Properties of J/ψ at T_c: QCD second-order Stark effect

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Starting from the temperature dependencies of the energy density and pressure from lattice QCD calculation, we extract the temperature dependencies of the electric and magnetic condensate near T_c. While the magnetic condensate hardly changes across T_c, we find that the electric condensate increases abruptly above T_c. This induces a small but equally abrupt decrease in the mass of J/ψ, which can be calculated through the second-order Stark effect. Combining the present result with the previously determined QCD sum rule constraint, we extract the thermal width of J/ψ above T_c, which also increases fast. These changes can be identified as the critical behavior of J/ψ across T_c associated with the phase transition. We find that the mass shift and width broadening of J/ψ at $1.05T_c$ will be around -100 MeV and 100 MeV, respectively.

Since the pioneering work by Hashimoto *et al.* [1] and the seminal work by Matsui and Satz [2], many experimental and theoretical works have been performed in the physics of J/ψ suppression in heavy ion collisions. The subject has recently evolved into a new stage as lattice calculations using maximum entropy methods (MEM) found the peak structure to survive up to almost $2T_c$ [3,4], which was speculated before [5] as lattice calculations showed that nonperturbative nature of QCD persists well above T_c [6,7]. This suggests that the sudden disappearance of J/ψ is not the direct signature of quark-gluon plasma (QGP) formation. Indeed, recent results from the Relativistic Heavy Ion Collider (RHIC) on the suppression factors at different rapidity and higher p_T seem even more confusing [8]. Hydrodynamic calculations suggest that the initial temperature of the QGP formed at RHIC is in the order of $2T_c$ and will last for 3 to 4 fm/c [9,10]. Therefore, considering the formation time of charmonium states after the creation of the $\bar{c}c$ pair, it is crucial to know the detailed properties of J/ψ near T_c to fully understand the suppression and/or enhancement of J/ψ in heavy ion collisions. Unfortunately, the present resolutions of the peak structure of J/ψ from the lattice calculations based on MEM are far from satisfactory [11]. In fact, the peak is too broad to even discriminate between J/ψ from ψ'. Moreover, the temperature region between T_c to $2T_c$ is known to be strongly interacting and, therefore, a nonperturbative method has to be implemented to consistently treat the charmonium at this temperature region. In a previous work [12], we have implemented QCD sum rules to investigate the properties of J/ψ near T_c. Although the results were nonperturbative, only a constraint on the combined mass decrease and width increase could be obtained. Here we point out that the critical behavior of QCD phase transition could be identified with a critical behavior of electric condensate at T_c,

and then by making use of the QCD second-order Stark effect, we show that such critical behavior can be translated to a sudden change in the mass of J/ψ across T_c.

We begin by characterizing the properties of the strongly interacting quark-gluon plasma (sQGP) of the pure gluon theory across T_c in terms of local operators. This is accomplished by making use of the energy-momentum tensor, which has a symmetric traceless part and a trace part via the trace anomaly,

$$T_{\alpha\beta} = -\mathcal{S}\mathcal{T}(G_{\alpha\mu}^a G_\beta^{a\mu}) + \frac{g_{\alpha\beta}}{4}\frac{\beta(g)}{2g}G_{\mu\nu}^a G^{a\mu\nu}. \quad (1)$$

Here, a and α, β are the color and Lorentz indices, respectively. The temperature dependence of the two independent parts of the energy-momentum tensor can be obtained from the lattice measurement of energy density and pressure at finite temperature.

$$\langle T_{\alpha\beta}\rangle_T = (\varepsilon + p)\left(u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta}\right) + (\varepsilon - 3p)\frac{g_{\alpha\beta}}{4}. \quad (2)$$

Here, u_α is the four velocity of the heat bath. Therefore, the temperature dependencies of gluonic operators can be identified with the pressure and energy density. To leading order in coupling, we can identify the trace part $-\frac{11}{8} \times [\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{a\mu\nu}\rangle_T - \langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{a\mu\nu}\rangle_0] = M_0(T)$ and the nontrace part $\langle-\mathcal{S}\mathcal{T}G_{\alpha\mu}^a G_\beta^{a\mu}\rangle_T = (u_\alpha u_\beta - \frac{1}{4}g_{\alpha\beta})M_2(T)$, where [12],

$$M_0(T) = (\varepsilon - 3p), \qquad M_2(T) = (\varepsilon + p), \quad (3)$$

and $\langle\frac{\alpha_s}{\pi}G_{\mu\nu}^a G^{a\mu\nu}\rangle_0 \equiv G_0^{\text{vac}}$ is the scalar gluon condensate in the vacuum. The lattice gauge theory result for ε and p in the pure SU(3) gauge theory was obtained from Ref. [13]. Figure 1 shows changes of M_0 and M_2 from their vacuum value scaled by their asymptotic temperature dependence of T^4. One notes that while M_2, which is also proportional to entropy density times temperature, moderately reaches the asymptotic temperature dependence M_0,

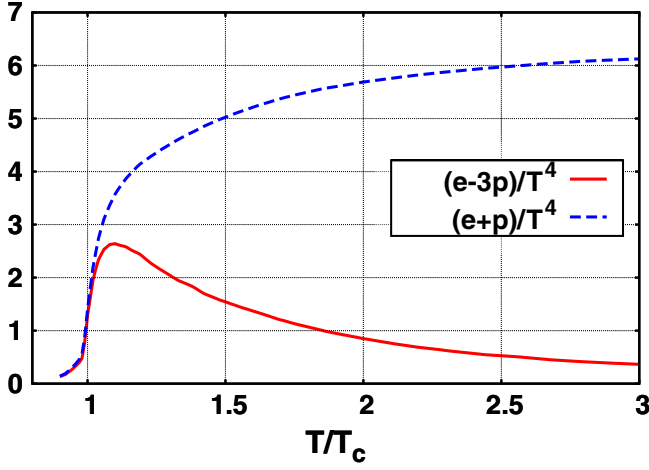*suhoung@phya.yonsei.ac.kr
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FIG. 1 (color online). $M_0 = (\varepsilon - 3p)$ and $M_2 = (\varepsilon + p)$ divided by T^4 as functions of temperature.

also known as the interaction measure or the gluon condensate, suddenly increases and then decreases at higher temperature. The strongly interacting nature of QGP is related to the large interaction measure [14], which takes its maximum value at around $1.1T_c$. It should be noted that the temperature dependence of the gluon condensate M_0 extracted here includes both the perturbative and the nonperturbative and thus the full temperature dependence. The sudden change near T_c is dominated by the sudden decrease of the nonperturbative part, which reduces to about half of its vacuum value [7], while at higher temperatures it is dominated by the perturbative contributions [13,15].

For the heat bath at rest, one can rewrite the thermal expectation values of the dimension four operators of the energy-momentum tensor in Eq. (1) in terms of electric and magnetic condensate [16]. This is possible after making the following identification.

$$\left\langle \frac{\alpha_s}{\pi} \mathcal{ST}(G^a_{\alpha\mu} G^{a\mu}_\beta) \right\rangle_T \equiv \frac{\alpha_s(T)}{\pi} \langle \mathcal{ST}(G^a_{\alpha\mu} G^{a\mu}_\beta) \rangle_T. \quad (4)$$

The scale dependence of the matrix element is transferred to the coupling constant. Therefore, we additionally need to know the temperature dependence of the coupling constant $\alpha_s(T)$. Since we will be using the matrix element in the operator product expansion (OPE) with the separation scale relevant for the heavy bound state, we will use the temperature-dependent running coupling constant extracted from the lattice computation of the heavy quark free energy [17]. Then we find,

$$\left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_T - \left\langle \frac{\alpha_s}{\pi} E^2 \right\rangle_0 = \frac{2}{11} M_0(T) + \frac{3}{4} \frac{\alpha_s(T)}{\pi} M_2(T),$$
$$(5)$$

$$\left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle_T - \left\langle \frac{\alpha_s}{\pi} B^2 \right\rangle = -\frac{2}{11} M_0(T) + \frac{3}{4} \frac{\alpha_s(T)}{\pi} M_2(T).$$
$$(6)$$

Figure 2 shows the temperature dependence of $\langle \frac{\alpha_s}{\pi} E^2 \rangle_T$ and $\langle \frac{\alpha_s}{\pi} B^2 \rangle_T$. One notes that there is a sudden increase in the electric condensate $\langle \frac{\alpha_s}{\pi} E^2 \rangle_T$, while the magnetic condensate $\langle \frac{\alpha_s}{\pi} B^2 \rangle_T$ hardly changes above T_c. This can be related to the fact that the area law behavior of the space-time Wilson loop changes to the perimeter law above T_c, while that of the space-space Wilson loop retains the area law behavior even above T_c [6]. The connection comes in as the nonperturbative behavior of a rectangular Wilson loop in the $S_1 S_2$ direction can be related to the nonvanishing gluon condensate $\langle \frac{\alpha_s}{\pi} G^2_{S_1 S_2} \rangle$ via the operator product expansion [18]. Hence, one can conclude that critical behaviors of QCD phase transition can be related to the sudden change in the electric condensate. Such local changes will induce critical behavior of a heavy quark system such as the J/ψ across the phase transition, which can be obtained through the QCD second-order Stark effect.

The perturbative QCD formalism for calculating the interaction between heavy quarkonium and partons was first developed by Peskin [19,20] in the nonrelativistic limit. The formula for the mass shift reduces to the second-order Stark effect in QCD, which was used previously to calculate the mass shift of charmonium in nuclear matter [16]. The information needed from the medium is the electric field square. As the dominant change across the phase transition is the electric condensate, one notes that the second-order Stark effect is the most natural formula to be used across the phase transition.

The second-order Stark effect for the ground state charmonium with momentum space wave function normalized as $\int \frac{d^3 p}{(2\pi)^3} |\psi(p)|^2 = 1$ is as follows,

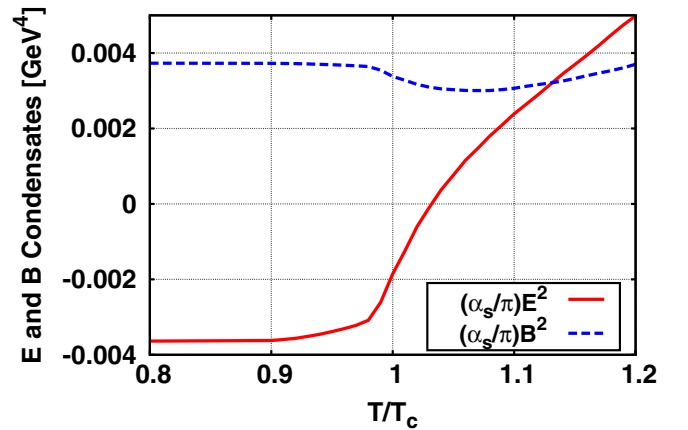FIG. 2 (color online). Electric and magnetic condensate near T_c as functions of temperature.

$$\Delta m_{J/\psi} = -\frac{1}{18} \int_0^\infty dk^2 \left| \frac{\partial \psi(k)}{\partial k} \right|^2 \frac{k}{k^2/m_c + \epsilon}$$

$$\times \left\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{E}^2 \right\rangle_T$$

$$= -\frac{7\pi^2}{18} \frac{a^2}{\epsilon} \left\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{E}^2 \right\rangle_T, \qquad (7)$$

where $k = |\boldsymbol{k}|$ and $\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{E}^2 \rangle_T$ denotes the value of change of the electric condensate from its vacuum value. The second line is obtained for the Coulomb wave function. Here, ϵ is the binding energy and m_c the charm quark mass. These parameters are fit to the size of the wave function obtained in the Cornell potential model [21], and to the mass of J/ψ assuming it to be a Coulombic bound state in the heavy quark limit [19]. The fit gives $m_c = 1704$ MeV, $a = 0.271$ fm and $\alpha_s = 0.57$. A few comments are in order. The minus sign in Eq. (7) is a model-independent result and follows from the fact that the second-order Stark effect is negative for the ground state. The factor of Bohr radius square a^2 follows from the dipole nature of the interaction, and the binding energy ϵ from the inverse propagator, characterizing the separation scale [19,22]. Therefore, the actual value of the mass shift does not depend much on the form of the wave function as long as the size of the wave function is fixed. The Bohr radius used in our calculation corresponds to $\langle r^2 \rangle^{1/2} = 0.47$ fm, which is the size of a more realistic wave function in the Cornell potential [21]. Therefore, the correction coming from using a more realistic wave function should be small.

The solid line in Fig. 3 shows the mass shift obtained from the second-order Stark effect. The second-order Stark effect formula is based on the operator product expansion (OPE) for bound state. As mentioned before, the formalism was first established by Peskin in 1979 and the separation scale is the binding $\epsilon = mg^4$. A more systematic derivation was developed recently by Brambila et al. [23] for the

bound state. Here, the relevant scales are mv and mv^2, where the former is related to the potential $1/r$ and the latter to the kinetic energy p^2/m. This scale mv^2 is the separation scale so that for effects with typical momentum larger than the separation scale should be taken into account through resummed perturbation by solving the Schrödinger equation [22], while that with smaller scale should be taken into account through the operator product expansion. The operators contain the nonperturbative physics of QCD, which is typically of order Λ_{QCD}. Therefore, the OPE for the bound state works best when $mv^2 \gg \Lambda_{\mathrm{QCD}}$ [23]. While there are concerns that this condition is marginal for charmonium, the approach should provide a quantitative description. Now the question is which approach should one take for the thermal interactions near T_c. Obviously, the effects of finite temperature involve new scales like the color screening. However, as has been known for some time, the temperature region from T_c to $2T_c$ is known to be strongly interaction and cannot be described by resumed perturbation [24]. Therefore, the approach we want to take is to calculate the nonperturbative temperature effect to the mass shift through the operator product expansion. The leading order contribution in this approach is coming from $\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{E}^2 \rangle_T$ as is given in Eq. (7), whose temperature dependence we extract directly from the lattice. The arguments for convergence of higher dimensional operators in our approach are twofold. First, we will restrict to the temperature region where the change in $\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{E}^2 \rangle_T$ is smaller than the vacuum value of $\langle \frac{\alpha_s}{\pi} \boldsymbol{E}^2 \rangle_0$ itself. We believe that then the OPE is under control as has been verified by the typical QCD sum rule approaches for heavy quark system in the vacuum. As can be seen in Fig. 2, this condition restricts our applicability to $1.05 T_c$, as in the QCD sum rule approach at finite temperature [12,25]. Second, a more direct evidence comes from the next term in the OPE correction, which comes from magnetic condensate. However, as can be seen from Fig. 2, the changes of $\langle \frac{\alpha_s}{\pi} \Delta \boldsymbol{B}^2 \rangle_T$ and hence the next term in the OPE should be small up to $1.05 T_c$ and slightly beyond. Therefore, we can conclude that the second-order Stark effect should be valid near T_c. As can be seen in Fig. 3, the results from the second-order Stark effect shows that the mass reduces abruptly above T_c and becomes smaller by about 100 MeV at $1.05 T_c$, reflecting the critical behavior of the QCD phase transition.

We put the present result in perspective with a nonperturbative result obtained before using the QCD sum rules [12,25]. The points in Fig. 3 represent the maximum mass shift obtained in Refs. [12,25]. As can be seen in the figure, the mass shift obtained from the second-order Stark effect is almost the same as the maximum mass shift obtained in the sum rule up to T_c and then becomes smaller. The mass shift at T_c is about -50 MeV. In the QCD sum rules, only a constraint for the combined mass shift and thermal width of $-\Delta m + \Gamma_T \simeq 80 + 17(T - T_c)$ MeV
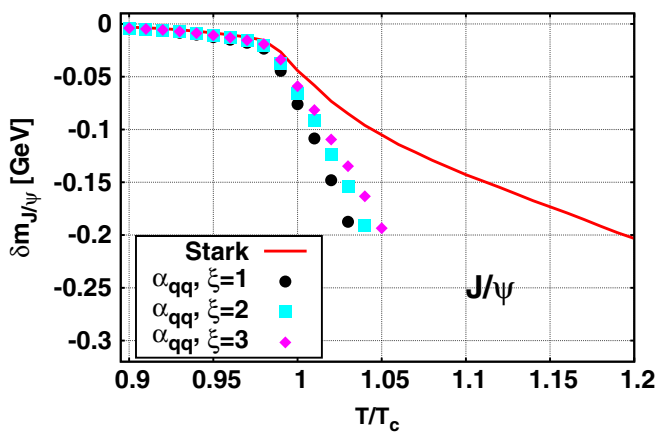

FIG. 3 (color online). Mass shift from the second-order Stark effect (solid line) and the maximal mass shift obtained from QCD sum rules from Ref. [12] (points).

could be obtained within the temperature range from T_c to $1.05T_c$. Therefore, the difference between the Stark effect and the maximum mass shift obtained from QCD sum rules above T_c in Fig. 3 could be attributed to the nonperturbative thermal width at finite temperature. In Fig. 4, we plot the thermal width obtained from combining the QCD sum rule constraint with the mass shift obtained from the QCD second-order Stark effect [26]. As can be seen in the figure, the thermal width at $1.05T_c$ becomes as large as 100 MeV. Such width slightly above T_c is larger than that estimated from a perturbative LO and NLO QCD method [27,28], but smaller than a recent phenomenological estimate [29]. The mass of quarkonium at finite temperature was also investigated in the potential models [30], where the mass was found to decrease at high temperature. However, the detailed potential has to be extracted from the lattice at each temperature and hence identifying the critical behavior near T_c will be difficult.

Finally, we comment that while the present result is obtained using lattice calculation in the pure gauge theory, including dynamical quark will not greatly modify the result. This follows from noting that the lattice result for the temperature dependence of pressure p/T^4 is independent of the number of flavors if scaled to their corresponding ideal gas limit [31]. Moreover, as was shown in Ref. [32], $G_0(T)$ extracted from a recent full lattice calculation of the interaction measure [33] after subtracting the quark contributions, and then dividing by a factor of $(1 + \frac{5}{12}n_f)$ appearing in the beta function, shows that the change of the magnitude near T_c is remarkably similar to that of the pure gauge theory. Hence the main input for our result does not change much even in the presence of dynamical quarks.

In summary, we have shown that the sudden increase in the energy density across the phase transition, which is a characteristic behavior of the QCD phase transition independent of the flavor, can be translated to a rapid increase in the electric condensate slightly above T_c. Using the QCD second-order Stark effect, this translates into an equally sudden decrease in the mass of J/ψ, which is around -50 MeV and -100 MeV, respectively, at T_c
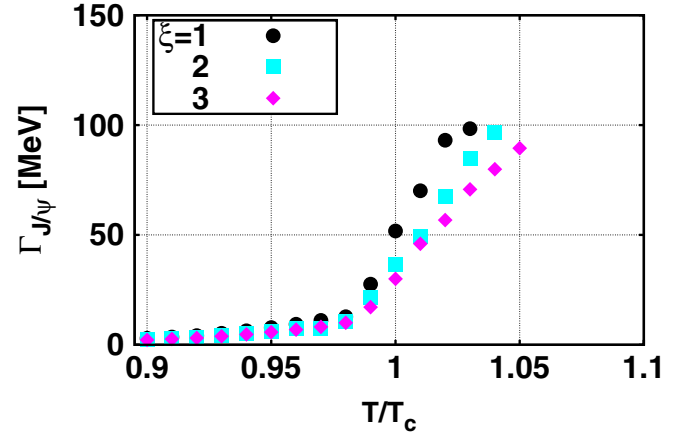

FIG. 4 (color online). Thermal width of J/ψ obtained from the second-order Stark effect and QCD sum rule constraint.

and $1.05T_c$. Combining with a QCD sum rule constraint, we obtain the thermal width of J/ψ slightly above T_c, and found it to be larger than previous perturbative estimates, and becoming as much as 100 MeV at $1.05T_c$. Hence, one can conclude that the critical behavior of J/ψ at T_c is not its sudden disappearance, but rather the abrupt changes of its mass and width. The mass shift is probably too small to be detected with present resolutions at RHIC. However, with the expected upgrades at RHIC and plans at LHC, such direct measurement could be possible. Indeed, the mass resolution of J/ψ for dimuon channel at LHC is 35 MeV for the CMS detector [34] and around 70 MeV for ALICE [35] and ATLAS [36]. It might be better for dielectron channel. Furthermore, the mass shift could also influence production rates within the statistical model [37]. The large width of 100–150 MeV already at $1.05T_c$ suggest that while the maximal entropy method shows a J/ψ peak structure surviving up to $2T_c$, the actual formation at heavy ion collisions might only be possible at lower temperatures where the width of the J/ψ becomes equal to its binding.

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