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Dilaton and axion as compensators coupled to $N = 1$ supergravity

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We present a locally $N = 1$ supersymmetric model of the dilaton φ and the two-form tensor (axion) $B_{\mu\nu}$ as compensators without propagation. This is a generalization of our previous model with global $N = 1$ supersymmetry to local $N = 1$ supersymmetry. The dilaton φ and the axion $B_{\mu\nu}$ are, respectively, absorbed into the vector A_μ and the three-form tensor $C_{\mu\nu\rho}$, where the latter is dual to the ordinary auxiliary field D in the usual vector multiplet. With local $N = 1$ supersymmetry, we have three multiplets: the multiplet of supergravity $(e_\mu{}^m, \psi_\mu)$, linear multiplet $(B_{\mu\nu}, \chi, \varphi)$, and the vector multiplet $(A_\mu, \lambda, C_{\mu\nu\rho})$. We find that the field strengths of B and C need the following particular Chern-Simons terms for consistency with local supersymmetry: $\mathcal{G} \equiv 3dB - 6B\mathcal{D}\varphi + 3mBA + mC$ and $\mathcal{H} = 4dC - 6BF + 4\mathcal{G}A + 8C\mathcal{D}\varphi - 4mCA$. The newly established supergravity couplings provide the supporting evidence of the consistency of our basic system of the dilaton and axion as compensators.

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I. INTRODUCTION

The two-form antisymmetric tensor field $B_{\mu\nu}$, generated as a moduli field in the Neveu-Schwarz sector in superstring [1] or naturally present in supergravity (SG) [2,3], has properties similar to the axion for solving the strong *CP* problem in QCD by the Peccei-Quinn mechanism [4,5]. The decay constant f of axions is constrained to be 10^{16} GeV $\lesssim f \lesssim 10^{19}$ GeV. This poses a serious problem, because these values deviate from the allowed range on axion couplings. Astrophysical data suggest $f \gtrsim 10^9$ GeV [6], implying that the axionlike field $B_{\mu\nu}$ must be very light and extremely weakly coupled. On the other hand, cosmological considerations on the overclosure of the Universe yields an upper bound of $f \lesssim 10^{12}$ GeV [7], not compatible with the light and weakly coupling features of the axion. These discrepancies can be circumvented if the axion is not a physically propagating field.

The massless dilaton with universal couplings [1,8] also suffers from a problem similar to the axion, because such a massless scalar particle in the low-energy spectrum induces discrepancies with cosmological observations. This is because such a massless dilaton will couple to many fields coherently, causing large-scale gravitational phenomena that are not observed [1].

One way to evade these problems might be to regard the axion and dilaton as compensators with *no* physical propagations via Stuckelberg-type mechanism [9,10]. In our previous paper [11], we have presented an $N = 1$ globally supersymmetric model with the linear multiplet (LM) with fields $(B_{\mu\nu}, \chi, \varphi)$ [12] and a vector multiplet (VM)[1] with fields $(A_\mu, \lambda, C_{\mu\nu\rho})$, where $C_{\mu\nu\rho}$ is *dual* to the ordinary auxiliary field D. In that model [11], the axion[2] $B_{\mu\nu}$ is absorbed into the longitudinal components of $C_{\mu\nu\rho}$, while the dilaton φ is absorbed into the longitudinal component of the vector A_μ. To be more quantitative, the original 1 degree of freedom of the dilaton φ is absorbed into 1 longitudinal component of A_μ, making it massive, while 1 on-shell (or 3 off-shell) degrees of freedom of $B_{\mu\nu}$ is absorbed into the longitudinal components of $C_{\mu\nu\rho}$, which originally has 0 on-shell (or 1 off-shell) degrees of freedom, and acquires 1 on-shell (or 4 off-shell) degrees of freedom [11].[3] The degrees of freedom of the B and C fields are summarized in Table I.

Since the dilaton and the two-form fields arise as the low-energy limit of superstring, it is natural to consider *local* $N = 1$ supersymmetry as the next step. It is also imperative to show that the previously established globally $N = 1$ supersymmetric system in [11] can be consistently coupled to SG, as the first nontrivial confirmation of the validity of the system. In the present paper, we carry out this objective, i.e., we couple the LM and the VM to SG, such that the dilaton and axion become compensators, in a way consistent with local $N = 1$ supersymmetry.

The coupling of LM [12] to SG has been formulated in the past in many different contexts [3,14–17]. Among others, in the low-energy limit of superstring [1], the dilaton field should appear in the string effective action [18] in exponents as "the string loop-counting factors" [1,19]. This becomes transparent in a special frame called "string frame" [18] or "beta-function-favored constraints (BFFC)" in superspace [20]. However, if we go to the canonical frame, the dilaton exponential couplings in front of the scalar curvature is absorbed by Weyl rescaling of the

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[1]We sometimes call this multiplet *dual* VM in this paper.

[2]In this paper, we refer to $B_{\mu\nu}$ as axion just for convenience.
[3]For more analyses for Stuckelberg formalism for general p-form tensors in general dimensions, see [13].

TABLE I. Degrees of freedom for $B_{\mu\nu}$ and $C_{\mu\nu\rho}$.

Degrees of freedom	$B_{\mu\nu}$	$C_{\mu\nu\rho}$	Massive $C_{\mu\nu\rho}$
On-shell	1	0	1
Off-shell	3	1	4

metric and other field redefinitions, and the dilaton exponents appear only with the axion, vector, and/or tensor fields. Explicit results of such effective actions for four-dimensional (4D) superstring are found in [17].

However, any exponential coupling to a vector field causes a problem for the following reason. The dilaton derivative $\partial_\mu \varphi$ should be covariantized as $\mathcal{D}_\mu \varphi \equiv \partial_\mu \varphi + mA_\mu$ which is gauge invariant $\mathcal{D}_\mu \varphi \rightarrow \mathcal{D}_\mu \varphi$ under the dilaton shift $\varphi \rightarrow \varphi - m\alpha$ with $A_\mu \rightarrow A_\mu + \partial_\mu \alpha$. For this purpose, A_μ should *not* transform exponentially, such as $A_\mu \rightarrow e^{m\alpha}(A_\mu + \partial_\mu \alpha)$. In other words, despite the SG couplings, we should avoid any exponential couplings of the dilaton to the vector field. However, from the usual string-based viewpoints [17], it seems impossible to get rid of the exponential couplings to vector fields in any theory based on superstring.

In our present paper, due to our objective of locally supersymmetric generalization of our previous model [11], we need to get couplings of the dilaton *without* exponential couplings to a vector field that absorbs the dilaton itself, as in the global case [11]. Therefore, we have to temporarily sacrifice possible links with string theory [1], and rely on a direct construction of SG couplings for local supersymmetry. Based on this guiding principle, we show that such couplings of the dilaton to the VM or SG are indeed possible.

We stress also that the three-form potential should be in the VM for our mechanism to work. Our formulation, therefore, differs also from past formulations such as in [15], where the three-form potential is described by superfields Ω and $\bar{\Omega}$ outside the VM. To our knowledge, our formulation in this paper is the first one, in which the three-form potential $C_{\mu\nu\rho}$ is replacing the usual D auxiliary field in the VM that is further coupled to SG. The local supersymmetric generalization of our global case [11] is easy to say but difficult to perform, and it has not been accomplished in the past.

In this paper, we do not seek the stringy origin of our dilaton couplings, for the reason described above. Instead, we need the dilaton in the LM, and the special three-form tensor inside the *dual* VM, all coupled to the multiplet of SG, as one generalization of our previous work [11]. In particular, we should avoid any exponential coupling of the dilaton to the vector field in the VM. We show below that such ideal couplings of the LM and VM to the SG multiplet really exist, by confirming the locally supersymmetric invariance of actions.

II. SG + LM + VM AT $m = 0$

We first establish the coupling of the multiplet of SG $(e_\mu{}^m, \psi_\mu)$, LM $(B_{\mu\nu}, \chi, \varphi)$, and VM $(A_\nu, \lambda, C_{\mu\nu\rho})$, when $m = 0$. Compared with the works [3,14,16,17] in the past, the new ingredients are the presence of the three-form field $C_{\mu\nu\nu}$ in the VM, and the absence of the exponential-dilaton couplings to the vector field in the VM, as explained in the Introduction.

The Lagrangian $\mathcal{L}_0$ for our total action $I_0 \equiv \int d^4x \mathcal{L}_0$, thus obtained, is[4]

$$
\begin{aligned}
e^{-1}\mathcal{L}_0 = & -\frac{1}{4}(R(\omega) - (\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega)\psi_\rho) - \frac{1}{12}(G_{[3]})^2 \\
& - \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{48}(H_{[4]})^2 - \frac{1}{4}(F_{\mu\nu})^2 \\
& + \frac{1}{2}(\bar{\chi} \not{D}(\omega)\chi) + \frac{1}{2}(\bar{\lambda}\not{D}(\omega)\lambda) + (\bar{\psi}_\mu \gamma^\nu \gamma^\mu \chi)\partial_\nu \varphi \\
& + \frac{1}{6}(\bar{\psi}_\mu \gamma^{[3]}\gamma^\mu \chi) G_{[3]} - \frac{1}{2}(\bar{\psi}_\mu \gamma^{\rho\sigma}\gamma^\mu \lambda)F_{\rho\sigma} \\
& - \frac{1}{24}(\bar{\psi}_\mu \gamma^{[4]}\gamma^\mu \lambda) H_{[4]} - \frac{1}{8}(\bar{\chi}\gamma^{[3]}\chi)G_{[3]} \\
& - \frac{1}{12}(\bar{\chi}\gamma^{[4]}\lambda)H_{[4]} - \frac{1}{24}(\bar{\lambda}\gamma^{[3]}\lambda)G_{[3]},
\end{aligned} \tag{2.1}
$$

up to quartic-fermion terms. Note that we have fixed the SG couplings in such a way that there arises no exponential coupling of the dilaton φ. The field strengths H, G, and F are defined by

$$
\begin{aligned}
H_{\mu\nu\rho\sigma} \equiv & +4\partial_{[\mu}C_{\nu\rho\sigma]} - 6B_{[\mu\nu}F_{\rho\sigma]} + 4G_{[\mu\nu\rho|}A_{|\sigma]} \\
& + 8C_{[\mu\nu\rho}\partial_{\sigma]}\varphi,
\end{aligned} \tag{2.2a}
$$

$$
G_{\mu\nu\rho} \equiv +3\partial_{[\mu}B_{\nu\rho]} - 6B_{[\mu\nu}\partial_{\rho]}\varphi, \tag{2.2b}
$$

$$
F_{\mu\nu} \equiv +2\partial_{[\mu}A_{\nu]}, \tag{2.2c}
$$

where H and G are nontrivial. In particular, the three different Chern-Simons (CS) terms in the H field strength proportional to BF, GA, and $Gd\varphi$ are needed for consistency with local supersymmetry and gauge symmetries.

The field strength $G_{\mu\nu\rho}$ in (2.2) satisfies the nontrivial Bianchi identity (BI)

$$
\partial_{[\mu|}G_{|\nu\rho\sigma]} \equiv -2G_{[\mu\nu\rho|}\partial_{|\sigma]}\varphi. \tag{2.3}
$$

Since we are in 4D, there is *no* such BI's as $\partial_{[\mu|}H_{|\nu\rho\sigma\tau]} \equiv 0$.

Our total action I_0 is invariant up to quartic-fermion terms under $N = 1$ local supersymmetry

[4]We use the signature $(\eta_{\mu\nu}) = \text{diag.}(-, +, +, +)$, Accordingly, $\epsilon_{\mu_1 \ldots \mu_{4-n}[n]}\epsilon^{[n]\rho_1 \ldots \rho_{4-n}} = -(-1)^n (n!)[(4-n)!] \times e^2 \delta_{[\mu_1}{}^{\rho_1} \ldots \delta^{\rho_{4-n}}_{\mu_{4-n}]}$, and $\epsilon^{[n][4-n]}\gamma_{[4-n]} = +i[(4-n)!] \times (-1)^{(n-1)(n-2)/2} e\gamma_5 \gamma^{[n]}$, where $[n]$ stands for the totally antisymmetric n indices $[\sigma_1 \sigma_2 \ldots \sigma_n]$.

$$\delta_Q e_\mu{}^m = -2(\bar\epsilon\gamma^m\psi_\mu), \tag{2.4a}$$

$$\delta_Q\psi_\mu = +D_\mu(\hat\omega)\epsilon - \frac{1}{12}(\gamma_\mu{}^{[3]}\epsilon)\hat G_{[3]}, \tag{2.4b}$$

$$\delta_Q B_{\mu\nu} = +(\bar\epsilon\gamma_{\mu\nu}\chi) + 2(\bar\epsilon\gamma_{[\mu}\psi_{\nu]}) + 2B_{\mu\nu}(\delta_Q\varphi) \equiv +\tilde\delta_Q B_{\mu\nu} + 2B_{\mu\nu}(\delta_Q\varphi), \tag{2.4c}$$

$$\delta_Q\chi = -(\gamma^\mu\epsilon)\hat D_\mu\varphi + \frac{1}{6}(\gamma^{[3]}\epsilon)\hat G_{[3]}, \tag{2.4d}$$

$$\delta_Q\varphi = +(\hat\epsilon\chi), \tag{2.4e}$$

$$\delta_Q A_\mu = +(\bar\epsilon\gamma_\mu\lambda), \tag{2.4f}$$

$$\delta_Q\lambda = +\frac{1}{2}(\gamma^{\mu\nu}\epsilon)\hat F_{\mu\nu} - \frac{1}{24}(\gamma^{[4]}\epsilon)\hat H_{[4]}, \tag{2.4g}$$

$$\delta_Q C_{\mu\nu\rho} = +(\hat\epsilon\gamma_{\mu\nu\rho}\lambda) + 3B_{[\mu\nu|}(\delta_Q A_{|\rho]}) - 3A_{[\mu|}(\tilde\delta_Q B_{|\nu\rho]}) + 2C_{\mu\nu\rho}(\delta_Q\varphi), \tag{2.4h}$$

up to quadratic-fermion terms. The second expression in (2.4c) defines $\tilde\delta_Q B_{\mu\nu}$ which is used in (2.4h). As is usual in SG theories [2], all the *hatted* field strengths are super-covariantizations of the *unhatted* ones:

$$\hat H_{\mu\nu\rho\sigma} \equiv H_{\mu\nu\rho\sigma} - 4(\bar\psi_{[\mu|}\gamma_{|\nu\rho\sigma]}\lambda), \tag{2.5a}$$

$$\hat G_{\mu\nu\rho} \equiv G_{\mu\nu\rho} - 3(\hat\psi_{[\mu|}\gamma_{|\nu\rho]}\chi) - 3(\bar\psi_{[\mu|}\gamma_{|\nu|}\psi_{|\rho]}), \tag{2.5b}$$

$$\hat F_{\mu\nu} \equiv F_{\mu\nu} - 2(\bar\psi_{[\mu|}\gamma_{|\nu]}\lambda), \tag{2.5c}$$

$$\hat D_\mu\varphi \equiv \partial_\mu\varphi - (\bar\psi_\mu\chi), \tag{2.5d}$$

and $\hat\omega_\mu{}^{rs}$ is the supercovariantized Lorentz connection [2] $\hat\omega_{mrs} \equiv (\hat C_{mrs} - \hat C_{msr} + \hat C_{srm})/2$ with $\hat C_{\mu\nu}{}^r \equiv 2\partial_{[\mu}e_{\nu]}{}^r + 2(\bar\psi_{[\mu}\gamma^r\psi_{\nu]})$..

There are certain ambiguities for the coefficients of the Lagrangian and transformation terms. However, we have fixed them in such a way that there arise *no* exponential factors in the Lagrangian. As explained in the Introduction, this is crucial for the *gauging* of the shift symmetry of the dilaton $\varphi \to \varphi + \text{const}$. We will come back to this point in the next section. Another ambiguity is the $(\bar\psi^\mu\gamma^\nu\psi^\rho)G_{\mu\nu\rho}$ term in the Lagrangian. This is because its (fermion) $\times$ (boson)^2-type variation is of the form ψGH. However, the gamma algebra involved is $\gamma^{\mu[3]}\gamma^\nu G_{[3]}G_{\mu\nu\rho}$ which is *identically zero*, as is easily seen in terms of the dual tensor $\tilde G^m \equiv (1/6)\epsilon^{mnrs}G_{nrs}$. Since the coefficient of the $(\bar\psi^\mu\gamma^\nu\psi^\rho)G_{\mu\nu\rho}$ term is arbitrary by the inspection of the (fermion) $\times$ (boson)^2-type variation, we have taken it to zero for simplicity.

The general variations of the field strengths H, G, and F are easily obtained as

$$\begin{aligned}\delta H_{\mu\nu\rho\sigma} = {} & +4\partial_{[\mu|}(\tilde\delta C_{|\nu\rho\sigma]}) + 8G_{[\mu\nu\rho|}(\delta A_{|\sigma]}) \\ & + 2H_{\mu\nu\rho\sigma}(\delta\varphi) - 12F_{[\mu\nu|}(\tilde\delta B_{|\rho\sigma]}) \\ & - 8(\partial_{[\mu|}\varphi)(\tilde\delta C_{|\nu\rho\sigma]}),\end{aligned} \tag{2.6a}$$

$$\begin{aligned}\delta G_{\mu\nu\rho} = {} & +3\partial_{[\mu|}(\tilde\delta B_{|\nu\rho]}) + 2G_{\mu\nu\rho}(\delta\varphi) \\ & - 6(\partial_{[\mu|}\varphi)(\tilde\delta B_{|\nu\rho]}),\end{aligned} \tag{2.6b}$$

$$\delta F_{\mu\nu} = +2\partial_{[\mu|}(\delta A_{|\nu]}), \tag{2.6c}$$

where

$$\begin{aligned}\tilde\delta C_{\mu\nu\rho} = {} & \delta C_{\mu\nu\rho} - 3B_{[\mu\nu|}(\delta A_{|\rho]}) + 3A_{[\mu|}(\tilde\delta B_{|\nu\rho]}) \\ & - 2C_{\mu\nu\rho}(\delta\varphi),\end{aligned} \tag{2.7a}$$

$$\tilde\delta B_{\mu\nu} = \delta B_{\mu\nu} - 2B_{\mu\nu}(\delta\varphi). \tag{2.7b}$$

As will be mentioned, this general variation rule can be also restricted to $N = 1$ local supersymmetry transformation rule.

As in SG theories in diverse dimensions [2], such as type IIA theory in ten dimensions (10D) [21], all of these *tilded* transformations are fixed in such a way that the general variations of the field strengths become covariant, i.e., no *bare* potential field must arise when written in terms of the *tilded* variations, as in (2.6).[5] Also, when these variations are restricted to supersymmetry transformations, all the *tilded* transformations should contain only the fermion-linear terms, but *no* CS-related terms.

Note that there is *no* modified supersymmetry transformation for A_μ. As desired, these terms are all manifestly gauge covariant. Equation (2.4c) is nothing but the special case of (2.7b) for local supersymmetry transformation.

We have one remark for the confirmation $\delta_Q I_0 = 0$. Even though most of the cancellation patterns are parallel to the globally supersymmetric case [11], there are also certain differences. For example, compared with the global case [11], there are additional CS terms in the field strengths (2.2). A typical example is the $G\partial\varphi$ term in the BI (2.3). This term causes a new contribution of the type $\chi G\partial\varphi$ in $\delta_Q\mathcal{L}_0$ compared with the global case [11]. However, due to the new term $\chi^2 G$ in the Lagrangian, as well as the G-linear term in $\delta_Q\psi_\mu$, they cancel each other with no problem.

III. GAUGING DILATON-SHIFT SYMMETRY BY COUPLING CONSTANT *m*

We have so far coupled to the LM and VM to SG without gauging the dilaton-shift symmetry: $\varphi \to \varphi + \text{const}$. We now gauge this symmetry as in the globally supersymmet-

[5] We will show all the gauge transformations (3.7) through (3.9) in the case of $m \neq 0$ in the next section.

ric case [11], so that the dilaton φ and the axion $B_{\mu\nu}$ will be absorbed into the longitudinal components of A_μ and $C_{\mu\nu\rho}$, making the latter fields massive.

The basic relationships are the covariantization of $\partial_\mu\varphi$, and certain nontrivial m-dependent terms added to the field strengths in (2.2)[6]:

$$\mathcal{D}_\mu\varphi \equiv +\partial_\mu\varphi + mA_\mu, \tag{3.1a}$$

$$\begin{aligned}\mathcal{G}_{\mu\nu\rho} \equiv & +3\partial_{[\mu|}B_{|\nu\rho]} - 6B_{[\mu\nu|}\mathcal{D}_{|\rho]}\varphi \\ & + 3mB_{[\mu\nu|}A_{|\rho]} + mC_{\mu\nu\rho},\end{aligned} \tag{3.1b}$$

$$\begin{aligned}\mathcal{H}_{\mu\nu\rho\sigma} \equiv & +4\partial_{[\mu|}C_{|\nu\rho\sigma]} - 6B_{\mu\nu|}F_{|\rho\sigma]} + 4\mathcal{G}_{[\mu\nu\rho|}A_{|\sigma]} \\ & + 8C_{\mu\nu\rho|}\mathcal{D}_{|\sigma]}\varphi - 4mC_{[\mu\nu\rho|}A_{|\sigma]}.\end{aligned} \tag{3.1c}$$

The F field strength stays the same as in (2.2c). Under $\varphi \rightarrow \varphi - m\alpha(x)$, we have the covariance $\delta_\alpha(\mathcal{D}_\mu\varphi) = 0$. Since $\alpha(x)$ has arbitrary space-time dependence in $\varphi \rightarrow \varphi - m\alpha(x)$, the dilaton φ is completely gauged away. To put it differently, (3.1a) implies that φ can be absorbed into the longitudinal component of A_μ, and similarly, (3.1b) implies that axion $B_{\mu\nu}$ is absorbed into the longitudinal component of $C_{\mu\nu\rho}$. These are qualitatively the same as in the globally supersymmetric case [11].

Compared with the $m = 0$ case (2.2), the new terms in (3.1) are the m-dependent bilinear terms, such as mCA in $\mathcal{H}$ and mBA in $\mathcal{G}$. The m-linear term mC in $\mathcal{H}$ is the same as in the globally supersymmetric case [11].

Note that the F field strength is *not* modified, and there is *no* $A\partial\varphi$ term in the field strength F. As has been mentioned, if there were such a term, it would be an obstruction to the gauging of the dilaton-shift symmetry $\varphi \rightarrow \varphi - m\alpha(x)$. This is because the term of the type $\xi A\partial\varphi$ in F can be attributed to the scaling of the gauge field, for the new frame $\tilde{A} \equiv Ae^{-\xi\varphi}$. Such a term would be an obstruction against the gauging of the dilaton symmetry, because the gauge field $\tilde{A}$ would *not* transform as $\tilde{A} \rightarrow \tilde{A} + \partial\alpha$.

All the m-dependent terms in (3.1) are required for consistency with supersymmetry. To be more specific, they can be justified due to the covariance of the general variations

$$\begin{aligned}\delta\mathcal{H}_{\mu\nu\rho\sigma} = & +4\partial_{[\mu|}(\tilde{\delta}C_{|\nu\rho\sigma]}) + 8\mathcal{G}_{[\mu\nu\rho|}(\delta A_{|\sigma]}) \\ & + 2\mathcal{H}_{\mu\nu\rho\sigma}(\delta\varphi) - 12F_{[\mu\nu|}(\tilde{\delta}B_{|\rho\sigma]}) \\ & - 8(\mathcal{D}_{[\mu|}\varphi)(\tilde{\delta}C_{|\nu\rho\sigma]}),\end{aligned} \tag{3.2a}$$

$$\begin{aligned}\delta\mathcal{G}_{\mu\nu\rho} = & +3\delta_{[\mu|}(\tilde{\delta}B_{|\nu\rho]}) + 2\mathcal{G}_{\mu\nu\rho}(\delta\varphi) \\ & - 6(\mathcal{D}_{[\mu|}\varphi)(\tilde{\delta}B_{|\nu\rho]}) + m(\tilde{\delta}C_{\mu\nu\rho}),\end{aligned} \tag{3.2b}$$

$$\delta F_{\mu\nu} = +2\partial_{[\mu|}(\delta A_{|\nu]}), \tag{3.2c}$$

where $\tilde{\delta}C$ and $\tilde{\delta}B$ are defined by (2.7) as in the $m = 0$ case. As compared with (2.6), the only explicitly m-dependent term is the last term in (3.2b) which is by itself covariant. All other terms are just the replacements of all the field strengths (2.2) and $\partial_\mu\varphi$ and $m = 0$ by those in (3.1) at $m \neq 0$. The only exception is $F_{\mu\nu}$ which is not modified, as has been mentioned. The covariance of each term in (3.2) also provides the supporting evidence for the total consistency of our system.

The field strengths $\mathcal{G}$ and $\mathcal{D}\varphi$ satisfy nontrivial BI's:

$$\partial_{[\mu|}\mathcal{G}_{|\nu\rho\sigma]} \equiv +\frac{1}{4}m\mathcal{H}_{\mu\nu\rho\sigma} - 2\mathcal{G}_{[\mu\nu\rho|}\mathcal{D}_{|\sigma]}\varphi, \tag{3.3a}$$

$$\partial_{[\mu|}\mathcal{D}_{|\nu]}\varphi \equiv +\frac{1}{2}mF_{\mu\nu}, \tag{3.3b}$$

while F satisfies the trivial one $\partial_{[\mu|}F_{|\nu\rho]} \equiv 0$.

At the Lagrangian level, the m couplings are introduced by replacing $\partial_\mu\varphi$, $G_{\mu\nu\rho}$, and $H_{\mu\nu\rho\sigma}$ respectively by $\mathcal{D}_\mu\varphi$, $\mathcal{G}_{\mu\nu\rho}$, and $\mathcal{H}_{\mu\nu\rho\sigma}$ in $\mathcal{L}_0$ (2.1), as well as adding an m-explicit term $m(\bar{\chi}\lambda)$, as in the globally supersymmetric case [11]. To be more explicit, our total Lagrangian $\mathcal{L}$ for our total action $I \equiv \int d^4x\mathcal{L}$ is

$$\begin{aligned}e^{-1}\mathcal{L} = & -\frac{1}{4}R(\omega) - (\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu(\omega)\psi_\rho) - \frac{1}{12}(\mathcal{G}_{[3]})^2 \\ & - \frac{1}{2}(\mathcal{D}_\mu\varphi)^2 - \frac{1}{48}(\mathcal{H}_{[4]})^2 - \frac{1}{4}(F_{\mu\nu})^2 + \frac{1}{2}(\bar{\chi}\not{D}(\omega)\chi) \\ & + \frac{1}{2}(\bar{\lambda}\not{D}(\omega)\lambda) + (\bar{\psi}_\mu\gamma^\nu\gamma^\mu\chi)\mathcal{D}_\nu\varphi \\ & + \frac{1}{6}(\bar{\psi}_\mu\gamma^{[3]}\gamma^\mu\chi)\mathcal{G}_{[3]} - \frac{1}{2}(\bar{\psi}_\mu\gamma^{\rho\sigma}\gamma^\mu\lambda)F_{\rho\sigma} \\ & - \frac{1}{24}(\bar{\psi}_\mu\gamma^{[4]}\gamma^\mu\lambda)\mathcal{H}_{[4]} - \frac{1}{8}(\bar{\chi}\gamma^{[3]}\chi)\mathcal{G}_{[3]} \\ & - \frac{1}{12}(\bar{\chi}\gamma^{[4]}\lambda)\mathcal{H}_{[4]} - \frac{1}{24}(\bar{\lambda}\gamma^{[3]}\lambda)\mathcal{G}_{[3]} + m(\bar{\chi}\lambda).\end{aligned} \tag{3.4}$$

Note that the only m-explicit term is the last one.

Accordingly, all the field strengths H, G, and $\partial\varphi$ in all transformations in (2.4) are replaced by $\mathcal{H}$, $\mathcal{G}$, and $\mathcal{D}\varphi$. Thus supersymmetry transformations, leaving our total action invariant $\delta_Q I = 0$ up to quartic-fermion terms, are

$$\delta_Q e_\mu{}^m = -2(\bar{\epsilon}\gamma^m\psi_\mu), \tag{3.5a}$$

$$\delta_Q\psi_\mu = +D_\mu(\hat{\omega})\epsilon - \frac{1}{12}(\gamma_\mu{}^{[3]}\epsilon)\hat{\mathcal{G}}_{[3]}, \tag{3.5b}$$

$$\delta_Q B_{\mu\nu} = +(\bar{\epsilon}\gamma_{\mu\nu}\chi) + 2(\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}) + 2B_{\mu\nu}(\delta_Q\varphi), \tag{3.5c}$$

$$\delta_Q\chi = -(\gamma^\mu\epsilon)\hat{\mathcal{D}}_\mu\varphi + \frac{1}{6}(\gamma^{[3]}\epsilon)\hat{\mathcal{G}}_{[3]}, \tag{3.5d}$$

$$\delta_Q\varphi = +(\bar{\epsilon}\chi), \tag{3.5e}$$

$$\delta_Q A_\mu = +(\bar{\epsilon}\gamma_\mu\lambda), \tag{3.5f}$$

$$\delta_Q\lambda = +\frac{1}{2}(\gamma^{\mu\nu}\epsilon)\hat{F}_{\mu\nu} - \frac{1}{24}(\gamma^{[4]}\epsilon)\hat{\mathcal{H}}_{[4]}, \tag{3.5g}$$

$$\begin{aligned}\delta_Q C_{\mu\nu\rho} = & +(\bar{\epsilon}\gamma_{\mu\nu\rho}\lambda) + 3B_{[\mu\nu|}(\delta_Q A_{|\rho]}) \\ & - 3A_{[\mu|}(\tilde{\delta}_Q B_{|\nu\rho]}) + 2C_{\mu\nu\rho}(\delta_Q\varphi),\end{aligned} \tag{3.5h}$$

[6]Equations (3.1b) and (3.1c) are reexpressed in terms of differential forms, as given in the Abstract of this paper. The normalization is self-explanatory there.

fixed up to quadratic-fermion terms. As in (2.5), all the *hatted* field strengths are for the supercovariantization of the *unhatted* ones.

The confirmation of the invariance $\delta_Q I = 0$ up to quartic-fermion terms under local supersymmetry is straightforward now, once the supersymmetry transformation rules for the field strengths are constructed from (3.2). This is because we can also restrict the general variation δ's in (3.2) to the local supersymmetry transformation δ_Q. In such a case, all the *tilded* variations $\tilde{\delta}_Q$ have only the linear-fermion part without CS-related terms, so that the gauge invariance of supersymmetry transformations of the field strengths constructed from (3.2) is manifest. Their explicit forms are

$$\tilde{\delta}_Q C_{\mu\nu\rho} = +(\bar{\epsilon}\gamma_{\mu\nu\rho}\lambda), \tag{3.6a}$$

$$\tilde{\delta}_Q B_{\mu\nu} = +(\bar{\epsilon}\gamma_{\mu\nu}\chi) + 2(\bar{\epsilon}\gamma_{[\mu}\psi_{\nu]}). \tag{3.6b}$$

This is crucial, when we confirm the invariance $\delta_Q I = 0$, because all the terms of the type (fermion) $\times$ (boson)^2 generated in $\delta_Q \mathcal{L}$ are gauge invariant, canceling other terms with gauge invariant field strengths. The covariant transformations in (3.2) for δ_Q, when (3.6) is used, also support the total consistency of our system.

Once the gauge covariance of the variation of all field strengths is established, the cancellation patterns of m-dependent terms in $\delta_Q I$ are similar to the globally supersymmetric case in [11]. There are only four sectors that contain m: (i) $m\lambda\mathcal{G}$, (ii) $m\lambda\mathcal{D}\varphi$, (iii) $m\chi\mathcal{H}$, and (iv) $m\chi F$. As in the globally supersymmetric case [11], the sectors (iii) and (iv) need the m-dependent terms in the BI's (3.3a) and (3.3b), respectively.

There are three different gauge transformations associated with our field strengths $\mathcal{D}\varphi$, F, $\mathcal{G}$, and $\mathcal{H}$. Let us call them α, β, and γ gauge transformations. Their definitions and special features are summarized as follows:

(i) α Transformation

$$\begin{aligned}
\delta_\alpha\varphi &= -m\alpha, & \delta_\alpha A_\mu &= +\partial_\mu\alpha,\\
\delta_\alpha B_{\mu\nu} &= 0, & \delta_\alpha C_{\mu\nu\rho} &= -3B_{[\mu\nu|}\partial_{|\rho]}\alpha,\\
\delta_\alpha(\mathcal{D}_\mu\varphi) &= 0, & \delta_\alpha F_{\mu\nu} &= 0,\\
\delta_\alpha \mathcal{G}_{\mu\nu\rho} &= 0, & \delta_\alpha \mathcal{H}_{\mu\nu\rho\sigma} &= 0.
\end{aligned} \tag{3.7}$$

(ii) β Transformation

$$\begin{aligned}
\delta_\beta\varphi &= 0, \qquad \delta_\beta A_\mu = 0,\\
\delta_\beta B_{\mu\nu} &= +2\partial_{[\mu|}\beta_{|\nu]} + 4\beta_{[\mu|}\mathcal{D}_{|\nu]}\varphi,\\
\delta_\beta C_{\mu\nu\rho} &= +6\beta_{[\mu|}F_{|\nu\rho]} - 6A_{[\mu|}\partial_{|\nu|}\beta_{|\rho]}\\
&\quad + 12\beta_{[\mu|}A_{|\nu|}\mathcal{D}_{|\rho]}\varphi,\\
\delta_\beta(\mathcal{D}_\mu\varphi) &= 0, \qquad \delta_\beta F_{\mu\nu} = 0,\\
\delta_\beta \mathcal{G}_{\mu\nu\rho} &= 0, \qquad \delta_\beta \mathcal{H}_{\mu\nu\rho\sigma} = 0.
\end{aligned} \tag{3.8}$$

(iii) γ Transformation[7]

$$\begin{aligned}
\delta_\gamma\varphi &= 0, \qquad \delta_\gamma A_\mu = 0,\\
\delta_\gamma B_{\mu\nu} &= -m\gamma_{\mu\nu},\\
\delta_\gamma C_{\mu\nu\rho} &= +3\partial_{[\mu|}\gamma_{|\nu\rho]} - 6\gamma_{[\mu\nu|}\mathcal{D}_{|\rho]}\varphi\\
&\quad + 3m\gamma_{[\mu\nu|}A_{|\rho]},\\
\delta_\gamma(\mathcal{D}_\mu\varphi) &= 0, \qquad \delta_\gamma F_{\mu\nu} = 0,\\
\delta_\gamma \mathcal{G}_{\mu\nu\rho} &= 0, \qquad \delta_\gamma \mathcal{H}_{\mu\nu\rho\sigma} = 0.
\end{aligned} \tag{3.9}$$

The α transformation is nothing but what we call dilaton-shift symmetry. Needless to say, our Lagrangian (3.4) is manifestly invariant under all the three symmetries: $\delta_\alpha I = \delta_\beta I = \delta_\gamma I = 0$.

The corresponding special case of $m = 0$, whose explicit form we skipped in the last section, is also recovered by these transformations. Even though we do not give the details, we can also confirm the invariances $\delta_\alpha I = 0$, $\delta_\beta I = 0$, and $\delta_\gamma I = 0$ in terms of *tilded* transformations in (2.7), as additional confirmation of the mutual consistency among the *tilded* transformations, the definitions of field strengths, and the gauge transformations. As is seen, these transformations involve different potentials and field strengths in a nontrivial way, and provide the supporting evidence for the validity of the definitions of field strengths, as well as transformations themselves.

As in the globally supersymmetric case [11], the vector field A_μ becomes massive because the kinetic term of φ is equivalent to the mass term for A_μ. This is nothing but the compensator mechanism [9,10] for the dilaton-shift symmetry $\varphi \rightarrow \varphi - m\alpha(x)$, where $\alpha(x)$ here is a finite parameter. Similarly, the originally nonpropagating $C_{\mu\nu\rho}$ field also becomes massive because of the kinetic term of $B_{\mu\nu}$ which is equivalent to the mass term of $C_{\mu\nu\rho}$. As Table I also shows, the $C_{\mu\nu\rho}$ field is no longer "auxiliary" but now is a propagating massive field with one physical degree of freedom. These features are also reflected in the infinitesimal transformations $\delta_\alpha\varphi$ in (3.7) and $\delta_\gamma B_{\mu\nu}$ in (3.9). These aspects are exactly the same as in the globally supersymmetric case [11], but now the consistency of the total system has been secured due to the coupling to SG.

IV. CONCLUDING REMARKS

In this paper, we have presented an $N = 1$ locally supersymmetric system of the dilaton φ and the axion $B_{\mu\nu}$ serving as compensators. Initially at $m = 0$, we established the SG coupling to the two multiplets of VM and LM. Subsequently, we introduced the coupling constant m that

[7] Even though the symbol $\gamma_{\mu\nu}$ is somewhat confusing with the γ matrices, we can limit these symbols only in this context of bosons, not mixing up with γ matrix computations.

controls the nontrivial Stuckelberg-type compensator mechanism [9,10] for the dilaton-shift symmetry $\varphi \rightarrow \varphi - m\alpha(x)$ with the vector A_μ in VM absorbing φ in LM. This is accompanied by its supersymmetric partner field $C_{\mu\nu\rho}$ in VM absorbing $B_{\mu\nu}$ in LM. Our result is a generalization of our previous result [11] with global to local $N = 1$ supersymmetry. Since the dilaton and the two-form fields arise as the low-energy limit of superstring, it is natural to consider local $N = 1$ supersymmetry as the next step.

As has been mentioned, the field strengths $\mathcal{H}$ of C and $\mathcal{G}$ of B have peculiar m-dependent as well as m-independent CS-terms as in (3.1). These highly nontrivial CS terms are required by supersymmetry as well as the covariant expressions for the general variations. These nontrivial CS terms arise not only in the $m = 0$ case (2.2), but also in the gauged case $m \neq 0$ (3.1). The correctness of these field strengths also has been reconfirmed by the three gauge invariances of our action under the infinitesimal δ_α, δ_β, and δ_γ transformations in (3.7) through (3.9).

In the conventional formulation of a VM, the auxiliary field D is just a (pseudo)scalar field, whose linear term breaks supersymmetry explicitly. In our case of the dual three-form potential $C_{\mu\nu\rho}$, however, its direct analog $\epsilon^{\mu\nu\rho\sigma}\mathcal{H}_{\mu\nu\rho\sigma}$ of the D-linear term is a total divergence at the lowest order, not affecting the C field equation. This indicates that the *dual* VM is not necessarily equivalent to the usual VM, when breaking supersymmetry. Moreover, since the C field becomes massive and propagating after the absorption of the $B_{\mu\nu}$ field [11], the difference between the usual and dual VM's is obviously nontrivial.

In this paper, we have not identified any string-theory based origin of our Stuckelberg-type mechanism, because such a link is very difficult to establish. For example, in the paper [22], the massive two-form potential, or massive vector field as its Poincaré dual has been discussed in the context of superstring. Unfortunately, however, these formulations are complimentary to our system, where the dilaton (zero-form) and the three-form potential fields become massive.

One can seek the stringy origin of the mechanism of the dilaton as a compensator in the Green-Schwarz formulation [1]. We start with the Fradkin-Tseytlin term [18]

$$I_{\Phi R} = \int d^2\sigma\sqrt{-g}\Phi(Z)R^{(2)}(\sigma) = -\int d^2\sigma\sqrt{-g}\Omega^i\Pi_i{}^A\nabla_A\Phi, \qquad (4.1)$$

where $\Phi(Z)$ is the dilaton superfield, and Z^M are the 10D superspace coordinates, while σ^i $(i = 0, 1)$ are the 2D curved coordinates. The Ω^i is related to the 2D scalar curvature $R^{(2)}(\sigma)$ by $\sqrt{-g}R^{(2)}(\sigma) = \partial_i(\sqrt{-g}\Omega^i)$. The indices $A, B, \ldots$ are 10D superspace coordinates, and $\nabla_A \equiv E_A{}^M\partial_M + (1/2)\phi_{Ab}{}^c\mathcal{M}_c{}^b$ is the usual 10D local Lorentz covariant superderivative. The second expression in (4.1) is possible due to the total divergence feature of $R^{(2)}$ mentioned above.

For a possible action invariant under the local dilaton shift $\Phi \rightarrow \Phi - m\alpha(Z)$, ∇_A should be covariantized to $\mathcal{D}_A\Phi \equiv \nabla_A\Phi + mA_A$ with the potential superfield A_A, so that $I_{\Phi R}$ in (4.1) is modified to

$$\tilde{I}_{\Phi R} \equiv -\int d^2\sigma\sqrt{-g}\Omega^i\Pi_i{}^A\mathcal{D}_A\Phi \equiv -\int d^2\sigma\sqrt{-g}\Omega^i\Pi_i{}^A(\nabla_A\Phi + mA_A). \qquad (4.2)$$

The invariance $\delta_\alpha\tilde{I}_{\Phi R} = 0$ is transparent under the dilaton-shift symmetry

$$\delta_\alpha\Phi = -m\alpha, \qquad \delta_\alpha A_A = \nabla_A\alpha \Rightarrow \delta_\alpha(\mathcal{D}_A\Phi) = 0. \quad (4.3)$$

The problem, however, is that the action $\tilde{I}_{\Phi R}$ breaks 2D local Lorentz symmetry. This can be seen under the infinitesimal local Lorentz transformation $\delta_\lambda\omega_i{}^{(k)(l)} = D_i\lambda^{(k)(l)}$, where $(k), (l), \ldots$ are 2D local Lorentz indices. This is equivalent to

$$\delta_\lambda(\sqrt{-g}\Omega^i) = -2\epsilon^{ij}\partial_j\tilde{\lambda}, \qquad (4.4)$$

by the relationship $\sqrt{-g}\Omega^i = -2\epsilon^{ij}\tilde{\omega}_j$, where $\tilde{\lambda} \equiv (1/2)\epsilon^{(i)(j)}\lambda_{(i)(j)}$ and $\tilde{\omega}_i \equiv -\epsilon^{(k)(l)}\omega_{i(k)(l)}$ are the dual of $\lambda_{(i)(j)}$ and $\omega_i{}^{(k)(l)}$, respectively. Using (4.4) in (4.2), we get

$$\delta_\lambda\tilde{I}_{\Phi R} = m\int d^2\sigma\epsilon^{ij}\tilde{\lambda}\Pi_i{}^A\Pi_j{}^BF_{BA} \neq 0. \qquad (4.5)$$

We cannot expect a cancellation by the Wess-Zumino-Novikov-Witten term, either, because we *cannot* let the axion field B_{AB} transform like $\delta_\lambda B_{AB} = -m\tilde{\lambda}F_{AB}$, because $\tilde{\lambda} = \tilde{\lambda}(\sigma)$ depends only on the 2D coordinates σ^i. This is the problem, when seeking the stringy origin of our formulation.

On the other hand, any Kaluza-Klein origin of the Stuckelberg formulation seems also difficult to establish. Even though there are many different types of *gauged* SG from Kaluza-Klein compactifications, to our knowledge, there has been *no* Stuckelberg-type compensator mechanism for the dilaton worked out, based on compactifications from higher dimensions.

Finally, we conclude with the following three remarks. First, it has been shown for the first time that dilaton-shift symmetry can be gauged with *local* supersymmetry, as the generalization of the globally supersymmetric case [11]. Ordinary gaugings or massive generalizations in SG theories [2] had been well-known in diverse dimensions, such as massive type IIA theory in 10D [21] or the gauging of R symmetry in 5D [23]. However, the important difference here is that we have succeeded in gauging the dilaton-shift symmetry $\varphi \rightarrow \varphi - m\alpha(x)$ consistently with local $N = 1$ supersymmetry in 4D. In addition to our globally supersymmetric version [11], we have now locally supersymmetric gauging of dilaton-shift symmetry, combined with axion symmetry.

Second, it has been established that the auxiliary field D in the VM can have its dual three-form potential $C_{\mu\nu\rho}$ with its field strength $H_{\mu\nu\rho\sigma}$ consistently with local $N = 1$ supersymmetry. Moreover, the $C_{\mu\nu\rho}$ is auxiliary and nonphysical for $m = 0$, but it starts propagating after absorbing the $B_{\mu\nu}$ field from the LM for $m \neq 0$. As in the globally supersymmetric case [11], even though the original C field is just the dual of the usual D auxiliary field, it is no longer auxiliary after the absorption of the B field. The dual auxiliary field $C_{\mu\nu\rho}$ of a VM is not only valid at the free-field level or global supersymmetry [11], but also at the interaction level with nontrivial CS terms (3.1) with SG.

Third, the formulation of a VM with the three-form potential $C_{\mu\nu\rho}$ is based on the important concept of the axion and dilaton regarded as simultaneous compensators in a locally supersymmetric theory, as a natural solution to the moduli problem. The dual VM with the auxiliary field $C_{\mu\nu\rho}$ has acquired its *raison d'etre* by the supersymmetric compensator mechanism for the axion and dilaton to be eliminated from the physical spectrum.

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