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Landau gauge gluon and ghost propagators in the refined Gribov-Zwanziger framework in 3 dimensions

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In previous works, we have constructed a refined version of the Gribov-Zwanziger action in 4 dimensions, by taking into account a novel dynamical effect. In this paper, we explore the 3-dimensional case. Analogously to 4 dimensions, we obtain a ghost propagator behaving like $1/p^2$ in the infrared, while the gluon propagator reaches a finite nonvanishing value at zero momentum. Simultaneously, a clear violation of positivity by the gluon propagator is also found. This behavior of the propagators turns out to be in agreement with the recent numerical simulations.

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I. INTRODUCTION

Lately, the infrared behavior of the gluon and the ghost propagator in $SU(N)$ Yang-Mills theories has been exhaustively investigated by many research groups. The low energy behavior of these propagators is of great interest as it might provide helpful information on various aspects of color confinement, which is still far from being understood. Previous results on the gluon and ghost propagators in the Landau gauge have reported an enhanced behavior for the ghost and a suppressed gluon propagator vanishing at zero momentum. This behavior was supported by both numerical simulations [1,2] and analytical studies [3–11].

Nevertheless, recent lattice data on bigger volumes point towards a ghost propagator which is no longer enhanced and a gluon propagator which attains a finite value at zero momentum [12–15]. Recently, several analytical approaches [16–20] have been worked out, being in agreement with these new data.

Among these approaches, we have settled for a framework within the Gribov-Zwanziger approach, successfully employed in the study of the propagators in 4D [17,18] in the Landau gauge, which is defined as $\partial_\mu A_\mu = 0$. We recall here that this condition does not uniquely fix the gauge freedom, as there can be configurations A'_μ, gauge equivalent to A_μ, which also fulfill $\partial_\mu A'_\mu = 0$ [7]. The Gribov-Zwanziger action allows for a (partial) resolution of this problem of gauge (Gribov) copies in a local and renormalizable setting [7–9].

Our refined framework was constructed as follows. We have added two extra terms to the ordinary Gribov-Zwanziger action, without destroying the renormalizability of the action. Let us already recall here that these additional terms precisely correspond to extra, yet unexplored, dynamical effects associated with the Gribov-Zwanziger action [17,18]. The first extra term corresponded to the introduction of a novel mass operator, while the second term represented an additional vacuum energy term which was required in order to remain within the Gribov region Ω. This region is defined as the set of field configurations fulfilling the Landau gauge condition, $\partial_\mu A^a_\mu = 0$, and for which the Faddeev-Popov operator,

$$\mathcal{M}^{ab} = -\partial_\mu(\partial_\mu \delta^{ab} + g f^{acb} A^c_\mu), \tag{1}$$

is strictly positive, namely,

$$\Omega \equiv \{A^a_\mu, \partial_\mu A^a_\mu = 0, \mathcal{M}^{ab} > 0\}. \tag{2}$$

This region is bounded by the horizon, $\partial\Omega$, where the first vanishing eigenvalue of $\mathcal{M}^{ab}$ appears. Doing so, a large number of gauge copies are already excluded, as their appearance is precisely related to the existence of zero modes for $\mathcal{M}^{ab}$ [7]. So far, we have only worked out the 4-dimensional case. The implementation of the Gribov-Zwanziger approach in 3D has not yet been carried out. The 3D case is also conceptually different from the 4D case due to the super-renormalizability, and there is no running of the coupling constant g^2. Hence, it is useful to study the 3D case in detail and make a comparison with the 4D case as well as with the available 3D lattice data.

Also in lattice simulations, one has to deal with the existence of gauge copies in the Landau gauge when studying the propagators. The Landau gauge is numerically implemented by minimizing the functional $\int d^3x A^2$ over its gauge orbit. Notice that the Gribov region Ω corre-

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sponds, in fact, to the set of all relative minima of $\int d^3xA^2$. An ideal implementation of the Landau gauge would correspond to finding the absolute minima of that functional. Even numerically, this is an extremely hard task to achieve, let alone to do it in the continuum formulation. Nevertheless, the gauge field configurations employed in the numerical evaluation correspond to relative minima of the functional $\int d^3xA^2$, so they belong to the region Ω. Let us also mention that, at present, it is unknown how to restrict in the continuum the path integration to gauge fields corresponding to absolute minima of $\int d^3xA^2$. That set of absolute minima forms a subset of the Gribov region Ω, known as the fundamental modular region Λ, $\Lambda \subset \Omega$. This means that Ω itself is still plagued by additional gauge copies [21–23]. Albeit it has been argued in [24] that these additional copies should not contribute to the correlation functions, so that there should be no difference between the regions Ω and Λ, we point out that the restriction to Ω via the local and renormalizable Gribov-Zwanziger approach is the best one can do for now in the continuum. Summarizing, both in the lattice and continuum studies of the gauge theory, one is looking at the same objects, e.g. gluon and ghost propagators (see Appendix C and Sec. V), in such a way that the issue of gauge copies has been at least partially handled. We refer to e.g. [12–15,25] for lattice works.

For the benefit of the reader, we review the original construction of the Gribov-Zwanziger action in Sec. II. Section III presents a detailed motivation of why we should include effects of a mass term like $S_{\bar{\varphi}\varphi} = M^2 \int d^3x(\bar{\varphi}\varphi - \bar{\omega}\omega)$ to the Gribov-Zwanziger action S_{GZ}. Also, the renormalizability of $S_{\mathrm{GZ}} + S_{\bar{\varphi}\varphi}$ is discussed. The requirement of renormalizability/consistency with the symmetries of the model puts rather severe restrictions on the possible additional operators which can be introduced in the theory. We also provide some details on the breaking of the Becchi-Rouet-Stora-Tyutin (BRST) symmetry. In Sec. IV we discuss the need for the inclusion of an extra vacuum term $S_{\mathrm{en}} = M^2 \int d^3x \frac{2(N^2-1)}{g^2N} \varsigma\lambda^2$. Again, we prove the renormalizability of the total action $S_{\mathrm{tot}} = S_{\mathrm{GZ}} + S_{\bar{\varphi}\varphi} + S_{\mathrm{en}}$. In Sec. V, we investigate in detail the gluon and ghost propagators, and already show that for $M^2 \neq 0$, the ghost propagator is not enhanced and the gluon propagator is finite at zero momentum, in accordance with the lattice data of [12–15]. In Sec. VI a variational technique is scrutinized in order to determine the value of M^2. This mass parameter M^2 is thus not added by hand to the original Gribov-Zwanziger action, but is determined in a self-consistent way. Using this variational technique, we shall also investigate the one loop ghost propagator in the full momentum range. Finally, our conclusion is given in Sec. VII.

II. THE ORIGINAL GRIBOV-ZWANZIGER ACTION

We summarize the construction of the Gribov-Zwanziger action in d dimensions, where it is understood that we actually work in dimensional regularization, with $d = 3 - 2\varepsilon$. We start from the following action [9]:

$$S_{\mathrm{start}} = S_{\mathrm{YM}} + S_{\mathrm{Landau}} - \gamma^4 g^2 \int d^dx f^{abc} A^b_\mu (\mathcal{M}^{-1})^{ad} f^{dec} A^e_\mu, \qquad (3)$$

where

$$S_{\mathrm{YM}} = \frac{1}{4}\int d^dx F^a_{\mu\nu} F^a_{\mu\nu} \qquad (4)$$

is the classical Yang-Mills action and

$$S_{\mathrm{Landau}} = \int d^dx (b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b) \qquad (5)$$

denotes the Landau gauge fixing and the ghost part. The part of expression (3) proportional to the Gribov mass parameter γ is the nonlocal horizon function, which implements the restriction to the first Gribov region, with the proviso that this γ is not free, but subject to the horizon condition [9]

$$\langle h(x)\rangle = d(N^2 - 1), \qquad (6)$$

where $h(x)$ is the so-called horizon function

$$h(x) = g^2 f^{abc} A^b_\mu (\mathcal{M}^{-1})^{ad} f^{dec} A^e_\mu. \qquad (7)$$

This nonlocal horizon function has also received attention from the lattice community; see for example [26], where a lattice formulation of the horizon function can be found.

In order to find a manageable local quantum field theory, one adds extra fields $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$ to localize the nonlocal part of the action (3). Doing so, the Gribov-Zwanziger action becomes [9,27]

$$S_{\mathrm{GZ}} = S_0 - \int d^dx(\gamma^2 g f^{abc} A^a_\mu \varphi^{bc}_\mu + \gamma^2 g f^{abc} A^a_\mu \bar{\varphi}^{bc}_\mu + d(N^2-1)\gamma^4), \qquad (8)$$

with

$$S_0 = S_{\mathrm{YM}} + \int d^dx (b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu (D_\mu c)^a) + \int d^dx(\bar{\varphi}^a_i \partial_\nu (D_\nu \varphi_i)^a - \bar{\omega}^a_i \partial_\nu (D_\nu \omega_i)^a - g(\partial_\nu \bar{\omega}^a_i) f^{abm} (D_\nu c)^b \varphi^m_i), \qquad (9)$$

whereby $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu)$ are a pair of complex conjugate bosonic fields, whereas $(\bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$ are anticommuting ghost fields. Based on a global $U(f)$ symmetry, $f = d(N^2 - 1)$, with respect to the composite index $i = (\mu, c)$ of the additional fields $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \bar{\omega}^{ac}_\mu, \omega^{ac}_\mu)$, we have introduced a more convenient notation $(\bar{\varphi}^{ac}_\mu, \varphi^{ac}_\mu, \bar{\omega}^{ac}_\mu, \omega^{ac}_\mu) = (\bar{\varphi}^a_\mu, \varphi^a_i, \bar{\omega}^a_i, \omega^a_i)$. Notice that for $d = 3$, $\dim(g^2) = 1$ and $\dim(\gamma^2) = 3/2$. Defining the quantum effective action Γ by means of

$$e^{-\Gamma} = \int d\Psi e^{-S}, \tag{10}$$

where Ψ is shorthand for all the fields, then it is easily shown that the horizon condition (6) is in fact equivalent to the requirement that

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0 \tag{11}$$

is fulfilled for $\gamma^2 \neq 0$, or

$$\langle g f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \rangle = -2d(N^2-1)\gamma^2. \tag{12}$$

This means that γ will become fixed in terms of the natural scale of the theory which, in 3D, is provided by the coupling itself. We thus expect to find $\gamma^2 \propto g^3$.

As it has been shown in [9,27,28], the action (8) is renormalizable to all orders, and is thus suitable for quantum computations.

III. THE GRIBOV-ZWANZIGER ACTION COMPLEMENTED WITH A NEW OPERATOR

A. Proposal

As carried out in the 4-dimensional case [17,18], we introduce the local composite operator (LCO) $\bar{\varphi}\varphi$ into the action (8). We shall couple this mass term to the action using a source J, and for renormalization purposes, we have to add this term in a BRST invariant way. Therefore, we shall see that the ω sector must gain the same mass. Hence, we consider the following action:

$$S' = S_{\rm GZ} + S_{\bar{\varphi}\varphi}, \qquad S_{\bar{\varphi}\varphi} = \int d^d x [-J(\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a)], \tag{13}$$

with $S_{\rm GZ}$ the original Gribov-Zwanziger action (8). The new source J has the dimension of a mass squared. Therefore, we shall often use the notation $M^2 = J$.

B. Motivation

Let us briefly repeat the motivation for adding an extra term to the Gribov-Zwanziger action. As pointed out in [17], the fields $(\bar{\varphi}_\mu^{ac}, \varphi_\mu^{ac}, \bar{\omega}_\mu^{ac}, \omega_\mu^{ac})$ introduced to localize the horizon function appearing in (3) are interacting fields corresponding, in fact, to the nonlocal dynamics associated with the horizon function. Therefore, these fields will develop their own quantum dynamics. We draw attention to the fact that the A and φ fields are intimately entangled, since there is a quadratic $A\varphi$-mixing term present in the tree-level action (8), namely, $g\gamma^2 f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc})$. Hence, we expect that any effect in the φ sector will immediately reflect in the gluon sector, altering, in particular, the behavior of the gluon propagator at zero momentum. This is nothing more than saying that the horizon strongly influences the gluon dynamics.

On top of the previous argument, we can give a second argument for why one should add the LCO $(\bar{\varphi}\varphi - \bar{\omega}\omega)$ to the action. The horizon condition corresponds to

$$\langle g f^{abc} A_\mu^a (\varphi_\mu^{bc} + \bar{\varphi}_\mu^{bc}) \rangle = -2d(N^2-1)\gamma^2, \tag{14}$$

i.e. to a condensate proportional to γ^2. One might then also expect that $(\bar{\varphi}\varphi - \bar{\omega}\omega)$ will be nonvanishing too when $\gamma^2 \neq 0$. Hence, one is almost obliged to incorporate the effects related to the operator $(\bar{\varphi}\varphi - \bar{\omega}\omega)$. To make this argument more explicit, let us compute the *perturbative* value of the condensate $\langle \bar{\varphi}\varphi - \bar{\omega}\omega \rangle$. We start from

$$\langle \bar{\varphi}\varphi - \bar{\omega}\omega \rangle_{\rm pert} = -\frac{\partial W(J)}{\partial J}\bigg|_{J=0}, \tag{15}$$

with $W(J)$ the generating functional defined in our case as

$$e^{-W(J)} = \int [d\Psi] e^{-S'} \tag{16}$$

and with S' the extended Gribov-Zwanziger action given in (13). At lowest order, one finds

$$W(J) = -d(N^2-1)\gamma^4 + \frac{N^2-1}{2}(d-1) \times \int \frac{d^d q}{(2\pi)^d} \ln\left(q^4 + q^2 \frac{2g^2 N\gamma^4}{q^2 + J}\right). \tag{17}$$

Making use of

$$\begin{aligned} &\int \frac{d^d q}{(2\pi)^d} \ln\left(q^4 + q^2 \frac{2g^2 N\gamma^4}{q^2+J}\right) \\ &\quad = \int \frac{d^d q}{(2\pi) d} \ln(q^4 + q^2 J + 2g^2 N\gamma^4) \\ &\qquad - \int \frac{d^d q}{(2\pi)^d} \ln(q^2 + J) \\ &\quad = \int \frac{d^d q}{(2\pi)^d} \ln(q^2 + \omega_+^2) + \int \frac{d^d q}{(2\pi)^d} \ln(q^2 + \omega_-^2) \\ &\qquad - \int \frac{d^d q}{(2\pi)^d} \ln(q^2 + J) \end{aligned} \tag{18}$$

with

$$\omega_\pm^2 = \frac{J \pm \sqrt{J^2 - 4\lambda^4}}{2}, \tag{19}$$

where $\lambda^4 = 2g^2 N\gamma^4$, and after employing a standard integral in dimensional regularization,

$$\int \frac{d^d \ell}{(2\pi)^d} \ln(\ell^2 + m^2) = -\frac{(m^2)^{d/2}}{(4\pi)^{d/2}} \Gamma(-d/2), \tag{20}$$

we find the following ultraviolet *finite* result:

$$W(J) = -3(N^2-1)\frac{\lambda^4}{2g^2 N} + \frac{N^2-1}{6\pi}(-\omega_+^3 - \omega_-^3 + J^{3/2}). \tag{21}$$

Using this explicit expression, we can easily obtain the perturbative value of the condensate (15), reading

$$\langle\bar{\varphi}\varphi - \bar{\omega}\omega\rangle_{\rm pert} = \sqrt{2}\frac{N^2-1}{8\pi}\lambda \approx 0.056(N^2-1)\lambda, \quad (22)$$

where λ is the nonzero solution of $\frac{\partial\Gamma(\lambda)}{\partial\lambda}=0$. Since at one loop

$$\Gamma(\lambda) = -d(N^2-1)\frac{\lambda^4}{2Ng^2} + \frac{N^2-1}{2}(d-1)\int\frac{d^dq}{(2\pi)^d}\times\ln(q^4+\lambda^4)$$
$$= -3(N^2-1)\frac{\lambda^4}{2g^2N} + \frac{\sqrt{2}}{6\pi}(N^2-1)\lambda^3, \quad (23)$$

we find that

$$\lambda = \frac{\sqrt{2}}{12\pi}g^2N, \quad (24)$$

with

$$E_{\rm vac} = g^6\frac{N^3(N^2-1)}{10\,368\pi^4} > 0. \quad (25)$$

We notice that the one loop vacuum energy corresponding to the Gribov-Zwanziger action is *positive*. The same feature was also observed in the one loop 4D case [18].

Besides this perturbative value of the condensate (22), it could also be possible that another, nonperturbative, value emerges. However, to calculate this nonperturbative value, one should calculate the Legendre transformation of the generating functional $W(J)$ to obtain the effective action $\Gamma(\sigma)$, where the absolute minimum configuration $\sigma_* \sim \bar{\varphi}\varphi - \bar{\omega}\omega$ would correspond to the true, energetically favored, vacuum. Here, we limit ourselves to observe that, similar to what was encountered in 4D [18], no such value has been found at one loop, meaning that higher order contributions have to be taken into account. For the interested reader, we have written down the corresponding details in Appendix A.

C. Renormalizability

To prove the renormalizability, let us start from the following action,

$$S'' = S' + S_{\rm LCO}, \qquad S_{\rm LCO} = \int d^dx\rho g^2 J, \quad (26)$$

with S' the action proposed in (13), and with the LCO parameter ρ a new dimensionless quantity. This term in ρ is, in principle, needed to take into account potential divergences proportional to g^2J, which are allowed by power counting and by the symmetries of the action.[1] This term would ensure multiplicative renormalizability of the functional $W(J)$. However, although this term cannot be ruled out at the level of the algebraic analysis, an analogous proof as in 4D can be written down allowing us to consistently set $\rho = 0$ [18].

To study the renormalizability of the Gribov-Zwanziger action, it is highly useful to embed it in an extended action, which reduces to the original model in a specific limit [9]. Doing so, we may have a larger number of Ward identities at our disposal, which are powerful tools to construct the most general possible counterterm. Therefore, proceeding as in the 4D case [18], we shall start with the following action:

$$\Sigma = S_0 + S_{\rm s} + S_{\rm ext} + S_{\bar{\varphi}\varphi} + S_{\rm LCO}, \quad (27)$$

with S_0 given in (8), $S_{\bar{\varphi}\varphi}$ in (13), and

$$S_{\rm s} = s\int {\rm d}^dx(-U^{ai}_\mu(D_\mu\varphi_i)^a - V^{ai}_\mu(D_\mu\bar{\omega}_i)^a - U^{ai}_\mu V^{ai}_\mu),$$
$$= \int d^dx(-M^{ai}_\mu(D_\mu\varphi_i)^a - gU^{ai}_\mu f^{abc}(D_\mu c)^b\varphi^c_i + U^{ai}_\mu(D_\mu\omega_i)^b - N^{ai}_\mu(D_\mu\bar{\omega}_i)^a - V^{ai}_\mu(D_\mu\bar{\varphi}_i)^a + gV^{ai}_\mu f^{abc}(D_\mu c)^b\bar{\omega}^c_i - M^{ai}_\mu V^{ai}_\mu + U^{ai}_\mu N^{ai}_\mu),$$
$$S_{\rm ext} = \int d^dx\left(-K^a_\mu(D_\mu c)^a + \frac{1}{2}gL^a f^{abc}c^bc^c\right). \quad (28)$$

We introduced new sources M^{ai}_μ, V^{ai}_μ, U^{ai}_μ, N^{ai}_μ, K^a_μ, and L^a, which are necessary to analyze the renormalization of the corresponding composite field operators in a BRST invariant fashion. The BRST operator s is defined through

$$\begin{aligned} &sA^a_\mu = -(D_\mu c)^a, \qquad sc^a = \tfrac{1}{2}gf^{abc}c^bc^c,\\ &s\bar{c}^a = b^a, \qquad sb^a = 0, \qquad s\varphi^a_i = \omega^a_i,\\ &s\omega^a_i = 0, \qquad s\bar{\omega}^a_i = \bar{\varphi}^a_i, \qquad s\bar{\varphi}^a_i = 0,\\ &sU^{ai}_\mu = M^{ai}_\mu, \qquad sM^{ai}_\mu = 0, \qquad sV^{ai}_\mu = N^{ai}_\mu,\\ &sN^{ai}_\mu = 0, \qquad sK^a_\mu = 0, \qquad sL^a = 0, \qquad sJ = 0, \end{aligned} \quad (29)$$

so that s is nilpotent, $s^2 = 0$. One can easily see that the action Σ is indeed BRST invariant, $s\Sigma = 0$. We underline the fact that the mass operator itself, $\bar{\varphi}\varphi - \bar{\omega}\omega$, is also BRST invariant. In Table I, we have summarized for all the fields and sources their mass dimension, ghost number, and $\mathcal{Q}_f$ charge, which is defined by means of the diagonal generator U_{ii} of the global $U(f)$ symmetry. We have chosen these mass dimensions such that, in any case, the action of the BRST transformation s raises the dimension by $1/2$.

At the end, we can give the sources the following physical values:

$$\begin{aligned} M^{ab}_{\mu\nu}|_{\rm phys} &= V^{ab}_{\mu\nu}|_{\rm phys} = \gamma^2\delta^{ab}\delta_{\mu\nu},\\ U^{ai}_\mu|_{\rm phys} &= N^{ai}_\mu|_{\rm phys} = K^a_\mu|_{\rm phys} = L^a|_{\rm phys} = 0, \end{aligned} \quad (30)$$

in order to recover the physically relevant action S'', given in (26). If the action Σ is renormalizable for any value of the sources, it will of course be for the specific values (30). This is an example of the fact that it can be highly useful to

[1] We refer to [29] for more details concerning the LCO formalism and the precise role of the LCO parameter.

TABLE I. Quantum numbers of the fields.

	A^a_μ	c^a	$\bar{c}^a$	b^a	φ^a_i	$\bar{\varphi}^a_i$	ω^a_i	$\bar{\omega}^a_i$	U^{ai}_μ	M^{ai}_μ	N^{ai}_μ	V^{ai}_μ	K^a_μ	L^a	J
Dimension	1/2	0	1	3/2	1/2	1/2	1	0	1	3/2	2	3/2	2	5/2	2
Ghost number	0	1	−1	0	0	0	1	−1	−1	0	1	0	−1	−2	0
Q_f charge	0	0	0	0	1	−1	1	−1	−1	−1	1	1	0	0	0

use a slightly more general version than the original action, whereby a more powerful set of Ward identities can be invoked to prove e.g. the renormalizability of S'. We have collected the details of the algebraic renormalization analysis in Appendix B. The main conclusion drawn is that the action Σ (27) is multiplicatively renormalizable to all orders of perturbation theory.

It is important to notice here that it is the Ward identities defining the extended Gribov-Zwanziger action (27) which dictate which terms can be added to the action, without jeopardizing the symmetry content, in general, and the renormalizability, in particular. From this perspective, the mass term (13) displays remarkable features. In fact, its introduction turns out to be compatible with both Ward identities and renormalizability, as originally proposed in [17].

D. The breaking of the BRST symmetry

We would like to draw attention to the fact that the actions S and S' are BRST invariant if $\gamma^2 = 0$, since then the extra fields can be integrated out, and we are left with the original Yang-Mills action in the Landau gauge. Evidently, we then have $\bar{\varphi}\varphi - \bar{\omega}\omega = -s(\bar{\omega}\varphi) = 0$. It is only when $\gamma^2 \neq 0$ that we can have such a nonvanishing condensate, since in that case the BRST symmetry generated by s is no longer preserved, since $sS' \neq 0$. More precisely, the "horizon terms" $\propto \gamma^2$ in Eq. (8) are not BRST invariant. It is an important observation that the Gribov-Zwanziger action is *not* BRST invariant: the BRST symmetry is explicitly broken by soft terms $\sim\gamma^2$. Nevertheless, it is worth emphasizing that this breaking can be kept under control at the quantum level [18]. In particular, the introduction of the local sources U, V, M, and N (see also [30]) nicely allows one to embed the action into the larger BRST invariant action, thereby allowing one to prove the renormalizability. When the sources attain their physical values, Eqs. (30), the exact Slavnov-Taylor identities of the larger action induce the corresponding softly broken identities for the physical action S''. The operators coupled to these sources are exactly those relevant for the discussion of the broken Slavnov-Taylor identity; see a similar discussion in [30].

The introduction of the horizon function thus *explicitly* breaks the BRST invariance of the Landau gauge fixed Yang-Mills action. In particular, in [18], it has been shown that the origin of this breaking is deeply related to the restriction to the Gribov region Ω. More precisely, it turns out that infinitesimal gauge transformations of field configurations belonging to the Gribov region Ω give rise to configurations lying outside of Ω. The appearance of the BRST breaking looks thus rather natural, when we keep in mind that the BRST transformation of the gauge field is inherited from its infinitesimal gauge transformation. Let us elaborate a bit on this here. The starting point is that we must restrict the domain of path integration to the region Ω. Let us consider the full set of fields present, A_μ, c, $\bar{c}$, b, and other potential fields. These fields can be seen as the coordinates of a space $\mathcal{F}$. We can define a manifold over $\mathcal{F}$, by means of the action functional,

$$S\colon A_\mu, c, \bar{c}, b, \ldots \in \mathcal{F} \to S(A_\mu, c, \bar{c}, b, \ldots) \in \mathbb{R}. \quad (31)$$

To be more precise, we must restrict ourselves to Ω, so we must consider the functional

$$S_{\text{restricted}}\colon A_\mu, c, \bar{c}, b, \ldots \in \mathcal{F}|A_\mu \in \Omega \to S(A_\mu, c, \bar{c}, b, \ldots) \in \mathbb{R}. \quad (32)$$

In a pictorial way, we can imagine this as some kind of "cylinder" in $\mathcal{F}$ with Ω as a ground surface, and the other unrestricted fields $(c, \bar{c}, \ldots)$ describing the height. Notice that we do not say what the action S precisely is; we only assume that it is BRST invariant. Now, we consider a particular point $(A^*_\mu, c^*, \bar{c}^*, b^*, \ldots)$ very close to the boundary of the cylinder; thus A^*_μ is located very close to the inner boundary of Ω, and $\partial_\mu A^*_\mu = 0$. We can decompose A^*_μ as

$$A^*_\mu = a_\mu + C_\mu, \quad (33)$$

with $C_\mu \in \partial\Omega$; thus C_μ lies on the Gribov horizon. The shift a_μ is a small (infinitesimal) perturbation. Obviously, $\partial_\mu C_\mu = \partial_\mu a_\mu = 0$. We then find

$$\tilde{A}_\mu = \underbrace{C_\mu + a_\mu}_{A^*_\mu} + D_\mu(C)\omega + \ldots \quad (34)$$

for the gauge transformed field. Since $C_\mu \in \partial\Omega$ and setting ω equal to the zero mode corresponding to C_μ yields

$$\partial_\mu \tilde{A}_\mu = \partial_\mu D_\mu(C)\omega = 0. \quad (35)$$

In addition, $\tilde{A}_\mu$ also lies very close to the boundary $\partial\Omega$, but on the other side of this boundary, as shown by Gribov in [7]. Let us now consider the infinitesimal BRST shift of the coordinate set $(A^*_\mu, c^*, \ldots)$, which is given by

$$A_\mu^* \to A_\mu^* + \theta D_\mu c^* \tag{36}$$

for the gauge field, with θ a Grassmann number. We did not specify yet the choice of our particular ghost coordinate. We take $c^* = \frac{\theta'}{\theta\theta'}\omega$, with θ' another Grassmann number,[2] and ω the zero mode. Since the BRST was supposed to be a symmetry of the cylinder, the transformed coordinate set of $(A_\mu^*, c^*, \ldots)$ should still be located within the cylinder. However, by construction of A_μ^* and c^*, we do end up outside of the cylinder. This contradiction means that maintaining the BRST symmetry is not possible when restricting to Ω.

One can also imagine a configuration $A_\mu^{**} \in \Omega$ not located close the boundary $\partial\Omega$; thus $\partial_\mu A_\mu^{**} = 0$ and $-\partial_\mu D_\mu(A^{**}) > 0$. If we are to assume that the BRST transformation of A_μ^{**},

$$A_\mu^{**} + \theta D_\mu c^{**}, \tag{37}$$

with c^{**} arbitrary, would remain within Ω, we are forced to conclude that $\partial_\mu D_\mu[A^{**}](\theta c^{**}) = 0$; thus θc^{**} would then be a zero mode, again in contradiction with the hypothesis that A_μ^{**} is not located on (or close to) the boundary $\partial\Omega$, meaning that there are no such zero modes.

An interesting property worth mentioning is that, despite the loss of the BRST symmetry, one can still use the associated broken Slavnov-Taylor identity to derive relations amongst several Green functions. This has exhaustively been studied in [18], and can be easily transported to the 3D case. We therefore refer the reader to [18] for any detail concerning the BRST breaking and its consequences. In particular, in [18] we have algebraically motivated that the (controlled) BRST breaking allows the Gribov parameter γ to become a physical parameter. If the BRST symmetry is preserved, γ would merely play a role akin to that of an unphysical gauge parameter.

We emphasize here that the soft breaking of the BRST is introduced in such a way that one keeps the nilpotency of the BRST operator. This is an important point, as it enables one to make use of the notion of the BRST cohomology. Indeed, as discussed earlier in this section, the proof of the renormalizability of the action is exactly possible by making use of the BRST cohomology in the extended model (27), which enjoys the BRST symmetry. In fact, one can define the physical operators in the Gribov-Zwanziger model as those obtained from the physical operators $\mathcal{O}_{\text{phys}}$ in the extended model (27), upon taking the physical limit (30) of these operators $\mathcal{O}_{\text{phys}}$, which are nothing more than the cohomology classes of the nilpotent BRST operator. The latter are precisely given by the gauge invariant singlet operators constructed with the field strength and its covariant derivative. In other words, due to the form of the BRST operator, the physical operator content of the theory is left unchanged, being identified with the colorless gauge invariant operators. As a consequence, both gluons and ghosts are excluded from the physical spectrum. Moreover, due to the presence of the Gribov parameter γ and of the mass M, the behavior of the correlation function of gauge invariant operators will also get modified in the infrared region, as it can be inferred from the expression of the resulting gluon propagator. For instance, although the gluon propagator turns out to exhibit positivity violations [see Sec. V, Eq. (50)], one might expect that the correlation functions of gauge invariant operators, like e.g. $\langle F^2(x)F^2(y)\rangle$, could display a real pole in momentum space, which would be related to the mass of a glueball bound state. This topic is currently under study. At asymptotically large momenta, one can neglect the soft BRST breaking term, in which case we are reduced to the normal Yang-Mills physical modes. The degrees of freedom corresponding to the fields φ, $\bar{\varphi}$, ω, and $\bar{\omega}$ will decouple from the physical spectrum, as these fields form BRST doublets [see (29)], and it is well known that these become trivial in the BRST cohomology [31].

E. A few more words about the intricacies of 3D gauge theories: Infrared problems and ultraviolet finiteness

In the previous subsection, we mentioned the case $\gamma^2 = 0$. Strictly speaking, the 3D theory will not be well defined in that case. In the absence of an infrared regulator, the perturbation theory of a super-renormalizable 3D gauge theory is ill defined due to severe infrared instabilities [32]. This can be intuitively understood, as the coupling constant g^2 carries the dimension of a mass. In the absence of an infrared regulator, the effective expansion parameter will look like g^2/p with p a certain external momentum or combination of external momenta. For $p \gg g^2$, very good ultraviolet behavior is apparent, but for $p \ll g^2$, infrared problems emerge. The presence of a dynamical mass scale (s) $m \propto g^2$ could ensure a sensible perturbation series, even for small p, as a natural expansion parameter is then provided by g^2/m. From this perspective, a nonvanishing Gribov mass γ^2 could also serve as an infrared cutoff. This feature is also explicitly seen from Eq. (24), from which an effective dimensionless expansion parameter can be derived as

$$\frac{g^2N}{(4\pi)^{3/2}\lambda} = \frac{3}{2\sqrt{2\pi}} \approx 0.6, \tag{38}$$

a quantity which is at least smaller than 1. The inverse factor $(4\pi)^{3/2}$ is the generic loop integration factor generated in 3D.

Although this is not the main concern of this paper, it might be interesting to perform higher order computations to effectively find out whether all infrared divergences are absent when $\gamma^2 \neq 0$.

In fact, it is possible to couple the regulating mass term $\frac{1}{2}m^2A^2$ to the Gribov-Zwanziger action. We did not con-

[2]Notice that $\theta\theta'$ is a normal number; thus we can divide by it.

sider this in the current paper, but in 4D this has been discussed in full detail in [18]. All Ward identities and relations between renormalization constants are maintained. Moreover, the form of the propagators is only quantitatively influenced by this additional mass m^2, whereby the main consequences of the mass related to $\bar{\varphi}\varphi - \bar{\omega}\omega$ are preserved (see below) [18]. In the presence of $\frac{1}{2}m^2A^2$, all Z factors are 1, or said otherwise, there are no ultraviolet divergences when computing Green functions [33,34]. Having a look at the relations (B18) and (B19), this also means that any other Z factor is 1, and hence the Gribov-Zwanziger theory is completely ultraviolet finite, including the vacuum functional, since there is no independent renormalization for it: the potential divergences related to γ^4 are killed by the already available Z factors, which are themselves trivial, and we already know that there are no divergences related to g^2J. It is understood that, if needed, the mass related to A^2 is brought back to zero, as the other mass parameters are expected to cure the theory in the infrared. This should be checked case by case. For the purposes of this paper, we shall later see that everything works out fine without a mass coupled to A^2.

IV. THE GRIBOV-ZWANZIGER ACTION SUPPLEMENTED BY AN EXTRA VACUUM TERM

A. Proposal

We propose to add an extra vacuum term to the action (13), i.e.

$$S_{\rm en} = 2\frac{d(N^2-1)}{\sqrt{2g^2N}}\int d^dx \varsigma\gamma^2 J, \qquad (39)$$

where the prefactor $2\frac{d(N^2-1)}{\sqrt{2g^2N}}$ is chosen for later convenience.

B. Motivation

We shall now explain the need for the inclusion of this vacuum term. So far, we have altered the original Gribov-Zwanziger action by adding a mass operator to the action. However, we have to be careful that with this addition we are not leaving the Gribov region when performing calculations. Initially, staying inside the Gribov horizon was assured by the horizon condition (11). This condition is equivalent to demanding that the Faddeev-Popov operator defined in (1) is positive, i.e. $-\partial_\mu D_\mu^{ab} > 0$. In turn, looking at the Landau gauge fixing (5), this is equivalent to demanding the positivity of the inverse ghost propagator, which can be deduced from Fig. 1, giving

FIG. 1. The one loop corrected ghost propagator.

$$\begin{aligned}\mathcal{G}^{ab}(k^2) &= \delta^{ab}\mathcal{G}(k^2)\\ &= \delta^{ab}\bigg(\frac{1}{k^2} + \frac{1}{k^2}\bigg[g^2\frac{N}{N^2-1}\int\frac{d^4q}{(2\pi)^4}\frac{(k-q)_\mu k_\nu}{(k-q)^2}\\ &\quad\times\langle A_\mu^a A_\nu^a\rangle\bigg]\frac{1}{k^2}\bigg) + \mathcal{O}(g^4)\\ &= \delta^{ab}\frac{1}{k^2}(1+\sigma(k^2)) + \mathcal{O}(g^4),\end{aligned} \qquad (40)$$

with

$$\sigma(k^2) = \frac{N}{N^2-1}\frac{g^2}{k^2}\int\frac{d^4q}{(2\pi)^4}\frac{(k-q)_\mu k_\nu}{(k-q)^2}\langle A_\mu^a A_\nu^a\rangle, \qquad (41)$$

and with $\langle A_\mu^a A_\nu^a\rangle$ the gluon propagator (see below). We can now rewrite the ghost propagator by performing a resummation as

$$\mathcal{G}(k^2) = \frac{1}{k^2}\frac{1}{1-\sigma(k^2)} + O(g^4) \qquad (42)$$

as we are only working up to order g^4. This corresponds to the usual resummation of a set of connected diagrams into the inverse of the one loop 1-particle-irreducible (1PI) ghost self-energy. We can therefore write the inverse ghost propagator as

$$\mathcal{G}^{-1}(k^2) = k^2(1-\sigma(k^2)) + O(g^4). \qquad (43)$$

Demanding the previous expression to be positive can be translated in the so-called Gribov no-pole condition [7],

$$\sigma(k^2) \leq 1, \qquad (44)$$

which is the key point of this investigation. Without the inclusion of this extra vacuum term, we shall demonstrate that it is impossible to satisfy the no-pole condition. We recall that this no-pole condition was the basis of the original Gribov paper [7].

We have introduced a new parameter ς which is still free and needs to be determined. We notice that this vacuum term is, in a sense, comparable to the vacuum term already present in the action (8), i.e. $-\int d^dx d(N^2-1)\gamma^4$. In fact, in the original Gribov-Zwanziger formulation, this term was also necessary to stay within the horizon. Analogously, as γ is fixed by a gap equation, we shall introduce a second gap equation to determine ς. Proceeding as in the 4D case, we shall impose that

$$\frac{\partial\sigma(0)}{\partial M^2}\bigg|_{M^2=0} = 0, \qquad (45)$$

which assures a smooth limit to the original Gribov-Zwanziger case when $M^2 \to 0$.

C. Renormalizability

The renormalizability of the following action,

$$S_{\rm tot} = S' + S_{\rm en} = S_{\rm GZ} + M^2 \int d^d x \Big[-(\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a) + 2\frac{d(N^2-1)}{\sqrt{2g^2N}} \varsigma \gamma^2 \Big], \tag{46}$$

can be proven analogously to [18]. The vacuum term $S_{\rm en}$ will not give rise to any additional counterterms; hence,

$$\frac{\varsigma_0 \gamma_0^2 J_0}{g_0} = \frac{\varsigma \gamma^2 J}{g}, \tag{47}$$

and consequently, adding this extra term does not give rise to a new renormalization factor,

$$Z_\varsigma = Z_g Z_{\gamma^2}^{-1} Z_J^{-1}. \tag{48}$$

V. THE GLUON AND GHOST PROPAGATORS

We shall use the following conventions for the gluon and the ghost propagator,

$$\begin{aligned} \langle A_\mu^a(-p) A_\nu^b(p) \rangle &= \delta^{ab} \mathcal{D}(p^2) \mathcal{P}_{\mu\nu}(p), \\ \langle c^a(-p) \bar{c}^b(p) \rangle &= \delta^{ab} \mathcal{G}(p^2), \end{aligned} \tag{49}$$

where $\mathcal{P}_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$ is the transverse projector.

A. The gluon propagator

The tree-level gluon propagator corresponding to the action (26) is given by

$$\mathcal{D}(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + \lambda^4}, \tag{50}$$

with

$$\lambda^4 = 2g^2 N \gamma^4. \tag{51}$$

This particular propagator (50) displays the following properties already at tree level:

(i) $\mathcal{D}(p^2)$ is infrared suppressed due to the presence of the mass scales M^2 and λ^4.

(ii) $\mathcal{D}(0) = \frac{M^2}{\lambda^4}$, i.e. the gluon propagator does not vanish at zero momentum if M^2 is different from zero.

These properties seem to be in qualitative accordance with the lattice data [12–14]. We also want to stress that the mass term related to $\bar{\varphi}\varphi - \bar{\omega}\omega$ plays a crucial role in having $\mathcal{D}(0) \neq 0$, since in the standard Gribov-Zwanziger scenario, the gluon propagator necessarily goes to zero.

It is instructive to determine the one loop value of the gluon propagator near zero momentum in order, for example, to produce a numerical estimate to compare with other methods. Again, this calculation is similar to that performed in 4D and we record those features which are different for our 3-dimensional analysis. First, we note that for this exercise we will follow [35], where the form of the gluon and φ field two-point mixing term is defined differently. For this intermediate calculation, here it is appropriate to use the conventions and method of [35] since they have been demonstrated to be consistent with the gluon suppression and ghost enhancement of the original Gribov analysis at two loops in $\overline{\rm MS}$ in 4D. At the end, we have been careful in converting back to the main conventions of this article in deriving the one loop freezing value of the gluon propagator. Our method involves computing the matrix of two-point functions comprising the gluon-φ field sector at one loop in the $\overline{\rm MS}$ scheme and then inverting this at one loop to determine the quantum corrections to the tree propagators. However, as we are ultimately only interested in the zero momentum limit, we restrict ourselves to evaluating the two-point functions in the zero momentum limit from the outset using the vacuum bubble expansion. In this way, all 14 contributing Feynman diagrams are expanded in powers of the external momentum p^2 but truncated at $O((p^2)^2)$. Unlike in 4D all the diagrams are ultraviolet finite and the renormalization of all the parameters is trivial. In other words, one simply replaces bare quantities by their corresponding renormalized ones. In effect, this is a trivial $\overline{\rm MS}$ renormalization. In addition, the basic one loop vacuum bubble integral is simple to compute and is given by differentiating (20) with respect to m^2. To determine the freezing value of the gluon propagator, we have adapted the computer programme used for the 4-dimensional computation [18], written in the symbolic manipulation language FORM [36], to the 3-dimensional case. As indicated already, this is the main reason for adopting the conventions of [35] here, and it requires the replacement of the basic one loop 4-dimensional integrals by their 3-dimensional counterparts in the FORM programme. While this may seem a trivial exercise, there is an important aspect to note. In performing automatic Feynman diagram computations, where the graphs are generated electronically with the QGRAF package [37], one always has to ensure the correctness of the result. Paradoxically an ultraviolet divergent 4-dimensional calculation is easier to check than a finite 3-dimensional one. The reason for this resides in the fact that for the former the correct nontrivial renormalization constants have to emerge, and in the case of a gauge theory, these have to satisfy the Slavnov-Taylor identities. In a finite calculation the luxury of this check is absent. However, in the present case the adaptation of a verified programme with minor changes gives us confidence in the eventual value we will derive. Moreover, a reasonable degree of consistency with other results, such as lattice methods, adds to our confidence in the result, as will become evident later.

For completeness, the Landau gauge propagators we use are, for an arbitrary color group,

$$\langle A^a_\mu(p)A^b_\nu(-p)\rangle = \frac{\delta^{ab}(p^2+M^2)}{[(p^2)^2+M^2p^2+C_A\gamma^4]}P_{\mu\nu}(p), \qquad \langle A^a_\mu(p)\bar{\varphi}^{bc}_\nu(-p)\rangle = -\frac{f^{abc}\gamma^2}{\sqrt{2}[(p^2)^2+M^2p^2+C_A\gamma^4]}P_{\mu\nu}(p),$$
$$\langle \varphi^{ab}_\mu(p)\bar{\varphi}^{cd}_\nu(-p)\rangle = -\frac{\delta^{ac}\delta^{bd}}{(p^2+M^2)}\eta_{\mu\nu} + \frac{f^{abe}f^{cde}\gamma^4}{(p^2+M^2)[(p^2)^2+M^2p^2+C_A\gamma^4]}P_{\mu\nu}(p), \tag{52}$$

where f^{abc} are the color group structure constants and the appearance of $1/\sqrt{2}$ derives from the conventions of [35]. We formally define the matrix of one loop corrections to the two-point functions as

$$\begin{pmatrix} p^2\delta^{ac} & -\gamma^2 f^{acd} \\ -\gamma^2 f^{cab} & -(p^2+M^2)\delta^{ac}\delta^{bd}\end{pmatrix} + \begin{pmatrix} X\delta^{ac} & Uf^{acd} \\ Nf^{cab} & Q\delta^{ac}\delta^{bd} + Wf^{ace}f^{bde} + Rf^{abe}f^{cde} + Sd_A^{abcd}\end{pmatrix}g^2 + O(g^4), \tag{53}$$

in the $\{\frac{1}{\sqrt{2}}A^a_\mu, \varphi^{ab}_\mu\}$ basis where the common tensor $P_{\mu\nu}(p)$ has been removed, d_A^{abcd} is defined by [38]

$$d_A^{abcd} = \frac{1}{6}\mathrm{Tr}(T^a_A T^{(b}_A T^c_A T^{d)}_A) \tag{54}$$

and $(T^a_A)_{bc} = -if^{abc}$ is the adjoint representation of the color group generators. Given the set of formal one loop two-point functions, it is easy to invert the matrix to one loop and formally derive the corresponding matrix of propagators as

$$\begin{pmatrix} \frac{(p^2+M^2)}{[(p^2)^2+M^2p^2+C_A\gamma^4]}\delta^{cp} & -\frac{\gamma^2}{[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{cpq} \\ -\frac{\gamma^2}{[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{pcd} & -\frac{1}{(p^2+M^2)}\delta^{cp}\delta^{dq} + \frac{\gamma^4}{(p^2+M^2)[(p^2)^2+M^2p^2+C_A\gamma^4]}f^{cdr}f^{pqr}\end{pmatrix}$$
$$+ \begin{pmatrix} A\delta^{cp} & Cf^{cpq} \\ Ef^{pcd} & G\delta^{cp}\delta^{dq} + Jf^{cpe}f^{dqe} + Kf^{cde}f^{pqe} + Ld_A^{cdpq}\end{pmatrix}g^2 + O(g^4). \tag{55}$$

We refer to Sec. 3 in [35] for a detailed discussion about the color structures emerging in this matrix.

As we are specifically interested in the gluon propagator, the only quantity of importance here is A, and it is formally the same as in [18]. In other words,

$$A = -\frac{1}{[(p^2)^2+M^2p^2+C_A\gamma^4]^2}\times\bigg[(p^2+M^2)^2X - C_A\gamma^2(N+U)(p^2+M^2) + C_A\gamma^4\Big(Q+C_AR+\frac{1}{2}C_AW\Big)\bigg]. \tag{56}$$

This concludes the derivation of the formal aspects of the computation of the one loop propagator corrections. The actual values of the two-point function contributions now need to be inserted from the vacuum bubble expansion.

Let us point out here that we are looking at the connected gluon two-point function, which is the relevant quantity, also measured on the lattice. The reader might notice that the lowest order part of the gluon propagator in (55) can also be obtained by integrating out the additional $(\bar{\varphi}, \varphi)$ fields in the quadratic approximation and can construct the tree-level gluon propagator in this fashion. In the case $M^2 = 0$, this also corresponds with the result for the gluon propagator from the original semiclassical approach by Gribov [7]. It is worth mentioning that this particular kind of propagator has been used frequently as an ansatz for a long time to do lattice fits; see e.g. [2].

A key difference in evaluating the relevant 14 Feynman diagrams derives from the dimensionality of the basic d-dimensional integral defined by

$$I(\mu) = \int\frac{d^dk}{(2\pi)^d}\frac{1}{(k^2+\mu^2)} \tag{57}$$

for a generic mass μ. On dimensional grounds $I(\mu)$ has mass dimensions of $(d-2)$. In 4 dimensions this of course corresponds to a mass dimension of 2. By contrast, in 3 dimensions this drops to 1. However, in each of the tree propagators there is a denominator factor which is quadratic in p^2 which gives two roots. In 4D this leads to a very simple partial fraction into two terms and hence the simple evaluation of the basic vacuum integral $I(\mu)$. In 3D the situation is more complicated. The same partial fraction emerges, but one has to choose the sign of the square root of the mass term appearing as μ^2 for the evaluation of the integral in terms of an object of mass dimension 1. In principle, this could lead to several analyses depending on the choice of sign in the square root. Another way of viewing this is to realize that in 3 dimensions one has to consider the common propagator factor as quartic in $\sqrt{p^2}$ and find four roots. For completeness, these are

$$m^\pm_+ = \pm\frac{1}{2}\bigg[\sqrt{M^2+2\sqrt{C_A}\gamma^2} + \sqrt{M^2-2\sqrt{C_A}\gamma^2}\bigg]$$
$$m^\pm_- = \pm\frac{1}{2}\bigg[\sqrt{M^2+2\sqrt{C_A}\gamma^2} - \sqrt{M^2-2\sqrt{C_A}\gamma^2}\bigg]. \tag{58}$$

In the limit where $M\to 0$, one recovers the four more recognizable poles of the usual Gribov propagator,

$$\lim_{M\to 0} m^\pm_+ = \pm\tfrac{1}{2}(1+i)\gamma, \qquad \lim_{M\to 0} m^\pm_- = \pm\tfrac{1}{2}(1-i)\gamma. \tag{59}$$

To resolve which is the correct choice of sign for the integral, we recall that the gap equation for the Gribov mass derives from a one loop calculation which also involves the basic integral $I(\mu)$. Examining that calculation in order to have a nontrivial solution with a *positive* coupling constant, one has to take m_+^+ and $m_-^\pm$. Interestingly, in our calculation the only square root which remains in the leading order vacuum bubble expansion relevant for the gluon propagator freezing is $\sqrt{M^2 + 2\sqrt{C_A}\gamma^2}$, which is always real. So there are no issues concerning the relative sizes of M^2 and γ^2, which could have led to a complex value in the square roots of (58). Given these considerations and being careful to revert to our general conventions, we finally find

$$\mathcal{D}(0) = \frac{M^2}{\lambda^4} + \frac{g^2N}{4\pi}\frac{M^4}{\lambda^8}\bigg[\frac{M^4}{\lambda^4}\sqrt{M^2+2\lambda^2} - \frac{M^5}{\lambda^4} - \frac{M^2}{\lambda^2} \times\sqrt{M^2+2\lambda^2} - \frac{17}{12}M^2\lambda^2\frac{\sqrt{M^2+2\lambda^2}}{M^4-4\lambda^4} + \frac{13}{4}\lambda^4\frac{\sqrt{M^2+2\lambda^2}}{M^4-4\lambda^4} - \frac{5}{3}\lambda^6M^2\frac{\sqrt{M^2+2\lambda^2}}{(M^4-4\lambda^4)^2} + \frac{10}{3}\lambda^8\frac{\sqrt{M^2+2\lambda^2}}{(M^4-4\lambda^4)^2} + \frac{7}{4}M - \frac{1}{4}\sqrt{M^2+2\lambda^2}\bigg]. \tag{60}$$

From the previous expression, we can still conclude that, for a nonzero value of M^2, $\mathcal{D}(0) \neq 0$.

B. The ghost propagator

We have found that the one loop corrected ghost propagator can be written as

$$\mathcal{G}(k^2) = \frac{1}{k^2}\frac{1}{1-\sigma(k^2)}, \tag{61}$$

where $\sigma(k^2)$ is the following momentum dependent function:

$$\sigma(k^2) = g^2N\frac{k_\mu k_\nu}{k^2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{(k-q)^2} \times\frac{q^2+M^2}{q^4+M^2q^2+\lambda^4}\mathcal{P}_{\mu\nu}(q). \tag{62}$$

Calculating this integral explicitly, we find

$$\sigma(k^2) = \frac{Ng^2}{32k^3\pi}\bigg\{\frac{1}{M^2-\sqrt{M^4-4\lambda^4}}\bigg(1+\frac{M^2}{\sqrt{M^4-4\lambda^4}}\bigg)\bigg[\sqrt{2}k^3\sqrt{M^2-\sqrt{M^4-4\lambda^4}} - \frac{k}{\sqrt{2}}\Big(M^2-\sqrt{M^4-4\lambda^4}\Big)^{3/2} - k^4\pi + \frac{1}{2}(2k^2+M^2-\sqrt{M^4-4\lambda^4})^2\arctan\frac{\sqrt{2}k}{\sqrt{M^2-\sqrt{M^4-4\lambda^4}}}\bigg] + \frac{1}{M^2+\sqrt{M^4-4\lambda^4}}\bigg(1-\frac{M^2}{\sqrt{M^4-4\lambda^4}}\bigg) \times\bigg[\sqrt{2}k^3\sqrt{M^2+\sqrt{M^4-4\lambda^4}} - \frac{k}{\sqrt{2}}(M^2+\sqrt{M^4-4\lambda^4})^{3/2} - k^4\pi + \frac{1}{2}(2k^2+M^2+\sqrt{M^4-4\lambda^4})^2\arctan\frac{\sqrt{2}k}{\sqrt{M^2+\sqrt{M^4-4\lambda^4}}}\bigg]\bigg\}. \tag{63}$$

In order to find the behavior of the ghost propagator near zero momentum, we take the limit $k^2 \to 0$ in Eq. (62),

$$\sigma(0) = g^2N\frac{2}{3}\int\frac{d^3q}{(2\pi)^3}\frac{1}{q^2}\frac{q^2+M^2}{q^4+M^2q^2+\lambda^4} = \frac{g^2N}{6\pi}\frac{M^2+\lambda^2}{\lambda^2\sqrt{M^2+2\lambda^2}}, \tag{64}$$

which can of course be obtained by taking the limit $k^2 \to 0$ in expression (63). Similarly, one can check that $\sigma \to 0$ for $k^2 \to \infty$ and/or $M^2 \to \infty$.

Before drawing any conclusions, we still need to have a look at the gap equations, which shall fix λ^2 as a function of M^2.

C. The gap equations

We begin with the first gap equation (11) in order to express λ as a function of M^2. The effective action at one loop order is given by

$$\Gamma_\gamma^{(1)} = -d(N^2-1)\gamma^4 + 2\frac{d(N^2-1)}{\sqrt{2g^2N}}\varsigma\gamma^2M^2 + \frac{(N^2-1)}{2} \times(d-1)\int\frac{d^dq}{(2\pi)^d}\ln\frac{q^4+M^2q^2+2g^2N\gamma^2}{q^2+M^2}. \tag{65}$$

With $\lambda^4 = 2g^2N\gamma^4$, we rewrite the previous expression,

$$\mathcal{E}^{(1)} = \frac{\Gamma_\gamma^{(1)}}{N^2-1}\frac{2g^2N}{d} = -\lambda^4 + 2\varsigma\lambda^2M^2 + g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d} \times \ln\frac{q^4+M^2q^2+\lambda^4}{q^2+M^2}, \tag{66}$$

and apply the gap equation (11),

$$0 = -1 + \varsigma\frac{M^2}{\lambda^2} + g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^4+M^2q^2+\lambda^4}. \tag{67}$$

In 3 dimensions, the integral in this gap equation is finite, resulting in

$$0 = -1 + \varsigma\frac{M^2}{\lambda^2} + \frac{g^2N}{6\pi}\frac{1}{\sqrt{M^2+2\lambda^2}}. \tag{68}$$

This expression will fix λ^2 as a function of M^2, i.e. $\lambda^2(M^2)$, once we have found an explicit value for ς.

This explicit value for ς will be provided by the second gap equation (45). From expression (62), one finds

$$\sigma(0) = g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2}\frac{q^2+M^2}{q^4+M^2q^2+\lambda^4} = g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^4+M^2q^2+\lambda^4} + M^2g^2N\frac{d-1}{d}\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2}\frac{1}{q^4+M^2q^2+\lambda^4}. \tag{69}$$

Therefore, we can rewrite the first gap equation (67) as

$$0 = \sigma(0) - 1 - M^2\frac{d-1}{d}g^2N\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2} \times \frac{1}{q^4+M^2q^2+\lambda^4} + \varsigma\frac{M^2}{\lambda^2}. \tag{70}$$

The second gap equation can then subsequently be obtained by acting with $\frac{\partial}{\partial M^2}$ on the previous expression and setting $M^2 = 0$. Doing so, we find

$$-\frac{d-1}{d}g^2N\int\frac{d^dq}{(2\pi)^d}\frac{1}{q^2}\frac{1}{q^4+\lambda^4(0)} + \varsigma\frac{1}{\lambda^2(0)} = 0 \tag{71}$$

by keeping (45) in mind. Setting $M^2 = 0$ in (68) yields

$$\sqrt{2\lambda^2(0)} = \frac{g^2N}{6\pi}. \tag{72}$$

Proceeding with Eq. (71), we find the following simple solution for ς,

$$\varsigma = \frac{1}{12\pi}\frac{2}{\sqrt{2}}\frac{g^2N}{\lambda(0)} = 1. \tag{73}$$

In summary, the following expression,

$$\frac{g^2N}{6\pi}\frac{1}{\sqrt{M^2+2\lambda^2}} = 1 - \frac{M^2}{\lambda^2}, \tag{74}$$

fixes $\lambda^2(M^2)$.

D. The ghost propagator at zero momentum

At this point, we have all the information we need to take a closer look at the ghost propagator at zero momentum. From Eqs. (64) and (74), we find

$$\sigma(0) = \left(\frac{M^2}{\lambda^2}+1\right)\left(1-\frac{M^2}{\lambda^2}\right) = 1 - \frac{M^4}{\lambda^4}. \tag{75}$$

From this expression we can make several observations. First, when $M^2 = 0$, we find that $\sigma(0) = 1$, which is exactly the result obtained in the original Gribov-Zwanziger action [7,9]. Consequently, the ghost propagator (61) is enhanced and behaves like $1/k^4$ in the low momentum region. Second, for any $M^2 > 0$, $\sigma(0)$ is smaller than 1. By contrast, without the inclusion of the extra vacuum term, $\sigma(0)$ would always be bigger than 1, which can be observed from expression (70). Therefore, it is absolutely necessary to include this term. With $\sigma(0)$ smaller than 1, the ghost propagator is not enhanced and behaves as $1/k^2$.

VI. STUDY OF THE DYNAMICAL EFFECTS RELATED TO M^2

A. Variational perturbation theory

In this section, we shall rely on variational perturbation theory in order to find a dynamical value for the hitherto arbitrary mass parameter M^2. Specifically, we follow [18,39] and introduce M^2 as a variational parameter into the theory by replacing the action (46) by

$$S_{\rm GZ} + (1-\ell^k)M^2\int d^dx\left[-(\bar{\varphi}_i^a\varphi_i^a - \bar{\omega}_i^a\omega_i^a) + 2\frac{d(N^2-1)}{\sqrt{2g^2N}}\varsigma\gamma^2\right], \tag{76}$$

where ℓ serves as the loop counting parameter, and formally $\ell = 1$ at the end [18]. In this fashion, it is clear that the original starting action $S_{\rm GZ}$ has not been changed. We have in fact added the terms in M^2, and subtracted them again at k orders higher in the loop expansion. We shall set $k = 2$, as we are working up to one loop. Taking $k = 1$ would destroy the effect of the vacuum term $S_{\rm en}$, which would be inconsistent as explained before.

We shall discuss a possible option to fix M^2. For this, we can expand any quantity $\mathcal{Q}^{(n)}$ (which is evaluated to a certain order n) in powers of ℓ, cut this series to the order n, and set $\ell = 1$. We can then require that $\frac{\partial\mathcal{Q}}{\partial M^2} = 0$, i.e. the principle of *minimal sensitivity* [40]. This latter requirement can be well motivated since an exact calculation of $\mathcal{Q}$ using the action (76), taking the full nonperturbative nature

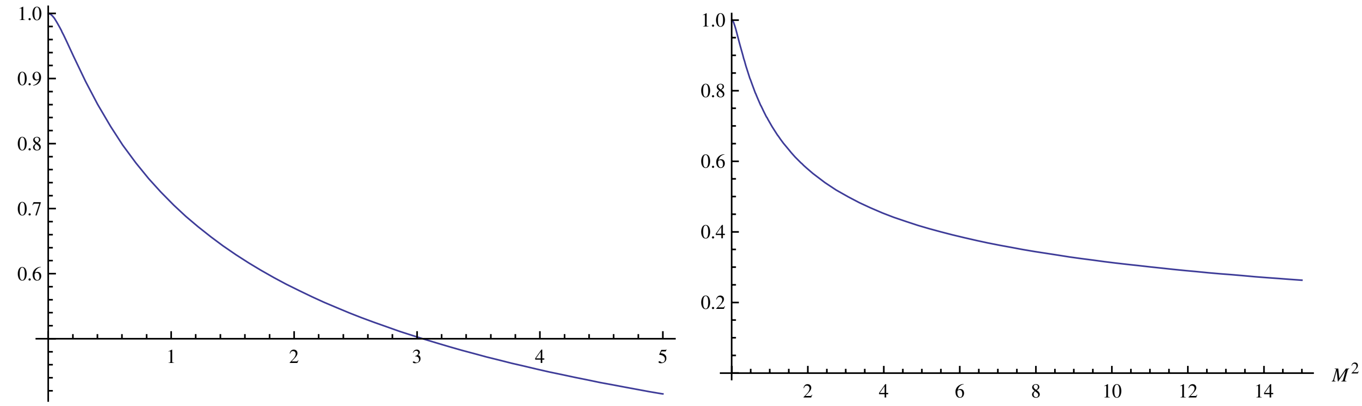

FIG. 2 (color online). $\sigma(0)$ as a function of M^2 in units of $\frac{g^2N}{6\pi} = 1$.

of the theory into account, would lead to an M^2-independent result, since $\ell = 1$ after all. At a finite order in ℓ however, some residual M^2 dependence will enter the result. In this way, we hope to capture some relevant nontrivial information yet at finite order, encoded in the parameter M^2. We then mimic the independence on M^2 of the exact result just by imposing $\frac{\partial Q}{\partial M^2} = 0$ and thereby fixing M^2. If, however, no optimal value for M^2 is found, one can impose that $\frac{\partial^2 Q}{(\partial M^2)^2} = 0$ [40].

B. The ghost propagator

In order to obtain a dynamical value for the ghost propagator in the presence of M^2, we shall now utilize the variational procedure outlined in the previous subsection. We start with the expression (62) for $\sigma(k^2)$ and replace g^2 with ℓg^2 and M^2 with $(1-\ell^2)M^2$, expand up to order ℓ, and set $\ell = 1$. Doing so, we recover again the same expression (62). Following an analogous procedure for the gap equations also results in the same expression (74). This latter equation determines $\lambda^2(M^2)$, which can be plugged into the expression of $\sigma(k^2)$, thereby making $\sigma(k^2)$ only a function of M^2, next to the momentum dependence.

First, let us investigate $\sigma(k^2)$ at zero momentum, which is the key point of this paper. Figure 2 displays $\sigma(0)$ as a function of M^2. We observe that $\sigma(0)$ is indeed smaller than 1 for all $M^2 > 0$ as already shown analytically in the previous section. We also find a smooth limit of $\sigma(0)$ for $M^2 \to 0$ required by the second gap equation (45), as can be seen from the left figure. According to the principle of minimal sensitivity, we have to search for an extremum, i.e. $\frac{\partial \sigma(0)}{\partial M^2} = 0$. Unfortunately, there is no such extremum present. Nevertheless, we do find a point of inflection, $\frac{\partial^2 \sigma(0)}{(\partial M^2)^2} = 0$ at $M^2 = 0.185(\frac{g^2N}{6\pi})^2$. Taking this value for M^2, we find

$$\sigma(0) = 0.94, \tag{77}$$

which is indeed smaller than 1 and results in a nonenhanced behavior of the ghost propagator. This value needs to be compared with the lattice value of $\sigma(0) = 0.79$, which can be extracted from the data in [15].

Second, let us have a look at this point of inflection, when "turning on" the momentum k^2. As shown in Table II, we observe that M^2 will decrease, until it vanishes at $k^2 \approx 0.55$. The corresponding $\sigma(k^2)$ is displayed in Fig. 3. We thus see that we find some kind of a momentum dependent effective mass $M^2(k^2)$, which disappears when k grows. This could have been anticipated, as we naturally expect that the deep ultraviolet sector should hardly be affected. Let us also notice here that $\sigma(k^2)$ is a decreasing function, as can be explicitly checked from Fig. 3. This of course means that we are staying within the horizon for any value of the momentum.

To compare our results with available lattice data, we must make the conversion to physical units of GeV. In [41] a continuum extrapolated value for the ratio $\sqrt{\sigma}/g^2$ was given for several gauge groups; in particular, $\sqrt{\sigma}/g^2 \approx 0.3351$ for $SU(2)$. Further, $\sqrt{\sigma}$ stands for the square root of the string tension. For this quantity, we used the input value of $\sqrt{\sigma} = 0.44$ GeV as in [2]. Therefore, for $SU(2)$, we find

$$\left(\frac{g^2N}{6\pi}\right)^2 \approx 0.0194\ \mathrm{GeV}^2. \tag{78}$$

In Fig. 4, we have plotted the lattice as well as our analytical result for the ghost dressing function, $k^2\mathcal{G}(k^2)$, in

TABLE II. Some $M^2_{\min}$ for different k^2 in units of $\frac{g^2N}{6\pi} = 1$.

k^2	0	0.05	0.1	0.15	0.2	0.25	0.30	0.35	0.40	0.45	0.50	0.55
$M^2_{\min}$	0.19	0.16	0.14	0.12	0.10	0.08	0.06	0.04	0.03	0.02	0.01	0

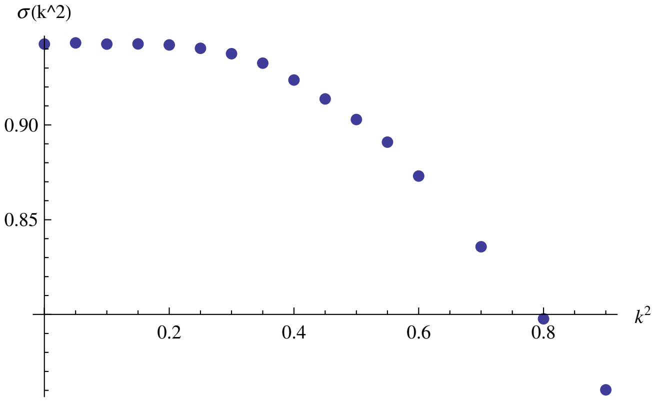

FIG. 3 (color online). The optimal $\sigma(k^2)$ as a function of k^2 in units of $\frac{g^2N}{6\pi} = 1$.

units of GeV. We used the numerical data of [15,42], adapted to our needs. We observe that for sufficiently large k^2, the lattice data and our analytical results converge. In this case, the novel mass M^2 becomes zero as advocated earlier, meaning that we are back in the usual Gribov-Zwanziger scenario. For smaller k^2, we found it more instructive to compare the lattice estimate of $\sigma(k^2)$ with our value, as the errors on $k^2\mathcal{G}(k^2) = \frac{1}{1-\sigma(k^2)}$ become large when looking at $\sigma(k^2)$ close to 1. Figure 5 displays $\sigma(k^2)$ in units of $\frac{g^2N}{6\pi} = 1$ up to $k^2 = 1 \times (\frac{g^2N}{6\pi})^2 = 0.0194\ \mathrm{GeV}^2$. We see that both results are in reasonable agreement, especially if we keep in mind that we have only calculated $\sigma(k^2)$ in a first order approximation.

C. The gluon propagator

We can apply an analogous procedure for the gluon propagator. This propagator at zero momentum, $\mathcal{D}(0)$, is displayed in Fig. 6. We immediately see that there is an extremum at $M^2 = 0.33(\frac{g^2N}{6\pi})^2$, resulting in

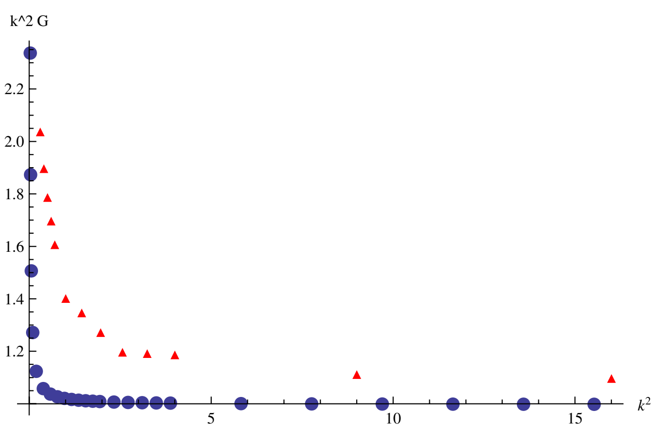

FIG. 4 (color online). The optimal $k^2\mathcal{G}(k^2)$ as a function of k^2 in units of GeV. The lattice (our analytical) results are indicated with triangles (dots). The error bars on the lattice data are roughly the size of the triangles.

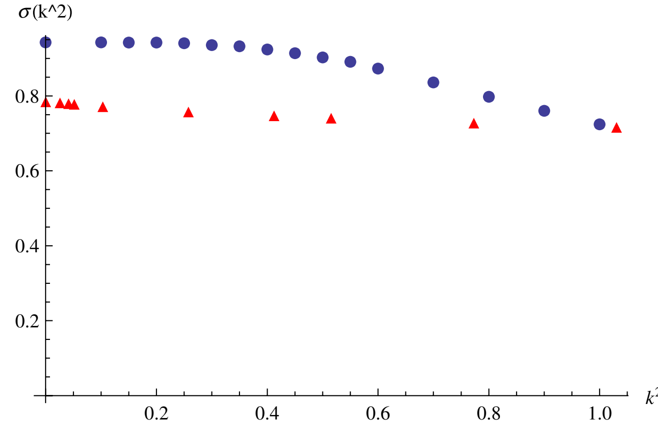

FIG. 5 (color online). The optimal $\sigma(k^2)$ as a function of k^2 in units of $\frac{g^2N}{6\pi} = 1$. The lattice (our analytical) results are indicated with triangles (dots).

$$\mathcal{D}(0) = \frac{0.24}{(\frac{g^2N}{6\pi})^2}. \tag{79}$$

Doing the conversion to physical units again, we find for $SU(2)$,

$$\mathcal{D}(0) \approx \frac{12}{\mathrm{GeV}^2}. \tag{80}$$

This value can be compared with the bounds derived from a partially numerical and partially analytical derivation [14]:

$$\frac{1.2}{\mathrm{GeV}^2} < \mathcal{D}(0) < \frac{12}{\mathrm{GeV}^2}. \tag{81}$$

We recall that our value $\mathcal{D}(0) = 12/\mathrm{GeV}^2$ is a first order approximation and is only of qualitative nature. Nevertheless, this value is still consistent with the boundaries of the lattice data set in [14].

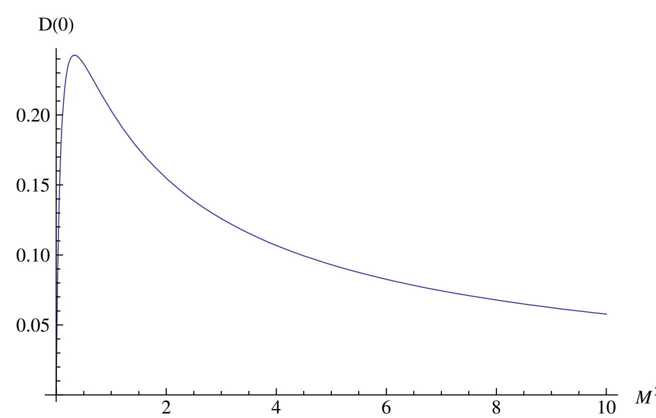

FIG. 6 (color online). $\mathcal{D}(0)$ in units of $\frac{g^2N}{6\pi} = 1$.

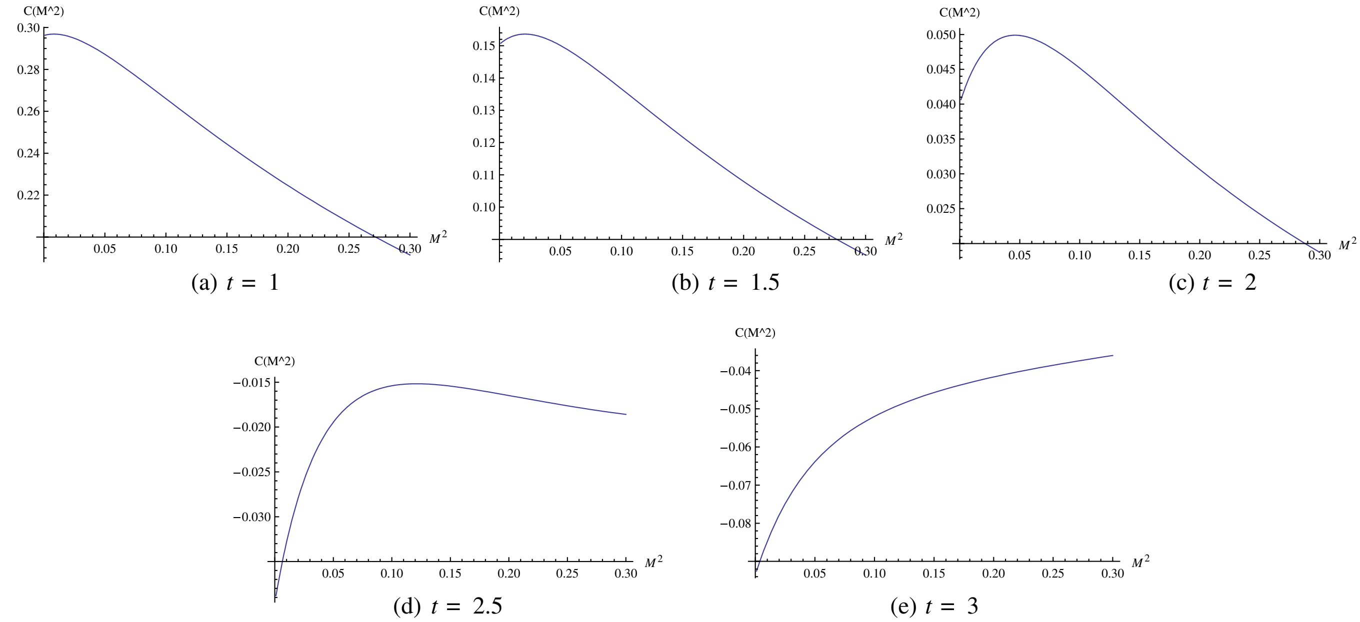

FIG. 7 (color online). $\mathcal{C}(t, M^2)$ for a few values of t as a function of M^2 in units of fm.

D. Violation of positivity

We shall investigate if a gluon propagator of the type (50) displays a violation of positivity, another fact which is reported by the lattice data [2]. Following the analysis of [2], the Euclidean gluon propagator can be expressed by a Källen-Lehmann representation as

$$\mathcal{D}(p^2) = \int_0^{+\infty} dM^2 \frac{\rho(M^2)}{p^2 + M^2}, \tag{82}$$

whereby the spectral density $\rho(M^2)$ should be positive to make possible the interpretation of the fields in term of stable particles. One can define the temporal correlator [2]

$$\mathcal{C}(t) = \int_0^{+\infty} dM \rho(M^2) \mathrm{e}^{-Mt}, \tag{83}$$

which is certainly positive for positive $\rho(M^2)$. However, if $\mathcal{C}(t)$ becomes negative for certain t, $\rho(M^2)$ cannot be positive for all M^2, indicating that the gluon is not a stable physical excitation. Since $\mathcal{C}(t)$ can be rewritten as

$$\mathcal{C}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{e}^{-ipt} \mathcal{D}(p^2) dp, \tag{84}$$

we must in fact calculate the $1D$ Fourier transformation of $\mathcal{D}(p^2)$. In Fig. 7 the Fourier transforms, $\mathcal{C}(t, M^2)$, are shown for different t in units of fm.

To determine M^2, we again rely on the variational setup. We observe that for small t, there is no real extremum. However, for a certain $t \sim 1$, an extremum emerges at $M^2 \sim 0$. This extremum starts to grow for increasing t, but at the same time, the curve flattens out. Therefore, starting from $t \sim 2.6$, the extremum disappears again.

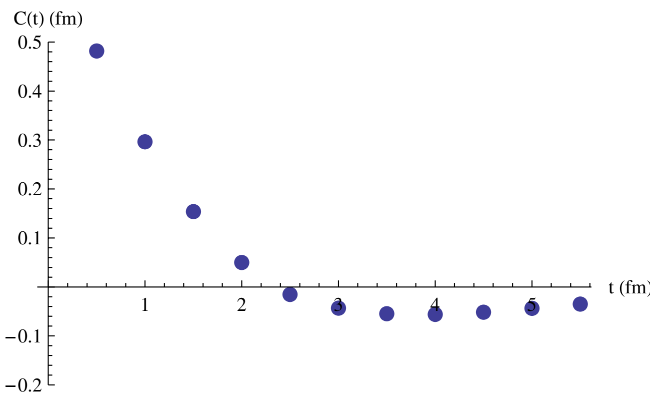

FIG. 8 (color online). $\mathcal{C}(t)$ in terms of t in units of fm in the refined GZ case.

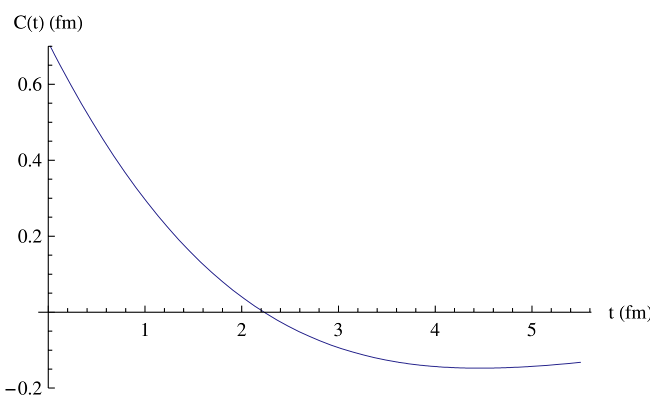

FIG. 9 (color online). $\mathcal{C}(t)$ in terms of t in units of fm in the pure GZ case.

Hence, we have taken[3] $M^2 = 0$ for $t < 1$, and $M^2 = 0.18$ for $t > 2.6$. The resulting temporal correlator is displayed in Fig. 8. We clearly observe a violation of positivity, which is in agreement with the lattice data (see Fig. 4 of [2]). Although this agreement is only at a qualitative level, the shape of $\mathcal{C}(t)$ is very similar.

It would be interesting to have a look at the temporal correlator in the pure Gribov-Zwanziger case ($M^2 = 0$). Therefore, $\mathcal{C}(t)$ is displayed in Fig. 9. We can conclude that this plot is *grosso modo* the same as in the refined Gribov-Zwanziger setup.

VII. CONCLUSION

In this paper, we have extended the 4D analysis of the Gribov-Zwanziger scenario to 3D. The motivations for our work were to include an additional nonperturbative contribution to the Gribov-Zwanziger action, related to a dynamical mass generation effect, and the recent lattice data of [12,14]. Two of the most striking features of these simulations were a finite nonzero value of the gluon propagator at zero momentum and the nonenhancement of the ghost, in both 3D and 4D cases.

We have started from the following renormalizable action,

$$S_{\rm GZ} + M^2 \int d^d x \bigg[-(\bar{\varphi}_i^a \varphi_i^a - \bar{\omega}_i^a \omega_i^a) + 2 \frac{d(N^2-1)}{\sqrt{2g^2 N}} \varsigma \gamma^2 \bigg], \tag{85}$$

which is an extension of the ordinary Gribov-Zwanziger action $S_{\rm GZ}$. Next to the local composite operator $\bar{\varphi}\varphi - \bar{\omega}\omega$, we have also added an extra vacuum term, necessary to ensure the restriction to the Gribov region Ω. In addition, the two parameters γ and ς are determined by the following gap equations,

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0, \qquad \frac{\partial \sigma(0)}{\partial M^2}\bigg|_{M^2=0} = 0. \tag{86}$$

Before fixing M^2, we have written down the explicit expression for the ghost and the gluon propagator. First, the one loop ghost propagator $\mathcal{G}(k^2)$ is given by

$$\mathcal{G}(k^2) = \frac{1}{k^2} \frac{1}{1 - \sigma(k^2)}, \tag{87}$$

with $\sigma(k^2)$ given by expression (63). At zero momentum, $\sigma(0)$ is given by

$$\sigma(0) = 1 - \frac{M^4}{\lambda^4}, \tag{88}$$

already after applying the gap equations. We see that for $M^2 \neq 0$, $\sigma(0)$ is smaller than 1, resulting in a nonenhanced ghost propagator. Second, the tree-level gluon propagator $\mathcal{D}(p^2)$ yields

$$\mathcal{D}(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + \lambda^4}, \tag{89}$$

with $\lambda^4 = 2g^2 N \gamma^4$. Therefore, already at tree level, we find that $\mathcal{D}(0) = \frac{M^2}{\lambda^4}$, which is nonzero for $M^2 \neq 0$. We have also presented the one loop gluon propagator at zero momentum in Eq. (60).

In the last part of this paper, we have discussed a variational method in order to obtain an explicit value for M^2 in the Gribov-Zwanziger model, which has provided a value for the zero momentum ghost propagator [or equivalently $\sigma(0)$] and the gluon propagator. With this method, we have found

$$\sigma(0) = 0.94, \qquad \mathcal{D}(0) = \frac{12}{\text{GeV}^2}. \tag{90}$$

Both values are in qualitative agreement with the lattice data. With this variational method, we also demonstrated the positivity violation of the gluon propagator, which is also confirmed by lattice data.

Let us also spend a few moments on the applicability of a weak coupling expansion. The reader will appreciate that we are no longer perturbing around a trivial vacuum, but are in fact considering a perturbative expansion around a nonperturbative vacuum characterized by a nonvanishing Gribov mass λ, and additionally $M^2 \neq 0$ in our refined framework. These parameters ensure that the no-pole condition (44) is fulfilled, which puts a nonperturbative restriction on the theory. Obviously, we do not claim that we have all the relevant nonperturbative dynamics enclosed in this formalism, but at least the results we have do qualitatively match the available lattice data. The validity of a systematic perturbative expansion around this vacuum state is reflected in a coupling constant which should be sufficiently small; see e.g. (38). This reasoning also applies to the 4D case; see [18].

In summary, we have presented a framework, which consistently accounts for the recent large volume lattice data in the infrared region in 3D and 4D. The question is what happens in 2D, as the most recent data in 2D keep predicting an enhanced ghost in combination with a vanishing zero momentum gluon propagator, contrasting with the higher-dimensional cases [14,15,43]. Details of the Gribov-Zwanziger framework in 2D shall be presented elsewhere, as the situation is rather different there [44].

Previous studies on the infrared behavior of the gluon and ghost propagators within the Schwinger-Dyson formalism in 4D, which were in compliance with the lattice data, can be found in [45–47]. In particular, we observe that in [47] a gluon propagator fit was given as $\mathcal{D}(p^2) = \frac{p^2 + m_0^2}{p^4 + m_0^2 p^2 + m_0^4}$, while the ghost propagator almost behaved like $\frac{1}{p^2}$. It is interesting to notice that this kind of gluon propagator is of the same type as the one found here, in a completely different way. Finally, let us also mention that

[3] $M^2 \sim 0.18$ is the maximal extremal value, corresponding to $t \sim 2.6$.

similar research in the maximal Abelian gauge [48] also gave results in qualitative agreement with the available lattice data in that gauge; see [49].

ACKNOWLEDGMENTS

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APPENDIX A: CONSTRUCTION OF THE EFFECTIVE ACTION $\Gamma(\sigma)$ FROM THE GENERATING FUNCTIONAL $W(J)$

1. General framework

We define the classical field $\sigma(x)$ conjugate to the source $J(x)$ as follows[4]:

$$\sigma(x) = \frac{\delta W(J)}{\delta J(x)}, \tag{A1}$$

with $W(J)$ the generating functional defined in (16). The effective action $\Gamma(\sigma)$ is then obtained in the usual way by a Legendre transformation

$$\Gamma(\sigma) = W(J) - \int d^d x J(x)\sigma(x), \tag{A2}$$

where $J(x)$ is understood to be a functional of $\sigma(x)$. It is easily derived that

$$\frac{\delta\Gamma}{\delta\sigma(x)} = -J(x). \tag{A3}$$

The original theory is recovered when the source J again attains a zero value. A source is nothing more than a tool to probe the theory. In our case, we are incorporating the effects induced by the operator $\bar{\varphi}\varphi - \bar{\omega}\omega$. At the end, a source should always become zero. Equation (A3), for example, corresponds to the quantum version of the classical equation of motion when $J = 0$.

In what follows, we shall limit ourselves to constant J and σ, as we are mainly interested in the (space time independent) vacuum expectation value of the operator coupled to the source J. The remaining task is to obtain the functional form of Γ in terms of σ. Usually, this is done using the background field formalism of Jackiw [50], when an effective potential $\Gamma(\phi_c)$ associated with an elementary field $\phi(x)$ is determined, whereby it is understood that $\phi(x) = \phi_c + \tilde{\phi}(x)$. The quantity $\tilde{\phi}(x)$ represents the quantum fluctuations around the vacuum expectation value of $\phi(x)$, i.e. $\langle\phi(x)\rangle = \phi_c$, $\langle\tilde{\phi}(x)\rangle = 0$. Unfortunately, when a composite operator is considered, this procedure is less useful, as there is no elementary field associated to the operator to begin with. Sometimes, techniques are at one's disposal to introduce such a field, e.g. by employing a Hubbard-Stratonovich transformation [29].

Let us thus proceed by explicitly performing the Legendre transformation, along the lines of [51,52]. The generating functional $W(J)$ will be obtained as a series in the coupling constant,

$$W(J) = W_0(J) + g^2 W_1(J) + \ldots. \tag{A4}$$

As a consequence, we may write

$$\sigma(J) = \sigma_0(J) + g^2\sigma_1(J) + \ldots, \quad \text{with } \sigma_n(J) = \frac{\partial W_n}{\partial J}. \tag{A5}$$

This last series can be inverted to give J as a function of σ,

$$J(\sigma) = J_0(\sigma) + g^2 J_1(\sigma) + \ldots. \tag{A6}$$

As a trivial consequence,

$$\sigma \equiv \sigma(J(\sigma)) = \sigma(J_0) + g^2(J_1\sigma_0'(J_0) + \sigma_1(J_0)) + \ldots. \tag{A7}$$

Since the field σ is supposed to be independent of the coupling constant at the level of the inversion,[5] we find an iterative inversion procedure,

$$\begin{aligned} \sigma &= \sigma_0(J_0) \Rightarrow J_0 = J_0(\sigma_0), \\ 0 &= J_1\sigma_0'(J_0) + \sigma_1(J_0) \Rightarrow J_1 = -\frac{\sigma_1(J_0)}{\sigma_0'(J_0)}, \\ &\vdots \end{aligned} \tag{A8}$$

In this fashion, every term in the series for J can be expressed in terms of J_0, which itself is a function of σ_0. Summarizing, everything can be written in terms of σ_0. Doing so, we can calculate the right-hand side of (A2) up to the desired order in g^2 by simply substituting the corresponding expressions for J_i in the left-hand side.

Once the inversion is performed, we must fix σ_0, which corresponds to the condensate of the operator coupled to J, by demanding that

$$\frac{\partial\{\Gamma_0(\sigma_0) + \Gamma_1(\sigma_0) + \ldots\}}{\partial\sigma_0} = 0 \Leftrightarrow J_0(\sigma_0) + J_1(\sigma_0) + \ldots = 0. \tag{A9}$$

[4]To avoid any confusion, this classical field $\sigma(x)$ has obviously nothing to do with the one loop correction to the ghost form factor $\sigma(k^2)$ first defined in Eq. (40).

[5]Only after imposing the gap equation, $\frac{\partial\Gamma}{\partial\sigma} = 0$, will σ collect a g^2 dependence.

2. Application to the LCO $\bar{\varphi}\varphi - \bar{\omega}\omega$

We shall now employ the previous method on the 3D Gribov-Zwanziger action, in the presence of the LCO $\bar{\varphi}\varphi - \bar{\omega}\omega$. The action we use is given by (26). Using (A5) and (A8) yields

$$\sigma_0(J_0) = \frac{N^2-1}{6\pi}\left\{\frac{3}{2}\sqrt{J_0} - \frac{3}{4}\left(\frac{J_0+\sqrt{J_0^2-4\lambda^4}}{2}\right)\left(1+\frac{J_0}{\sqrt{J_0^2-4\lambda^4}}\right) - \frac{3}{4}\left(\frac{J_0-\sqrt{J_0^2-4\lambda^4}}{2}\right)\left(1-\frac{J_0}{\sqrt{J_0^2-4\lambda^4}}\right)\right\}. \tag{A10}$$

Defining $\tilde{\sigma} \equiv \frac{6\pi}{N^2-1}\sigma_0$, we found six different branches for the inverse function $J_0(\tilde{\sigma})$. However, only one of these was real valued and gave rise to a solution of the gap equation, so we focus our attention on this particular solution,

$$J_0(\tilde{\sigma}) = \frac{36\lambda^2\tilde{\sigma}^2 - \tilde{\sigma}^4}{27\tilde{\sigma}^2} + \frac{1}{432\sqrt[3]{2\tilde{\sigma}^2}}\Big\{\tilde{\sigma}^4[8192\tilde{\sigma}^8 + 442\,368\lambda^2\tilde{\sigma}^6 + 6\,469\,632\lambda^4\tilde{\sigma}^4 + 20\,901\,888\lambda^6\tilde{\sigma}^2 + 45\,349\,632\lambda^8 + 186\,624\sqrt{3}\lambda^5(9\lambda^2+4\tilde{\sigma}^2)\sqrt{243\lambda^2+8\tilde{\sigma}^2}]\Big\}^{1/3} + \frac{256\tilde{\sigma}^8 + 9216\lambda^2\tilde{\sigma}^6 + 51\,840\lambda^4\tilde{\sigma}^4}{2162^{2/3}\tilde{\sigma}^2}\Big\{\tilde{\sigma}^4[8192\tilde{\sigma}^8 + 442\,368\lambda^2\tilde{\sigma}^6 + 6\,469\,632\lambda^4\tilde{\sigma}^4 + 20\,901\,888\lambda^6\tilde{\sigma}^2 + 45\,349\,632\lambda^8 + 186\,624\sqrt{3}\lambda^5(9\lambda^2+4\tilde{\sigma}^2)\sqrt{243\lambda^2+8\tilde{\sigma}^2}]\Big\}^{1/3}.$$

We have displayed a plot of J_0/λ^2 in terms of $\tilde{\sigma}/\lambda$ in Fig. 10. Solving the gap equation $J_0(\tilde{\sigma}) = 0$ numerically leads to

$$\sigma_0 \approx 1.06\frac{N^2-1}{6\pi}\lambda \approx 0.056(N^2-1)\lambda. \tag{A11}$$

This numerical solution is consistent with the analytically derived solution (22). Let us now determine the Gribov mass λ. Therefore, we first compute the effective action using its definition (A2). We did not write down the eventual result in terms of λ and $\tilde{\sigma}$, as the expression is quite lengthy. We subsequently solved the gap equation (horizon condition) $\frac{\partial\Gamma}{\partial\lambda} = 0$ (see Fig. 11) numerically in the case $N = 3$, and we found

$$\lambda \approx 0.113g^2, \tag{A12}$$

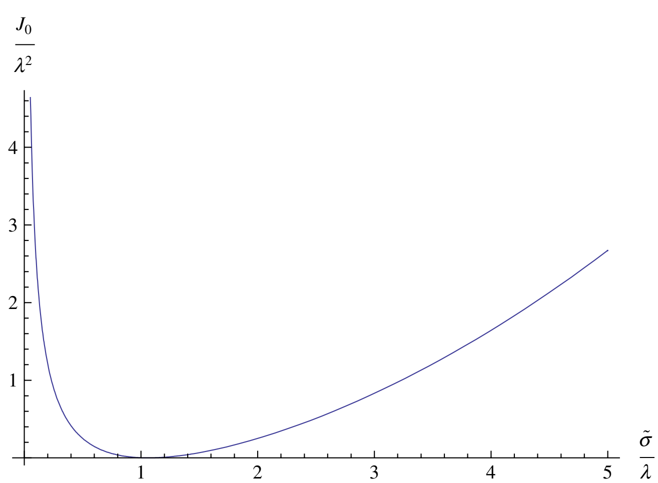

FIG. 10 (color online). A plot of $\frac{J_0}{\lambda^2}$ in terms of $\frac{\tilde{\sigma}}{\lambda}$.

which is also consistent with the analytical result (24). The corresponding vacuum energy is given by

$$E_{\text{vac}} \approx 0.000\,214g^6, \tag{A13}$$

again equivalent to (25).

The gap equation derived from the effective action $\Gamma(\sigma_0, \lambda)$ should of course give back this perturbative solution, next to a potential nonperturbative solution, which can never be discovered by simply using (15). The whole point of the inversion (Legendre transformation) is just that there might be multiple solutions to the equation $J = 0$, next to the perturbative one. In our case, the situation is interesting because there is already a nontrivial scale in the game, namely, the Gribov mass λ. This allows us to obtain a nonvanishing value for the condensate already at the perturbative level.

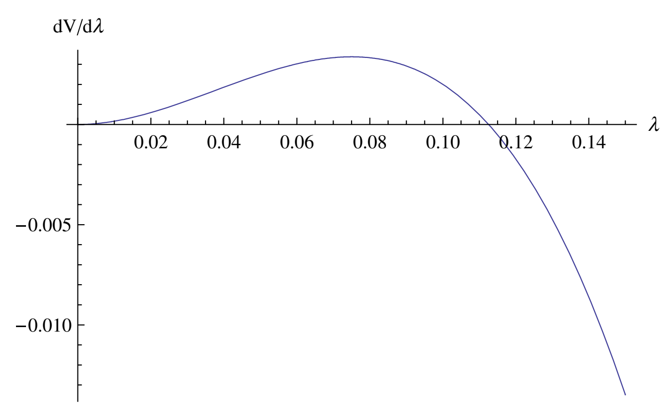

FIG. 11 (color online). The horizon function $\frac{\partial\Gamma}{\partial\lambda}$ for $N = 3$ in units of $g^2 = 1$.

APPENDIX B: THE RENORMALIZABILITY OF THE EXTENDED GRIBOV-ZWANZIGER ACTION: ALGEBRAIC ANALYSIS

1. The Ward identities

The action (27) exhibits several Ward identities which we have listed below.

(i) The global $U(f)$ invariance:

$$U_{ij}\Sigma = 0, \quad U_{ij} = \int d^dx\Big(\varphi_i^a \frac{\delta}{\delta\varphi_j^a} - \bar{\varphi}_j^a \frac{\delta}{\delta\bar{\varphi}_i^a} + \omega_i^a \frac{\delta}{\delta\omega_j^a} - \bar{\omega}_j^a \frac{\delta}{\delta\bar{\omega}_i^a}\Big). \tag{B1}$$

(ii) The Slavnov-Taylor identity:

$$\mathcal{S}(\Sigma) = 0, \quad \mathcal{S}(\Sigma) = \int d^dx\Big(\frac{\delta\Sigma}{\delta K_\mu^a}\frac{\delta\Sigma}{\delta A_\mu^a} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta\Sigma}{\delta c^a} + b^a\frac{\delta\Sigma}{\delta\bar{c}^a} + \bar{\varphi}_i^a\frac{\delta\Sigma}{\delta\bar{\omega}_i^a} + \omega_i^a\frac{\delta\Sigma}{\delta\varphi_i^a} + M_\mu^{ai}\frac{\delta\Sigma}{\delta U_\mu^{ai}} + N_\mu^{ai}\frac{\delta\Sigma}{\delta V_\mu^{ai}}\Big). \tag{B2}$$

(iii) The Landau gauge condition and the antighost equation:

$$\frac{\delta\Sigma}{\delta b^a} = \partial_\mu A_\mu^a, \tag{B3}$$

$$\frac{\delta\Sigma}{\delta\bar{c}^a} + \partial_\mu\frac{\delta\Sigma}{\delta K_\mu^a} = 0. \tag{B4}$$

(iv) The ghost Ward identity:

$$\mathcal{G}^a\Sigma = \Delta_{\rm cl}^a, \quad \mathcal{G}^a = \int d^dx\Big(\frac{\delta}{\delta c^a} + gf^{abc}\Big(\bar{c}^b\frac{\delta}{\delta b^c} + \varphi_i^b\frac{\delta}{\delta\omega_i^c} + \bar{\omega}_i^b\frac{\delta}{\delta\bar{\varphi}_i^c} + V_\mu^{bi}\frac{\delta}{\delta N_\mu^{ci}} + U_\mu^{bi}\frac{\delta}{\delta M_\mu^{ci}}\Big)\Big),$$
$$\Delta_{\rm cl}^a = g\int d^dx f^{abc}(K_\mu^b A_\mu^c - L^b c^c). \tag{B5}$$

Notice that $\Delta_{\rm cl}^a$ is a classical breaking, since it is linear in the quantum fields.

(v) The linearly broken local constraints:

$$\frac{\delta\Sigma}{\delta\bar{\varphi}^{ai}} + \partial_\mu\frac{\delta\Sigma}{\delta M_\mu^{ai}} = gf^{abc}A_\mu^b V_\mu^{ci} + J\varphi_i^a, \tag{B6}$$

$$\frac{\delta\Sigma}{\delta\omega^{ai}} + \partial_\mu\frac{\delta\Sigma}{\delta N_\mu^{ai}} - gf^{abc}\bar{\omega}^{bi}\frac{\delta\Sigma}{\delta b^c} = gf^{abc}A_\mu^b U_\mu^{ci} + J\bar{\omega}_i^a, \tag{B7}$$

$$\frac{\delta\Sigma}{\delta\bar{\omega}^{ai}} + \partial_\mu\frac{\delta\Sigma}{\delta U_\mu^{ai}} - gf^{abc}V_\mu^{bi}\frac{\delta\Sigma}{\delta K_\mu^c} = -gf^{abc}A_\mu^b N_\mu^{ci} - J\omega_i^a, \tag{B8}$$

$$\frac{\delta\Sigma}{\delta\varphi^{ai}} + \partial_\mu\frac{\delta\Sigma}{\delta V_\mu^{ai}} - gf^{abc}\bar{\varphi}^{bi}\frac{\delta\Sigma}{\delta b^c} - gf^{abc}\bar{\omega}^{bi}\frac{\delta\Sigma}{\delta\bar{c}^c} - gf^{abc}U_\mu^{bi}\frac{\delta\Sigma}{\delta K_\mu^c} = gf^{abc}A_\mu^b M_\mu^{ci} + J\bar{\varphi}_i^a. \tag{B9}$$

(vi) The exact $\mathcal{R}_{ij}$ symmetry:

$$\mathcal{R}_{ij}\Sigma = 0, \quad \mathcal{R}_{ij} = \int d^dx\Big(\varphi_i^a\frac{\delta}{\delta\omega_j^a} - \bar{\omega}_j^a\frac{\delta}{\delta\bar{\varphi}_i^a} - V_\mu^{ai}\frac{\delta}{\delta N_\mu^{aj}} + U_\mu^{aj}\frac{\delta}{\delta M_\mu^{ai}}\Big). \tag{B10}$$

We shall now follow the algebraic renormalization procedure [31], according to which the most general allowed invariant counterterm Σ^c is an integrated local polynomial in the fields and sources with dimension bounded by 3, with vanishing ghost number and Q_f-charge, and subject to the following constraints:

$$U_{ij}\Sigma^c = 0, \quad \mathcal{B}_\Sigma\Sigma^c = 0, \quad \frac{\delta\Sigma}{\delta\bar{c}^a} + \partial_\mu\frac{\delta\Sigma}{\delta K_\mu^a} = 0, \quad \frac{\delta\Sigma^c}{\delta b^a} = 0, \quad \mathcal{G}^a\Sigma^c = 0,$$
$$\frac{\delta\Sigma^c}{\delta\varphi^{ai}} + \partial_\mu\frac{\delta\Sigma^c}{\delta V_\mu^{ai}} - gf^{abc}\bar{\omega}^{bi}\frac{\delta\Sigma^c}{\delta\bar{c}^c} - gf^{abc}U_\mu^{bi}\frac{\delta\Sigma^c}{\delta K_\mu^c} = 0, \quad \frac{\delta\Sigma^c}{\delta\bar{\omega}^{ai}} + \partial_\mu\frac{\delta\Sigma^c}{\delta U_\mu^{ai}} - gf^{abc}V_\mu^{bi}\frac{\delta\Sigma^c}{\delta K_\mu^c} = 0, \tag{B11}$$
$$\frac{\delta\Sigma^c}{\delta\omega^{ai}} + \partial_\mu\frac{\delta\Sigma^c}{\delta N_\mu^{ai}} = 0, \quad \frac{\delta\Sigma}{\delta\bar{\varphi}^{ai}} + \partial_\mu\frac{\delta\Sigma}{\delta M_\mu^{ai}} = 0, \quad \mathcal{R}_{ij}\Sigma^c = 0.$$

The operator B_Σ is the nilpotent linearized Slavnov-Taylor operator,

$$\mathcal{B}_\Sigma = \int d^4x \bigg(\frac{\delta\Sigma}{\delta K^a_\mu}\frac{\delta}{\delta A^a_\mu} + \frac{\delta\Sigma}{\delta A^a_\mu}\frac{\delta}{\delta K^a_\mu} + \frac{\delta\Sigma}{\delta L^a}\frac{\delta}{\delta c^a} + \frac{\delta\Sigma}{\delta c^a}\frac{\delta}{\delta L^a} + b^a\frac{\delta}{\delta\bar{c}^a} + \bar{\varphi}^a_i\frac{\delta}{\delta\bar{\omega}^a_i} + \omega^a_i\frac{\delta}{\delta\varphi^a_i} + M^{ai}_\mu\frac{\delta}{\delta U^{ai}_\mu} + N^{ai}_\mu\frac{\delta}{\delta V^{ai}_\mu}\bigg),$$

$$\mathcal{B}_\Sigma\mathcal{B}_\Sigma = 0. \tag{B12}$$

One can show that Σ^c does not depend on the Lagrange multiplier b^a, and that the antighost $\bar{c}^a$ and the i-valued fields φ^a_i, ω^a_i, $\bar{\varphi}^a_i$, $\bar{\omega}^a_i$ can only enter through the combinations [9]

$$\begin{aligned}
\tilde{K}^a_\mu &= K^a_\mu + \partial_\mu\bar{c}^a - gf^{abc}\tilde{U}^{bi}_\mu\varphi^{ci} - gf^{abc}V^{bi}_\mu\bar{\omega}^{ci},\\
\tilde{U}^{ai}_\mu &= U^{ai}_\mu + \partial_\mu\bar{\omega}^{ai}, \qquad \tilde{V}^{ai}_\mu = V^{ai}_\mu + \partial_\mu\varphi^{ai},\\
\tilde{N}^{ai}_\mu &= N^{ai}_\mu + \partial_\mu\omega^{ai}, \qquad \tilde{M}^{ai}_\mu = V^{ai}_\mu + \partial_\mu\bar{\varphi}^{ai}.
\end{aligned} \tag{B13}$$

Imposing the previous constraints arising from the Ward identities, the most general counterterm can be brought in the following compact form:

$$\Sigma^c = a_0S_{\rm YM} + a_1\int d^dx\bigg(A^a_\mu\frac{\delta S_{\rm YM}}{\delta A^a_\mu} + \tilde{K}^a_\mu\partial_\mu c^a + \tilde{V}^{ai}_\mu\tilde{M}^{ai}_\mu - \tilde{U}^{ai}_\mu\tilde{N}^{ai}_\mu\bigg) + a_2\int d^dx\rho g^2J, \tag{B14}$$

with a_0, a_1, and a_2 three arbitrary parameters. It is an impressive feature of the action Σ that only three independent parameters can enter the counterterm, despite the presence of many fields and sources.

2. Stability of the action and the renormalization constants

As the most general counterterm Σ^c compatible with the Ward identities has now been constructed, it still remains to be checked whether the starting action Σ is stable, i.e. that Σ^c can be reabsorbed into Σ through a renormalization of the parameters, fields, and sources appearing in Σ.

According to expression (B14), Σ^c contains three parameters a_0, a_1, and a_2, which correspond to a multiplicative renormalization of the gauge coupling constant g, the parameter ρ, the fields $\phi = (A^a_\mu, c^a, \bar{c}^a, b^a, \varphi^a_i, \omega^a_i, \bar{\varphi}^a_i, \bar{\omega}^a_i)$, and the sources $\Phi = (K^{a\mu}, L^a, M^{ai}_\mu, N^{ai}_\mu, V^{ai}_\mu, U^{ai}_\mu, J)$, according to

$$\Sigma(g, \rho, \phi, \Phi) + \eta\Sigma^c = \Sigma(g_0, \rho_0, \phi_0, \Phi_0) + O(\eta^2). \tag{B15}$$

This is possible, provided that we define the bare quantities in terms of the renormalized quantities as

$$g_0 = Z_g g, \qquad \rho_0 = Z_\rho\rho, \qquad \phi_0 = Z^{1/2}_\phi\phi, \qquad \Phi_0 = Z_\Phi\Phi, \tag{B16}$$

with

$$Z_g = 1 + \eta\frac{a_0}{2}, \qquad Z^{1/2}_A = 1 + \eta\bigg(a_1 - \frac{a_0}{2}\bigg), \qquad Z_\rho = 1 + \eta(a_2 - a_1 - a_0). \tag{B17}$$

These are the only independent renormalization constants. For the rest of the fields, we have

$$Z_c = Z_{\bar{c}} = Z_\varphi = Z_{\bar{\varphi}} = Z_\omega = Z_{\bar{\omega}} = Z_g^{-1}Z_A^{-1/2}, \tag{B18}$$

while for the renormalization of the sources $(M^{ai}_\mu, N^{ai}_\mu, V^{ai}_\mu, U^{ai}_\mu, J)$,

$$Z_M = Z_N = Z_V = Z_U = Z_g^{-1/2}Z_A^{-1/4}, \qquad Z_J = Z_gZ_A^{1/2}. \tag{B19}$$

We see thus that the LCO $\bar{\varphi}\varphi - \bar{\omega}\omega$ does not renormalize independently, as is evident from (B19). The only new parameter entering the game corresponds to the renormalization of the vacuum functional, expressed by ρ and its renormalization factor Z_ρ. As discussed in the main body of this paper, this parameter turns out to be equal to zero anyhow.

APPENDIX C: PROPAGATORS IN LATTICE AND CONTINUUM FORMULATION

For the benefit of the reader, in this appendix we discuss how our results compare with the corresponding lattice data in the case of the gluon propagator. The quantity which is evaluated in both cases is the gluon propagator, namely, the connected gluon two-point function $\langle A^a_\mu(x)A^b_\nu(y)\rangle$, where the gauge field configurations A^a_μ are restricted to the Gribov region Ω. We shall also show that the gluon and the ghost propagators are color diagonal.

1. Continuum formulation

In the continuum formulation, the gluon propagator is given by the connected gluon two-point function, and expressible by means of

$$\langle A^a_\mu(x)A^b_\nu(y)\rangle = \frac{\delta^2 Z^c(J)}{\delta J^a_\mu(x)\delta J^b_\nu(y)}\bigg|_{J=0}, \tag{C1}$$

with $Z^c(J)$ the generating functional of the connected gluon n-point functions, which in our case will read

$$\mathrm{e}^{-Z^c(J)} \equiv \mathrm{e}^{-Z^c(J,J_\varphi,J_{\bar{\varphi}})} = \int[d\Psi]\mathrm{e}^{-(S_{\rm tot}+\int d^4x(J^a_\mu A^a_\mu + J^{ab}_{\varphi,\mu}\varphi^{ab}_\mu + J^{ab}_{\bar{\varphi},\mu}\bar{\varphi}^{ab}_\mu))}, \tag{C2}$$

where $S_{\rm tot}$ is the improved Gribov-Zwanziger action (46). This amounts to considering the Landau gauge fixing, such that the relevant gluon configurations belong to the Gribov region; i.e. these are (local) minima of $\int d^3xA^2$ along the gauge orbit.

As proven in [18], the gluon propagator (C1) is transverse. In a condensed notation, one shall find

$$
\mathrm{e}^{-Z^c(J,J_\varphi,J_{\bar\varphi})} = \mathrm{e}^{-(J \quad J_\varphi \quad J_{\bar\varphi})\mathcal{M}\begin{pmatrix} J \\ J_\varphi \\ J_{\bar\varphi}\end{pmatrix} + \text{higher order terms in } J,J_\varphi,J_{\bar\varphi}}, \tag{C3}
$$

where $\mathcal{M}$ is the matrix propagator, as written down in (55) up to first order. The upper left corner of this matrix $\mathcal{M}$ corresponds precisely to the gluon propagator, as is apparent by taking the second derivative with respect to the source J^a_μ.

If one is interested in the 1PI two-point functions, one should look at the corresponding generator, which is the effective action $\Gamma[A_\mu, \varphi, \bar\varphi]$. As is well known, the corresponding 1PI two-point function will be the inverse of the connected two-point function (or propagator). Said otherwise, the corresponding matrices will be each other's inverse. This is also explained in the main body of the text, with the 1PI two-point function matrix written down in (53), again up to first order.

As we have already stressed earlier in the paper, the extra fields $(\bar\varphi^{ac}_\mu, \varphi^{ac}_\mu, \bar\omega^{ac}_\mu, \omega^{ac}_\mu)$ are introduced in order to obtain a local, manageable field theory that is capable of restricting the gauge field configurations to the Gribov region, which is a rather nontrivial operation in the continuum. In principle, one could opt to work in an effective field theory fashion by again integrating out the extra fields. Clearly, this will give rise to a very complicated (nonlocal) action, written solely in terms of the original Yang-Mills fields. In this formulation, the gluon propagator is directly related to the inverse of the 1PI two-point function, due to the absence of mixing. Anyhow, the result for the propagator itself will be the same as the one already obtained before in the preferable local and manifestly renormalizable formulation with the extra fields, when looking at the same order in g^2. This can be easily checked at tree level: integrating out the auxiliary fields in (46) leads to the following quadratic (nonlocal) effective action,

$$
S_{\text{quad}} = \int d^3x\Big[\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \frac{1}{2\alpha}(\partial_\mu A^a_\mu)^2 - N\gamma^4 g^2 A^a_\mu \frac{1}{\partial^2 - M^2}A^a_\mu + \ldots\Big], \tag{C4}
$$

where the limit $\alpha \to 0$ is understood in order to recover the Landau gauge, and where we skipped the irrelevant constant terms. The tree-level gluon propagator in momentum space is, in this case, $1/\mathcal{Q}_2$, with $\mathcal{Q}_2$ the quadratic form appearing in (C4), when expressed in momentum space. Clearly, this leads back to the lowest order approximation in the upper left corner of (55).

Concerning the ghost propagator, a similar formalism applies.

Let us discuss the issue of color diagonality. The global color symmetry guarantees us that the gluon and the ghost propagators are color diagonal. This property is encoded in the global $SU(N)$ Ward identity, which reads at the classical level

$$
\int d^dx\Big((\delta^b_{\text{adj}}A^a_\mu)\frac{\delta\Sigma}{\delta A^a_\mu} + \sum_\phi(\delta^b_{\text{adj}}\phi)\frac{\delta\Sigma}{\delta\phi}\Big) = 0, \tag{C5}
$$

with Σ the classical action and ϕ all the other fields. In particular,

$$
\delta^b_{\text{adj}}A^a_\mu = f^{abc}A^c_\mu, \tag{C6}
$$

and similar relations for the other fields ϕ. Therefore, we find

$$
\int d^dx\Big(f^{abc}A^c_\mu\frac{\delta\Sigma}{\delta A^a_\mu} + \ldots\Big) = 0. \tag{C7}
$$

This identity can be upgraded to the quantum level,[6]

$$
\int d^dx\Big(f^{abc}A^c_\mu\frac{\delta\Gamma}{\delta A^a_\mu} + \ldots\Big) = 0, \tag{C8}
$$

with Γ the generator of 1PI correlators. Performing the Legendre transformation leads to the analogous Ward identity for the generator Z^c of connected correlators,

$$
\int d^dx\Big(f^{abc}J^a_\mu\frac{\delta Z^c}{\delta J^c_\mu}\Big) = 0. \tag{C9}
$$

We shall concentrate on the gluon sector, so we have already set all other sources equal to zero. Taking a derivative with respect to $J^d_\kappa(y)$ leads to

$$
f^{dbc}\frac{\delta Z^c}{\delta J^c_\kappa(y)} + \int d^dx\Big(f^{abc}J^a_\mu(x)\frac{\delta^2 Z^c}{\delta J^c_\mu(x)\delta J^d_\kappa(y)}\Big) = 0. \tag{C10}
$$

Next, taking a derivative with respect to $J^\ell_\lambda(z)$ and setting $J = 0$ at the end gives the following relationship,

$$
f^{dbc}\frac{\delta^2 Z^c}{\delta J^\ell_\lambda(z)\delta J^c_\kappa(y)}\Big|_{J=0} + f^{\ell bc}\frac{\delta^2 Z^c}{\delta J^c_\lambda(z)\delta J^d_\kappa(y)}\Big|_{J=0} = 0, \tag{C11}
$$

or equivalently

$$
f^{dbc}\langle A^c_\kappa(y)A^\ell_\lambda(z)\rangle + f^{\ell bc}\langle A^d_\kappa(y)A^c_\lambda(z)\rangle = 0. \tag{C12}
$$

This relation expresses nothing more than the fact that the gluon propagator is an $SU(N)$ invariant rank-two tensor. Therefore,

[6] We refer to [31] for the explicit proof.

$$\langle A^c_\kappa(y) A^\ell_\lambda(z)\rangle \propto \delta^{c\ell}, \tag{C13}$$

since $\delta^{c\ell}$ is the unique invariant rank-two tensor.

Obviously, all available explicit loop results obtained with the (modified) Gribov-Zwanziger action are compatible with the general proof. Notice also that the proof is the same as the one we would use in the case of normal $SU(N)$ gauge theories.

An analogous result can be derived for the ghost propagator.

2. Lattice formulation

In a lattice formulation, one also calculates the connected two-point function, by taking the Monte Carlo average of the discrete version of the operator $\langle A^a_\mu(x) A^b_\nu(y)\rangle$. The statistical weight for this simulation is given by the exponential of the discretized version of the Yang-Mills gauge action, e.g. the Wilson action. The Landau gauge fixing is numerically implemented by minimizing a suitable functional along the gauge orbits, which corresponds to minimizing $\int d^3xA^2$ in the continuum. As we have already explained in the Introduction, this amounts to numerically selecting a gauge configuration within the Gribov region, equivalent to what we did in the continuum. We refer the interested reader to Sec. 2 of [53] for the explicit expressions of the discrete action, gauge fields, and minimizing functional. In particular, we refer to Sec. 2.4 in which the continuum and lattice versions of the gluon propagator are written down. The lattice gluon propagator also turns out to be transverse. Moreover, both the gluon and ghost propagators are found to be color diagonal.

We emphasize that lattice simulations thus never directly calculate any 1PI two-point function, but, we repeat, do calculate the (connected) two-point correlator, i.e. the propagator itself.

[1] A. Sternbeck, E. M. Ilgenfritz, M. Muller-Preussker, and A. Schiller, Nucl. Phys. B, Proc. Suppl. **140**, 653 (2005).
[2] A. Cucchieri, T. Mendes, and A. R. Taurines, Phys. Rev. D **71**, 051902 (2005).
[3] R. Alkofer and L. von Smekal, Phys. Rep. **353**, 281 (2001).
[4] C. Lerche and L. von Smekal, Phys. Rev. D **65**, 125006 (2002).
[5] J. M. Pawlowski, D. F. Litim, S. Nedelko, and L. von Smekal, Phys. Rev. Lett. **93**, 152002 (2004).
[6] R. Alkofer, W. Detmold, C. S. Fischer, and P. Maris, Phys. Rev. D **70**, 014014 (2004).
[7] V. N. Gribov, Nucl. Phys. **B139**, 1 (1978).
[8] D. Zwanziger, Nucl. Phys. **B323**, 513 (1989).
[9] D. Zwanziger, Nucl. Phys. **B399**, 477 (1993).
[10] D. Zwanziger, Phys. Rev. D **65**, 094039 (2002).
[11] J. A. Gracey, Phys. Lett. B **632**, 282 (2006).
[12] A. Cucchieri and T. Mendes, Proc. Sci., LAT2007 (2007) 297.
[13] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Proc. Sci., LAT2007 (2007) 290.
[14] A. Cucchieri and T. Mendes, Phys. Rev. Lett. **100**, 241601 (2008).
[15] A. Cucchieri and T. Mendes, Phys. Rev. D **78**, 094503 (2008).
[16] A. C. Aguilar, D. Binosi, and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).
[17] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. D **77**, 071501 (2008).
[18] D. Dudal, J. A. Gracey, S. P. Sorella, N. Vandersickel, and H. Verschelde, Phys. Rev. D **78**, 065047 (2008).
[19] Ph. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, and J. Rodriguez-Quintero, J. High Energy Phys. 06 (2008) 099.
[20] H. H. Matevosyan, A. P. Szczepaniak, and P. O. Bowman, Phys. Rev. D **78**, 014033 (2008).
[21] M. A. Semenov-Tian-Shanskii and V. A. Franke, Zap. Nauchn. Semin. LOMI **120**, 159 (1982) [J. Sov. Math. **34**, 1999 (1986)].
[22] G. Dell'Antonio and D. Zwanziger, Commun. Math. Phys. **138**, 291 (1991).
[23] G. Dell'Antonio and D. Zwanziger, in *Proceedings of the NATO Advanced Research Workshop on Probabilistic Methods in Quantum Field Theory and Quantum Gravity, Cargèse, 1989*, edited by Damgaard and Hueffel (Plenum Press, New York, 1989), p. 107.
[24] D. Zwanziger, Phys. Rev. D **69**, 016002 (2004).
[25] A. Cucchieri and T. Mendes, arXiv:0809.2777.
[26] S. Furui and H. Nakajima, Few-Body Syst. **40**, 101 (2006).
[27] D. Dudal, R. F. Sobreiro, S. P. Sorella, and H. Verschelde, Phys. Rev. D **72**, 014016 (2005).
[28] N. Maggiore and M. Schaden, Phys. Rev. D **50**, 6616 (1994).
[29] K. Knecht and H. Verschelde, Phys. Rev. D **64**, 085006 (2001).
[30] M. A. L. Capri, D. Dudal, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, R. Thibes, and H. Verschelde, Eur. Phys. J. C **52**, 459 (2007).
[31] O. Piguet and S. P. Sorella, Lect. Notes Phys. **M28**, 1 (1995).
[32] R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981).
[33] D. Dudal, J. A. Gracey, R. F. Sobreiro, S. P. Sorella, and H. Verschelde, Phys. Rev. D **75**, 061701 (2007).
[34] D. Dudal, J. A. Gracey, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, and H. Verschelde, Ann. Phys. (N.Y.) **317**, 203 (2005).
[35] J. A. Gracey, J. High Energy Phys. 05 (2006) 052.
[36] J. A. M. Vermaseren, arXiv:math-ph/00010025.
[37] P. Nogueira, J. Comput. Phys. **105**, 279 (1993).

[38] T. vanRitbergen, A. N. Schellekens, and J. A. M. Vermaseren, Int. J. Mod. Phys. A **14**, 41 (1999).

[39] R. Jackiw and S. Y. Pi, Phys. Lett. B **368**, 131 (1996).

[40] P. M. Stevenson, Phys. Rev. D **23**, 2916 (1981).

[41] B. Lucini and M. Teper, Phys. Rev. D **66**, 097502 (2002).

[42] A. Cucchieri, A. Maas, and T. Mendes, Phys. Rev. D **77**, 094510 (2008).

[43] A. Maas, Phys. Rev. D **75**, 116004 (2007).

[44] D. Dudal, S. P. Sorella, N. Vandersickel, and H. Verschelde, arXiv:0808.3379 .

[45] Ph. Boucaud, Th. Bruntjen, J. P. Leroy, A. Le Yaouanc, A. Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero, and C. Roiesnel, arXiv:hep-lat/0602006.

[46] Ph. Boucaud, J. P. Leroy, A. Le Yaouanc, A. Y. Lokhov, J. Micheli, O. Pene, J. Rodriguez-Quintero, and C. Roiesnel, Eur. Phys. J. A **31**, 750 (2007).

[47] A. C. Aguilar and A. A. Natale, J. High Energy Phys. 08 (2004) 057.

[48] M. A. L. Capri, V. E. R. Lemes, R. F. Sobreiro, S. P. Sorella, and R. Thibes, Phys. Rev. D **77**, 105023 (2008).

[49] T. Mendes, A. Cucchieri, and A. Mihara, AIP Conf. Proc. **892**, 203 (2007).

[50] R. Jackiw, Phys. Rev. D **9**, 1686 (1974).

[51] S. Yokojima, Phys. Rev. D **51**, 2996 (1995).

[52] K. Okumura, Int. J. Mod. Phys. A **11**, 65 (1996).

[53] J. C. R. Bloch, A. Cucchieri, K. Langfeld, and T. Mendes, Nucl. Phys. **B687**, 76 (2004).