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Are $Y(4260)$ and $Z_2^+(4250)$ D_1D or D_0D^* hadronic molecules?

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In this work, we have investigated whether $Y(4260)$ and $Z_2^+(4250)$ could be D_1D or D_0D^* molecules in the framework of meson exchange model. The off-diagonal interaction induced by π exchange plays a dominant role. The σ exchange has been taken into account, which leads to diagonal interaction. The contribution of σ exchange is not favorable to the formation of the molecular state with $I^G(J^{PC}) = 0^-(1^{--})$; however, it is beneficial to the binding of the molecule with $I^G(J^P) = 1^-(1^-)$. Light vector meson exchange leads to diagonal interaction as well. For $Z_2^+(4250)$, the contribution from ρ and ω exchange almost cancels each other. For the currently allowed values of the effective coupling constants and a reasonable cutoff Λ in the range 1–2 GeV, we find that $Y(4260)$ could be accommodated as a D_1D and D_0D^* molecule, whereas the interpretation of $Z_2^+(4250)$ as a D_1D or D_0D^* molecule is disfavored. The bottom analog of $Y(4260)$ and $Z_2^+(4250)$ may exist, and the most promising channels to discover them are $\pi^+\pi^-\Upsilon$ and $\pi^+\chi_{b1}$, respectively.

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I. INTRODUCTION

In the past years, a number of charmoniumlike X, Y, Z states have been observed, which stimulate a lot of discussion about the structures and properties of these resonances. In particular, the $Z^+(4430)$ observed in the $\pi^+\psi'$ invariant spectrum [1] carries one unit electric charge. Consequently, it cannot be simple charmonium. Recently, two new resonancelike structures $Z_1^+(4051)$ and $Z_2^+(4250)$ in the $\pi^+\chi_{c1}$ mass distribution in exclusive $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ have been reported by the Belle collaboration [2]. Their masses and widths are determined to be $M_1 = (4051 \pm 14^{+20}_{-41})$ MeV, $\Gamma_1 = (82^{+21+47}_{-17-22})$ MeV, $M_2 = (4248^{+44+180}_{-29-35})$ MeV, and $\Gamma_2 = (177^{+54+316}_{-39-61})$ MeV, respectively, with the product branching fractions $\mathcal{B}(\bar{B}^0 \to K^-Z_{1,2}^+) \times \mathcal{B}(Z_{1,2}^+ \to \pi^+\chi_{c1}) = (3.0^{+1.5+3.7}_{-0.8-1.6}) \times 10^{-5}$ and $(4.0^{+2.3+19.7}_{-0.9-0.5}) \times 10^{-5}$, respectively. Both $Z_1^+(4051)$ and $Z_2^+(4250)$ carry one unit electric charge like $Z^+(4430)$, hence they must be states beyond quark model, if these states are confirmed in future. Since π^+ is an isovector with negative G parity, and χ_{c1} is a isospin singlet with positive G parity, the quantum numbers of $Z_1^+(4051)$ and $Z_2^+(4250)$ are $I^G = 1^-$. It is remarkable that some states are in the vicinity of the S-wave threshold of two charmed mesons, e.g., $X(3872)$ and $Z^+(4430)$ are very close to the thresholds of D^*D and D_1D^*, respectively, therefore it is tempting to interpret these states as molecular states [3,4]. Particularly, $Y(4260)$ and $Z_2^+(4250)$ are close to the D_1D and D_0D^* thresholds, which inspires the theoretical interpretations of $Y(4260)$ as a D_0D^* molecule [5] and $Z_2^+(4250)$ as a D_1D molecule [6].

$Y(4260)$ was reported by the *BABAR* collaboration in the $\pi^+\pi^-J/\psi$ invariant spectrum of the reaction $e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^-J/\psi$ [7], which has been confirmed by both the CLEO and the Belle collaboration [8,9]. A fit to the peak with a single Breit-Wigner resonance shape yields a mass $M = (4259 \pm 10)$ MeV and the full width $\Gamma = (88 \pm 24)$ MeV. Evidently the state is a vector with $c\bar{c}$ flavor, and its quantum numbers are determined to be $I^G(J^{PC}) = 0^-(1^{--})$. Although it is above the threshold for decaying into $D\bar{D}$, $D\bar{D}^*(D^*\bar{D})$, or $D^*\bar{D}^*$ meson pairs, there is no evidence for $Y(4260)$ in these channels [10–12]. Therefore $Y(4260)$ appears not to be a canonical charmonium.

The observation of the $Y(4260)$ has sparked many theoretical speculations. It has variously been identified as a conventional $\psi(4S)$ based on a relativistic quark model [13], a tetraquark $c\bar{c}s\bar{s}$ state [14] which decays predominantly into $D_s\bar{D}_s$, or a charmonium hybrid [15]. The data on $e^+e^- \to D_s\bar{D}_s$ show a peaking above threshold around 4 GeV but no evidence of affinity for a structure at 4.26 GeV [16]. If these data are confirmed, then the interpretation of $Y(4260)$ as a $c\bar{c}s\bar{s}$ tetraquark would be ruled out. Moreover, dynamical calculation of tetraquark states indicated that $Y(4260)$ cannot be interpreted as a P-wave 1^{--} state of charm-strange diquark-antidiquark, because the corresponding mass is found to be 200 MeV heavier [17]. Although the charmonium hybrid is a very attractive interpretation, the lattice QCD simulations predict that the lightest charmonium hybrid is about 4.4 GeV [18], which is very close to the new charmoniumlike state $Y(4360)$ [19]. As has been proposed in Ref. [20], a possible resolution to this issue is that $Y(4360)$ is the candidate of the charmonium hybrid, while $Y(4260)$ is a D_1D hadronic molecule.

In Ref. [21], Swanson emphasized that we should examine the D_1D molecular interpretation before finally concluding that $Y(4260)$ is a charmonium hybrid. Furthermore, he pointed out that π exchange does not lead to a diagonal interaction in the D_1D channel, and certain a novel mechanism such as off-diagonal interaction may be required. In Refs. [22,23], Close showed that parity conservation requires the π vertex to link $D \leftrightarrow D^*$ and

$D_1 \leftrightarrow D_0$, then the π exchange gives an off-diagonal potential linking $D_1\bar{D} \leftrightarrow D_0\bar{D}^*$ or $D\bar{D}_1 \leftrightarrow D^*\bar{D}_0$. This π exchange attraction possibly results in a 1^{--} hadronic molecule near the $D_1 D$ threshold. In this work, we shall investigate whether $Y(4260)$ and $Z_2^+(4250)$ could be a hadronic molecule due to the off-diagonal π exchange effect in the framework of heavy quark effective theory. The contribution of σ exchange has been considered, which results in diagonal interaction. The light vector mesons ρ and ω exchange is discussed as well.

The paper is organized as follows. In Sec. II, we present the formalism to include both heavy meson and anti-meson fields in the heavy meson chiral perturbation theory (HMχPT), and the complete Lagrangian is written out explicitly. Section III illustrates the systematic procedure for converting a general T-matrix into an equivalent potential operator. Later we follow this to derive the effective potential. In Sec. IV, we present both the diagonal and nondiagonal potential related with $Y(4260)$ and $Z_2^+(4250)$. In Sec. V, we investigate the possible bound states of the $D_1 D$ and $D_0 D^*$ system by solving the coupled-channel Schrödinger equations, and the structures of $Y(4260)$ and $Z_2^+(4250)$ are discussed. Moreover, the bottom analog of $Y(4260)$ and $Z_2^+(4250)$ is studied. We present our conclusions and some relevant discussions in Sec. VI. Finally, the potential from ρ and ω exchange is shown in the Appendix.

II. FORMALISM FOR THE SYSTEM CONTAINING BOTH MESON AND ANTI-MESON FIELDS IN HMχPT

The strong interaction between pseudo-Goldstone bosons and the mesons containing a heavy quark is described by the so-called heavy meson chiral perturbation theory (HMχPT) [24–26]. The heavy meson chiral perturbation theory is constructed starting from the spin-flavor symmetry occurring in QCD in the infinite heavy quark mass limit, and from the chiral symmetry valid in the massless limit for the light quarks. In HMχPT, the heavy-light meson field appears in a covariant form, which is represented by a 4×4 Dirac-type matrix. The negative and positive parity doublets containing a heavy quark Q and a light antiquark of flavor a, can be, respectively, described by the superfields H_a, S_a, and T_a^μ as follows:

$$\begin{aligned} H_a^{(Q)} &= \frac{1+\not{v}}{2}[P_a^{*(Q)\mu}\gamma_\mu - P_a^{(Q)}\gamma_5] \\ S_a^{(Q)} &= \frac{1+\not{v}}{2}[P_{1a}^{\prime(Q)\mu}\gamma_\mu\gamma_5 - P_{0a}^{*(Q)}] \\ T_a^{(Q)\mu} &= \frac{1+\not{v}}{2}\bigg[P_{2a}^{*(Q)\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}P_{1a\nu}^{(Q)}\gamma_5 \\ &\quad \times\bigg(g^{\mu\nu} - \frac{1}{3}\gamma^\nu(\gamma^\mu - v^\mu)\bigg)\bigg]. \end{aligned} \tag{1}$$

The above various operators annihilate mesons of four-velocity v which is conserved in strong interaction processes. The heavy field operators contain a factor $\sqrt{M_P}$ and have dimension $3/2$. Under a heavy quark spin $SU(2)$ transformation S and a generic light flavor transformation U [i.e., $U \in SU(3)$],

$$\begin{aligned} H_a^{(Q)} \to SH_b^{(Q)}U_{ba}^\dagger, \qquad S_a^{(Q)} \to SS_b^{(Q)}U_{ba}^\dagger, \\ T_a^{(Q)\mu} \to ST_b^{(Q)\mu}U_{ba}^\dagger. \end{aligned} \tag{2}$$

The conjugate field, which creates heavy-light mesons containing a heavy quark Q and a light antiquark of flavor a, is defined as

$$\begin{aligned} \bar{H}_a^{(Q)} = \gamma_0 H_a^{(Q)\dagger}\gamma_0, \qquad \bar{S}_a^{(Q)} = \gamma_0 S_a^{(Q)\dagger}\gamma_0, \\ \bar{T}_a^{(Q)\mu} \equiv \gamma_0 T_a^{(Q)\mu\dagger}\gamma_0 \end{aligned} \tag{3}$$

which transforms under S and U as

$$\begin{aligned} \bar{H}_a^{(Q)} \to U_{ab}\bar{H}_b^{(Q)}S^\dagger, \qquad \bar{S}_a^{(Q)} \to U_{ab}\bar{S}_b^{(Q)}S^\dagger, \\ \bar{T}_a^{(Q)\mu} \to U_{ab}\bar{T}_b^{(Q)\mu}S^\dagger. \end{aligned} \tag{4}$$

The octet of light pseudoscalar mesons can be introduced using the nonlinear representation $\Sigma = \xi^2$ and $\xi = \exp(i\mathcal{M}/f_\pi)$ with $f_\pi = 132$ MeV. The matrix $\mathcal{M}$ contains π, K, η fields, which is a 3×3 Hermitian and traceless matrix

$$\mathcal{M} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \tag{5}$$

Under the chiral symmetry, the field ξ transforms as

$$\xi \to g_L \xi U^\dagger = U\xi g_R^\dagger, \tag{6}$$

where g_L and g_R are left-handed and right-handed global $SU(3)$ transformation, respectively.

The effective QCD Lagrangian is constructed by imposing invariance under both heavy quark spin-flavor transformation and chiral transformation; it is [24–28]

$$\begin{aligned} \mathcal{L}_P &= ig\langle H_b^{(Q)}\mathcal{A}_{ba}\gamma_5\bar{H}_a^{(Q)}\rangle + ik\langle T_b^{(Q)\mu}\mathcal{A}_{ba}\gamma_5\bar{T}_{a\mu}^{(Q)}\rangle \\ &\quad + i\tilde{k}\langle S_b^{(Q)}\mathcal{A}_{ba}\gamma_5\bar{S}_a^{(Q)}\rangle + \bigg[ih\langle S_b^{(Q)}\mathcal{A}_{ba}\gamma_5\bar{H}_a^{(Q)}\rangle \\ &\quad + i\tilde{h}\langle T_b^{(Q)\mu}\mathcal{A}_{\mu ba}\gamma_5\bar{S}_a^{(Q)}\rangle \\ &\quad + i\frac{h_1}{\Lambda_\chi}\langle T_b^{(Q)\mu}(D_\mu\mathcal{A})_{ba}\gamma_5\bar{H}_a^{(Q)}\rangle \\ &\quad + i\frac{h_2}{\Lambda_\chi}\langle T_b^{(Q)\mu}(\not{D}\mathcal{A}_\mu)_{ba}\gamma_5\bar{H}_a^{(Q)}\rangle + \text{H.c.}\bigg], \end{aligned} \tag{7}$$

where $\langle\cdots\rangle$ means trace over the 4×4 matrices, the covariant derivative $D_\mu = \partial_\mu + \mathcal{V}_\mu$, the vector current $\mathcal{V}_\mu$, and the axial current $\mathcal{A}_\mu$ are defined by

$$\mathcal{V}_\mu = \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$$
$$\mathcal{A}_\mu = \tfrac{1}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger). \tag{8}$$

In order to describe mesons containing heavy antiquark $\bar{Q}$, we have to introduce six new fields $P^{*(\bar{Q})}_{a\mu}$, $P^{(\bar{Q})}_a$, $P'^{(\bar{Q})}_{1a\mu}$, $P^{*(\bar{Q})}_{0a}$, $P^{*(\bar{Q})}_{2a\mu\nu}$, and $P^{(\bar{Q})}_{1a\mu}$ which destroy mesons containing a heavy antiquark $\bar{Q}$. The phase of the field $P^{*(\bar{Q})}_{a\mu}$ relative to $P^{*(Q)}_{a\mu}$, $P^{(\bar{Q})}_a$ to $P^{(Q)}_a$ etc. can be fixed by the following charge conjugation convention:

$$P^{*(\bar{Q})}_{a\mu} = -\mathcal{C}P^{*(Q)}_{a\mu}\mathcal{C}^{-1}, \qquad P^{(\bar{Q})}_a = \mathcal{C}P^{(Q)}_a\mathcal{C}^{-1},$$
$$P'^{(\bar{Q})}_{1a\mu} = \mathcal{C}P'^{(Q)}_{1a\mu}\mathcal{C}^{-1}, \qquad P^{*(\bar{Q})}_{0a} = \mathcal{C}P^{*(Q)}_{0a}\mathcal{C}^{-1},$$
$$P^{*(\bar{Q})}_{2a\mu\nu} = -\mathcal{C}P^{*(Q)}_{2a\mu\nu}\mathcal{C}^{-1}, \qquad P^{(\bar{Q})}_{1a\mu} = \mathcal{C}P^{(Q)}_{1a\mu}\mathcal{C}^{-1}, \tag{9}$$
$$\mathcal{C}\xi\mathcal{C}^{-1} = \xi^T, \qquad \mathcal{C}\mathcal{V}_\mu\mathcal{C}^{-1} = -\mathcal{V}^T_\mu,$$
$$\mathcal{C}\mathcal{A}_\mu\mathcal{C}^{-1} = \mathcal{A}^T_\mu.$$

The mesons containing a heavy antiquark $\bar{Q}$ and a light quark of flavor a can be included into the theory by applying the charge conjugation operation to the above heavy-light meson superfields $H^{(Q)}_a$, $S^{(Q)}_a$, and $T^{(Q)}_{a\mu}$ [29]:

$$H^{(\bar{Q})}_a = C(\mathcal{C}H^{(Q)}_a\mathcal{C}^{-1})^TC^{-1} = [P^{*(\bar{Q})\mu}_a\gamma_\mu - P^{(\bar{Q})}_a\gamma_5]\frac{1-\not{v}}{2}$$
$$S^{(\bar{Q})}_a = C(\mathcal{C}S^{(Q)}_a\mathcal{C}^{-1})^TC^{-1} = [P'^{(\bar{Q})\mu}_{1a}\gamma_\mu\gamma_5 - P^{*(\bar{Q})}_{0a}]\frac{1-\not{v}}{2}$$
$$T^{(\bar{Q})\mu}_a = C(\mathcal{C}T^{(Q)}_{a\mu}\mathcal{C}^{-1})^TC^{-1} = \Big[P^{(\bar{Q})\mu\nu}_{2a}\gamma_\nu - \sqrt{\frac{3}{2}}P^{(\bar{Q})}_{1a\nu}\gamma_5 \times\Big(g^{\mu\nu} - \frac{1}{3}(\gamma^\mu - v^\mu)\gamma^\nu\Big)\Big]\frac{1-\not{v}}{2}. \tag{10}$$

The matrix C is the charge conjugation matrix for Dirac spinors with $C = i\gamma^2\gamma^0$, and the transpose is on the spinor matrix indices. Under the heavy quark spin transformation S and light quark $SU(3)$ flavor symmetry U,

$$H^{(\bar{Q})}_a \to U_{ab}H^{(\bar{Q})}_bS^\dagger, \qquad S^{(\bar{Q})}_a \to U_{ab}S^{(\bar{Q})}_bS^\dagger,$$
$$T^{(\bar{Q})}_{a\mu} \to U_{ab}T^{(\bar{Q})}_{b\mu}S^\dagger. \tag{11}$$

Similarly the Hermitian conjugate fields are defined by

$$\bar{H}^{(\bar{Q})}_a = \gamma_0H^{(\bar{Q})\dagger}_a\gamma_0, \qquad \bar{S}^{(\bar{Q})}_a = \gamma_0S^{(\bar{Q})\dagger}_a\gamma_0,$$
$$\bar{T}^{(\bar{Q})}_{a\mu} = \gamma_0T^{(\bar{Q})\dagger}_{a\mu}\gamma_0. \tag{12}$$

Under the symmetry transformation S and U,

$$\bar{H}^{(\bar{Q})}_a \to S\bar{H}^{(\bar{Q})}_bU^\dagger_{ba}, \qquad \bar{S}^{(\bar{Q})}_a \to S\bar{S}^{(\bar{Q})}_bU^\dagger_{ba},$$
$$\bar{T}^{(\bar{Q})}_{a\mu} \to S\bar{T}^{(\bar{Q})}_{b\mu}U^\dagger_{ba}. \tag{13}$$

For the system including both a heavy meson and a heavy anti-meson field in the HMχPT, the total effective Lagrangian should be invariant under the charge conjugation transformation. The interaction between the pseudo-Goldstone bosons and the meson containing one heavy antiquark can be obtained from Eq. (7) by applying the charge conjugation operator,

$$\begin{aligned}\mathcal{L}'_P = {}& ig\langle\bar{H}^{(\bar{Q})}_a\mathcal{A}_{ab}\gamma_5H^{(\bar{Q})}_b\rangle + ik\langle\bar{T}^{(\bar{Q})\mu}_a\mathcal{A}_{ab}\gamma_5T^{(\bar{Q})}_{b\mu}\rangle \\ &+ i\tilde{k}\langle\bar{S}^{(\bar{Q})}_a\mathcal{A}_{ab}\gamma_5S^{(\bar{Q})}_b\rangle + \Big[ih\langle\bar{H}^{(\bar{Q})}_a\mathcal{A}_{ab}\gamma_5S^{(\bar{Q})}_b\rangle \\ &+ i\tilde{h}\langle\bar{S}^{(\bar{Q})}_a\mathcal{A}_{\mu ab}\gamma_5T^{(\bar{Q})\mu}_b\rangle \\ &+ i\frac{h_1}{\Lambda_\chi}\langle\bar{H}^{(\bar{Q})}_a(\mathcal{A}\overleftarrow{D}'_\mu)_{ab}\gamma_5T^{(\bar{Q})\mu}_b\rangle \\ &+ i\frac{h_2}{\Lambda_\chi}\langle\bar{H}^{(\bar{Q})}_a(\mathcal{A}_\mu\overleftarrow{\not{D}}')_{ab}\gamma_5T^{(\bar{Q})\mu}_b\rangle + \text{H.c.}\Big],\end{aligned} \tag{14}$$

where $D'_\mu = \partial_\mu - \mathcal{V}_\mu$. After expanding the effective Lagrangian in Eq. (7) and (14) to the leading order of the pseudo-Goldstone field, we can obtain the following effective interactions, which is needed in our work:

$$\begin{aligned}\mathcal{L}_{DD^*P} &= g_{DD^*P}D_b(\partial_\mu\mathcal{M})_{ba}D^{*\mu\dagger}_a + g_{DD^*P}D^{*\mu}_b(\partial_\mu\mathcal{M})_{ba}D^\dagger_a + g_{\bar{D}\bar{D}^*P}\bar{D}^{*\mu\dagger}_a(\partial_\mu\mathcal{M})_{ab}\bar{D}_b + g_{\bar{D}\bar{D}^*P}\bar{D}^\dagger_a(\partial_\mu\mathcal{M})_{ab}\bar{D}^{*\mu}_b \\ \mathcal{L}_{D_0D_1P} &= g_{D_0D_1P}D^\mu_{1b}(\partial_\mu\mathcal{M})_{ba}D^\dagger_{0a} + g_{\bar{D}_0\bar{D}_1P}\bar{D}^\dagger_{0a}(\partial_\mu\mathcal{M})_{ab}\bar{D}^\mu_{1b} + \text{H.c.} \\ \mathcal{L}_{DD_0P} &= ig_{DD_0P}(D_{0b}\overleftrightarrow{\partial}_\mu D^\dagger_a)\partial_\mu\mathcal{M}_{ba} + ig_{\bar{D}\bar{D}_0P}(\bar{D}_{0b}\overleftrightarrow{\partial}_\mu\bar{D}^\dagger_a)\partial^\mu\mathcal{M}_{ab} + \text{H.c.} \\ \mathcal{L}_{D^*D_1P} &= g_{D^*D_1P}\Big[3D^\mu_{1b}(\partial_\mu\partial_\nu\mathcal{M})_{ba}D^{*\nu\dagger}_a - D^\mu_{1b}(\partial^\nu\partial_\nu\mathcal{M})_{ba}D^{*\dagger}_{a\mu} + \frac{1}{M_{D^*}M_{D_1}}\partial^\nu D^\mu_{1b}(\partial_\nu\partial_\tau\mathcal{M})_{ba}\partial^\tau D^{*\dagger}_{a\mu}\Big] \\ &\quad + g_{\bar{D}^*\bar{D}_1P}\Big[3\bar{D}^{*\mu\dagger}_a(\partial_\mu\partial_\nu\mathcal{M})_{ab}\bar{D}^\nu_{1b} - \bar{D}^{*\mu\dagger}_a(\partial^\nu\partial_\nu\mathcal{M})_{ab}\bar{D}_{1b\mu} + \frac{1}{M_{D^*}M_{D_1}}\partial^\nu\bar{D}^{*\mu\dagger}_a(\partial_\nu\partial_\tau\mathcal{M})_{ab}\partial^\tau\bar{D}_{1b\mu}\Big] + \text{H.c.}\end{aligned} \tag{15}$$

In the chiral and heavy quark limit, the above coupling constants are

$$\begin{aligned} g_{DD^*P} &= -g_{\bar{D}\bar{D}^*P} = -\frac{2g}{f_\pi}\sqrt{M_D M_{D^*}} \\ g_{D_0D_1P} &= g_{\bar{D}_0\bar{D}_1P} = -\frac{2\sqrt{6}}{3}\frac{\tilde{h}}{f_\pi}\sqrt{M_{D_0}M_{D_1}} \\ g_{DD_0P} &= g_{\bar{D}\bar{D}_0P} = -\frac{h}{f_\pi} \\ g_{D^*D_1P} &= g_{\bar{D}^*\bar{D}_1P} = -\frac{\sqrt{6}}{3}\frac{h_1+h_2}{\Lambda_\chi f_\pi}\sqrt{M_{D^*}M_{D_1}}. \end{aligned} \tag{16}$$

We would like to stress that the DD^*P coupling constant is the negative of the $\bar{D}\bar{D}^*P$ coupling constant, because of the phase convention for charge conjugation chosen in Eq. (9). The effective Lagrangian between σ and heavy meson (anti-meson) are [30]

$$\begin{aligned} \mathcal{L}_\sigma &= g_\sigma\langle H_a^{(Q)}\sigma\bar{H}_a^{(Q)}\rangle + g'_\sigma\langle S_a^{(Q)}\sigma\bar{S}_a^{(Q)}\rangle \\ &\quad + g''_\sigma\langle T_a^{(Q)\mu}\sigma\bar{T}_{a\mu}^{(Q)}\rangle + \left[\frac{h_\sigma}{f_\pi}\langle S_a^{(Q)}\gamma^\mu(\partial_\mu\sigma)\bar{H}_a^{(Q)}\rangle\right. \\ &\quad \left. + \frac{h'_\sigma}{f_\pi}\langle T_a^{(Q)\mu}(\partial_\mu\sigma)\bar{H}_a^{(Q)}\rangle + \text{H.c.}\right] + g_\sigma\langle\bar{H}_a^{(\bar{Q})}\sigma H_a^{(\bar{Q})}\rangle \\ &\quad + g'_\sigma\langle\bar{S}_a^{(\bar{Q})}\sigma S_a^{(\bar{Q})}\rangle + g''_\sigma\langle\bar{T}_a^{(\bar{Q})\mu}\sigma T_{a\mu}^{(\bar{Q})}\rangle \\ &\quad + \left[-\frac{h_\sigma}{f_\pi}\langle\bar{H}_a^{(\bar{Q})}\gamma^\mu(\partial_\mu\sigma)S_a^{(\bar{Q})}\rangle\right. \\ &\quad \left. + \frac{h'_\sigma}{f_\pi}\langle\bar{H}_a^{(\bar{Q})}(\partial_\mu\sigma)T_b^{(\bar{Q})\mu}\rangle + \text{H.c.}\right]. \end{aligned} \tag{17}$$

The coupling constants are estimated as follows [30]:

$$g_\sigma = -\frac{g_\pi}{2\sqrt{6}}, \qquad g'_\sigma = -\frac{g_\pi}{2\sqrt{6}}, \qquad h_\sigma = \frac{g_A}{\sqrt{3}}, \tag{18}$$

where $g_\pi = 3.73$ and $g_A = 0.6$. As in Ref. [31], we take $|g''_\sigma| = |g_\sigma|$ and $|h'_\sigma| = |h_\sigma|$ approximately when performing the numerical analysis. Expanding the Lagrangian $\mathcal{L}_\sigma$, we get the interactions associated with σ:

$$\begin{aligned} \mathcal{L}_{DD\sigma} &= g_{DD\sigma}D_aD_a^\dagger\sigma + g_{\bar{D}\bar{D}\sigma}\bar{D}_a\bar{D}_a^\dagger\sigma \\ \mathcal{L}_{D_1D_1\sigma} &= g_{D_1D_1\sigma}D_{1a}^\mu D_{1a\mu}^\dagger\sigma + g_{\bar{D}_1\bar{D}_1\sigma}\bar{D}_{1a}^\mu\bar{D}_{1a\mu}^\dagger\sigma \\ \mathcal{L}_{DD_1\sigma} &= g_{DD_1\sigma}D_{1a}^\mu D_a^\dagger\partial_\mu\sigma + g_{\bar{D}\bar{D}_1\sigma}\bar{D}_{1a}^\mu\bar{D}_a^\dagger\partial_\mu\sigma + \text{H.c.} \\ \mathcal{L}_{D^*D^*\sigma} &= g_{D^*D^*\sigma}D_a^{*\mu}D_{a\mu}^{*\dagger}\sigma + g_{\bar{D}^*\bar{D}^*\sigma}\bar{D}_a^{*\mu}\bar{D}_{a\mu}^{*\dagger}\sigma \\ \mathcal{L}_{D_0D_0\sigma} &= g_{D_0D_0\sigma}D_{0a}D_{0a}^\dagger\sigma + g_{\bar{D}_0\bar{D}_0\sigma}\bar{D}_{0a}\bar{D}_{0a}^\dagger\sigma \\ \mathcal{L}_{D^*D_0\sigma} &= g_{D^*D_0\sigma}D_{0a}D_a^{*\mu\dagger}\partial_\mu\sigma + g_{\bar{D}^*\bar{D}_0\sigma}\bar{D}_{0a}\bar{D}_a^{*\mu\dagger}\partial_\mu\sigma \\ &\quad + \text{H.c.} \end{aligned} \tag{19}$$

The relevant coupling constants are

$$\begin{aligned} g_{DD\sigma} &= g_{\bar{D}\bar{D}\sigma} = -2g_\sigma M_D \\ g_{D_1D_1\sigma} &= g_{\bar{D}_1\bar{D}_1\sigma} = -2g''_\sigma M_{D_1} \\ g_{DD_1\sigma} &= g_{\bar{D}\bar{D}_1\sigma} = -\frac{2\sqrt{6}}{3}\frac{h'_\sigma}{f_\pi}\sqrt{M_DM_{D_1}} \\ g_{D^*D^*\sigma} &= g_{\bar{D}^*\bar{D}^*\sigma} = 2g_\sigma M_{D^*} \\ g_{D_0D_0\sigma} &= g_{\bar{D}_0\bar{D}_0\sigma} = 2g'_\sigma M_{D_0} \\ g_{D^*D_0\sigma} &= -g_{\bar{D}^*\bar{D}_0\sigma} = -\frac{2h_\sigma}{f_\pi}\sqrt{M_{D^*}M_{D_0}}. \end{aligned} \tag{20}$$

III. CONVERTING THE T-MATRIX INTO THE EFFECTIVE POTENTIAL

The T-matrix for the $A(\mathbf{p_1})B(\mathbf{p_2}) \to C(\mathbf{p}'_1)D(\mathbf{p}'_2)$ scattering process can be represented by an equivalent Born-order potential operator $V_{bn}(\mathbf{r_1}-\mathbf{r_2}, \nabla_1, \nabla_2)$ between pointlike particles; the definition of this potential operator is [32,33]

$$\begin{aligned} &\delta^3(\mathbf{p}'_1+\mathbf{p}'_2-\mathbf{p_1}-\mathbf{p_2})T_{fi}(\mathbf{p_1},\mathbf{p_2},\mathbf{p}'_1,\mathbf{p}'_2) \\ &= \frac{1}{(2\pi)^3}\iint d^3\mathbf{r_1}d^3\mathbf{r_2}e^{-i(\mathbf{p}'_1\cdot\mathbf{r_1}+\mathbf{p}'_2\cdot\mathbf{r_2})} \\ &\quad \times V_{bn}(\mathbf{r_1}-\mathbf{r_2},\nabla_1,\nabla_2)e^{i(\mathbf{p_1}\cdot\mathbf{r_1}+\mathbf{p_2}\cdot\mathbf{r_2})}, \end{aligned} \tag{21}$$

where $T_{fi}(\mathbf{p_1},\mathbf{p_2},\mathbf{p}'_1,\mathbf{p}'_2)$ is the T-matrix for the process $A(\mathbf{p_1})B(\mathbf{p_2}) \to C(\mathbf{p}'_1)D(\mathbf{p}'_2)$. In general, T_{fi} depends on all the involved momentum $\mathbf{p_1}$, $\mathbf{p_2}$, $\mathbf{p}'_1$, and $\mathbf{p}'_2$. For convenience, we introduce

$$\begin{aligned} \mathbf{P_1} &\equiv \tfrac{1}{2}(\mathbf{p_1}+\mathbf{p}'_1), \qquad \mathbf{P_2} \equiv \tfrac{1}{2}(\mathbf{p_2}+\mathbf{p}'_2), \\ \mathbf{q} &\equiv \mathbf{p}'_1 - \mathbf{p_1} \equiv \mathbf{p_2} - \mathbf{p}'_2. \end{aligned} \tag{22}$$

In the center of mass frame $\mathbf{P_1} = -\mathbf{P_2}$. The amplitude T_{fi} can be expanded as a power series in P_{1i} and P_{2i}:

$$\begin{aligned} T_{fi}(\mathbf{p_1},\mathbf{p_2},\mathbf{p}'_1,\mathbf{p}'_2) &= T^{(0)}(\mathbf{q}) + T_i^{(1,0)}(\mathbf{q})P_{1i} + T_i^{(0,1)}(\mathbf{q})P_{2i} \\ &\quad + T_{ij}^{(1,1)}(\mathbf{q})P_{1i}P_{2j} + \cdots. \end{aligned} \tag{23}$$

This procedure produces the full Breit-Fermi Hamiltonian when it is applied to the photon exchanged electron-electron scattering amplitude expanded to $\mathcal{O}(P^2)$. The leading term $T^{(0)}(\mathbf{q})$ is a function of $\mathbf{q}$ only; its Fourier transformation gives us a local potential $V(\mathbf{r})$ that is a function of $\mathbf{r_1}-\mathbf{r_2}\equiv\mathbf{r}$ only. The relation between $T^{(0)}(\mathbf{q})$ and $V(\mathbf{r})$ is

$$V(\mathbf{r}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}T^{(0)}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}}, \tag{24}$$

For the higher terms of the T-matrix expansion, P_{1i} and P_{2i} are replaced by left and right gradients in the equivalent potential operator defined implicitly by Eq. (21) [32]. Following this systematic procedure, we can convert a general T-matrix into an equivalent potential operator. In this work, we obtain the local potential by Fourier transforming the leading terms $T^{(0)}(\mathbf{q})$ of the scattering amplitude T_{fi}, which is common in the potential model [34,35].

Since the propagators are off shell, we introduce a form factor at each vertex when writing out the scattering amplitude, the usual form factor is expressed as [36,37]

$$F(q) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \tag{25}$$

where Λ is an adjustable constant within a reasonable range of 1–2 GeV, which models the off-shell effects at the vertices due to the internal structure of the meson. m and q are the mass and the four momentum of the exchanged meson, respectively.

IV. THE EFFECTIVE POTENTIALS RELATED WITH $Y(4260)$ AND $Z_2^+(4250)$

Recently, the meson exchange model based on the HMχPT has been used to study possible heavy flavor molecule [31,38]. In this section, we will follow the general procedure shown above to derive the effective potential associated with $Y(4260)$ and $Z_2^+(4250)$ in the framework of HMχPT. From the effective interaction in Eqs. (15) and (19), we can write down the corresponding scattering amplitude for each diagram, including the form factor at each vertex. Then we get the equivalent potential in momentum space following the general formalism presented in Sec. III. Finally, we make a Fourier transformation to derive the potentials in coordinate space. Because of parity conservation, pseudoscalar π and η exchange only contributes to the off-diagonal interaction, whereas σ exchange and light vector mesons ρ, ω exchange result in diagonal interaction only. The corresponding scattering diagrams are shown in Fig. 1.

Under the ansatz of $Y(4260)$ as a D_1D or D_0D^* hadronic molecule, we can write down its flavor wave function:

$$\begin{aligned} |Y(4260)\rangle &= \tfrac{1}{2}[|D_1^0\bar{D}^0\rangle + |D_1^+D^-\rangle - |D^0\bar{D}_1^0\rangle - |D^+D_1^-\rangle] \\ |Y'(4260)\rangle &= \tfrac{1}{2}[|D_0^0\bar{D}^{*0}\rangle + |D_0^+D^{*-}\rangle + |D^{*0}\bar{D}_0^0\rangle \\ &\quad + |D^{*+}D_0^-\rangle]. \end{aligned} \tag{26}$$

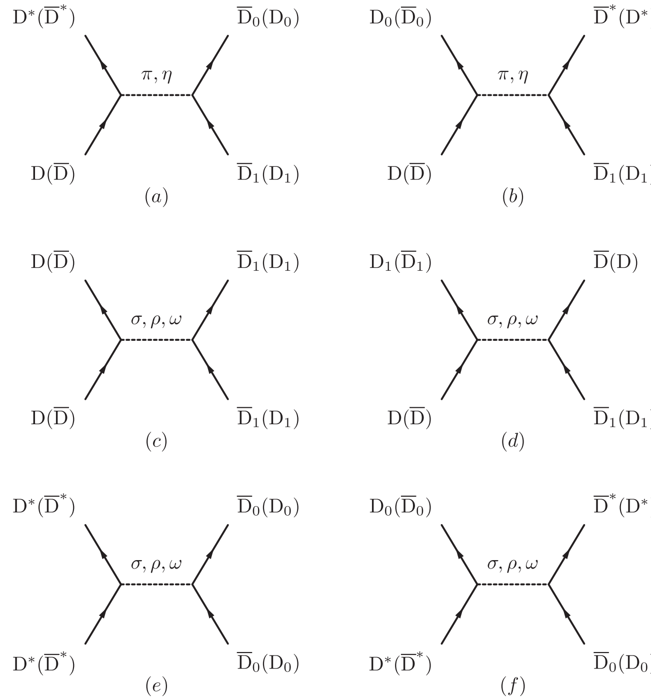

FIG. 1. The scattering diagrams with pseudoscalars π, η exchange, σ exchange, and light vector mesons ρ, ω exchange.

We stress that the phase convention under charge conjugation is consistent with Eq. (9). In the same way, the flavor wave function of $Z_2^+(4250)$ is

$$\begin{aligned} |Z_2^+(4250)\rangle &= \frac{1}{\sqrt{2}}[|D_1^+\bar{D}^0\rangle + |D^+\bar{D}_1^0\rangle] \\ |Z_2'^+(4250)\rangle &= \frac{1}{\sqrt{2}}[|D_0^+\bar{D}^{*0}\rangle - |D^{*+}\bar{D}_0^0\rangle]. \end{aligned} \tag{27}$$

In this case, its quantum number is $I^G(J^P) = 1^-(1^-)$. Following the procedure discussed above, we can calculate the effective potential in momentum space; it is a lengthy and tedious calculation.

For $Y(4260)$, the exchange potential in momentum space is

$$\begin{aligned} V_{12}(\mathbf{q}) = V_{21}(\mathbf{q}) &= \frac{\sqrt{6}}{6}\frac{g\tilde{h}}{f_\pi^2}\left(\frac{\Lambda^2 - m_\pi^2}{\mathbf{q}^2 + X_1^2}\right)^2\frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_1^2} + \frac{\sqrt{6}}{54}\frac{g\tilde{h}}{f_\pi^2}\left(\frac{\Lambda^2 - m_\eta^2}{\mathbf{q}^2 + X_1^2}\right)^2\frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_2^2} \\ V_{11}(\mathbf{q}) &= \left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + \Lambda^2}\right)^2\frac{g_\sigma g''_\sigma}{\mathbf{q}^2 + m_\sigma^2} + \frac{2h_\sigma'^2}{9f_\pi^2}\left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + X_2^2}\right)^2\frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_3^2} \\ V_{22}(\mathbf{q}) &= \left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + \Lambda^2}\right)^2\frac{g_\sigma g'_\sigma}{\mathbf{q}^2 + m_\sigma^2} + \frac{h_\sigma^2}{3f_\pi^2}\left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + X_3^2}\right)^2\frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_4^2}, \end{aligned} \tag{28}$$

where we have included the monopole form factor in Eq. (25) to regularize the potential. The diagonal potential $V_{11}(\mathbf{q})$ and $V_{22}(\mathbf{q})$ is induced by σ exchange, and the nondiagonal potential $V_{12}(\mathbf{q})$ [or $V_{21}(\mathbf{q})$] arises from the pseudo-Goldstone bosons π and η exchange. The effective potential from ρ, ω exchange is shown in the Appendix. The potential for $Z_2^+(4250)$ in momentum space is

$$V_{12}(\mathbf{q}) = V_{21}(\mathbf{q}) = -\frac{\sqrt{6}}{18}\frac{g\tilde{h}}{f_\pi^2}\left(\frac{\Lambda^2 - m_\pi^2}{\mathbf{q}^2 + X_1^2}\right)^2 \frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_1^2} + \frac{\sqrt{6}}{54}\frac{g\tilde{h}}{f_\pi^2}\left(\frac{\Lambda^2 - m_\eta^2}{\mathbf{q}^2 + X_1^2}\right)^2 \frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_2^2}$$
$$V_{11}(\mathbf{q}) = \left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + \Lambda^2}\right)^2 \frac{g_\sigma g''_\sigma}{\mathbf{q}^2 + m_\sigma^2} - \frac{2h'^2_\sigma}{9f_\pi^2}\left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + X_2^2}\right)^2 \frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_3^2} \tag{29}$$
$$V_{22}(\mathbf{q}) = \left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + \Lambda^2}\right)^2 \frac{g_\sigma g'_\sigma}{\mathbf{q}^2 + m_\sigma^2} - \frac{h_\sigma^2}{3f_\pi^2}\left(\frac{\Lambda^2 - m_\sigma^2}{\mathbf{q}^2 + X_3^2}\right)^2 \frac{\mathbf{q}^2}{\mathbf{q}^2 + \mu_4^2}.$$

The various parameters appearing in the above formulas are defined as follows:

$$X_1^2 = \Lambda^2 - (M_{D^*} - M_D)(M_{D_1} - M_{D_0}) \qquad X_2^2 = \Lambda^2 - (M_{D_1} - M_D)^2 \qquad X_3^2 = \Lambda^2 - (M_{D_0} - M_{D^*})^2$$
$$\mu_1^2 = m_\pi^2 - (M_{D^*} - M_D)(M_{D_1} - M_{D_0}) \qquad \mu_2^2 = m_\eta^2 - (M_{D^*} - M_D)(M_{D_1} - M_{D_0}) \tag{30}$$
$$\mu_3^2 = m_\sigma^2 - (M_{D_1} - M_D)^2 \qquad \mu_4^2 = m_\sigma^2 - (M_{D_0} - M_{D^*})^2.$$

After performing Fourier transformation, we obtain the potential forms in configuration space. For $Y(4260)$, the potential in coordinate space is

$$V_{12}(r) = V_{21}(r) = \frac{\sqrt{6}}{6}\frac{g\tilde{h}}{f_\pi^2} Z(\Lambda, X_1, \mu_1, m_\pi, r) + \frac{\sqrt{6}}{54}\frac{g\tilde{h}}{f_\pi^2} Z(\Lambda, X_1, \mu_2, m_\eta, r)$$
$$V_{11}(r) = g_\sigma g''_\sigma H(\Lambda, m_\sigma, r) + \frac{2h'^2_\sigma}{9f_\pi^2} Z(\Lambda, X_2, \mu_3, m_\sigma, r) \qquad V_{22}(r) = g_\sigma g'_\sigma H(\Lambda, m_\sigma, r) + \frac{h_\sigma^2}{3f_\pi^2} Z(\Lambda, X_3, \mu_4, m_\sigma, r). \tag{31}$$

The potential in coordinate space for $Z_2^+(4250)$ is

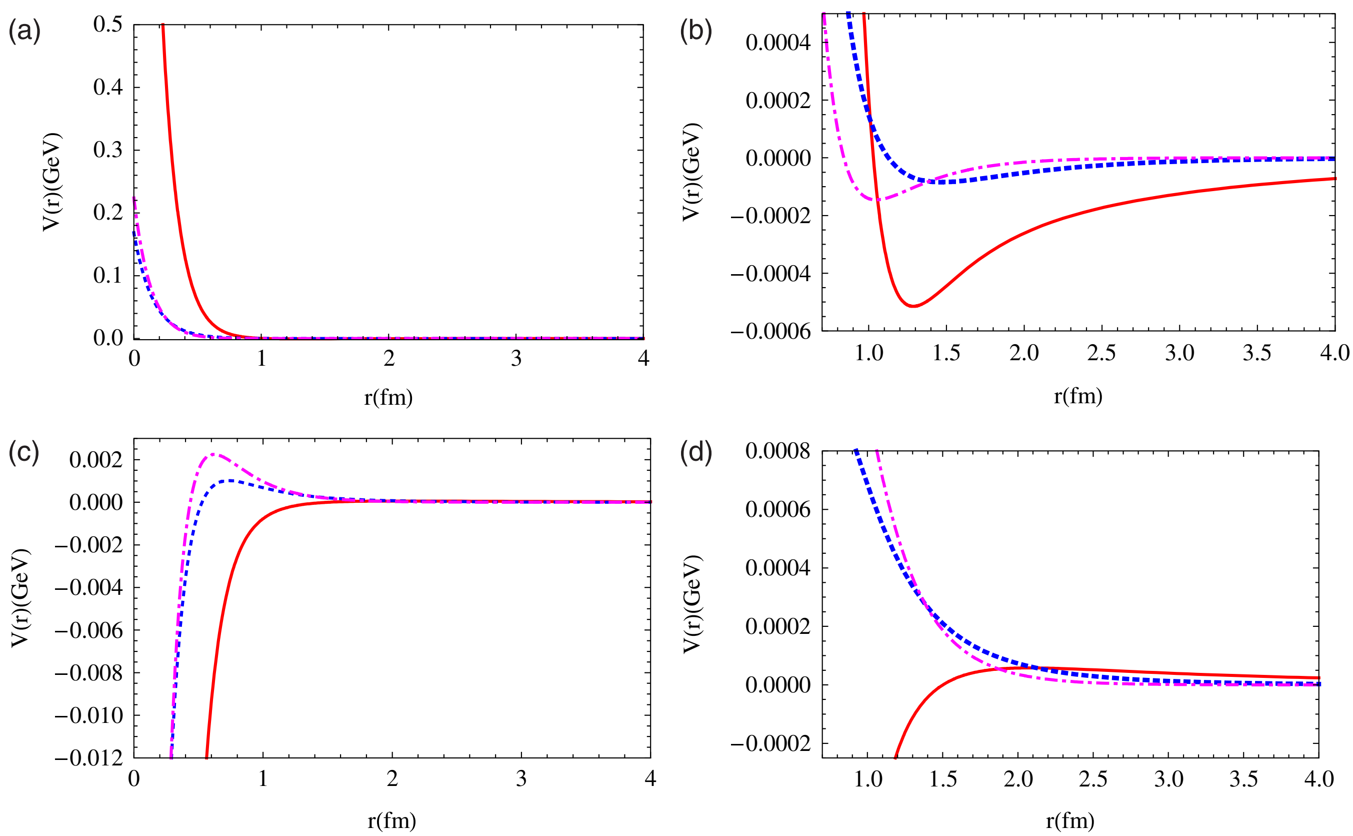

FIG. 2 (color online). The effective potential for the D_1D and D_0D^* system from pseudoscalar π, η exchange, and scalar σ exchange. The solid line represents the nondiagonal potential $V_{12}(r)$ [or $V_{21}(r)$], short dashed and dash dotted lines respectively correspond to the diagonal potential $V_{11}(r)$ and $V_{22}(r)$. (a) and (b) are related with $Y(4260)$, and (b) shows the long range behavior of the potential. (c) and (d) are related with $Z_2^+(4250)$, and (d) is the long rang shape of the potential.

$$V_{12}(r)=V_{21}(r)=-\frac{\sqrt{6}}{18}\frac{g\tilde{h}}{f_\pi^2}Z(\Lambda,X_1,\mu_1,m_\pi,r)+\frac{\sqrt{6}}{54}\frac{g\tilde{h}}{f_\pi^2}Z(\Lambda,X_1,\mu_2,m_\eta,r)$$
$$V_{11}(r)=g_\sigma g''_\sigma H(\Lambda,m_\sigma,r)-\frac{2h'^2_\sigma}{9f_\pi^2}Z(\Lambda,X_2,\mu_3,m_\sigma,r)\qquad V_{22}(r)=g_\sigma g'_\sigma H(\Lambda,m_\sigma,r)-\frac{h_\sigma^2}{3f_\pi^2}Z(\Lambda,X_3,\mu_4,m_\sigma,r). \tag{32}$$

Here the functions $H(\Lambda, m, r)$ and $Z(\Lambda, X, \mu, m, r)$ are defined as

$$H(\Lambda,m,r)=\frac{1}{4\pi}\frac{1}{r}(e^{-mr}-e^{-\Lambda r})-\frac{\Lambda^2-m^2}{8\pi\Lambda}e^{-\Lambda r}$$
$$Z(\Lambda,X,\mu,m,r)=\frac{1}{4\pi}\frac{1}{r}(X^2e^{-Xr}-\mu^2e^{-\mu r})+\frac{\Lambda^2-m^2}{8\pi}\left(X-\frac{2}{r}\right)e^{-Xr}. \tag{33}$$

We take the typical values of the coupling constants $g\tilde{h}=0.85$, $g_\sigma g'_\sigma=0.58$, $g_\sigma g''_\sigma=0.58$, and $|h_\sigma|=|h'_\sigma|=0.35$, and $\Lambda=1.5$ GeV is chosen for an illustration, the variation of the effective potential with respect to r is shown in Fig. 2. It is obvious that the magnitude of the diagonal potentials from σ exchange is smaller than that of the off-diagonal potential from π and η exchange, this is mainly because m_π is small than m_σ. Moreover, the magnitude of the off-diagonal potential related with $Y(4260)$ is larger than associated with $Z_2^+(4250)$, the latter is about one-third of the former. This is consistent with results from the chiral quark model [36]; consequently, the $I^G(J^{PC})=0^-(1^{--})$ configuration is easier to bind than the $I^G(J^P)=1^-(1^-)$ configuration.

V. THE STRUCTURES OF $Y(4260)$ AND $Z_2^+(4250)$ AND THE BOTTOM ANALOG

A. The bound states of the D_1D and D_0D^* system with the structure of $Y(4260)$ and $Z_2^+(4250)$

With the above effective potential, we shall explore whether there are bound states with $I^G(J^{PC})=0^-(1^{--})$ or $I^G(J^P)=1^-(1^-)$ in the D_1D and D_0D^* system, by means of solving the two channels coupled Schrödinger equation. There are various methods to integrate the coupled-channel Schrödinger equation numerically. In this work we shall employ two packages, MATSCS [39] and FESSDE2.2 [40], to perform the numerical calculation so that the results obtained by one program can be checked by another. The first package is a MATLAB software, and the second is written in FORTRAN77. Both packages can fastly and accurately solve the eigenvalue problem for systems of coupled Schrödinger equations, and the results obtained by two codes are the same within error.

The masses of the involved mesons are taken from PDG [41]: $M_D=1869.3$ MeV, $M_{D^*}=2006.7$ MeV, $M_{D_1}=2422$ MeV, $M_{D_0}=2308$ MeV, $m_\pi=135$ MeV, $m_\eta=547.5$ MeV, $m_\sigma=600$ MeV, $m_\rho=775.5$ MeV, and $m_\omega=782.65$ MeV. The effective coupling constants in HMχPT have been studied from various phenomenological and theoretical approaches, and the estimates for g, $\tilde{h}$ are listed in Table I. It is obvious that there are still large uncertainties in their values. In the following, we shall first consider whether one pseudoscalar π and η exchange can result in a bound state in the D_1D and D_0D^* system, then the contribution of σ exchange is included.

The numerical results with only one pseudoscalar exchange are presented in Table II. For several typical values of $g\tilde{h}$, we vary the cutoff Λ from a small value until we find a solution which lies below the D_1D threshold. Here the mass M is measured with respect to the D_1D threshold $M_{D_1}+M_D\simeq 4291.3$ MeV, $r_{\rm rms}$ is the root of mean square radius, and R denotes the ratio between the D_1D and D_0D^* components in the bound state solutions. By comparing the results with different Λ for the same value of the parameter

TABLE I. Summary of theoretical estimates for the effective coupling g and $\tilde{h}$.

Reference	g	Remark
[42]	$0.59 \pm 0.07 \pm 0.01$	Combining CLEO's results on D^* decay width
[43]	0.46 ± 0.04	Through a constituent quark-meson model
[44]	0.53	Including one loop corrections without positive parity states
[44]	0.65	Including one loop corrections with positive parity states
[27]	0.44 ± 0.16	From QCD sum rule
[45]	0.39 ± 0.16	From QCD sum rule
[46]	0.32 ± 0.02	
[28]	0.75	From nonrelativistic quark model
Reference	$\tilde{h}$	**Remark**
[28]	$\lvert\tilde{h}\rvert = 0.87$	From nonrelativistic quark
[47]	$0.91^{+0.5}_{-0.3}$	In a constituent quark-meson model in soft pion limit

TABLE II. The mass, the root of mean square radius (rms), and the ratio (R) between the DD_1 and D^*D_0 components for the bound state solutions of the DD_1 and D^*D_0 system with one pseudoscalar exchange, and the mass is measured with respect to the D_1D threshold $M_D + M_{D_1} \simeq 4291.3$ MeV.

	Y_{cc} with $I^G(J^{PC}) = 0^-(1^{--})$				Z^+_{cc} with $I^G(J^P) = 1^-(1^-)$			
$g\tilde{h}$	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
	3.3	−4.04	1.39	2.63	14.5	−2.40	1.76	3.27
0.23	3.4	−12.20	0.82	1.71	14.6	−7.53	0.98	2.02
	3.5	−23.86	0.63	1.41	14.7	−14.71	0.71	1.60
	2.3	−3.29	1.56	2.90	9.6	−3.53	1.45	2.76
0.35	2.4	−11.32	0.86	1.74	9.7	−9.24	0.89	1.87
	2.5	−24.95	0.63	1.40	9.8	−16.99	0.68	1.54
	1.6	−1.79	2.17	3.88	6.3	−5.53	1.17	2.29
0.54	1.7	−10.33	0.95	1.84	6.4	−12.13	0.80	1.71
	1.8	−24.66	0.67	1.41	6.5	−20.82	0.63	1.45
	1.2	−7.43	1.15	2.11	4.0	−3.45	1.50	2.81
0.85	1.3	−22.69	0.76	1.46	4.1	−9.28	0.93	1.88
	1.4	−46.18	0.57	1.24	4.2	−17.42	0.71	1.53

$g\tilde{h}$, one notes that the magnitude of M increases with Λ, whereas the reverse is true for $r_{\rm rms}$ and R. The bound state mass is sensitive to the parameter $g\tilde{h}$ as well, larger $g\tilde{h}$ is helpful to form a molecular state. From the numerical results in Table II, we see that one can get a molecular state consistent with $Y(4260)$, given an appropriate value for $g\tilde{h}$ and a reasonable cutoff Λ in the range 1–2 GeV. However, the existence of a bound state with $I^G(J^P) = 1^-(1^-)$ requires that the value of Λ should be at least larger than 4 GeV. The cutoff parameter Λ is a typical hadronic scale, which is generally expected to be in the range 1–2 GeV. If Λ is required to be much larger than 2 GeV in order to form a bound state, we tend to conclude that such a bound state should not exist. Therefore, it is not appropriate to assign $Z^+_2(4250)$ as a D_1D or D_0D^* molecule, if only the nondiagonal interaction from π and η exchange is considered.

Then we include the contribution coming from σ exchange, which leads to only the diagonal interaction. The corresponding numerical results are shown in Table III. The radial wave functions $\chi(r) = rR(r)$ for certain parameter values are shown in Fig. 3. The wave function corresponding to other solutions in Tables III and IV has similar shape with that in Fig. 3. We find that the σ exchange interaction has significant effects; the variations of M, $r_{\rm rms}$, and R with respect to Λ have the same pattern as those in the only pseudoscalar exchange case. Varying the parameters $g\tilde{h}$, $g_\sigma g'_\sigma$, $g_\sigma g''_\sigma$, and h_σ in the reasonable

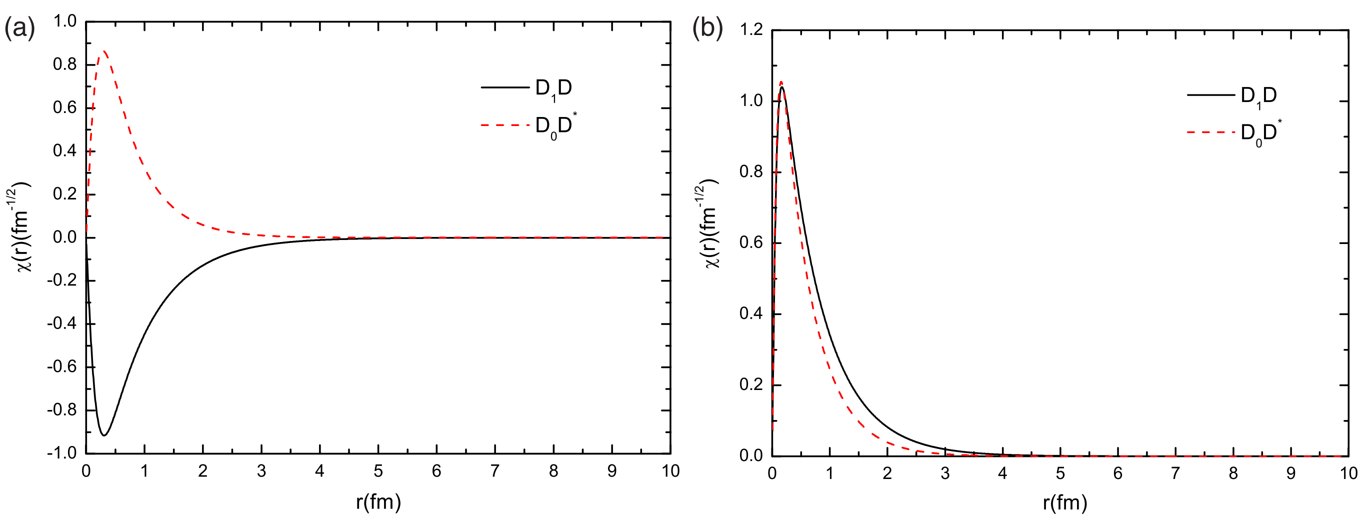

FIG. 3 (color online). The radial wave function $\chi(r) = rR(r)$ for the molecular states with $I^G(J^{PC}) = 0^-(1^{--})$ and $I^G(J^P) = 1^-(1^-)$, respectively. (a) corresponds to the former state, and (b) is for the latter. We have taken $g\tilde{h} = 0.85$, $g_\sigma g'_\sigma = 0.58$, $g_\sigma g''_\sigma = 0.58$, and $|h_\sigma| = h'_\sigma = 0.35$; Λ is chosen to be 1.4 and 3.4 GeV, respectively.

TABLE III. The mass, the root of mean square radius (rms), and the ratio (R) between the DD_1 and D^*D_0 components for the bound state solutions of the DD_1 and D^*D_0 system with both one pseudoscalar exchange and σ exchange, and the mass is measured with respect to the D_1D threshold $M_D + M_{D_1} \simeq 4291.3$ MeV.

				Y_{cc} with $I^G(J^{PC}) = 0^-(1^{--})$				Z_{cc}^+ with $I^G(J^P) = 1^-(1^-)$			
$g\tilde{h}$	$g_\sigma g'_\sigma$	$g_\sigma g''_\sigma$	h_σ	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
0.23	0.58	0.58	0.35	4.7	−3.43	1.48	2.81	6.1	−5.43	1.09	1.47
				4.8	−10.87	0.84	1.81	6.2	−15.82	0.66	1.00
				4.9	−21.42	0.62	1.50	6.3	−29.69	0.51	0.84
			0.50	8.7	−5.49	1.16	2.43	4.0	−5.49	1.06	1.19
				8.8	−12.52	0.78	1.82	4.1	−18.45	0.62	0.74
				8.9	−21.75	0.61	1.58	4.2	−36.23	0.48	0.59
	0.58	−0.58	0.35	4.5	−3.65	1.50	3.49	5.9	−4.22	1.33	2.52
				4.6	−9.70	0.93	2.31	6.0	−11.84	0.80	1.59
				4.7	−18.27	0.70	1.86	6.1	−22.48	0.60	1.25
			0.50	8.4	−8.62	0.98	2.62	3.9	−5.62	1.14	1.94
				8.5	−15.03	0.75	2.14	4.0	−16.19	0.70	1.17
				8.6	−23.09	0.62	1.88	4.1	−31.14	0.53	0.90
	−0.58	0.58	0.35	4.6	−7.02	1.02	1.80	5.9	−8.06	0.87	0.95
				4.7	−15.40	0.72	1.41	6.0	−18.32	0.61	0.73
				4.8	−26.34	0.58	1.25	6.1	−31.34	0.50	0.64
			0.50	8.4	−3.81	1.37	2.32	3.9	−11.21	0.74	0.65
				8.5	−9.37	0.88	1.67	4.0	−25.48	0.54	0.50
				8.6	−16.74	0.68	1.42	4.1	−43.76	0.45	0.43
	−0.58	−0.58	0.35	4.4	−6.47	1.11	2.32	5.7	−5.35	1.15	1.69
				4.5	−13.31	0.80	1.78	5.8	−12.98	0.75	1.17
				4.6	−22.33	0.64	1.53	5.9	−23.10	0.59	0.96
			0.50	8.0	−3.13	1.60	3.37	3.8	−8.99	0.88	1.12
				8.1	−7.05	1.06	2.36	3.9	−20.74	0.62	0.78
				8.2	−12.35	0.82	1.92	4.0	−36.29	0.50	0.64
0.35	0.58	0.58	0.35	2.8	−2.15	1.91	3.38	5.2	−8.56	0.90	1.48
				2.9	−9.48	0.93	1.85	5.3	−19.71	0.62	1.13
				3.0	−20.76	0.66	1.46	5.4	−34.24	0.50	0.98
			0.50	3.7	−6.18	1.12	2.19	3.6	−3.77	1.34	1.87
				3.8	−14.57	0.76	1.64	3.7	−14.71	0.72	1.07
				3.9	−25.91	0.59	1.42	3.8	−30.46	0.53	0.85
	0.58	−0.58	0.35	2.8	−9.66	0.95	2.15	5.0	−6.18	1.12	2.38
				2.9	−19.82	0.70	1.70	5.1	−14.42	0.76	1.68
				3.0	−33.45	0.56	1.48	5.2	−25.58	0.58	1.38
			0.50	3.5	−3.96	1.45	3.20	3.5	−4.65	1.29	2.55
				3.6	−10.06	0.93	2.18	3.6	−13.97	0.77	1.54
				3.7	−18.66	0.70	1.77	3.7	−27.47	0.57	1.18
	−0.58	0.58	0.35	2.8	−8.07	0.99	1.74	5.0	−7.24	0.95	1.25
				2.9	−18.21	0.70	1.36	5.1	−16.94	0.66	0.94
				3.0	−31.84	0.56	1.21	5.2	−29.51	0.53	0.82
			0.50	3.6	−7.83	0.99	1.77	3.5	−5.98	1.03	1.19
				3.7	−16.23	0.72	1.42	3.6	−17.51	0.65	0.80
				3.8	−27.23	0.59	1.26	3.7	−33.14	0.51	0.66
	−0.58	−0.58	0.35	2.7	−8.08	1.03	2.05	4.9	−11.84	0.81	1.44
				2.8	−17.20	0.74	1.59	5.0	−21.48	0.63	1.17
				2.9	−29.48	0.59	1.38	5.1	−33.64	0.52	1.03
			0.50	3.4	−5.08	1.27	2.51	3.4	−5.68	1.14	1.80
				3.5	−11.34	0.87	1.84	3.5	−15.32	0.72	1.16
				3.6	−19.83	0.69	1.55	3.6	−28.71	0.56	0.93

TABLE III. (*Continued*)

$g\tilde{h}$	$g_\sigma g'_\sigma$	$g_\sigma g''_\sigma$	h_σ	Y_{cc} with $I^G(J^{PC}) = 0^-(1^{--})$ Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Z^+_{cc} with $I^G(J^P) = 1^-(1^-)$ Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
0.54	0.58	0.58	0.35	1.8	−2.42	1.84	3.21	4.2	−9.99	0.86	1.59
				1.9	−11.03	0.91	1.75	4.3	−21.32	0.62	1.25
				2.0	−24.76	0.66	1.39	4.4	−36.09	0.50	1.10
			0.50	2.1	−8.24	1.02	1.92	3.2	−12.62	0.78	1.38
				2.2	−19.25	0.71	1.47	3.3	−26.99	0.57	1.08
				2.3	−34.54	0.57	1.30	3.4	−45.95	0.47	0.95
	0.58	−0.58	0.35	1.8	−6.29	1.19	2.42	4.1	−13.88	0.78	1.83
				1.9	−16.88	0.78	1.69	4.2	−24.83	0.61	1.50
				2.0	−32.31	0.60	1.43	4.3	−38.79	0.50	1.32
			0.50	2.0	−5.65	1.25	2.51	3.1	−11.78	0.85	1.87
				2.1	−14.59	0.82	1.77	3.2	−24.09	0.62	1.43
				2.2	−27.44	0.64	1.49	3.3	−40.53	0.51	1.21
	−0.58	0.58	0.35	1.8	−5.35	1.25	2.11	4.1	−13.94	0.74	1.18
				1.9	−15.93	0.78	1.44	4.2	−25.68	0.58	1.00
				2.0	−31.41	0.60	1.23	4.3	−40.44	0.48	0.91
			0.50	2.0	−4.61	1.33	2.20	3.1	−12.19	0.79	1.16
				2.1	−13.49	0.83	1.50	3.2	−25.70	0.59	0.92
				2.2	−26.33	0.64	1.27	3.3	−43.37	0.48	0.81
	−0.58	−0.58	0.35	1.8	−9.78	0.98	1.88	3.9	−8.33	0.98	1.84
				1.9	−22.04	0.70	1.46	4.0	−16.85	0.71	1.42
				2.0	−39.09	0.56	1.28	4.1	−28.01	0.58	1.22
			0.50	2.0	−9.66	0.98	1.87	3.0	−10.72	0.87	1.60
				2.1	−20.31	0.72	1.48	3.1	−22.23	0.64	1.22
				2.2	−34.80	0.58	1.31	3.2	−37.53	0.52	1.04
0.85	0.58	0.58	0.35	1.3	−11.86	0.93	1.72	3.2	−10.84	0.86	1.67
				1.4	−28.73	0.67	1.34	3.3	−22.29	0.64	1.34
				1.5	−53.33	0.53	1.20	3.4	−37.36	0.52	1.19
			0.50	1.3	−5.27	1.31	2.29	2.6	−9.21	0.94	1.76
				1.4	−16.97	0.81	1.52	2.7	−21.74	0.65	1.32
				1.5	−34.96	0.62	1.28	2.8	−38.91	0.52	1.13
	0.58	−0.58	0.35	1.3	−14.66	0.87	1.71	3.1	−11.85	0.86	1.99
				1.4	−32.74	0.64	1.38	3.2	−22.30	0.66	1.61
				1.5	−58.63	0.52	1.24	3.3	−35.98	0.54	1.41
			0.50	1.3	−7.58	1.14	2.13	2.5	−7.56	1.07	2.38
				1.4	−20.52	0.76	1.53	2.6	−17.94	0.73	1.70
				1.5	−39.78	0.60	1.31	2.7	−32.49	0.57	1.40
	−0.58	0.58	0.35	1.3	−14.13	0.87	1.56	3.1	−10.85	0.85	1.44
				1.4	−32.23	0.64	1.26	3.2	−21.69	0.64	1.17
				1.5	−58.16	0.52	1.13	3.3	−35.80	0.53	1.05
			0.50	1.3	−7.04	1.15	1.95	2.5	−6.58	1.08	1.74
				1.4	−19.98	0.76	1.38	2.6	−17.47	0.71	1.23
				1.5	−39.26	0.59	1.20	2.7	−32.64	0.56	1.03
	−0.58	−0.58	0.35	1.2	−4.75	1.40	2.51	3.0	−11.33	0.87	1.75
				1.3	−16.97	0.82	1.56	3.1	−21.18	0.67	1.42
				1.4	−36.26	0.62	1.29	3.2	−33.99	0.56	1.25
			0.50	1.3	−9.47	1.03	1.87	2.5	−13.90	0.81	1.62
				1.4	−23.58	0.72	1.41	2.6	−26.68	0.62	1.30
				1.5	−44.09	0.58	1.23	2.7	−43.54	0.51	1.14

range results in a large change of the predictions, which indicates that the results are sensitive to the effective coupling constants. We can see that large $g\tilde{h}$, negative $g_\sigma g'_\sigma$ and $g_\sigma g''_\sigma$ are favorable to binding the molecular states. Comparing the results in Tables II, III, and IV, we find that σ exchange is against the formation of the bound state with $I^G(J^{PC}) = 0^-(1^{--})$; nevertheless, it is beneficial to the formation of the $I^G(J^P) = 1^-(1^-)$ molecular state.

TABLE IV. The mass, the root of mean square radius (rms), and the ratio (R) between the BB_1 and B^*B_0 components for the bound state solutions of the BB_1 and B^*B_0 system with one pseudoscalar exchange, and the mass is measured with respect to the B_1B threshold $M_B + M_{B_1} \simeq 11004$ MeV.

	Y_{bb} with $I^G(J^{PC}) = 0^-(1^{--})$				Z_{bb}^+ with $I^G(J^P) = 1^-(1^-)$			
$g\tilde{h}$	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
	1.8	−5.60	0.81	3.08	6.2	−7.90	0.63	2.53
0.23	1.9	−14.96	0.53	2.03	6.3	−13.45	0.49	2.04
	2.0	−28.27	0.42	1.63	6.4	−20.22	0.40	1.76
	1.4	−8.76	0.70	2.57	4.2	−5.17	0.88	3.08
0.35	1.5	−21.93	0.49	1.79	4.3	−10.31	0.57	2.29
	1.6	−40.65	0.39	1.48	4.4	−16.95	0.45	1.89
	1.1	−11.29	0.66	2.35	2.9	−6.55	0.73	2.81
0.54	1.2	−29.05	0.47	1.65	3.0	−12.79	0.54	2.13
	1.3	−54.93	0.38	1.38	3.1	−20.87	0.44	1.77
	0.8	−3.49	1.13	4.14	2.0	−7.38	0.72	2.72
0.85	0.9	−18.98	0.58	1.94	2.1	−14.97	0.53	2.03
	1.0	−46.04	0.43	1.46	2.2	−25.03	0.44	1.69

As for $Y(4260)$, the conclusion reached with only pseudoscalar exchange remains. $Y(4260)$ could be accommodated as a molecule state for appropriate effective coupling constants and cutoff. A $I^G(J^P) = 1^-(1^-)$ bound state around 4250 MeV requires Λ should be at least 3 GeV, therefore we conclude that the interpretation of $Z_2^+(4250)$ as a D_1D or D_0D^* molecule is disfavored. This conclusion is consistent with the general observations from the chiral quark model. It is found that the isoscalar channel is easier to bind than the isovector channel for the same components [48].

B. The bottom analog of $Y(4260)$ and $Z_2^+(4250)$

The bottom analog Y_{bb} and Z_{bb}^+, respectively, denote the states obtained by replacing both the charm quark and antiquark with bottom quark and antiquark in $Y(4260)$ and $Z_2^+(4250)$. The above calculation can be easily extended to study these states. The shape of both the diagonal and the nondiagonal potential is similar to that of the charm system, except that the former is larger than the latter in magnitude. Furthermore, since the kinetic energy is greatly reduced because of the heavier mass of the B meson, a molecular state is more easily formed. We choose the same set of parameters as in the previous section. The numerical results with only pseudoscalar π, η exchange are shown in Table IV, and the results with both pseudoscalar and σ exchange are listed in Tables V and VI. As is expected, the magnitude M of the bottom analog is larger than that of the corresponding charmed state for the same parameters. The variation of M, $r_{\rm rms}$, and R with Λ is the same as the charm system, large $g\tilde{h}$, negative $g_\sigma g'_\sigma$ and $g_\sigma g''_\sigma$ are beneficial to molecule formation as well. From the results in Tables V and VI, we note that both the bottom analog Y_{bb} and Z_{bb}^+ may exist.

Since $Y(4260)$ has a large branch ratio into $\pi^+\pi^-J/\psi$, the bottom analog Y_{bb} should be searched for in the $\pi^+\pi^-\Upsilon$ channel. Specifically, the state Y_{bb} can be searched for at B factories and future Super B factory via initial state radiation (ISR) $e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^-\Upsilon$ or by $e^+e^- \to \pi^+\pi^-\Upsilon$ direct scan [49]. Furthermore, Y_{bb} may be searched for at Tevatron via $p\bar{p} \to Y_{bb} \to \pi^+\pi^-\Upsilon$, and LHC is more promising. Similarly, for the bottom analog Z_{bb}^+, the most hopeful discovery channel would be $Z_{bb}^+ \to \pi^+\chi_{b1}$, where χ_{b1} is in turn detected by its decay into $\gamma\Upsilon$ [41]. Because of the large mass of this state, it is difficult to produce such a state via decay of a certain particle [i.e., $Z_2^+(4250)$ is produced in B decay [2]], consequently large hadron collides such as Tevatron and LHC are a good place to search for this state.

VI. CONCLUSION AND DISCUSSIONS

In this work, we have performed a dynamical study of $Y(4260)$ and $Z_2^+(4250)$ simultaneously to see whether they could be a D_1D or a D_0D^* hadronic molecule. We have employed the HMχPT, which combines the heavy quark symmetry and the chiral symmetry. Since both the heavy meson and heavy anti-meson are involved, the interaction related with the heavy anti-meson has been included explicitly, and the total effective Lagrangian is invariant under the charge conjugation transformation.

The off-diagonal interaction from pseudoscalar π, η exchange plays a dominant role, which is a straightforward support to the off-diagonal interaction mechanism proposed by Swanson and Close. σ exchange leads to only diagonal interaction; its contribution has been taken into account in this work. We find that σ exchange is not favorable to the formation of the molecular state with $I^G(J^{PC}) = 0^-(1^{--})$, whereas it is helpful to the binding of the molecule with $I^G(J^P) = 1^-(1^-)$. For an appropriate value of the effective coupling constants and a reasonable cutoff Λ, $Y(4260)$ could be accommodated as a D_1D and D_0D^* molecule. However, the existence of a molecule

TABLE V. The mass, the root of mean square radius (rms), and the ratio (R) between the BB_1 and B^*B_0 components for the bound state solutions of the BB_1 and B^*B_0 system with both one pseudoscalar exchange and σ exchange, and the mass is measured with respect to the B_1B threshold $M_B + M_{B_1} \simeq 11004$ MeV.

				Y_{bb} with $I^G(J^{PC}) = 0^-(1^{--})$				Z_{bb}^+ with $I^G(J^P) = 1^-(1^-)$			
$g\tilde{h}$	$g_\sigma g'_\sigma$	$g_\sigma g''_\sigma$	h_σ	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
0.23	0.58	0.58	0.35	2.4	−8.16	0.65	2.47	3.1	−4.84	0.76	2.21
				2.5	−17.22	0.47	1.89	3.2	−14.28	0.59	1.41
				2.6	−29.14	0.39	1.63	3.3	−27.06	0.36	1.11
			0.50	3.8	−4.40	0.87	3.44	2.3	−11.53	0.52	1.42
				3.9	−9.54	0.59	2.50	2.4	−27.04	0.38	0.94
				4.0	−16.34	0.46	2.07	2.5	−47.66	0.31	0.74
	0.58	−0.58	0.35	2.2	−5.63	0.84	3.87	2.9	−5.32	0.83	4.34
				2.3	−12.07	0.59	2.73	3.0	−11.82	0.57	2.78
				2.4	−20.93	0.47	2.19	3.1	−20.94	0.44	2.05
			0.50	3.5	−7.67	0.73	3.76	2.2	−11.16	0.60	2.84
				3.6	−12.17	0.58	3.02	2.3	−23.19	0.44	1.78
				3.7	−17.83	0.49	2.57	2.4	−39.83	0.35	1.29
	−0.58	0.58	0.35	2.3	−8.15	0.65	2.17	3.0	−10.11	0.52	1.16
				2.4	−16.87	0.48	1.69	3.1	−21.07	0.39	0.88
				2.5	−28.20	0.40	1.46	3.2	−34.78	0.33	0.75
			0.50	3.6	−5.80	0.75	2.61	2.2	−10.04	0.54	1.08
				3.7	−10.92	0.56	2.05	2.3	−24.96	0.38	0.71
				3.8	−17.38	0.45	1.76	2.4	−44.59	0.32	0.56
	−0.58	−0.58	0.35	2.1	−5.02	0.88	3.59	2.8	−7.04	0.71	2.64
				2.2	−11.16	0.61	2.48	2.9	−14.49	0.51	1.78
				2.3	−19.60	0.48	1.98	3.0	−24.44	0.41	1.38
			0.50	3.3	−8.23	0.70	3.10	2.1	−8.20	0.67	2.43
				3.4	−12.63	0.57	2.56	2.2	−19.40	0.46	1.40
				3.5	−18.04	0.49	2.21	2.3	−35.11	0.37	0.99
0.35	0.58	0.58	0.35	1.6	−8.48	0.68	2.46	2.7	−8.38	0.61	2.01
				1.7	−19.80	0.49	1.81	2.8	−19.05	0.43	1.46
				1.8	−35.44	0.39	1.53	2.9	−33.04	0.35	1.21
			0.50	1.9	−8.79	0.66	2.43	2.1	−11.60	0.54	1.72
				2.0	−17.95	0.49	1.89	2.2	−26.56	0.40	1.19
				2.1	−30.10	0.41	1.63	2.3	−46.60	0.33	0.96
	0.58	−0.58	0.35	1.5	−5.95	0.84	3.48	2.5	−6.02	0.79	3.99
				1.6	−15.06	0.57	2.32	2.6	−13.16	0.55	2.65
				1.7	−28.17	0.45	1.85	2.7	−23.07	0.44	2.02
			0.50	1.8	−9.31	0.69	2.89	2.0	−10.45	0.62	2.98
				1.9	−17.68	0.52	2.23	2.1	−22.30	0.45	1.95
				2.0	−28.77	0.43	1.89	2.2	−38.74	0.37	1.47
	−0.58	0.58	0.35	1.6	−12.71	0.58	1.94	2.6	−10.35	0.55	1.44
				1.7	−25.80	0.44	1.54	2.7	−21.35	0.41	1.10
				1.8	−43.15	0.37	1.35	2.8	−35.32	0.35	0.94
			0.50	1.8	−6.49	0.76	2.51	2.0	−8.08	0.62	1.59
				1.9	−14.61	0.54	1.85	2.1	−21.54	0.42	1.02
				2.0	−25.57	0.43	1.56	2.2	−39.85	0.35	0.80
	−0.58	−0.58	0.35	1.5	−8.80	0.71	2.68	2.4	−6.03	0.78	3.09
				1.6	−19.49	0.51	1.94	2.5	−13.31	0.54	2.05
				1.7	−34.19	0.42	1.61	2.6	−23.26	0.43	1.58
			0.50	1.7	−6.61	0.80	3.08	1.9	−6.54	0.76	3.03
				1.8	−13.96	0.58	2.23	2.0	−16.80	0.50	1.77
				1.9	−23.92	0.47	1.84	2.1	−31.57	0.40	1.27

TABLE VI. The mass, the root of mean square radius (rms), and the ratio (R) between the BB_1 and B^*B_0 components for the bound state solutions of the BB_1 and B^*B_0 system with both one pseudoscalar exchange and σ exchange, and the mass is measured with respect to the B_1B threshold $M_B + M_{B_1} \simeq 11\,004$ MeV.

				Y_{bb} with $I^G(J^{PC}) = 0^-(1^{--})$				Z_{bb}^+ with $I^G(J^P) = 1^-(1^-)$			
$g\tilde{h}$	$g_\sigma g'_\sigma$	$g_\sigma g''_\sigma$	h_σ	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R	Λ (GeV)	M (MeV)	$r_{\rm rms}$ (fm)	R
0.54	0.58	0.58	0.35	1.1	−4.16	0.99	3.49	2.2	−6.67	0.70	2.45
				1.2	−15.49	0.58	2.00	2.3	−16.59	0.48	1.70
				1.3	−33.34	0.44	1.56	2.4	−30.12	0.38	1.39
			0.50	1.2	−6.93	0.79	2.73	1.8	−6.41	0.73	2.56
				1.3	−18.44	0.53	1.87	1.9	−18.58	0.47	1.60
				1.4	−35.15	0.43	1.55	2.0	−36.01	0.37	1.24
	0.58	−0.58	0.35	1.1	−6.49	0.84	3.19	2.1	−8.66	0.68	3.21
				1.2	−19.23	0.55	2.03	2.2	−17.54	0.50	2.29
				1.3	−38.42	0.43	1.62	2.3	−29.56	0.41	1.83
			0.50	1.2	−10.20	0.69	2.60	1.7	−5.05	0.88	4.32
				1.3	−23.12	0.51	1.90	1.8	−14.36	0.56	2.48
				1.4	−41.24	0.41	1.60	1.9	−28.38	0.43	1.79
	−0.58	0.58	0.35	1.1	−5.38	0.88	2.98	2.1	−5.49	0.76	2.23
				1.2	−17.94	0.55	1.81	2.2	−14.76	0.50	1.49
				1.3	−37.11	0.43	1.44	2.3	−27.44	0.40	1.20
			0.50	1.2	−8.84	0.71	2.35	1.8	−12.06	0.56	1.62
				1.3	−21.63	0.51	1.69	1.9	−27.02	0.41	1.16
				1.4	−39.68	0.41	1.42	2.0	−47.05	0.34	0.96
	−0.58	−0.58	0.35	1.1	−7.79	0.78	2.81	2.0	6.63	0.76	3.06
				1.2	−21.69	0.52	1.85	2.1	−14.74	0.54	2.07
				1.3	−42.15	0.42	1.50	2.2	−25.89	0.43	1.62
			0.50	1.2	−12.19	0.65	2.30	1.7	−8.44	0.69	2.75
				1.3	−26.32	0.49	1.73	1.8	−20.06	0.49	1.76
				1.4	−45.74	0.40	1.49	1.9	−36.41	0.39	1.35
0.85	0.58	0.58	0.35	0.9	−15.10	0.63	2.09	1.8	−14.35	0.54	1.93
				1.0	−37.85	0.46	1.53	1.9	−27.75	0.42	1.53
				1.1	−71.44	0.37	1.31	2.0	−45.31	0.35	1.32
			0.50	0.9	−12.39	0.67	2.24	1.5	−5.80	0.80	2.90
				1.0	−31.80	0.48	1.60	1.6	−17.64	0.51	1.79
				1.1	−60.44	0.39	1.36	1.7	−35.09	0.40	1.39
	0.58	−0.58	0.35	0.9	−16.45	0.62	2.10	1.7	−11.83	0.62	2.67
				1.0	−40.19	0.45	1.56	1.8	−23.08	0.47	2.00
				1.1	−74.91	0.37	1.35	1.9	−38.13	0.39	1.66
			0.50	0.9	−13.70	0.65	2.25	1.5	−11.33	0.64	2.73
				1.0	−34.09	0.48	1.64	1.6	−24.92	0.47	1.91
				1.1	−63.85	0.39	1.40	1.7	−43.85	0.38	1.53
	−0.58	0.58	0.35	0.9	−15.93	0.62	2.01	1.7	−9.30	0.64	2.04
				1.0	−39.56	0.45	1.47	1.8	−20.77	0.47	1.49
				1.1	−74.25	0.37	1.27	1.9	−36.20	0.39	1.25
			0.50	0.9	−13.17	0.66	2.15	1.5	−9.18	0.65	2.11
				1.0	−33.43	0.48	1.54	1.6	−23.12	0.46	1.44
				1.1	−63.13	0.39	1.31	1.7	−42.63	0.38	1.16
	−0.58	−0.58	0.35	0.9	−17.27	0.61	2.02	1.6	−7.05	0.77	3.02
				1.0	−41.88	0.45	1.51	1.7	−16.42	0.54	2.04
				1.1	−77.70	0.37	1.30	1.8	−29.47	0.43	1.61
			0.50	0.9	−14.47	0.64	2.16	1.5	−14.85	0.57	2.14
				1.0	−35.71	0.47	1.57	1.6	−30.27	0.44	1.57
				1.1	−66.52	0.38	1.35	1.7	−51.10	0.36	1.30

around 4250 MeV with $I^G(J^P) = 1^-(1^-)$ requires that Λ should be at least 3 GeV, given the currently allowed values of the coupling constants. Consequently, the interpretation of $Z_2^+(4250)$ as a D_1D or D_0D^* molecule is disfavored. Its structure should be studied further. Through calculating the masses of excited heavy tetraquarks with hidden charm in the diquark-antidiquark picture, the authors in Ref. [50] suggested that $Z_2^+(4250)$ could be the charged partner of the 1^- $1P$ state $S\bar{S}$ or as the 0^- $1P$ state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark. QCD sum rule analysis for $Z_2^+(4250)$ is performed in Ref. [51].

The effective potential from vector meson ρ, ω exchange has been presented analytically. Because of the accidental coincidence of m_ρ and m_ω, the contribution from ρ and ω exchange almost cancels in the potential related with $Z_2^+(4250)$. For $Y(4260)$, the situation is not the same. A number of effective coupling constants are involved. Because some of them have not been determined so far, we cannot give a quantitative estimate about the contribution from vector meson exchange. Qualitatively, it should be smaller than the contribution coming from pseudoscalar and σ exchange in magnitude. It is necessary and interesting to examine the effect of vector meson exchange on $Y(4260)$ in the future.

The bottom analog of $Y(4260)$ and $Z_2^+(4250)$ denoted by Y_{bb} and Z_{bb}^+, respectively, may exist. Y_{bb} can be searched for in $e^+e^- \to \gamma_{\rm ISR}\pi^+\pi^-\Upsilon$ or by $e^+e^- \to \pi^+\pi^-\Upsilon$ direct scan. The direct production of Y_{bb} at Tevatron or LHC via $p\bar{p} \to Y_{bb} \to \pi^+\pi^-\Upsilon$ is a hopeful approach as well. For Z_{bb}^+, the most promising discovery channel is $Z_{bb}^+ \to \pi^+\chi_{b1}$.

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APPENDIX: THE POTENTIAL FROM LIGHT VECTOR MESONS ρ AND ω EXCHANGE

The light vector mesons nonet can be introduced by using the hidden gauge symmetry approach, and the Lagrangian containing these particles is as follows: [27,52,53]

$$\begin{aligned}\mathcal{L}_V &= i\beta\langle H_b^{(Q)} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{H}_a^{(Q)}\rangle + i\lambda\langle H_b^{(Q)}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\rangle + i\beta_1\langle S_b^{(Q)} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{S}_a^{(Q)}\rangle \\ &\quad + i\lambda_1\langle S_b^{(Q)}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\bar{S}_a^{(Q)}\rangle + i\beta_2\langle T_b^{(Q)\lambda} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{T}_{a\lambda}^{(Q)}\rangle + i\lambda_2\langle T_b^{(Q)\lambda}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ba}\bar{T}_{a\lambda}^{(Q)}\rangle \\ &\quad + [i\zeta\langle H_b^{(Q)}\gamma^\mu(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{S}_a^{(Q)}\rangle + i\mu\langle H_b^{(Q)}\sigma^{\lambda\nu}F_{\lambda\nu}(\rho)_{ba}\bar{S}_a^{(Q)}\rangle + i\zeta_1\langle T_b^{(Q)\mu}(\mathcal{V}_\mu - \rho_\mu)_{ba}\bar{H}_a^{(Q)}\rangle \\ &\quad + \mu_1\langle T_b^{(Q)\mu}\gamma^\nu F_{\mu\nu}(\rho)_{ba}\bar{H}_a^{(Q)}\rangle + \text{H.c.}], \end{aligned} \tag{A1}$$

where $F_{\mu\nu}(\rho) = \partial_\mu\rho_\nu - \partial_\nu\rho_\mu + [\rho_\mu, \rho_\nu]$, and ρ_μ is defined as

$$\rho_\mu = i\frac{g_V}{\sqrt{2}}V_\mu. \tag{A2}$$

V_μ is a Hermitian 3×3 matrix analogous to Eq. (5) containing ρ, K^*, ω, and ϕ:

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \tag{A3}$$

By imposing the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin relations, one obtains $g_V \simeq 5.8$. For the same reason, the interaction between the light vector resonances and heavy anti-mesons should be included via applying the charge conjugation transformation:

$$\begin{aligned}\mathcal{L}'_V &= -i\beta\langle \bar{H}_a^{(\bar{Q})} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ab}H_b^{(\bar{Q})}\rangle + i\lambda\langle \bar{H}_a^{(\bar{Q})}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ab}H_b^{(\bar{Q})}\rangle - i\beta_1\langle \bar{S}_a^{(\bar{Q})} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ab}S_b^{(\bar{Q})}\rangle \\ &\quad + i\lambda_1\langle \bar{S}_a^{(\bar{Q})}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ab}S_b^{(\bar{Q})}\rangle - i\beta_2\langle \bar{T}_{a\lambda}^{(\bar{Q})} v^\mu(\mathcal{V}_\mu - \rho_\mu)_{ab}T_b^{(\bar{Q})\lambda}\rangle + i\lambda_2\langle \bar{T}_{a\lambda}^{(\bar{Q})}\sigma^{\mu\nu}F_{\mu\nu}(\rho)_{ab}T_b^{(\bar{Q})\lambda}\rangle \\ &\quad + [i\zeta\langle \bar{S}_a^{(\bar{Q})}\gamma^\mu(\mathcal{V}_\mu - \rho_\mu)_{ab}H_a^{(\bar{Q})}\rangle + i\mu\langle \bar{S}_a^{(\bar{Q})}\sigma^{\lambda\nu}F_{\lambda\nu}(\rho)_{ab}H_b^{(\bar{Q})}\rangle - i\zeta_1\langle \bar{H}_a^{(\bar{Q})}(\mathcal{V}_\mu - \rho_\mu)_{ab}T_b^{(\bar{Q})\mu}\rangle \\ &\quad + \mu_1\langle \bar{H}_a^{(\bar{Q})}\gamma^\nu F_{\mu\nu}(\rho)_{ab}T_b^{(\bar{Q})\mu}\rangle + \text{H.c.}], \end{aligned} \tag{A4}$$

where we have used the property $\mathcal{C}V_\mu\mathcal{C}^{-1} = -V_\mu^T$. Then the effective interactions relevant to the concerned tree level scattering diagrams are as follows:

$$
\begin{aligned}
\mathcal{L}_{DDV} &= ig_{DDV}(D_b \overleftrightarrow{\partial}_\mu D_a^\dagger)V_{ba}^\mu + ig_{\bar{D}\bar{D}V}(\bar{D}_b \overleftrightarrow{\partial}_\mu \bar{D}_a^\dagger)V_{ab}^\mu \\
\mathcal{L}_{D_1D_1V} &= ig_{D_1D_1V}(D_{1b}^\nu \overleftrightarrow{\partial}_\mu D_{1a\nu}^\dagger)V_{ba}^\mu + ig'_{D_1D_1V}(D_{1b}^\mu D_{1a}^{\nu\dagger} - D_{1a}^{\mu\dagger}D_{1b}^\nu)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba} + ig_{\bar{D}_1\bar{D}_1V}(\bar{D}_{1b\nu} \overleftrightarrow{\partial}_\mu \bar{D}_{1a}^{\nu\dagger})V_{ab}^\mu \\
&\quad + ig'_{\bar{D}_1\bar{D}_1V}(\bar{D}_{1b}^\mu \bar{D}_{1a}^{\nu\dagger} - \bar{D}_{1a}^{\mu\dagger}\bar{D}_{1b}^\nu)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} \\
\mathcal{L}_{DD_1V} &= g_{DD_1V}D_{1b}^\mu V_{\mu ba}D_a^\dagger + g'_{DD_1V}(D_{1b}^\mu \overleftrightarrow{\partial}^\nu D_a^\dagger)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba} + g_{\bar{D}\bar{D}_1V}\bar{D}_a^\dagger V_{\mu ab}\bar{D}_{1b}^\mu \\
&\quad + g'_{\bar{D}\bar{D}_1V}(\bar{D}_{1b}^\mu \overleftrightarrow{\partial}^\nu \bar{D}_a^\dagger)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} + \text{H.c.} \\
\mathcal{L}_{D^*D^*V} &= ig_{D^*D^*V}(D_{b\nu}^* \overleftrightarrow{\partial}_\mu D_a^{*\nu\dagger})V_{ba}^\mu + ig'_{D^*D^*V}(D_b^{*\mu}D_a^{*\nu\dagger} - D_a^{*\mu\dagger}D_b^{*\nu})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba} + ig_{\bar{D}^*\bar{D}^*V}(\bar{D}_{b\nu}^* \overleftrightarrow{\partial}_\mu \bar{D}_a^{*\nu\dagger})V_{ab}^\mu \\
&\quad + ig'_{\bar{D}^*\bar{D}^*V}(\bar{D}_b^{*\mu}\bar{D}_a^{*\nu\dagger} - \bar{D}_a^{*\mu\dagger}\bar{D}_b^{*\nu})(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} \\
\mathcal{L}_{D_0D_0V} &= ig_{D_0D_0V}(D_{0b} \overleftrightarrow{\partial}_\mu D_{0a}^\dagger)V_{ba}^\mu + ig_{\bar{D}_0\bar{D}_0V}(\bar{D}_{0b} \overleftrightarrow{\partial}_\mu \bar{D}_{0a}^\dagger)V_{ab}^\mu \\
\mathcal{L}_{D^*D_0V} &= g_{D^*D_0V}D_b^{*\mu}V_{\mu ba}D_{0a}^\dagger + g'_{D^*D_0V}(D_b^{*\nu}\overleftrightarrow{\partial}^\mu D_{0a}^\dagger - D_b^{*\mu}\overleftrightarrow{\partial}^\nu D_{0a}^\dagger)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ba} + g_{\bar{D}^*\bar{D}_0V}\bar{D}_{0a}^\dagger V_{\mu ab}\bar{D}_b^{*\mu} \\
&\quad + g'_{\bar{D}^*\bar{D}_0V}(\bar{D}_b^{*\nu}\overleftrightarrow{\partial}^\mu \bar{D}_{0a}^\dagger - \bar{D}_b^{*\mu}\overleftrightarrow{\partial}^\nu \bar{D}_{0a}^\dagger)(\partial_\mu V_\nu - \partial_\nu V_\mu)_{ab} + \text{H.c.}
\end{aligned} \tag{A5}
$$

The coupling constants are as follows [54]:

$$
\begin{gathered}
g_{DDV} = -g_{\bar{D}\bar{D}V} = \frac{1}{\sqrt{2}}\beta g_V \qquad g_{D_1D_1V} = -g_{\bar{D}_1\bar{D}_1V} = \frac{1}{\sqrt{2}}\beta_2 g_V \qquad g'_{D_1D_1V} = -g'_{\bar{D}_1\bar{D}_1V} = \frac{5\lambda_2 g_V}{3\sqrt{2}}M_{D_1} \\
g_{DD_1V} = -g_{\bar{D}\bar{D}_1V} = -\frac{2}{\sqrt{3}}\zeta_1 g_V\sqrt{M_D M_{D_1}} \qquad g'_{DD_1V} = -g'_{\bar{D}\bar{D}_1V} = \frac{1}{\sqrt{3}}\mu_1 g_V \\
g_{D^*D^*V} = -g_{\bar{D}^*\bar{D}^*V} = -\frac{1}{\sqrt{2}}\beta g_V \qquad g'_{D^*D^*V} = -g'_{\bar{D}^*\bar{D}^*V} = -\sqrt{2}\lambda g_V M_{D^*} \qquad g_{D_0D_0V} = -g_{\bar{D}_0\bar{D}_0V} = -\frac{1}{\sqrt{2}}\beta_1 g_V \\
g_{D^*D_0V} = g_{\bar{D}^*\bar{D}_0V} = -\zeta g_V\sqrt{2M_{D^*}M_{D_0}} \qquad g'_{D^*D_0V} = g'_{\bar{D}^*\bar{D}_0V} = -\frac{1}{\sqrt{2}}\mu g_V.
\end{gathered} \tag{A6}
$$

From the above effective interactions, following the general procedure presented in Sec. III, we can calculate the effective potential from ρ and ω exchange. For $Y(4260)$, the potential in coordinate space is

$$
\begin{aligned}
V_{12}^{\rho,\omega}(r) &= V_{21}^{\rho,\omega}(r) = 0 \\
V_{11}^{\rho,\omega}(r) &= \frac{1}{4}\beta\beta_2 g_V^2[3H(\Lambda, m_\rho, r) + H(\Lambda, m_\omega, r)] + \frac{\beta\beta_2 g_V^2(M_D^2 + M_{D_1}^2)}{32M_D^2M_{D_1}^2}[3G(\Lambda, m_\rho, r) + G(\Lambda, m_\omega, r)] \\
&\quad - \frac{1}{6}g_V^2\bigg[\zeta_1 + \frac{\mu_1(M_D^2 - M_{D_1}^2)}{2\sqrt{M_D M_{D_1}}}\bigg]^2[3Y(\Lambda, X_2, \mu_5, m_\rho, r) + Y(\Lambda, X_2, \mu_6, m_\omega, r)] + \frac{g_V^2\mu_1^2(M_D + M_{D_1})^2}{72M_D M_{D_1}} \\
&\quad \times [3Z(\Lambda, X_2, \mu_5, m_\rho, r) + Z(\Lambda, X_2, \mu_6, m_\omega, r)] - \frac{g_V^2\zeta_1^2}{18}\bigg[\frac{3}{m_\rho^2}Z(\Lambda, X_2, \mu_5, m_\rho, r) + \frac{1}{m_\omega^2}Z(\Lambda, X_2, \mu_6, m_\omega, r)\bigg] \\
V_{22}^{\rho,\omega}(r) &= \frac{1}{4}\beta\beta_1 g_V^2[3H(\Lambda, m_\rho, r) + H(\Lambda, m_\omega, r)] + \frac{\beta\beta_1 g_V^2(M_{D^*}^2 + M_{D_0}^2)}{32M_{D^*}^2M_{D_0}^2}[3G(\Lambda, m_\rho, r) + G(\Lambda, m_\omega, r)] \\
&\quad - \frac{1}{4}g_V^2\bigg[\zeta - \frac{\mu(M_{D^*}^2 - M_{D_0}^2)}{\sqrt{M_{D^*}M_{D_0}}}\bigg]^2[3Y(\Lambda, X_3, \mu_7, m_\rho, r) + Y(\Lambda, X_3, \mu_8, m_\omega, r)] + \frac{g_V^2\mu^2(M_{D^*} + M_{D_0})^2}{12M_{D^*}M_{D_0}} \\
&\quad \times [3Z(\Lambda, X_3, \mu_7, m_\rho, r) + Z(\Lambda, X_3, \mu_8, m_\omega, r)] - \frac{g_V^2\zeta^2}{12}\bigg[\frac{3}{m_\rho^2}Z(\Lambda, X_3, \mu_7, m_\rho, r) + \frac{1}{m_\omega^2}Z(\Lambda, X_3, \mu_8, m_\omega, r)\bigg].
\end{aligned} \tag{A7}
$$

The potential in coordinate space for $Z_2^+(4250)$ is

$$V_{12}^{\rho,\omega}(r) = V_{21}^{\rho,\omega}(r) = 0$$

$$\begin{aligned} V_{11}^{\rho,\omega}(r) = & -\frac{1}{4}\beta\beta_2 g_V^2[H(\Lambda, m_\rho, r) - H(\Lambda, \omega, r)] - \frac{\beta\beta_2 g_V^2(M_D^2 + M_{D_1}^2)}{32M_D^2 M_{D_1}^2}[G(\Lambda, m_\rho, r) - G(\Lambda, m_\omega, r)] \\ & - \frac{1}{6}g_V^2\left[\zeta_1 + \frac{\mu_1(M_D^2 - M_{D_1}^2)}{2\sqrt{M_D M_{D_1}}}\right]^2[Y(\Lambda, X_2, \mu_5, m_\rho, r) - Y(\Lambda, X_2, \mu_6, m_\omega, r)] + \frac{g_V^2\mu_1^2(M_D + M_{D_1})^2}{72M_D M_{D_1}} \\ & \times[Z(\Lambda, X_2, \mu_5, m_\rho, r) - Z(\Lambda, X_2, \mu_6, m_\omega, r)] - \frac{g_V^2\zeta_1^2}{18}\left[\frac{1}{m_\rho^2}Z(\Lambda, X_2, \mu_5, m_\rho, r) - \frac{1}{m_\omega^2}Z(\Lambda, X_2, \mu_6, m_\omega, r)\right] \\ V_{22}^{\rho,\omega}(r) = & -\frac{1}{4}\beta\beta_1 g_V^2[H(\Lambda, m_\rho, r) - H(\Lambda, m_\omega, r)] - \frac{\beta\beta_1 g_V^2(M_{D^*}^2 + M_{D_0}^2)}{32M_{D^*}^2 M_{D_0}^2}[G(\Lambda, m_\rho, r) - G(\Lambda, m_\omega, r)] \\ & - \frac{1}{4}g_V^2\left[\zeta - \frac{\mu(M_{D^*}^2 - M_{D_0}^2)}{\sqrt{M_{D^*}M_{D_0}}}\right]^2[Y(\Lambda, X_3, \mu_7, m_\rho, r) - Y(\Lambda, X_3, \mu_8, m_\omega, r)] + \frac{g_V^2\mu^2(M_{D^*} + M_{D_0})^2}{12M_{D^*}M_{D_0}} \\ & \times[Z(\Lambda, X_3, \mu_7, m_\rho, r) - Z(\Lambda, X_3, \mu_8, m_\omega, r)] - \frac{g_V^2\zeta^2}{12}\left[\frac{1}{m_\rho^2}Z(\Lambda, X_3, \mu_7, m_\rho, r) - \frac{1}{m_\omega^2}Z(\Lambda, X_3, \mu_8, m_\omega, r)\right], \end{aligned} \tag{A8}$$

where the parameters μ_i $(i = 5, 6, 7, 8)$ are given by

$$\begin{aligned} \mu_5^2 &= m_\rho^2 - (M_{D_1} - M_D)^2 \\ \mu_6^2 &= m_\omega^2 - (M_{D_1} - M_D)^2 \\ \mu_7^2 &= m_\rho^2 - (M_{D_0} - M_{D^*})^2 \\ \mu_8^2 &= m_\omega^2 - (M_{D_0} - M_{D^*})^2. \end{aligned} \tag{A9}$$

The new functions $G(\Lambda, m, r)$ and $Y(\Lambda, X, \mu, m, r)$ are defined as follows:

$$\begin{aligned} G(\Lambda, m, r) &= \frac{1}{4\pi}\frac{1}{r}(\Lambda^2 e^{-\Lambda r} - m^2 e^{-mr}) + \frac{\Lambda^2 - m^2}{8\pi}\left(\Lambda - \frac{2}{r}\right)e^{-\Lambda r} \\ Y(\Lambda, X, \mu, m, r) &= \frac{1}{4\pi}\frac{1}{r}(e^{-\mu r} - e^{-Xr}) - \frac{\Lambda^2 - m^2}{8\pi X}e^{-Xr}. \end{aligned} \tag{A10}$$

As is demonstrated in Eqs. (A7) and (A8), light vector mesons ρ and ω exchange leads to diagonal interaction, and the off-diagonal components of the effective potential are zero because of parity conservation. For $Z_2^+(4250)$, it is obvious that the potential coming from ρ exchange almost cancels that from ω exchange, because of the accidental coincidence of m_ρ and m_ω, i.e., $m_\rho \simeq 775.5$ MeV and $m_\omega \simeq 782.7$ MeV [41].

There are a number of parameters β, β_1, β_2, μ, μ_1, ζ, and ζ_1 involved in the effective potential. The information about the effective coupling constants between the heavy meson and the light vector mesons is very scarce until now, especially those related with the P-wave heavy mesons. By vector meson dominance, β is estimated to be about 0.9 [42]. Reference [27] gives $\mu = -0.1$ GeV^{-1} and $\zeta = 0.1$. The remaining parameters have not been determined as far as we know, and we even do not know the ranges which they are in. So at present we cannot give a quantitative estimate about the vector meson exchange contribution to the potential associated with $Y(4260)$. Since the light vector meson mass m_ρ, m_ω is larger than m_π, m_η, and m_σ, we expect that the potential induced by vector meson exchange should be smaller than that due to pseudoscalar and scalar exchange in magnitude. In principle, we can determine these coupling constants following the methods of QCD sum rule, nonrelativistic potential model, and so on, by means of which certain coupling constants in HMχPT have been estimated. In the future, if we could get a reliable estimate about these coupling constants from both phenomenological and theoretical approaches, the effective potential arising from ρ, ω exchange and its effect on the structure of $Y(4260)$ could be analyzed in the same way as in Sec. V.

[1] S.-K. Choi *et al.* (Belle Collaboration), Phys. Rev. Lett. **100**, 142001 (2008).

[2] R. Mizuk *et al.* (Belle Collaboration), Phys. Rev. D **78**, 072004 (2008).

[3] N. A. Tornqvist, Phys. Lett. B **590**, 209 (2004); F. E. Close and P. R. Page, Phys. Lett. B **578**, 119 (2004); C. Y. Wong, Phys. Rev. C **69**, 055202 (2004); E. S. Swanson, Phys. Lett. B **588**, 189 (2004); C. E. Thomas and F. E. Close, Phys. Rev. D **78**, 034007 (2008).
[4] G. J. Ding, arXiv:0711.1485; G. J. Ding, W. Huang, J. F. Liu, and M. L. Yan, arXiv:0805.3822; J. L. Rosner, Phys. Rev. D **76**, 114002 (2007); C. Meng and K. T. Chao, arXiv:0708.4222; X. Liu, Y. R. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D **77**, 094015 (2008).
[5] R. M. Albuquerque and M. Nielsen, arXiv:0804.4817.
[6] S. H. Lee, K. Morita, and M. Nielsen, arXiv:0808.0690; Phys. Rev. D **78**, 076001 (2008).
[7] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **95**, 142001 (2005).
[8] Q. He *et al.* (CLEO Collaboration), Phys. Rev. D **74**, 091104 (2006).
[9] C. Z. Yuan *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 182004 (2007).
[10] G. Pakhlova *et al.* (Belle Collaboration), Phys. Rev. Lett. **98**, 092001 (2007).
[11] G. Pakhlova *et al.* (Belle Collaboration), Phys. Rev. D **77**, 011103 (2008).
[12] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **76**, 111105 (2007).
[13] F. J. Llanes-Estrada, Phys. Rev. D **72**, 031503 (2005).
[14] L. Maiani, V. Riquer, F. Piccinini, and A. D. Polosa, Phys. Rev. D **72**, 031502 (2005).
[15] S. L. Zhu, Phys. Lett. B **625**, 212 (2005); E. Kou and O. Pene, Phys. Lett. B **631**, 164 (2005); F. E. Close and P. R. Page, Phys. Lett. B **628**, 215 (2005).
[16] M. S. Dubrovin (CLEO Collaboration), arXiv:0705.3476.
[17] D. Ebert, R. N. Faustov, and V. O. Galkin, Phys. Lett. B **634**, 214 (2006).
[18] C. W. Bernard *et al.* (MILC Collaboration), Phys. Rev. D **56**, 7039 (1997); Z. H. Mei and X. Q. Luo, Int. J. Mod. Phys. A **18**, 5713 (2003); G. S. Bali, Eur. Phys. J. A **19**, 1 (2004).
[19] X. L. Wang *et al.* (Belle Collaboration), Phys. Rev. Lett. **99**, 142002 (2007).
[20] G. J. Ding, J. J. Zhu, and M. L. Yan, Phys. Rev. D **77**, 014033 (2008).
[21] E. Swanson, AIP Conf. Proc. **814**, 203 (2006); Int. J. Mod. Phys. A **21**, 733 (2006).
[22] F. E. Close, arXiv:0801.2646.
[23] F. E. Close, in Proceedings of 5th Flavor Physics and *CP* Violation Conference (FPCP 2007), Bled, Slovenia, 2007, pp. 020 (arXiv:0706.2709).
[24] G. Burdman and J. F. Donoghue, Phys. Lett. B **280**, 287 (1992).
[25] M. B. Wise, Phys. Rev. D **45**, R2188 (1992).
[26] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin, and H. L. Yu, Phys. Rev. D **46**, 1148 (1992); **55**, 5851(E) (1997).
[27] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Rep. **281**, 145 (1997).
[28] A. F. Falk and M. E. Luke, Phys. Lett. B **292**, 119 (1992).
[29] B. Grinstein, E. E. Jenkins, A. V. Manohar, M. J. Savage, and M. B. Wise, Nucl. Phys. **B380**, 369 (1992).
[30] W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D **68**, 054024 (2003).
[31] X. Liu, Y. R. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D **77**, 094015 (2008).
[32] T. Barnes and G. I. Ghandour, Phys. Lett. B **118**, 411 (1982).
[33] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon Press, New York, 1982).
[34] T. Barnes, N. Black, D. J. Dean, and E. S. Swanson, Phys. Rev. C **60**, 045202 (1999).
[35] T. Barnes, N. Black, and E. S. Swanson, Phys. Rev. C **63**, 025204 (2001).
[36] N. A. Tornqvist, Z. Phys. C **61**, 525 (1994).
[37] M. P. Locher, Y. Lu, and B. S. Zou, Z. Phys. A **347**, 281 (1994); X. Q. Li, D. V. Bugg, and B. S. Zou, Phys. Rev. D **55**, 1421 (1997).
[38] Y. R. Liu, X. Liu, W. Z. Deng, and S. L. Zhu, Eur. Phys. J. C **56**, 63 (2008); X. Liu, Y. R. Liu, W. Z. Deng, and S. L. Zhu, Phys. Rev. D **77**, 034003 (2008); X. Liu, Z. G. Luo, Y. R. Liu, and S. L. Zhu, arXiv:0808.0073.
[39] V. Ledoux, M. Van Daele, and G. Vanden Berghe, Comput. Phys. Commun. **176**, 191 (2007).
[40] A. G. Abrashkevich, D. G. Abrashkevichg, M. S. Kaschiev, and I. V. Puzynin, Comput. Phys. Commun. **85**, 40 (1995); **85**, 65 (1995); **115**, 90 (1998).
[41] W.-M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[42] C. Isola, M. Ladisa, G. Nardulli, and P. Santorelli, Phys. Rev. D **68**, 114001 (2003).
[43] A. Deandrea, R. Gatto, G. Nardulli, and A. D. Polosa, J. High Energy Phys. 02 (1999) 021.
[44] S. Fajfer and J. F. Kamenik, Phys. Rev. D **74**, 074023 (2006).
[45] P. Colangelo, G. Nardulli, A. Deandrea, N. Di Bartolomeo, R. Gatto, and F. Feruglio, Phys. Lett. B **339**, 151 (1994).
[46] V. M. Belyaev, V. M. Braun, A. Khodjamirian, and R. Ruckl, Phys. Rev. D **51**, 6177 (1995).
[47] A. D. Polosa, arXiv:hep-ph/9909371.
[48] E. S. Swanson, Phys. Rep. **429**, 243 (2006).
[49] G. W. S. Hou, arXiv:hep-ph/0611153.
[50] D. Ebert, R. N. Faustov, and V. O. Galkin, arXiv:0808.3912.
[51] Z. G. Wang, arXiv:0807.4592; arXiv:0807.2581.
[52] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **292**, 371 (1992).
[53] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio, and G. Nardulli, Phys. Lett. B **299**, 139 (1993).
[54] The following effective Lagrangian are obtained by expanding Eq. (A1) and (A4) term by term, then they are checked by the program FEYNCALC [55].
[55] http://www.feyncalc.org/.