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$B_s^0 - \bar{B}_s^0$ mixing and $b \to s$ transitions in an isosinglet down quark model

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The recent observation of the mass difference in the B_s system seems to be not in complete agreement with the corresponding standard model value. We consider the model with an extra vectorlike down quark to explain this discrepancy and obtain the constraints on the new physics parameters. Thereafter, we show that with these new constraints this model can successfully explain other observed deviations associated with $b \to s$ transitions, namely, $B_s \to \psi\phi$, $B \to K\pi$, and $B \to \phi K_s$.

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I. INTRODUCTION

The results of the currently running two asymmetric B factories confirmed the fact that the phenomenon of CP violation in the standard model (SM) is due to the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [1]. The observed data are almost in the line of the SM expectations and there is no clear indication of new physics so far. However, there are some interesting deviations from that of the SM expectations which could provide us an indirect signal of new physics. Here we are concentrating on few such deviations which are associated with the CP violation parameters of flavor-changing neutral current (FCNC) mediated $b \to s$ transitions. A partial list includes:

(i) The observed mass difference measured between heavy and light B_s mesons [2] seems to be inconsistent with its SM value with a deviation of few sigma.

(ii) The observed discrepancy between the measured $S_{\phi K_s}$ and $S_{\psi K_s}$ [3] already gave an indication of the possible existence of NP in the $B \to \phi K_s$ decay amplitude. Within the SM, these CP symmetries are expected to be the same with a deviation of about 5% [4].

(iii) The recent observation of a very large $S_{\psi\phi}$ by the CDF collaboration [5] is in contrast to its expected SM value, i.e., $S_{\psi\phi} \approx 0$. This may be considered as a clear signal of new physics in the $b \to s$ transitions.

(iv) There appears to be some disagreement between the direct CP asymmetry parameters of $B^- \to \pi^0 K^-$ and that of the $\bar{B}^0 \to \pi^+ K^-$. $\Delta A_{CP}(K\pi)$, which is the difference of these two parameters, is found to be around 15% [3], whereas the SM expectation is vanishingly small. This constitutes what is called the πK puzzle in the literature and is believed to be an indication of the existence of new physics.

(v) The $B_s \to \mu^+\mu^-$ problem has been widely discussed in the literature. The SM value is quite small (we have only the upper limit for the branching ratio) and it is a very clean mode so if we have any smoking gun signal of new physics elsewhere in $b \to s$ transitions it is quite likely that it could also be found in this mode. Therefore, $B_s \to \mu^+\mu^-$ is a golden mode to detect new physics.

In this paper, we would like to see the effect of the extra vectorlike down quark [6] in explaining the above mentioned observed discrepancies. It is a simple model beyond the standard model with an enlarged matter sector due to an additional vectorlike down quark D_4. Isosinglet quarks appear in many extensions of the SM like the low energy limit of the E_6 grand unified theory models [7]. The mixing of this singlet type down quark with the three SM down type quarks provides a framework to study the deviations of the unitarity constraint of the 3×3 CKM matrix. To be more explicit, the presence of an additional down quark implies a 4×4 matrix $V_{i\alpha}$ ($i = u, c, t, 4$, $\alpha = d, s, b, b'$) would diagonalize the down quark mass matrix. Because of this, some new features appear in the low energy phenomenology. The charged currents are unchanged except that the $V_{\rm CKM}$ is now the 3×4 upper submatrix of V. However, the distinctive feature of this model is that the FCNC interaction enters at tree level in the neutral current Lagrangian of the left-handed down quarks as [6]

$$\mathcal{L}_Z = \frac{g}{2\cos\theta_W}[\bar{u}_{Li}\gamma^\mu u_{Li} - \bar{d}_{L\alpha}U_{\alpha\beta}\gamma^\mu d_{L\beta} - 2\sin^2\theta_W J^\mu_{em}]Z_\mu, \quad (1)$$

with

$$U_{\alpha\beta} = \sum_{i=u,c,t} V^\dagger_{\alpha i}V_{i\beta} = \delta_{\alpha\beta} - V^*_{4\alpha}V_{4\beta}, \quad (2)$$

where U is the neutral current mixing matrix for the down sector, which is given above. As V is not unitary, $U \neq \mathbf{1}$. In particular the nondiagonal elements do not vanish

$$U_{\alpha\beta} = -V^*_{4\alpha}V_{4\beta} \neq 0 \quad \text{for } \alpha \neq \beta. \quad (3)$$

Since the various $U_{\alpha\beta}$ are nonvanishing they would signal new physics and the presence of FCNC at the tree level, which can substantially modify the predictions of SM for the FCNC processes. Of course, these low energy couplings are severely restricted by the low energy results available on different FCNC processes, i.e., $\mathrm{Br}(K_L \to \mu\bar{\mu})_{\mathrm{SD}}$, $\mathrm{Br}(K^+ \to \pi^+ \nu\bar{\nu})$, ϵ_K, ΔM_K, ΔM_{B_d}, ΔM_{B_s}, $\mathrm{Br}(B \to X_{d,s} l^+ l^-)$ etc. [8] Nevertheless, it is well-known that even fulfilling these strong constraints there could still be large effects on B factory experiments on CP violation. The implications of the FCNC mediated Z boson effect has been extensively studied in the context of b physics [9–11].

II. $B_s - \bar{B}_s$ MIXING

We will first concentrate on the mass difference between the neutral B_s meson mass eigenstates (ΔM_s) that characterizes the $B_s - \bar{B}_s$ mixing phenomena. In the SM, $B_s - \bar{B}_s$ mixing occurs at the one-loop level by flavor-changing weak interaction box diagrams and hence is very sensitive to new physics effects.

In the SM, the effective Hamiltonian describing the $\Delta B = 2$ transition, induced by the box diagram, is given by [12]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F^2}{16\pi^2} \lambda_t^2 M_W^2 S_0(x_t) \eta_t (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} \quad (4)$$

where $\lambda_t = V_{tb} V_{ts}^*$, η_t is the QCD correction factor and $S_0(x_t)$ is the loop function

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3}{2} \frac{\log x_t x_t^3}{(1-x_t)^3}, \quad (5)$$

with $x_t = m_t^2/M_W^2$. Thus, the $B_s - \bar{B}_s$ mixing amplitude in the SM can be written as

$$M_{12}^{\mathrm{SM}} = \frac{1}{2M_{B_s}} \langle \bar{B}_s | \mathcal{H}_{\mathrm{eff}} | B_s \rangle = \frac{G_F^2}{12\pi^2} M_W^2 \lambda_t^2 \eta_t B_s f_{B_s}^2 M_{B_s} S_0(x_t), \quad (6)$$

where the vacuum insertion method has been used to evaluate the matrix element

$$\langle \bar{B}_s | (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} | B_s \rangle = \tfrac{8}{3} B_s f_{B_s}^2 M_{B_s}^2. \quad (7)$$

The corresponding mass difference is related to the mixing amplitude through $\Delta M_s = 2|M_{12}|$.

Recently, Lenz and Nierste [13] updated the theoretical estimation of the B_s mass difference in the SM, with the value $(\Delta M_{B_s})^{\mathrm{SM}} = (19.30 \pm 6.68)\ \mathrm{ps}^{-1}$ (for Set-I parameters) and $(\Delta M_{B_s})^{\mathrm{SM}} = (20.31 \pm 3.25)\ \mathrm{ps}^{-1}$ (Set-II).

The CDF [2] and D0 [14] collaborations have also recently reported new results for the $B_s - \bar{B}_s$ mass difference

$$\begin{aligned} &\Delta M_{B_s} = (17.77 \pm 0.10 \pm 0.07)\ \mathrm{ps}^{-1} && \text{(CDF)} \\ &17\ \mathrm{ps}^{-1} < \Delta M_{B_s} < 21\ \mathrm{ps}^{-1} && 90\%\text{C.L.(D0)}. \end{aligned} \quad (8)$$

Although the experimental results appear to be consistent with the standard model prediction, but they do not completely exclude the possible new physics effects in $\Delta B = 2$ transitions. In the literature, there have already been many discussions both in model independent [15–17] and model dependent ways [18] regarding the implications of these new measurements. In this work, we would like to see the effect of the extended isosiglet down quark model on the mass difference of the B_s system and its possible implications for the other $b \to s$ transition processes.

In the model with an extra vectorlike down quark there will be two additional contributions to the $B_s - \bar{B}_s$ mixing amplitude. The first one is induced by a tree level FCNC mediated Z boson, with two nonstandard (flavor-changing) $Z - b - s$ couplings as shown in Fig. 1(a) and the second contribution contains one nonstandard $Z - b - s$ coupling and one SM loop-induced $Z - b - s$ coupling as depicted in Fig. 1(b). With these new contributions, the mass difference between B_s^H and B_s^L deviates significantly from its SM value.

To evaluate these two additional contributions, one can write from Eq. (1) the effective FCNC mediated Lagrangian for Zbs interaction as

$$\mathcal{L}_{\mathrm{FCNC}}^{Z} = -\frac{g}{2\cos\theta_W} U_{sb} \bar{s}_L \gamma^\mu b_L Z_\mu. \quad (9)$$

This gives the effective Hamiltonian induced by the tree level FCNC mediated Z boson (Fig. 1(a)) as

$$\mathcal{H}_{\mathrm{eff}}^{Z} = \frac{G_F}{\sqrt{2}} U_{sb}^2 \eta_Z (\bar{s}_L \gamma^\mu b_L)(\bar{s}_L \gamma_\mu b_L), \quad (10)$$

where $\eta_Z = (\alpha_s(m_Z))^{6/23}$ is the QCD correction factor. Using the matrix elements as defined in Eq. (7), we obtain

$$M_{12}^{Z} = \frac{G_F}{3\sqrt{2}} U_{sb}^2 \eta_Z B_s f_{B_s}^2 M_{B_s}. \quad (11)$$

The effective Hamiltonian induced by the SM penguin at one vertex and Z mediated FCNC coupling on the other (Fig. 1(b)) is given as

$$\mathcal{H}_{\mathrm{eff}}^{\mathrm{SM}+Z} = \frac{G_F^2}{4\pi^2} \lambda_t \eta_{Zt} M_W^2 U_{sb} C_0(x_t) (\bar{s}b)_{V-A} (\bar{s}b)_{V-A} \quad (12)$$

where η_{Zt} is the QCD correction factor and

$$C_0(x_t) = \frac{x_t}{8}\left(\frac{x_t - 6}{x_t - 1} + \frac{3x_t + 2}{(x_t - 1)^2} \log x_t\right). \quad (13)$$

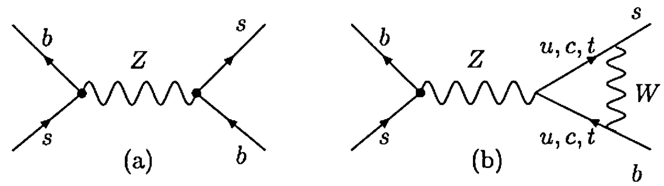

FIG. 1. Feynman diagrams for $B_s - \bar{B}_s$ mixing in the model with an extra vectorlike down quark, where the blob represents the tree level flavor-changing vertex.

This gives

$$M_{12}^{\mathrm{SM}+Z} = \frac{G_F^2}{3\pi^2}\lambda_t U_{sb}\eta_{Zt} M_W^2 C_0(x_t) B_s f_{B_s}^2 M_{B_s}. \quad (14)$$

Thus, the mass difference ΔM_s in this model can be given as

$$\Delta M_s = 2|M_{12}^{\mathrm{SM}} + M_{12}^Z + M_{12}^{\mathrm{SM}+Z}| = \Delta M_s^{\mathrm{SM}} \left| 1 + a\left(\frac{U_{sb}}{\lambda_t}\right) + b\left(\frac{U_{sb}}{\lambda_t}\right)^2 \right| \quad (15)$$

with

$$a = 4\frac{C_0(x_t)}{S_0(x_t)}, \qquad b = \frac{2\sqrt{2}\pi^2}{G_F M_W^2 S_0(x_t)}, \quad (16)$$

where we have assumed $\eta_t \approx \eta_Z \approx \eta_{Zt}$. The coupling U_{sb} characterizing the $Z - b - s$ strength is in general complex and can be parametrized as $U_{sb} = |U_{sb}|e^{i\phi_s}$, where ϕ_s is the new weak phase. The constraints on these parameters can be obtained using the recent measurement on ΔM_s.

Since $V_{tb}V_{ts}^* = -|V_{tb}V_{ts}|e^{i\beta_s}$, we parametrize

$$\frac{U_{sb}}{V_{tb}V_{ts}^*} = -\left|\frac{U_{sb}}{V_{tb}V_{ts}}\right| e^{i(\phi_s - \beta_s)} \equiv -xe^{i(\phi_s - \beta_s)}. \quad (17)$$

For numerical evaluation, we use the CKM elements as $|V_{tb}| = 0.999\,176^{+0.000\,031}_{-0.000\,044}$, $|V_{ts}| = 0.039\,72^{+0.001\,15}_{-0.000\,77}$ [19], $\beta_s = -1.1°$, the masses of W boson and t quark as $M_W = 80.4$ GeV, $m_t = 168$ GeV. For ΔM_s, we use the CDF result [2] $\Delta M_s = 17.77 \pm 0.12\ \mathrm{ps}^{-1}$ and for $\Delta M_s^{\mathrm{SM}} = 19.30 \pm 6.68\ \mathrm{ps}^{-1}$ [13], which yields $\Delta M_s/\Delta M_s^{\mathrm{SM}} = 0.92 \pm 0.32$. Varying $(\Delta M_s/\Delta M_s^{\mathrm{SM}})$ within its $1-\sigma$ range the allowed parameter space in the $\phi_s - |U_{sb}|$ plane is shown in Fig. 5. From the figure it can be seen that for higher value of $|U_{sb}|$ the phase ϕ_s is very tightly constrained. However, for $|U_{sb}| \leq 0.0015$ there is no constraint on the new weak phase ϕ_s, i.e., the whole range $0 - 2\pi$ is allowed. The constraint on $|U_{sb}|$ obtained from

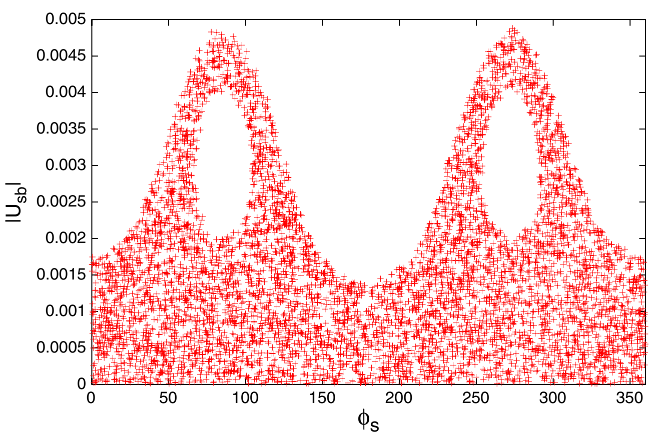

FIG. 2 (color online). The $1-\sigma$ allowed range of $(\Delta M_s/\Delta M_s^{\mathrm{SM}})$ in the $\phi_s - |U_{sb}|$ plane.

$B \to X_s l^+ l^-$, i.e., $|U_{sb}| \leq 0.002$, [8] is consistent with the constraint obtained from $B_s - \bar{B}_s$ mixing. We now use the allowed values of $|U_{sb}|$ (i.e., we use $|U_{sb}| \leq 0.002$ so that constraints coming from both the observables will be satisfied) and ϕ_s to study some anomalies associated with $b \to s$ transitions. In particular, we would like to see whether the constraints obtained above in the extended isosinglet down quark model, consistent with $B_s - \bar{B}_s$ mixing, can also explain the discrepancies in the modes $B_s \to \psi\phi$, $B_s \to \mu^+\mu^-$, $B \to \pi K$, and $B \to \phi K_s$.

III. MIXING INDUCED *CP* ASYMMETRY IN $B_s \to J/\psi\phi(S_{\psi\phi})$

We now consider the effect of the isosinglet down quark on the mixing induced *CP* asymmetry in $B_s \to J/\psi\phi$ mode. Recently a very largish *CP* asymmetry has been measured by the CDF collaboration [5] in the tagged analysis of $B_s \to J/\psi\phi$ with value $S_{\psi\phi} \in [0.23, 0.97]$.

Within the SM, this asymmetry is expected to be vanishingly small, which comes basically from the $B_s - \bar{B}_s$ mixing phase. Since this mode receives dominant contribution from $b \to c\bar{c}s$ tree level transition, the NP contribution to its decay amplitude is naively expected to be negligible. Therefore, the observed large *CP* asymmetry is believed to be originating from the new *CP* violating phase in $B_s - \bar{B}_s$ mixing.

Now parametrizing the new physics contribution to the $B_s - \bar{B}_s$ mixing amplitude as

$$M_{12} = M_{12}^{\mathrm{SM}} + M_{12}^Z + M_{12}^{\mathrm{SM}+Z} = M_{12}^{\mathrm{SM}} C_{B_s} e^{2i\theta_s}, \quad (18)$$

one can obtain

$$S_{\psi\phi} = -\eta_{\psi\phi}\sin(2\beta_s + 2\theta_s), \quad (19)$$

where β_s is the phase of $V_{ts} = -|V_{ts}|e^{-i\beta_s}$ and $\eta_{\psi\phi}$ is the *CP* parity of the $\psi\phi$ final state. Taking $\eta_{\psi\phi} = +1$ and $\beta_s \approx -1.1°$ we obtain the mixing induced *CP* asymmetry as

$$S_{\psi\phi} = \sin(2|\beta_s| - 2\theta_s). \quad (20)$$

Now substituting the expressions for M_{12}^{SM}, M_{12}^Z, and $M_{12}^{\mathrm{SM}+Z}$ from Equations (6), (11), and (14) in Eq. (18), we obtain the new *CP*-odd phase of $B_s - \bar{B}_s$ mixing as

$$2\theta_s = \arctan\left(\frac{-ax\sin(\phi_s + |\beta_s|) + bx^2\sin(2\phi_s + 2|\beta_s|)}{1 - ax\cos(\phi_s + |\beta_s|) + bx^2\cos(2\phi_s + 2|\beta_s|)}\right), \quad (21)$$

where a, b, and x are defined in Eqs. (16) and (17), respectively. In Fig. 3, we show the variation of $S_{\psi\phi}$ (20) with the new weak phase ϕ_s for two representative values of $|U_{sb}|$. From the figure it can be seen that the observed largish $S_{\psi\phi}$ can be explained in the model with an extra vectorlike down quark for $|U_{sb}| \geq 0.0015$.

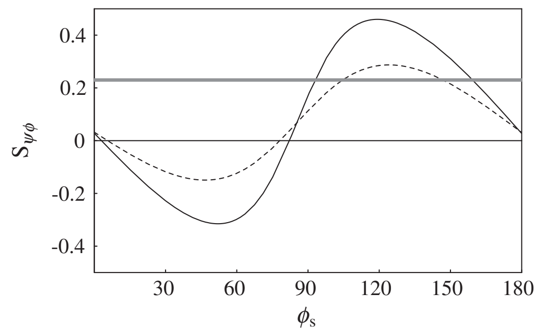

FIG. 3. Variation of $S_{\psi\phi}$ with the new weak phase ϕ_s where the solid and dotted lines are for $|U_{sb}| = 0.002$ and 0.0015, respectively. The horizontal line represents the lower limit of the experimental value.

IV. $B_s \to \mu^+\mu^-$

Now let us consider the FCNC mediated leptonic transition $B_s \to \mu^+\mu^-$. This decay mode has attracted a lot of attention recently since it is very sensitive to the structure of the SM and potential source of new physics beyond the SM. Furthermore, this process is very clean and the only nonperturbative quantity involved is the decay constant of the B_s meson which can be reliably calculated by the well-known nonperturbative methods such as QCD sum rules, lattice gauge theory, etc. Therefore, it provides a good hunting ground to look for new physics. The recent updated branching ratio $\text{Br}(B_s \to \mu^+\mu^-) = (3.35 \pm 0.32) \times 10^{-9}$ in the SM [16] is well below the present experimental upper limit [3]

$$\text{Br}(B_s \to \mu^+\mu^-) < 4.7 \times 10^{-8}. \tag{22}$$

This decay has been analyzed in many beyond the SM scenarios in a number of papers [20]. Let us start by recalling the result for $B_s \to \mu^+\mu^-$ in the standard model. The effective Hamiltonian describing this process is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}V_{tb}V_{ts}^*\bigg[C_9(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu\mu) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu\gamma_5\mu) - \frac{2C_7 m_b}{q^2} \times (\bar{s}i\sigma_{\mu\nu}q^\nu P_R b)(\bar{\mu}\gamma^\mu\mu)\bigg], \tag{23}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$ and q is the momentum transfer. C_i's are the Wilson coefficients evaluated at the b quark mass scale in next-to-leading-logarithmic order with values [21]

$$C_7 = -0.308, \qquad C_9 = 4.154, \qquad C_{10} = -4.261. \tag{24}$$

To evaluate the transition amplitude one can generally adopt the vacuum insertion method, where the form factors of the various currents are defined as follows

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 b|B_s^0\rangle = if_{B_s}p_B^\mu, \qquad \langle 0|\bar{s}\gamma_5 b|B_s^0\rangle = if_{B_s}m_{B_s}, \qquad \langle 0|\bar{s}\sigma^{\mu\nu}P_R b|B_s^0\rangle = 0. \tag{25}$$

Since $p_B^\mu = p_+^\mu + p_-^\mu$, the contribution from the C_9 term in Eq. (23) will vanish upon contraction with the lepton bilinear, C_7 will also give zero by (25), and the remaining C_{10} term will get a factor of $2m_\mu$.

Thus, the transition amplitude for the process is given as

$$\mathcal{M}(B_s \to \mu^+\mu^-) = i\frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}V_{tb}V_{ts}^* f_{B_s}C_{10}m_\mu(\bar{\mu}\gamma_5\mu), \tag{26}$$

and the corresponding branching ratio is given as

$$\text{Br}(B_s \to \mu^+\mu^-) = \frac{G_F^2\tau_{B_s}}{16\pi^3}\alpha^2 f_{B_s}^2 m_{B_s}m_\mu^2|V_{tb}V_{ts}^*|^2C_{10}^2 \times \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}. \tag{27}$$

Helicity suppression is reflected by the presence of m_μ^2 in (27) which gives a very small branching ratio of $(3.35 \pm 0.32) \times 10^{-9}$ for $\mu^+\mu^-$ [16].

Now let us analyze the decay modes $B_s \to \mu^+\mu^-$ in the model with the Z mediated FCNC occurring at the tree level. The effective Hamiltonian for $B_s \to \mu^+\mu^-$ is given as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}U_{sb}[\bar{s}\gamma^\mu(1-\gamma_5)b][\bar{\mu}(C_V^\mu\gamma_\mu - C_A^\mu\gamma_\mu\gamma_5)\mu], \tag{28}$$

where C_V^μ and C_A^μ are the vector and axial vector $Z\mu^+\mu^-$ couplings, which are given as

$$C_V^\mu = -\tfrac{1}{2} + 2\sin^2\theta_W, \qquad C_A^\mu = -\tfrac{1}{2}. \tag{29}$$

Since, the structure of the effective Hamiltonian (28) in this model is the same form as that of the SM, like $\sim(V-A)\times(V-A)$ form, therefore its effect on the various decay observables can be encoded by replacing the SM Wilson coefficients C_9 and C_{10} by

$$C_9^{\text{eff}} = C_9 + \frac{2\pi}{\alpha}\frac{U_{sb}C_V^\mu}{V_{tb}V_{ts}^*}, \qquad C_{10}^{\text{eff}} = C_{10} - \frac{2\pi}{\alpha}\frac{U_{sb}C_A^\mu}{V_{tb}V_{ts}^*}. \tag{30}$$

Thus, one can obtain the branching ratio including the NP contributions by substituting C_{10}^{eff} from (30) in (27). Now varying the value $|U_{sb}|$ between 0 and 1.5×10^{-3} and the phase ϕ_s between $(0° - 360°)$ the branching ratio for $B_s \to \mu^+\mu^-$ is shown in Fig. 4. From the figure one can conclude that the branching ratio of $B_s \to \mu^+\mu^-$ in this model can be significantly enhanced from its SM value. Observation of this mode in the upcoming experiments will provide additional constraints on the new physics parameters.

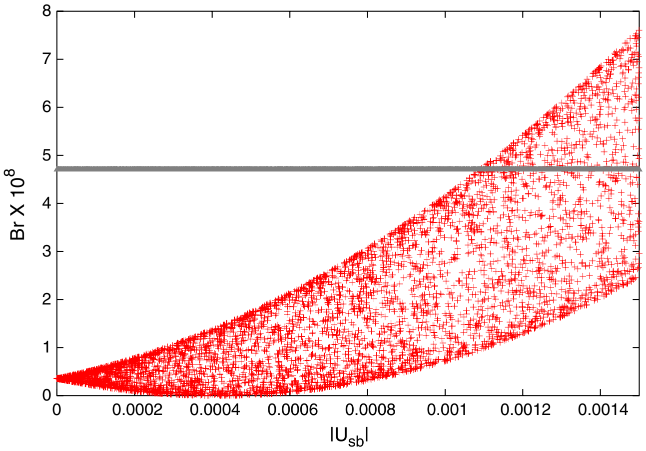

FIG. 4 (color online). The allowed range of the branching ratio for the $B_s \to \mu^+\mu^-$ process in the $\mathrm{Br} - |U_{sb}|$ plane. The horizontal line represents the experimental upper limit.

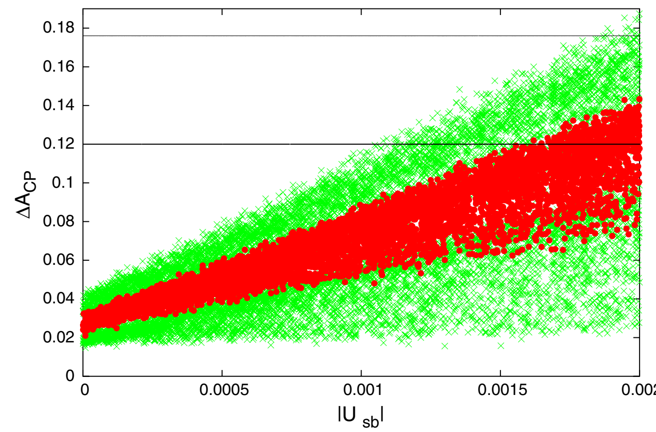

FIG. 5 (color online). The allowed range of the CP asymmetry difference (ΔA_{CP}) in the ($\Delta A_{CP} - |U_{sb}|$) plane as shown by the points. The 30% error bars are due to hadronic uncertainties and shown by × bands. The horizontal lines correspond to the experimentally allowed $1 - \sigma$ range.

V. $\Delta A_{CP}(K\pi)$ PUZZLE

The $\Delta A_{CP}(K\pi)$ puzzle refers to the difference in direct CP asymmetries in $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$ modes. These two modes receive similar dominating contributions from tree and QCD penguin diagrams and hence one would naively expect that these two channels will have the same direct CP asymmetries, i.e., $\mathcal{A}_{\pi^0 K^-} = \mathcal{A}_{\pi^+ K^-}$. In the QCD factorization approach, the difference between these asymmetries is found to be [22]

$$\Delta A_{CP} = \mathcal{A}_{K^-\pi^0} - \mathcal{A}_{K^-\pi^+} = (2.5 \pm 1.5)\% \tag{31}$$

whereas the corresponding experimental value is [3]

$$\Delta A_{CP} = (14.8 \pm 2.8)\%, \tag{32}$$

which yields nearly 4σ deviation.

In the SM, the relevant effective Hamiltonian describing the decay modes $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$ is given by

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}}\left[V_{ub}V_{us}^*(C_1 O_1 + C_2 O_2) - V_{tb}V_{ts}^* \sum_{i=3}^{10} C_i O_i\right], \tag{33}$$

where C_i's are the Wilson coefficients evaluated at the b quark mass scale and O_i's are the four-quark current operators.

Thus, one can obtain the transition amplitudes in the QCD factorization approach as [23], where the CKM unitarity $\lambda_u + \lambda_c + \lambda_t = 0$ has been used

$$\sqrt{2}A(B^- \to \pi^0 K^-) = \lambda_u(A_{\pi\bar{K}}(\alpha_1 + \beta_2) + A_{\bar{K}\pi}\alpha_2) + \sum_{q=u,c} \lambda_q \Big(A_{\pi\bar{K}}(\alpha_4^q + \alpha_{4,EW}^q + \beta_3^q + \beta_{3,EW}^q) + \frac{3}{2} A_{\bar{K}\pi}\alpha_{3,EW}^q \Big) \tag{34}$$

and

$$A(\bar{B}^0 \to \pi^+ K^-) = \lambda_u(A_{\pi\bar{K}}\alpha_1) + \sum_{q=u,c} \lambda_q A_{\pi\bar{K}} \Big(\alpha_4^q + \alpha_{4,EW}^q + \beta_3^q - \frac{1}{2}\beta_{3,EW}^q \Big), \tag{35}$$

where

$$A_{\pi\bar{K}} = i\frac{G_F}{\sqrt{2}} M_B^2 F_0^{B\to\pi}(0) f_K \quad \text{and} \quad A_{\bar{K}\pi} = i\frac{G_F}{\sqrt{2}} M_B^2 F_0^{B\to K}(0) f_\pi. \tag{36}$$

The parameters α_i's and β_i's are related to the Wilson coefficients C_i's and the corresponding expressions can be found in [23].

To account for this discrepancy here we consider the effect of the extra isosinglet down quark. As discussed earlier, in this model the Z mediated FCNC interaction is introduced at the tree level as shown in Eq. (9). Because of the new interactions the effective Hamiltonian describing the $b \to s\bar{s}s$ process receives the additional contribution given as [10],

$$\mathcal{H}_{\rm eff}^{Z} = -\frac{G_F}{\sqrt{2}}[\tilde{C}_3 O_3 + \tilde{C}_7 O_7 + \tilde{C}_9 O_9], \tag{37}$$

where the four-quark operators O_3, O_7, and O_9 have the same structure as the SM QCD and electroweak penguin operators and the new Wilson coefficients $\tilde{C}_i$'s at the M_Z scale are given by

$$\tilde{C}_3(M_Z) = \tfrac{1}{6}U_{sb}, \qquad \tilde{C}_7(M_Z) = \tfrac{2}{3}U_{sb}\sin^2\theta_W, \qquad \tilde{C}_9(M_Z) = -\tfrac{2}{3}U_{sb}(1 - \sin^2\theta_W). \tag{38}$$

These new Wilson coefficients will be evolved from the

M_Z scale to the m_b scale using the renormalization group equation given in [24], as

$$\vec{C}_i(m_b) = U_5(m_b, M_W, \alpha)\vec{C}(M_W), \tag{39}$$

where $\vec{C}$ is the 10×1 column vector of the Wilson coefficients and U_5 is the five flavor 10×10 evolution matrix. The explicit forms of $\vec{C}(M_W)$ and $U_5(m_b, M_W, \alpha)$ are given in [24], as described earlier. Because of the renormalization group evolution these three Wilson coefficients generate a new set of Wilson coefficients $\tilde{C}_i(i = 3, \cdots, 10)$ at the low energy regime (i.e., at the m_b scale) as presented in Table I, where we have used $\sin^2\theta_W = 0.231$.

As discussed earlier, due to the presence of the additional isosinglet down quark the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t = U_{sb}$. Thus, replacing $\lambda_t = U_{sb} - (\lambda_u + \lambda_c)$, one can write the transition amplitudes including the new contributions as

$$\begin{aligned}\sqrt{2}A(B^- \to \pi^0 K^-) = {}& \lambda_u(A_{\pi\bar{K}}(\alpha_1 + \beta_2) + A_{\bar{K}\pi}\alpha_2) \\ &+ \sum_{q=u,c} \lambda_q\Big(A_{\pi\bar{K}}(\alpha_4^q + \alpha_{4,EW}^q + \beta_3^q \\ &+ \beta_{3,EW}^q) + \frac{3}{2}A_{\bar{K}\pi}\alpha_{3,EW}^q\Big) \\ &- U_{sb}\Big(A_{\pi\bar{K}}(\Delta\alpha_4 + \Delta\alpha_{4,EW} + \Delta\beta_3 \\ &+ \Delta\beta_{3,EW}) + \frac{3}{2}A_{\bar{K}\pi}\Delta\alpha_{3,EW}\Big)\end{aligned} \tag{40}$$

and

$$\begin{aligned}A(\bar{B}^0 \to \pi^+ K^-) = {}& \lambda_u(A_{\pi\bar{K}}\alpha_1) + \sum_{q=u,c}\lambda_q A_{\pi\bar{K}}\Big(\alpha_4^q + \alpha_{4,EW}^q \\ &+ \beta_3^q - \frac{1}{2}\beta_{3,EW}^q\Big) - U_{sb}A_{\pi\bar{K}}\Big(\Delta\alpha_4 \\ &+ \Delta\alpha_{4,EW} + \Delta\beta_3 - \frac{1}{2}\Delta\beta_{3,EW}\Big),\end{aligned} \tag{41}$$

where $\Delta\alpha_i$'s and $\Delta\beta_i$'s are related to the modified Wilson coefficients $\Delta C_i = \tilde{C}_i(m_b) + C_i^t(m_b)$, where $C_i^t(m_b)$'s are the values of the Wilson coefficients at the m_b scale due to the t quark exchange.

Thus, including the new contributions, one can symbolically represent these amplitudes as

$$\text{Amp} = \lambda_u A_u + \lambda_c A_c - U_{sb}A_{\text{new}}. \tag{42}$$

λ's and U_{bs} contain the weak phase information and A_i's are associated with the strong phases. Thus one can explicitly separate the strong and weak phases and write the amplitudes as

$$\text{Amp} = \lambda_c A_c[1 + rae^{i(\delta_1-\gamma)} - r'be^{i(\delta_2+\phi_s)}], \tag{43}$$

where $a = |\lambda_u/\lambda_c|$, $b = |U_{sb}/\lambda_c|$, $-\gamma$ is the weak phase of V_{ub}, and ϕ_s is the weak phase of U_{sb}. $r = |A_u/A_c|$, $r' = |A_{\text{new}}/A_c|$, and δ_1 (δ_2) are the relative strong phases between A_u and A_c (A_{new} and A_c). Thus from the above amplitudes one can obtain the direct CP asymmetry parameter as

$$A_{CP} = \frac{2[ra\sin\delta_1\sin\gamma + r'b\sin\delta_2\sin\phi_s + rr'ab\sin(\delta_2-\delta_1)\sin(\gamma+\phi_s)]}{[\mathcal{R} + 2(ra\cos\delta_1\cos\gamma - 2r'b\cos\phi_s\cos\delta_2 - 2rr'ab\cos(\gamma+\phi_s)\cos(\delta_2-\delta_1))]} \tag{44}$$

where $\mathcal{R} = 1 + (ra)^2 + (r'b)^2$.

For numerical evaluation, we use input parameters as given in the S4 scenario of the QCD factorization approach. For the CKM matrix elements, we use the values from [25], extracted from direct measurements and $\gamma = (67^{+32}_{-25})^\circ$ [19]. The particle masses are taken from [25]. We vary the $|U_{bs}|$ in the range $0 \le |U_{sb}| \le 0.002$ and the corresponding phase between $30^\circ \le \phi_s \le 150^\circ$ and the allowed region in the ΔA_{CP} and $|U_{sb}|$ plane is shown in Fig. 2. From the figure it can be seen that the observed ΔA_{CP} can be accommodated in the vectorlike down quark model.

VI. $S_{\phi K_s}$

Next we consider the decay mode $\bar{B}^0 \to \phi K^0$. In the SM, it proceeds through the quark level transition $b \to s\bar{s}s$ and hence the mixing induced CP asymmetry in this mode ($S_{\phi K}$) is expected to give the same value as that of the $B \to J/\psi K_s$ with an uncertainty of around 5%. However, the present world average of this parameter is $S_{\phi K} = 0.44^{+0.17}_{-0.18}$ [3], which has nearly a 2.4σ deviation from the corresponding $S_{\psi K_s}$, with $S_{\phi K_s} < S_{\psi K_s}$. We would like to see whether the model with an extra vectorlike down quark can account for this discrepancy.

TABLE I. Values of the new Wilson coefficients at the m_b scale.

$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$	$\tilde{C}_7$	$\tilde{C}_8$	$\tilde{C}_9$	$\tilde{C}_{10}$
$0.19U_{sb}$	$-0.066U_{sb}$	$0.009U_{sb}$	$-0.031U_{sb}$	$0.145U_{sb}$	$0.053U_{sb}$	$-0.566U_{sb}$	$0.127U_{sb}$

In this model, one can write the amplitude for this process, analogous to $B \to \pi K$ processes, as

$$A(\bar{B}^0 \to \bar{K}^0 \phi) = A_{\bar{K}\phi}\Bigg[\sum_{q=u,c} \lambda_q\Big(\alpha_3^q + \alpha_4^q + \beta_3^q - \frac{1}{2}(\alpha_{3,EW}^q + \alpha_{4,EW}^q + \beta_{3,EW}^q)\Big) - U_{sb}\Big(\Delta\alpha_3 + \Delta\alpha_4 + \Delta\beta_3 - \frac{1}{2}(\Delta\alpha_{3,EW} + \Delta\alpha_{4,EW} + \Delta\beta_{3,EW})\Big)\Bigg], \quad (45)$$

with $A_{\bar{K}^0\phi} = -2m_\phi(\epsilon_\phi \cdot p_B)F_+^{B\to K}(0)f_\phi$, which again can be expressed as

$$A(\bar{B}^0 \to \bar{K}^0 \phi) = \lambda_u A'_u + \lambda_c A'_c - U_{sb}A'_{\rm new} = \lambda_c A'_c[1 + r_1 e^{i(\delta-\gamma)} - r'_1 b e^{i\phi_s} e^{i\delta'}], \quad (46)$$

where

$$r_1 = |A'_u/A'_c|, \qquad \delta = {\rm Arg}(A'_u/A'_c), \qquad r'_1 = |A'_{\rm new}/A'_c|, \qquad \delta' = {\rm Arg}(A'_{\rm new}/A'_c). \quad (47)$$

Thus, one can obtain the expression for mixing the induced CP asymmetry parameter as

$$S_{\phi K} = \frac{X}{\mathcal{R}' + 2r_1 a\cos\delta\cos\gamma - 2r'_1 b\cos\delta'\cos\phi_s - 2r_1 r'_1 ab\cos(\delta-\delta')\cos(\gamma+\phi_s)}, \quad (48)$$

where $\mathcal{R}' = 1 + (r_1 a)^2 + (r'_1 b)^2$ and

$$X = \sin 2\beta + 2r_1 a\cos\delta\sin(2\beta+\gamma) - 2r'_1 b\cos\delta'\sin(2\beta-\phi_s) + (r_1 a)^2\sin(2\beta+2\gamma) + (r'_1 b)^2\sin(2\beta-2\phi_s) - 2r_1 r'_1 ab\cos(\delta-\delta') \times \sin(2\beta+\gamma-\phi_s). \quad (49)$$

For numerical evaluation, we use the input parameters as given in the S4 scenario of QCD factorization. Using the CKM elements, as discussed earlier, along with $\beta = (21.1 \pm 0.9)°$ [3], the variation of $S_{\phi K}$ with ϕ_s for different values of $|U_{sb}|$ is shown in Fig. 6. From the figure it can be seen that the experimental value of $S_{\phi K}$ can be accommodated in this model.

VII. SUMMARY AND CONCLUSION

Recent results of $B_s - \bar{B}_s$ mixing have created a lot of attention in B decays and furthermore it is also claimed in the literature that it could be the first evidence of physics beyond the SM in the b sector. Of course, there are many candidates beyond the SM scenarios which can explain such a discrepancy, but here we will employ the model with an extended isosinglet down quark to study the same and explore whether other seemingly problematic deviations in the $b \to s$ sector, as indicated by the data at present, can also be explained simultaneously.

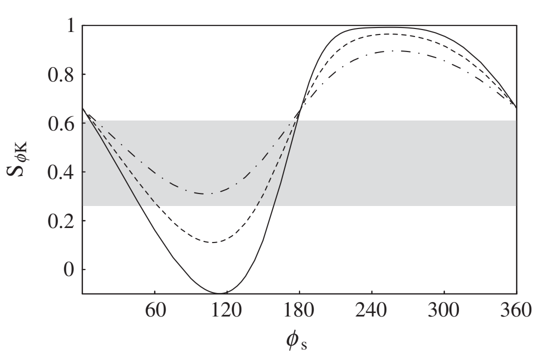

FIG. 6. The variation of $S_{\phi K}$ (in the S4 scenario) with the new weak phase ϕ_s, where the dot-dashed, short-dashed, and solid curves are for $|U_{sb}| = 0.001$, 0.0015, and 0.002. The horizontal band corresponds to experimental allowed 1σ range.

A minimal extension of the SM with only the addition of an extra isosinglet down quark in a vectorlike representation of the SM gauge group that induces FCNC couplings in the Z boson couplings. These models naturally arise for instance as the low energy limit of an E_6 grand unified theory. From the phenomenological point of view models with isosinglet quarks provide the simplest self-consistent framework to study deviations of 3×3 unitarity of the CKM matrix as well as flavor-changing neutral currents at the tree level.

As stated earlier, we impose the extended isosinglet down quark model to explain the deviation of $B_s - \bar{B}_s$ mixing from that of the SM expectation and obtained the constraints on the parameters of the new physics model and checked whether these severely constrained parameters still can explain other $b \to s$ processes, which appear to be not in agreement with the SM expectations.

Recently, CDF observed that the mixing induced parameter ($S_{\psi\phi}$) for the decay mode $B_s \to \psi\phi$ appears to be not in agreement with the SM expectation. In the SM, the value of $B_s \to \psi\phi$ is vanishingly small but the experiment has found a rather large value which might be an indication of new physics. We applied the constraints of the new physics model, obtained from the $B_s - \bar{B}_s$ mixing, to see whether one can explain the same. It can be seen from Fig. 3 that one can explain the discrepancy in the NP model under consideration.

Next we consider the decay mode $B_s \to \mu^+\mu^-$, which is believed to be a very clean mode and only the upper limit ($< 4.7\times 10^{-8}$) on its branching ratio has been obtained so far which is much larger than the SM value. We used the constraints of the isosinglet down quark model and see that

(Fig. 4) a huge enhancement can be possible due to its effect and can reach the upper limit obtained by the experiment.

Thereafter, we consider the πK puzzle, which is basically the difference of direct CP asymmetry parameters, represented by $\Delta A_{CP}(K\pi)$, of the modes $B^- \to \pi^0 K^-$ and $\bar{B}^0 \to \pi^+ K^-$. In the SM the value of $\Delta A_{CP}(K\pi)$ is expected to be close to zero whereas the experimental value is found to be around 15%. Invoking the new physics constraints, obtained before, we have shown that the observed asymmetry can be obtained in this scenario.

Finally, we consider the long standing problem of $S_{\phi K_s}$ corresponding to the decay mode $B \to \phi K_s$, which has about a 2.5 sigma deviation from that of the $S_{\psi K_s}$. This large deviation is believed to be due to the beyond the SM physics. We employed the NP model under consideration and found that it can easily explain such a discrepancy (Fig. 6).

To conclude, in this paper we employed the model with an extended isosinglet down quark to constrain the parameters of the model using the $B_s - \bar{B}_s$ mixing result. Thereafter, we checked whether deviations in other $b \to s$ modes, namely, $B_s \to \psi\phi$, $B_s \to \mu^+\mu^-$, $B \to \pi K$, and $B \to \phi K_s$ can also be understood in this model and found that the new physics parameters allowed by the $B_s - \bar{B}_s$ mixing result can explain these discrepancies successfully. With more data in the future we will have a better understanding of these problems and possibly we shall be able to ascertain the nature of the new physics or else rule out some of the existing beyond the SM scenarios, which appear to be allowed at present.

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