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Generic Friedberg-Lee symmetry of Dirac neutrinos

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We write out the generic Dirac neutrino mass operator which possesses the Friedberg-Lee symmetry and find that its corresponding neutrino mass matrix is asymmetric. Following a simple way to break the Friedberg-Lee symmetry, we calculate the neutrino mass eigenvalues and show that the resultant neutrino mixing pattern is nearly tri-bimaximal. Imposing the Hermitian condition on the neutrino mass matrix, we also show that the simplified ansatz is consistent with current experimental data and favors the normal neutrino mass hierarchy.

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Recent solar, atmospheric, reactor, and accelerator neutrino experiments have provided us with very convincing evidence that neutrinos are slightly massive and lepton flavors are significantly mixed [1]. The flavor mixing of three lepton families can be described by a 3×3 unitary matrix U [2], which is usually parametrized as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12$, 13 and 23), and δ is the CP-violating phase. If neutrinos are Majorana particles, U should contain two more CP-violating phases, which are referred to as the Majorana phases and have nothing to do with neutrino oscillations. The latest global analysis of current neutrino oscillation data yields $30.9° \leq \theta_{12} \leq 37.8°$, $35.1° \leq \theta_{23} \leq 53.4°$ and $0° \leq \theta_{13} < 12.4°$ with 3σ uncertainty [3], but the phase δ remains entirely unconstrained. While the absolute mass scale of three neutrinos is not yet fixed, their two mass-squared differences have already been determined to quite a good degree of accuracy [3]: $\Delta m^2_{21} \equiv m^2_2 - m^2_1 = (7.14 \cdots 8.19) \times 10^{-5}\ \mathrm{eV}^2$ and $\Delta m^2_{32} \equiv m^2_3 - m^2_2 = \pm(2.06 \cdots 2.81) \times 10^{-3}\ \mathrm{eV}^2$ with 3σ uncertainty.

Many theoretical and phenomenological attempts have been made to interpret the smallness of three neutrino masses and the largeness of two neutrino mixing angles [4]. Among them, the flavor symmetry approach is, in particular, simple and predictive. A new and intriguing flavor symmetry is the one proposed by Friedberg and Lee (FL) [5]. In the basis where the mass eigenstates of three charged leptons are identified with their flavor eigenstates, the Dirac neutrino mass operator can be written as

$$\mathcal{L}_{\rm FL} = \sum_{\alpha,\beta} Y_{\alpha\beta}(\bar{\nu}_\alpha - \bar{\nu}_\beta)(\nu_\alpha - \nu_\beta), \tag{2}$$

where α and β run over e, μ, and τ. The FL symmetry means that $\mathcal{L}_{\rm FL}$ is invariant under the translational transformations $\nu_e \to \nu_e + z$, $\nu_\mu \to \nu_\mu + z$, and $\nu_\tau \to \nu_\tau + z$, where z is a constant element of the Grassmann algebra independent of space and time [5]. The corresponding neutrino mass matrix is a symmetric matrix,

$$M_{\rm FL} = \begin{pmatrix} b + c & -b & -c \\ -b & a + b & -a \\ -c & -a & a + c \end{pmatrix}, \tag{3}$$

where $a = Y_{\mu\tau} + Y_{\tau\mu}$, $b = Y_{e\mu} + Y_{\mu e}$, and $c = Y_{\tau e} + Y_{e\tau}$. Note that the determinant of $M_{\rm FL}$ is vanishing [i.e., $\mathrm{Det}(M_{\rm FL}) = 0$], and thus one of the neutrinos must be massless. One may explicitly break the FL symmetry of $\mathcal{L}_{\rm FL}$ to make realistic predictions for both neutrino masses and flavor mixing angles. So far some interesting works have been done to apply the FL symmetry to the Majorana neutrino mass operator [6–8], to combine the FL symmetry with the seesaw mechanism [9,10], to extend the FL symmetry to the quark sector [11], and to generalize the FL symmetry in a specific model containing some scalar fields [12].

Here let us clarify two points associated with the FL symmetry itself and its applications. First, the FL symmetry is not a symmetry of the full Lagrangian of electroweak interactions $\mathcal{L}_{\rm EW}$. Although the kinetic term of $\mathcal{L}_{\rm EW}$ is invariant under the transformations $\nu_\alpha \to \nu_\alpha + z$ (for $\alpha = e, \mu, \tau$) with z being independent of space and time, other terms of $\mathcal{L}_{\rm EW}$ do not have such a symmetry. In this sense, the FL symmetry is actually a phenomenological constraint or assumption imposed on the neutrino mass term of $\mathcal{L}_{\rm EW}$ (after electroweak symmetry breaking) in order to organize the texture of the neutrino mass matrix. Second, a similar constraint can be imposed on the mass term of charged

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leptons, but this treatment will in general give rise to small lepton flavor mixing angles. The reason is simply that the lepton flavor mixing matrix U is a measure of the mismatch between two unitary matrices U_l and U_ν used, respectively, to diagonalize the charged lepton and neutrino mass matrices; i.e., $U = U_l^\dagger U_\nu$ [13]. If both mass matrices had the same symmetry (or equivalently, the parallel textures), a large cancellation would occur between $U_l^\dagger$ and U_ν such that U would be very close to the identity matrix [14]. It is therefore a purely phenomenological assumption to impose the FL symmetry only on the neutrino sector so as to make a sufficient mismatch between two sectors. In the chosen basis, where the charged lepton mass matrix is diagonal and positive, U_l turns out to be the identity matrix and thus $U = U_\nu$ holds. The latter is then possible to accommodate large neutrino mixing angles if the neutrino mass matrix takes the form of $M_{\rm FL}$ or its slight variations, as already shown in Refs. [5–8].

We notice that $\mathcal{L}_{\rm FL}$ in Eq. (3) is not the most generic mass operator of Dirac neutrinos which obeys the FL symmetry. The Dirac neutrino mass operator

$$\mathcal{L}'_{\rm FL} = \sum_{\alpha,\beta}\sum_{\alpha',\beta'} Y^{\alpha\beta}_{\alpha'\beta'}(\bar\nu_\alpha - \bar\nu_\beta)(\nu_{\alpha'} - \nu_{\beta'}), \qquad (4)$$

where the Greek superscripts and subscripts run over e, μ, and τ, is more general than $\mathcal{L}_{\rm FL}$ and also invariant under the translational transformations $\nu_e \to \nu_e + z$, $\nu_\mu \to \nu_\mu + z$, and $\nu_\tau \to \nu_\tau + z$. Its corresponding neutrino mass matrix $M'_{\rm FL}$ takes the form

$$M'_{\rm FL} = \begin{pmatrix} B+C & -B-D & -C+D \\ -B+D & A+B & -A-D \\ -C-D & -A+D & A+C \end{pmatrix}, \qquad (5)$$

where

$$\begin{aligned} A = \tfrac{1}{2}[&-(Y^{\tau\mu}_{\mu e} + Y^{\mu\tau}_{e\mu} - Y^{\tau\mu}_{e\mu} - Y^{\mu\tau}_{\mu e}) + (Y^{\mu e}_{e\tau} + Y^{e\mu}_{\tau e} - Y^{\mu e}_{\tau e} - Y^{e\mu}_{e\tau}) - (Y^{e\tau}_{\tau\mu} + Y^{\tau e}_{\mu\tau} - Y^{e\tau}_{\mu\tau} - Y^{\tau e}_{\tau\mu}) - (Y^{\tau\mu}_{e\tau} + Y^{\mu\tau}_{\tau e} \\ &- Y^{\tau\mu}_{\tau e} - Y^{\mu\tau}_{e\tau}) - (Y^{\mu e}_{\tau\mu} + Y^{e\mu}_{\mu\tau} - Y^{\mu e}_{\mu\tau} - Y^{e\mu}_{\tau\mu}) + (Y^{e\tau}_{\mu e} + Y^{\tau e}_{e\mu} - Y^{e\tau}_{e\mu} - Y^{\tau e}_{\mu e})] + (Y^{\tau\mu}_{\tau\mu} + Y^{\mu\tau}_{\mu\tau} - Y^{\tau\mu}_{\mu\tau} - Y^{\mu\tau}_{\tau\mu}); \\ B = \tfrac{1}{2}[&-(Y^{\tau\mu}_{\mu e} + Y^{\mu\tau}_{e\mu} - Y^{\tau\mu}_{e\mu} - Y^{\mu\tau}_{\mu e}) - (Y^{\mu e}_{e\tau} + Y^{e\mu}_{\tau e} - Y^{\mu e}_{\tau e} - Y^{e\mu}_{e\tau}) + (Y^{e\tau}_{\tau\mu} + Y^{\tau e}_{\mu\tau} - Y^{e\tau}_{\mu\tau} - Y^{\tau e}_{\tau\mu}) + (Y^{\tau\mu}_{e\tau} + Y^{\mu\tau}_{\tau e} \\ &- Y^{\tau\mu}_{\tau e} - Y^{\mu\tau}_{e\tau}) - (Y^{\mu e}_{\tau\mu} + Y^{e\mu}_{\mu\tau} - Y^{\mu e}_{\mu\tau} - Y^{e\mu}_{\tau\mu}) - (Y^{e\tau}_{\mu e} + Y^{\tau e}_{e\mu} - Y^{e\tau}_{e\mu} - Y^{\tau e}_{\mu e})] + (Y^{\mu e}_{\mu e} + Y^{e\mu}_{e\mu} - Y^{\mu e}_{e\mu} - Y^{e\mu}_{\mu e}); \\ C = \tfrac{1}{2}[&(Y^{\tau\mu}_{\mu e} + Y^{\mu\tau}_{e\mu} - Y^{\tau\mu}_{e\mu} - Y^{\mu\tau}_{\mu e}) - (Y^{\mu e}_{e\tau} + Y^{e\mu}_{\tau e} - Y^{\mu e}_{\tau e} - Y^{e\mu}_{e\tau}) - (Y^{e\tau}_{\tau\mu} + Y^{\tau e}_{\mu\tau} - Y^{e\tau}_{\mu\tau} - Y^{\tau e}_{\tau\mu}) - (Y^{\tau\mu}_{e\tau} + Y^{\mu\tau}_{\tau e} \\ &- Y^{\tau\mu}_{\tau e} - Y^{\mu\tau}_{e\tau}) + (Y^{\mu e}_{\tau\mu} + Y^{e\mu}_{\mu\tau} - Y^{\mu e}_{\mu\tau} - Y^{e\mu}_{\tau\mu}) - (Y^{e\tau}_{\mu e} + Y^{\tau e}_{e\mu} - Y^{e\tau}_{e\mu} - Y^{\tau e}_{\mu e})] + (Y^{e\tau}_{e\tau} + Y^{\tau e}_{\tau e} - Y^{e\tau}_{\tau e} - Y^{\tau e}_{e\tau}); \\ D = \tfrac{1}{2}[&(Y^{\tau\mu}_{\mu e} + Y^{\mu\tau}_{e\mu} - Y^{\tau\mu}_{e\mu} - Y^{\mu\tau}_{\mu e}) + (Y^{\mu e}_{e\tau} + Y^{e\mu}_{\tau e} - Y^{\mu e}_{\tau e} - Y^{e\mu}_{e\tau}) + (Y^{e\tau}_{\tau\mu} + Y^{\tau e}_{\mu\tau} - Y^{e\tau}_{\mu\tau} - Y^{\tau e}_{\tau\mu}) - (Y^{\tau\mu}_{e\tau} + Y^{\mu\tau}_{\tau e} \\ &- Y^{\tau\mu}_{\tau e} - Y^{\mu\tau}_{e\tau}) - (Y^{\mu e}_{\tau\mu} + Y^{e\mu}_{\mu\tau} - Y^{\mu e}_{\mu\tau} - Y^{e\mu}_{\tau\mu}) - (Y^{e\tau}_{\mu e} + Y^{\tau e}_{e\mu} - Y^{e\tau}_{e\mu} - Y^{\tau e}_{\mu e})]. \end{aligned} \qquad (6)$$

We see that $M'_{\rm FL}$ is an asymmetric matrix and its asymmetry is characterized by nonvanishing D. Given $D = 0$, $M'_{\rm FL}$ turns out to be equivalent to $M_{\rm FL}$.

Based on the above observation, we are going to focus our interest on the phenomenological implications of $\mathcal{L}'_{\rm FL}$ for Dirac neutrinos. We shall follow a simple way to break the FL symmetry of $\mathcal{L}'_{\rm FL}$ and obtain the neutrino mass matrix $M_\nu = M'_{\rm FL} + m_0\mathbf{1}$ with $\mathbf{1}$ being the identity matrix. Then we shall show that a nearly tri-bimaximal neutrino mixing pattern, which is favored by current neutrino oscillation data, can always be obtained from M_ν. A simpler and Hermitian form of M_ν will also be discussed in detail.

Although $M'_{\rm FL}$ in Eq. (5) is asymmetric, one can easily verify that its determinant vanishes as $M_{\rm FL}$ does. Hence one of the mass eigenvalues of $M'_{\rm FL}$ must be zero. For simplicity, here we follow Ref. [5] to break the FL symmetry of $\mathcal{L}'_{\rm FL}$:

$$\mathcal{L}_\nu = \mathcal{L}'_{\rm FL} + m_0 \sum_\alpha \bar\nu_\alpha \nu_\alpha, \qquad (7)$$

where m_0 is in general a complex parameter, and α runs over e, μ, and τ. Corresponding to $\mathcal{L}_\nu$, the Dirac neutrino mass matrix reads

$$\begin{aligned} M_\nu &= M'_{\rm FL} + m_0\mathbf{1} \\ &= \begin{pmatrix} B+C & -B-D & -C+D \\ -B+D & A+B & -A-D \\ -C-D & -A+D & A+C \end{pmatrix} \\ &\quad + m_0\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \qquad (8)$$

We see that $\mathcal{L}_\nu$ or M_ν can possess the exact μ-τ symmetry only when both $B = C$ and $D = 0$ are satisfied. To derive the neutrino mass spectrum and the flavor mixing pattern from M_ν, we consider the following unitary transformation: $U^\dagger M_\nu M_\nu^\dagger U = {\rm Diag}\{m_1^2, m_2^2, m_3^2\}$, where m_i (for $i =$ 1, 2, 3) stand for three neutrino masses. Because we have taken the basis in which the mass and flavor eigenstates of three charged leptons are identical, the unitary matrix U is just the neutrino mixing matrix linking the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$.

A salient feature of M_ν is that the sum of three elements in its any row or column equals m_0, implying that one of its three eigenvalues must be m_0. For this reason, the unitary transformation U used to diagonalize the Hermitian matrix

$M_\nu M_\nu^\dagger$ must have an eigenvector which contains three equal components $1/\sqrt{3}$. It is then possible to express U as a production of the tri-bimaximal mixing matrix U_0 [15] and a complex rotation matrix U_θ in the (1,3) plane:

$$U = U_0 \otimes U_\theta = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \otimes \begin{pmatrix} \cos\theta & 0 & \sin\theta e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta e^{i\delta} & 0 & \cos\theta \end{pmatrix}, \quad (9)$$

in which δ signifies CP violation and is equivalent to the one defined in Eq. (1). After a straightforward calculation, we obtain

$$\delta = -\arg(T_{13}), \qquad \theta = \frac{1}{2}\arctan\left(\frac{2|T_{13}|}{T_{33} - T_{11}}\right), \quad (10)$$

where

$$\begin{aligned} T_{11} &= 3(|B|^2 + |C|^2 + \mathrm{Re}[B^*C] + |D|^2 - \mathrm{Re}[(C-B)D^*]) \\ &\quad + 3\,\mathrm{Re}[(B+C)m_0^*] + |m_0|^2, \\ T_{33} &= |B|^2 + |C|^2 - \mathrm{Re}[B^*C] + 4|A|^2 + 2\,\mathrm{Re}[(B+C)A^*] \\ &\quad + 3\,\mathrm{Re}[(C-B)D^*] + 3|D|^2 + 4\,\mathrm{Re}[Am_0^*] \\ &\quad + \mathrm{Re}[(B+C)m_0^*] + |m_0|^2, \\ T_{13} &= \sqrt{3}(|C|^2 - |B|^2 - i\,\mathrm{Im}[B^*C]) + \sqrt{3}\,\mathrm{Re}[(B+C)D^*] \\ &\quad + 2\sqrt{3}i\,\mathrm{Im}[(B+C)D^*] + \sqrt{3}(C-B)A^* - 2\sqrt{3}A^*D \\ &\quad + \sqrt{3}\,\mathrm{Re}[(C-B)m_0^*] - 2\sqrt{3}i\,\mathrm{Im}[Dm_0^*]. \end{aligned} \quad (11)$$

Furthermore, three mass eigenvalues of M_ν are found to be $m_2 = |m_0|$ and

$$\begin{aligned} m_1 &= \sqrt{\tfrac{1}{2}(T_{11} + T_{33}) - \tfrac{1}{2}(T_{33} - T_{11})\cos 2\theta - |T_{13}|\sin 2\theta}, \\ m_3 &= \sqrt{\tfrac{1}{2}(T_{11} + T_{33}) + \tfrac{1}{2}(T_{33} - T_{11})\cos 2\theta + |T_{13}|\sin 2\theta}. \end{aligned} \quad (12)$$

Comparing between Eqs. (1) and (9), one may easily arrive at the analytical results of three mixing angles:

$$\begin{aligned} \sin\theta_{12} &= \frac{1}{\sqrt{2 + \cos 2\theta}}, \\ \sin\theta_{23} &= \frac{\sqrt{2 + \cos 2\theta - \sqrt{3}\sin 2\theta\cos\delta}}{\sqrt{2(2 + \cos 2\theta)}}, \\ \sin\theta_{13} &= \frac{2}{\sqrt{6}}|\sin\theta|. \end{aligned} \quad (13)$$

In addition, we find that the Jarlskog invariant of leptonic CP violation [16] is given by $\mathcal{J} = \sin 2\theta \sin\delta/(6\sqrt{3})$ in this phenomenological scenario of Dirac neutrino mixing.

Given the asymmetric form of M_ν in Eq. (8), the Hermitian relation $M_\nu^\dagger = M_\nu$ can be achieved if and only if A, B, C, and m_0 are all real and D is purely imaginary (i.e., $D^* = -D$). Let us define $D = iD'$ and rewrite M_ν as

$$M_\nu = \begin{pmatrix} B + C & -B - iD' & -C + iD' \\ -B + iD' & A + B & -A - iD' \\ -C - iD' & -A + iD' & A + C \end{pmatrix} + m_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (14)$$

where A, B, C, D', and m_0 are all real. Now M_ν is Hermitian and only contains five free parameters. We are going to show that this interesting texture of M_ν is actually compatible with current neutrino oscillation data.

With the help of Eqs. (10)–(13), it is easy to obtain three neutrino masses and three flavor mixing angles from Hermitian M_ν given in Eq. (14). First, we have $m_2 = |m_0|$ and

$$\begin{aligned} m_1 &= |(A + B + C + m_0) \\ &\quad \mp \sqrt{(A^2 + B^2 + C^2) - (AB + BC + CA) + 3D'^2}|, \\ m_3 &= |(A + B + C + m_0) \\ &\quad \pm \sqrt{(A^2 + B^2 + C^2) - (AB + BC + CA) + 3D'^2}|. \end{aligned} \quad (15)$$

Second, the expressions of $\sin\theta_{12}$, $\sin\theta_{23}$, and $\sin\theta_{13}$ are the same as those shown in Eq. (13) with δ and θ being now given by

$$\delta = \arctan\left(\frac{2D'}{C - B}\right), \quad \theta = \frac{1}{2}\arctan\left[\frac{\sqrt{3[(C-B)^2 + 4D'^2]}}{2A - B - C}\right]. \quad (16)$$

Note that δ is just the CP-violating phase of U, and θ has been restricted to the range $-\pi/4 \leq \theta \leq \pi/4$. Note also that $\theta > 0$ and $\theta < 0$ correspond to the options of "$\mp$" signs in the expression of m_1 (or the options of "$\pm$" signs in the expression of m_3) in Eq. (15). Taking account of current experimental constraints on three mixing angles [3], we obtain $|\theta| < 18°$. The smallness of $|\theta|$ implies that U is a nearly tri-bimaximal mixing pattern.

If both $B = C$ and $D' = 0$ hold, then M_ν possesses the exact μ-τ symmetry which gives rise to the exact tri-bimaximal neutrino mixing pattern U_0. There are two simpler ways to produce the deviation of U from U_0:

(1) $B \neq C$ and $D' = 0$. Then we have $\theta_{13} \neq 0°$ and $\theta_{23} \neq 45°$ together with $\delta = 0°$.

(2) $B = C$ and $D' \neq 0$. Then we have $\delta = \pm\pi/2$, $\theta_{23} = 45°$ and $\theta_{13} \neq 0°$.

The second possibility is more interesting in the sense that $|\mathcal{J}| = \sin 2\theta/(6\sqrt{3})$ can be as large as a few percent for $|\theta| \geq 3°$ and may lead to observable CP-violating effects in long-baseline neutrino oscillations.

To illustrate, we carry out a simple numerical analysis of the parameter space of Hermitian M_ν by using current neutrino oscillation data on $(\Delta m^2_{21}, \Delta m^2_{32})$ and

$(\theta_{12}, \theta_{13}, \theta_{23})$ as the inputs. Without loss of generality, we assume $m_0 > 0$. Our numerical results indicate that only the normal neutrino mass hierarchy (i.e., $\Delta m^2_{32} > 0$) is favored in this Hermitian ansatz. The allowed regions of A, B, C, D', and m_0 are shown in Fig. 1, where $m_0 \lesssim 0.2$ eV has been taken as a generous upper bound on the absolute neutrino mass scale [17]. Because of $m_0 = m_2$, the lower bound of m_0 is $m_0 > \sqrt{\Delta m^2_{21}} \approx 0.09$ eV as one can see from Fig. 1. The Jarlskog invariant $\mathcal{J}$ may vary from 0 to 0.057 in the obtained parameter space.

Let us make some concluding remarks. This work is a simple but useful generalization of the original FL symmetry for Dirac neutrinos. Such a generic FL symmetry can be applied to the quark sector to obtain generic (or Hermitian) quark mass matrices. But it will have no influence on the neutrino mass matrix if massive neutrinos are Majorana particles, because a Majorana neutrino mass matrix must always be symmetric.

Moreover, the FL symmetry is essentially a phenomenological constraint on the texture of the neutrino mass matrix at low energies. One may certainly consider imposing such a symmetry on the neutrino mass matrix at a superhigh energy scale. In this case, however, one has to carefully take account of radiative corrections to the results of neutrino masses and flavor mixing angles when they are confronted with current experimental data. It is generally expected that radiative corrections are negligible if three neutrino masses have a normal and strong hierarchy, and they may be significant if three neutrino masses are nearly degenerate [18]. For the ansatz discussed in this paper, which only favors the normal neutrino mass hierarchy, the same behaviors of radiative corrections can be expected if it is prescribed at a high energy scale instead of low scales.

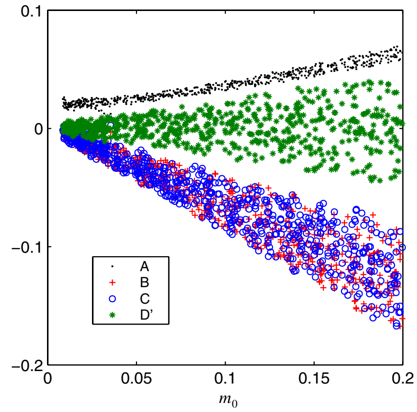

FIG. 1 (color online). The parameter space of A, B, C, and D' versus m_0 (all of them in unit of eV) in the scenario of Hermitian M_ν, where only the normal neutrino mass hierarchy (i.e., $\Delta m^2_{32} > 0$) is allowed.

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[1] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[2] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
[3] G. L. Fogli *et al.*, Phys. Rev. D **78**, 033010 (2008).
[4] H. Fritzsch and Z. Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); Altarelli and F. Feruglio, New J. Phys. **6**, 106 (2004); R. N. Mohapatra and A. Yu. Smirnov, Annu. Rev. Nucl. Part. Sci. **56**, 569 (2006); A. Strumia and F. Vissani, arXiv:hep-ph/0606054.
[5] R. Friedberg and T. D. Lee, High Energy Phys. Nucl. Phys. **30**, 591 (2006).
[6] Z. Z. Xing, H. Zhang, and S. Zhou, Phys. Lett. B **641**, 189 (2006); Int. J. Mod. Phys. A **23**, 3384 (2008).
[7] S. Luo and Z. Z. Xing, Phys. Lett. B **646**, 242 (2007).
[8] Z. Z. Xing, Int. J. Mod. Phys. E **16**, 1361 (2007).
[9] C. Jarlskog, Phys. Rev. D **77**, 073002 (2008).
[10] W. Chao, S. Luo, and Z. Z. Xing, Phys. Lett. B **659**, 281 (2008).
[11] R. Friedberg and T. D. Lee, Ann. Phys. (N.Y.) **323**, 1087 (2008); **323**, 1677 (2008).
[12] C. S. Huang, T. J. Li, W. Liao, and S. H. Zhu, Phys. Rev. D **78**, 013005 (2008).
[13] Z. Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004); Phys. Lett. B **618**, 141 (2005).
[14] See, e.g., H. Fritzsch and Z. Z. Xing, Phys. Lett. B **372**, 265 (1996); **440**, 313 (1998); Phys. Rev. D **61**, 073016 (2000).
[15] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. B **530**, 167 (2002); Z. Z. Xing, Phys. Lett. B **533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. B **535**, 163 (2002).
[16] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); H. Fritzsch and Z. Z. Xing, Nucl. Phys. **B556**, 49 (1999).
[17] E. Komatsu *et al.* (WMAP Collaboration), arXiv:0803.0547, where $\sum m_i < 0.61$ eV (95% C.L.) has been presented.
[18] M. Lindner, M. Ratz, and M. A. Schmidt, J. High Energy Phys. 09 (2005) 081; Z. Z. Xing and H. Zhang, Commun. Theor. Phys. **48**, 525 (2007).