

Primordial non-Gaussianity: Large-scale structure signature in the perturbative bias model

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I compute the effect on the power spectrum of tracers of the large-scale mass-density field (e.g., galaxies) of primordial non-Gaussianity of the form $\Phi = \phi + f_{\text{NL}}(\phi^2 - \langle \phi^2 \rangle) + g_{\text{NL}}\phi^3 + \dots$, where Φ is proportional to the initial potential fluctuations and ϕ is a Gaussian field, using beyond-linear-order perturbation theory. I find that the need to eliminate large higher-order corrections necessitates the addition of a new term to the bias model, proportional to ϕ , i.e., $\delta_g = b_\delta \delta + b_\phi f_{\text{NL}} \phi + \dots$, with all the consequences this implies for clustering statistics, e.g., $P_{gg}(k) = b_\delta^2 P_{\delta\delta}(k) + 2b_\delta b_\phi f_{\text{NL}} P_{\phi\delta}(k) + b_\phi^2 f_{\text{NL}}^2 P_{\phi\phi}(k) + \dots$. This result is consistent with calculations based on a model for dark matter halo clustering, showing that the form is quite general, not requiring assumptions about peaks, or the formation or existence of halos. The halo model plays the same role it does in the usual bias picture, giving a prediction for b_ϕ for galaxies known to sit in a certain type of halo. Previous projections for future constraints based on this effect have been very conservative—there is enough volume at $z \lesssim 2$ to measure f_{NL} to $\sim \pm 1$, with much more volume at higher z . As a prelude to the bias calculation, I point out that the beyond-linear (in ϕ) corrections to the power spectrum of mass-density perturbations are naively infinite, so it is dangerous to assume they are negligible; however, the infinite part can be removed by a renormalization of the fluctuation amplitude, with the residual k -dependent corrections negligible for models allowed by current constraints.

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I. INTRODUCTION

Inflation [1–4] has been tested successfully mainly through measurements of the power spectrum of primordial density perturbations [5,6]. These perturbations are expected to be nearly Gaussian and nearly scale invariant [7–11]. Testing the Gaussianity of the perturbations with increasing accuracy will be a major goal of future work aimed at distinguishing different models (see [12] for a review of non-Gaussianity from inflation). The simplest models of inflation predict that non-Gaussianity will be undetectably small [12–18], but multifield models [19–24], models where non-Gaussianity is generated during reheating [25,26] or preheating [27–30], bouncing/ekpyrotic/cyclic models [31–34], or inflation models based on nonlocal field theory [35,36] can predict levels of non-Gaussianity near the present detection limits ($f_{\text{NL}} \sim 100$, as defined below). While there have been some hints of non-Gaussianity in the cosmic microwave background (CMB) [37,38], the general consensus seems to be that nothing convincingly primordial has been detected [6,39,40].

Recently, [41–43] showed that there should be a distinctive signature of non-Gaussianity in the large-scale power spectrum of dark matter halos, observable as galaxies, in the local model for non-Gaussianity with curvature perturbations proportional to

$$\Phi = \phi + f_{\text{NL}}(\phi^2 - \langle \phi^2 \rangle) + g_{\text{NL}}\phi^3 + \dots \quad (1)$$

[12,44–46], where I include the 3rd order term, which is necessary to compute the power spectrum to 4th order in

the small, Gaussian, perturbations ϕ . This form is only a special case of non-Gaussianity, but it is simple, and calculations using it should point the way to generalizations. The idea of [43], to use the power spectrum, was a departure from the previously standard approach of studying non-Gaussianity in large-scale structure (LSS) using the bispectrum [47–55] or other higher-order statistics [56–61]. It was somewhat surprising that this signal would be detectable, because the nonlinear corrections to the power spectrum in this model were generally assumed to be negligibly small (because fluctuations in ϕ are of order 10^{-5}). Reference [62] verified and further explored this idea and applied it to real galaxy and quasar data sets. They found observational constraints $-1(-23) < f_{\text{NL}} < +70(+86)$, at 95% (99.7%) confidence, by combining LSS and CMB data. The projected rms error from the Planck satellite, measuring the CMB anisotropy, is $\sigma_{f_{\text{NL}}} \sim 5$ [44,63].

The calculations of [41–43,62] were all based on models for the clustering of dark matter halos. The purpose of this paper is to investigate the result using a different approach—the renormalized perturbative bias model of [64]. Instead of focusing on dark matter halos, this approach starts by assuming that the tracer density field is a completely general, unknown, function of the local mass-density field and Taylor expanding this function in the mass-density fluctuations, leading to the form [65]

$$\delta_g(\delta) = c_1 \delta + \frac{1}{2} c_2 (\delta^2 - \sigma^2) + \frac{1}{6} c_3 \delta^3 + \epsilon + \mathcal{O}(\delta_1^4), \quad (2)$$

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where δ is the mass-density perturbation, $\sigma^2 = \langle \delta^2 \rangle$, and δ_g is the tracer density perturbation (I will usually refer to the tracer as galaxies, although there are many other possibilities like quasars [62], the Ly α forest [66–68], galaxy cluster/Sunyaev-Zel’dovich effect measurements [69], and possibly future 21 cm surveys [70,71]). The unknown coefficients of the Taylor series have become the bias parameters c_i and a random noise variable ϵ has been added to allow for stochasticity in the relation (e.g., from shot noise, at least). This type of bias model has been explored in many papers [72–79]. The new development in [64] was to show that large higher-order corrections in this perturbative approach can be eliminated by redefining the bias parameters to absorb them, motivated by the way that masses and coupling constants are redefined to absorb divergent loop corrections in quantum field theory [80]. I argued that this renormalization approach is an improvement over the previous approach of defining the galaxy density to be a function of a smoothed mass-density field (so the higher-order terms in the Taylor series can be kept explicitly small [65,73]), because the smoothing needed is so extreme that it directly affects the scales of interest [81,82], and because, in the unrenormalized approach, the higher-order corrections modify the power on arbitrarily large, ideally truly linear, scales [73]. Reference [83] found extremely good agreement between the renormalized model and the clustering of galaxies in numerical simulations. Generally, the value of perturbation theory (PT) for describing LSS will only increase as observational measurements become more precise, because this increases the range of scales where corrections to linear theory are important but are still small enough to be treated perturbatively. The value of PT has been further enhanced recently by the introduction of several renormalization methods applicable to the mass-power spectrum calculation [84–93].

To complete the relation between primordial non-Gaussianity and final density, note that the initial density field is related to Φ through the transfer function from primordial to late-time linear fluctuations, $T(k)$, and the Poisson equation

$$\delta_1(\mathbf{k}, a) = \frac{2}{3} \frac{c^2 D(a)}{\Omega_{m,0} H_0^2} k^2 T(k) \Phi_{\mathbf{k}} \equiv M(k, a) \Phi_{\mathbf{k}}, \quad (3)$$

using the definitions of [55], where I will use the subscript on δ to indicate the order in the initial density perturbations, not the Gaussian field ϕ (I will use δ_L to indicate the fully linear, including in ϕ , density perturbation). The growth factor is normalized so $D(a) = a$ in the matter dominated era. The transfer function is time independent and normalized by $T(k \rightarrow 0) \rightarrow 1$. These definitions make $\Phi_{\mathbf{k}}$ time independent and close to scale invariant. The perturbative density field is written as a Taylor series in δ_1 , i.e., $\delta = \delta_1 + \delta_2 + \delta_3 + \dots$, with $\delta_i \sim \mathcal{O}(\delta_1^i)$, e.g.,

$$\delta_2(\mathbf{k}) = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \delta_1(\mathbf{q}) \delta_1(\mathbf{k} - \mathbf{q}) J_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}), \quad (4)$$

where

$$J_S^{(2)}(\mathbf{k}_1, \mathbf{k}_2) = \frac{5}{7} + \frac{1}{2} \frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2} \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + \frac{2}{7} \left(\frac{\mathbf{k}_1 \cdot \mathbf{k}_2}{k_1 k_2} \right)^2 \quad (5)$$

is given by standard LSS perturbation theory (see [94] for a review).

The rest of the paper goes as follows: To put the perturbative bias calculation on a firm foundation, in Sec. II, I compute the mass power spectrum in the non-Gaussian model. In Sec. III, I calculate the power spectrum of a biased tracer. Finally, in Sec. IV, I discuss the results.

II. MASS-DENSITY POWER SPECTRUM

In this section, I compute the power spectrum of mass-density fluctuations in the non-Gaussian model. I note in advance that infrared divergences will appear, i.e., sensitivity to arbitrarily large scales, which suggests that a more sophisticated calculation than the usual, essentially Newtonian, LSS perturbation theory might be required. The point of this section is simply to show that the usual LSS PT can in fact be pursued self-consistently, without ignoring any divergences, imposing arbitrary cutoffs, or integrating into a regime where the calculations are not valid. All of the sensitivity to very large (and very small) scales can be absorbed into the overall normalization of the power spectrum, so there is no need to worry about exactly where a physical large-scale cutoff might appear, as long as it is much larger than the scale of the observations. For more physical discussions of infrared divergences in inflation, see [95–102]. Note that there are no divergences in standard LSS PT calculations of the mass power spectrum starting from Gaussian primordial perturbations, for a typical Λ CDM power spectrum (divergences for pure power law linear power spectra are discussed in [103–105]).

The power spectrum to 4th order in δ_1 is

$$P_{\delta\delta}(k) = P_{\delta_1\delta_1}(k) + 2P_{\delta_1\delta_2}(k) + 2P_{\delta_1\delta_3}(k) + P_{\delta_2\delta_2}(k) + \dots \quad (6)$$

The most interesting term turns out to be the first one,

$$P_{\delta_1\delta_1}(k) = M^2(k, a) P_{\Phi\Phi}(k), \quad (7)$$

with

$$P_{\Phi\Phi}(k) = P_{\phi\phi}(k) + 2g_{\text{NL}} P_{\phi\phi^3} + f_{\text{NL}}^2 P_{\phi^2\phi^2}(k) + \dots \quad (8)$$

The first term here is simply the usual lowest order power spectrum, which I assume is given by $P_{\phi\phi}(k) = Ak^{n_s-4}$. The 2nd term is divergent, in a very simple way:

$$2g_{\text{NL}} P_{\phi\phi^3}(k) = 6g_{\text{NL}} \sigma_{\phi\phi}^2 P_{\phi\phi}(k), \quad (9)$$

where

$$\sigma_{\phi\phi}^2 = \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\phi\phi}(q). \quad (10)$$

For $n_s < 1$, $\sigma_{\phi\phi}^2 = A(\epsilon^{n_s-1} - \Lambda^{n_s-1})/2\pi^2(1 - n_s)$ is infrared divergent, where ϵ is the minimum q and Λ is the maximum q (which we would like to take to zero and infinity, respectively). The solution to this divergence is obvious: the divergent term just renormalizes the amplitude of the power spectrum, i.e., $A \rightarrow A + 6g_{\text{NL}}\sigma_{\phi\phi}^2$.

The third term in Eq. (8) is slightly more subtle:

$$f_{\text{NL}}^2 P_{\phi^2\phi^2}(k) = 2f_{\text{NL}}^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|) \xrightarrow{\epsilon/k \rightarrow 0} [4f_{\text{NL}}^2 \sigma_{\phi\phi}^2] P_{\phi\phi}(k). \quad (11)$$

There is again an infrared divergence for $n_s < 1$, but it does not look like a simple k -independent renormalization of the

$$\begin{aligned} f_{\text{NL}}^2 P_{\phi^2\phi^2}(k) &= P_{\phi\phi}(k) \left(4f_{\text{NL}}^2 \sigma_{\phi\phi}^2 + 2f_{\text{NL}}^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[\frac{P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|)}{P_{\phi\phi}(k_*)} - P_{\phi\phi}(q) - P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|) \right] \right) \\ &\quad + 2f_{\text{NL}}^2 P_{\phi\phi}(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} \left[\frac{P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|)}{P_{\phi\phi}(k)} - P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|) - \frac{P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|)}{P_{\phi\phi}(k_*)} \right. \\ &\quad \left. + P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|) \right]. \end{aligned} \quad (13)$$

I have simply added and subtracted $P_{\phi\phi}(k)$ times some constant piece, with the constant chosen to cancel the correction term at some arbitrary scale k_* . Note that we could drop the terms in the integral that are simply integrals over $-P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|)$ and $P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|)$, because they exactly cancel. This would leave us with the form we would have found by starting with the second form of renormalization (involving k_*) instead of going through the initial step of subtracting the truly infinite part, i.e., the first renormalization was not strictly necessary. I leave the equation in the more complicated looking form because the simpler one would involve cancellation between pieces of the integral that diverge at different values of q (i.e., k and k_*), which makes the integral less straightforward to evaluate.

The power spectrum of Φ is now simply

$$\begin{aligned} P_{\Phi\Phi}(k) &= P_{\phi\phi}(k) + 2f_{\text{NL}}^2 P_{\phi\phi}(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} \\ &\quad \times \left[\frac{P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|)}{P_{\phi\phi}(k)} - P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|) \right. \\ &\quad \left. - \frac{P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|)}{P_{\phi\phi}(k_*)} + P_{\phi\phi}(|\mathbf{k}_* - \mathbf{q}|) \right], \end{aligned} \quad (14)$$

where $P_{\phi\phi}$ now has the renormalized amplitude, essen-

tially disconnected from the original amplitude. Inserting $P_{\phi\phi}(k) = Ak^{n_s-4}$, we find, at least to a very good approximation

$$\begin{aligned} f_{\text{NL}}^2 P_{\phi^2\phi^2}(k) &= [4f_{\text{NL}}^2 \sigma_{\phi\phi}^2] P_{\phi\phi}(k) + 2f_{\text{NL}}^2 \int \frac{d^3\mathbf{q}}{(2\pi)^3} \\ &\quad \times [P_{\phi\phi}(q) P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|) \\ &\quad - P_{\phi\phi}(q) P_{\phi\phi}(k) - P_{\phi\phi}(k) P_{\phi\phi}(|\mathbf{k} - \mathbf{q}|)], \end{aligned} \quad (12)$$

where I have written the two equivalent negative terms in the integral to emphasize that half of the divergence comes from $q \rightarrow 0$ and half from $q \rightarrow k$.

Subtracting the divergent part has now introduced a new source of trouble, unfortunately, in that the subtracted terms very nearly diverge as $q \rightarrow \infty$, e.g., for $n_s = 0.96$ a cutoff $\Lambda \gtrsim 10^{50}k$ is required for convergence to 1% accuracy. The solution is to pull out another factor renormalizing the amplitude, as follows:

tially disconnected from the original amplitude. Inserting $P_{\phi\phi}(k) = Ak^{n_s-4}$, we find, at least to a very good approximation

$$\begin{aligned} P_{\Phi\Phi}(k) &\simeq P_{\phi\phi}(k) + f_{\text{NL}}^2 A^2 \frac{2}{\pi^2(n_s - 1)} \\ &\quad \times (k^{2n_s-5} - k^{n_s-4} k_*^{n_s-1}) \\ &= P_{\phi\phi}(k) \left(1 + \frac{4f_{\text{NL}}^2}{(n_s - 1)} \frac{A(k^{n_s-1} - k_*^{n_s-1})}{2\pi^2} \right). \end{aligned} \quad (15)$$

Note that this result is well behaved in the scale-invariant limit, $(n_s - 1) \rightarrow 0$, when $(k^{n_s-1} - k_*^{n_s-1})/(n_s - 1) \rightarrow \ln(k/k_*)$ (this would have been a disaster without the second renormalization, introducing k_* , of the UV near divergence, which becomes a true divergence in this limit).

Now we can estimate the change in apparent slope,

$$\begin{aligned} \Delta n_s &= d \ln \left(1 + \frac{4f_{\text{NL}}^2}{(n_s - 1)} \frac{A(k^{n_s-1} - k_*^{n_s-1})}{2\pi^2} \right) / d \ln k \\ &\simeq 4f_{\text{NL}}^2 \Delta_{\phi\phi}^2(k), \end{aligned} \quad (16)$$

where $\Delta_{\phi\phi}^2(k) = Ak^{n_s-1}/2\pi^2$, and change in the running of the slope

$$\Delta \alpha_s = \frac{d\Delta n_s}{d \ln k} \simeq (n_s - 1) \Delta n_s. \quad (17)$$

For a realistic model $\Delta_{\phi\phi}^2(k) \simeq 8 \times 10^{-10}$ so the change in slope is quite small, e.g., for an unrealistically large $f_{\text{NL}} = 1000$, $\Delta n_s \simeq 0.0033$, which is roughly the expected precision of a measurement of the slope using Planck data [106,107]. The bottom line of this calculation is that we can safely ignore the non-Gaussian contribution to $P_{\delta_1\delta_1}$, for realistic models. Note that, even if the new contribution to the slope or running had been significant, it would not necessarily have been distinguishable from a change in the underlying inflation model, i.e., the bare slope or running. It would still need to be kept in mind, however, in order to correctly interpret measurements of the power spectrum as constraints on the inflation model.

The rest of the calculation of the mass power spectrum is less interesting,

$$2P_{\delta_1\delta_2}(k) = 4f_{\text{NL}} \int \frac{d^3\mathbf{q}}{(2\pi)^3} J^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) P_{\phi\delta_L}(q) \times [2P_{\phi\delta_L}(k)M(|\mathbf{k} - \mathbf{q}|) + P_{\phi\delta_L}(|\mathbf{k} - \mathbf{q}|)M(k)], \quad (18)$$

where $P_{\phi\delta_L}(k) \equiv M(k)P_{\phi\phi}(k)$. For realistic power spectra, $P_{\delta_1\delta_2}(k)$ is comfortably convergent in both limits, and produces a subpercent effect for $f_{\text{NL}} < 10^5$. The terms 4th order in δ_1 are of course just the usual nonlinear correction to the Gaussian linear theory power spectrum.

To be clear, the conclusion of this section is that the non-Gaussianity of the initial conditions in the local model does not significantly affect the mass power spectrum, verifying what has been previously assumed. Note that [88] considers a different form of renormalized PT with non-Gaussianity, but the results are not directly comparable because their specific example is a different form of non-Gaussianity (local in density).

III. BIAS

This section contains the primary calculation of this paper: the power spectrum of biased tracers of the density field. I start by assuming the bias takes the form of a local Taylor series in density

$$\delta_g \equiv \rho_g/\bar{\rho}_g - 1 = c_\delta\delta + \frac{1}{2}c_{\delta^2}(\delta^2 - \sigma^2) + \frac{1}{3!}c_{\delta^3}\delta^3 + \epsilon + \mathcal{O}(\delta_1^4). \quad (19)$$

The first, and only, new nontrivial term related to non-Gaussianity is the linear-quadratic cross-term

$$c_\delta c_{\delta^2} P_{\delta\delta^2}(k) = 2c_\delta c_{\delta^2} f_{\text{NL}} \left[M(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\phi\delta_L}(q) \times P_{\phi\delta_L}(|\mathbf{k} - \mathbf{q}|) + 2P_{\phi\delta_L}(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} \times P_{\phi\delta_L}(q)M(|\mathbf{k} - \mathbf{q}|) \right] + f_{\text{NL}} - \text{independent parts}. \quad (20)$$

The second term in the brackets is not well behaved in the limit $q \rightarrow \infty$, where it becomes $[c_\delta c_{\delta^2} 4f_{\text{NL}} \sigma_{\delta\delta}^2] P_{\phi\delta_L}(k)$ (this term comes from the part of $\delta(\mathbf{x})\delta^2(\mathbf{y})$ proportional to $\delta(\mathbf{x})\delta(\mathbf{y})\phi(\mathbf{y})\nabla^2\phi(\mathbf{y})$). It nearly diverges, as discussed above, in the same sense as the unsmoothed linear density variance nearly diverges, i.e., the cutoff-dependent limit goes like Λ^{n_s-1} (at least until one hits the cold dark matter free streaming scale, which actually happens before the integral would otherwise converge [108]—the fact that that scale enters this discussion should make it clear why something needs to be done about this term). It is not immediately obvious how to deal with this term. Ignoring it is not a good option, because it can become large relative to the leading order term, at small k , i.e.,

$$\frac{c_\delta c_{\delta^2} 4f_{\text{NL}} \sigma_{\delta\delta}^2 P_{\phi\delta_L}(k)}{c_\delta^2 P_{\delta_L\delta_L}(k)} = \frac{c_{\delta^2}}{c_\delta} 4f_{\text{NL}} \sigma_{\delta\delta}^2 M^{-1}(k) \sim 29 \left(\frac{f_{\text{NL}}}{100} \right) \left(\frac{c_{\delta^2}}{c_\delta} \right) \left(\frac{\sigma_{\delta\delta}^2}{100} \right) \times \left(\frac{k}{0.01h \text{ Mpc}^{-1}} \right)^{-2}. \quad (21)$$

Taking this result at face value is not an option, both because it is a large correction in what is supposed to be a perturbative expansion, and because it is ridiculous in LSS perturbation theory to trust the quantity $\sigma_{\delta\delta}^2$, which requires integration many orders of magnitude into the nonlinear regime for convergence. We cannot remove this term by renormalizing the linear bias parameter, because the k dependence of the linear term, proportional to $P_{\delta_L\delta_L}(k)$, is not consistent with the k dependence of this term, proportional to $P_{\phi\delta_L}(k)$ (we could always cancel the value of this term at one point k_* , but the differing k dependence would mean that the term would become large again very quickly as we went away from k_*).

The solution seems to be to introduce a new term in the bias formula $c_\phi f_{\text{NL}} \phi$, i.e.,

$$\delta_g \equiv \rho_g/\bar{\rho}_g - 1 = c_\delta\delta + c_\phi f_{\text{NL}} \phi + \frac{1}{2}c_{\delta^2}(\delta^2 - \sigma^2) + \frac{1}{3!}c_{\delta^3}\delta^3 + \epsilon + \mathcal{O}(\delta_1^4). \quad (22)$$

Then the power spectrum will contain the term $2c_\delta c_\phi f_{\text{NL}} P_{\phi\delta_L}(k)$, which is needed to absorb the divergence, i.e., the bare value of c_ϕ (which might be zero) will be renormalized to $b_\phi = c_\phi + 2c_{\delta^2} \sigma_{\delta\delta}^2$.

Inevitably, we now also obtain a contribution to the power spectrum $c_\phi^2 f_{\text{NL}}^2 P_{\phi\phi}(k)$, i.e., the linear theory power spectrum is now

$$P_{gg}(k) = c_\delta^2 P_{\delta_L \delta_L}(k) + 2c_\delta c_\phi f_{\text{NL}} P_{\phi\delta_L}(k) + c_\phi^2 f_{\text{NL}}^2 P_{\phi\phi}(k) + \dots \quad (23)$$

Note that the decision to write $c_\phi f_{\text{NL}}$ instead of including the f_{NL} factor in c_ϕ is suggestive but ultimately arbitrary, since c_ϕ can in principle depend on f_{NL} . The divergent correction to c_ϕ is of generally the same order of magnitude, i.e., proportional to $\sigma_{\delta\delta}$, as the correction to c_δ introduced by [64], which gives us some reason to expect b_δ and b_ϕ to be of similar order (I use b in place of c to indicate the renormalized parameter). Figure 1 shows an example of this power spectrum.

Equation (23) is really the bottom line result of the paper, but for completeness I compute all the higher-order terms arising from the new term

$$\begin{aligned} & 2c_\phi f_{\text{NL}} \left[\frac{c_{\delta^2}}{2} P_{\phi\delta^2}(k) + \frac{c_{\delta^3}}{6} P_{\phi\delta^3}(k) \right] \\ &= 4c_\phi c_{\delta^2} f_{\text{NL}}^2 P_{\phi\phi}(k) \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\phi\delta_L}(q) M(|\mathbf{k} - \mathbf{q}|) \\ &+ c_\phi f_{\text{NL}} \left(\frac{68}{21} c_{\delta^2} + c_{\delta^3} \right) P_{\phi\delta_L}(k) \sigma_{\delta\delta}^2. \end{aligned} \quad (24)$$

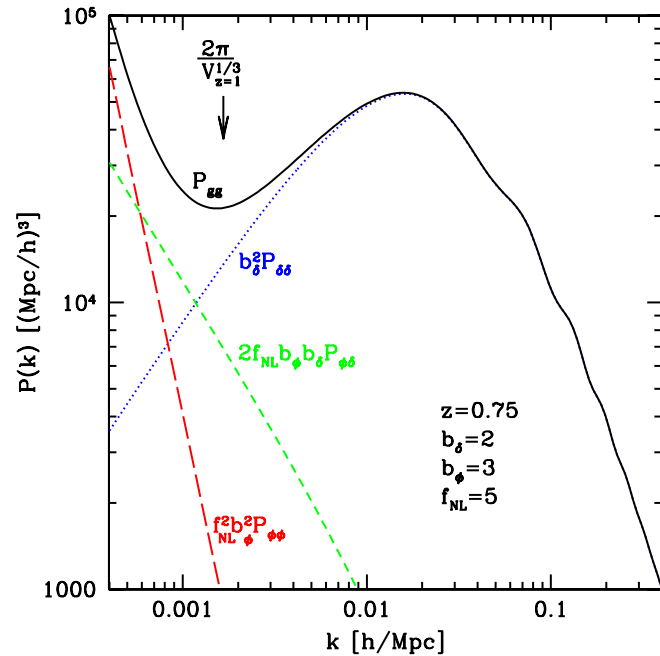


FIG. 1 (color online). Real-space power spectrum for $f_{\text{NL}} = 5$, at $z = 0.75$, of galaxies with $b_\delta = 2$, $b_\phi = 3$ (black, solid line). Also shown are the components of the power spectrum, $b_\delta^2 P_{\delta\delta}(k)$ (blue, dotted line), $2f_{\text{NL}} b_\phi b_\delta P_{\phi\delta}(k)$ (green, short-dashed), and $f_{\text{NL}}^2 b_\phi^2 P_{\phi\phi}(k)$ (red, long-dashed line). The arrow indicates the approximate minimum k probed by an all-sky survey out to $z = 1$.

Note that each piece here has a near-divergent part, however, each of these is neatly canceled by the renormalizations we are already doing. The first term has the same form as the ones we dealt with in Eq. (21), so one can see that the redefinition $b_\phi = c_\phi + 2c_{\delta^2} \sigma_{\delta\delta}^2$ will cancel the divergent part of the first term in Eq. (24). Similarly, the original renormalization of c_δ in [64], $b_\delta = c_\delta + (\frac{34}{21} c_{\delta^2} + \frac{c_{\delta^3}}{2}) \sigma_{\delta\delta}^2$, cancels the rest of Eq. (24), when $b_\delta \delta$ is cross multiplied with $c_\phi f_{\text{NL}} \phi$. Things work out so neatly simply because $P_{\phi X}(k) = M^{-1}(k) P_{\delta X}(k)$ (where X could be anything), so there is no nontrivial new behavior when calculating cross terms with ϕ instead of δ . The cross terms between the new linear ϕ term and higher-order gravitational corrections to δ are negligible because the latter only contribute at small scales, while ϕ is only significant at large scales (note, however, that if one were trying to exploit the very small non-Gaussian signal that is present at relatively high k , these terms might come into play).

This linear-quadratic ($\delta\delta^2$) cross term is the only new interesting one because the linear-linear term is of course just the usual linear bias times the nonlinear power spectrum (shown to be unaffected by non-Gaussianity in the previous section), while the quadratic-quadratic and linear-cubic terms are already 4th order in ϕ , without the non-Gaussian part.

After the divergent pieces are subtracted from Eqs. (21) and (24), the k -dependent residuals are tiny for any reasonable f_{NL} , so I drop them. The final power spectrum is thus

$$\begin{aligned} P_{gg}(k) &= b_\delta^2 P_{\delta\delta}(k) + 2b_\delta b_\phi f_{\text{NL}} P_{\phi\delta}(k) + b_\phi^2 f_{\text{NL}}^2 P_{\phi\phi}(k) \\ &+ \frac{b_{\delta^2}^2}{2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\delta\delta}(q) [P_{\delta\delta}(|\mathbf{k} - \mathbf{q}|) - P_{\delta\delta}(q)] \\ &+ 2b_{\delta^2} \int \frac{d^3\mathbf{q}}{(2\pi)^3} P_{\delta\delta}(q) P_{\delta\delta}(|\mathbf{k} - \mathbf{q}|) \\ &\times J_S^{(2)}(\mathbf{q}, \mathbf{k} - \mathbf{q}) + N, \end{aligned} \quad (25)$$

where the last three terms are those found in [64], unrelated to primordial non-Gaussianity.

IV. CONCLUSIONS AND DISCUSSION

In Sec. II, we found that the corrections to the mass power spectrum arising from the non-Gaussian initial conditions are naively infinite, but the divergence can be removed by a renormalization of the amplitude of perturbations, after which there is no significant effect of non-Gaussianity for realistic models. It is interesting to note that a sufficiently large numerical simulation would have to deal with this issue explicitly, because the most natural method of implementing initial conditions effectively implements the unrenormalized version [43]; however, the dynamical range of typical simulations is small enough that this effect is unlikely to be noticeable (the box size and

finite resolution provide large and small-scale cutoffs). The issues discussed in Sec. II should be revisited when considering other forms of non-Gaussianity.

In Sec. III, I computed the power spectrum of biased tracers of mass density and found a large higher-order perturbative correction that can only be removed by renormalization if we add a term to the standard linear bias model, proportional to ϕ , producing the model

$$\delta_g = c_\delta \delta + c_\phi f_{\text{NL}} \phi + \frac{1}{2} c_{\delta^2} (\delta^2 - \sigma^2) + \frac{1}{3!} c_{\delta^3} \delta^3 + \epsilon + \mathcal{O}(\delta_1^4). \quad (26)$$

This new term will propagate into the computation of all statistics, not just the power spectrum. Note that ϕ should not be confused with the Newtonian potential at the time of observation—it is defined in terms of δ by Eq. (3).

One might ask at this point: “once there is a linear term, why did we not add higher-order terms in ϕ ?” The short answer is simply that we did not need to. We added the minimal term necessary to produce a well-behaved calculation. A calculation to even higher order would no doubt require the addition of higher-order counter terms, and guide their form. It seems likely that the higher-order terms would be completely negligible, like the corrections to the mass power spectrum, but, as we have learned, one should really compute them instead of assuming.

Measuring f_{NL} accurately depends on predicting b_ϕ for an observable in just the same way as measuring the amplitude of the matter power spectrum depends on a prediction of the usual bias b_δ . By comparison to the results of [41,43,62], we can determine the value of b_ϕ predicted by the halo model. As emphasized by [62], however, it should be kept in mind that this kind of prediction makes some assumptions that are not guaranteed to be true, e.g., the galaxy population in a halo might depend on something other than halo mass alone, like merging history, which affects the clustering of the population. Halo model predictions will give a good estimate for the size of effect we expect for a given f_{NL} and type of galaxy, but if we are fortunate enough to make a detection it will be very difficult to measure f_{NL} to high precision using only this power spectrum effect. With that caveat, Eq. (18) of [62] leads to

$$b_\phi = 2(b_\delta - 1)\delta_c = 3.372(b_\delta - 1). \quad (27)$$

Reference [43] computed that a very modest future survey, extending only to $z = 0.7$ (while the bulk of the volume of the universe is at higher redshift) can constrain f_{NL} to $\sim \pm 10$. It is interesting to know at least roughly how well a larger survey can do. For a well-sampled survey (i.e., no shot noise), the signal to noise of a single mode is

$$\frac{S}{N}_{\text{mode}} = \frac{[1 + (\frac{k_*}{k})^2]^2 - 1}{[1 + (\frac{k_*}{k})^2]^2}, \quad (28)$$

(note that a null test would not include the signal in the denominator, but this makes very little difference, as I will show) where

$$k_* = \left(\frac{3b_\phi f_{\text{NL}} \Omega_{m,0}}{2b_\delta D(a)} \right)^{1/2} \frac{h \text{ Mpc}^{-1}}{3000} \quad (29)$$

I am assuming $T(k) \approx 1$ on the relevant scales, as discussed below. I will ignore redshift space, geometric, and evolutionary distortions, and any confounding systematic errors or parameter degeneracies. The total signal to noise of a survey is approximately

$$\left(\frac{S}{N} \right)^2 \approx \frac{V}{4\pi^2} \int_{k_{\min}}^{k_{\max}} dk k^2 \left[\frac{S}{N}(k) \right]^2, \quad (30)$$

where $k_{\min} \sim 2\pi/V^{1/3}$ (the detection limit is not very sensitive to k_{\min} , as long as k_{\min} is basically set by the survey volume). k_{\max} should be something less than ∞ , but, for interesting k_* the integral converges within the linear regime (the integral would not converge if the k dependence of the transfer function was included, however, the information gained this way would come from a tiny change in signal on scales where the signal would not be uniquely distinguishable from changes in other parameters, and linearity cannot be safely assumed, so a projection including it would be unreliable). The bulk of the signal comes from the range $2k_* \lesssim k \lesssim 10k_*$. Consequently, the bulk of the signal comes from the $f_{\text{NL}} \phi \delta$ cross term, rather than the $f_{\text{NL}}^2 \phi \phi$ term (cross correlating with an unbiased

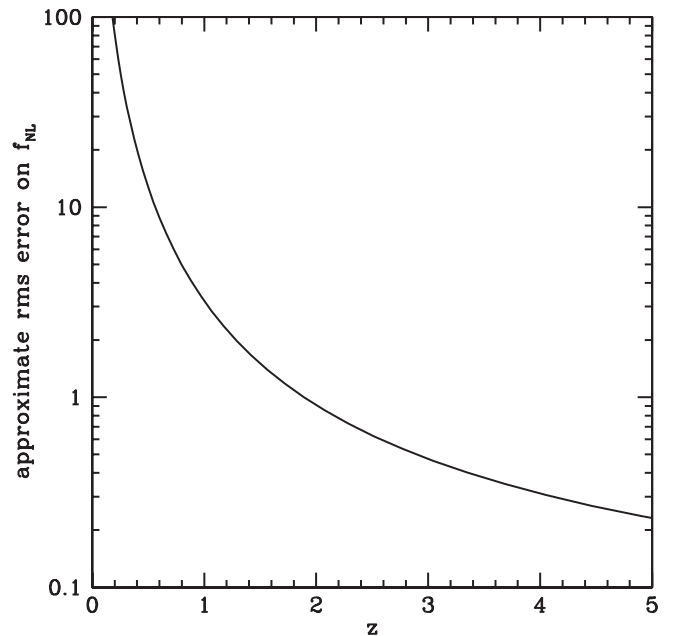


FIG. 2. Rms error on f_{NL} for a well-sampled all-sky survey out to z , ignoring redshift-space, geometric, and evolutionary distortions. Rms detection significance is essentially linear in f_{NL} (out to a $\sim 5\sigma$ level of detection), and scales roughly as volume $^{2/3}$ (as long as the volume remains compact).

tracer is worse than autocorrelation, but essentially only because of the factor of 2 that is lost in the cross term). To a very good approximation, $S/N = 1$ when $k_* \sim \pi V^{-1/3} \sim k_{\min}/2$ (e.g., this would be $\sim 0.8\pi V^{-1/3}$ if I used the null test S/N per mode and set k_{\min} based on the diameter of a sphere instead of the side length of a cube, which makes it slightly smaller), which gives $f_{\text{NL}} b_\phi / b_\delta D(a) \simeq 200 (\text{Gpc}/h)^2 / V^{2/3}$ (for $\Omega_m = 0.3$). Assuming Eq. (27) and $b_\delta D(a) = 2$, we have finally that $S/N = 1$ when $f_{\text{NL}} \sim 125 [(\text{Gpc}/h)^2 / V^{2/3}] / [2/D(a) - 1]$. I plot the value of this function for a survey covering all volume up to z in Fig. 2. Note that the assumption $b_\delta D(a) = 2$ appears to make b_δ unreasonably large at high redshift, however, the value of b_δ cancels in b_ϕ / b_δ if b_δ is very large, so the result is not actually very sensitive to this assumption. We see that there is plenty of volume in the Universe to measure f_{NL} even if it is less than 1, although this will of course take a huge amount of work. Reference [42] found similar results, using a variety of slightly different assumptions, which testifies to the robustness of the projection (which ultimately just amounts to the statement that the survey should be large enough to resolve the k where the non-Gaussian-induced power becomes roughly equal to the Gaussian power). Twenty-one cm intensity mapping [71,109] may be a route to surveying all of the post-reionization volume.

Even if one's tracer is weakly biased, so the halo model predicts little signal, it may be possible to make a nonlinear

transformation of the density field to enhance the bias; however, it is not clear that any such transformation would be better than simply measuring the bispectrum. It is interesting to note that the power spectrum measurement proposed by [43] and discussed in this paper can be seen as nature's implementation of the poor person's bispectrum measurement suggested by [110], i.e., the cross correlation of the field with the square of the field, where here the squaring is done for us, as part of the formation of biased structure. The significance of the difference between the natural and artificial versions should not be underestimated—exploiting the natural version we can observe the effect on large scales even if limited resolution or noise prevents us from probing the small scales that would be required to perform a similar transformation artificially. References [110,111] found that f_{NL} may be measurable to ~ 0.01 using the bispectrum of high redshift 21 cm observations and probing down to very small scales. In this paper, I have excluded the possibility of measuring the signal using small scales (by assuming the transfer function is 1 in the signal-to-noise ratio (S/N) calculation), but it may be useful, especially at high redshift, to explore the possibility of using smaller scales and combining the bispectrum and power spectrum.

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