Large gauge invariant nonstandard neutrino interactions

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Theories beyond the standard model must necessarily respect its gauge symmetry. This implies strict constraints on the possible models of nonstandard neutrino interactions, which we analyze. The focus is set on the effective low-energy dimension six and eight operators involving four leptons, decomposing them according to all possible tree-level mediators, as a guide for model building. The new couplings are required to have sizable strength, while processes involving four charged leptons are required to be suppressed. For nonstandard interactions in matter, only diagonal tau-neutrino interactions can escape these requirements and can be allowed to result from dimension six operators. Large nonstandard neutrino interactions from dimension eight operators alone are phenomenologically allowed in all flavor channels and are shown to require at least two new mediator particles. The new couplings must obey general cancellation conditions both at the dimension six and dimension eight levels, which result from expressing the operators obtained from the mediator analysis in terms of a complete basis of operators. We illustrate with one example how to apply this information to model building.

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I. INTRODUCTION

The experimental observation of neutrino masses and mixings is the first evidence for physics beyond the standard model (SM)—maybe together with the indication for dark matter—and points to the existence of a new, yet unknown, physics scale. The tiny masses of the neutrinos, which are orders of magnitude lighter than those of other fermions, suggest a large new physics scale leading to very suppressed effects. Since neutrinos have only weak interactions with the SM particles, they may even constitute an excellent window into the new physics underlying the "dark sectors" of the Universe, i.e., dark matter and dark energy. Therefore, new physics may very well appear next in the form of exotic couplings involving neutrinos, which are often called *nonstandard neutrino interactions* (NSI) [1–5]. This possibility is being actively explored and will be the subject of the present work. In particular, we will, in a model-independent way, discuss the connection between NSI and the possible tree-level mediators of new physics inducing them. If NSI are detected, this study will serve as a guide for the model builder.

Note that the very tiny neutrino mass differences have only been detectable because the masses affect the neutrino propagation by inducing small phase shifts, which can be compensated by the very long distances traveled in neutrino oscillation experiments. In contrast, nonstandard couplings are short distance (local) effects, which usually do

not benefit from such an enhancement—unless they affect the propagation in matter, one of the several possibilities explored below. Notice, though, that neutrino oscillation experiments may well turn out to be the best *arena* to detect NSI, as they can affect oscillation amplitudes linearly, instead of quadratically as in most charged lepton flavor violation processes.

On general grounds, whatever the nature of the new putative couplings is, observable effects will only be expected for a new physics scale Λ near the present experimental limits, i.e., above the electroweak symmetry breaking (EWSB) scale. An example of NSI is given by the dimension six ($d = 6$) operator in

$$\frac{1}{\Lambda^2}(\bar{\nu}_\alpha \gamma^\rho P_L \nu_\beta)(\bar{\ell}_\gamma \gamma_\rho \ell_\delta). \tag{1}$$

In this expression, spinor indices are omitted, Greek letters denote flavor indices, $P_L = (1 - \gamma_5)/2$ is the left-handed chiral projection operator, and ν and ℓ label the SM neutrino and charged lepton fields, respectively.

The operator in Eq. (1) is not a singlet of the SM gauge group, $SU(3) \times SU(2) \times U(1)$, while the high-energy theory has to contain and encompass the SM gauge group. For instance, the coupling in Eq. (1) could result from the following gauge invariant operator:

$$\frac{1}{\Lambda^2}(\bar{L}_\alpha \gamma^\rho L_\beta)(\bar{L}_\gamma \gamma_\rho L_\delta), \tag{2}$$

where L denotes the leptonic $SU(2)$ doublets. Equation (2) illustrates the consequences of electroweak gauge invariance: The coupling in Eq. (1) is necessarily accompanied by other charged lepton transitions for which stringent

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limits may exist. As an example, for $\beta = \mu$ and $\alpha = \gamma = \delta = e$, Eq. (2) would imply $\mu \to 3e$ transitions with the same strength as the interaction in Eq. (1).

In order to discuss such constraints coming from the gauge invariant framework, it is convenient to rephrase the problem in terms of a generic low-energy effective theory. Effective theories allow rather model-independent analyses based on the fundamental symmetries, while only the coefficients of the effective operators are model dependent. The impact of the heavy fields present in the high-energy theory can be parametrized, without loss of generality, by the addition to the Lagrangian of a tower of *nonrenormalizable* operators $\mathcal{O}^d$ of dimension $d > 4$, made out of the SM fields and invariant under the SM gauge group. The operator coefficients are weighted by inverse powers of the high scale Λ:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \delta\mathcal{L}_{\text{eff}}^{d=5} + \delta\mathcal{L}_{\text{eff}}^{d=6} + \cdots,$$

with $\quad \delta\mathcal{L}_{\text{eff}}^d \propto \dfrac{1}{\Lambda^{d-4}} \mathcal{O}^d.$ $\qquad$ (3)

After EWSB, some of the effective operators may result in corrections to the low-energy SM parameters. In addition, new exotic couplings may result, such as those in Eq. (1).

The only possible $d = 5$ operator is the famous Weinberg operator [6], which leads, after EWSB, to Majorana neutrino masses. We will not need to consider it for the present study of NSI.

At $d = 6$, some operators modify the low-energy standard couplings, among which nonunitary corrections to the leptonic Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix are especially relevant to the case under study. Departures from unitarity are a general feature of beyond-the-SM models involving exotic fermions [2,7–15]. All fermions of the same charge will generically mix through the mass matrix, leading in those theories to unitary mixing matrices of dimension larger than three, while the effective 3×3 submatrix—relevant at low energies—is not unitary. In the effective Lagrangian formalism, the effect appears technically at leading order through $d = 6$ gauge invariant operators involving only *two* fermions, which induce noncanonical corrections to the fermion kinetic terms [7–11]. Such operators are, for instance, typical of fermion-mediated seesaw scenarios. The trademark of nonunitarity is that the coefficients of the NSI operators induced by it and contributing to neutrino production, detection, and matter effects are not independent but related. Barring fine-tuned cancellations, the stringent bounds and future signals on nonunitarity [16–18] apply as well to NSI, except for those NSI operators exclusively affecting the propagation in matter. Recently, the value of the elements of the PMNS matrix have been extracted from data without assuming a unitary mixing matrix [19], and new related *CP*-odd signals have been proposed as well [20–22]. For a detailed discussion of the NSI-nonunitarity relationship, see [23]. We will leave nonunitarity out of the main line of this work, discussing only its qualitative implications.

Effective interactions, such as the NSI in Eq. (1), obviously require one to consider operators made out of *four* leptonic fields, plus Higgs fields in the case of operators with $d > 6$ [24–26]. There is a plethora of $d = 6$ [27] and $d = 8$ [28] operators,[1] with different classes of models resulting in different operators and operator coefficients. Those among them relevant for NSI can affect neutrino production or detection processes, or modify the matter effects in their propagation, depending on the operator or combination of operators considered. Notice, though, that the coefficient of a $d = 8$ operator is expected to be suppressed by a factor v^2/Λ^2 with respect to $d = 6$ operator coefficients (where v is the vacuum expectation value of the Higgs field $\langle H^0 \rangle = v/\sqrt{2} = 174$ GeV), and thus negligible unless the new scale is very close to the electroweak one.

Much effort has been dedicated to analyzing the experimental constraints and future detection prospects of NSI [24,30,31]. This encompasses their impact on weak decays, solar and atmospheric neutrino physics [32–39], astrophysics [40,41], early universe processes [42], collider and neutrino scattering experiments [28,43–46], and past and future neutrino oscillation experiments including neutrino factories [47–69]. Overall, the signals involving the ν_τ field are, at present, the least constrained ones.

We emphasize that we study "large" NSI, where large refers to being potentially observable at future neutrino oscillation experiments. For example, for the flavor-changing NSI interfering with standard oscillations, bounds 10^{-2} to 10^{-3} (relative to the SM four-fermion interaction coupling) on the operator in Eq. (1) can be expected from a neutrino factory (see, e.g., Ref. [65]). After EWSB, the four-fermion interactions from Eq. (3) will be suppressed by roughly $(v/\Lambda)^{d-4}$ with respect to the SM four-fermion interactions, which means that Λ is allowed to be largest for the $d = 6$ effective operators, whereas for $d = 8$ operators, it has to be very close to the electroweak scale in order to produce a sizable effect. Therefore, we focus on the $d = 6$ operators first, and then increase the level of complexity. However, from this simple comparison, one can already read off that new physics above the TeV scale will be very difficult to be observed at future neutrino oscillation experiments, since the suppression with respect to the SM is roughly 10^{-2} (for $d = 6$) and 10^{-4} (for $d = 8$), respectively—especially if the $d = 6$ operators turn out to be not good enough.

Most of the literature deals mainly with NSI in experiments, and does not discuss the models behind them—with some exceptions [3,5,23,25,26,70–73]. In this work, we will classify all $d = 6$ and $d = 8$, $SU(3) \times SU(2) \times U(1)$

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[1]$d = 7, 9 \ldots$ operators [29] are odd under baryon and lepton number and not relevant for the present discussion.

invariant, leptonic NSI operators in terms of the heavy mediators inducing them. This can be achieved in a model-independent way, decomposing each operator into all possible products of currents. The SM charges of the corresponding field combinations will be the SM charges of the putative heavy mediators. The study will thus be confined to the case in which the tower of nonrenormalizable operators has been produced by the *tree-level* exchange of new heavy fields, whose precise nature other than their SM charges we do not need to know.

We will first emphasize the case in which the mediators exchanged only couple to SM bilinear field combinations. In this study, we refer to "SM bilinears" as fundamental interactions of *exactly two* SM fields with one or two exotic fields, where the latter possibility amounts to couplings between two exotic bosons and two Higgs doublets. Other than that, there can be, in addition, new exotic couplings involving only *one* SM field, which will also be addressed later on.

The decomposition in terms of SM bilinears, initiated in [25,26], immediately leads to correlations between previously uncorrelated effective NSI operator coefficients, with very fruitful physics consequences, as we will show below. For instance, it has been realized that the lowest dimension operators, that lead to NSI without simultaneously inducing dangerous transitions among four charged leptons, are $d = 8$ operators. The first example proposed [24,74] is of the form

$$\mathcal{O}_{\mathrm{NSI}} = (\bar{L}^i H_i)\gamma^\rho(H^{\dagger i} L_i)(\bar{E}\gamma_\rho E), \qquad (4)$$

where i, L, E, and H denote the $SU(2)_L$ index, leptonic doublet, leptonic singlet, and Higgs doublet, respectively [note that we use the convention with $Y = -1/2$ for the Higgs doublet, $H^T = (\frac{1}{\sqrt{2}}H^0, H^-)$]. We will illustrate below, though, that any realistic model responsible for it will also induce other dangerous $d = 6$ operators and/or some other low-energy effects for which stringent experimental bounds may exist [i.e. nonunitary corrections to the PMNS matrix, corrections to the electroweak precision data, flavor-changing neutral currents (FCNC), etc.]. Equation (4) is only one example of the NSI sought. We will determine in this work several other independent $d = 8$ operators which lead to large NSI and no charged lepton processes. The symbol $\mathcal{O}_{\mathrm{NSI}}$ will be extended to denote generically any element in this ensemble.

In our general analysis, after determining all possible mediators, the resulting correlations between the possible $d = 6$ and $d = 8$ operators will be systematically studied. We will then establish which mediators or combinations of mediators can lead to large NSI, without inducing experimentally excluded leptonic charged flavor-changing transitions, and/or other undesired phenomenological consequences.

Our main motivation in this study is to determine the minimum level of complexity needed for a viable model of

NSI. As an illustration for model building, a particular simple toy model will be developed in which the operator $\mathcal{O}_{\mathrm{NSI}}$ above is induced unaccompanied by any leptonic $d = 6$ operator. The aim is to show the generic price to pay at the theoretical level for allowing observable NSI effects at future experiments.

Note that we focus in this study on the necessary conditions to build a model with large NSI, while for any given model additional limitations may arise. Supplementary constraints which could arise from a phenomenological analysis at one loop are not considered here either but should be addressed when considering a particular model. From the experimental point of view, we will not make any explicit statement about how likely it is to observe large NSI. We leave the interpretation of this likeliness, by judging the necessary conditions for a viable model, to the reader. Finally, possible NSI involving quark fields are not included in this study either.

II. EFFECTIVE OPERATOR FORMALISM

The SM Lagrangian is extended to accommodate the tower of effective operators

$$\delta\mathcal{L}_{\mathrm{eff}} = \frac{1}{\Lambda^2}\sum_i^{d=6}\mathcal{C}_i\mathcal{O}_i^{d=6} + \frac{1}{\Lambda^4}\sum_k^{d=8}\mathcal{C}_k\mathcal{O}_k^{d=8}, \qquad (5)$$

where the two terms run over all possible $d = 6$ and $d = 8$ operators relevant for purely leptonic NSI. The flavor composition will be made explicit in each coefficient and operator, i.e., $(\mathcal{C}_i)_{\beta\delta}^{\alpha\gamma}(\mathcal{O}_i)_{\alpha\gamma}^{\beta\delta}$. All distinct flavor combinations for the same operator structure will be taken into account, as they correspond, in fact, to independent operators.

A. Effective operator basis

In order to find all possible $d = 6$ and $d = 8$ effective operators leading to purely leptonic NSI, we will use the following bases:
(i) $d = 6$ *operators.*—A complete basis of $d = 6$ operators invariant under the SM gauge group and made out of the SM light fields was proposed by Buchmüller and Wyler (BW) [27]. The four fermion operator structures relevant to our problem are

$$(\mathcal{O}_{LE})_{\alpha\gamma}^{\beta\delta} = (\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha), \qquad (6)$$

$$(\mathcal{O}_{LL}^{\mathbf{1}})_{\alpha\gamma}^{\beta\delta} = (\bar{L}^\beta\gamma^\rho L_\alpha)(\bar{L}^\delta\gamma_\rho L_\gamma), \qquad (7)$$

$$(\mathcal{O}_{LL}^{\mathbf{3}})_{\alpha\gamma}^{\beta\delta} = (\bar{L}^\beta\gamma^\rho\vec{\tau}L_\alpha)(\bar{L}^\delta\gamma_\rho\vec{\tau}L_\gamma), \qquad (8)$$

$$(\mathcal{O}_{EE})_{\alpha\gamma}^{\beta\delta} = (\bar{E}^\beta\gamma^\rho E_\alpha)(\bar{E}^\delta\gamma_\rho E_\gamma), \qquad (9)$$

where L (E) refers to the $SU(2)$ leptonic doublet (singlet). We will refer to the coefficient matrices

for these operators by $(\mathcal{C}_{LE})^{\alpha\gamma}_{\beta\delta}$, $(\mathcal{C}^1_{LL})^{\alpha\gamma}_{\beta\delta}$, $(\mathcal{C}^3_{LL})^{\alpha\gamma}_{\beta\delta}$, and $(\mathcal{C}_{EE})^{\alpha\gamma}_{\beta\delta}$, respectively. The operators $\mathcal{O}_{EE}$ do not produce NSI directly, but will play a role when considering charged lepton flavor violation [since they share some mediators with the operators in Eqs. (6)–(8)].

In addition to the above, there are two $d = 6$ operator structures, including two lepton doublets L and two Higgs doublets H,

$$(\mathcal{O}^1_{LH})^\beta_\alpha = (\bar{L}^\beta H)\mathrm{i}\slashed{\partial}(H^\dagger L_\alpha), \qquad (10)$$

$$(\mathcal{O}^3_{LH})^\beta_\alpha = (\bar{L}^\beta \vec{\tau} H)\mathrm{i}\slashed{\partial}(H^\dagger \vec{\tau} L_\alpha), \qquad (11)$$

and an operator with two E's and two H's,

$$(\mathcal{O}_{EH})^\beta_\alpha = (H^\dagger \mathrm{i} D^\rho H)(\bar{E}^\beta \gamma_\rho E_\alpha), \qquad (12)$$

where D denotes the SM covariant derivative. These three operators belong to the class which, after EWSB, correct the parameters of the SM Lagrangian. In particular, they renormalize the kinetic energy of neutrinos and/or charged leptons [9–11]. As previously mentioned, they result in nonunitary corrections to the leptonic mixing matrix and/or correct the charged and neutral electroweak currents [27], and will not be further developed in this work. We include them above only for the sake of completeness (see also Sec. IV B 2).

(ii) $d = 8$ *operators.*—A basis was discussed by Berezhiani and Rossi (BR) [28], with the relevant operators given by

$$(\mathcal{O}^1_{LEH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho L_\alpha)(\bar{E}^\delta \gamma_\rho E_\gamma)(H^\dagger H), \qquad (13)$$

$$(\mathcal{O}^3_{LEH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho \vec{\tau} L_\alpha)(\bar{E}^\delta \gamma_\rho E_\gamma)(H^\dagger \vec{\tau} H), \qquad (14)$$

$$(\mathcal{O}^{111}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho L_\alpha)(\bar{L}^\delta \gamma_\rho L_\gamma)(H^\dagger H), \qquad (15)$$

$$(\mathcal{O}^{331}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho \vec{\tau} L_\alpha)(\bar{L}^\delta \gamma_\rho \vec{\tau} L_\gamma)(H^\dagger H), \qquad (16)$$

$$(\mathcal{O}^{133}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho L_\alpha)(\bar{L}^\delta \gamma_\rho \vec{\tau} L_\gamma)(H^\dagger \vec{\tau} H), \qquad (17)$$

$$(\mathcal{O}^{313}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\bar{L}^\beta \gamma^\rho \vec{\tau} L_\alpha)(\bar{L}^\delta \gamma_\rho L_\gamma)(H^\dagger \vec{\tau} H), \qquad (18)$$

$$(\mathcal{O}^{333}_{LLH})^{\beta\delta}_{\alpha\gamma} = (-\mathrm{i}\epsilon^{abc})(\bar{L}^\beta \gamma^\rho \tau^a L_\alpha)(\bar{L}^\delta \gamma_\rho \tau^b L_\gamma) \times (H^\dagger \tau^c H), \qquad (19)$$

$$(\mathcal{O}_{EEH})^{\beta\delta}_{\alpha\gamma} = (\bar{E}\gamma^\rho E)(\bar{E}\gamma_\rho E)(H^\dagger H). \qquad (20)$$

In these operators, subscripts correspond to a shortcut notation for their SM field composition, whereas superscripts denote the corresponding $SU(2)$ charges of the field combinations. Once again, although the

operators $\mathcal{O}_{EEH}$ cannot induce NSI by themselves, they will come to play a related role, as they induce charged lepton flavor violating transitions.

Strictly speaking, not all of the above operators are independent when the full flavor structure is taken into account, as

$$(\mathcal{O}^{313}_{LLH})^{\beta\delta}_{\alpha\gamma} = (\mathcal{O}^{133}_{LLH})^{\delta\beta}_{\gamma\alpha}. \qquad (21)$$

However, the expressions below will look much simpler if both operators are used.

Notice that the phenomenologically interesting $\mathcal{O}_{\mathrm{NSI}}$ operator in Eq. (4) can be expressed as a combination of the first two operators in the list above,

$$\mathcal{O}_{\mathrm{NSI}} = \tfrac{1}{2}(\mathcal{O}^1_{LEH} + \mathcal{O}^3_{LEH}). \qquad (22)$$

This means, for instance, that if a model only induces at $d = 8$ the operators $\mathcal{O}^1_{LEH}$ and $\mathcal{O}^3_{LEH}$ with similar weights and no $d = 6$ NSI operator, it could be an optimal candidate for viable large NSI. We will explore later some examples of this kind.

B. Decomposition in terms of $SU(2)$ field components

After EWSB, the contributions from the $d = 6$ and $d = 8$ gauge invariant operators result in two very simple sets of operators. From the $\bar{L}L\bar{E}E$-type operators, Eqs. (6), (13), and (14), we find

$$\begin{aligned}
\delta\mathcal{L}_{\mathrm{eff}} = \frac{1}{\Lambda^2}&\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}^1_{LEH} + \mathcal{C}^3_{LEH})\right)^{\alpha\gamma}_{\beta\delta} \\
&\times (\bar{\nu}^\beta \gamma^\rho \mathrm{P}_L \nu_\alpha)(\bar{\ell}^\delta \gamma_\rho \mathrm{P}_R \ell_\gamma) \\
+ \frac{1}{\Lambda^2}&\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2}(\mathcal{C}^1_{LEH} - \mathcal{C}^3_{LEH})\right)^{\alpha\gamma}_{\beta\delta} \\
&\times (\bar{\ell}^\beta \gamma^\rho \mathrm{P}_L \ell_\alpha)(\bar{\ell}^\delta \gamma_\rho \mathrm{P}_R \ell_\gamma) + \mathrm{H.c.}
\end{aligned} \qquad (23)$$

The first term in this equation produces the relevant NSI, whereas the second term leads to the (unwanted) four charged lepton contributions. The NSI in the first term involve only right-handed charged leptons. In consequence, their effect at the neutrino source will be chirally suppressed.[2]

From the operators involving four lepton doublets, Eqs. (7), (8), and (15)–(19), the results is that[3]

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[2]At detection, the effect of these NSI is subdominant because of the dominance of the neutrino-nucleon cross section.

[3]Here we do not show the interactions among four neutrinos which these operators also induce. See Appendix A for a discussion of these interactions.

$\delta\mathcal{L}_{\text{eff}}$

$$= \frac{1}{\Lambda^2} (\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta\delta}^{\alpha\gamma} (\bar{\nu}^\beta \gamma^\rho P_L \nu_\alpha)(\bar{\ell}^\delta \gamma^\rho P_L \ell_\gamma)$$

$$+ \frac{1}{\Lambda^2} \left(\mathcal{C}_{LL}^1 + \mathcal{C}_{LL}^3 + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LLH}^{111} + \mathcal{C}_{LLH}^{331} - \mathcal{C}_{LLH}^{133} - \mathcal{C}_{LLH}^{313}) \right)_{\beta\delta}^{\alpha\gamma}$$

$$\times (\bar{\ell}^\beta \gamma^\rho P_L \ell_\alpha)(\bar{\ell}^\delta \gamma^\rho P_L \ell_\gamma) + \text{H.c.}, \qquad (24)$$

where

$(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta\delta}^{\alpha\gamma}$

$$= \left(\mathcal{C}_{LL}^1 - \mathcal{C}_{LL}^3 + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LLH}^{111} - \mathcal{C}_{LLH}^{331} - \mathcal{C}_{LLH}^{133} + \mathcal{C}_{LLH}^{313}) \right)_{\beta\delta}^{\alpha\gamma}$$

$$+ \left(\mathcal{C}_{LL}^1 - \mathcal{C}_{LL}^3 + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LLH}^{111} - \mathcal{C}_{LLH}^{331} + \mathcal{C}_{LLH}^{133} - \mathcal{C}_{LLH}^{313}) \right)_{\delta\beta}^{\gamma\alpha}$$

$$+ \left(2\mathcal{C}_{LL}^3 + \frac{v^2}{\Lambda^2} (\mathcal{C}_{LLH}^{331} - \mathcal{C}_{LLH}^{333}) \right)_{\delta\beta}^{\alpha\gamma}$$

$$+ \left(2\mathcal{C}_{LL}^3 + \frac{v^2}{\Lambda^2} (\mathcal{C}_{LLH}^{331} + \mathcal{C}_{LLH}^{333}) \right)_{\beta\delta}^{\gamma\alpha}. \qquad (25)$$

Note the different flavor structure in the four terms in Eq. (25). In addition, note that the term relevant for the NSI, i.e., the first line in Eq. (24), couples to left-handed charged leptons, which means that source NSI can be generated as well. In summary, matter NSI are (not) correlated with source and production NSI for $\bar{L}L\bar{L}L$ ($\bar{L}L\bar{E}E$)-type operators.

C. Connection to NSI and phenomenology

Let us first consider NSI in matter. The phenomenology of neutrino propagation under these conditions is customarily described in terms of the Hamiltonian in the flavor basis [4,24,51],

$$\mathcal{H}_F = \frac{1}{2E} \left\{ U \begin{pmatrix} 0 & & \\ & \Delta m_{21}^2 & \\ & & \Delta m_{31}^2 \end{pmatrix} U^\dagger \right.$$

$$\left. + a_{\text{CC}} \begin{pmatrix} 1 + \epsilon_{ee}^m & \epsilon_{e\mu}^m & \epsilon_{e\tau}^m \\ (\epsilon_{e\mu}^m)^* & \epsilon_{\mu\mu}^m & \epsilon_{\mu\tau}^m \\ (\epsilon_{e\tau}^m)^* & (\epsilon_{\mu\tau}^m)^* & \epsilon_{\tau\tau}^m \end{pmatrix} \right\}, \qquad (26)$$

where a_{CC} is the usual matter effect term defined as $a_{\text{CC}} \equiv 2\sqrt{2}EG_F N_e$ (with N_e the electron number density in Earth matter).

From Eqs. (23) and (24) it follows that

$$\epsilon_{\beta\alpha}^{m,L} = \frac{v^2}{2\Lambda^2} (\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta e}^{\alpha e},$$

$$\epsilon_{\beta\alpha}^{m,R} = \frac{v^2}{2\Lambda^2} \left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LEH}^1 + \mathcal{C}_{LEH}^3) \right)_{\beta e}^{\alpha e}, \qquad (27)$$

with $\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L}$ as defined in Eq. (25). These two parameters in matter lead to a total

$$\epsilon_{\beta\alpha}^m = \epsilon_{\beta\alpha}^{m,L} + \epsilon_{\beta\alpha}^{m,R}, \qquad (28)$$

because matter effects are only sensitive to the vector component.

In addition to the propagation in matter, the production or detection processes can be affected by NSI. For the specific case of a neutrino factory and considering just the purely leptonic NSI under discussion, only effects at the source are relevant, since the detection interactions involve quarks.[4] They are customarily parametrized in terms of $\epsilon_{\alpha\beta}^s$, which describes an effective source state $|\nu_\alpha^s\rangle$ as [4,48,75]

$$|\nu_\alpha^s\rangle = |\nu_\alpha\rangle + \sum_{\gamma=e,\mu,\tau} \epsilon_{\alpha\gamma}^s |\nu_\gamma\rangle. \qquad (29)$$

In this case, the muon decay rate could be modified by the NSI interaction in Eq. (24), with the largest effect resulting from the coherent contribution to the state at the source [48,53]. It appears as an admixture of a given flavor ν_α with all other flavors, encoded by ν_γ in Eq. (29). Two types of contributions are possible,

$$\epsilon_{\mu\beta}^s = \frac{v^2}{2\Lambda^2} (\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta e}^{e\mu} \quad \text{or} \quad \epsilon_{e\beta}^s = \frac{v^2}{2\Lambda^2} (\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta\mu}^{\mu e}. \quad (30)$$

The second possibility will affect the golden $\nu_e \rightarrow \nu_\mu$ appearance channel, where the effect might be easiest to observe. If the coefficients in Eq. (25) are known for a specific model, one can easily calculate the connection between source and propagation effects via Eqs. (27) and (30), a connection which does not hold for $\bar{L}L\bar{E}E$-type operators above, as explained earlier.

D. Conditions to suppress charged lepton processes

Let us discuss now potentially dangerous contributions to charged lepton flavor violation processes, possible modifications of G_F, and the constraints on lepton universality. The focus is set on pure charged lepton processes at tree level. These interactions can result from the second terms in Eqs. (23) and (24). They should be very suppressed in any phenomenologically viable model. In order to cancel those terms, the putative beyond-the-SM theory has to satisfy, to a high degree of accuracy, the following constraints:

$$\left(-\frac{1}{2}\mathcal{C}_{LE} + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LEH}^1 - \mathcal{C}_{LEH}^3) \right)_{\beta\delta}^{\alpha\gamma} = 0, \qquad (31)$$

$$\left(\mathcal{C}_{LL}^1 + \mathcal{C}_{LL}^3 + \frac{v^2}{2\Lambda^2} (\mathcal{C}_{LLH}^{111} + \mathcal{C}_{LLH}^{331} - \mathcal{C}_{LLH}^{133} - \mathcal{C}_{LLH}^{313}) \right)_{\beta\delta}^{\alpha\gamma} = 0,$$
$$(32)$$

for all possible values of the flavor indices (Greek letters). A possibility suggested by these equations is that there could be cancellations among $d = 6$ and $d = 8$ operator coefficients. However, we will not discuss such a possibil-

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[4]Superbeams, for instance, use hadronic interactions for neutrino production, which are not affected by purely leptonic NSI to first order.

ity in this study, as it would correspond to fine-tuning the scale Λ. We will therefore require that the $d = 6$ and $d = 8$ operator coefficients in Eqs. (31) and (32) cancel independently.

For the $d = 6$ operator coefficients, we get (omitting flavor indices)

$$\mathcal{C}_{LE} = 0, \qquad \mathcal{C}_{LL}^{\mathbf{1}} = -\mathcal{C}_{LL}^{\mathbf{3}}, \qquad \mathcal{C}_{EE} = 0, \qquad (33)$$

which implies that only $\bar{L}L\bar{L}L$-type operators can induce large NSI. One possibility for its implementation is the antisymmetric operator mediated by an $SU(2)$ singlet scalar in Ref. [26], which turns out to be the only $d = 6$ possibility requiring just one tree-level mediator, as we shall explicitly demonstrate.

For the $d = 8$ operator coefficients, the cancellation conditions read

$$\mathcal{C}_{LEH}^{\mathbf{1}} = \mathcal{C}_{LEH}^{\mathbf{3}}, \qquad \mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} - \mathcal{C}_{LLH}^{\mathbf{133}} - \mathcal{C}_{LLH}^{\mathbf{313}} = 0,$$
$$\mathcal{C}_{LLH}^{\mathbf{333}} \text{ arbitrary}, \qquad \mathcal{C}_{EEH} = 0, \qquad (34)$$

where the first condition corresponds to operators of the type $\mathcal{O}_{\text{NSI}}$ in Eq. (4); see Eq. (22). In the following, we will refer to operators satisfying Eq. (34) as $\mathcal{O}_{\text{NSI}}$; i.e., we define the class of potential nonstandard neutrino interaction operators in mass dimension eight as the one which does not introduce any harmful $d = 8$ processes with four charged leptons. Equation (4) is (apart from Fierz rearrangements) the only such possibility with two right-handed charged leptons involved. When considering leptonic NSI involving four left-handed fields, several new operators of this kind will be determined later on.

As far as the possible NSI in terms of $SU(2)$ field components are concerned, not all flavor structures can be generated from the $d = 6$ effective gauge invariant operators if charged lepton processes are suppressed. Applying the $d = 6$ cancellation conditions in Eq. (33) to Eq. (25), the result is that the $d = 6$ contribution to the coefficient $(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta\delta}^{\alpha\gamma}$ is antisymmetric in the flavor index exchanges $(\alpha, \gamma) \rightarrow (\gamma, \alpha)$ and $(\beta, \delta) \rightarrow (\delta, \beta)$, which means that $\alpha \neq \gamma$ and $\beta \neq \delta$ for viable NSI. As regards matter effects, this implies that only $\epsilon_{\mu\mu}^m$, $\epsilon_{\mu\tau}^m$, and $\epsilon_{\tau\tau}^m$—defined in Eq. (27)—can be generated from $d = 6$ operators, and the connection with the source effects is given by

$$\epsilon_{\mu\mu}^m = -\epsilon_{ee}^s = -\epsilon_{\mu\mu}^s, \qquad (35)$$

$$\epsilon_{\mu\tau}^m = -(\epsilon_{\mu\tau}^s)^*. \qquad (36)$$

In contrast, $\epsilon_{\tau\tau}^m$ is not connected to the source effects at the effective operator level.[5] Notice that, for instance, the NSI in Eq. (35) contribute to the G_F measurement coherently

(i.e., the interference with SM couplings contributes linearly to the rates), for which quite stringent bounds exist. These results hold, in general, for any purely leptonic NSI $d = 6$ operator with suppressed interactions among four charged leptons, i.e., Eq. (33). Furthermore, for the particular case of a neutrino factory, the antisymmetry conditions and constraints described above imply that the only possible non-negligible NSI source terms induced by $d = 6$ operators are $\epsilon_{e\tau}^s$ and $\epsilon_{\mu\tau}^s$.

III. MODEL ANALYSIS OF $d = 6$ OPERATORS

In this section, we discuss the model-building implications of requesting large $d = 6$ NSI induced by theories of physics beyond the standard model. We specifically highlight the basic principles, which can be found in the $d = 8$ case as well. However, as we shall see later, the $d = 8$ case is technically somewhat more challenging.

In order to shed light on model building, let us analyze the operators according to the possible tree-level mediator. This is most efficiently done by listing all possible SM bilinear field combinations, and combining them in all possible ways [25,26].

We therefore show in Table I the possible bilinears constructed from leptons only, which can lead to the $d = 6$ NSI operators in Eqs. (6)–(9). It is obvious from the table that the bilinears carry the mediator information and that they can therefore be directly associated with specific models (as illustrated). The mediators are denoted—throughout this paper—by $\mathbf{X}_Y^{\mathcal{L}}$, where

(i) $\mathbf{X}$ denotes the $SU(2)$ nature, i.e., singlet $\mathbf{1}$, doublet $\mathbf{2}$, or triplet $\mathbf{3}$.

(ii) $\mathcal{L}$ refers to the Lorentz nature, i.e., scalar (s), vector (v), left-handed (L), or right-handed (R) fermion.[6]

(iii) Y refers to the hypercharge $Y = Q - I_3^W$.

Table II shows, in turn, all possible $d = 6$ operators which can be constructed from the SM bilinear field combinations in Table I. The coefficients of the $d = 6$ operators obtained by this procedure are denoted by $(c^{\mathbf{X}\mathcal{L}})_{\beta\delta}^{\alpha\gamma}$ and $(f^{\mathbf{X}\mathcal{L}})_{\beta\delta}^{\alpha\gamma}$, where c (f) indicates that the corresponding operator results from the exchange of particles carrying lepton number two (zero), and $\mathbf{X}$ and $\mathcal{L}$ refer again to the $SU(2)$ and Lorentz nature, respectively. Any subscript refers to the combination of bilinears involved.

At this point it is important to note that the operators obtained from the mediators do not constitute a basis. Instead they are not independent, but linear combinations of those in the BW basis, Eqs. (6)–(9). Therefore, it might be more accurate to call them ''mediator operators'' or ''operator combinations.'' We will not make this special distinction, but the reader should keep that in mind. Rewriting the individual effective operators from Table II

---

[5]There can also be subdominant effects in detection chains. For example, in OPERA, the taus resulting from hadronic interactions decay into muons or electrons. This implies, for instance, $\epsilon_{\tau\tau}^m = -\epsilon_{\tau\tau}^s$, which means that tau decay into electrons is in this case connected with matter NSI. Note that NSI and SM contributions add coherently to the $\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e$ width.

[6]Fermionic mediators will appear explicitly later on, when discussing $d = 8$ effective interactions.

TABLE I.   Possible SM bilinear field combinations involving only leptons. ΔL refers to the lepton number of the mediator; $SU(2)_L$ to electroweak singlets (**1**), doublets (**2**), or triplets (**3**); $U(1)_Y$ to the hypercharge $Y \equiv Q - I_3^W$; and "Lorentz" to the Lorentz nature. The mediator notation is $X_Y^{\mathcal{L}}$, where X, $\mathcal{L}$, and Y denote its $SU(2)$, Lorentz, and Y properties, respectively. The mediators which carry two units of lepton number were studied in Ref. [76].

ΔL	$SU(2)_L$	$U(1)_Y$	Lorentz	Mediator	Bilinear(s)	Models [Refs.]
2	**1**	-1	Scalar	$\mathbf{1}_{-1}^s$	$\bar{L}^c i\tau^2 L$	Zee model [77–79], $\not{R}_p$SUSY [80]
		-2	Scalar	$\mathbf{1}_{-2}^s$	$\bar{E}^c E$	
	3	-1	Scalar	$\mathbf{3}_{-1}^s$	$\bar{L}^c i\tau^2 \tau^a L$	Left-right sym. [81–84]
	2	$-3/2$	Vector	$\mathbf{2}_{-3/2}^v$	$\bar{E}^c \gamma^\rho L$	331 model [85–87]
0	**1**	0	Vector	$\mathbf{1}_0^v$	$\bar{L}\gamma^\rho L$, $\bar{E}\gamma^\rho E$	Models with Z' [31]
	3	0	Vector	$\mathbf{3}_0^v$	$\bar{L}\gamma^\rho \tau^a L$	Models with W' [31]
	2	$1/2$	Scalar	$\mathbf{2}_{1/2}^s$	$\bar{E}L$	$\not{R}_p$SUSY [80]

in the BW basis, we find the coefficients given in the last four columns of Table II. For example, the first line of the second group, mediated by $\mathbf{1}_{-1}^s$, reads (including flavor indices)

$$
\begin{aligned}
\delta\mathcal{L}_{\text{eff}}^{d=6} &= \frac{(c_{LL}^{\mathbf{1}s})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2}((\bar{L}^c)_\alpha i\tau^2 L_\gamma)(\bar{L}^\beta i\tau^2 (L^c)^\delta) \\
&= \frac{1}{4}\frac{(c_{LL}^{\mathbf{1}s})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2}(\mathcal{O}_{LL}^{\mathbf{1}})_{\alpha\gamma}^{\beta\delta} - \frac{1}{4}\frac{(c_{LL}^{\mathbf{1}s})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2}(\mathcal{O}_{LL}^{\mathbf{3}})_{\alpha\gamma}^{\beta\delta}. \quad (37)
\end{aligned}
$$

Conversely, the decomposition of the operator $\mathcal{O}_{LL}^{\mathbf{1}}$ of the BW basis can be read off from the column labeled $\mathcal{C}_{LL}^{\mathbf{1}}$, in terms of the relative weights of the mediator operators:

$$
(\mathcal{C}_{LL}^{\mathbf{1}})_{\beta\delta}^{\alpha\gamma} = \frac{1}{4}(c_{LL}^{\mathbf{1}s})_{\beta\delta}^{\alpha\gamma} - \frac{3}{4}(c^{\mathbf{3}s})_{\beta\delta}^{\alpha\gamma} + (f_{LL}^{\mathbf{1}v})_{\beta\delta}^{\alpha\gamma}. \quad (38)
$$

Note that the flavor indices in the first column of Table II are arranged such that the flavor indices of all coefficients and of the BW operators are the same as in Eqs. (37) and (38). Therefore, we show the flavor indices explicitly only in the first column.

In order to have large NSI without four charged lepton interactions, the $d = 6$ cancellation conditions Eq. (33) must now be implemented. One can directly read off now from Table II that these conditions can be rewritten as

$$
2c^{\mathbf{2}v} - 2f_{LE}^{\mathbf{1}v} + f^{\mathbf{2}s} = 0 \qquad (\text{from } \mathcal{C}_{LE} = 0), \quad (39)
$$

$$
-c^{\mathbf{3}s} + f_{LL}^{\mathbf{1}v} + f^{\mathbf{3}v} = 0 \qquad (\text{from } \mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}} = 0), \quad (40)
$$

$$
c_{EE}^{\mathbf{1}s} + 2f_{EE}^{\mathbf{1}v} = 0 \qquad (\text{from } \mathcal{C}_{EE} = 0), \quad (41)
$$

in the mediator picture. The operators contributing to the first equation will not produce any NSI [since $\mathcal{C}_{LE} = 0$ in Eq. (23)], while the operators present in the second equation lead to NSI if $\mathcal{C}_{LL}^{\mathbf{1}} = -\mathcal{C}_{LL}^{\mathbf{3}} \neq 0$ [cf., Eq. (24)].

One reason to use Table II is to discuss departures from the SM couplings. For example, for a hypothetical experimental departure pointing towards a four-lepton coupling such as that in operator $\mathcal{O}_{LL}^{\mathbf{3}}$ in Eq. (8), Table II indicates directly that a new heavy scalar triplet could induce it at tree level, while a scalar doublet would not.

TABLE II.   Possible $d = 6$ operators obtained by combining the lepton bilinears in Table I. The coefficients are labeled $c^{\mathbf{X}\mathcal{L}}$ ($\Delta L = 2$) or $f^{\mathbf{X}\mathcal{L}}$ ($\Delta L = 0$), where the **X** and $\mathcal{L}$ are defined as in Table I and the subscripts refer to the combination of bilinears, in an obvious notation. The last four columns contain their contribution to the $d = 6$ operator coefficients in the BW basis in Eqs. (6)–(9). The flavor structure for any coefficient in the table is understood to be $()_{\beta\delta}^{\alpha\gamma}$; see main text for further explanations.

$d = 6$ operators	Mediator	$\mathcal{C}_{LE}$	$\mathcal{C}_{LL}^{\mathbf{1}}$	$\mathcal{C}_{LL}^{\mathbf{3}}$	$\mathcal{C}_{EE}$
$\bar{L}E\bar{E}L$					
$(c^{\mathbf{2}v}/\Lambda^2)((\bar{E}^c)_\gamma \gamma^\rho L_\alpha)(\bar{L}^\beta \gamma_\rho (E^c)^\delta)$	$\mathbf{2}_{-3/2}^v$	$2c^{\mathbf{2}v}$			
$(f_{LE}^{\mathbf{1}v}/\Lambda^2)(\bar{L}^\beta \gamma^\rho L_\alpha)(\bar{E}^\delta \gamma_\rho E_\gamma)$	$\mathbf{1}_0^v$	$-2f_{LE}^{\mathbf{1}v}$			
$(f^{\mathbf{2}s}/\Lambda^2)(\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha)$	$\mathbf{2}_{1/2}^s$	$f^{\mathbf{2}s}$			
$\bar{L}L\bar{L}L$					
$(c_{LL}^{\mathbf{1}s}/\Lambda^2)((\bar{L}^c)_\alpha i\tau^2 L_\gamma)(\bar{L}^\beta i\tau^2 (L^c)^\delta)$	$\mathbf{1}_{-1}^s$		$\frac{1}{4}c^{\mathbf{1}s}$	$-\frac{1}{4}c^{\mathbf{1}s}$	
$(c^{\mathbf{3}s}/\Lambda^2)((\bar{L}^c)_\alpha i\tau^2 \vec{\tau} L_\gamma)(\bar{L}^\beta \vec{\tau} i\tau^2 (L^c)^\delta)$	$\mathbf{3}_{-1}^s$		$-\frac{3}{4}c^{\mathbf{3}s}$	$-\frac{1}{4}c^{\mathbf{3}s}$	
$(f_{LL}^{\mathbf{1}v}/\Lambda^2)(\bar{L}^\beta \gamma^\rho L_\alpha)(\bar{L}^\delta \gamma_\rho L_\gamma)$	$\mathbf{1}_0^v$		$f_{LL}^{\mathbf{1}v}$		
$(f^{\mathbf{3}v}/\Lambda^2)(\bar{L}^\beta \gamma^\rho \vec{\tau} L_\alpha)(\bar{L}^\delta \gamma_\rho \vec{\tau} L_\gamma)$	$\mathbf{3}_0^v$			$f^{\mathbf{3}v}$	
$\bar{E}E\bar{E}E$					
$(c_{EE}^{\mathbf{1}s}/\Lambda^2)((\bar{E}^c)_\alpha E_\gamma)(\bar{E}^\beta (E^c)^\delta)$	$\mathbf{1}_{-2}^s$				$\frac{1}{2}c_{EE}^{\mathbf{1}s}$
$(f_{EE}^{\mathbf{1}v}/\Lambda^2)(\bar{E}^\beta \gamma^\rho E_\alpha)(\bar{E}^\delta \gamma_\rho E_\gamma)$	$\mathbf{1}_0^v$				$f_{EE}^{\mathbf{1}v}$

From the model-building perspective, it is illustrative to consider again the case of the operator mediated by $\mathbf{1}^s_{-1}$ leading to Eq. (37). The table shows that it is the only $d = 6$ possibility using only one mediator which directly satisfies the cancellation condition of pure charged lepton interactions, Eq. (33) [or their tree-level equivalent, Eqs. (39)–(41)]. This antisymmetric combination of the basis elements was first found in Ref. [26].

This example serves to illuminate the power of the mediator analysis (see Ref. [26] and also Ref. [23]). The $\mathbf{1}^s_{-1}$ exchange leading to the originally proposed operator is depicted in Fig. 1, left diagram. Once a certain mediator is assumed for a certain operator, contributions to other operators are simultaneously induced. This is illustrated in the center and right diagrams of Fig. 1, i.e.,

$$\frac{|(c^{\mathbf{1}s}_{LL})^{e\mu}_{e\tau}|^2}{\Lambda^4} = \frac{|\lambda_{e\mu}|^2|\lambda_{e\tau}|^2}{M^4_{\mathbf{1}s}} = \frac{|(c^{\mathbf{1}s}_{LL})^{e\mu}_{e\mu}||(c^{\mathbf{1}s}_{LL})^{e\tau}_{e\tau}|}{\Lambda^4}, \quad (42)$$

where $\lambda_{\alpha\beta}$ is the coupling for the lepton-bilepton interaction and $M_{\mathbf{1}s}$ is the mass of the bilepton. A coherent contribution to G_F and a violation of the lepton universality are then induced by the middle and right diagrams of Fig. 1. From the strict experimental bounds on these quantities, $\epsilon^m_{\mu\tau}$ has been constrained to $|\epsilon^m_{\mu\tau}| \lesssim 1.9 \times 10^{-3}$ (90% CL), using this particular mediator [23]. The bound from a neutrino factory on $|\epsilon^m_{\mu\tau}|$ would be 1.8×10^{-2} for complex $\epsilon^m_{\mu\tau}$ [65]. If it was assumed to be real, which does not describe the most general class of models, the bound would be 3.7×10^{-4} (90% CL) [65]. However, since this is a model-dependent assumption, we do not use this bound.

The antisymmetric operator discussed in the previous paragraphs is not, however, the only possibility to build a model satisfying Eqs. (39)–(41). For example, one may choose bosonic triplets $\mathbf{3}^s_{-1}$ and $\mathbf{3}^v_0$, for which the coefficients can be chosen independently, in order to satisfy Eq. (40) without suppressing completely the $d = 6$ NSI operator coefficient. In particular, if the simplest possibility is experimentally constrained, one may consider models with more than one mediator.

At this point, we would like to clarify that cancellations or fine-tuning of operator coefficients cannot be arguments by themselves for judging the naturalness and complexity
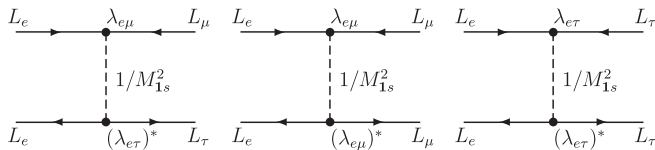
of a model. These depend on the field content and the symmetries of the model. Consider for instance, once again, the antisymmetric operator on the left-hand side of Eq. (37), induced at tree level by the exchange of just one mediator, $\mathbf{1}^s_{-1}$, illustrated in Fig. 1, left diagram. That equation shows that, in the BW basis, the antisymmetric operator appears to be constructed from the combination of two BW operators with specific (fine-tuned?) coefficients. In the effective operator picture, "fine-tuning" is thus a basis- and model-dependent qualification. We therefore define the simplest model to be the one with the fewest mediators. In the $d = 6$ case, it is the antisymmetric operator in Eq. (37) with only one mediator. In the case that the NSI come only through $d = 8$ (or higher dimension) effective operators, we will demonstrate that the simplest viable models require at least two mediators. Once the field content is chosen, any relative precise adjustment of the couplings of the mediators can be considered as fine-tuning, unless the symmetries of the model enforce this adjustment. It will be left to the model builder to eventually explore possible symmetries, whenever such cancellations turn out to be required below for phenomenologically viable NSI.

In general, it is easy to show that all NSI from $d = 6$ operators are strongly constrained when the possible mediators are taken into account. There is, however, one exception. The present experimental constraints allow the condition $\mathcal{C}_{LE} = 0$ in Eq. (33), which cancels interactions among four charged leptons, to be substantially violated for certain combinations of flavor indices. In particular, the coefficient of the flavor conserving (BW) operator $(\bar{L}^\tau E_e)(\bar{E}^e L_\tau)$ is not very strongly constrained [28,74]. The mediators $\mathbf{2}^v_{-3/2}$ or $\mathbf{2}^s_{1/2}$ (cf., Table II) can generate such an operator, leading to the following effective interactions [cf., Eq. (23)]:

$$\delta\mathcal{L}^{d=6}_{\text{eff}} = -\frac{(\mathcal{C}_{LE})^{\tau e}_{\tau e}}{2\Lambda^2}((\bar{\nu}^\tau\gamma^\rho P_L \nu_\tau)(\bar{e}\gamma_\rho P_R e)$$
$$- (\bar{\tau}\gamma^\rho P_L \tau)(\bar{e}\gamma_\rho P_R e)) + \text{H.c.} \quad (43)$$

The coefficient is constrained by [see Eq. (14) in Ref. [28]]

$$|\epsilon^m_{\tau\tau}| = \frac{v^2}{4\Lambda^2}|(\mathcal{C}_{LE})^{\tau e}_{\tau e}| = |\kappa_{\tau R}| \lesssim 0.1. \quad (44)$$

If the possibility of large $SU(2)_L$ breaking effects was considered in addition, a possible gain of almost an order of magnitude could be allowed for the NSI $\epsilon^m_{\tau\tau}$ strength [26]. In conclusion, large (order unity) values for $\epsilon^m_{\tau\tau}$ resulting from $d = 6$ effective interactions are not excluded.

Table II also shows that the relationship between mediator and coefficient is unique at the $d = 6$ level, except for $\mathbf{1}^v_0$. If a model uses this mediator, then there will be three different $d = 6$ operator contributions, which are independent in the BW basis. In particular, one cannot

FIG. 1. Diagrams mediated by a bilepton $\mathbf{1}^s_{-1}$. The effective dimension six operator results from the first diagram at energies below the mediator mass $M_{\mathbf{1s}}$. However, the fundamental interaction will also create the middle and right diagrams, and the couplings will be related.

neglect $\mathcal{O}_{EE}$, which can induce physics effects while not resulting in NSI.

IV. MODEL ANALYSIS OF $d = 8$ OPERATORS

We consider all possible $d = 8$ operators which can induce purely leptonic NSI, analyzing them from the point of view of their possible tree-level mediators. We will focus on the systematic analysis of all possible products of SM bilinears, which may result from exchanging mediators which only couple to pairs of SM fields. Such an analysis was performed for $d = 6$ operators in Refs. [25,26], and we extend it here to the $d = 8$ case. Other scenarios leading to some of the $d = 8$ operators will be briefly analyzed afterwards.

A convenient basis of linearly independent $d = 8$ operators has been given in Eqs. (13)–(20), i.e., the BR basis. In order to suppress four charged lepton interactions, both the cancellation conditions for $d = 8$ operators in Eq. (34) and the cancellation conditions for $d = 6$ operators in Eq. (33) are now required to be satisfied. Under these conditions, if any $d = 6$ operator remains, it is expected to dominate the new physics, and as discussed in the previous section, only effects related to $\epsilon_{\tau\tau}^m$ are then allowed to be experimentally sizable. In this section, we instead focus on NSI which stem exclusively from $d = 8$ (and higher) operators and their implications for model building. In particular, we are interested in the $\mathcal{O}_{\mathrm{NSI}}$ without four charged lepton interactions, i.e., satisfying Eq. (34), which has been the object of intense speculations in the literature.

When the mediators couple only to SM bilinears, we have the following options with respect to the undesired $d = 6$ operators:

(1) The required mediators do not induce any $d = 6$ operator involving four leptons (in other words, the mediators differ from those in Table II).

(2) The $d = 6$ couplings induced by different mediators turn out to explicitly cancel among themselves.

As we will illustrate later, there is no simple possibility for which the first option works. For the second option to happen, the coefficients for the BW operators in Eqs. (6)–(9) have to vanish independently, because they constitute a basis:

$$\mathcal{C}_{LE} = 0, \qquad \mathcal{C}_{LL}^{\mathbf{1}} = 0, \qquad \mathcal{C}_{LL}^{\mathbf{3}} = 0, \qquad \mathcal{C}_{EE} = 0. \tag{45}$$

Their implementation in the mediator picture can be read off from the columns in Table II. They are given by Eqs. (39) and (41), together with

$$\frac{1}{4} c_{LL}^{\mathbf{1}s} - \frac{3}{4} c^{\mathbf{3}s} + f_{LL}^{\mathbf{1}v} = 0 \qquad \text{(from } \mathcal{C}_{LL}^{\mathbf{1}} = 0), \tag{46}$$

$$-\frac{1}{4} c_{LL}^{\mathbf{1}s} - \frac{1}{4} c^{\mathbf{3}s} + f^{\mathbf{3}v} = 0 \qquad \text{(from } \mathcal{C}_{LL}^{\mathbf{3}} = 0), \tag{47}$$

which replace Eq. (40) of that set. For example, if a model

introduces two bosonic doublets $\mathbf{2}_{-3/2}^v$ and $\mathbf{2}_{1/2}^s$, one can satisfy Eq. (39) [to which Eq. (45) simplifies in this case] by achieving $2c^{\mathbf{2}v} + f^{\mathbf{2}s} = 0$.

Note that the introduction of exotic fermions potentially leads to the additional $d = 6$ operators in Eqs. (10)–(12), which are made out of two lepton fields and two Higgs doublets. In accordance with the main line of this section, we do not consider constraints from those operators, which means that, unless explicitly stated otherwise, when mentioning $d = 6$ operators in this section we refer exclusively to those in Eqs. (6)–(9).

A. A toy model

In order to estimate the theoretical price to pay for obtaining large NSI from exotic particles coupling to SM bilinears, without large charged lepton flavor violation, we show here a toy model in a bottom-up fashion, which precisely generates the $d = 8$ operator $\mathcal{O}_{\mathrm{NSI}}$ in Eq. (4) and no $d = 6$ operator. Then we will provide a systematic analysis, from which we will recover the toy model as the simplest possibility in a top-down approach.

Consider the following toy Lagrangian for the underlying theory, which adds both a new scalar doublet ($\mathbf{2}_{1/2}^s$) Φ and a vector doublet ($\mathbf{2}_{-3/2}^v$) V_μ to the SM Lagrangian, with general couplings to the SM fields y, g, and λ's,

$$\begin{aligned} \mathcal{L} = \mathcal{L}_{\mathrm{SM}} &- (y)_\beta^\gamma (\bar{L}^\beta)^i E_\gamma \Phi_i - (g)_{\beta\delta}(\bar{L}^\beta)^i \gamma^\rho (E^c)^\delta (V_\rho)_i \\ &+ \lambda_{\mathbf{1}s}(H^\dagger H)(\Phi^\dagger \Phi) + \lambda_{\mathbf{3}s}(H^\dagger \vec{\tau} H)(\Phi^\dagger \vec{\tau}\Phi) \\ &+ \lambda_{\mathbf{1}v}(H^\dagger H)(V_\rho^\dagger V^\rho) + \lambda_{\mathbf{3}v}(H^\dagger \vec{\tau} H)(V_\rho^\dagger \vec{\tau} V^\rho) \\ &+ \mathrm{H.c.} + \ldots, \end{aligned} \tag{48}$$

where the dots refer to other bosonic interactions not relevant for this work. After integrating out the intermediate particles, the following $d = 6$ effective interactions involving leptons are induced (see Table II):

$$\begin{aligned} \delta \mathcal{L}_{\mathrm{eff}}^{d=6} = &\frac{(c^{\mathbf{2}v})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2} (\bar{E}_\gamma^c \gamma^\rho L_\alpha)(\bar{L}^\beta \gamma_\rho E^{c\delta}) \\ &+ \frac{(f^{\mathbf{2}s})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2} (\bar{L}^\beta E_\gamma)(\bar{E}^\delta L_\alpha), \end{aligned} \tag{49}$$

where now

$$\frac{(c^{\mathbf{2}v})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2} = -\frac{(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta}}{M_V^2}, \qquad \frac{(f^{\mathbf{2}s})_{\beta\delta}^{\alpha\gamma}}{\Lambda^2} = \frac{(y^\dagger)_\delta{}^\alpha (y)_\beta{}^\gamma}{M_\Phi^2}. \tag{50}$$

For simplicity of notation and illustrative purposes, we can assume $M_\Phi \simeq M_V \equiv M(= \Lambda)$. The $d = 6$ cancellation conditions on four charged lepton transitions in Eq. (33), or its equivalent in the mediator picture, Eq. (39), translate into

$$- 2(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + (y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma = 0. \qquad (51)$$

The relevant effective $d = 8$ Lagrangian induced reads

$$
\begin{aligned}
\delta\mathcal{L}_{\text{eff}}^{d=8} = \frac{1}{M^4}[&\lambda_{\mathbf{1}s}(\bar{L}yE)(\bar{E}y^\dagger L)(H^\dagger H)\\
&+ \lambda_{\mathbf{3}s}(\bar{L}yE)\vec{\tau}(\bar{E}y^\dagger L)(H^\dagger\vec{\tau}H)\\
&+ \lambda_{\mathbf{1}v}(\bar{L}g\gamma^\rho E^c)(\bar{E}^c\gamma_\rho g^\dagger L)(H^\dagger H)\\
&+ \lambda_{\mathbf{3}v}(\bar{L}g\gamma^\rho E^c)\vec{\tau}(\bar{E}^c\gamma_\rho g^\dagger L)(H^\dagger\vec{\tau}H)], \quad (52)
\end{aligned}
$$

where flavor indices have been omitted and each expression in brackets is to be understood as a flavor singlet. Equation (52) can be rewritten in terms of the operators of the BR basis in Eqs. (13) and (14), as

$$\delta\mathcal{L}_{\text{eff}}^{d=8} = -\frac{1}{\Lambda^4}(\mathcal{C}_{LEH}^{\mathbf{1}}\mathcal{O}_{LEH}^{\mathbf{1}} + \mathcal{C}_{LEH}^{\mathbf{3}}\mathcal{O}_{LEH}^{\mathbf{3}}), \qquad (53)$$

where

$$\mathcal{C}_{LEH}^{\mathbf{1}} = \lambda_{\mathbf{1}v}(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + \tfrac{1}{2}\lambda_{\mathbf{1}s}(y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma, \qquad (54)$$

$$\mathcal{C}_{LEH}^{\mathbf{3}} = \lambda_{\mathbf{3}v}(g^\dagger)^{\gamma\alpha}(g)_{\beta\delta} + \tfrac{1}{2}\lambda_{\mathbf{3}s}(y^\dagger)_\delta{}^\alpha(y)_\beta{}^\gamma. \qquad (55)$$

In order not to produce interactions between four charged leptons, it is necessary to satisfy Eq. (34), i.e., the condition $\mathcal{C}_{LEH}^{\mathbf{1}} = \mathcal{C}_{LEH}^{\mathbf{3}} \neq 0$, so that the effective $d = 8$ interaction in Eq. (53) reduces precisely to $\mathcal{O}_{\text{NSI}}$ in Eq. (22). Together with the $d = 6$ cancellation condition, Eq. (51), it is finally required that

$$\lambda_{\mathbf{1}s} + \lambda_{\mathbf{1}v} = \lambda_{\mathbf{3}s} + \lambda_{\mathbf{3}v} \neq 0. \qquad (56)$$

As a consequence, the NSI in matter can be substantial for all flavors. While source and detection NSI cannot be created from our toy model, the epsilon matter parameter reads

$$|\epsilon_{\beta\alpha}^{m,R}| = \frac{v^4}{2M^4}|(\lambda_{\mathbf{1}s} + \lambda_{\mathbf{1}v})(g^\dagger)^{e\alpha}(g)_{\beta e}|. \qquad (57)$$

In summary, by adding both an $SU(2)$ doublet scalar and a doublet vector to the SM content, and imposing two relations to their couplings, Eqs. (51) and (56), a toy model for viable large NSI has resulted. The model interactions are visualized in Fig. 2, where the first two effective interactions in Eq. (52) correspond to the diagram on the left—mediated by $\mathbf{2}_{1/2}^s$—and the last two interactions to the diagram on the right—mediated by $\mathbf{2}_{-3/2}^v$. In fact, other combinations of just one of the first two operators in Eq. (52) together with one of the last two operators in that equation would have been enough for this purpose.[7] As we will demonstrate below, our toy model is the most general possible model involving only two mediators, when the exotic particles couple only to SM bilinears.

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[7]For instance, a combination involving $\lambda_{\mathbf{1}s}$ and $\lambda_{\mathbf{3}v}$, or alternatively $\lambda_{\mathbf{3}s}$ and $\lambda_{\mathbf{1}v}$, would be suitable.
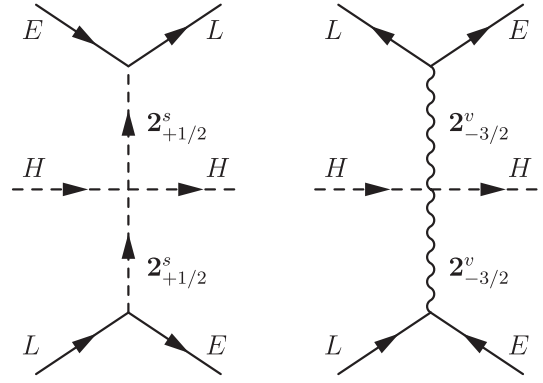

FIG. 2.   Dimension eight operator decomposed into dimension four interactions.

We keep dubbing the construction above as "toy" because, to begin with, the presence of a vector field, which is not a gauge boson, implies that it is nonrenormalizable. The toy Lagrangian, Eq. (48), can thus only be considered as an effective theory of some larger construction, such as models of extra dimensions in which the vector doublet could be a component of a higher dimensional gauge theory.

Moreover, its phenomenological analysis is beyond the scope of the present work: the constraints from electroweak precision tests need to be analyzed for each specific model, in particular, the *oblique* corrections [88–90] it may induce. The new couplings may also have a relevant impact on other flavor-changing transitions at the loop level, although considering large values for the quartic couplings λ and small values for the elements of the g and y flavor matrices, it will probably remain phenomenologically safe.

The toy model demonstrates that it is possible to achieve the desired $d = 8$ interactions, without simultaneously inducing $d = 6$ ones, by fixing the coefficients of the new fields in the Lagrangian. It requires *ad hoc* cancellations though, and it is left as an open question for the model builder whether some symmetry can justify them.

B. Systematic analysis

In this subsection, a systematic analysis of all possible effective NSI $d = 8$ operators is performed. The full decomposition of any combination of $d = 8$ operators, constructed from combining bilinear combinations of SM fields, leads to a large number of possibilities. We will first consider the cases which are conceptually similar to the toy model above, i.e., new fundamental interactions involving exactly *two* SM fields, which are the SM bilinears according to our earlier definition. Then we will discuss new interactions involving only one SM field.

1. Mediators coupling to SM bilinears

We summarize these possibilities for the $\bar{L}L\bar{E}E$-type operators in Table III and for the $\bar{L}L\bar{L}L$-type operators in

TABLE III. Complete list of $\bar{L}L\bar{E}E$-type $d=8$ interactions which involve two SM fields at any possible vertex of interaction (field bilinears within brackets). The columns show an ordinal for each operator, the $d=8$ interaction, the corresponding combination of interactions in the BR basis, whether $\mathcal{O}_{NSI}$ is satisfied, and the necessary mediators, respectively. Those mediators also leading to $d=6$ operators in Table II are in boldface. The superscript L/R indicates massive vector fermions. The flavor structure is to be understood as $\bar{L}^\beta L_\alpha \bar{E}^\delta E_\gamma$.

#	Dimension eight operator	$\mathcal{C}^1_{LEH}$	$\mathcal{C}^3_{LEH}$	$\mathcal{O}_{NSI}$?	Mediators
Combination $\bar{L}L$					
1	$(\bar{L}\gamma^\rho L)(\bar{E}\gamma_\rho E)(H^\dagger H)$	1			$\mathbf{1}_0^v$
2	$(\bar{L}\gamma^\rho L)(\bar{E}H^\dagger)(\gamma_\rho)(HE)$	1			$\mathbf{1}_0^v + 2^{L/R}_{-3/2}$
3	$(\bar{L}\gamma^\rho L)(\bar{E}H^T)(\gamma_\rho)(H^*E)$	1			$\mathbf{1}_0^v + 2^{\bar{L}/R}_{-1/2}$
4	$(\bar{L}\gamma^\rho \vec{\tau}L)(\bar{E}\gamma_\rho E)(H^\dagger \vec{\tau}H)$		1		$3_0^v + \mathbf{1}_0^v$
5	$(\bar{L}\gamma^\rho \vec{\tau}L)(\bar{E}H^\dagger)(\gamma_\rho \vec{\tau})(HE)$		1		$3_0^v + 2^{L/R}_{-3/2}$
6	$(\bar{L}\gamma^\rho \vec{\tau}L)(\bar{E}H^T)(\gamma_\rho \vec{\tau})(H^*E)$		1		$3_0^v + 2^{\bar{L}/R}_{-1/2}$
Combination $\bar{E}L$					
7	$(\bar{L}E)(\bar{E}L)(H^\dagger H)$	$-1/2$			$2^s_{+1/2}$
8	$(\bar{L}E)(\vec{\tau})(\bar{E}L)(H^\dagger \vec{\tau}H)$		$-1/2$		$2^s_{+1/2}$
9	$(\bar{L}H)(H^\dagger E)(\bar{E}L)$	$-1/4$	$-1/4$	√	$2^s_{+1/2} + 1^R_0 + 2^{L/R}_{-1/2}$
10	$(\bar{L}\vec{\tau}H)(H^\dagger E)(\vec{\tau})(\bar{E}L)$	$-3/4$	$1/4$		$2^s_{+1/2} + 3^{L/R}_0 + 2^{L/R}_{-1/2}$
11	$(\bar{L}i\tau^2 H^*)(H^T E)(i\tau^2)(\bar{E}L)$	$1/4$	$-1/4$		$2^s_{+1/2} + 1^{L/R}_{-1} + 2^{\bar{L}/R}_{-3/2}$
12	$(\bar{L}\vec{\tau}i\tau^2 H^*)(H^T E)(i\tau^2\vec{\tau})(\bar{E}L)$	$3/4$	$1/4$		$2^s_{+1/2} + 3^{\bar{L}/R}_{-1} + 2^{\bar{L}/R}_{-3/2}$
Combination $\bar{E}^c L$					
13	$(\bar{L}\gamma^\rho E^c)(\bar{E}^c \gamma_\rho L)(H^\dagger H)$	-1			$2^v_{-3/2}$
14	$(\bar{L}\gamma^\rho E^c)(\vec{\tau})(\bar{E}^c \gamma_\rho L)(H^\dagger \vec{\tau}H)$		-1		$2^v_{-3/2}$
15	$(\bar{L}H)(\gamma^\rho)(H^\dagger E^c)(\bar{E}^c \gamma_\rho L)$	$-1/2$	$-1/2$	√	$2^v_{-3/2} + 1^R_0 + 2^{L/R}_{+3/2}$
16	$(\bar{L}\vec{\tau}H)(\gamma^\rho)(H^\dagger E^c)(\vec{\tau})(\bar{E}^c \gamma_\rho L)$	$-3/2$	$1/2$		$2^v_{-3/2} + 3^{L/R}_0 + 2^{L/R}_{+3/2}$
17	$(\bar{L}i\tau^2 H^*)(\gamma^\rho)(H^T E^c)(i\tau^2)(\bar{E}^c \gamma_\rho L)$	$-1/2$	$1/2$		$2^v_{-3/2} + 1^{L/R}_{-1} + 2^{\bar{L}/R}_{+1/2}$
18	$(\bar{L}\vec{\tau}i\tau^2 H^*)(\gamma^\rho)(H^T E^c)(i\tau^2\vec{\tau})(\bar{E}^c \gamma_\rho L)$	$-3/2$	$-1/2$		$2^v_{-3/2} + 3^{L/R}_{-1} + 2^{\bar{L}/R}_{+1/2}$
Combination $H^\dagger L$					
19	$(\bar{L}E)(\bar{E}H)(H^\dagger L)$	$-1/4$	$-1/4$	√	$2^s_{+1/2} + 1^R_0 + 2^{L/R}_{-1/2}$
20	$(\bar{L}E)(\vec{\tau})(\bar{E}H)(H^\dagger \vec{\tau}L)$	$-3/4$	$1/4$		$2^s_{+1/2} + 3^{L/R}_0 + 2^{L/R}_{-1/2}$
21	$(\bar{L}H)(\gamma^\rho)(H^\dagger L)(\bar{E}\gamma_\rho E)$	$1/2$	$1/2$	√	$1^v_0 + 1^R_0$
22	$(\bar{L}\vec{\tau}H)(\gamma^\rho)(H^\dagger \vec{\tau}L)(\bar{E}\gamma_\rho E)$	$3/2$	$-1/2$		$1^v_0 + 3^{L/R}_0$
23	$(\bar{L}\gamma^\rho E^c)(\bar{E}^c H)(\gamma^\rho)(H^\dagger L)$	$-1/2$	$-1/2$	√	$2^v_{-3/2} + 1^R_0 + 2^{L/R}_{+3/2}$
24	$(\bar{L}\gamma^\rho E^c)(\bar{E}^c H)(\gamma^\rho)(H^\dagger L)$	$-3/2$	$1/2$		$2^v_{-3/2} + 3^{L/R}_0 + 2^{L/R}_{+3/2}$
Combination HL					
25	$(\bar{L}E)(i\tau^2)(\bar{E}H^*)(H^T i\tau^2 L)$	$1/4$	$-1/4$		$2^s_{+1/2} + 1^{L/R}_{-1} + 2^{L/R}_{-3/2}$
26	$(\bar{L}E)(\vec{\tau}i\tau^2)(\bar{E}H^*)(H^T i\tau^2\vec{\tau}L)$	$3/4$	$1/4$		$2^s_{+1/2} + 3^{\bar{L}/R}_{-1} + 2^{\bar{L}/R}_{-3/2}$
27	$(\bar{L}i\tau^2 H^*)(\gamma^\rho)(H^T i\tau^2 L)(\bar{E}\gamma_\rho E)$	$-1/2$	$1/2$		$1^v_0 + 1^{L/R}_{-1}$
28	$(\bar{L}\vec{\tau}i\tau^2 H^*)(\gamma^\rho)(H^T i\tau^2\vec{\tau}L)(\bar{E}\gamma_\rho E)$	$-3/2$	$-1/2$		$1^v_0 + 3^{L/R}_{-1}$
29	$(\bar{L}\gamma^\rho E^c)(i\tau^2)(\bar{E}^c H^*)(\gamma_\rho)(H^T i\tau^2 L)$	$1/2$	$-1/2$		$2^v_{-3/2} + 1^{L/R}_{-1} + 2^{L/R}_{+1/2}$
30	$(\bar{L}\gamma^\rho E^c)(\vec{\tau}i\tau^2)(\bar{E}^c H^*)(\gamma_\rho)(H^T i\tau^2\vec{\tau}L)$	$3/2$	$1/2$		$2^v_{-3/2} + 3^{\bar{L}/R}_{-1} + 2^{\bar{L}/R}_{+1/2}$

Table IV, which are some of the main results of this study. The notation used has been described in Sec. III. The tables show, from left to right in each row,

(i) an ordinal assigned to each operator,
(ii) the operator itself,
(iii) the value of the operator coefficients of the BR basis needed to reconstruct it,
(iv) whether the $d=8$ cancellation conditions in Eq. (34) are directly fulfilled ("$\mathcal{O}_{NSI}$?"),
(v) and the required mediators, with those inducing additional $d=6$ interactions of four charged leptons (Table II) highlighted in boldface.

Obviously, the number of possible mediators of $d=8$ interactions is much larger than for the $d=6$ case in Table II. In particular, fermions are now possible mediators, unlike for $d=6$. We illustrate the operator decomposition for operator #2 from Table III, showing the corresponding Feynman diagram in Fig. 3.

Notice that only the minimal mediator content necessary to obtain each possible $d=8$ operator is shown in Tables III and IV. In other words, although there is always a particular set of exotic particles whose exchange induces at tree level the $d=8$ operators considered, this set might not be unique. Nevertheless, for each operator, the particle

TABLE IV. Same as Table III, but for the $\bar{L}L\bar{L}L$-type operators. Note that in this case the relationship between the flavor structure and the symbol is not unique. We show the flavor structure for each group separately.

| #	Dimension eight operator	$\mathcal{C}_{LLH}^{111}$	$\mathcal{C}_{LLH}^{331}$	$\mathcal{C}_{LLH}^{133}$	$\mathcal{C}_{LLH}^{313}$	$\mathcal{C}_{LLH}^{333}$	$\mathcal{O}_{NSI}$?	Mediators
Combination $(\bar{L}^\beta L_\alpha)(\bar{L}^\delta L_\gamma)(H^\dagger H)$								
31	$(\bar{L}\gamma^\rho L)(\bar{L}\gamma_\rho L)(H^\dagger H)$	1						1_0^v
32	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}\gamma_\rho\vec{\tau}L)(H^\dagger H)$		1					3_0^v
33	$(\bar{L}\gamma^\rho L)(\bar{L}\gamma_\rho\vec{\tau}L)(H^\dagger\vec{\tau}H)$			1				$1_0^v + 3_0^v$
34	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}\gamma_\rho L)(H^\dagger\vec{\tau}H)$				1			$1_0^v + 3_0^v$
35	$(-i\epsilon^{abc})(\bar{L}\gamma^\rho\tau^a L)\times$ $(\bar{L}\gamma_\rho\tau^b L)(H^\dagger\tau^c H)$					1	√	3_0^v
Combination $(\bar{L}^\beta L_\alpha)(\bar{L}^\delta H)(H^\dagger L_\gamma)$								
36	$(\bar{L}\gamma^\rho L)(\bar{L}H)(\gamma_\rho)(H^\dagger L)$	1/2		1/2			√	$1_0^v + 1_0^R$
37	$(\bar{L}\gamma^\rho L)(\bar{L}\vec{\tau}H)(\gamma_\rho)(H^\dagger\vec{\tau}L)$	3/2		-1/2				$1_0^v + 3_0^{L/R}$
38	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}\vec{\tau}H)(\gamma_\rho)(H^\dagger L)$		1/2		1/2	1/2	√	$1_0^v + 1_0^R + 3_0^{L/R}$
39	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}H)(\gamma_\rho)(H^\dagger\vec{\tau}L)$		1/2		1/2	-1/2	√	$1_0^v + 1_0^R + 3_0^{L/R}$
40	$(-i\epsilon^{abc})(\bar{L}\gamma^\rho\tau^a)\times(\bar{L}\tau^b H)(\gamma_\rho)(H^\dagger\tau^c L)$		1		-1			$3_0^v + 1_0^R + 3_0^{L/R}$
Combination $(\bar{L}^\beta L_\alpha)(\bar{L}^\delta H^\dagger)(L_\gamma H)$								
41	$(\bar{L}\gamma^\rho L)(\bar{L}i\tau^2 H^*)(\gamma_\rho)(H^T i\tau^2 L)$	-1/2		1/2				$1_0^v + 1_{-1}^{L/R}$
42	$(\bar{L}\gamma^\rho L)(\bar{L}\vec{\tau}i\tau^2 H^*)(\gamma_\rho)(H^T i\tau^2\vec{\tau}L)$	-3/2		-1/2				$1_0^v + 3_{-1}^{L/R}$
43	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}\vec{\tau}i\tau^2 H^*)(\gamma_\rho)(H^T i\tau^2 L)$			-1/2	1/2	1/2		$3_0^v + 1_{-1}^{L/R} + 3_{-1}^{L/R}$
44	$(\bar{L}\gamma^\rho\vec{\tau}L)(\bar{L}i\tau^2 H^*)(\gamma_\rho)(H^T i\tau^2\vec{\tau}L)$			-1/2	1/2	-1/2		$3_0^v + 1_{-1}^{\bar{L}/R} + 3_{-1}^{\bar{L}/R}$
45	$(-i\epsilon^{abc})(\bar{L}\gamma^\rho\tau^a L)\times(\bar{L}\tau^b i\tau^2 H^*)(\gamma_\rho)(H^T i\tau^2\tau^c L)$			-1	-1		√	$3_0^v + 3_{-1}^{L/R}$
Combination $(\bar{L}^\beta (L^c)^\delta)((\bar{L}^c)_\alpha L_\gamma)(H^\dagger H)$								
46	$(\bar{L}i\tau^2 L^c)(\bar{L}^c i\tau^2 L)(H^\dagger H)$	1/4	-1/4				√	1_{-1}^s
47	$(\bar{L}\vec{\tau}i\tau^2 L^c)(\bar{L}^c i\tau^2\vec{\tau}L)(H^\dagger H)$	-3/4	-1/4					3_{-1}^s
48	$(\bar{L}i\tau^2 L^c)(\bar{L}^c i\tau^2\vec{\tau}L)(H^\dagger\vec{\tau}H)$			1/4	-1/4	-1/4	√	$1_{-1}^s + 3_{-1}^s$
49	$(\bar{L}\vec{\tau}i\tau^2 L^c)(\bar{L}^c i\tau^2 L)(H^\dagger\vec{\tau}H)$			-1/4	1/4	-1/4	√	$1_{-1}^s + 3_{-1}^s$
50	$(-i\epsilon^{abc})(\bar{L}\tau^a i\tau^2 L^c)\times(\bar{L}^c i\tau^2\tau^b L)(H^\dagger\tau^c H)$			-1/2	-1/2			3_{-1}^s
Combination $(\bar{L}^\beta H^\dagger)((L^c)^\delta)((\bar{L}^c)_\alpha L_\gamma)$								
51	$(\bar{L}i\tau^2 H^*)(H^T L^c)(\bar{L}^c i\tau^2 L)$	1/8	-1/8	1/8	-1/8	1/8	√	$1_{-1}^s + 1_0^L + 1_{-1}^{L/R}$
52	$(\bar{L}\vec{\tau}i\tau^2 H^*)(H^T L^c\vec{\tau})(\bar{L}^c i\tau^2 L)$	-3/8	3/8	1/8	-1/8	1/8	√	$1_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
53	$(\bar{L}\vec{\tau}i\tau^2 H^*)(H^T L^c)(\bar{L}^c i\tau^2\vec{\tau}L)$	-3/8	-1/8	-3/8	-1/8	1/8		$3_{-1}^s + 1_0^L + 3_0^{\bar{L}/R}$
54	$(\bar{L}i\tau^2 H^*)(H^T\vec{\tau}L^c)(\bar{L}^c i\tau^2\vec{\tau}L)$	3/8	1/8	-1/8	-3/8	-1/8		$3_{-1}^s + 3_0^{L/R} + 1_{-1}^{L/R}$
55	$(-i\epsilon^{abc})(\bar{L}\tau^a i\tau^2 H^*)\times(H^T\tau^b L^c)(\bar{L}^c i\tau^2\tau^c L)$	3/4	1/4	-1/4	1/4	1/4		$3_{-1}^s + 3_0^{L/R} + 1_{-1}^{\bar{L}/R}$
Combination $(\bar{L}^\beta (L^c)^\delta)(H^\dagger (\bar{L}^c)_\alpha)(L_\gamma H)$								
56	$(\bar{L}i\tau^2 L^c)(\bar{L}^c H^*)(H^T i\tau^2 L)$	1/8	-1/8	-1/8	1/8	1/8	√	$1_{-1}^s + 1_0^L + 1_{-1}^{L/R}$
57	$(\bar{L}\vec{\tau}i\tau^2 L^c)(\bar{L}^c\vec{\tau}H^*)(H^T i\tau^2 L)$	3/8	1/8	-3/8	-1/8	-1/8		$3_{-1}^s + 3_0^{L/R} + 1_{-1}^{\bar{L}/R}$
58	$(\bar{L}i\tau^2 L^c)(\bar{L}^c\vec{\tau}H^*)(H^T i\tau^2\vec{\tau}L)$	-3/8	3/8	-1/8	1/8	1/8	√	$1_{-1}^s + 3_0^{L/R} + 3_0^{L/R}$
59	$(\bar{L}\vec{\tau}i\tau^2 L^c)(\bar{L}^c H^*)(H^T i\tau^2\vec{\tau}L)$	-3/8	-1/8	-1/8	-3/8	1/8	√	$3_{-1}^s + 1_0^L + 3_{-1}^{L/R}$
60	$(-i\epsilon^{abc})(\bar{L}\tau^a i\tau^2 L^c)\times(\bar{L}^c\tau^b H^*)(H^T i\tau^2\tau^c L)$	3/4	1/4	1/4	-1/4	1/4		$3_{-1}^s + 3_0^{L/R} + 3_{-1}^{L/R}$
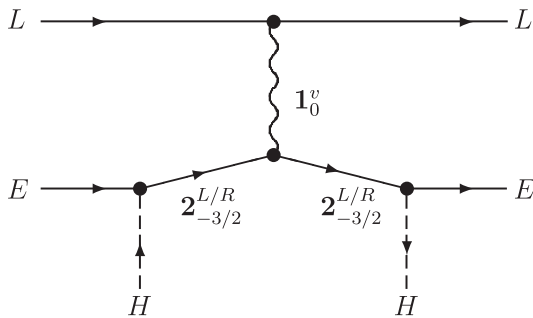
FIG. 3. Example for a fully decomposed operator. The diagram corresponds to #2 of Table III.

content shown in the tables is contained in all other possible sets of mediators leading to it.

From both Tables III and IV, and from Table II, one can easily read off the following key results for the operators considered:

(i) There is no way to write down a $d = 8$ operator without involving a mediator (pinpointed in boldface) which also generates $d = 6$ four-lepton interactions.

(ii) In order to build $\mathcal{O}_{NSI}$ *and* to cancel the dangerous (or all) NSI $d = 6$ contributions, at least two new fields are needed.

This implies that fine-tuning—or hopefully symmetries—will be required if all $d = 6$ NSI are to be canceled,

Eq. (45).[8] For model building, one may use the tables as follows: In order to generate a pure $\mathcal{O}_{\text{NSI}}$-type operator, it is necessary to choose effective operators such that Eq. (34) is fulfilled, i.e., interactions with four charged leptons are suppressed, and that Eq. (45) is satisfied, i.e., the NSI contributions from $d = 6$ operators cancel. The two simplest methods to build a model leading to a pure $\mathcal{O}_{\text{NSI}}$ interaction are as follows:

(1) To select from the tables those effective operators marked as $\mathcal{O}_{\text{NSI}}$.

(2) To linearly combine the effective operators in either of the tables to obtain an $\mathcal{O}_{\text{NSI}}$ structure. One possibility is to choose any combination of at least two non-$\mathcal{O}_{\text{NSI}}$ operators which are linearly independent in the BR basis (not considering $\mathcal{O}_{LLH}^{\mathbf{333}}$).[9]

The necessary mediators can then be directly read off from the tables; as the next step, the $d = 6$ cancellation conditions should be translated into relations among the couplings.

Note that, in addition, there might be flavor-dependent conditions and other constraints, which means that our tables can only serve as hints on how to build the simplest models. For example, one may have to worry about electroweak precision data, flavor-changing neutral currents, nonunitarity of the PMNS matrix, loop constraints, and chiral anomalies if exotic fermions are introduced.[10] Also, vectorial scalar $SU(2)$ doublets call for a deeper theory when present, as discussed earlier.

Other such constraints can result from interactions of the $\bar{E}E\bar{E}E$ type, which we show in Table V. Although these interactions do not produce NSI, care is mandatory when one introduces mediators which could induce such inter-

TABLE V. Effective $d = 8$ operators of the $\bar{E}E\bar{E}E$ type. The columns show an ordinal for each operator, the corresponding coefficient in the BR basis, and the tree-level mediators, respectively. The flavor structure is given in the first and fourth rows. Although these operators are not NSI operators, they share with the latter some common mediators which may induce charged lepton flavor violation.

#	Dimension eight operator	$\mathcal{C}_{EEH}$	Mediators
61	$(\bar{E}^\beta \gamma^\rho E_\alpha)(\bar{E}^\delta \gamma_\rho E_\gamma)(H^\dagger H)$	1	$\mathbf{1}_0^v$
62	$(\bar{E}\gamma^\rho E)(\bar{E}H^T)(\gamma_\rho)(H^*E)$	1	$\mathbf{1}_0^v + \mathbf{2}_{+1/2}^{L/R}$
63	$(\bar{E}\gamma^\rho E)(\bar{E}H^\dagger)(\gamma_\rho)(HE)$	1	$\mathbf{1}_0^v + \mathbf{2}_{-3/2}^{\bar{L}/\bar{R}}$
64	$(\bar{E}^\beta E^{c\delta})(\bar{E}_\alpha^c E_\gamma)(H^\dagger H)$	1/2	$\mathbf{1}_{-2}^s$
65	$(\bar{E}H^\dagger)(E^c H)(\bar{E}^c E)$	1/2	$\mathbf{1}_{-2}^s + \mathbf{2}_{-3/2}^{L/R} + \mathbf{2}_{+1/2}^{L/R}$
66	$(\bar{E}E^c)(H^\dagger \bar{E}^c)(EH)$	1/2	$\mathbf{1}_{-2}^s + \mathbf{2}_{-3/2}^{\bar{L}/\bar{R}} + \mathbf{2}_{+1/2}^{\bar{L}/\bar{R}}$

actions. For example, operator #36 not only produces NSI, but will also lead to potential nonunitarity [through the operator in Eq. (10)] and other unwanted $d = 6$ effects, and operator #61 from Table V potentially leads to charged lepton flavor violation.

For the operators in Table III, our toy model is seen to be the only possibility using only two new fields, namely, $\mathbf{2}_{+1/2}^s$ and $\mathbf{2}_{-3/2}^v$. It combines operators #7, #8, #13, and #14, which correspond to the four effective interactions in Eq. (52) in our toy model. The table also allows us to conclude that it is also the most general version of the model with only two fields, while a simpler version might, for instance, only include #7 and #14. Recall that source and detection NSI cannot be created from our toy model, while matter NSI for all flavors are allowed. All $\mathcal{O}_{\text{NSI}}$ operators obtained in Table III correspond to the combination of operators of the BR basis in Eq. (22) which are thus equivalent to Eq. (4)

In Table IV, the simplest possibility to build a pure $\mathcal{O}_{\text{NSI}}$ and no $d = 6$ interaction requires at least three fields, namely, $\mathbf{1}_{-1}^s$, $\mathbf{3}_0^v$, and $\mathbf{3}_{-1}^s$, which may come from a large number of possible operator combinations. For example, one may combine operators #35 and #48. As discussed in Sec. II, such a model could have correlations between source and matter NSI. Note that neither these models nor our toy model involve fermions,[11] which means that they cannot generate corrections to the unitarity of the PMNS matrix [through contributions to the operators in Eqs. (10) and (11)] nor to electroweak data [through contributions to the operators in Eqs. (11) and (12)], or at least not at leading order.

2. New interactions involving only one SM field

Beyond the operators in the tables above, a much larger number of effective operators is obtained if, in addition to the interactions with SM bilinears, couplings between one

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[8]Recall that this condition ensures that, in addition to avoiding lepton flavor violation among four charged fermions, other putatively dangerous $d = 6$ couplings are suppressed, such as possible contributions to the very precise measurement of G_F determined from muon decay. Note as well that, in principle, one could avoid imposing such a strong cancellation condition by assuming very large couplings among the new heavy fields, and very small values for the couplings between those heavy fields and the SM fields which induce $d = 6$ operators. However, since the product between these two types of couplings will be present in the $d = 8$ operator (as in our toy model), the $d = 8$ couplings would be effectively suppressed as well and extreme fine-tuning would be needed.

[9]In short, the linear combination of two vectors involves only one free parameter (aside from the normalization). The condition in Eq. (34) amounts then to a linear equation with only one parameter, which can always be solved for. Since the vectors are linearly independent, they cannot cancel each other, which means that there will be nonvanishing NSI. If, on the other hand, one chooses linearly dependent vectors, there will be no $d = 8$ operator at all—neither $\mathcal{O}_{\text{NSI}}$ nor the harmful one.

[10]This concerns, for instance, several examples in Table III. In general, in order to cancel the chiral anomaly, new vectorlike fermions may be introduced. In the tables we just show the smallest number of mediators which can induce the $d = 8$ operators.

[11]More precisely, they do not involve Yukawa couplings linking the exotic and standard fermions.

SM field and two exotic fields are allowed in the fundamental theory [23]. The resulting $d = 8$ operators are diagrammatically illustrated in Fig. 4 and fall into three categories, which contain the following SM bilinears at the external vertices:

(1) (LE)- or (LL)-type interactions with new fields. At least one of the mediators will necessarily induce some of the $d = 6$ interactions among four leptons discussed earlier (corresponding to the external vertices in the figure), and the couplings will thus be subject to the corresponding constraints. The fundamental interactions describing the internal vertices, however, may not be related to the previously discussed $d = 6$ interactions.

(2) (LH)-type interactions. In this case, the mediators do not necessarily induce any dangerous $d = 6$ operator involving four leptons, even if there are some common mediators. The connections previously studied linking $d = 6$ and $d = 8$ operators do not need to hold. Nevertheless, these types of interactions involve exotic fermions [$SU(2)$ singlets or triplets] and are constrained by nonunitary contributions to the PMNS matrix, and some of them also by electroweak precision data (see, e.g., Ref. [23]): Figure 4, center diagram, illustrates that this class of diagrams is connected to one of the $d = 6$ operators in Eqs. (10) and (11), or a combination of them.

(3) (EH)-type interactions. These types of interactions are suggestive. The mediators may not induce dangerous four-fermion $d = 6$ operators. Furthermore, they do not introduce corrections to the PMNS matrix at leading order. They involve exotic leptons, however, which are typically strongly constrained by electroweak precision tests [15]. Figure 4, right diagram, illustrates that this class of diagrams is connected to the $d = 6$ operators in Eq. (12).

Possible "mixed" diagrams, that is, diagrams involving two different SM bilinear couplings, will combine the corresponding properties. For instance, a model containing both (LE) and (LH) couplings to exotic mediators will simultaneously induce some of the $d = 6$ operators in Table II *and* some of the operators in Eqs. (10)–(12) which induce nonunitarity.

It is easy to show that the vertex involving just one SM field (L, E, or H) requires that the two exotic particles attached to it have different $SU(2) \times U(1)$ charges. Indeed, we have explicitly checked that *all* of these possibilities require at least two new fields to be phenomenologically viable; i.e., they are not simpler than the cases discussed prior to this subsection.

The scenarios in diagram #2 and especially #3 in Fig. 4 are appealing alternatives, as neither of them is correlated to harmful $d = 6$ interactions (i.e., four charged-fermion lepton couplings), and #3 does not induce nonunitarity either. Furthermore, these two examples are $\mathcal{O}_{\mathrm{NSI}}$ operators. Indeed, the exchange of a singlet fermion $\mathbf{1}_0^R$ and a charged scalar $\mathbf{1}_{-1}^s$ shown in #2 gives schematically

$$(\bar{L}H)(E)(\bar{E})(H^\dagger L) = -\tfrac{1}{4}(\mathcal{O}_{LEH}^{\mathbf{1}}) - \tfrac{1}{4}(\mathcal{O}_{LEH}^{\mathbf{3}}). \qquad (58)$$

Here the projection onto the BR basis shows that it complies with the $d = 8$ cancellation conditions, Eq. (34). The mediator $\mathbf{1}_{-1}^s$ could induce in addition $d = 6$ effective interactions if also couples to SM lepton doublets, as shown in Table II, but such couplings are not mandatory. In contrast, the PMNS unitarity constraints should be relevant, as a singlet exotic fermion is involved.

Turning now to type #3 and the scenario with an exotic doublet fermion $\mathbf{2}_{-3/2}^{L/R}$ and a charged scalar $\mathbf{1}_{-1}^s$, the resulting effective operator for this example is of the form

$$(EH)(\bar{L})(L)(H^\dagger \bar{E}) = -\tfrac{1}{4}(\mathcal{O}_{LEH}^{\mathbf{1}}) - \tfrac{1}{4}(\mathcal{O}_{LEH}^{\mathbf{3}}), \qquad (59)$$

and is thus again of the $\mathcal{O}_{\mathrm{NSI}}$ type. Furthermore, in this case the interactions neither lead to nonunitarity, nor do they induce any $d = 6$ operator in Table II if the charged scalar does not couple to SM lepton doublets [in other words, the $d = 6$ complete cancellation conditions in Eq. (45) can be implemented as well]. Other scenarios of the kind just discussed do not necessarily have to lead by themselves to $\mathcal{O}_{\mathrm{NSI}}$ structures: for them, cancellations similar to those in our toy model could be considered. However, it remains to be explored how difficult it is to circumvent the constraints which electroweak precision tests impose on exotic leptons, and whether the necessary cancellations are feasible without running into extreme fine-tunings, for instance, enlarging the scalar sector of the theory.
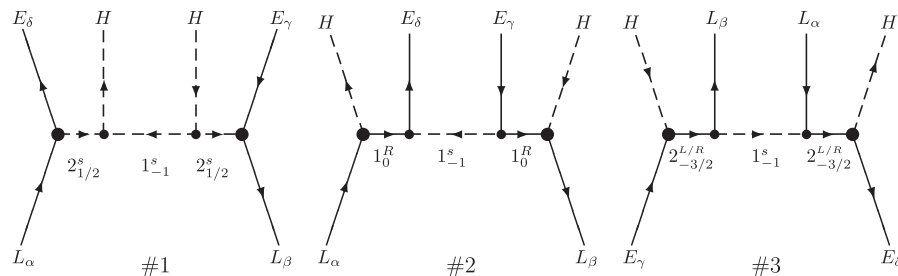

FIG. 4. Examples for each category of diagrams which lead to $d = 8$ operators and require couplings of the new fields *both* to SM bilinears and to only one SM field.

During the completion of this work, Ref. [23] appeared. It explores (but is not limited to) the possible exchange of exotic fields which in our notation have quantum numbers of a scalar $\mathbf{1}^{s}_{-1}$ (to obtain $d = 6$ NSI) and of a fermion $\mathbf{1}^{R}_{0}$ (to obtain $d = 8$ NSI). The latter also induces $d = 6$ interactions, which lead to nonunitary contributions to the PMNS matrix, as is well known and is further explored in that reference. Reference [23] performs a systematic topological scan of the $d = 8$ operators, based on Feynman diagrams, trying to obtain the interaction $\mathcal{O}_{\mathrm{NSI}}$ directly from just one Feynman diagram while avoiding *any* harmful $d = 6$ and $d = 8$ contributions. Our tables correspond to topologies 2 and 3 in this reference, whereas the previous paragraph in this subsection would correspond to their topology 1. Since all possibilities in our tables contain at least one mediator leading to harmful $d = 6$ effects if one does not allow for cancellations, Ref. [23] effectively excludes topologies 2 and 3 in their scan (apart from our #46, which does not induce harmful $d = 6$ four charged lepton interactions, but the mediator $\mathbf{1}^{s}_{-1}$ is constrained otherwise, as we and Ref. [23] discussed before). Therefore, our work is complementary to that reference. Note that they find that the NSI in matter and the NSI at the source or detector are correlated in all of their examples by the nonunitary effects of the heavy fermions, whereas it is easy to see that uncorrelated scenarios are achievable when one combines different operators from our Table III (such as #7, #8, #13, and #14). As the most important difference, we relate the operators obtained from mediator exchanges to a complete basis of independent operators, which allows us to deduce the general cancellation conditions.

V. SUMMARY AND CONCLUSION

In this study, we have discussed the possibility of large NSI in the neutrino sector. Since any model of new physics has to recover the standard model at low energies, we have required gauge invariance under the SM gauge group and studied the possible effective theories. The focus is set on purely leptonic NSI, that is, on operators in which the only fermion fields appearing are leptons. Our analysis has been based on the full (analytical) decomposition of all possible dimension six and eight effective operators, which can be induced at tree level by any hypothetical beyond-the-SM theory. Special focus has been set on the scenario in which the exotic mediators couple to SM bilinear field combinations.

Our aim is to gauge the theoretical price of achieving phenomenologically viable large neutrino NSI, and to establish the minimal constraints that models have to respect for this purpose. Our main requirements are as follows:

(i) Interactions with four charged leptons have to be absent or highly suppressed, since these would lead to charged lepton flavor violation or corrections to G_F.

(ii) When analyzing NSI from $d = 8$ operators, any $d = 6$ contribution among four leptons is not allowed or has to be very suppressed, since this would either be the dominating NSI (if harmless) or would lead to unacceptably strong interactions among four charged leptons (if harmful).

The NSI operators obtained have been expanded in a complete basis of independent operators, which has allowed us to consistently consider cancellations among the contributions of different operators. This new approach has established the general cancellation conditions which the model parameters have to fulfill, to avoid four charged lepton interactions when the exotic mediators couple to SM bilinears.

We then studied the required complexity of any realistic model, such as with respect to the number of mediators or type of cancellations needed. In short, we have demonstrated that it is not possible to create such NSI at $d = 6$ for all flavor channels. For $d = 8$, we have constructed a simple toy model in which the necessary cancellations occur. It introduces two new $SU(2)$ doublet mediators, a Lorentz scalar and a vector, which induce the desired large NSI without any dangerous $d = 6$ flavor-changing transitions among four charged leptons or nonunitarity corrections. It also allows for matter NSI uncorrelated with source or detector effects. Furthermore, we have obtained and analyzed the general classification of $d = 8$ interactions in a systematic way.

More precisely, for the $d = 6$ operators it is shown at the effective operator level that, if the four charged lepton contributions have to be exactly canceled, it is not possible to obtain $\epsilon^{m}_{e\tau}$ under the above assumptions. In addition, there are certain connections between the source NSI at a neutrino factory and the matter NSI, such as $\epsilon^{m}_{\mu\tau} = -(\epsilon^{s}_{\mu\tau})^{*}$. We have shown that there is only one viable possibility for a $d = 6$ interaction with *one mediator only*, which is the well-known antisymmetric operator from Ref. [26]. Nevertheless, there are other options with more than one mediator in order to cancel all related interactions involving four charged leptons. Altogether, if the full decomposition of the operators is taken into account along with the current bounds, only a large $\epsilon^{m}_{\tau\tau}$ might be viable with only one mediator and it can be as large as order unity. In this case, the current bounds do not require that the four charged lepton contributions cancel exactly.

For the $d = 8$ operators, we have shown that at least two new fields are required to avoid the undesired $d = 6$ and $d = 8$ interactions involving four charged leptons. In fact, when the mediators of a $d = 8$ effective operator couple only to SM bilinears, there will always be at least one field leading as well to $d = 6$ contributions as well. These have to be canceled in each case by fine-tuning or symmetries. This result can be directly seen from Tables III and IV, which list the possible dimension eight operators, including the mediators, for fundamental interactions involving

two SM fields. In those tables, the mediators highlighted in boldface lead to dimension six interactions as well.

Our simple toy model, with two bosonic $SU(2)$ doublets added to the SM content, leads directly to the desired $d = 8$ NSI and fulfills our minimal requirements. Notice, however, that we have not considered constraints from loop effects, neither for this toy model nor for the general operator analysis, as it is beyond the scope of the present work. These corrections could be especially relevant for some models when considering $d = 8$ interactions, as the one-loop corrections could spoil the $d = 6$ cancellation conditions. They should be taken into account in any model which aims to be realistic.

Similar considerations apply to the scenarios in which exotic couplings to *both* one SM field and SM bilinears are simultaneously allowed. In order to then induce large $d = 8$ NSI and no $d = 6$ couplings among four leptons, a minimum of two exotic mediators is once again needed. Some simple candidate models may not even require (strong) cancellation conditions, and they deserve further exploration. This is the case, for instance, when a singlet scalar and a fermionic doublet are added to the SM content.

As far as the connection between source and matter NSI is concerned, we have demonstrated that it depends on the operators used. For example, several of the $d = 8$ operators in Table III, or combinations of them, will only induce matter NSI, while those requiring singlet or triplet fermionic mediators may induce correlations (through nonunitary corrections to the PMNS matrix). On the other hand, all $d = 8$ operators in Table IV will, in principle, allow for a connection between source and matter NSI independent of the mediators used. Therefore, it might be very well possible to detect matter NSI without source or detector effects, in the absence of fermions as exotic particles, such as is illustrated by our toy model. Note, as well, that the models based on Table IV require at least three new fields, which means that a source and matter NSI connection might be more easily achieved through nonunitary corrections to the PMNS matrix.

In conclusion, we have demonstrated that the minimum complexity of a realistic model leading to large NSI and no charged lepton flavor violation requires at least two new fields inducing $d = 8$ NSI couplings. We have determined the possible SM charges of those mediators and the cancellation conditions for the dimension six interactions among four leptons that they simultaneously induce in most cases. These cancellation conditions translate into precise relations among model parameters. One exception might be $\epsilon_{\tau\tau}^m$, which might be created at the dimension six level. Our results imply a number of constraints such that the observational prospects do not seem bright, especially since we did not identify some symmetry which would account for them. On the other side, we showed that large NSI are not excluded, and we found out which conditions need to be satisfied in order for any model to be viable. We

agree that those conditions should be justified by symmetries or other arguments for the model to be credible. Until such justification is found in some model, we leave it up to the reader to decide on the perspective for large NSI.

APPENDIX A: ON NONSTANDARD FOUR NEUTRINO INTERACTIONS

Although interactions among four neutrinos hardly contribute to laboratory processes, there has been some interest in the literature in the context of flavor oscillations in astrophysical environments, such as dense neutrino gases; see, e.g., Ref. [91] and references therein. The direct laboratory bounds on these interactions are naturally extremely weak; see Refs. [92,93]. In this appendix, we discuss these four neutrino interactions in our gauge invariant framework.

1. Effective operator formalism

Since the four neutrino interactions require interactions with four lepton doublets, they only appear for the $\bar{L}L\bar{L}L$ operators. In this case, Eq. (24) reads, including the four neutrino interactions,

$$
\begin{aligned}
\delta\mathcal{L}_{\text{eff}} &= \frac{1}{\Lambda^2}(\mathcal{C}_{\text{NSI}}^{\bar{L}L\bar{L}L})_{\beta\delta}^{\alpha\gamma}(\bar{\nu}^\beta\gamma^\rho P_L\nu_\alpha)(\bar{\ell}^\delta\gamma^\rho P_L\ell_\gamma) \\
&+ \frac{1}{\Lambda^2}\left(\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}(\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} - \mathcal{C}_{LLH}^{\mathbf{133}} - \mathcal{C}_{LLH}^{\mathbf{313}})\right)_{\beta\delta}^{\alpha\gamma} \\
&\times (\bar{\ell}^\beta\gamma^\rho P_L\ell_\alpha)(\bar{\ell}^\delta\gamma^\rho P_L\ell_\gamma) \\
&+ \frac{1}{\Lambda^2}\left(\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}} + \frac{v^2}{2\Lambda^2}(\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}})\right)_{\beta\delta}^{\alpha\gamma} \\
&\times (\bar{\nu}^\beta\gamma^\rho P_L\nu_\alpha)(\bar{\nu}^\delta\gamma^\rho P_L\nu_\gamma) + \text{H.c.} \quad (\text{A1})
\end{aligned}
$$

The first point one notices is that the four charged lepton and four neutrino interactions share for $d = 6$ the same coefficient $\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}}$. This means that for $d = 6$, any bound from charged lepton flavor violation etc. can be

directly translated into the four neutrino interactions. This is illustrated here with one example. For $\beta = \mu$ and $\alpha = \gamma = \delta = e$, the bound from $\mu \to eee$ can, apart from some $SU(2)$ symmetry breaking effects, be directly transferred to the four neutrino interactions. In our notation, one has

$$\mathrm{Br}\left(\mu \to 3e\right) = \frac{1}{G_F^2}\left(\frac{\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}}}{\Lambda^2}\right)^2 = \frac{F^2}{G_F^2}, \qquad \text{(A2)}$$

where the nonstandard parameter is defined as $F \equiv (\mathcal{C}_{LL}^{\mathbf{1}} + \mathcal{C}_{LL}^{\mathbf{3}})/\Lambda^2$—as often done in the literature. The current bound $\mathrm{Br}(\mu \to 3e) < 10^{-12}$ (90% CL) [94] then directly translates into $F \lesssim 10^{-6} G_F$, which is far below any laboratory bound or even the bound from primordial nucleosynthesis. Of course, it is dependent on the participating flavors and is somewhat looser for combinations involving the τ, but this procedure illustrates the generic argument. Note that the bound for a vector mediated interaction, such as often discussed in the literature, turns out to be the same in this case.

As discussed in Sec. II, Eq. (33) should be satisfied for any realistic model in order to avoid these bounds. As we can read off from Eq. (A1), however, the $d = 6$ coefficients for the four charged lepton and four neutrino interactions are exactly the same, which means that there will not be any four neutrino interactions in that case. As a consequence, one has to go to $d = 8$ with the interactions being suppressed by Λ^4.

For $d = 8$, the corresponding Eq. (34) to suppress the harmful interactions among four charged fermions can be implemented in qualitatively different ways. For example, if $\mathcal{C}_{LLH}^{\mathbf{111}} = -\mathcal{C}_{LLH}^{\mathbf{331}}$ and $\mathcal{C}_{LLH}^{\mathbf{133}} = -\mathcal{C}_{LLH}^{\mathbf{313}}$, there will be no four neutrino interactions but there will be NSI, whereas for $\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} = \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}} \neq 0$, there will be both four neutrino interactions and NSI. As it is demonstrated below, both possibilities can be realized within the model framework in this study.

2. Model analysis

In order to find models for large four neutrino interactions at $d = 8$, the same argumentation as in Sec. IV is needed. First of all, Eq. (34) has to be satisfied to suppress the four charged lepton processes. Second, the $d = 6$ contributions to the NSI have to be canceled, since there are strong bounds; i.e., Eq. (45) has to be satisfied. As an additional condition, one can *not* have [cf., Eq. (A1)]

$$\mathcal{C}_{LLH}^{\mathbf{111}} + \mathcal{C}_{LLH}^{\mathbf{331}} + \mathcal{C}_{LLH}^{\mathbf{133}} + \mathcal{C}_{LLH}^{\mathbf{313}} = 0 \qquad \text{(A3)}$$

because such an operator will not contribute to the four neutrino interactions. The relevant decomposed operators can be found in Table IV, where one can easily read off if Eq. (A3) is satisfied. Furthermore, note that operators which only induce $\mathcal{C}_{LLH}^{\mathbf{333}}$ will not be useful for the four neutrino interactions. We find from the table that operators #35, #40, #41, #43, #44, #46, #48, #49, #51, #52, #54, #56, #57, and #58 do not contribute to the four neutrino interactions. This implies that the possibility pointed out in the main text, i.e., to combine #35 and #48, does not lead to four neutrino interactions. One has to use more complicated combinations by combining different operators, such as #32 and #50 to satisfy Eq. (34), and #48 [which satisfies Eq. (34)] to introduce an additional mediator to cancel the $d = 6$ NSI. Then the four neutrino interactions can be constructed with three different mediators, where only #32 and #50 contribute to the four neutrino interactions. Constructions with less mediators are, under the assumptions in this study, not possible, which is different from the NSI, which can be generated from two mediators.

As soon as a specific model is known, the relationship among source and production NSI, matter NSI, and four neutrino interactions can be easily calculated using Sec. II and Eq. (A1).

In summary, for the $d = 6$ four neutrino interactions, gauge invariance implies that they face the stringent bounds from charged lepton flavor violation, such as from μ to three electrons. Therefore, large four neutrino interactions have to come from $d = 8$ effective operators. From the model point of view, having four neutrino interactions is even more complicated than having large NSI, since at least three different mediators are needed in the framework discussed in this study.

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