η_b and η_c radiative decays in the Salpeter model with the AdS/QCD inspired potential

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The decay constants and the radiative decay widths of $\eta_b(nS)$ and $\eta_c(nS)$ are computed within a semirelativistic quark model, using a potential found through the AdS/QCD correspondence. For η_c, the results are in agreement with experimental data, while in the case of η_c' a discrepancy is found and the possible reasons are discussed.

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The recent observation of the bottomonium ground state $\eta_b(1S)$ by the *BABAR* Collaboration, in the $\Upsilon(3S)$ radiative decay mode: $\Upsilon(3S) \to \gamma\eta_b$ [1], has led to a new interest in this particle and its decays. The measured mass is

$$M_{\eta_b} = 9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})\ \text{MeV} \qquad (1)$$

corresponding to a hyperfine splitting $M_{\Upsilon(1S)} - M_{\eta_b(1S)} = 71.4^{+2.3}_{-3.1}(\text{stat}) \pm 2.7(\text{syst})$ MeV [1].

On the other hand, the subsequent decay of η_b has not been observed. The meson η_b is expected to decay mainly to hadrons; other possible modes are the radiative transitions.

Motivated by this new experimental observation, in this paper a calculation of the η_b decay constant and of the width relative to the decay $\eta_b \to \gamma\gamma$ is presented, in the framework of the model proposed in Ref. [2]. Moreover, the calculation has been extended to the radial excitations and to the charmonium corresponding states, since the model can be properly employed for heavy quarkonium states. This can be useful, since in many cases the experimental values are not known and since there are some discrepancies among the predictions of different theoretical models and the experimental values. For example, there is only one measurement, by the CLEO Collaboration, of the $\eta_c' \to \gamma\gamma$ radiative decay width: $\Gamma_{\gamma\gamma}(\eta_c') = 1.3 \pm 0.6$ KeV [3], a result not reproduced by most of theoretical predictions which suggest larger values. Within this model, the decay constants and the leptonic decay widths of vector $b\bar{b}$ and $c\bar{c}$ mesons are also evaluated and a comparison with the experimental results is carried out.

In the model introduced in Ref. [2] the meson spectrum is computed solving a semirelativistic wave equation, the Salpeter equation:

$$\left(\sqrt{m_1^2 - \nabla^2} + \sqrt{m_2^2 - \nabla^2} + V(r)\right)\psi(\mathbf{r}) = M\psi(\mathbf{r}), \qquad (2)$$

where m_1 and m_2 are the masses of the constituent quarks and M and $\psi(\mathbf{r})$ are the mass and the wave function of the meson, respectively. The $\ell = 0$ case is considered. The potential $V(r)$ comprises three terms:

$$V(r) = V_{\text{AdS/QCD}}(r) + V_{\text{spin}}(r) + V_0. \qquad (3)$$

The main feature of the model is that the static potential $V_{\text{AdS/QCD}}(r)$ is obtained evaluating the expectation value of the Wilson loop in the AdS/QCD framework [4], which provides a holographic model able to describe some aspects of QCD, namely, linear confinement, Regge trajectories, glueball spectrum, the light meson spectrum, and decay constants [5]. The static $q\bar{q}$ potential is obtained, in this framework, as a parametric function [4]:

$$V_{\text{AdS/QCD}}(\lambda) = \frac{g}{\pi}\sqrt{\frac{c}{\lambda}}\left(-1 + \int_0^1 dv\, v^{-2} \times [e^{\lambda v^2/2}(1 - v^4 e^{\lambda(1-v^2)})^{-1/2} - 1]\right),$$
$$r(\lambda) = 2\sqrt{\frac{\lambda}{c}}\int_0^1 dv\, v^2 e^{\lambda(1-v^2)/2}(1 - v^4 e^{\lambda(1-v^2)})^{-1/2}, \qquad (4)$$

where r is the interquark distance and λ varies in the range: $0 \le \lambda < 2$. The potential $V_{\text{AdS/QCD}}(r)$, therefore, depends on two parameters, g and c; it is depicted in Fig. 1 for the two values of c and g employed in the present analysis.

The term of the potential $V(r)$ in Eq. (3) accounting for the spin-interaction is given by

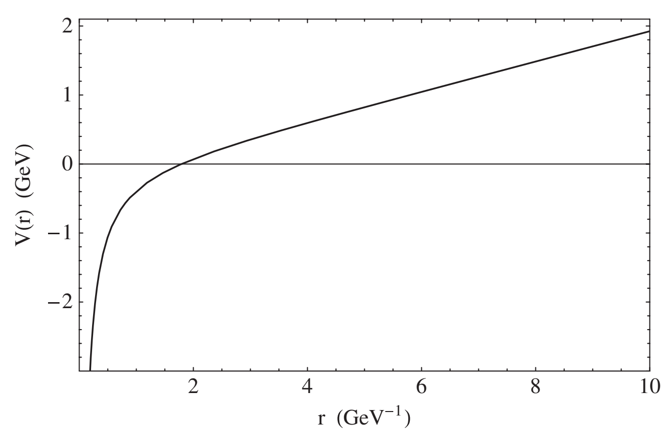

FIG. 1. The $q\bar{q}$ potential $V_{\text{AdS/QCD}}(r)$ obtained from the AdS/QCD correspondence, with $c = 0.4$ GeV^2 and $g = 2.50$.

$$V_{\rm spin}(r) = A\frac{\tilde{\delta}(r)}{m_1 m_2}\mathbf{S_1}\cdot\mathbf{S_2} \quad \text{with} \quad \tilde{\delta}(r) = \left(\frac{\sigma}{\sqrt{\pi}}\right)^3 e^{-\sigma^2 r^2}, \tag{5}$$

and involves the parameters σ, together with A_b and A_c for the cases of beauty and charm, respectively. The constant term V_0 is the last parameter fixing the potential in the Salpeter equation.

The singularity of the wave function at $r = 0$ is regulated introducing a value $r_{\rm min}$, so that at smaller distances the potential is constant and equal to $V(r_{\rm min})$. At odds with the analysis in [2], where $r_{\rm min}$ is an additional input parameter in the fit of the meson spectrum, here we fix $r_{\rm min}$ according to a QCD duality argument [6]: $r_{\rm min} = \frac{4\pi}{3M}$, where M is the mass of the meson, so that the input set of parameters includes $c = 0.4\ \text{GeV}^2$, $g = 2.50$, and $V_0 = -0.47$ GeV for the AdS/QCD potential, $A_c = 14.56$, $A_b = 6.49$, and $\sigma = 0.47$ GeV for the spin term, and the constituent quark masses $m_c = 1.59$ GeV and $m_b = 5.02$ GeV; the values of all the parameters are fixed by a best fit of the meson masses computed in the model to the observed heavy meson masses [7].

Within the Salpeter model, the decay constants f_P and f_V of a pseudoscalar and a vector meson, defined by

$$\begin{aligned}\langle 0|A^\mu_{ij}|P(k)\rangle &= ik^\mu Q_{ij} f_P,\\ \langle 0|V^\mu_{ij}|V(k,\lambda)\rangle &= \epsilon(\lambda)^\mu Q_{ij} m_V f_V,\end{aligned} \tag{6}$$

respectively, where k is the momentum, λ the helicity, and ϵ the polarization vector of the meson, are given by [8]

$$\begin{aligned} f_P &= \sqrt{3}\frac{1}{2\pi M}\int_0^{+\infty} dk k \tilde{u}_0(k) N^{1/2} \\ &\quad\times\left[1 - \frac{k^2}{(E_i+m_i)(E_j+m_j)}\right],\\ f_V &= \sqrt{3}\frac{1}{2\pi M}\int_0^{+\infty} dk k \tilde{u}_0(k) N^{1/2}\\ &\quad\times\left[1 + \frac{k^2}{3(E_i+m_i)(E_j+m_j)}\right],\end{aligned} \tag{7}$$

with

$$N = \frac{(E_i+m_i)(E_j+m_j)}{E_j E_i}.$$

In (6), A^μ_{ij} is the axial current $\bar{q}_i\gamma_5\gamma^\mu q_j$, V^μ_{ij} is the vector current $\bar{q}_i\gamma^\mu q_j$, and Q_{ij} is the meson flavor matrix. In (7), M is the mass of the meson, m_i is the mass of the constituent quark i and E_i its energy, $\tilde{u}(k)$ is the meson reduced wave function in momentum space, obtained by Fourier transforming the reduced radial wave function $u(r) = r\psi(r)$; and k is the momentum of the constituent quark in the rest frame of the meson.

The obtained spectrum and decay constants of $c\bar{c}$ and $b\bar{b}$ S-wave mesons are collected in Table I; in Fig. 2 the corresponding wave functions are depicted. It is interesting to notice that f_{η_c} turns out to be compatible with a determination obtained by the CLEO Collaboration: $f_{\eta_c} = 335 \pm 75$ MeV [9].

Using the computed values of f_P and f_V, it is possible to determine the widths $\Gamma_{\gamma\gamma}$ of the radiative decays $\eta_{b,c}(nS) \to \gamma\gamma$, and the widths $\Gamma_{\ell^+\ell^-}$ of the processes $\psi(nS) \to \ell^+\ell^-$ and $\Upsilon(nS) \to \ell^+\ell^-$. The widths can be easily computed using the effective Lagrangians [10,11]:

TABLE I. Masses of pseudoscalar and vector $c\bar{c}$ and $b\bar{b}$ states compared to the experimental data. In the third column the decay constants, computed using (7), are reported.

Particle	Th. mass (MeV)	Exp. mass (MeV) [7]	Decay const. (MeV)
η_c	3025.3	2980.3 ± 1.2	342
η_c'	3603.5	3637.0 ± 4	266
η_c''	4039.3		195
J/ψ	3079.8	3096.916 ± 0.011	356
ψ'	3624.3	3686.09 ± 0.04	237
ψ''	4057.0	4039 ± 1	185
η_b	9433.9	$9388.9^{+3.1}_{-2.3}(\text{stat}) \pm 2.7(\text{syst})$ [1]	637
η_b'	9996.8		430
η_b''	10347.5		367
Υ	9438.3	9460.30 ± 0.26	686
$\Upsilon(2S)$	9998.6	10023.26 ± 0.31	484
$\Upsilon(3S)$	10348.8	10355.2 ± 0.5	335
$\Upsilon(4S)$	10622.3	10579.4 ± 1.2	301

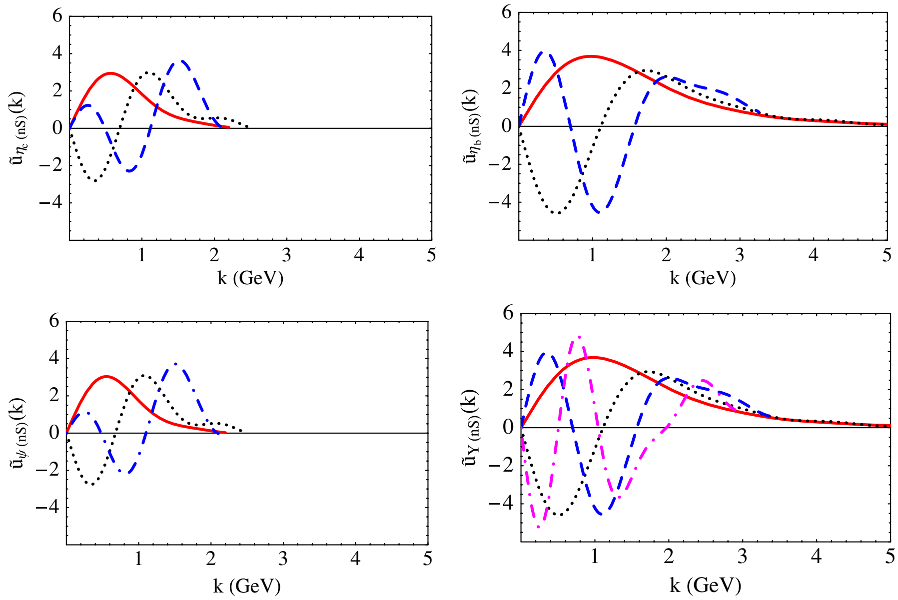

FIG. 2 (color online). The momentum wave functions of the first three states of $\eta_c(nS)$ (top left), $\eta_b(nS)$ (top right), $J/\psi(nS)$ (bottom left), and $\Upsilon(nS)$ (bottom right). The continuos line represents the 1S state, the dotted line represents the 2S state, the dashed line represents the 3S state, and the dashed-dotted line represents the 4S state. The wave functions are dimensionless: they are normalized as $\int dk|\tilde{u}(k)|^2 = 2M$.

$$\mathcal{L}^{\gamma\gamma}_{\rm eff} = -ic_1(\bar{q}\gamma^\sigma\gamma^5 q)\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}A^\rho, \qquad \mathcal{L}^{\ell\bar{\ell}}_{\rm eff} = -c_2(\bar{q}\gamma^\mu q)(\ell\gamma_\mu\bar{\ell}), \tag{8}$$

where

$$c_1 = \frac{Q^2 4\pi\alpha_{em}}{(M^2 + E_b M)}, \qquad c_2 = \frac{Q4\pi\alpha_{em}}{M^2}. \tag{9}$$

One obtains

$$\Gamma_{\gamma\gamma} = \frac{4\pi Q^4\alpha^2_{em}M^3 f^2_P}{(M^2 + E_b M)^2}, \qquad \Gamma_{\ell^+\ell^-} = \frac{4\pi Q^2\alpha^2_{em} f^2_V}{3M}, \tag{10}$$

where Q is the electric charge (in units of e) of the constituent quark and $E_b = 2m - M$ is the binding energy.

The values obtained for the pseudoscalar mesons are shown in Table II, together with recent theoretical results. The prediction for the η_c radiative decay width is compatible with the experimental value within the error; in the case of η'_c, the measurement by the CLEO Collaboration [3] is smaller (or marginally compatible) than the obtained theoretical prediction and that in other calculations [15].

Concerning the $b\bar{b}$ pseudoscalar meson, the theoretical models in Table II predict, for the $\eta_b \to \gamma\gamma$ decay width, values in the range 230–560 eV; the result obtained in this paper points towards small values in this range.

For vector mesons, the predicted and the experimental values of the leptonic decay widths are reported in Table III. There is an overall agreement, excluding a discrepancy in the $\Upsilon(3S)$ that could be attributed to a possible D-wave component in this meson.

In conclusion, the decay constants and the radiative decay widths of $b\bar{b}$ and $c\bar{c}$ pseudoscalar mesons, computed within a semirelativistic quark model which uses a potential inspired by the AdS/QCD correspondence, are compatible with the experimental data, in particular, in the case of f_{η_c} and $\Gamma_{\gamma\gamma}(\eta_c)$. The measurement of $\Gamma_{\gamma\gamma}(\eta'_c)$ carried out by the CLEO Collaboration [3] is not reproduced, since

TABLE II. Decay widths $\Gamma_{\gamma\gamma}$ (in KeV) of pseudoscalar states in two photons. The value denoted by * is reported by the PDG [7] as a datum not included in the summary tables.

Particle	This paper	Lansberg *et al.*[11]	Lakhina *et al.*[12]	Kim *et al.*[13]	Ebert *et al.*[14]	Exp.
η_c	4.252	7.46	7.18	7.14 ± 0.95	5.5	$7.2 \pm 0.7 \pm 2.0$ *
η'_c	3.306	4.1	1.71	4.44 ± 0.48	1.8	1.3 ± 0.6 [3]
η''_c	1.992		1.21			
η_b	0.313	0.560	0.230	0.384 ± 0.047	0.350	
η'_b	0.151	0.269	0.070	0.191 ± 0.025	0.150	
η''_b	0.092	0.208	0.040		0.100	

TABLE III. Decay widths $\Gamma_{\ell^+\ell^-}$ (in KeV) of vector mesons.

Particle	This paper	Exp. [7]
J/ψ	4.080	$5.55 \pm 0.14 \pm 0.02$
ψ'	2.375	2.38 ± 0.04
ψ''	0.836	0.86 ± 0.07
Υ	1.237	1.340 ± 0.018
$\Upsilon(2S)$	0.581	0.612 ± 0.011
$\Upsilon(3S)$	0.270	0.443 ± 0.008
$\Upsilon(4S)$	0.212	0.272 ± 0.029

the obtained result differs by more than 2σ. In this respect, our result follows most theoretical models [10,12–15], which predict higher values for $\Gamma_{\gamma\gamma}(\eta_c')$, although in some cases within the experimental error. This might suggest that the disagreement could be attributed to the systematics of the experimental measurement, namely, to the assumption that η_c and η_c' have the same branching fractions to the final state $K_S K \pi$. As for η_b, the prediction of the $\eta_b \to \gamma\gamma$ decay width suggests that this decay mode could be observed in the forthcoming experimental analyses.

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