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Scalar field dark energy perturbations and their scale dependence

Sanil Unnikrishnan,[1,*] H. K. Jassal,[2,†] and T. R. Seshadri[1,‡]

[1]*Department of Physics and Astrophysics, University of Delhi, Delhi: 110007, India*

[2]*Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India*

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We estimate the amplitude of perturbation in dark energy at different length scales for a quintessence model with an exponential potential. It is shown that on length scales much smaller than Hubble radius, perturbation in dark energy is negligible in comparison to that in dark matter. However, on scales comparable to the Hubble radius ($\lambda_p > 1000$ Mpc) the perturbation in dark energy in general cannot be neglected. As compared to the ΛCDM model, the large-scale matter power spectrum is suppressed in a generic quintessence dark energy model. We show that on scales $\lambda_p < 1000$ Mpc, this suppression is primarily due to different background evolution compared to the ΛCDM model. However, on much larger scales perturbation in dark energy can affect the matter power spectrum significantly. Hence this analysis can act as a discriminator between the ΛCDM model and other generic dark energy models with $w_{\rm de} \neq -1$.

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I. INTRODUCTION

Cosmological observations suggest that about 70% of the content of our universe is made of a form of matter which drives the accelerated expansion of the Universe [1]. These observations include Supernova type Ia observations [2], observations of cosmic microwave background (CMB) [3–5] and large scale structure [6–8]. The accelerated expansion of the Universe can of course be explained by introducing a cosmological constant Λ in the Einstein's equation [9,10]. However, the cosmological constant model is plagued by the fine turning problem [9]. This has motivated the study of dark energy models to explain the current accelerated expansion of the Universe. The simplest model as an alternative to the cosmological constant model is to assume that this accelerated expansion is driven by a canonical scalar field with a potential $V(\phi)$. This class of dark energy models is known as the quintessence model and the scalar field is known as a quintessence field. Various quintessence models have been studied in the literature [11–13]. There exists another class of string theory inspired scalar field dark energy models known as tachyon models [14,15]. Models of dark energy which allow $w < -1$ are known as phantom models [16]. Phantom type dark energy can also be realized in a scalar-tensor theory of gravitation. (See, for example, Ref. [17].) Other scalar field models include k-essence field [18], branes [19] and Chaplygin gas model and its generalizations [20]. There are also some phenomenological models [21], field theoretical and reorganization group based models (see e.g. [22]), models that unify dark matter and dark energy [23], holographic dark energy models [24] and many others like those based on horizon thermodynamics (e.g. see [25]). For reviews of dark energy models see for instance Ref. [26], and for constraining parameters using observations see Ref. [27].

Homogeneous dark energy distribution leads to accelerated expansion of the Universe which, in turn, governs the luminosity distance and angular diameter distance. The rate of expansion also influences the growth of density perturbations in the universe. This is evident from the abundance of rich clusters of galaxies and their evolution and the integrated Sachs Wolfe (ISW) effect [28].

In this paper we present a set of arguments which lead to the conclusion that inhomogeneous dark matter with homogeneous dark energy at all length scales is inconsistent with Einstein's equation if $p_{\rm de} \neq -\rho_{\rm de}$. We further analyze how the dark matter power spectrum is influenced by perturbation in dark energy with an evolving equation of state parameter.

Dark energy perturbations have been extensively studied in the linear approximation [29–31]. It was shown in Ref. [29] that dark energy perturbations affect the low l quadrapole in the CMB angular power spectrum through the ISW effect. This analysis was done for a constant equation-of-state parameter. For models with $w > -1$ this effect is enhanced while for phantomlike models it is suppressed. In these models dark matter perturbations and dark energy perturbations are anticorrelated for large effective sound speeds. This anticorrelation is a gauge-dependent effect [30]. Detailed studies in dark energy perturbations also include [32,33]. Clustering of dark energy within matter over density and voids were studied by Mota *et al* [34].

In this paper we study the evolution of perturbation in dark energy in a quintessence model which results in an evolving equation-of-state parameter different from that considered in Refs. [29,35]. We use a specific model of

*sanil@physics.du.ac.in
†hkj@mri.ernet.in
‡trs@physics.du.ac.in

scalar field dark energy with an exponential potential. We find that although for scales much smaller than the Hubble radius the perturbation in dark energy is small, for scales $\geq H^{-1}$ the perturbation in dark energy can be comparable to that in matter. Hence, although on small scales ($\ll H^{-1}$) the dark energy can be treated as homogeneous, one has to take into account the perturbation in dark energy over scales $\sim H^{-1}$ if $w_{\rm de} \neq -1$. Clearly, in the specific case of $w_{\rm de} = -1$ the dark energy is homogeneous at all scales.

We choose to work in the longitudinal gauge since in that case we can directly relate the metric perturbation Φ to the gravitational potential perturbation. For a specific model, we investigate how quintessence dark energy influences the matter power spectrum. We show that on scales $\lambda_p < 1000$ Mpc, the matter power spectrum is not significantly affected whether or not we include fluctuations in the quintessence field in perturbation equations. However, on much larger scales, including or excluding fluctuations in the quintessence field does result in significant changes in the matter power spectrum.

This paper is organized as follows. In Sec. II we discuss the background cosmology for matter and the scalar field system and describe the cosmological perturbation equation in longitudinal gauge for this system. In Sec. III we obtain the numerical solution of the perturbation equation to determine the ratio of the perturbations in dark energy to the perturbations in matter at various length scales. Section IV summarizes the results.

II. INHOMOGENEOUS MATTER AND DARK ENERGY

We shall consider a system of minimally coupled matter (dark + baryonic) and canonical scalar field[1] with a Lagrangian of the form:

$$\mathcal{L}_\phi = \tfrac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi). \tag{1}$$

As a simple example, we consider a scalar field potential of the form [36–39]:

$$V(\phi) = V_o \exp\left[-\sqrt{\lambda}\frac{\phi}{M_p}\right]. \tag{2}$$

Here λ and V_o are two parameters of the potential, and $M_p = (8\pi G)^{-1/2}$ is the Planck mass. This potential leads to scaling solutions [12,13,38]. However, for treating the exponential potential as a possible candidate for dark energy, we require that the scalar field should not enter the scaling regime. This is possible to achieve by restricting the choice of the parameter in the exponential potential [38].

[1]We shall denote scalar field by ϕ and the metric perturbation in the longitudinal gauge by Φ.

A spatially flat homogeneous and isotropic line element is described by the Friedman-Robertson-Walker (FRW) metric of the form:

$$ds^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]. \tag{3}$$

For the system of pressureless matter and quintessence with exponential potential the background evolution $a(t)$ is completely determined by solving the following Friedmann equation and the Klein Gordon equation:

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\left[\rho_m a^{-3} + \frac{1}{2}\dot{\phi}^2 + V_o \exp\left(-\sqrt{\lambda}\frac{\phi}{M_p}\right)\right] \tag{4}$$

and

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\sqrt{\lambda}}{M_p}V_o \exp\left[-\sqrt{\lambda}\frac{\phi}{M_p}\right] = 0. \tag{5}$$

A. Equations for perturbations

For analyzing perturbations in scalar fields and matter we work in the longitudinal gauge [40–42] (For a recent pedagogical review, see [43]). For scalar field and for pressureless matter (described as a perfect fluid), the perturbed energy-momentum tensor has no anisotropic term, i.e., $\delta T^i{}_j$ is diagonal (where $i, j = 1, 2, 3$).[2] Einstein's equations would then imply that the scalar-metric perturbations in this gauge are completely described by a single scalar variable Φ. For such a system, a perturbed FRW metric in longitudinal gauge attains the form [42]:

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Phi)[dx^2 + dy^2 + dz^2]. \tag{6}$$

It is a good approximation to treat dark matter, baryonic matter, etc. as perfect fluids. The energy-momentum tensor of a perfect fluid is described as

$$T^\mu{}_\nu = (\rho + p)u^\mu u_\nu - p\delta^\mu{}_\nu. \tag{7}$$

Perturbations in the energy density ρ, pressure p, and the four velocity u^μ are defined as

$$\rho(t, \vec{x}) = \rho_o(t) + \delta\rho(t, \vec{x}) \tag{8}$$

$$p(t, \vec{x}) = p_o(t) + \delta p(t, \vec{x}) \tag{9}$$

$$u^\mu = {}^{(o)}u^\mu + \delta u^\mu, \tag{10}$$

where ${}^{(o)}u^\mu = \{1, 0, 0, 0\}$; $\rho_o(t)$ and $p_o(t)$ are average values of the energy density and pressure, respectively, on a constant time hypersurface. The peculiar velocity is given by δu^i. Substituting Eqs. (8)–(10), in Eq. (7) and neglecting second and higher order perturbation terms we get

$$\delta T^0{}_0 = \delta\rho \tag{11}$$

[2]Please note that through out this paper $\mu, \nu = 0, 1, 2, 3$ and $i, j = 1, 2, 3$

$$\delta T^i{}_0 = (\rho_o + p_o)\delta u^i \tag{12}$$

$$\delta T^i{}_j = -\delta p \delta^i{}_j. \tag{13}$$

For a scalar (quintessence) field with a Lagrangian of the form Eq. (1), we define the perturbations as

$$\phi(\vec{x}, t) = \phi_o(t) + \delta\phi(\vec{x}, t), \tag{14}$$

where $\phi_o(t)$ is the average value of the scalar field on the constant time hypersurface. The energy-momentum tensor for the scaler field is given by

$$T^\mu{}_\nu = \partial^\mu \phi \partial_\nu \phi - \mathcal{L}_\phi \delta^\mu{}_\nu. \tag{15}$$

Substituting Eq. (14) in Eq. (15) and subtracting the homogeneous part in the energy-momentum tensor we get

$$\delta T^0{}_0 = \delta\rho_\phi = \dot{\phi}_o \dot{\delta\phi} - \Phi\dot{\phi}_o^2 + V'(\phi_o)\delta\phi \tag{16}$$

$$\delta T^i_j = -\delta p_\phi \delta^i_j = -[\dot{\phi}_o \dot{\delta\phi} - \Phi\dot{\phi}_o^2 - V'\delta\phi]\delta^i_j \tag{17}$$

$$\delta T^0{}_i = (\rho_{\phi_o} + p_{\phi_o})\delta u_{i_{(\phi)}} = \dot{\phi}_o \delta\phi_{,i}. \tag{18}$$

B. Inhomogeneous dark energy

The existence of inhomogeneity in nonrelativistic matter (dark + baryonic) is evident from direct observations. We ask: is it reasonable to assume that dark energy is in general homogeneous given the observational fact that matter (dark + baryonic) is clustered? Dark energy has equation of state $p < -\frac{1}{3}\rho$, leading to the accelerated expansion of the Universe. The fluid with such an equation of state behaves gravitationally as a repulsive form of matter and opposes gravitational clustering.

Our assumption that matter and scalar field are minimally coupled implies that the energy-momentum tensor for both matter and scalar field are individually conserved. This would then imply that for both of these components the variables defining the perturbations $\delta\rho$, δp, and δu^i would satisfy the following two equations[3]:

$$\dot{\delta\rho} + (\rho_o + p_o)\vec{\nabla}.\vec{\delta u} + 3\frac{\dot{a}}{a}(\delta\rho + \delta p) - 3\dot{\Phi}(\rho_o + p_o) = 0 \tag{19}$$

$$(\rho_o + p_o)\dot{\delta u_i} + \dot{p}_o \delta u_i - \delta p_{,i} - \Phi_{,i}(\rho_o + p_o) = 0. \tag{20}$$

If dark energy with $w \neq -1$ were to be homogeneously distributed, i.e. if $\delta T^\mu{}_{\nu(\mathrm{DE})} = 0$, then, Eqs. (19) and (20) imply that the gravitational potential does not depend on space and time (i.e. $\Phi =$ constant). If this is the case then in the line element (6), we can rescale the time and space coordinate such that Eq. (6) becomes the FRW metric (3). This would then imply that $\delta\rho_m = 0$. Hence homogeneously distributed dark energy at all length scales would naturally imply that matter is also distributed homogeneously. As we see structures over different scales, this is an observational evidence that the matter in the universe is clearly not homogeneously distributed. Hence, if we assume that dark energy with $w \neq -1$ is homogeneously distributed at all length scales, then it is inconsistent with the observed features of the Universe [29–31].

C. Linearized Einstein's equation

The perturbed Einstein's equation about a flat FRW metric are given by $\delta G^\mu{}_\nu = 8\pi G \delta T^\mu{}_\nu$. In our case $\delta T^\mu{}_\nu = \delta T^\mu{}_{\nu(\mathrm{matter})} + \delta T^\mu{}_{\nu(\phi)}$. Since matter has negligible pressure we set $\delta T^i{}_{j(\mathrm{matter})} = 0$. Fluctuation in pressure is contributed only by the scalar field. Calculating the perturbed Einstein's tensor $\delta G^\mu{}_\nu$ from the line element (6) and substituting $\delta T^\mu{}_\nu$ from Eqs. (11)–(13) and from Eqs. (16) and (17), we obtain the following linearized Einstein's equations:

$$3\frac{\dot{a}^2}{a^2}\Phi + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2\Phi}{a^2} = -4\pi G[\delta\rho_m + \dot{\phi}_o\dot{\delta\phi} - \Phi\dot{\phi}_o^2 + V'(\phi_o)\delta\phi], \tag{21}$$

$$\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\Phi = 4\pi G[\dot{\phi}_o\dot{\delta\phi} - \Phi\dot{\phi}_o^2 - V'(\phi_o)\delta\phi], \tag{22}$$

$$\dot{\Phi} + \frac{\dot{a}}{a}\Phi = 4\pi G(\rho_o a^{-3} v_m + \dot{\phi}_o \delta\phi), \tag{23}$$

where $V'(\phi_o) = \partial V(\phi_o)/\partial\phi_o$ and v_m is the potential for the matter peculiar velocity such that $\delta u_i = \nabla_i v_m$. Since these equations are linear, we have Fourier decomposed the perturbed quantities such as Φ, $\delta\phi$, $\delta\rho_m$, and v_m and replaced ∇^2 by $-k^2$, where k is the wave number defined as $k = 2\pi/\lambda_p$ and λ_p is the comoving length scale of perturbation. In these equations all the perturbed quantities correspond to the amplitude of perturbations in the $k$thmode.

Equation (22) is the dynamical equation for the metric perturbation Φ and the perturbation in the scalar field turns out to be the driving term. The unknown variables are Φ, $\delta\phi$, and $\delta\rho_m$. Once $\Phi(t)$ and $\delta\phi(t)$ are known then the potential for matter peculiar velocity v_m can be determined using Eq. (23). Also it is interesting to note that once $\Phi(t)$ and $\delta\phi(t)$ are known then even the matter density perturbation $\delta\rho_m$ can be determined using Eq. (21). Hence for such a system of pressureless matter and scalar field, the dynamics of perturbations is uniquely determined if we know the solution $\Phi(t)$ and $\delta\phi(t)$. Hence we need just two second order equations connecting $\Phi(t)$, $\delta\phi(t)$. We choose Eq. (22) as one of these equations. In addition to this, the

[3] Note that for the scalar field $\delta T^0{}_i = \dot{\phi}_o\delta\phi_{,i}$. This could also be written as $\delta T^0{}_i = (\rho_{\phi_o} + p_{\phi_o})\delta u_{i_{(\phi)}}$, where $\delta u_{i_{(\phi)}} = \dot{\phi}^{-1}\delta\phi_{,i}$.

dynamical equation for the perturbations in the scalar field $\delta\phi(t)$ is obtained from the scalar field Lagrangian (1) and this is given by

$$\ddot{\delta\phi} + 3\frac{\dot{a}}{a}\dot{\delta\phi} + \frac{k^2\delta\phi}{a^2} + 2\Phi V'(\phi_o) - 4\dot{\Phi}\dot{\phi}_o + V''(\phi_o)\delta\phi = 0. \quad (24)$$

For any quintessence potential $V(\phi)$, in a system of matter and the scalar field, one can solve the coupled Eqs. (22) and (24) to study the behavior of the perturbed system. Once the solutions $\Phi(t)$ and $\delta\phi(t)$ are obtained, we can then calculate the fractional density perturbation defined as

$$\delta = \frac{\delta\rho}{\rho_o}, \quad (25)$$

for both matter as well as the scalar field from Eqs. (16) and (21). This is given by

$$\delta_\phi = \frac{1}{\frac{1}{2}\dot{\phi}_o^2 + V(\phi_o)}[\dot{\phi}_o\dot{\delta\phi} - \Phi\dot{\phi}_o^2 + V'(\phi_o)\delta\phi] \quad (26)$$

$$\delta_m = -\frac{1}{4\pi G\rho_{m_o}a^{-3}}\left\{3\frac{\dot{a}^2}{a^2}\Phi + 3\frac{\dot{a}}{a}\dot{\Phi} + \frac{k^2\Phi}{a^2}\right\} + \frac{\delta_\phi}{\rho_{m_o}a^{-3}}\left[\frac{1}{2}\dot{\phi}_o^2 + V(\phi_o)\right]. \quad (27)$$

Using these two equations we shall calculate δ_m and δ_ϕ for a system consisting of dark matter with negligible pressure and quintessence dark energy with exponential potential.

III. NUMERICAL SOLUTIONS

For solving the background equations [Eqs. (4) and (5)], we introduce the following dimensionless variables:

$$y = \frac{\phi_o - \phi_{oi}}{M_p} \quad (28)$$

$$s = \frac{a}{a_i} \quad (29)$$

$$x = H_i(t - t_i), \quad (30)$$

where a_i, ϕ_{oi}, and H_i are the values of the scale factor, scalar field, and the Hubble parameter at some initial time $t = t_i$.

In terms of these new variables, the two equations [Eqs. (4) and (5)] describing the background cosmology become:

$$\frac{s'^2}{s^2} - \Omega_{mi}s^{-3} - \frac{1}{3}\left[\frac{y'2}{2} + \bar{V}\exp(-\sqrt{\lambda}y)\right] = 0 \quad (31)$$

$$y'' + 3\frac{s'}{s}y' - \sqrt{\lambda}\bar{V}\exp(-\sqrt{\lambda}y) = 0, \quad (32)$$

where prime "0000′" corresponds to the derivative with respect to x, $\bar{V} = V_oH_i^{-2}M_p^{-2}\exp(-\sqrt{\lambda}M_p^{-1}\phi_{oi})$ and Ω_{mi} is the dimensionless matter density parameter at the initial epoch $t = t_i$.

Since $\Omega_{\rm total} = \Omega_{\rm mi} + \Omega_{\phi_i} = 1$, it follows that

$$\bar{V} = \tfrac{1}{2}(1 - \Omega_{mi})(1 - w_i) \quad (33)$$

$$y_i' = \sqrt{3(1 - \Omega_{mi})(1 + w_i)}, \quad (34)$$

where w_i is the value of the equation of state parameter of the scalar field ϕ at $t = t_i$. Choosing w_i to be very close to -1 at a redshift of $z_i = 1000$, we solve the two background equations [Eqs. (4) and (5)].

Figure 1 shows the plot of Ω_m and Ω_ϕ as a function of the scale factor. This figure gives the value of the scale factor at the matter dark energy equality $a_{\rm eq} = 0.68$. The corresponding redshift of matter dark energy equality is $z_{\rm eq} = 0.46$. The redshift at which the universe underwent a transition from a decelerated expansion phase to an accelerated expansion phase turns out to be $z_{\rm acc} = 0.81$. Figure 2 shows the evolution of the equation-of-state parameter w_ϕ as a function of the scale factor. For the choice $\lambda = 1$, this gives the value of the w_ϕ at the present epoch to be $w_{\phi_o} = -0.83$. If $\lambda = 0.1$, then $w_{\phi_o} = -0.98$.

For numerically solving the two perturbation equations Eqs. (22) and (24), we introduce the following two dimensionless variables:

$$\Phi_N = \frac{\Phi}{\Phi_i} \quad (35)$$

$$\delta y = \frac{\delta\phi}{\Phi_i M_p}. \quad (36)$$

Here Φ_N is the normalized gravitational potential with Φ_i being the value of the metric potential at the initial time $t = t_i$. In terms of these two new variables, the Eqs. (22) and (24) can be rewritten as

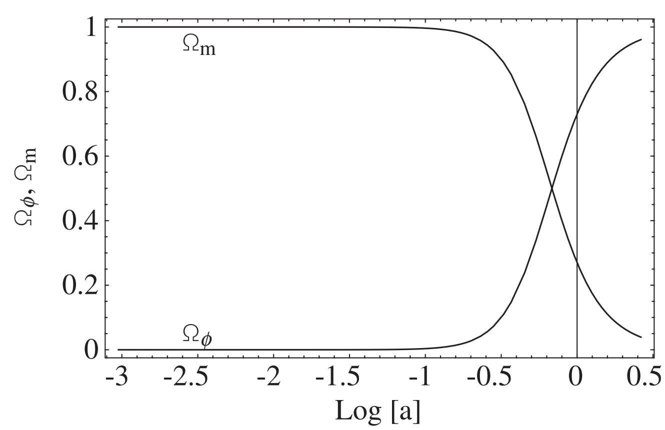

FIG. 1. This plot shows the variation of Ω_m and Ω_ϕ as a function of scale factor. We have chosen $\lambda = 1$. In the x-axis, "Log" refers to the logarithm to base 10.

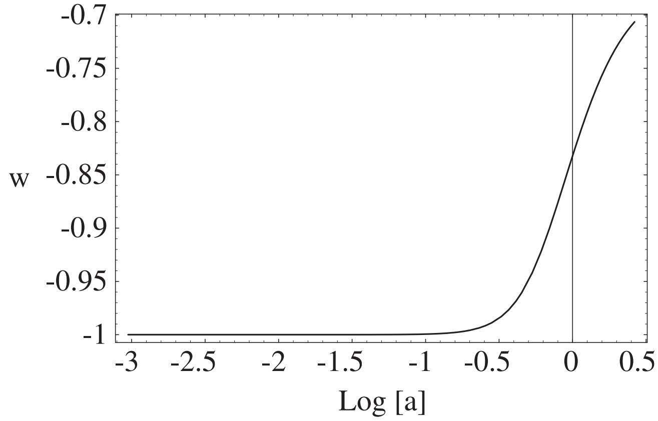

FIG. 2. This plot shows the variation of equation-of-state parameter w_ϕ as a function of scale factor. We have chosen $\lambda = 1$. In the x-axis, Log refers to the logarithm to base 10.

$$\Phi_N'' + 4\frac{s'}{s}\Phi_N' + \left(2\frac{s''}{s} + \frac{s'^2}{s^2}\right)\Phi_N - \frac{1}{2}[y'\delta y' - \Phi_N y'^2 + \sqrt{\lambda}\bar{V}\exp(-\sqrt{\lambda}y)\delta y] = 0 \quad (37)$$

$$\delta y'' + 3\frac{s'}{s}\delta y' + (\lambda\delta y - 2\Phi_N\sqrt{\lambda})\bar{V}\exp(-\sqrt{\lambda}y) - 4\Phi_N' y' + \frac{\bar{k}^2\delta y}{\beta s^2} = 0. \quad (38)$$

Here $\bar{k} = k/H_o$, where H_o is the Hubble parameter at the present epoch, is the wave number scaled with respect to the Hubble radius. In Eq. (38) the constant $\beta = \frac{1}{s_o'^2}$, where s_o' is the value of s' at the present epoch. This can be determined numerically from Eq. (31).

We assume that the perturbation in the scalar field in the matter-dominated epoch at $z \approx 1000$ is negligibly small compared to other perturbed quantities such as Φ, δ_m etc. The scalar field can then be treated as initially homogeneous at $t = t_i$. This corresponds to setting the initial condition $\delta y_i = 0$ and $\delta y_i' = 0$. The only initial condition that needs to be determined is the value of Φ_{N_i}' at $t = t_i$. In the matter-dominated epoch ($z \approx 1000$), the linearized Einstein's equation (22) can be analytically solved to obtain the solution $\dot{\Phi}(t) \propto t^{-8/3}$. Hence $\dot{\Phi}(t)$ decays to zero in the matter-dominated epoch for all values of wave number k, and we can set the initial condition $\Phi_{N_i}'(k) = 0$.

After solving the two perturbation equations using the above initial conditions we can now determine the dimensionless density perturbations δ_m and δ_ϕ defined in Eqs. (26) and (27). In terms of the dimensionless variable defined in Eqs. (35) and (36), we can express δ_m and δ_ϕ in the following form:

$$\frac{\delta_\phi}{\Phi_i} = \frac{y'\delta y' - \Phi_N y'^2 - \sqrt{\lambda}\delta y\bar{V}\exp\{-\sqrt{\lambda}y\}}{\frac{y'^2}{2} + \bar{V}\exp\{-\sqrt{\lambda}y\}} \quad (39)$$

$$\frac{\delta_m}{\Phi_i} = -\frac{1}{\Omega_{mi}s^{-3}}\Big[2\frac{s'^2}{s^2}\Phi_N + 2\frac{s'}{s}\Phi_N' + 2\frac{\bar{k}^2}{3s^2\beta}\Phi_N + \frac{1}{3}(y'\delta y' - \Phi_N y'^2 - \sqrt{\lambda}\delta y\bar{V}\exp\{-\sqrt{\lambda}y\})\Big]. \quad (40)$$

In Fig. 3 we have plotted Φ_N as a function of the scale factor for the value of the parameter $\lambda = 1$. The dotted line in this figure shows $\Phi_N(a)$ for the model of dark energy considered in this paper. We can see that in the matter-dominated epoch $\Phi_N = 1$ and it is constant. Once the dark energy dominated epoch begins, the gravitational potential starts to decay. For comparison, in the same figure we had plotted [see bold line in Fig. 3] the form of $\Phi_N(a)$ in the ΛCDM model. This implies that the behavior of Φ_N in the matter-dominated era is the same in both the ΛCDM model and the model of dark energy considered in this paper. But once the dark energy dominated phase begins, $\Phi_N(a)$ decays faster than the corresponding $\Phi_N(a)$ in the ΛCDM model. In this figure the wavelength of perturbation was fixed to $\lambda_p = 10^3$ Mpc. With $H_o = 73$ Km/s $\times$ Mpc^{-1} [44], this value of λ_p corresponds to $\bar{k} = k/H_o \approx 26$.

In Fig. 4, we have plotted the matter perturbations δ_m as a function of the scale factor for the dark energy potential parameter $\lambda = 1$. In this plot the solid line corresponds to $\delta_m(a)$ for the ΛCDM model and the dotted line in the same plot shows $\delta_m(a)$ for the dark energy model considered in this paper. Matter perturbation $\delta_m(a)$ initially grows linearly with the scale factor in the matter-dominated epoch, however, once the Universe undergoes transition from the decelerated expansion phase to accelerated expansion phase, the growth of $\delta_m(a)$ is suppressed. In fact the growth of $\delta_m(a)$ is suppressed more in the dark energy model with exponential potential than the corresponding $\delta_m(a)$ in the ΛCDM model. At the present epoch, for the same set of

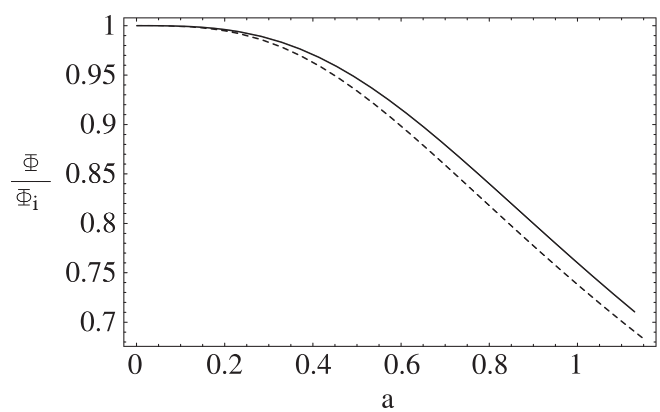

FIG. 3. This plot shows the how the gravitational potential evolves with time. The bold line shows how gravitational potential $\Phi_N(a)$ evolves with the scale factor in the ΛCDM model, while the dotted lines shows $\Phi_N(a)$ for the dark energy model considered in this paper. Here the parameter $\lambda = 1$. In this plot the wavelength of perturbation $\lambda_p = 10^3$ Mpc.

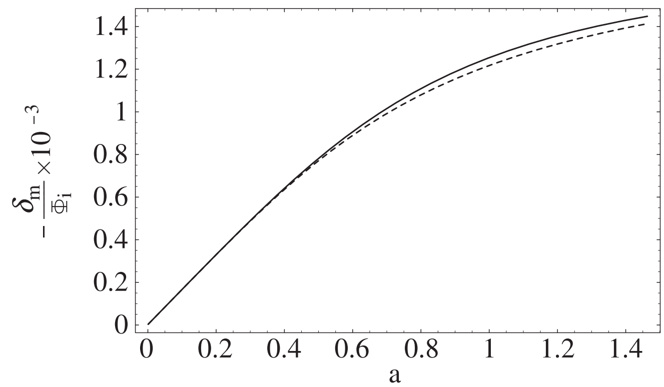

FIG. 4. This plot shows how matter perturbation δ_m evolves with scale factor. The bold lines on this plot correspond to the ΛCDM model, while the dotted lines in the same plot show how the matter perturbations evolve with scale factor for the dark energy model considered in this paper. Here the parameter $\lambda = 1$ and the wavelength of perturbation $\lambda_p = 10^3$ Mpc.

initial conditions we find that

$$^{\rm QCDM}\delta_m^2(z=0) = 0.96\ ^{(\Lambda\rm CDM)}\delta_m^2(z=0), \qquad (41)$$

where QCDM corresponds to the quintessence + cold dark matter. Hence, the matter power spectrum is suppressed in the QCDM model considered in this paper than the corresponding value in the ΛCDM model. This nearly 4% suppression corresponds to the length scale of perturbation $\lambda_p = 10^3$ Mpc and for the value of the parameter $\lambda = 1$. By 4% suppression, we mean that $[(P(k)_{\Lambda\rm CDM} - P(k)_{\rm QCDM})/P(k)_{\Lambda\rm CDM}] \times 100 = 4$. [Here $P(k)$ is the matter power spectrum which by definition is proportional to δ_m^2.] In Fig. 5, we plot the ratio $P(k)_{\rm QCDM}/P(k)_{\Lambda\rm CDM}$ as a function of the length scale of perturbation λ_p. This figure implies that the greater the length scale λ_p and the greater the value of the parameter λ, the higher is the percentage of suppression. In fact at $\lambda_p = 10^5$ Mpc, the matter power spectrum is suppressed by about 15% if the potential parameter $\lambda = 1$. This implies that if $\lambda = 1$ then at $\lambda_p = 10^5$ Mpc, $P(k)_{\Lambda\rm CDM} - P(k)_{\rm QCDM} = 0.15 \times P(k)_{\Lambda\rm CDM}$. Since $P(k)_{\Lambda\rm CDM}$ at 10^3 Mpc is at least an order of magnitude greater that the corresponding value at 10^5 Mpc [45], it follows that although the percentage of suppression is larger at larger scales, the actual difference $P(k)_{\Lambda\rm CDM} - P(k)_{\rm QCDM}$ is smaller at larger scales.

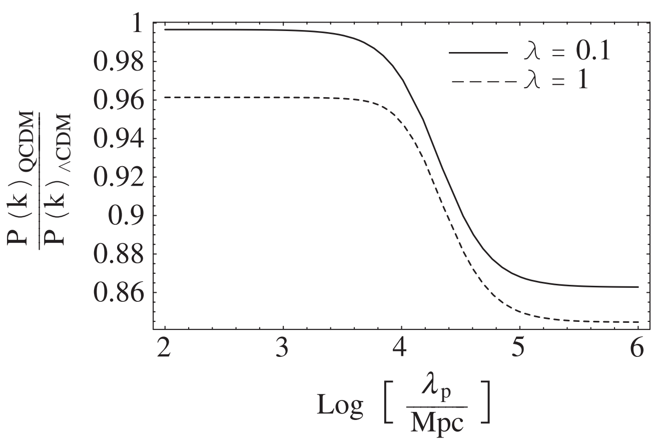

FIG. 5. The ratio of power spectrums $P(k)_{\rm QCDM}/P(k)_{\Lambda\rm CDM}$ as a function of length scale of perturbation λ_p. If the potential parameter $\lambda = 1$, then it turns out that at the present epoch, the equation-of-state parameter $w_0 = -0.83$, and if $\lambda = 0.1$ then $w_0 = -0.98$. In the x-axis, Log refers to the logarithm to base 10.

In Fig. 6, we have plotted the perturbation in dark energy δ_ϕ as a function of the scale factor for $\lambda = 1$. We can see that initially, at around $z = 1000$, perturbation δ_ϕ is almost zero. Once the dark energy dominated epoch begins the perturbations in dark energy grow.

A. Dependence of the ratio δ_ϕ/δ_m on length scales

In Figs. 4 and 6 we have scaled δ_m and δ_ϕ with respect to Φ_i, the initial value of the metric perturbation. From Figs. 4 and 6, we find that at the present epoch [z = 0], the ratio of dark energy perturbations to matter perturbations at $\lambda_p = 10^3$ Mpc and $\lambda = 1$ is given by

$$\left(\frac{\delta_\phi}{\delta_m}\right)_{(z=0)} = 10^{-4}. \qquad (42)$$

Given a fixed λ, the ratio δ_ϕ/δ_m would depend on the length scale of perturbation λ_p. In Fig. 7, we plot the value of this ratio at the present epoch as a function of length scale λ_p. This figure shows that at small scales [$\lambda_p <$ 1000 Mpc], the perturbations in dark energy can be neglected in comparison with the perturbations in matter. This is because in these length scales $\delta_\phi \simeq 10^{-5}\delta_m$. And since δ_m itself is small, this value of δ_ϕ corresponds to a higher order term. In the linear regime, for scales $\lambda_p <$ 1000 Mpc we can neglect the perturbations in dark energy

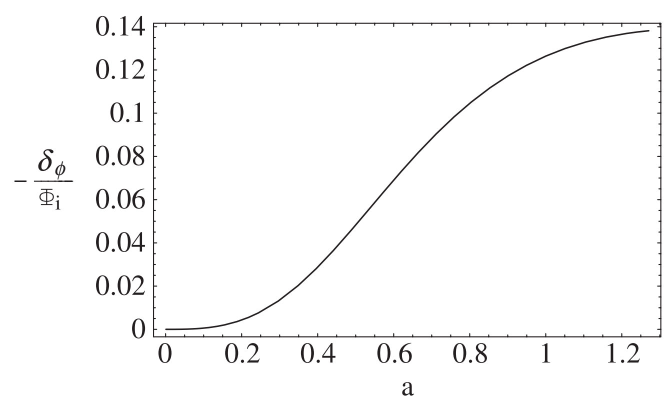

FIG. 6. This plot shows how perturbations in dark energy δ_ϕ evolve with scale factor. Here the parameter $\lambda = 1$ and the wavelength of perturbation $\lambda_p = 10^3$ Mpc.

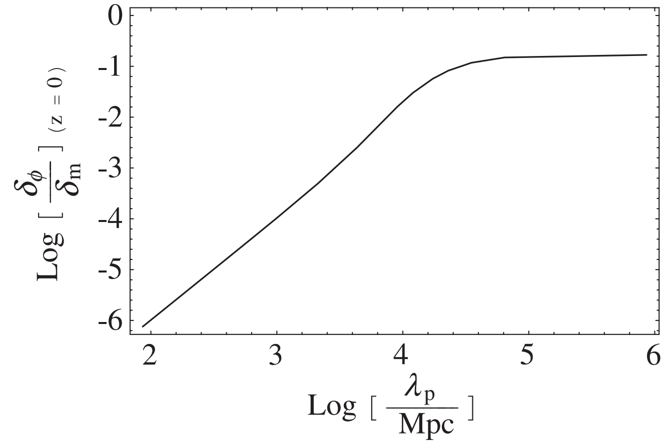

FIG. 7. This plot shows the dependence of the ratio δ_ϕ/δ_m at the present epoch on the wavelength of perturbations λ_p. In this plot the parameter $\lambda = 1$. In both the axis, Log refers to the logarithm to base 10.

(for this model) and we can treat dark energy to be homogeneous. The effect on the matter perturbation by dark energy on these scales would be through background $a(t)$.

On large scales [for $\lambda_p > 1000$ Mpc], the dark energy perturbations can become comparable to δ_m. In Fig. 7, for $\lambda_p = 10^5$ Mpc, we find that $(\delta_\phi/\delta_m)_{(z=0)} = 0.17$. Even on large scales, the perturbations in dark energy can be neglected if the equation-of-state parameter at the present epoch is very close to -1. This matches with the fact that in a pure cosmological constant model of dark energy with $w = -1$, its energy density is distributed homogeneously at all length scales.

In Fig. 8, we have plotted the variation of the ratio δ_ϕ/δ_m at the present epoch as a function of w_o which is the equation of state parameter at the present epoch. Each value of λ would result in a specific value of the equation-of-state parameter w_o as determined by the background equations (4) and (5) for a fixed set of initial conditions. This figure shows that this ratio $\delta_\phi/\delta_m \to 0$ when $w_0 \to -1$. This result is true for all length scales of perturbations and it is consistent with the argument presented in Sec. II that perturbation in matter implies perturbation in dark energy if $w_{\text{de}} \neq -1$.

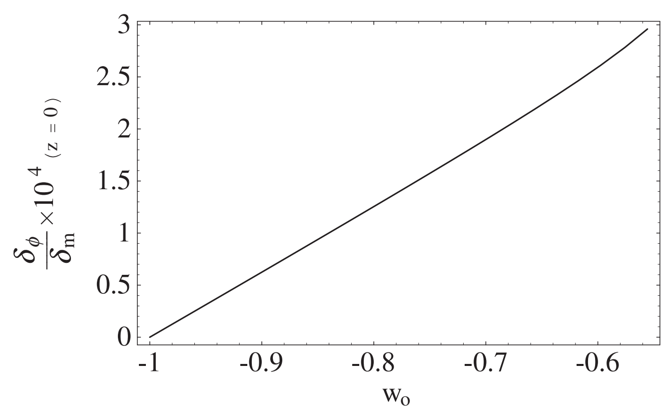

FIG. 8. This plot shows the dependence of the ratio δ_ϕ/δ_m at the present epoch on the equation-of-state parameter w_o. It is the value of the parameter λ in the potential $V(\phi)$ which determines the present value of the equation-of-state parameter w_o. In this plot the wavelength of perturbation $\lambda_p = 10^3$ Mpc.

The two plots in Fig. 9, show the ratio δ_ϕ/δ_m as a function of λ at a length scale of perturbations of 500 Mpc and 10^5 Mpc, respectively. Note that the figure on the left is scaled by a factor 10^5. These figures imply that on large scales the dependence on the parameter λ is stronger than on small scales.

B. The role of quintessence on the matter power spectrum

It is evident from Eq. (41) and Fig. 5 that matter perturbation is suppressed relative to the ΛCDM model. On length scales $\lambda_p < 1000$ Mpc, the perturbation in dark energy is negligibly small compared to the perturbation in matter (see Fig. 7). However, even on these scales the matter power spectrum is suppressed relative to that in the ΛCDM model. This is evident from Fig. 5. It is therefore necessary to distinguish the role of background evolution $a(t)$ and perturbation in dark energy on the suppression of the matter power spectrum relative to ΛCDM.

In order to address this issue we evaluate the suppression of the matter power spectrum relative to ΛCDM if we treat dark energy as homogeneous. In such a scenario, this suppression would be solely a consequence of different background evolution relative to ΛCDM. Since we are considering a quintessence model of dark energy, by "*homogeneous dark energy*" we mean that we are assuming that quintessence field is homogeneous in the longitudinal gauge. There exists a gauge known as the uniform field gauge where by definition the quintessence field is homogeneous. However, the coordinate transformation from the uniform field gauge to the longitudinal gauge would necessarily result in nonzero fluctuation in the quintessence field. Here our aim is to calculate the suppression of the matter power spectrum relative to ΛCDM if we forcefully impose the assumption that the quintessence field is homogeneous at all length scales and compare the same without imposing this assumption.

If the quintessence field is homogeneous, then the evolution of the metric perturbation Φ is determined by the following equation:

$$\ddot{\Phi} + 4\frac{\dot{a}}{a}\dot{\Phi} + \left[\frac{\ddot{a}}{a} + \left(2 - \frac{3}{2}\Omega_m(a)\right)\frac{\dot{a}^2}{a^2}\right]\Phi = 0. \quad (43)$$

This equation follows from Eq. (22). Consequently the evolution of the matter perturbation is determined by the following equation:

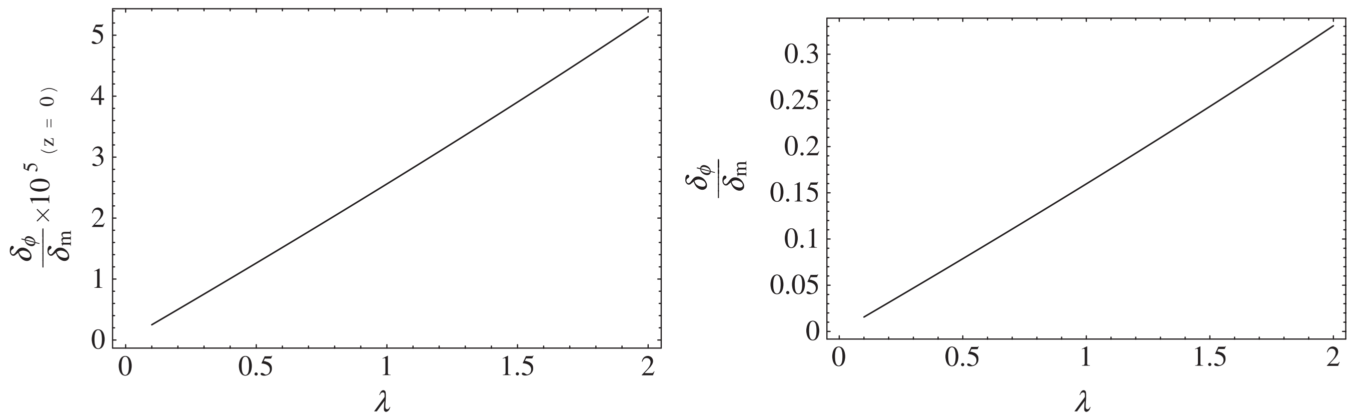

FIG. 9. The left plot shows the ratio of perturbation in dark energy to the matter perturbation at the present epoch for a length scale of perturbation of 500 Mpc as a function of potential parameter λ. At this scale the ratio remains at almost 10^{-5}. The plot on the right shows the ratio at the present epoch for a length scale of perturbation of 10^5 Mpc as a function of parameter λ. At this length scale, for different values of the parameter λ between 0.1 and 2, the ratio varies from 0.01–0.33.

$$
{}^{(\mathrm{HQCDM})}\delta_m^2 = -\frac{2}{3H^2\Omega_m(a)}\left\{3\frac{\dot a}{a}\dot\Phi + \frac{k^2\Phi}{a^2} + \left[\frac{\ddot a}{a} + \left(2 + \frac{3}{2}\Omega_m(a)\right)\frac{\dot a^2}{a^2}\right]\Phi\right\}. \quad (44)
$$

Here the superscript "HQCDM" stands for homogeneous quintessence + cold dark matter. The above equation [Eq. (44)] follows from Eqs. (21) and (22).

In Fig. 10, we have plotted the evolution of ${}^{(\mathrm{HQCDM})}\delta_m(a)$ with the scale factor $a(t)$. For comparison the bold line in the same figure shows ${}^{(\mathrm{QCDM})}\delta_m(a)$, which corresponds to matter perturbations if we include quintessence fluctuations in our calculations. From this figure, it follows that although matter perturbation is suppressed relative to ΛCDM, it is in fact enhanced in comparison to matter perturbation obtained by treating the quintessence field as homogeneous. Hence perturbation in dark energy actually enhances matter perturbation. This is also intuitively expected as both dark matter and dark energy clusters do not anticluster in the model considered in this paper. This is evident from Figs. 4 and 6 since both δ_m and δ_ϕ have the same sign. The consequent effect of this on the gravitational potential results in this enhancement of matter perturbation relative to the homogeneous dark energy case. The accelerated expansion of the of Universe determined by $a(t)$ dilutes the metric perturbation Φ and consequently the overall growth of matter perturbation δ_m is suppressed.

In Fig. 11, we plot the ratio $P(k)_{\mathrm{QCDM}}/P(k)_{\mathrm{HQCDM}}$ at the present epoch as a function of the length scale of perturbation for values of the parameter $\lambda = 0.1$ and $\lambda = 1$. On length scales $\lambda_p < 1000$ Mpc, we find that $P(k)_{\mathrm{QCDM}} \approx P(k)_{\mathrm{HQCDM}}$ and this is independent of the choice of pa-

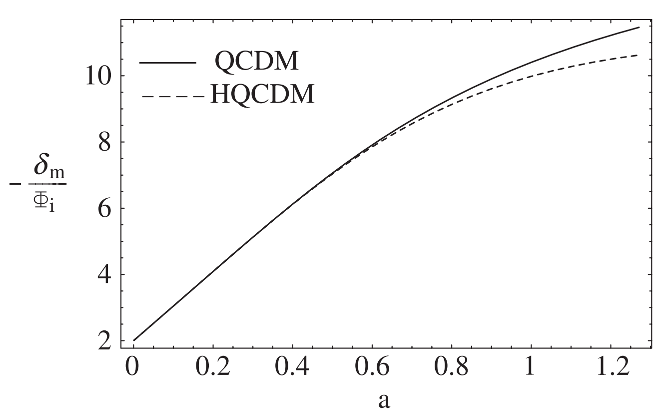

FIG. 10. In this plot the bold line (labeled as QCDM) shows how matter perturbation δ_m evolves with the scale factor if we include perturbations in the quintessence field. For comparison the dashed line (labeled as HQCDM) corresponds to $\delta_m(a)$ assuming a homogeneous quintessence field. In this plot $k = 5 \times 10^{-4}$ Mpc^{-1} and parameter $\lambda = 1$.

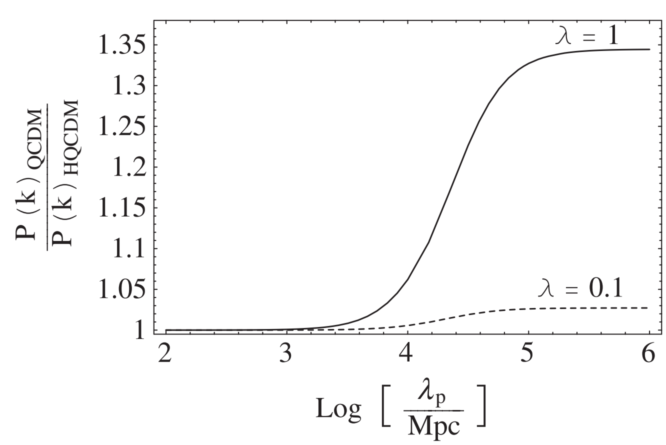

FIG. 11. This plot shows the ratio $P(k)_{\mathrm{QCDM}}/P(k)_{\mathrm{HQCDM}}$ at the present epoch for different length scales and for two different values of the parameter λ. In the x-axis, Log refers to the logarithm to base 10.

rameter λ. This implies that on these scales, including or excluding quintessence fluctuation in the perturbation equation does not influence the matter power spectrum significantly. The suppression of the matter power spectrum on these scales (as shown in Fig. 5) is therefore primarily due to different background evolution relative to that in the ΛCDM model.

However, on large scales $\lambda_p > 1000$ Mpc, $P(k)_{\mathrm{QCDM}}$ deviates significantly from $P(k)_{\mathrm{HQCDM}}$ for larger values of the parameter λ (see Fig. 11). Hence on these scales, including or excluding quintessence fluctuation in the perturbation equation does influence the matter power spectrum significantly.

Figure 5 implies that $P(k)_{\mathrm{QCDM}} < P(k)_{\Lambda\mathrm{CDM}}$. However, Fig. 11 implies that $P(k)_{\mathrm{QCDM}} > P(k)_{\mathrm{HQCDM}}$. This means that $P(k)_{\mathrm{HQCDM}} < P(k)_{\mathrm{QCDM}} < P(k)_{\Lambda\mathrm{CDM}}$. This implies that the matter power spectrum is suppressed relative to that in the ΛCDM model even if we treat the quintessence field as homogeneous. This also implies that although large-scale matter perturbation is suppressed in the generic quintessence dark energy model compared to that in ΛCDM, perturbations in dark energy (in quintessence) *enhance* matter perturbation relative to the corresponding matter perturbation obtained by treating the quintessence field as homogeneous. This enhancement is significant on large scales i.e. for $\lambda_p > 1000$ Mpc (see Fig. 11).

We compare our results with a different scalar field potential $V(\phi) = \frac{1}{2}m^2\phi^2$. In Fig. 12 we plot the ratio of dark energy perturbations to matter perturbations as a function of length scale. The parameter m is fixed to $m = 0.94H_0$ in natural units and the present day equation-of-state parameter is $w = -0.87$. The results are consistent with those shown in Fig. 7. Hence our result that quintessence dark energy can be treated as homogeneous at scales $\lambda_p < 1000$ Mpc is a generic result.

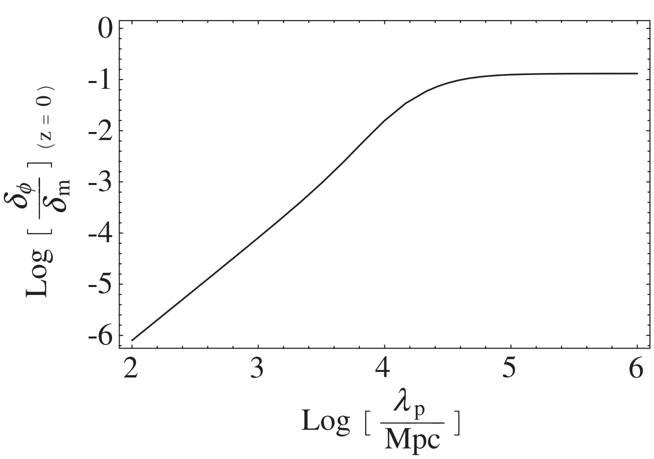

FIG. 12. In the figure we plot the ratio of dark energy perturbations to dark matter perturbations for $V(\phi) = 1/2m^2\phi^2$. The results agree with those shown in Fig. 7. In both the axis, Log refers to the logarithm to base 10.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have investigated the perturbations in dark energy. This is motivated by the fact that the assumption that the distribution of dark energy (with $w \neq -1$) is homogeneous at all length scales is inconsistent with the observational fact that dark matter is distributed inhomogeneously. On length scales comparable to or greater than the Hubble radius ($\lambda_p > 1000$ Mpc), the perturbations in dark energy can become comparable to perturbations in matter if $w_{\mathrm{de}} \neq -1$. The model parameters we have chosen correspond to $w \approx -0.8$ and $w \approx -0.9$, which are within the range allowed by Supernova observations and WMAP5 observations. Given this range, the evolution of perturbations differs significantly. For scales $\lambda_p < 1000$ Mpc, the perturbation in dark energy δ_ϕ can be neglected in comparison with the perturbation in matter δ_m at least in the linear regime. We have demonstrated this using an exponential potential for the quintessence field. This result agrees with those presented in Ref. [34] on sub-Hubble scales.

We have further demonstrated that quintessence dark energy results in suppression of the matter power spectrum relative to the ΛCDM model. We found that at a length scale of $\lambda_p = 1000$ Mpc and for the value of the parameter $\lambda = 1$, the matter power spectrum is suppressed by about 4% compared to its value in the ΛCDM model for the same set of initial conditions. However, at $\lambda_p = 10^5$ Mpc, the matter power spectrum is suppressed by about 15% compared to its value in the ΛCDM model. We have demonstrated that on scales $\lambda_p < 1000$ Mpc this suppression is primarily due to different background evolution relative to the ΛCDM model. The resultant matter power spectrum is nearly invariant even if we assume that the quintessence field is homogeneous on these scales. However, on the much larger scale $\lambda_p > 1000$ Mpc, including or excluding fluctuations in the quintessence field results in significant changes in the matter power spectrum.

All these results emphasize that dark energy can indeed be treated as nearly homogeneous on scales $\lambda_p < 1000$ Mpc. However, on much larger scales ($\lambda_p > 1000$ Mpc), if the equation-of-state parameter deviates from -1, then perturbations in dark energy do influence the matter power spectrum significantly. If a definitive detection of perturbations in dark energy is made, it will certainly rule out the cosmological constant at least as the sole candidate of dark energy.

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