{D5e}$$

For the spatial component T_{ab}^i, the integrand is identical to the integrand in T_{ab}, except that the integrand in T_{ab}^i contains an additional factor k^i. By making use of Eqs. (D2)–(D4), we find that

$$T_{02Q}^i = \frac{i}{4} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{k^i}{(m^2 + \boldsymbol{k}^2)^{3/2}}, \tag{D6a}$$

$$T_{11g}^i = \frac{i}{4m} \int_{\boldsymbol{k}} \frac{k^i}{\boldsymbol{k}^2}, \tag{D6b}$$

$$T_{11Q}^i = -\frac{i}{4m} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{k^i}{\boldsymbol{k}^2}\left(1 - \frac{m}{\sqrt{m^2 + \boldsymbol{k}^2}}\right), \tag{D6c}$$

$$T_{12g}^i = -\frac{i}{8m^2} \int_{\boldsymbol{k}} \frac{k^i}{|\boldsymbol{k}|^3}, \tag{D6d}$$

$$T_{12Q}^i = \frac{i}{8m^2} \mathcal{N}\!\!\int_{\boldsymbol{k}} \frac{k^i}{(m^2 + \boldsymbol{k}^2)^{3/2}}. \tag{D6e}$$

Expanding the integrands in Eqs. (D2)–(D6) in powers of $\boldsymbol{k}^2/m^2$, we find that all of the terms in the expansions yield scaleless, power-divergent integrals, with the exception of the integral T_{12g} in Eq. (D4a). Therefore,

$$T_{11} = T_{02} = T_{02}^{\mu} = T_{11}^{\mu} = T_{12}^{\mu} = 0 \tag{D7}$$

and

$$T_{12} = -\frac{i}{32\pi^2 m^2}\left(\frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}}\right), \tag{D8}$$

where we have used Eq. (C2).

[1] G. T. Bodwin, H. S. Chung, D. Kang, J. Lee, and C. Yu, Phys. Rev. D **77**, 094017 (2008).
[2] R. Van Royen and V. F. Weisskopf, Nuovo Cimento A **50**, 617 (1967); **51**, 583(E) (1967).
[3] R. Barbieri, R. Gatto, R. Kogerler, and Z. Kunszt, Phys. Lett. **57B**, 455 (1975).
[4] W. Celmaster, Phys. Rev. D **19**, 1517 (1979).
[5] G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D **51**, 1125 (1995); **55**, 5853(E) (1997).
[6] G. T. Bodwin and A. Petrelli, Phys. Rev. D **66**, 094011 (2002).
[7] A. Czarnecki and K. Melnikov, Phys. Rev. Lett. **80**, 2531 (1998).
[8] M. Beneke, A. Signer, and V. A. Smirnov, Phys. Rev. Lett. **80**, 2535 (1998).
[9] M. E. Luke and M. J. Savage, Phys. Rev. D **57**, 413 (1998).
[10] M. Beneke and V. A. Smirnov, Nucl. Phys. **B522**, 321 (1998).

[11] G. T. Bodwin and Y. Q. Chen, Phys. Rev. D **60**, 054008 (1999).
[12] E. Braaten and S. Fleming, Phys. Rev. D **52**, 181 (1995).
[13] N. Brambilla, E. Mereghetti, and A. Vairo, J. High Energy Phys. 08 (2006) 039.
[14] G. T. Bodwin, D. Kang, and J. Lee, Phys. Rev. D **74**, 014014 (2006).
[15] M. Gremm and A. Kapustin, Phys. Lett. B **407**, 323 (1997).
[16] J. Lee, H. K. Noh, and C. Yu, J. Korean Phys. Soc. **50**, 403 (2007).