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Automatized full one-loop renormalization of the MSSM: The Higgs sector, the issue of $\tan\beta$ and gauge invariance

N. Baro,[1] F. Boudjema,[1] and A. Semenov[2]

[1]*LAPTH, Université de Savoie, CNRS, BP 110, F-74941 Annecy-le-Vieux Cedex, France*
[2]*Joint Institute of Nuclear Research, JINR, 141980 Dubna, Russia*

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We give an extensive description of the renormalization of the Higgs sector of the minimal supersymmetric model in SLOOPS. SLOOPS is an automatized code for the computation of one-loop processes in the MSSM. In this paper, the first in a series, we study in detail the nongauge invariance of some definitions of $\tan\beta$. We rely on a general nonlinear gauge-fixing constraint to make the gauge parameter dependence of different schemes for $\tan\beta$ explicit at one loop. In so doing, we update, within these general gauges, an important Ward-Slavnov-Taylor identity on the mixing between the pseudoscalar Higgs, A^0, and Z^0. We then compare the $\tan\beta$ scheme dependence of a few observables. We find that the best $\tan\beta$ scheme is the one based on the decay $A^0 \rightarrow \tau^+\tau^-$ because of its gauge invariance, because it is unambiguously defined from a physical observable, and because it is numerically stable. The oft used $\overline{\rm DR}$ scheme performs almost as well on the last count, but is usually defined from nongauge-invariant quantities in the Higgs sector. The use of the heavier scalar Higgs mass in *lieu* of $\tan\beta$, though related to a physical parameter, induces radiative corrections that are too large in many instances, and is therefore not recommended.

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I. INTRODUCTION

Were it not for the radiative corrections to the lightest Higgs mass [1], the minimal supersymmetric model or MSSM would have been a forgotten elegant model a long time ago. Indeed, at tree level the mass of the lightest Higgs is predicted to be less than the mass of the Z^0 boson, M_{Z^0}. That would have been a real pity from a model whose most appealing and foremost motivation was to solve the hierarchy problem and make the Higgs more natural, besides providing a very good dark matter candidate. The renormalization of the Higgs sector of the MSSM is therefore important. It is also important because it provides a link to the other parameters of the standard model, namely, all the masses of the particles. It also encodes another parameter that can describe the relative scale of the two vacuum expectation values needed for each Higgs doublet of the SM, often referred to as $\tan\beta$, and which permeates all the other sectors of the MSSM: the gaugino/Higgsino sector and the sfermion sector. Many renormalization schemes or definitions of this parameter are unsatisfactory, as we will see, mainly because they lack a direct physical interpretation or do not correspond to a physical and gauge independent parameter.

The aim of this paper is to give an extensive description of the renormalization of the Higgs sector in SLOOPS at one loop. SLOOPS is a fully automated code for the one-loop calculation of any cross section or decay in the MSSM at one loop. Although there have been a few studies of the renormalization of the Higgs sector (see [2,3] for a recent review of the Higgs in supersymmetry), some performed even beyond the one-loop approximation, especially concerning the mass of the lightest CP-even Higgs [4,5], looking at the problem afresh while keeping the issue of gauge invariance in mind will prove rewarding. Moreover, our motivation in developing SLOOPS was also to have a *full* one-loop renormalization of all the sectors of the MSSM in a coherent way; therefore, the study of the Higgs sector is a first step. We will point to the nongauge invariance of some definitions of $\tan\beta$. Although this has been known (see for example [6]) and pointed out at two loops in the usual linear gauge [7], most practitioners have kept the usage of some nongauge-invariant definitions of $\tan\beta$ because of their simplicity at the technical level, being based on definitions involving two-point-function self-energies. With the automatization of the loop calculations, considerations and definitions of $\tan\beta$ based on three-point functions (decays) are hardly more involved than those based solely on two-point functions describing self-energies, including transitions.

In the approach adopted within SLOOPS, we strive for an on-shell, OS, renormalization scheme, in particular, for $\tan\beta$. We rely on a general nonlinear gauge-fixing constraint to make the gauge parameter dependence of different schemes explicit for $\tan\beta$ at one loop. In so doing, we rederive and update the Ward-Slavnov-Taylor identity on the $A^0Z^0/H^\pm W^\pm$ mixing in the nonlinear gauge. We then compare qualitatively and quantitatively the $\tan\beta$ scheme dependence of a few observables. A^0 is the CP-odd Higgs scalar and $H^\pm$ are the charged Higgses. We find that the best $\tan\beta$ scheme is the one based on the decay $A^0 \rightarrow \tau^+\tau^-$ because of its gauge invariance, being unambigu-

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ously defined from a physical observable, and because it is numerically stable. The oft used $\overline{\rm DR}$ scheme performs almost as well on the last count, but is usually defined from nongauge-invariant quantities in the Higgs sector. The use of the heavier CP-even scalar Higgs mass in *lieu* of $\tan\beta$, though related to a physical parameter, induces radiative corrections that are too large in many instances and is therefore not acceptable. It has been argued that the definitions within the Higgs sector may be considered universal compared to a definition involving a particular Higgs decay, for example. However, as stressed in [8], staying within the confines of the Higgs sector and the Higgs potential, one faces the issue that many definitions may be basis dependent; as we will see, this will translate at one loop into issues about gauge invariance for these definitions. Concerning the application to the corrections to the lightest Higgs mass, our one-loop treatment is certainly not up to date; however, our motivation is to stress the gauge dependence issues and compare the impact of the scheme dependence for $\tan\beta$ for many observables, starting with those directly related to the properties of the Higgses of the MSSM, before reviewing in our forthcoming studies [9] the impact of $\tan\beta$ on observables in the chargino/neutralino as well as the sfermion sectors. We feel that this issue is of importance, as is a consistent one-loop OS implementation.

The present paper is organized as follows. In Sec. II we review the Higgs sector of the MSSM at tree level. This may, by now, be considered trivial but it is a necessary step before we embark on the renormalization procedure. We also detail this part in order to show what might qualify as a physical basis independent observable. Section III presents the nonlinear gauge-fixing condition that we use. This includes eight gauge-fixing parameters which are crucial in studying many issues related to gauge invariance that are not easily uncovered when one works within the usual linear gauge. Section IV constitutes the theoretical core of our analyses and deals with renormalization, introducing counterterms for the Lagrangian parameters and the field renormalization constants. We expose our renormaliszation conditions and update the Slavnov-Taylor identities involving the $A^0 - Z^0$ and $H^\pm - W^\pm$ transitions. Section V is devoted to defining $\tan\beta$. We consider a few schemes. Before turning to applications and numerical results, we briefly describe how our automatic code is set up in Sec. VI. In Sec. VII a numerical investigation of the scheme dependence and gauge dependence of these schemes is studied, taking as examples loop corrections to Higgs masses, and decays of the Higgses to fermions and to gauge and Higgs bosons. Section VIII gives our conclusions. The paper contains two appendixes. Appendix A details the derivation of Slavnov-Taylor identities for the $A^0 - Z^0$ transition. Field renormalization may be introduced at the level of the *unphysical* fields before rotation to the physical fields is performed; Appendix B relates these field renormalization constants on the Higgs fields to the one we introduce directly after the physical fields are defined. This may help in comparing different approaches in the literature.

To avoid clutter we use some abbreviations for the trigonometric functions. For example, for an angle θ, $\cos\theta$ will be abbreviated as c_θ, etc., so that we will from now on use t_β for $\tan\beta$.

II. THE HIGGS SECTOR OF THE MSSM AT TREE LEVEL

A. The Higgs potential

As is known (see, for instance, [3]), the MSSM requires two Higgs doublets H_1 and H_2 with opposite hypercharge. The Higgs potential in the MSSM is given by

$$\begin{aligned} V &= m_1^2|H_1|^2 + m_2^2|H_2|^2 + m_{12}^2(H_1 \wedge H_2 + \text{H.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 \\ &\quad \text{with} \quad H_1 \wedge H_2 = H_1^a H_2^b \epsilon_{ab} \\ &\quad (\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{ii} = 0). \end{aligned} \tag{2.1}$$

The mass terms are all soft masses, even if both m_1^2 and m_2^2 contain the supersymmetry (SUSY) preserving $|\mu|^2$-term which originates from the F-terms. g, g' are, respectively, the $SU(2)_W$ and $U(1)_Y$ gauge couplings. Decomposing each Higgs doublet field $H_{1,2}$ in terms of its components,

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} (v_1 + \phi_1^0 - i\varphi_1^0)/\sqrt{2} \\ -\phi_1^- \end{pmatrix}, \tag{2.2}$$

$$H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ (v_2 + \phi_2^0 + i\varphi_2^0)/\sqrt{2} \end{pmatrix}, \tag{2.3}$$

the tree-level Higgs potential can be written as

$$V = V_{\text{const}} + V_{\text{linear}} + V_{\text{mass}} + V_{\text{cubic}} + V_{\text{quartic}}, \tag{2.4}$$

where

$$\begin{aligned} V_{\text{linear}} &= \Big(m_1^2 v_1 + m_{12}^2 v_2 + \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)v_1\Big)\phi_1^0 \\ &\quad + \Big(m_2^2 v_2 + m_{12}^2 v_1 - \frac{g^2 + g'^2}{8}(v_1^2 - v_2^2)v_2\Big)\phi_2^0 \\ &\equiv T_{\phi_1^0}\phi_1^0 + T_{\phi_2^0}\phi_2^0 \end{aligned} \tag{2.5}$$

and

$$V_{\rm mass} = \frac{1}{2}(\varphi_1^0 \quad \varphi_2^0)\underbrace{\begin{pmatrix} m_1^2 + \frac{g^2+g'^2}{8}(v_1^2 - v_2^2) & m_{12}^2 \\ m_{12}^2 & m_2^2 - \frac{g^2+g'^2}{8}(v_1^2 - v_2^2)\end{pmatrix}}_{M^2_{\varphi^0}}\begin{pmatrix}\varphi_1^0 \\ \varphi_2^0\end{pmatrix}$$
$$+ \frac{1}{2}(\phi_1^0 \quad \phi_2^0)\underbrace{\left(M^2_{\varphi^0} + \frac{g^2+g'^2}{4}\begin{pmatrix} v_1^2 & -v_1v_2 \\ -v_1v_2 & v_2^2\end{pmatrix}\right)}_{M^2_{\phi^0}}\begin{pmatrix}\phi_1^0 \\ \phi_2^0\end{pmatrix}$$
$$+ (\phi_1^- \quad \phi_2^-)\underbrace{\left(M^2_{\varphi^0} + \frac{g^2}{4}\begin{pmatrix} v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2\end{pmatrix}\right)}_{M^2_{\phi^\pm}}\begin{pmatrix}\phi_1^+ \\ \phi_2^+\end{pmatrix}. \tag{2.6}$$

It is illuminating to express the mass matrices in terms of the tadpoles, especially for the pseudoscalar states

$$M^2_{\varphi^0} = \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2}\end{pmatrix} - \frac{m_{12}^2}{v_1v_2}N_{GP} \quad \text{with} \quad N_{GP} = \begin{pmatrix} v_2^2 & -v_1v_2 \\ -v_1v_2 & v_1^2\end{pmatrix}, \qquad M^2_{\phi^c} = \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2}\end{pmatrix} - \left(\frac{m_{12}^2}{v_1v_2} - \frac{g^2}{4}\right)N_{GP}. \tag{2.7}$$

The requirement that v_1 and v_2 correspond to the true vacua is a requirement on the vanishing of the tadpoles. The tadpoles, by the way, are also a trade-off for m_1^2 and m_2^2. Indeed, note that expressing everything in terms of $T_{\phi^0_{1,2}}$, all explicit dependence on m_1^2 and m_2^2 has disappeared, even in the scalar (*CP*-even) sector. Note that once the tadpole condition has been imposed,

$$T_{\phi^0_{1,2}} = 0, \tag{2.8}$$

we immediately find that, in both the charged sector and pseudoscalar sector, there is a Goldstone boson (i.e. a zero mass eigenvalue). This is immediate from the fact that

$$\det(N_{GP}) = 0. \tag{2.9}$$

The masses of the physical charged Higgs, $M_{H^\pm}$, and the pseudoscalar Higgs, M_{A^0}, are then just set from *the invariant* obtained from

$${\rm Tr}\,(N_{GP}) = v_1^2 + v_2^2 = v^2, \tag{2.10}$$

which is another way of seeing that the combination v is a proper "observable." Indeed, after gauging we will find that the masses of the weak gauge bosons are

$$M^2_{W^\pm} = \tfrac{1}{4}g^2v^2, \qquad M^2_{Z^0} = \tfrac{1}{4}(g^2 + g'^2)v^2. \tag{2.11}$$

Then

$$M^2_{A^0} = {\rm Tr}(M^2_{\varphi^0}) = -m_{12}^2\frac{v^2}{v_1v_2} = m_1^2 + m_2^2, \tag{2.12}$$

$$M^2_{H^\pm} = M^2_{A^0} + M^2_{W^\pm}. \tag{2.13}$$

In Eq. (2.12), the first equality does show an implicit dependence on the ratio of vacuum expectation values, or vev's (t_β), but not through $m_1^2 + m_2^2$. The latter must be basis independent, as is the *combination* m_{12}^2/v_1v_2. This is to be kept in mind.

It is also interesting to note that for the scalar Higgses, there is a simple sum rule that does not involve any ratio of vev's. Indeed, taking the trace of $M^2_{\phi^0}$ and calling the two physical *CP*-even Higgses h^0, with mass M_{h^0}, and H^0, with mass M_{H^0}, which would be obtained after rotation, we get the sum rule

$$M^2_{h^0} + M^2_{H^0} = M^2_{A^0} + M^2_{Z^0}. \tag{2.14}$$

h^0 will denote the lightest *CP*-even Higgs. Let us, as a *bookkeeping device*, introduce the angle β. At the moment, this is just to help keep easy notations:

$$c_\beta = \frac{v_1}{v}, \qquad s_\beta = \frac{v_2}{v} \quad \text{with} \quad v = \sqrt{v_1^2 + v_2^2}. \tag{2.15}$$

The determinant of the scalar Higgses, on the other hand, gives

$$M^2_{h^0}M^2_{H^0} = M^2_{A^0}M^2_{Z^0}c^2_{2\beta}. \tag{2.16}$$

This shows that if we take M_{H^0}, M_{A^0}, M_{Z^0} as input parameters, we first derive M_{h^0} from Eq. (2.14), and then $c^2_{2\beta}$ from Eq. (2.16). In general, with a set of input parameters M_{H^0}, M_{A^0}, M_{Z^0}, $c^2_{2\beta} \leq 1$ is not, however, guaranteed. We could, of course, fix $c^2_{2\beta}$ (t_β) and derive M_{H^0} *and* M_{h^0}, which is what is usually done.

The soft SUSY breaking mass parameters $m^2_{1,2,12}$ can be expressed in terms of the physical quantities M_{A^0}, M_{Z^0}, and c_β [as, for example, derived from Eqs. (2.14), (2.15), and (2.16)]:

$$m_1^2 = s_\beta^2 M_{A^0}^2 - \tfrac{1}{2} c_{2\beta} M_{Z^0}^2, \tag{2.17}$$

$$m_{12}^2 = -\tfrac{1}{2} s_{2\beta} M_{A^0}^2, \tag{2.18}$$

$$m_2^2 = c_\beta^2 M_{A^0}^2 + \tfrac{1}{2} c_{2\beta} M_{Z^0}^2. \tag{2.19}$$

B. Basis and rotations

So far the properties of the physical fields, like their masses, have been derived without reverting to a specific basis. The angle β defined in Eq. (2.15) was just a bookkeeping device. Still, to go from the fields at the Lagrangian level to the physical fields, one needs to perform a rotation. This should have no effect on physical observables. This naive observation is important, especially when we move to one loop. The rotations we will perform will get rid of field mixing. With the tadpole condition set to zero, it is clear that the pseudoscalar and charged scalar eigenstates are diagonalized through the same unitary matrix. At tree level this is defined precisely through the same angle β as in Eq. (2.15),

$$N_{GP} = U(-\beta)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} U(\beta), \qquad U(\beta) = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix},$$
$$U^\dagger(\beta) = U(-\beta). \tag{2.20}$$

Call $\mathcal{T}_v$ the tadpole matrix, defined as

$$\mathcal{T}_v = \begin{pmatrix} \frac{T_{\phi_1^0}}{v_1} & 0 \\ 0 & \frac{T_{\phi_2^0}}{v_2} \end{pmatrix}. \tag{2.21}$$

The tadpole is, of course, set to zero. But we will leave this zero there in the notation, as we will need this when we go to the one-loop counterterms. Then the mass matrices for the CP-even, CP-odd, and charged scalars can be written as

$$M_{\varphi^0}^2 = \mathcal{T}_v + M_{A^0}^2 N_{GP}, \tag{2.22}$$

$$M_{\phi^\pm}^2 = \mathcal{T}_v + (M_{A^0}^2 + M_{W^\pm}^2) N_{GP}, \tag{2.23}$$

$$M_{\phi^0}^2 = \mathcal{T}_v + M_{A^0}^2 N_{GP} + M_{Z^0}^2 U(\beta)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} U(-\beta). \tag{2.24}$$

The neutral Higgs is diagonalized through a rotation α such that

$$\begin{aligned} U(\alpha) M_{\phi^0}^2 U(-\alpha) &= \begin{pmatrix} M_{H^0}^2 & 0 \\ 0 & M_{h^0}^2 \end{pmatrix} \\ &= U(\alpha)\mathcal{T}_v U(-\alpha) + M_{A^0}^2 U(\alpha - \beta) \\ &\quad \times \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} U(\beta - \alpha) \\ &\quad + M_{Z^0}^2 U(\alpha + \beta)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \\ &\quad \times U(-(\alpha + \beta)). \end{aligned} \tag{2.25}$$

The diagonalization procedure also produces other, sometimes useful, constraints and relations:

$$M_{H^0}^2 = M_{A^0}^2 s_{\alpha-\beta}^2 + M_{Z^0}^2 c_{\alpha+\beta}^2, \tag{2.26}$$

$$M_{h^0}^2 = M_{A^0}^2 c_{\alpha-\beta}^2 + M_{Z^0}^2 s_{\alpha+\beta}^2, \tag{2.27}$$

$$M_{A^0}^2 s_{2(\alpha-\beta)} = M_{Z^0}^2 c_{2(\alpha+\beta)}, \tag{2.28}$$

$$t_{2\alpha} = t_{2\beta} \frac{M_{A^0}^2 + M_{Z^0}^2}{M_{A^0}^2 - M_{Z^0}^2}. \tag{2.29}$$

Note that in the decoupling limit, $M_{A^0} \gg M_{Z^0}$, one has in effect decoupled one of the Higgs doublets, and the other has the properties of the SM Higgs doublet. The decoupling parameter is also measured with the parameter $c_{\beta-\alpha} \to M_{Z^0}^2/M_{A^0}^2$ for $M_{A^0} \gg M_{Z^0}$.

Therefore, the mass eigenstates in the Higgs sector are given by

$$\begin{aligned} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} &= U(\beta)\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}, \\ \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} &= U(\beta)\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}, \\ \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} &= U(\alpha)\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} = \begin{pmatrix} c_\alpha & s_\alpha \\ -s_\alpha & c_\alpha \end{pmatrix}\begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \end{aligned} \tag{2.30}$$

C. Counting parameters

Before we embark on the technicalities of renormalization and the choice of judicious input parameters, it is best to review how to proceed, in general, and how to make contact with the renormalization of the SM. This will help clarify what the fundamental parameters are and which physical parameters can be used for a legitimate renormalization scheme. Moreover, since some observables *belong* to the SM, like the $W^\pm$, Z^0 masses and the electromagnetic coupling constant e which are used as physical input parameters in the OS scheme; isolating these three parameters means that their renormalization will proceed exactly as within the OS renormalization of the SM; see [10] for details.

In the SM, the fundamental parameters at the Lagrangian level for the gauge sector are g and g'. The Higgs potential with the Higgs doublet $\mathcal{H}$,

$$V(\mathcal{H}) = -\mu^2 \mathcal{H}^\dagger \mathcal{H} + \lambda(\mathcal{H}^\dagger \mathcal{H})^2 \quad \text{with}$$
$$|\langle 0|\mathcal{H}|0\rangle|^2 = \frac{v^2}{2} \neq 0, \tag{2.31}$$

furnishes the following: μ^2 (the "Higgs mass"), λ (the Higgs self-coupling), and v (the vacuum expectation value). We thus have at Lagrangian level, five parameters between the Higgs sector and the gauge sector. μ^2, λ, v are not all independent. v, the vev, is defined as the minimum of the potential; this is equivalent to requiring no tadpoles. The no-tadpole requirement amounts to no terms linear in the scalar Higgs. With the tadpole defined as T, we have at tree level

$$T = v(\mu^2 - \lambda v^2) \to 0. \tag{2.32}$$

This requirement is to be carried to any loop level. Out of this constraint, the five physical parameters in the OS scheme are e, $M_{W^\pm}$, M_{Z^0}, M_{H^0}, T. At all orders one defines $c_W = M_{W^\pm}/M_{Z^0}$. The latter is not an independent physical parameter. Therefore, in the SM a one-to-one mapping between the physical set e, $M_{W^\pm}$, M_{Z^0}, M_{H^0}, T and the Lagrangian parameters g, g', v, μ, λ is made.

In the MSSM, the Higgs sector furnishes m_1^2, m_2^2, m_{12}^2 (the Higgs doublets soft masses) and v_1, v_2 (the vev's of the Higgs doublets). The gauge sector is still governed by the $U(1)_Y$ and $SU(2)_W$ gauge couplings g, g'. The requirement of no tadpoles from both Higgs doublets, and hence any linear combination of them, is also a strong constraint. From these seven parameters in all, the physical parameters are usually split between the SM physical on-shell parameters

$$e,\ M_{W^\pm},\ M_{Z^0}, \tag{2.33}$$

which are a trade-off for g, g', $v^2 = v_1^2 + v_2^2$, and the MSSM Higgs parameters

$$M_{A^0},\ T_{\phi_1^0},\ T_{\phi_2^0};\ \text{“}t_\beta\text{”}, \tag{2.34}$$

which are a trade-off for m_1^2, m_2^2, m_{12}^2, v_2/v_1. At tree level we can set $t_\beta = v_2/v_1$ but this is, as yet, not directly related to an observable. While v can directly be expressed as a physical gauge boson mass, the ratio v_2/v_1 within the Higgs sector does not have an immediate simple physical interpretation, hence the difficulty with this Lagrangian parameter. One possibility is to trade it with the mass of one of the CP-even neutral Higgs through Eq. (2.16).

III. NONLINEAR GAUGE FIXING

In SLOOPS we have generalized the usual 't Hooft linear gauge condition to a more general nonlinear gauge that involves, thanks to the extra scalars in the Higgs sector, eight extra parameters ($\tilde{\alpha}$, $\tilde{\beta}$, $\tilde{\delta}$, $\tilde{\omega}$, $\tilde{\kappa}$, $\tilde{\rho}$, $\tilde{\epsilon}$, $\tilde{\gamma}$). Such gauges within the standard model proved useful and powerful [10,11] to check the correctness of the calculation. We have also exploited these gauges in the one-loop calculation of $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \gamma\gamma, Z^0\gamma$ [12] and in corrections to the relic density in [13]. A seven-parameter nonlinear gauge-fixing Lagrangian based on the one we introduce here is used in [14]. We can extend this nonlinear gauge fixing so that the gauge-fixing function involves the sfermions also. We refrain, in this paper, from working through this generalization. We will take these gauge-fixing terms to be *renormalized*. In particular, the gauge functions involve the physical fields. Although this will not make all Green's functions finite, it is enough to make all S-matrix elements finite. The gauge fixing can be written

$$\mathcal{L}^{\rm GF} = -\frac{1}{\xi_W} F^+ F^- - \frac{1}{2\xi_Z}|F^Z|^2 - \frac{1}{2\xi_\gamma}|F^\gamma|^2, \tag{3.1}$$

where

$$\begin{aligned} F^+ &= \left(\partial_\mu - ie\tilde{\alpha}\gamma_\mu - ie\frac{c_W}{s_W}\tilde{\beta}Z_\mu\right)W^{\mu +} \\ &\quad + i\xi_W \frac{e}{2s_W}(v + \tilde{\delta}h^0 + \tilde{\omega}H^0 + i\tilde{\rho}A^0 + i\tilde{\kappa}G^0)G^+, \\ F^Z &= \partial_\mu Z^\mu + \xi_Z \frac{e}{s_{2W}}(v + \tilde{\epsilon}h^0 + \tilde{\gamma}H^0)G^0, \\ F^\gamma &= \partial_\mu \gamma^\mu. \end{aligned} \tag{3.2}$$

The ghost Lagrangian $\mathcal{L}^{\rm Gh}$ is derived by requiring that the full effective Lagrangian, $\mathcal{L}^Q$, be invariant under the Becchi-Rouet-Stora-Tyutin (BRST) transformation. As discussed in [10], this is a much more appropriate procedure than the usual Fadeev-Popov approach, especially when dealing with the quantum symmetries of the generalized nonlinear gauges we are using. $\delta_{\rm BRS}\mathcal{L}^Q = 0$ therefore implies $\delta_{\rm BRS}\mathcal{L}^{\rm GF} = -\delta_{\rm BRS}\mathcal{L}^{\rm Gh}$.

It is very useful to also introduce the auxiliary B-field formulation of the gauge-fixing Lagrangian $\mathcal{L}^{\rm GF}$, especially from the perspective of deriving some Ward identities. The gauge fixing can then be expressed as

$$\begin{aligned} \mathcal{L}^{\rm GF} &= \xi_W B^+ B^- + \frac{\xi_Z}{2}|B^Z|^2 + \frac{\xi_\gamma}{2}|B^\gamma|^2 + B^- F^+ \\ &\quad + B^+ F^- + B^Z F^Z + B^\gamma F^\gamma. \end{aligned} \tag{3.3}$$

From the equations of motion for the B fields, we recover the usual $\mathcal{L}^{\rm GF}$ together with the condition $B^i = -\frac{F^i}{\xi_i}$ ($\xi_i = \{\xi_W, \xi_Z, \xi_\gamma\}$). The antighost $\bar{c}^i$ is defined from the gauge-fixing functions; we write

$$\delta_{\rm BRS}\bar{c}^i = B^i. \tag{3.4}$$

Then the ghost Lagrangian can be written as

$$\begin{aligned} \mathcal{L}^{\rm Gh} &= -(\bar{c}^+ \delta_{\rm BRS}F^+ + \bar{c}^- \delta_{\rm BRS}F^- + \bar{c}^Z \delta_{\rm BRS}F^Z \\ &\quad + \bar{c}^\gamma \delta_{\rm BRS}F^\gamma)\delta_{\rm BRS}\tilde{\mathcal{L}}^{\rm Gh}. \end{aligned} \tag{3.5}$$

The Fadeev-Popov prescription is therefore readily recovered, $\mathcal{L}^{\rm FP}$, but only up to an overall function, $\delta_{\rm BRS}\tilde{\mathcal{L}}^{\rm Gh}$, which is BRST invariant. The latter is set to zero for one-loop calculations. Our code SLOOPS implements this prescription automatically, leading to the automatic generation of the whole set of Feynman rules for the ghost sector.

For all applications we set the Feynman parameters $\xi_{W,Z,\gamma}$ to 1. This allows one to use the minimum set of libraries for the tensor reduction. Indeed, $\xi_{W,Z,\gamma} \neq 1$ can generate high rank tensor loop functions that would take much time to reduce to the set of scalar functions.

It is important to stress, once more, that since we do not seek to have all Green's functions finite but only the S-matrix elements, we take the gauge-fixing Lagrangian as being renormalized.

Judicious choices of the nonlinear gauge parameters can lead to simplifications like the vanishing of certain vertices. For example, with $\tilde{\alpha} = 1$, the $W^{+\mu}G^-\gamma_\mu$ vertex cancels. More examples can be found in Appendix A for the vanishing of some ghost couplings to Higgses.

IV. RENORMALIZATION

Our renormalization procedure is within the spirit of the on-shell scheme, borrowing as much as possible from the programme carried strictly within the standard model in [10]. For the gauge sector and the fermion sector, besides the electromagnetic coupling which we fix from the Thomson limit, we therefore take the same set of physical input parameters, namely, the masses of the $W^\pm$ and Z^0 together with the masses of all the standard model fermions. To define the Higgs sector parameters, the set in Eq. (2.34) looks most appropriate were it not for the ill-defined t_β. Indeed, the mass of the pseudoscalar M_{A^0} within the MSSM with CP conservation is a physical parameter. As within the standard model, we also take the tadpole. For t_β the aim of this paper is to review, propose, and compare different schemes. Renormalization of these parameters would then lead to finite S-matrix elements. For the mass eigenstates and thus a proper identification of the physical particles that appear as external legs in our processes, field renormalization is needed. S-matrix elements obtained from these rescaled Green's functions will lead to external legs with unit residue and will avoid mixing. Therefore one also needs wave-function renormalization of the fields. Especially for the unphysical sector of the theory, the precise choice of the fields redefinition is not essential if one is only interested in S-matrix elements of physical processes. It has to be stressed that one can do without this if one is willing to include loop corrections on the external legs. In the MSSM and in the Higgs sector, in particular, mixing effects, especially at one loop, are a nuisance and have introduced some confusion especially in defining t_β with the help of wave-function renormalization constants or equivalently from two-point functions describing particle mixing. For the Higgs sector one needs to be wary of mixing of the Goldstones with the CP-odd Higgs or, almost equivalently, between the Z^0 and the CP-odd Higgs or the $W^\pm$ and the charged Higgs. These two-point functions are related through gauge invariance and impose strong constraints on the wave-function renormalization constants. We will derive Ward-Slavnov-Taylor identities relating these two-point functions, and hence their associated counterterms, before imposing any *ad-hoc* conditions.

A. Shifts in mass parameters and gauge couplings

All fields and parameters introduced so far are considered as bare parameters with the exception of the gauge-fixing Lagrangian which we choose to write in terms of *renormalized fields*. Care should then be exercised when we split the tree-level contributions and the counterterms. Shifts are then introduced for the Lagrangian parameters and the fields with the notation that a bare quantity is labeled as X_0. It will split in terms of renormalized quantities X and counterterms δX,

$$g_0 = g + \delta g, \qquad g'_0 \rightarrow g' + \delta g', \tag{4.1}$$

$$\begin{aligned} m_{i0}^2 &= m_i^2 + \delta m_i^2 \quad \text{for } i = 1, 2, \\ m_{120}^2 &= m_{12}^2 + \delta m_{12}^2, \end{aligned} \tag{4.2}$$

$$\begin{aligned} v_{i0} &= v_i - \delta v_i \quad \text{for } i = 1, 2 \\ &\text{hence } \frac{\delta t_\beta}{t_\beta} = \frac{\delta v_1}{v_1} - \frac{\delta v_2}{v_2}. \end{aligned} \tag{4.3}$$

In our approach the angles defining the rotation matrices, β and α in Eq. (2.30), are defined as *renormalized* quantities. For example, the relation between the Goldstone boson/pseudoscalar Higgs boson and the fields $\varphi_{1,2}^0$ is maintained at all orders. Indeed,

$$\begin{aligned} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 &= U(\beta)\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}_0 \quad \text{also implies} \\ \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} &= U(\beta)\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}. \end{aligned} \tag{4.4}$$

Since in our approach we will always perform a field renormalization, there is no need for inducing more shifts from $U(\alpha, \beta)$. Therefore $U(\alpha, \beta)$ is taken as renormalized. For example, if we perform a field renormalization in the $\varphi_{1,2}^0$ basis

$$\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}_0 = Z_{\varphi^0}\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix} = \begin{pmatrix} Z_{\varphi_1^0}^{1/2} & Z_{\varphi_1^0\varphi_2^0}^{1/2} \\ Z_{\varphi_2^0\varphi_1^0}^{1/2} & Z_{\varphi_2^0}^{1/2} \end{pmatrix}\begin{pmatrix} \varphi_1^0 \\ \varphi_2^0 \end{pmatrix}, \tag{4.5}$$

this will imply

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = U(\beta) Z_{\varphi^0} U(-\beta) \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = Z_P \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} Z^{1/2}_{G^0G^0} & Z^{1/2}_{G^0A^0} \\ Z^{1/2}_{A^0G^0} & Z^{1/2}_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix}. \tag{4.6}$$

For the field renormalization we can perform this either at the level of the φ_i^0, i.e. before any rotation on the field in the Lagrangian is made, through Z_{φ^0} as is done in [14–16], or in a much more efficient way directly in the basis G^0A^0 since the latter are directly related to our renormalization conditions on the physical fields as we will see later. For instance, there is no need for $Z_{G^0G^0}$ in our approach since we will not be dealing with Goldstone bosons in the external legs.

B. Tadpole terms

We start with the terms linear in the Higgs fields which will lead to renormalization of the tadpoles. With the tree-level condition on the tadpoles $T_{\phi_1^0} = T_{\phi_2^0} = 0$, field normalization, if it were performed, does not contribute; we therefore have

$$V_{\rm linear}|_0 = (\delta T_{\phi_1^0} \phi_1^0 + \delta T_{\phi_2^0} \phi_2^0), \tag{4.7}$$

with

$$\frac{\delta T_{\phi_1^0}}{v_1} = \frac{M^2_{Z^0}}{2} c_{2\beta} \frac{\delta g^2 + \delta g'^2}{g^2 + g'^2} + \delta m_1^2 + t_\beta \delta m_{12}^2 - \left(m_1^2 + \frac{M^2_{Z^0}}{2} c_{2\beta} + M^2_{Z^0} c^2_\beta\right) \frac{\delta v_1}{v_1} + \left(-m_{12}^2 + \frac{M^2_{Z^0}}{2} s_{2\beta}\right) t_\beta \frac{\delta v_2}{v_2}, \tag{4.8}$$

$$\frac{\delta T_{\phi_2^0}}{v_2} = \frac{\delta T_{\phi_1^0}}{v_1} (v_1 \leftrightarrow v_2, m_1 \leftrightarrow m_2). \tag{4.9}$$

The minimum condition requires that the one-loop tadpole contribution generated by one-loop diagrams, $T^{\rm loop}_{\phi_i^0}$, is canceled by the tadpole counterterm. This imposes

$$\delta T_{\phi_i^0} = -T^{\rm loop}_{\phi_i^0}. \tag{4.10}$$

$T^{\rm loop}_{\phi_i^0}$ is calculated from the one-loop tadpole amplitude for H^0, $T^{\rm loop}_{H^0}$ and h^0, $T^{\rm loop}_{H^0}$ by simply moving to the physical basis

$$\begin{pmatrix} T^{\rm loop}_{\phi_1^0} \\ T^{\rm loop}_{\phi_2^0} \end{pmatrix} = \begin{pmatrix} c_\alpha & -s_\alpha \\ s_\alpha & c_\alpha \end{pmatrix} \begin{pmatrix} T^{\rm loop}_{H^0} \\ T^{\rm loop}_{h^0} \end{pmatrix}. \tag{4.11}$$

C. Mass counterterms in the Higgs sector

We now move to the mass counterterms induced by shifts in the Lagrangian parameters. We need to consider all terms bilinear in the fields. From the bare matrices $M^2_{\varphi^0}$, $M^2_{\phi^\pm}$, and $M^2_{\phi^0}$ [Eqs. (2.6), (2.22), (2.23), and (2.24)], we find the corresponding counterterms in matrix form in the basis $\varphi^0_{1,2}$, $\phi^0_{1,2}$, and $\phi^\pm_{1,2}$,

$$\delta M^2_{\varphi^0} = \begin{pmatrix} \delta m_1^2 + \frac{1}{2} c_{2\beta} \delta M^2_{Z^0} - \frac{M^2_{Z^0}}{2} s^2_{2\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 \\ \delta m_{12}^2 & \delta m_2^2 - \frac{1}{2} c_{2\beta} \delta M^2_{Z^0} + \frac{M^2_{Z^0}}{2} s^2_{2\beta} \frac{\delta t_\beta}{t_\beta} \end{pmatrix},$$

$$\delta M^2_{\phi^\pm} = \begin{pmatrix} \delta m_1^2 + \frac{1}{2} c_{2\beta} \delta M^2_{Z^0} + s^2_\beta \delta M^2_{W^\pm} - \frac{M^2_{Z^0}}{2} s^2_{2\beta} s^2_W \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M^2_{W^\pm} - \frac{M^2_{W^\pm}}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} \\ \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M^2_{W^\pm} - \frac{M^2_{W^\pm}}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_2^2 - \frac{1}{2} c_{2\beta} \delta M^2_{Z^0} + c^2_\beta \delta M^2_{W^\pm} + \frac{M^2_{Z^0}}{2} s^2_{2\beta} s^2_W \frac{\delta t_\beta}{t_\beta} \end{pmatrix},$$

$$\delta M^2_{\phi^0} = \begin{pmatrix} \delta m_1^2 + \frac{1}{2}(4c^2_\beta - 1) \delta M^2_{Z^0} - M^2_{Z^0} s^2_{2\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M^2_{Z^0} - \frac{M^2_{Z^0}}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} \\ \delta m_{12}^2 - \frac{1}{2} s_{2\beta} \delta M^2_{Z^0} - \frac{M^2_{Z^0}}{4} s_{4\beta} \frac{\delta t_\beta}{t_\beta} & \delta m_2^2 + \frac{1}{2}(4s^2_\beta - 1) \delta M^2_{Z^0} + M^2_{Z^0} s^2_{2\beta} \frac{\delta t_\beta}{t_\beta} \end{pmatrix}.$$

It is then straightforward to move to the physical fields through the rotation matrices $U(\alpha)$ and $U(\beta)$, to find the mass counterterms $\delta M^2_{A^0}$, $\delta M^2_{H^\pm}$, $\delta M^2_{h^0}$, $\delta M^2_{H^0}$ for, respectively, the pseudoscalar Higgs A^0, the charged Higgs $H^\pm$, and the two CP-even Higgses h^0, H^0. A mass mixing between these two Higgses, $\delta M^2_{H^0h^0}$, is also induced

$$\begin{aligned}
\delta M_{A^0}^2 &= s_\beta^2 \delta m_1^2 + c_\beta^2 \delta m_2^2 - s_{2\beta}\delta m_{12}^2 - \frac{1}{2}c_{2\beta}^2 \delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}^2 c_{2\beta}\frac{\delta t_\beta}{t_\beta},\\
\delta M_{H^\pm}^2 &= \delta M_{A^0}^2 + \delta M_{W^\pm}^2,\\
\delta M_{H^0}^2 &= c_\alpha^2 \delta m_1^2 + s_\alpha^2 \delta m_2^2 + s_{2\alpha}\delta m_{12}^2 + \frac{1}{2}(4(c_\alpha^2 c_\beta^2 + s_\alpha^2 s_\beta^2 - c_\alpha s_\alpha c_\beta s_\beta) - 1)\delta M_{Z^0}^2 - \frac{M_{Z^0}^2}{2} s_{2\beta}(2c_{2\alpha}s_{2\beta} + s_{2\alpha}c_{2\beta})\frac{\delta t_\beta}{t_\beta},\\
\delta M_{h^0}^2 &= s_\alpha^2 \delta m_1^2 + c_\alpha^2 \delta m_2^2 - s_{2\alpha}\delta m_{12}^2 + \frac{1}{2}(4(c_\alpha^2 s_\beta^2 + s_\alpha^2 c_\beta^2 + c_\alpha s_\alpha c_\beta s_\beta) - 1)\delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}(2c_{2\alpha}s_{2\beta} + s_{2\alpha}c_{2\beta})\frac{\delta t_\beta}{t_\beta},\\
\delta M_{H^0 h^0}^2 &= c_{2\alpha}\delta m_{12}^2 + \frac{1}{2}s_{2\alpha}(\delta m_2^2 - \delta m_1^2) - \frac{1}{2}(2s_{2\alpha}c_{2\beta} + s_{2\beta}c_{2\alpha})\delta M_{Z^0}^2 + \frac{M_{Z^0}^2}{2} s_{2\beta}(2s_{2\alpha}s_{2\beta} - c_{2\alpha}c_{2\beta})\frac{\delta t_\beta}{t_\beta}.
\end{aligned} \tag{4.12}$$

A mass term seems to be induced for the Goldstone bosons as well as a mixing between the Goldstones and the corresponding CP-odd Higgs,

$$\delta M_{G^0}^2 = c_\beta^2 \delta m_1^2 + s_\beta^2 \delta m_2^2 + s_{2\beta}\delta m_{12}^2 + \frac{1}{2}c_{2\beta}^2 \delta M_{Z^0}^2 - \frac{1}{2}M_{Z^0}^2 s_{2\beta}^2 c_{2\beta}\frac{\delta t_\beta}{t_\beta}, \tag{4.13}$$

$$\delta M_{G^\pm}^2 = \delta M_{G^0}^2, \tag{4.14}$$

$$\delta M_{G^0A^0}^2 = c_{2\beta}\delta m_{12}^2 + c_\beta s_\beta(\delta m_2^2 - \delta m_1^2) - \frac{1}{2}c_{2\beta}s_{2\beta}\delta M_{Z^0}^2 + M_{Z^0}^2 s_{2\beta}^2 c_\beta s_\beta \frac{\delta t_\beta}{t_\beta}, \tag{4.15}$$

$$\delta M_{G^\pm H^\pm}^2 = \delta M_{G^0A^0}^2 - M_{W^\pm}^2 c_\beta s_\beta \frac{\delta t_\beta}{t_\beta}. \tag{4.16}$$

It is much more transparent to reexpress these mass counterterms by trading-off $\delta m_{1,2}$ and δm_{12} with our input parameters $\delta T_{\phi^0_{1,2}}$, $\delta M_{A^0}^2$, δt_β through

$$\begin{aligned}
\delta m_1^2 &= c_\beta^2(s_\beta^2 + 1)\frac{\delta T_{\phi_1^0}}{v_1} - c_\beta^2 s_\beta^2 \frac{\delta T_{\phi_2^0}}{v_2} + s_\beta^2 \delta M_{A^0}^2 - \frac{1}{2}c_{2\beta}\delta M_{Z^0}^2 + \frac{1}{2}s_{2\beta}^2(M_{A^0}^2 + M_{Z^0}^2)\frac{\delta t_\beta}{t_\beta},\\
\delta m_{12}^2 &= \frac{1}{2}s_{2\beta}\left(s_\beta^2 \frac{\delta T_{\phi_1^0}}{v_1} + c_\beta^2 \frac{\delta T_{\phi_2^0}}{v_2} - \delta M_{A^0}^2 - c_{2\beta}M_{A^0}^2 \frac{\delta t_\beta}{t_\beta}\right),\\
\delta m_2^2 &= -c_\beta^2 s_\beta^2 \frac{\delta T_{\phi_1^0}}{v_1} + s_\beta^2(c_\beta^2 + 1)\frac{\delta T_{\phi_2^0}}{v_2} + c_\beta^2 \delta M_{A^0}^2 + \frac{1}{2}c_{2\beta}\delta M_{Z^0}^2 - \frac{1}{2}s_{2\beta}^2(M_{A^0}^2 + M_{Z^0}^2)\frac{\delta t_\beta}{t_\beta}.
\end{aligned} \tag{4.17}$$

In terms of $\delta T_{\phi^0_{1,2}}$, $\delta M_{A^0}^2$, δt_β, the mass counterterms of Eqs. (4.12) and (4.16) can be written as

$$\begin{aligned}
\delta M_{G^0}^2 &= \delta M_{G^\pm}^2 = \frac{1}{v}(c_{\alpha-\beta}\delta T_{H^0} - s_{\alpha-\beta}\delta T_{h^0}),\\
\delta M_{G^0A^0}^2 &= \frac{1}{v}(s_{\alpha-\beta}\delta T_{H^0} + c_{\alpha-\beta}\delta T_{h^0}) - s_{2\beta}\frac{M_{A^0}^2}{2}\frac{\delta t_\beta}{t_\beta},\\
\delta M_{G^\pm H^\pm}^2 &= \frac{1}{v}(s_{\alpha-\beta}\delta T_{H^0} + c_{\alpha-\beta}\delta T_{h^0}) - s_{2\beta}\frac{M_{H^\pm}^2}{2}\frac{\delta t_\beta}{t_\beta},\\
\delta M_{H^\pm}^2 &= \delta M_{A^0}^2 + \delta M_{W^\pm}^2,\\
\delta M_{h^0}^2 &= -\frac{1}{v}(c_{\alpha-\beta}s_{\alpha-\beta}^2 \delta T_{H^0} + s_{\alpha-\beta}(1 + c_{\alpha-\beta}^2)\delta T_{h^0}) + c_{\alpha-\beta}^2 \delta M_{A^0}^2 + s_{\alpha+\beta}^2 \delta M_{Z^0}^2 + s_{2\beta}s_{2(\alpha+\beta)}M_{Z^0}^2 \frac{\delta t_\beta}{t_\beta},\\
\delta M_{H^0}^2 &= \frac{1}{v}(c_{\alpha-\beta}(1 + s_{\alpha-\beta}^2)\delta T_{H^0} + s_{\alpha-\beta}c_{\alpha-\beta}^2 \delta T_{h^0}) + s_{\alpha-\beta}^2 \delta M_{A^0}^2 + c_{\alpha+\beta}^2 \delta M_{Z^0}^2 - s_{2\beta}s_{2(\alpha+\beta)}M_{Z^0}^2 \frac{\delta t_\beta}{t_\beta},\\
\delta M_{H^0h^0} &= -\frac{1}{v}s_{\alpha-\beta}^3 \delta T_{H^0} + \frac{1}{v}c_{\alpha-\beta}^3 \delta T_{h^0} + \frac{1}{2}s_{2(\alpha-\beta)}\delta M_{A^0}^2 - \frac{1}{2}s_{2(\alpha+\beta)}\delta M_{Z^0}^2 - \frac{s_{2\beta}}{2}(M_{A^0}^2 c_{2(\alpha-\beta)} + M_{Z^0}^2 c_{2(\alpha+\beta)})\frac{\delta t_\beta}{t_\beta}.
\end{aligned} \tag{4.18}$$

It is very satisfying to see that $\delta M^2_{G^0} = \delta M^2_{G^\pm}$ is accounted for totally by the tadpole counterterms.

D. Field renormalization

We can now introduce field renormalization at the level of the physical fields without the need to first go through field renormalization in the basis $\phi^0_{1,2}$, $\varphi^0_{1,2}$, $\phi^\pm_{1,2}$. In most generality we can write, as in Eq. (4.6),

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix}_0 = Z_P \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \equiv \begin{pmatrix} Z^{1/2}_{G^0} & Z^{1/2}_{G^0A^0} \\ Z^{1/2}_{A^0G^0} & Z^{1/2}_{A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix},$$
$$\begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix}_0 = Z_C \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} \equiv \begin{pmatrix} Z^{1/2}_{G^\pm} & Z^{1/2}_{G^\pm H^\pm} \\ Z^{1/2}_{H^\pm G^\pm} & Z^{1/2}_{H^\pm} \end{pmatrix} \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix},$$
$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix}_0 = Z_S \begin{pmatrix} H^0 \\ h^0 \end{pmatrix} \equiv \begin{pmatrix} Z^{1/2}_{H^0} & Z^{1/2}_{H^0h^0} \\ Z^{1/2}_{h^0H^0} & Z^{1/2}_{h^0} \end{pmatrix} \begin{pmatrix} H^0 \\ h^0 \end{pmatrix}. \quad (4.19)$$

It is always possible to move to another basis through Eq. (4.6). Field renormalization will help get rid of mixing between physical fields when these are on shell and set the residue to 1.

E. Self-energies in the Higgs sector

Collecting the contribution of all the counterterms, including wave-function renormalization, the renormalized self-energies can be written as

$$\begin{aligned}
\hat{\Sigma}_{G^0G^0}(q^2) &= \Sigma_{G^0G^0}(q^2) + \delta M^2_{G^0} - q^2\delta Z_{G^0},\\
\hat{\Sigma}_{G^0A^0}(q^2) &= \Sigma_{G^0A^0}(q^2) + \delta M^2_{G^0A^0} - \frac{1}{2}q^2\delta Z_{G^0A^0}\\
&\quad - \frac{1}{2}(q^2 - M^2_{A^0})\delta Z_{A^0G^0},\\
\hat{\Sigma}_{A^0A^0}(q^2) &= \Sigma_{A^0A^0}(q^2) + \delta M^2_{A^0} - (q^2 - M^2_{A^0})\delta Z_{A^0},\\
\hat{\Sigma}_{G^\pm G^\pm}(q^2) &= \Sigma_{G^\pm G^\pm}(q^2) + \delta M^2_{G^\pm} - q^2\delta Z_{G^\pm},\\
\hat{\Sigma}_{G^\pm H^\pm}(q^2) &= \Sigma_{G^\pm H^\pm}(q^2) + \delta M^2_{G^\pm H^\pm} - \frac{1}{2}q^2\delta Z_{G^\pm H^\pm}\\
&\quad - \frac{1}{2}(q^2 - M^2_{H^\pm})\delta Z_{H^\pm G^\pm},\\
\hat{\Sigma}_{H^\pm H^\pm}(q^2) &= \Sigma_{H^\pm H^\pm}(q^2) + \delta M^2_{H^\pm} - (q^2 - M^2_{H^\pm})\delta Z_{H^\pm},\\
\hat{\Sigma}_{H^0H^0}(q^2) &= \Sigma_{H^0H^0}(q^2) + \delta M^2_{H^0} - (q^2 - M^2_{H^0})\delta Z_{H^0},\\
\hat{\Sigma}_{H^0h^0}(q^2) &= \Sigma_{H^0h^0}(q^2) + \delta M^2_{H^0h^0} - \frac{1}{2}(q^2 - M^2_{H^0})\delta Z_{H^0h^0}\\
&\quad - \frac{1}{2}(q^2 - M^2_{h^0})\delta Z_{h^0H^0},\\
\hat{\Sigma}_{h^0h^0}(q^2) &= \Sigma_{h^0h^0}(q^2) + \delta M^2_{h^0} - (q^2 - M^2_{h^0})\delta Z_{h^0}.
\end{aligned}$$

Note that, as we stressed all along, since we are only interested in having finite S-matrix transitions and not finite Green's functions, there is no need to try to make all two-point functions finite. For instance, the diagonal Goldstone self-energies $\hat{\Sigma}_{G^0G^0}(q^2)$ and $\hat{\Sigma}_{G^\pm G^\pm}(q^2)$ do not need any field renormalization. Therefore we can set, for example, $\delta Z_{G^0} = \delta Z_{G^\pm} = 0$ for simplicity. $\delta Z_{A^0G^0}$ is also not needed as it is only involved in the transition of the Goldstone boson to the pseudoscalar Higgs.

F. A^0Z^0 and $H^\pm W^\pm$ transitions

The (massive) gauge bosons and the pseudoscalar mix. This originates from the same part of the gauge Lagrangian where the gauge bosons, at tree level, mix with the Goldstone bosons as in the standard model; see for example [10]. The latter is eliminated through the usual 't Hooft gauge fixing. To wit, from

$$\begin{aligned}
\mathcal{L}^{GV}_0 &= \frac{g}{2}i(v_1\partial^\mu\phi^-_1 + v_2\partial^\mu\phi^-_2)W^+_\mu + \text{H.c.}\\
&\quad - \frac{g}{2c_W}(v_1\partial^\mu\varphi^0_1 + v_2\partial^\mu\varphi^0_2)Z^0_\mu|_0,
\end{aligned} \quad (4.20)$$

we end up with

$$\begin{aligned}
\mathcal{L}^{GV}_0 &= \mathcal{L}^{GV} + \frac{1}{2}\left(\delta Z_{G^\pm} + \delta Z_{W^\pm} + \frac{\delta M^2_{W^\pm}}{M^2_{W^\pm}}\right)\\
&\quad \times (iM_{W^\pm}\partial^\mu G^- W^+_\mu + \text{H.c.})\\
&\quad - \frac{1}{2}\left(\delta Z_{G^0} + \delta Z_{Z^0Z^0} + \frac{\delta M^2_{Z^0}}{M^2_{Z^0}}\right)M_{Z^0}\partial^\mu G^0 Z^0_\mu\\
&\quad - \frac{1}{2}\delta Z_{Z^0\gamma}M_{Z^0}\partial^\mu G^0\gamma_\mu + \frac{1}{2}\left(\delta Z_{G^\pm H^\pm} + s_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)\\
&\quad \times (iM_{W^\pm}\partial^\mu H^- W^+_\mu + \text{H.c.})\\
&\quad - \frac{1}{2}\left(\delta Z_{G^0A^0} + s_{2\beta}\frac{\delta t_\beta}{t_\beta}\right)M_{Z^0}\partial^\mu A^0 Z^0_\mu.
\end{aligned} \quad (4.21)$$

For the sake of completeness, we have also kept in Eq. (4.21) the wave-function renormalization constants of the gauge bosons, namely, $\delta Z_{W^\pm}$, $\delta Z_{Z^0Z^0}$, and $\delta Z_{Z^0\gamma}$ (for the $Z^0 \to \gamma$ transition); see [10]. The conditions on the latter are the same as in the standard model; details are found in [10].

The novelty, however, is that now we have $A^0 - Z^0$ and $H^\pm - W^\pm$ transitions whose self-energies can be written as

$$\hat{\Sigma}_{A^0Z^0}(q^2) = \Sigma_{A^0Z^0}(q^2) + \frac{M_{Z^0}}{2}\left(\delta Z_{G^0A^0} + s_{2\beta}\frac{\delta t_\beta}{t_\beta}\right), \quad (4.22)$$

$$\hat{\Sigma}_{H^\pm W^\pm}(q^2) = \Sigma_{H^\pm W^\pm}(q^2) + \frac{M_{W^\pm}}{2}\left(\delta Z_{G^\pm H^\pm} + s_{2\beta}\frac{\delta t_\beta}{t_\beta}\right). \quad (4.23)$$

Note that apart from δt_β the same counterterm $\delta Z_{G^0A^0}$ appears in the G^0A^0 transition. In fact, there is a Ward identity relating these two transitions. Contrary to what one might see in some papers [17–19], the relation is much more complicated for $q^2 \neq M^2_{A^0}$ and gets more subtle in the case of the nonlinear gauge. This identity is very important especially since in many approaches the transition has been used as a *definition* for δt_β. The identity can be most easily derived by considering the BRST transformation on the ("ghost") operator $\langle 0|\bar{c}^Z(x)A^0(y)|0\rangle = 0$. Full details are given in Appendix A. We have the constraint

$$\begin{aligned} &q^2\hat{\Sigma}_{A^0Z^0}(q^2) + M_{Z^0}\hat{\Sigma}_{A^0G^0}(q^2) \\ &\quad = (q^2 - M^2_{Z^0})\frac{1}{(4\pi)^2}\frac{e^2 M_{Z^0}}{s^2_{2W}} s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2) + \frac{M_{Z^0}}{2}(q^2 \\ &\qquad - M^2_{A^0})\bigg(\frac{1}{(4\pi)^2}\frac{2e^2}{s^2_{2W}}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(q^2) + s_{2\beta}\frac{\delta t_\beta}{t_\beta} - \delta Z_{A^0G^0}\bigg). \end{aligned} \tag{4.24}$$

$\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2)$ and $\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(q^2)$ are functions defined in Appendix A. They vanish in the linear gauge with $\tilde{\epsilon} = \tilde{\gamma} = 0$. The constraint shows that even in the linear gauge $q^2\hat{\Sigma}_{A^0Z^0}(q^2) + M_{Z^0}\hat{\Sigma}_{A^0G^0}(q^2)$ is zero only for $q^2 = M^2_{A^0}$ and not for *any* q^2. We will get back to the exploitation of this constraint later. A similar constraint also relates $\hat{\Sigma}_{H^\pm W^\pm}(q^2)$ and $\hat{\Sigma}_{G^\pm H^\pm}(q^2)$,

$$\begin{aligned} &q^2\hat{\Sigma}_{H^\pm W^\pm}(q^2) + M_{W^\pm}\hat{\Sigma}_{H^\pm G^\pm}(q^2) \\ &\quad = (q^2 - M^2_{W^\pm})\frac{1}{(4\pi)^2}\frac{e^2 M_{W^\pm}}{s^2_{2W}}\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(q^2) \\ &\qquad + \frac{M_{W^\pm}}{2}(q^2 - M^2_{H^\pm})\bigg(\frac{1}{(4\pi)^2}\frac{2e^2}{s^2_{2W}}\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{cc}(q^2) \\ &\qquad + s_{2\beta}\frac{\delta t_\beta}{t_\beta} - \delta Z_{H^\pm G^\pm}\bigg). \end{aligned}$$

$\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(q^2)$ and $\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{cc}(q^2)$ are defined in Eq. (A26); see Appendix A.

G. Renormalization conditions

1. Pole masses, residues, and mixing

Masses are defined as pole masses from the propagator. Moreover, this propagator must have residue 1 at the pole mass. In the case of particle mixing, the mixing must vanish at the pole mass of any physical particle. In general, in the case of mixing this requires solving a system of an inverse propagator matrix with solutions given by the pole masses. For a two-particle mixing one has to deal with the determinant of a 2×2 matrix which is a quadratic form in the self-energies whose solutions are the corrected masses. The equation reads

$$\begin{aligned} &[(q^2 - M^2_{h^0,\text{tree}} - \hat{\Sigma}_{h^0h^0}(q^2))(q^2 - M^2_{H^0,\text{tree}} - \hat{\Sigma}_{H^0H^0}(q^2)) \\ &\qquad - (\hat{\Sigma}_{h^0H^0}(q^2))^2] = 0. \end{aligned} \tag{4.25}$$

$M_{h^0,\text{tree}}$ refers to the tree-level mass. This equation simplifies considerably at one loop since one only has to keep the linear term, or first order in the loop expansion, in the equation. In principle, the argument that appears in the self-energy two-point functions is the pole mass which might get a correction from its value at tree level. To get the corrections one can proceed through iteration, starting from the tree-level masses as an argument of the two-point function. Higher order terms in the expansion will appear as higher orders in the loop expansion, and we do not count them as being part of the one-loop correction. A genuine one-loop correction results for the pole mass, $M_{i,1\text{ loop}}$, starting from a tree-level mass $M_{i,\text{tree}}$ with $\hat{\Sigma}_{ii}(q^2)$ the diagonal renormalized self-energy. Therefore the solution of

$$q^2 - M^2_{i,\text{tree}} - \text{Re}\hat{\Sigma}_{ii}(q^2) = 0 \quad \text{at} \quad q^2 = M^2_{i,1\text{ loop}}, \tag{4.26}$$

which in the one-loop approximation means

$$\begin{aligned} M^2_{i,1\text{ loop}} &= M^2_{i,\text{tree}} + \text{Re}\hat{\Sigma}_{ii}(M^2_{i,\text{tree}}) \\ &= M^2_{i,\text{tree}} + \delta M^2_{ii} + \text{Re}\Sigma_{ii}(M^2_{i,\text{tree}}). \end{aligned} \tag{4.27}$$

The latter condition will constrain the Lagrangian parameters, with δM^2_{ii} a gauge-invariant quantity. Likewise, at one loop, the requirement of a residue equal to 1, for the diagonal propagator and vanishing mixing when the physical particle is on shell, leads to

$$\begin{aligned} &\text{Re}\,\hat{\Sigma}'_{ii}(M^2_{i,\text{tree}}) = 0 \quad \text{with} \quad \frac{\partial\hat{\Sigma}_{ii}(q^2)}{\partial q^2} = \hat{\Sigma}'_{ii}(q^2), \\ &\text{Re}\hat{\Sigma}'_{ij}(M^2_{i,\text{tree}}) = \text{Re}\hat{\Sigma}'_{ij}(M^2_{j,\text{tree}}) = 0, \qquad i \neq j. \end{aligned} \tag{4.28}$$

In our renormalization programme, Eqs. (4.28) set the field renormalization constants and avoid having to include corrections on the external legs. The field renormalization constants are therefore not necessarily gauge invariant nor gauge parameter independent.

2. Renormalization conditions and corrections on the mass parameters

As we have explained earlier, one needs to fix the counterterms for $\delta M^2_{A^0}$ and δt_β once tadpole renormalization has been carried through to arrive at finite and gauge-invariant S-matrix elements. Taking M_{A^0} as an input parameter means that its mass is fixed the same at all orders; we therefore set

$$\delta M^2_{A^0} = -\text{Re}\Sigma_{A^0A^0}(M^2_{A^0}). \tag{4.29}$$

Finding a condition to fix δt_β is an arduous task that has been debated for some time. We will study many schemes for δt_β in Sec. V.

The charged Higgs mass is independent of t_β; it gets a finite correction at one loop once M_{A^0} is used as an input parameter,

$$M^2_{H^\pm,1\,\mathrm{loop}} = M^2_{H^\pm,\mathrm{tree}} + \mathrm{Re}\Sigma_{H^\pm H^\pm}(M^2_{H^\pm,\mathrm{tree}}) - \mathrm{Re}\Sigma_{A^0A^0}(M^2_{A^0}) - \mathrm{Re}\Pi^T_{W^\pm}(M^2_{W^\pm}). \quad (4.30)$$

We have used $\delta M^2_{W^\pm} = \mathrm{Re}\Pi^T_{W^\pm}(M^2_{W^\pm})$, where $\Pi^T_{W^\pm}(q^2)$ is the transverse two-point function of the $W^\pm$ following the same implementation as performed in [10]. The finiteness of the corrected charged Higgs mass is the first nontrivial check on the code concerning the Higgs sector.

The sum rule involving the CP-even Higgs masses, Eq. (2.14), is also independent of t_β. This sum rule gets corrected at one loop,

$$\begin{aligned} M^2_{h^0,1\,\mathrm{loop}} + M^2_{H^0,1\,\mathrm{loop}} = {} & M^2_{A^0} + M^2_{Z^0} + \mathrm{Re}\Sigma_{h^0h^0}(M^2_{h^0}) \\ & + \mathrm{Re}\Sigma_{H^0H^0}(M^2_{H^0}) \\ & + \frac{g}{2M_{W^\pm}}(c_{\alpha-\beta}\delta T_{H^0} - s_{\alpha-\beta}\delta T_{h^0}) \\ & - \mathrm{Re}\Sigma_{A^0A^0}(M^2_{A^0}) - \mathrm{Re}\Pi^T_{Z^0Z^0}(M^2_{Z^0}). \end{aligned} \quad (4.31)$$

Here also we have used $\delta M^2_{Z^0} = \mathrm{Re}\Pi^T_{Z^0Z^0}(M^2_{Z^0})$, where $\Pi^T_{Z^0Z^0}(q^2)$ is the transverse two-point function of the Z^0 boson; see [10]. Otherwise, to predict $M^2_{h^0,1\,\mathrm{loop}}$ or $M^2_{H^0,1\,\mathrm{loop}}$ one needs a prescription on δt_β; see Eq. (4.18). Obviously fixing one of these masses, for instance M_{H^0} in particular in analogy with M_{A^0}, is a scheme for t_β. In this scheme, therefore, $\mathrm{Re}\hat{\Sigma}_{H^0H^0}(M^2_{H^0}) = 0$, which sets a gauge-invariant counterterm for t_β; see Eq. (5.13).

H. Constraining the field renormalization constants

We have introduced through the field renormalization matrices Z_P, Z_C, Z_S a total of 12 such constants; see Eq. (4.19). However, as argued repeatedly, some of these constants are only involved in the transition involving an external Goldstone bosons, i.e. in situations that do not correspond to a physical process. Therefore we can give the constants δZ_{G^0}, $\delta Z_{G^\pm}$, $\delta Z_{A^0G^0}$, $\delta Z_{H^\pm G^\pm}$ any value; S-matrix elements will not depend on these constants. It is therefore easiest to set these four constants to 0 in actual calculations and give them arbitrary values in preliminary tests of a calculation of a physical process.

For the transitions involving physical Higgs particles, we just go along the general lines described in Sec. IV G 1, in order to avoid loop corrections on the external legs. In the following, in order to avoid too much clutter the masses that will appear as arguments are the tree-level masses (or the input mass for M_{A^0}). The conditions read

$$\mathrm{Re}\,\hat{\Sigma}'_{A^0A^0}(M^2_{A^0}) = 0, \quad (4.32)$$

$$\mathrm{Re}\,\hat{\Sigma}'_{H^\pm H^\pm}(M^2_{H^\pm}) = 0, \quad (4.33)$$

$$\mathrm{Re}\,\hat{\Sigma}'_{H^0H^0}(M^2_{H^0}) = 0, \quad (4.34)$$

$$\mathrm{Re}\,\hat{\Sigma}'_{h^0h^0}(M^2_{h^0}) = 0, \quad (4.35)$$

$$\mathrm{Re}\,\hat{\Sigma}_{H^0h^0}(M^2_{H^0}) = 0, \quad \mathrm{Re}\hat{\Sigma}_{H^0h^0}(M^2_{h^0}) = 0. \quad (4.36)$$

From these we immediately derive six out of the eight field renormalization constants in the Higgs sector,

$$\delta Z_{A^0} = \mathrm{Re}\Sigma'_{A^0A^0}(M^2_{A^0}), \quad (4.37)$$

$$\delta Z_{H^\pm} = \mathrm{Re}\Sigma'_{H^\pm H^\pm}(M^2_{H^\pm}), \quad (4.38)$$

$$\delta Z_{H^0} = \mathrm{Re}\Sigma'_{H^0H^0}(M^2_{H^0}), \quad (4.39)$$

$$\delta Z_{h^0} = \mathrm{Re}\Sigma'_{h^0h^0}(M^2_{h^0}), \quad (4.40)$$

$$\delta Z_{h^0H^0} = 2\frac{\mathrm{Re}\Sigma_{H^0h^0}(M^2_{H^0}) + \delta M^2_{H^0h^0}}{M^2_{H^0} - M^2_{h^0}}, \quad (4.41)$$

$$\delta Z_{H^0h^0} = 2\frac{\mathrm{Re}\Sigma_{H^0h^0}(M^2_{h^0}) + \delta M^2_{H^0h^0}}{M^2_{h^0} - M^2_{H^0}}. \quad (4.42)$$

When considering a process with A^0 as an external leg,[1] in principle it involves the $A^0 \to A^0$ transition but also the $A^0 \to Z^0$ and the $A^0 \to G^0$ transitions. The field renormalization constant δZ_{A^0} [see Eq. (4.37)] allows one to set the $A^0 \to A^0$ transition to 0 and moves its effect to a vertex counterterm correction. One therefore would be tempted by setting $\hat{\Sigma}_{A^0Z^0}(M^2_{A^0}) = 0$ together with $\hat{\Sigma}_{A^0G^0}(M^2_{A^0}) = 0$, as is done almost everywhere in the literature. In our case this would mean that the remaining constant $\delta Z_{G^0A^0}$ could be derived equivalently from one of these conditions. However, the Ward identity we derived in Eq. (4.24) imposes a very important constraint. It shows that in a general nonlinear gauge we cannot impose *both* $\hat{\Sigma}_{A^0Z^0}(M^2_{A^0}) = 0$ and $\hat{\Sigma}_{A^0G^0}(M^2_{A^0}) = 0$. It seems, at first sight, that this requires that one introduces loop corrections on the external legs when considering, for example, processes with the pseudoscalar Higgs as an external leg. In the linear gauge, on the other hand, this is possible since $\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2) = 0$; we could then adjust $\delta Z_{G^0A^0}$ *and* $\delta Z_{A^0G^0}$ to have $\hat{\Sigma}_{A^0Z^0}(M^2_{A^0}) = 0$ *and* $\hat{\Sigma}_{A^0G^0}(M^2_{A^0}) = 0$. Note, however, that contrary to what we encounter in some publications (see for example [17,18]), $q^2\hat{\Sigma}_{A^0Z^0}(q^2) + M_{Z^0}\hat{\Sigma}_{A^0G^0}(q^2)$ does not vanish for any value of q^2 but only for $q^2 = M^2_{A^0}$.[2]

[1]The argument with the charged Higgs is exactly the same; therefore we will not make explicit the detailed derivation of the field renormalization constant $\delta Z_{G^\pm H^\pm}$ but only quote the result.

[2]The charged counterpart of this identity is also not valid for *any* q^2, as is assumed sometimes; see [19].

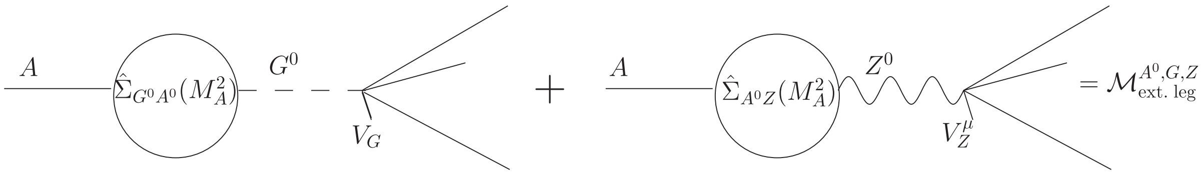

FIG. 1. The combined contribution of the $A^0 - Z^0$ and $A^0 - G^0$ transitions.

Let us show how, despite the constraint in Eq. (4.24), we can still avoid one-loop corrections and counterterms in the external legs associated with an external pseudoscalar A^0. Of concern to us are the transitions $A^0 - Z^0$ and $A^0 - G^0$. The idea is that, although we cannot make both $\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2) = 0$ and $\hat{\Sigma}_{A^0G^0}(M_{A^0}^2) = 0$, we will try to make the combined contribution to the external leg vanish. This combined contribution is pictured in Fig. 1.

To the tree-level coupling of the A^0 to some vertex V, at one loop the transition $A^0 - G^0$ involves the coupling of the tree-level neutral Goldstone to this vertex, V_G, while the Z^0 transition involves the corresponding vertex V_Z^μ. The total contribution of Fig. 1 for A^0 with momentum q on shell with $q^2 = M_{A^0}^2$ can be written as

$$\mathcal{M}^{A^0,G,Z}_{\text{ext.leg}} = \frac{\hat{\Sigma}_{A^0G^0}(M_{A^0}^2)V_G + q.V_Z\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2)}{M_{A^0}^2 - M_{Z^0}^2}$$
$$= \frac{V_G}{M_{A^0}^2 - M_{Z^0}^2}(\hat{\Sigma}_{A^0G^0}(M_{A^0}^2) + M_{Z^0}\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2)). \tag{4.43}$$

In the second step of Eq. (4.43) we used another identity that can be readily derived at tree level from the invariance of the Lagrangian under gauge transformations.[3] Therefore, in order not to deal with any correction on the external pseudoscalar leg, we require

$$\hat{\Sigma}_{A^0G^0}(M_{A^0}^2) + M_{Z^0}\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2) = 0. \tag{4.44}$$

For this requirement Eq. (4.44), which is a renormalization condition, to be consistent with the Ward identity in Eq. (4.24), leads to

$$\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2) = -\frac{1}{M_{Z^0}}\hat{\Sigma}_{A^0G^0}(M_{A^0}^2)$$
$$= \frac{1}{(4\pi)^2}\frac{e^2M_{Z^0}}{s_{2W}^2}s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(M_{A^0}^2). \tag{4.45}$$

[3] Consider the part of the Lagrangian with the Z^0 and the neutral Goldstone G^0. Before gauge fixing this Lagrangian is invariant under the transformation $Z^0_\mu \to Z^0_\mu + i\partial_\mu\omega$, $G^0 \to G^0 + M_{Z^0}\omega$. If the Z^0 (vector) current is V_Z^α and the Goldstone current V_G, that is, we have the interaction $Z^0.V_Z + G^0V_G$, invariance of the Lagrangian implies $-i\partial_\alpha V_Z^\alpha + M_{Z^0}V_G = 0$. In Eq. (4.43), this implies $q.V_Z = M_{Z^0}V_G$ where q is the Z^0 momentum.

In particular, with $\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(M_{A^0}^2) = 0$ in the linear gauge, we can make $\hat{\Sigma}_{A^0Z^0}(M_{A^0}^2) = \hat{\Sigma}_{A^0G^0}(M_{A^0}^2) = 0$. This condition readily gives

$$\delta Z_{G^0A^0} = -s_{2\beta}\frac{\delta t_\beta}{t_\beta} - 2\frac{\Sigma^{\text{tad}}_{A^0Z^0}(M_{A^0}^2)}{M_{Z^0}} + \frac{2}{(4\pi)^2}$$
$$\times \frac{e^2}{s_{2W}^2}s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(M_{A^0}^2). \tag{4.46}$$

Since $\delta Z_{A^0G^0}$ only enters in off-shell processes, A^0 off shell or an external Goldstone boson, there is no need to constrain it through some other renormalization condition. Our aim, as stressed repeatedly, is not to renormalize all Green's functions, but only S-matrix elements without the need for external leg corrections. The Ward identities that we derived in this section were, numerically, checked extensively in our code for various values of q^2, including $q^2 = M_{A^0}^2$ and $q^2 = M_{Z^0}^2$, and for different values of the nonlinear gauge parameters. Moreover, it is due to the $\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(M_{A^0}^2)$ contribution in $\delta Z_{G^0A^0}$ that we are able to obtain finite and gauge-invariant results for processes involving A^0 as an external particle. For $\delta Z_{G^\pm H^\pm}$ a similar derivation gives

$$\delta Z_{G^\pm H^\pm} = -s_{2\beta}\frac{\delta t_\beta}{t_\beta} - 2\frac{\Sigma^{\text{tad}}_{H^\pm W^\pm}(M_{H^\pm}^2)}{M_{W^\pm}}$$
$$+ \frac{2}{(4\pi)^2}\frac{e^2}{s_{2W}^2}\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(M_{H^\pm}^2). \tag{4.47}$$

With $\delta Z_{G^0A^0}$ (and $\delta Z_{G^\pm H^\pm}$) all our field renormalization constants are set and defined.

V. DEFINITIONS OF t_β AND THE t_β SCHEMES

A. Dabelstein-Chankowski-Pokorski-Rosiek scheme (DCPR)

This scheme, which we will refer to as the DCPR scheme, has been quite popular and is based on an OS renormalization scheme in the Higgs sector [15,16], working in the usual linear gauge. The definition of t_β, however, is difficult to reconcile with an on-shell quantity that represents a direct interpretation in terms of a physical observable. One first introduces a wave-function renormalization constant, δZ_{H_i}, for each Higgs doublet H_i, i.e. before rotation

$$H_i \to (1 + \tfrac{1}{2}\delta Z_{H_i})H_i, \qquad i = 1, 2. \tag{5.1}$$

To make contact with our approach and parameters, concerning wave-function renormalization, we refer to Appendix B. The vacuum expectation values are also shifted such that the counterterm for each v_i can be written

$$v_i \to v_i\left(1 - \frac{\tilde{\delta} v_i}{v_i} + \frac{1}{2}\delta Z_{H_i}\right), \tag{5.2}$$

giving

$$\frac{\delta t_\beta}{t_\beta} = \frac{\tilde{\delta} v_1}{v_1} - \frac{\tilde{\delta} v_2}{v_2} - \frac{1}{2}(\delta Z_{H_1} - \delta Z_{H_2}). \tag{5.3}$$

The DCPR scheme takes $\frac{\tilde{\delta} v_1}{v_1} = \frac{\tilde{\delta} v_2}{v_2}$ such that, in effect,

$$\frac{\delta t_\beta}{t_\beta} = \frac{1}{2}(\delta Z_{H_2} - \delta Z_{H_1}). \tag{5.4}$$

t_β is defined by requiring that the (renormalized) A^0Z^0 transition vanish at $q^2 = M^2_{A^0}$; therefore from

$$\mathrm{Re}\,\hat{\Sigma}_{A^0Z^0}(M^2_{A^0}) = 0, \tag{5.5}$$

with

$$\hat{\Sigma}_{A^0Z^0}(q^2) = \Sigma_{A^0Z^0}(q^2) + \frac{M_{Z^0}}{4} s_{2\beta}\left(\delta Z_{H_2} - \delta Z_{H_1} + 2\frac{\delta t_\beta}{t_\beta}\right), \tag{5.6}$$

one obtains that

$$\frac{\delta t_\beta}{t_\beta}_{\mathrm{DCPR}} = -\frac{1}{M_{Z^0} s_{2\beta}} \mathrm{Re}\Sigma_{A^0Z^0}(M^2_{A^0}). \tag{5.7}$$

This definition is clearly not directly related to an observable. Moreover, δt_β is expressed in terms of wave-function renormalization constants; see Eq. (5.4).

B. $\overline{\mathrm{DR}}$ scheme ($\overline{\mathrm{DR}}$)

In this scheme the counterterm for t_β is taken to be a pure divergence proportional to the ultraviolet (UV) factor in dimensional reduction, C_{UV},

$$C_{\mathrm{UV}} = 2/(4 - n) - \gamma_E + \ln(4\pi), \tag{5.8}$$

where n is the dimensionality of space-time. In this scheme the finite part of the counterterm is therefore set to zero:

$$\frac{\delta t_\beta^{\mathrm{fin}^{\overline{\mathrm{DR}}}}}{t_\beta} = 0. \tag{5.9}$$

The divergent part can be related to a few quantities not necessarily directly related to an observable. In the vein of the DCPR approach within the linear gauge, where δt_β is defined in Eq. (5.4), solving for $\delta Z_{H_2} - \delta Z_{H_1}$ leads to the HHW prescription of Hollik, Heinemeyer, and Weiglein [20] [see also Eq. (B15)],

$$\frac{\delta t_\beta^{\overline{\mathrm{DR}}-\mathrm{HHW}}}{t_\beta} = \frac{1}{2c_{2\alpha}}(\mathrm{Re}\Sigma'_{h^0h^0}(M^2_{h^0}) - \mathrm{Re}\Sigma'_{H^0H^0}(M^2_{H^0}))^\infty. \tag{5.10}$$

The superscript ∞ means that only the infinite C_{UV} part in dimensional reduction is taken into account. A more satisfactory $\overline{\mathrm{DR}}$ scheme can be based on a physical observable. Pierce and Papadopoulos (PP) [21] have defined δt_β by relating it to the *divergent* part of $M^2_{H^0} - M^2_{h^0}$. Note that the sum $M^2_{H^0} + M^2_{h^0}$ does not depend on t_β, as can be seen from the tree-level sum rule in Eq. (2.14). Hence [see also Eq. (4.31)],

$$\begin{aligned}\frac{\delta t_\beta^{\overline{\mathrm{DR}}-\mathrm{PP}}}{t_\beta} = \frac{1}{2s_{2\beta}s_{2(\alpha+\beta)}M^2_{Z^0}}\Bigg(&\frac{1}{v}(c_{\alpha-\beta}(1+2s^2_{\alpha-\beta})\delta T_{H^0} \\ &+ s_{\alpha-\beta}(1+2c^2_{\alpha-\beta})\delta T_{h^0}) + \mathrm{Re}\Sigma_{H^0H^0}(M^2_{H^0}) \\ &- \mathrm{Re}\Sigma_{h^0h^0}(M^2_{h^0}) + c_{2(\alpha+\beta)}\mathrm{Re}\Sigma_{A^0A^0}(M^2_{A^0}) \\ &- c_{2(\alpha+\beta)}\mathrm{Re}\Pi^T_{Z^0Z^0}(M^2_{Z^0})\Bigg)^\infty.\end{aligned} \tag{5.11}$$

C. An on-shell scheme (OS_{M_H}) with M_{H^0} as an input

In this scheme one takes M_{H^0}, the largest of the two scalar Higgs masses, as an input parameter. This trade-off is operative in the Higgs sector independently of any process. Therefore M_{H^0} is no longer a prediction but is extracted from a measurement together with M_{A^0}. As such, it does not receive a correction at any loop order; δt_β is defined from the constraint

$$\mathrm{Re}\,\hat{\Sigma}_{H^0H^0}(M^2_{H^0}) = 0, \tag{5.12}$$

which leads to

$$\begin{aligned}\frac{\delta t_\beta^{\mathrm{OS}_{M_H}}}{t_\beta} = \frac{1}{s_{2\beta}s_{2(\alpha-\beta)}M^2_{A^0}}\Bigg(&(c^2_\alpha - s^2_\beta s^2_{\alpha-\beta})\frac{\delta T_{\phi^0_1}}{v_1} \\ &+ (s^2_\alpha - c^2_\beta s^2_{\alpha-\beta})\frac{\delta T_{\phi^0_2}}{v_2} + \mathrm{Re}\Sigma_{H^0H^0}(M^2_{H^0}) \\ &- s^2_{\alpha-\beta}\mathrm{Re}\Sigma_{A^0A^0}(M^2_{A^0}) - c^2_{\alpha+\beta}\mathrm{Re}\Pi^T_{Z^0Z^0}(M^2_{Z^0})\Bigg).\end{aligned} \tag{5.13}$$

This scheme has been advocated in [14,18] and is one of the schemes implemented in SLOOPS. At tree level, t_β is extracted from the relation defined in Eq. (2.16),

$$c^2_{2\beta} = \frac{(M^2_{A^0} + M^2_{Z^0} - M^2_{H^0})M^2_{H^0}}{M^2_{A^0}M^2_{Z^0}}. \tag{5.14}$$

In our numerical examples the input parameters are such that the requirement $c^2_{2\beta} \le 1$ is always met. In fact, given a set M_{A^0}, M_{Z^0}, we *generate* M_{H^0} through a given value of t_β. The value M_{H^0} is taken as the physical mass at all loop orders; in particular, at one loop it does not receive a correction. As pointed out in Sec. II, in general, with a set M_{H^0}, M_{A^0}, M_{Z^0}, $c_{2\beta}^2 \leq 1$ is not guaranteed. With this important proviso, we extract $\tan\beta$ (with $\tan\beta > 1$) as

$$t_\beta = \sqrt{\frac{M_{A^0}M_{Z^0} + M_{H^0}\sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}{M_{A^0}M_{Z^0} - M_{H^0}\sqrt{M_{A^0}^2 + M_{Z^0}^2 - M_{H^0}^2}}}. \tag{5.15}$$

That this choice might lead to large corrections and large uncertainties can already be guessed by considering the uncertainty on $\tan\beta$ given an uncertainty on M_{H^0}, M_{A^0}, M_{Z^0} with, respectively, δM_{H^0}, δM_{A^0}, δM_{Z^0}. For clarity, let us take $\delta M_{Z^0} = 0$ as would be fit from an experimental point of view since M_{Z^0} is known with an excellent precision from the LEP measurements. We find

$$\frac{\delta t_\beta}{t_\beta} = \frac{M_{A^0}^2}{M_{H^0}^2 - M_{A^0}^2}\frac{M_{H^0}^2}{M_{H^0}^2 - M_{Z^0}^2}\left(-\frac{M_{H^0}^2 - M_{Z^0}^2}{M_{A^0}^2}\frac{\delta M_{A^0}^2}{M_{A^0}^2} + \frac{M_{H^0}^2}{M_{A^0}^2}\frac{2M_{H^0}^2 - M_{A^0}^2 - M_{Z^0}^2}{M_{H^0}^2}\frac{\delta M_{H^0}^2}{M_{H^0}^2}\right). \tag{5.16}$$

With typical input parameters in the decoupling limit $M_{A^0} \gg M_{Z^0}$ with $M_{A^0}/M_{H^0} \sim 1$, a large uncertainty ensues, to wit

$$\frac{\delta t_\beta}{t_\beta} \simeq \frac{1}{M_{H^0}^2/M_{A^0}^2 - 1}\left(-\frac{\delta M_{A^0}^2}{M_{A^0}^2} + \frac{\delta M_{H^0}^2}{M_{H^0}^2}\right). \tag{5.17}$$

Therefore, although δt_β is manifestly gauge invariant, one should expect large uncertainties from loop corrections. This scheme is similar to the one considered in [6] based on Eq. (5.14).

D. $A_{\tau\tau}$ as an input parameter ($\mathrm{OS}_{A_{\tau\tau}}$)

β, which appears in the Higgs sector, relies on the assumption of a basis; only quantities which are basis independent are physical quantities [8,22]. The Higgs potential of the MSSM appears as a general two-Higgs doublet model if one restricts oneself solely to the Higgs sector. The degeneracy is lifted when defining the Yukawa Higgs coupling to fermions. This picks up a specific direction. One should therefore define $\tan\beta$ from the Higgs couplings to fermions. Since M_{A^0} is used as an input parameter, assuming one has had access to the pseudoscalar Higgs, it seems natural to take a coupling $A^0 f\bar{f}$. Since couplings to quarks are subject to large QCD radiative corrections, the best choice is to consider the $A_{\tau\tau}$ coupling, which is the largest coupling to leptons,

$$\mathcal{L}_{A_{\tau\tau}}^0 = i\frac{m_\tau}{v_1}s_\beta\bar{\tau}\gamma_5\tau A^0 = i\frac{gm_\tau}{2M_{W^\pm}}t_\beta\bar{\tau}\gamma_5\tau A^0 \quad \text{with} \quad v_1 = vc_\beta. \tag{5.18}$$

This coupling can be extracted from the measurement of the width $\Gamma_{A_{\tau\tau}}$ with m_τ the mass of the τ. Note also that $\delta\Gamma_{A_{\tau\tau}} = 2\delta t_\beta/t_\beta$ so that, contrary to the on-shell scheme based on M_{H^0}, OS_{M_H}, this scheme should therefore not introduce additional large uncertainties, assuming of course that this decay can be large and be measured precisely. This scheme therefore appears very natural; however, it has not been used in practice because one has considered it as being a *process-dependent* definition set outside the purely Higgs sector, which, moreover, implies that fixing the counterterm involves a three-point function. This last argument is unjustified; take, for example, the G_μ scheme in the SM where muon decay is used as a trade-off for $M_{W^\pm}$, taking advantage of the fact that G_μ has long been so much better measured than $M_{W^\pm}$. The G_μ scheme involves four-point functions. We find that, technically, this scheme is not more difficult to implement than a scheme based on two-point functions. The full counterterm to $A_{\tau\tau}$ involves the $G^0 \to A^0$ shift, the A^0 and $\tau^\pm$ wave-function renormalization constants, among other things; we get

$$\begin{aligned}\delta\mathcal{L}_{A_{\tau\tau}} &= \mathcal{L}_{A_{\tau\tau}}^0\left(\delta_{\mathrm{CT}}^{A_{\tau\tau}} + \frac{\delta t_\beta}{t_\beta}\right) \quad \text{with} \\ \delta_{\mathrm{CT}}^{A_{\tau\tau}} &= \left(\frac{\delta m_\tau}{m_\tau} + \frac{\delta e}{e} + \frac{c_W^2}{2s_W^2}\frac{\delta M_{W^\pm}^2}{M_{W^\pm}^2} - \frac{1}{2s_W^2}\frac{\delta M_{Z^0}^2}{M_{Z^0}^2} + \frac{1}{2}\delta Z_{A^0A^0} - \frac{1}{2t_\beta}\delta\tilde{Z}_{G^0A^0} + \frac{1}{2}(\delta Z_L^\tau + \delta Z_R^\tau)\right), \\ -\frac{1}{2t_\beta}\delta\tilde{Z}_{G^0A^0} &= \frac{1}{t_\beta}\frac{\Sigma_{A^0Z^0}(M_{A^0}^2)}{M_{Z^0}} - \frac{1}{1+t_\beta^2}\frac{\alpha}{2\pi}M_{Z^0}\mathcal{F}_{GA}^{\tilde{\epsilon},\tilde{\gamma}}(M_{Z^0}^2).\end{aligned} \tag{5.19}$$

δm_τ (the τ mass counterterm), δe (the electromagnetic coupling counterterm), $\delta M_{W^\pm,Z^0}$ (the gauge boson mass counterterms), and the τ wave-function renormalization constant $\delta Z_{L,R}^\tau$ counterterms are defined on shell exactly as in the SM [10]. The full one-loop virtual corrections consist of the vertex corrections $\delta_{\mathrm{V}}^{A_{\tau\tau}}$, which contribute a one-loop vertex correction to the decay rate as

$$\delta\Gamma_1^{\mathrm{vertex}} = 2\Gamma_0\delta_{\mathrm{V}}^{A_{\tau\tau}}. \tag{5.20}$$

The latter are made UV finite by the addition of the counterterm in Eq. (5.19). These virtual QED corrections, both vertex and counterterm (from δm_τ and $\delta Z_{L,R}^\tau$), include genuine QED corrections through photon exchange which are infrared divergent. In our case the infrared divergence can be trivially regularized through the introduction of a small fictitious mass, λ, for the photon. As known, the fictitious mass dependence is canceled when photon bremsstrahlung is added. Taking into account the latter may depend on the experimental setup that often requires cuts on the additional photon kinematical variables. Therefore, it is much more appropriate to take as an observable a quantity devoid of such cuts, knowing that hard/soft radiation can be easily added. Fortunately, for a *neutral* decay such as this one, which is of an Abelian nature, the virtual QED correction constitutes a gauge-invariant subset that can be trivially calculated separately. The virtual QED corrections to the decay width $A^0 \to \tau^+\tau^-$ are known [23]; they contribute a one-loop correction

$$\delta\Gamma_1^{\rm QED} = 2\Gamma_0\delta_v^{\rm QED} \quad \text{with}$$

$$\delta_v^{\rm QED} = \frac{\alpha}{2\pi}\bigg(-\bigg(\frac{1+\beta^2}{2\beta}\ln\frac{1+\beta}{1-\beta}-1\bigg)\ln\frac{m_\tau^2}{\lambda^2}-1 + \frac{1+\beta^2}{\beta}\bigg[{\rm Li}_2\bigg(\frac{1-\beta}{1+\beta}\bigg)+\ln\frac{1+\beta}{2\beta}\ln\frac{1+\beta}{1-\beta} - \frac{1}{4}\ln^2\frac{1+\beta}{1-\beta}+\frac{\pi^2}{3}\bigg]\bigg), \tag{5.21}$$

$$\beta = \sqrt{1-\frac{4m_\tau^2}{M_{A^0}^2}}, \tag{5.22}$$

$${\rm Li}_2(x) = -\int_0^x \frac{dt}{t}\ln(1-t). \tag{5.23}$$

This QED correction only depends on M_{A^0}, e, m_τ, as it should, and does not involve any other (MSSM) parameter. Subtracting this QED correction from the full one-loop virtual correction in Eq. (5.19) will give the genuine SUSY non-QED contribution that does not depend on any fictitious photon mass nor any experimental cut. Our scheme is to require that δt_β is such that this contribution vanishes and that, therefore, $A^0 \to \tau^+\tau^-$ is only subject to QED corrections. This gives

$$\frac{\delta t_\beta^{{\rm OS}_{A\tau\tau}}}{t_\beta} = -(\delta_{\rm V}^{A_{\tau\tau}} + \delta_{\rm CT}^{A_{\tau\tau}} - \delta_v^{\rm QED}). \tag{5.24}$$

This definition is independent of the fictitious mass of the photon λ used as a regulator. We have checked this explicitly within SLOOPS.

VI. SETUP OF THE AUTOMATIC CALCULATION OF THE CROSS SECTIONS

All the steps necessary for the renormalization of the Higgs sector as presented here together with a complete definition of the MSSM have been implemented in SLOOPS. As we will discuss in a forthcoming publication [9], the other sectors have also been implemented and results relying on the complete renormalization of the MSSM have been given in [13]. Since even the calculation of a single two-point function in the MSSM requires the calculation of hundreds of diagrams, some automatization is unavoidable. Even in the SM, one-loop calculations of $2 \to 2$ processes involve hundreds of diagrams and calculation by hand is almost impracticable. Efficient automatic codes for any generic $2 \to 2$ process, that have now been exploited for many $2 \to 3$ [24,25] and even some $2 \to 4$ [26,27] processes, are almost unavoidable for such calculations. For the electroweak theory these are the GRACE-LOOP [10] code and the bundle of packages based on FEYNARTS [28], FORMCALC [29], and LOOPTOOLS [30], which we will refer to as FFL for short.

With its much larger particle content, a far greater number of parameters, and more complex structure, the need for an automatic code at one loop for the minimal supersymmetric standard model is even more important. A few parts that are needed for such a code have been developed based on an extension of [31] but, as far as we know, no complete code exists or is, at least publicly, available. GRACE-SUSY [32] is now also being developed at one loop, and many results exist [14]. One of the main difficulties that has to be tackled is the implementation of the model file, since this requires that one enters the thousands of vertices that define the Feynman rules. On the theory side a proper renormalization scheme needs to be set up, which then means extending many of these rules to include counterterms. When this is done, one can just use, or hope to use, the machinery developed for the SM, in particular, the symbolic manipulation part and, most importantly, the loop integral routines including tensor reduction algorithms or any other efficient set of basis integrals.

SLOOPS combines LANHEP [33] (originally part of the package COMPHEP [34]) with the FFL bundle but with an extended and adapted LOOPTOOLS [12]. LANHEP is a very powerful routine that *automatically* generates all the sets of Feynman rules of a given model, the latter being defined in a simple and compact format very similar to the canonical coordinate representation. Use of multiplets and the superpotential is built in to minimize human error. The ghost Lagrangian is derived directly from the BRST transformations. The LANHEP module also allows one to shift fields and parameters and thus generates counterterms most efficiently. Understandably, the LANHEP output file must be in the format of the model file of the code it is interfaced with. In the case of FEYNARTS both the *generic* (Lorentz structure) and *classes* (particle content) files had to be given. Moreover, because we use a nonlinear gauge-fixing condition [10] (see below), the FEYNARTS default *generic* file had to be extended.

VII. t_β SCHEME DEPENDENCE OF PHYSICAL OBSERVABLES, GAUGE INVARIANCE: A NUMERICAL INVESTIGATION

In this first investigation we will restrict ourselves to Higgs observables. Other observables involving other supersymmetric particles require that we first expose and detail our renormalization procedure of the chargino/neutralino and the sfermion sector. This will be presented in [9]. We have, however, presented some results on the $\tan\beta$

scheme dependence of a few cross sections that are needed for the calculation of the relic density in the MSSM [13].

A. Parameters

To make contact with the analysis of [6] and also allow comparisons, we will consider the three sets of benchmarks points for the Higgs based on [35]. The three sets of parameters, called *mhmax*, large μ, and *nomix*, are as in [35] except that we set a common trilinear A_f to all sfermions for convenience. For each set there are two values of t_β, $t_\beta = 3$, 50; see Table I.

B. Gauge independence and the finite part of t_β

If t_β is defined as a physical parameter, then δt_β must be gauge invariant and gauge parameter independent. Our nonlinear gauge fixing allows us to check the gauge parameter independence of δt_β and hence t_β. Even when two schemes are gauge parameter independent, the values of δt_β are not expected to be the same. It is therefore also interesting to inquire how much two schemes differ from each other. Naturally, since δt_β is not ultraviolet finite we split this contribution into a finite part and an infinite part, the latter being regularized in dimensional reduction, such that

$$\delta t_\beta = \delta t_\beta^{\rm fin} + \delta t_\beta^{\infty} C_{\rm UV}. \tag{7.1}$$

The $\overline{\rm DR}$ schemes have, by definition, $\delta t_\beta^{\rm fin} = 0$. When calculating observables in this scheme, we will also need to specify a scale $\bar{\mu}$ which we associate with the scale introduced by dimensional reduction. For the latter our default value is $\bar{\mu} = M_{A^0}$. Our set of nonlinear gauge parameters is defined as nlgs $= (\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\omega}, \tilde{\rho}, \tilde{\kappa}, \tilde{\epsilon}, \tilde{\gamma})$.

The usual linear gauge, nlgs $= 0$, corresponds to all these parameters set to 0. For the gauge parameter independence we will compare the results of the linear gauge to a nonlinear gauge where all the nonlinear gauge parameters have been set to 10, referring to this as nlgs $= 10$.

To make the point about the gauge parameter dependence, it is enough to consider only one of the benchmark points.

As expected, we see from Table II that only the schemes based on a physical definition of t_β are gauge parameter independent. Therefore neither DCPR nor a $\overline{\rm DR}$ manifestation of it based on [20] is gauge independent. Within the physical definitions, note that although the divergent part is, as expected, the same for all the schemes in all gauges, the finite parts are quite different from each other; in particular, the OS_{M_H} scheme introduces a "correction" of about 30% to t_β. This is just an indication that this scheme might induce large corrections on observables. However, one needs to be cautious; in the same way that the $C_{\rm UV}$ part cancels in observables, a large finite correction could, in principle, also be absorbed when we consider a physical process. Our rather extensive analysis will show that this is, after all, not the case. Schemes where the finite part of δt_β is large do, generally, induce large corrections. It is important to note that for the linear gauge all schemes give the same $C_{\rm UV}$ part. Having made the point about the gauge parameter dependence, we will now work purely in the linear gauge since some of the schemes introduced in the literature are *acceptable* only within the linear gauge.

TABLE I. The set of SM and MSSM parameters for the benchmark points. All mass parameters are in GeV. We take M_1 according to the so-called gaugino mass unification with $M_1 = \frac{5 s_W^2 M_2}{3 c_W^2}$.

Parameter	Value
s_W	0.480 76
e	0.313 45
g_s	1.238
M_{Z^0}	91.1884
m_e	0.000 511
m_μ	0.1057
m_τ	1.777
m_u	0.046
m_d	0.046
m_c	1.42
m_s	0.2
m_t	174.3
m_b	3
M_{A^0}	500
t_β	3;50
mhmax	Value
μ	-200
M_2	200
M_3	800
$M_{\tilde{F}_L}$	1000
$M_{\tilde{f}_R}$	1000
A_f	$2000 + \mu/t_\beta$
nomix	Value
μ	-200
M_2	200
M_3	800
$M_{\tilde{F}_L}$	1000
$M_{\tilde{f}_R}$	1000
A_f	μ/t_β
large μ	Value
μ	1000
M_2	400
M_3	200
$M_{\tilde{F}_L}$	400
$M_{\tilde{f}_R}$	400
A_f	$-300 + \mu/t_\beta$

TABLE II. Gauge dependence of δt_β at the scale $\bar{\mu} = M_{A^0}$ for the set *mhmax* at $t_\beta = 3$.

δt_β^∞	nlgs = 0	nlgs = 10
DCPR	-3.19×10^{-2}	-1.04×10^{-1}
OS_{M_H}	-3.19×10^{-2}	-3.19×10^{-2}
$OS_{A_{\tau\tau}}$	-3.19×10^{-2}	-3.19×10^{-2}
$\overline{DR}$-HHW	-3.19×10^{-2}	$+5.32 \times 10^{-2}$
$\overline{DR}$-PP	-3.19×10^{-2}	-3.19×10^{-2}

δt_β^{fin}	nlgs = 0	nlgs = 10
DCPR	−0.10	−0.27
OS_{M_H}	+0.92	+0.92
$OS_{A_{\tau\tau}}$	−0.10	−0.10
$\overline{DR}$-HHW	0	0
$\overline{DR}$-PP	0	0

Therefore in this case the results for $\overline{DR}$-HHW and $\overline{DR}$-PP are the same and will be denoted as $\overline{DR}$ in what follows.

C. δt_β^{fin}

First of all, let us mention that our numerical results concerning the DCPR and $\overline{DR}$ schemes agree quite well with those of [6] concerning the shifts in t_β and the lightest *CP*-even Higgs mass. Our results for OS_{M_H} follow sensibly the same trend as the scheme defined as the Higgs mass scheme in [6]. We see that for small t_β DCPR and $OS_{A_{\tau\tau}}$ give sensibly the same result with a finite relative shift of a few percent; see Table III. For larger t_β the difference is much larger, and we notice that $OS_{A_{\tau\tau}}$ gives much smaller shifts. On the other hand, the OS_{M_H} gives huge corrections for $t_\beta = 50$, well above 100%. As we will see, this will have an impact on the radiative corrections on some observables based on this scheme.

D. Higgs masses and their scheme dependence

We start with the one-loop correction to the lightest *CP*-even Higgs; see Table IV. Of course, this has now been calculated beyond one loop, as the one-loop correction is large; however, a study of the scheme dependence is important. Moreover, this study represents a direct application of the code that can be compared to results in the literature. We note that all schemes, apart from OS_{M_H}, are in very good agreement with each other for both values of t_β. Leaving aside the case of $t_\beta = 50$ in the large μ scenario, despite the very large shifts we observed in δt_β^{fin} for the OS_{M_H} scheme, the t_β dependence is very suppressed such that the OS_{M_H} scheme compares favorably with the other schemes. In the case of the correction to the heaviest *CP*-even Higgs at one loop, by definition there is no correction in the OS_{M_H} scheme, and the other schemes agree with each other at a very high level of precision; see Table V. Moreover, especially at high t_β, the correction is very small.

TABLE III. δt_β^{fin} for the Higgs benchmark points.

$t_\beta = 3$	*mhmax*	large μ	*nomix*
DCPR	−0.10	−0.06	−0.08
OS_{M_H}	+0.92	−1.31	+0.64
$OS_{A_{\tau\tau}}$	−0.10	−0.06	−0.08
$\overline{DR}$	0	0	0

$t_\beta = 50$	*mhmax*	large μ	*nomix*
DCPR	+3.42	+14.57	+0.48
OS_{M_H}	−385.53	−2010.84	−290.18
$OS_{A_{\tau\tau}}$	+0.12	−4.72	+0.16
$\overline{DR}$	0	0	0

The mass of the charged Higgs does not depend on δt_β; therefore the correction is scheme independent, with the counterterm $\delta M_{H^\pm}^2 = \delta M_{W^\pm}^2 + \delta M_{A^0}^2$.

E. Higgs decays to SM particles and their scheme dependence

1. $A^0 \to \tau^+\tau^-$, the non-QED one-loop corrections

We now study the non-QED corrections to the decay width $A^0 \to \tau^+\tau^-$; see Sec. V D for our benchmark points. By definition there is no correction in the $OS_{A_{\tau\tau}}$ scheme. Many interesting and important conclusions can be drawn from Table VI. First of all, we note that the scheme dependence is quite large here. After all, this is an observable which is directly proportional to δt_β. In fact, the difference between schemes can be accounted for by $2\delta t_\beta$ read off from Table III. For this decay, the OS_{M_H} scheme is totally unsuitable; for $t_\beta = 3$ the corrections are of order 100%, whereas for $t_\beta = 50$ the one-loop correc-

TABLE IV. Mass of the lightest *CP*-even Higgs at one loop in different schemes. All masses are in GeV.

$t_\beta = 3$, $M_{h^0}^{TL} = 72.51$	*mhmax*	large μ	*nomix*
DCPR	134.28	97.57	112.26
OS_{M_H}	140.25	86.68	117.37
$OS_{A_{\tau\tau}}$	134.25	97.59	112.27
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	134.87	98.10	112.86
$\overline{DR}$ $\bar{\mu} = m_t$	134.47	97.55	112.38

$t_\beta = 50$, $M_{h^0}^{TL} = 91.11$	*mhmax*	large μ	*nomix*
DCPR	144.50	35.88	124.80
OS_{M_H}	143.76	13.21	124.16
$OS_{A_{\tau\tau}}$	144.50	35.73	124.80
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	144.50	35.77	124.80
$\overline{DR}$ $\bar{\mu} = m_t$	144.50	35.77	124.80

TABLE V. Mass of the heaviest *CP*-even Higgs at one loop in different schemes. All masses are in GeV.

$t_\beta = 3$ $M^{TL}_{H^0} = 503.05$	*mhmax*	large μ	*nomix*
DCPR	504.68	501.05	504.21
OS_{M_H}	503.05	503.05	503.05
$\mathrm{OS}_{A_{\tau\tau}}$	504.68	501.05	504.21
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	504.52	500.95	504.08
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	504.63	501.05	504.19

$t_\beta = 50$ $M^{TL}_{H^0} = 500.01$	*mhmax*	large μ	*nomix*
DCPR	499.80	498.90	499.85
OS_{M_H}	500.01	500.01	500.01
$\mathrm{OS}_{A_{\tau\tau}}$	499.80	498.91	499.85
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	499.80	498.91	500.01
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	499.80	498.91	499.85

tion is an order of magnitude, at least, larger than tree level. Especially for $t_\beta = 3$, in $\overline{\mathrm{DR}}$ the scale dependence is not negligible. For example, with $\bar{\mu} = m_t$ in $\overline{\mathrm{DR}}$ the correction is of order $\sim 1\%$ and 5% for $\bar{\mu} = M_{A^0}$. The corrections are much smaller in DCPR being at the permil level. The scale dependence is much smaller for $t_\beta = 30$ and the corrections in $\overline{\mathrm{DR}}$ are now smaller than in DCPR. Note also that in the large μ scenario the corrections are large.

2. $H^0 \to \tau^+\tau^-$, the non-QED one-loop corrections

Similar conclusions can be drawn from the study of the non-QED corrections to $H^0 \to \tau^+\tau^-$; see Table VII. The QED corrections for this decay can be implemented as in [23]. The only difference is that now there is also a correction in the case of the $\mathrm{OS}_{A_{\tau\tau}}$ scheme. But, as expected, this correction is very small for both values of t_β. Note that for $t_\beta = 50$ the DCPR scheme gives very large corrections in the large μ scenario. For this process we have not taken into account the one-loop correction to M_{H^0} since, as we have seen, this correction is very small for all schemes and also because one is much too far from the $\tau\tau$ threshold, $M_{H^0} \sim 500$ GeV $\gg 2m_\tau$, where this effect can play a role.

3. $H^0 \to Z^0Z^0$ and $A^0 \to Z^0h^0$

$H^0 \to Z^0Z^0$ was studied in [21] where a large correction was found. We confirm here (see Table VIII) that a large

TABLE VI. Corrections to the decay $A^0 \to \tau^+\tau^-$ at one loop without the universal QED correction. All widths are in GeV.

$t_\beta = 3$ $\Gamma^{TL} = 9.40 \times 10^{-3}$	*mhmax*	large μ	*nomix*
DCPR	$+3.56 \times 10^{-5}$	-8.71×10^{-6}	-7.37×10^{-6}
OS_{M_H}	$+6.41 \times 10^{-3}$	-7.82×10^{-3}	$+4.56 \times 10^{-3}$
$\mathrm{OS}_{A_{\tau\tau}}$	0	0	0
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	$+6.51 \times 10^{-4}$	$+3.94 \times 10^{-4}$	$+5.18 \times 10^{-4}$
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	$+2.30 \times 10^{-4}$	-2.66×10^{-5}	$+9.67 \times 10^{-5}$

$t_\beta = 50$ $\Gamma^{TL} = 2.61 \times 10^{0}$	*mhmax*	large μ	*nomix*
DCPR	$+3.45 \times 10^{-1}$	$+2.01 \times 10^{0}$	$+3.35 \times 10^{-2}$
OS_{M_H}	-4.03×10^{1}	-2.09×10^{2}	-3.03×10^{1}
$\mathrm{OS}_{A_{\tau\tau}}$	0	0	0
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	-1.21×10^{-2}	$+4.92 \times 10^{-1}$	-1.66×10^{-2}
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	-3.00×10^{-2}	$+4.75 \times 10^{-1}$	-3.44×10^{-2}

TABLE VII. Corrections to the decay $H^0 \to \tau^+\tau^-$ at one loop without the universal QED correction. All widths are in GeV.

$t_\beta = 3$ $\Gamma^{TL} = 9.35 \times 10^{-3}$	*mhmax*	large μ	*nomix*
DCPR	-1.09×10^{-4}	-7.96×10^{-5}	-1.09×10^{-4}
OS_{M_H}	$+6.28 \times 10^{-3}$	-7.91×10^{-3}	$+4.47 \times 10^{-3}$
$OS_{A_{\tau\tau}}$	-1.45×10^{-4}	-7.09×10^{-5}	-1.01×10^{-4}
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	$+5.08 \times 10^{-4}$	$+3.24 \times 10^{-4}$	$+4.17 \times 10^{-4}$
$\overline{DR}$ $\bar{\mu} = m_t$	$+8.57 \times 10^{-5}$	-9.75×10^{-5}	-4.52×10^{-6}
$t_\beta = 50$ $\Gamma^{TL} = 2.61 \times 10^{0}$	*mhmax*	large μ	*nomix*
DCPR	$+3.54 \times 10^{-1}$	$+2.02 \times 10^{0}$	$+4.31 \times 10^{-2}$
OS_{M_H}	-4.03×10^{1}	-2.09×10^{2}	-3.03×10^{1}
$OS_{A_{\tau\tau}}$	$+9.52 \times 10^{-3}$	$+1.94 \times 10^{-3}$	$+9.55 \times 10^{-3}$
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	-2.59×10^{-3}	$+4.94 \times 10^{-1}$	-7.00×10^{-3}
$\overline{DR}$ $\bar{\mu} = m_t$	-2.04×10^{-2}	$+4.76 \times 10^{-1}$	-2.49×10^{-2}

correction is indeed induced with the one-loop result of the same order if not exceeding both at $t_\beta = 3$ and $t_\beta = 50$ the tree-level result. This larger correction is not due to the scheme dependence, since in this process the latter is very small whereas one sees a large correction with all the schemes. The correction is large because the benchmark points with $M_{A^0} = 500$ GeV are in the decoupling regime where $H^0 \to Z^0Z^0$ practically vanishes at tree level. $H^0Z^0Z^0$ is proportional to $c_{\beta-\alpha} \sim M_{Z^0}/M_{A^0}$; the coupling is therefore almost induced at one loop without the $1/M_{A^0}$ suppression. Here, again, because $M_{H^0} \gg 2M_{Z^0}$ the one-loop correction on M_{H^0} is negligible. Very similar results and conclusions can be drawn for the process $A^0 \to Z^0h^0$; see Table IX.

VIII. CONCLUSIONS

The use of the nonlinear gauge has allowed us, for the first time, to quantitatively and qualitatively study different proposals for the ubiquitous parameter tanβ and its effect on the Higgs observables, both the physical Higgs masses and their decays. Our first preliminary conclusion is that the scheme based on the extraction and definition of tanβ from a decay such as $A^0 \to \tau^+\tau^-$ is by far the most satisfactory. Not only is this definition directly related to a physical observable and therefore gauge independent, the functional dependence of the physical width in tanβ is linear and is the same independently of the value of the pseudoscalar Higgs mass. Moreover, the definition is clean once we subtract the universal gauge-invariant QED cor-

TABLE VIII. Corrections to the decay $H^0 \to Z^0Z^0$ at one loop. All widths are in GeV.

$t_\beta = 3$ $\Gamma^{TL} = 8.97 \times 10^{-3}$	*mhmax*	large μ	*nomix*
DCPR	$+1.59 \times 10^{-2}$	-6.32×10^{-3}	$+8.47 \times 10^{-3}$
OS_{M_H}	$+1.40 \times 10^{-2}$	-4.00×10^{-3}	$+7.12 \times 10^{-3}$
$OS_{A_{\tau\tau}}$	$+1.59 \times 10^{-2}$	-6.32×10^{-3}	$+8.47 \times 10^{-3}$
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	$+1.57 \times 10^{-2}$	-6.44×10^{-3}	$+8.32 \times 10^{-3}$
$\overline{DR}$ $\bar{\mu} = m_t$	$+1.58 \times 10^{-2}$	-6.32×10^{-3}	$+8.44 \times 10^{-3}$
$t_\beta = 50$ $\Gamma^{TL} = 6.40 \times 10^{-5}$	*mhmax*	large μ	*nomix*
DCPR	$+2.18 \times 10^{-5}$	-5.14×10^{-4}	$+3.89 \times 10^{-5}$
OS_{M_H}	$+1.01 \times 10^{-2}$	$+4.66 \times 10^{-3}$	$+7.81 \times 10^{-4}$
$OS_{A_{\tau\tau}}$	$+3.02 \times 10^{-5}$	-4.65×10^{-4}	$+3.97 \times 10^{-5}$
$\overline{DR}$ $\bar{\mu} = M_{A^0}$	$+3.05 \times 10^{-5}$	-4.77×10^{-4}	$+4.01 \times 10^{-5}$
$\overline{DR}$ $\bar{\mu} = m_t$	$+3.09 \times 10^{-5}$	-4.76×10^{-4}	$+4.05 \times 10^{-5}$

TABLE IX. Corrections to the decay $A^0 \to Z^0 h^0$ at one loop. All widths are in GeV.

$t_\beta = 3$ $\Gamma^{TL} = 9.03 \times 10^{-3}$	*mhmax*	large μ	*nomix*
DCPR	$+2.42 \times 10^{-2}$	$+3.86 \times 10^{-3}$	$+1.68 \times 10^{-2}$
OS_{M_H}	$+2.23 \times 10^{-2}$	$+6.20 \times 10^{-3}$	$+1.55 \times 10^{-2}$
$\mathrm{OS}_{A_{\tau\tau}}$	$+2.50 \times 10^{-2}$	$+3.86 \times 10^{-3}$	$+1.64 \times 10^{-2}$
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	$+2.48 \times 10^{-2}$	$+3.74 \times 10^{-3}$	$+1.67 \times 10^{-2}$
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	$+2.41 \times 10^{-2}$	3.87×10^{-3}	$+1.68 \times 10^{-2}$
$t_\beta = 50$ $\Gamma^{TL} = 6.30 \times 10^{-5}$	*Mhmax*	large μ	*Nomix*
DCPR	$+2.39 \times 10^{-5}$	$+8.75 \times 10^{-4}$	$+4.31 \times 10^{-5}$
OS_{M_H}	$+1.00 \times 10^{-3}$	$+5.97 \times 10^{-3}$	$+7.74 \times 10^{-4}$
$\mathrm{OS}_{A_{\tau\tau}}$	$+3.48 \times 10^{-5}$	$+9.26 \times 10^{-4}$	$+4.39 \times 10^{-5}$
$\overline{\mathrm{DR}}$ $\bar{\mu} = M_{A^0}$	$+3.51 \times 10^{-5}$	$+9.12 \times 10^{-4}$	$+4.43 \times 10^{-5}$
$\overline{\mathrm{DR}}$ $\bar{\mu} = m_t$	$+3.30 \times 10^{-5}$	$+9.12 \times 10^{-4}$	$+4.47 \times 10^{-5}$

rection. The scheme is also the most pleasing and satisfactory since it is the one where the observables we have studied show the least corrections, therefore leading to a stable prediction. On this last count, the $\overline{\mathrm{DR}}$ scheme performs almost just as well. However, the widely used $\overline{\mathrm{DR}}$ scheme extracted from the A^0Z^0 transition is not gauge invariant and is therefore terribly unsatisfactory from a theoretical point of view. In the nonlinear gauge with a general gauge-fixing set of parameters, the parameter gauge dependence shows up already at one loop, whereas it has been known that the scheme fails even in the linear gauge but at two loops [7]. A gauge independent $\overline{\mathrm{DR}}$ scheme such as the one proposed in [21] is the most satisfactory. A scheme based on the usage of M_{H^0} as an independent parameter from the Higgs sector leads to corrections that are too large in most of the observables we considered so far. We therefore propose that the decay $A^0 \to \tau^+\tau^-$ be used as a definition of $\tan\beta$. This choice assumes that this decay will one day be measured with high enough precision, but this depends much on the spectrum of the MSSM. Were it not for the unambiguous extraction of the full QED corrections, the decay of the charged Higgs to $\tau\nu$ may also qualify as a suitable input parameter; see [36] for prospects on the measurement of this decay. Apart from the discussion on gauge invariance and the issue of the scheme dependence for $\tan\beta$, we have shown how a complete one-loop renormalization of the MSSM can be automatized and we have given results and details concerning the Higgs sector, which is the first step in a successful implementation of this programme.

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APPENDIX A: THE WARD-SLAVNOV-TAYLOR IDENTITY FOR THE TRANSITIONS A^0Z^0 AND A^0G^0

There is an identity relating the A^0Z^0 and A^0G^0 transitions. This is most useful for $q^2 = M^2_{A^0}$. Contrary to what one might see in some papers, the relation is much more complicated for $q^2 \neq M^2_{A^0}$ and gets more subtle in the case of the nonlinear gauge.

The identity can be most easily derived by considering the BRST transformation on the (ghost) operator $\langle 0|\bar{c}^Z(x)A^0(y)|0\rangle = 0$. We find

$$\delta_{\mathrm{BRS}}\langle 0|\bar{c}^Z(x)A^0(y)|0\rangle = \langle 0|(\delta_{\mathrm{BRS}}\bar{c}^Z(x))A^0(y)|0\rangle - \langle 0|\bar{c}^Z(x)(\delta_{\mathrm{BRS}}A^0(y))|0\rangle = 0, \tag{A1}$$

with

$$\delta_{\mathrm{BRS}}A^0 = -\frac{g}{2}(c^+H^- + c^-H^+) + \frac{e}{s_{2W}}c^Z(c_{\alpha-\beta}h^0 + s_{\alpha-\beta}H^0) \tag{A2}$$

and

$$\delta_{\mathrm{BRS}}\bar{c}^Z = B^Z. \tag{A3}$$

Therefore,

$$\langle 0|B^Z(x)A^0(y)|0\rangle + \frac{g}{2}(\langle 0|\bar{c}^Z(x)c^+(y)H^-(y)|0\rangle + \langle 0|\bar{c}^Z(x)c^-(y)H^+(y)|0\rangle) - \frac{e}{s_{2W}}(c_{\alpha-\beta}\langle 0|\bar{c}^Z(x)c^Z(y)h^0(y)|0\rangle + s_{\alpha-\beta}\langle 0|\bar{c}^Z(x)c^Z(y)H^0(y)|0\rangle) = 0. \quad \text{(A4)}$$

At tree level, there is no vertex involving $\bar{c}^Z c^\pm H^\pm$. Using the equation of motion of the B field, we obtain a relation for the following Green's functions (external legs are not amputated):

$$\partial_x\langle 0|Z^0(x)A^0(y)|0\rangle + M_{Z^0}\langle 0|G^0(x)A^0(y)|0\rangle + \frac{e}{s_{2W}}(\tilde{\epsilon}\langle 0|h^0(x)G^0(x)A^0(y)|0\rangle + \tilde{\gamma}\langle 0|H^0(x)G^0(x)A^0(y)|0\rangle) + \frac{e}{s_{2W}}(c_{\alpha-\beta}\langle 0|\bar{c}^Z(x)c^Z(y)h^0(y)|0\rangle + s_{\alpha-\beta}\langle 0|\bar{c}^Z(x)c^Z(y)H^0(y)|0\rangle) = 0. \quad \text{(A5)}$$

In a diagrammatic form, we have

$$\begin{aligned}&\frac{1}{q^2-M^2_{Z^0}}\frac{1}{q^2-M^2_{A^0}}\left(iq_\mu \times Z^\mu \dashrightarrow\circlearrowleft\dashrightarrow A^0 + M_{Z^0}\times G^0 \dashrightarrow\circlearrowleft\dashrightarrow A^0\right)\\ &= -\frac{i}{q^2-M^2_{A^0}}\frac{e}{s_{2W}}\left(\tilde{\epsilon}\times \circlearrowleft^{G^0}_{h^0}\dashrightarrow A^0 + \tilde{\gamma}\times \circlearrowleft^{G^0}_{H^0}\dashrightarrow A^0\right)\\ &+ \frac{i}{q^2-M^2_{Z^0}}\frac{e}{s_{2W}}\left(c_{\alpha-\beta}\times \bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{h^0} + s_{\alpha-\beta}\times \bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{H^0}\right), \end{aligned} \quad \text{(A6)}$$

and obtain the relation

$$\begin{aligned} q^2\Sigma_{A^0Z^0}(q^2) + M_{Z^0}\Sigma_{A^0G^0}(q^2) &= -(q^2-M^2_{Z^0})\frac{ie}{s_{2W}}\left(\tilde{\epsilon}\times \circlearrowleft^{G^0}_{h^0}\dashrightarrow A^0 + \tilde{\gamma}\times \circlearrowleft^{G^0}_{H^0}\dashrightarrow A^0\right)\\ &+ (q^2-M^2_{A^0})\frac{ie}{s_{2W}}\left(c_{\alpha-\beta}\times \bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{h^0} + s_{\alpha-\beta}\times \bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{H^0}\right). \end{aligned} \quad \text{(A7)}$$

With the following vertices,

$$\mathcal{L} \supset -\frac{eM_{Z^0}}{s_{2W}}s_{2\beta}(s_{\alpha+\beta}h^0 - c_{\alpha+\beta}H^0)A^0G^0, \quad \text{(A8)}$$

$$\mathcal{L}^{\mathrm{Gh}} \supset \frac{eM_{Z^0}}{s_{2W}}((s_{\alpha-\beta}-\tilde{\epsilon})h^0 - (c_{\alpha-\beta}+\tilde{\gamma})H^0)\bar{c}^Zc^Z, \quad \text{(A9)}$$

we calculate all the "lollipops,"

$$\circlearrowleft^{G^0}_{h^0}\dashrightarrow A^0 = -i\frac{eM_{Z^0}}{s_{2W}}s_{2\beta}s_{\alpha+\beta}B_0(q^2, M^2_{h^0}, M^2_{Z^0}), \quad \text{(A10)}$$

$$\circlearrowleft^{G^0}_{H^0}\dashrightarrow A^0 = i\frac{eM_{Z^0}}{s_{2W}}s_{2\beta}c_{\alpha+\beta}B_0(q^2, M^2_{H^0}, M^2_{Z^0}), \quad \text{(A11)}$$

$$\bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{h^0} = i\frac{eM_{Z^0}}{s_{2W}}(s_{\alpha-\beta}-\tilde{\epsilon})B_0(q^2, M^2_{h^0}, M^2_{Z^0}), \quad \text{(A12)}$$

$$\bar{c}^Z \dashrightarrow\circlearrowleft^{c^Z}_{H^0} = -i\frac{eM_{Z^0}}{s_{2W}}(c_{\alpha-\beta}+\tilde{\gamma})B_0(q^2, M^2_{H^0}, M^2_{Z^0}), \quad \text{(A13)}$$

with

$$B_0(q^2, M_1^2, M_2^2) = C_{\mathrm{UV}} - \int_0^1 dx\,\ln(\Delta(q^2, M_1^2, M_2^2)), \quad \text{(A14)}$$

$$\Delta(q^2, M_1^2, M_2^2) = q^2x^2 - (q^2 + M_2^2 - M_1^2)x + M_2^2. \quad \text{(A15)}$$

We finally obtain the identity

$$\begin{aligned} q^2\Sigma_{A^0Z^0}(q^2) + M_{Z^0}\Sigma_{A^0G^0}(q^2) &= \frac{1}{(4\pi)^2}\frac{e^2M_{Z^0}}{s^2_{2W}}((q^2-M^2_{Z^0})s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2) + (q^2-M^2_{A^0})\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(q^2)),\\ \text{with}\quad \mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2) &= \tilde{\gamma}c_{\alpha+\beta}B_0(q^2, M^2_{H^0}, M^2_{Z^0}) - \tilde{\epsilon}s_{\alpha+\beta}B_0(q^2, M^2_{h^0}, M^2_{Z^0}),\\ \mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(q^2) &= \tilde{\epsilon}c_{\alpha-\beta}B_0(q^2, M^2_{h^0}, M^2_{Z^0}) + \tilde{\gamma}s_{\alpha-\beta}B_0(q^2, M^2_{H^0}, M^2_{Z^0})\\ &+ \frac{1}{2}s_{2(\alpha-\beta)}(B_0(q^2, M^2_{H^0}, M^2_{Z^0}) - B_0(q^2, M^2_{h^0}, M^2_{Z^0})). \end{aligned} \quad \text{(A16)}$$

To implement this formula into SLOOPS and check it numerically, we need to introduce the tadpole part in FORMCALC, and we define $\Sigma^{\rm tad}$ as the self-energy without a tadpole:

$$q^2\Sigma^{\rm tad}_{A^0Z^0}(q^2) + M_{Z^0}\Sigma^{\rm tad}_{A^0G^0}(q^2) + M_{Z^0}\delta T = \frac{1}{(4\pi)^2}\frac{e^2M_{Z^0}}{s^2_{2W}}((q^2 - M^2_{Z^0})s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA} + (q^2 - M^2_{A^0})\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}),$$
$$\text{where } \delta T = \frac{e}{s_{2W}M_{Z^0}}(s_{\alpha-\beta}\delta T_{H^0} + c_{\alpha-\beta}\delta T_{h^0}). \tag{A17}$$

We remark on some simplifications in the functions $\mathcal{F}$ for specific choices of the nonlinear gauge parameters,

$$\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(\tilde{\epsilon} = 0, \tilde{\gamma} = 0) = 0, \tag{A18}$$

$$\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(\tilde{\epsilon} = c_{\alpha+\beta}, \tilde{\gamma} = s_{\alpha+\beta}) = \frac{1}{2}s_{2(\alpha+\beta)}\int_0^1 dx\ln\left(\frac{\Delta(q^2, M^2_{h^0}, M^2_{Z^0})}{\Delta(q^2, M^2_{H^0}, M^2_{Z^0})}\right), \tag{A19}$$

$$\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(\tilde{\epsilon} = s_{\alpha-\beta}, \tilde{\gamma} = -c_{\alpha-\beta}) = 0, \tag{A20}$$

$$\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(\tilde{\epsilon} = 0, \tilde{\gamma} = 0) = \frac{1}{2}s_{2(\alpha-\beta)}\int_0^1 dx\ln\left(\frac{\Delta(q^2, M^2_{h^0}, M^2_{Z^0})}{\Delta(q^2, M^2_{H^0}, M^2_{Z^0})}\right). \tag{A21}$$

In terms of renormalized self-energies,

$$\hat{\Sigma}_{A^0Z^0}(q^2) = \Sigma^{\rm tad}_{A^0Z}(q^2) + \frac{M_{Z^0}}{2}\left(\delta Z_{G^0A^0} + s_{2\beta}\frac{\delta t_\beta}{t_\beta}\right), \tag{A22}$$

$$\hat{\Sigma}_{A^0G^0}(q^2) = \Sigma^{\rm tad}_{A^0G^0}(q^2) + \delta M^2_{A^0G^0} - \tfrac{1}{2}q^2\delta Z_{G^0A^0} - \tfrac{1}{2}(q^2 - M^2_{A^0})\delta Z_{A^0G^0}, \tag{A23}$$

with [Eq. (4.18)]

$$\delta M^2_{A^0G^0} = \delta T - \frac{1}{2}s_{2\beta}M^2_{A^0}\frac{\delta t_\beta}{t_\beta}, \tag{A24}$$

we obtain the following constraint on the *renormalized* two-point functions:

$$\begin{aligned} q^2\hat{\Sigma}_{A^0Z^0}(q^2) + M_{Z^0}\hat{\Sigma}_{A^0G^0}(q^2) &= (q^2 - M^2_{Z^0})\frac{1}{(4\pi)^2}\frac{e^2M_{Z^0}}{s^2_{2W}}s_{2\beta}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{GA}(q^2) \\ &\quad + \frac{M_{Z^0}}{2}(q^2 - M^2_{A^0})\left(\frac{1}{(4\pi)^2}\frac{2e^2}{s^2_{2W}}\mathcal{F}^{\tilde{\epsilon},\tilde{\gamma}}_{cc}(q^2) + s_{2\beta}\frac{\delta t_\beta}{t_\beta} - \delta Z_{AG}\right). \end{aligned} \tag{A25}$$

Note that in this identity δT and, more importantly, $\delta Z_{G^0A^0}$ drop out.

The derivation of the identity for the charged Higgses follows along the same steps. We only quote the result,

$$\begin{aligned} q^2\hat{\Sigma}_{H^+W^+}(q^2) + M_{W^\pm}\hat{\Sigma}_{H^+W^+}(q^2) &= (q^2 - M^2_{W^\pm})\frac{1}{(4\pi)^2}\frac{e^2M_{W^\pm}}{s^2_{2W}}\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(q^2) \\ &\quad + \frac{M_{W^\pm}}{2}(q^2 - M^2_{H^\pm})\left(\frac{1}{(4\pi)^2}\frac{2e^2}{s^2_{2W}}\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{cc}(q^2) + s_{2\beta}\frac{\delta t_\beta}{t_\beta} - \delta Z_{H^\pm G\pm}\right), \end{aligned}$$

with the functions $\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(q^2)$ and $\mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{cc}(q^2)$ defined as

$$\begin{aligned} \mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{HW}(q^2) &= \tilde{\delta}(s_{2\beta}s_{\alpha+\beta} - c^2_Wc_{\alpha-\beta})B_0(q^2, M^2_{W^\pm}, M^2_{h^0}) \\ &\quad - \tilde{\omega}(c_{2\beta}s_{\alpha+\beta} - s^2_Ws_{\alpha-\beta})B_0(q^2, M^2_{W^\pm}, M^2_{H^0}) \\ &\quad + \tilde{\rho}c^2_WB_0(q^2, M^2_{W^\pm}, M^2_{A^0}), \\ \mathcal{G}^{\tilde{\rho},\tilde{\omega},\tilde{\delta}}_{cc}(q^2) &= c_{\alpha-\beta}(s_{\alpha-\beta} - \tilde{\delta})c^2_WB_0(q^2, M^2_{W^\pm}, M^2_{h^0}) \\ &\quad - s_{\alpha-\beta}(c_{\alpha-\beta} + \tilde{\omega})c^2_WB_0(q^2, M^2_{W^\pm}, M^2_{H^0}) \\ &\quad - \tilde{\rho}c^2_WB_0(q^2, M^2_{W^\pm}, M^2_{A^0}). \end{aligned} \tag{A26}$$

APPENDIX B: WAVE-FUNCTION RENORMALIZATION CONSTANTS BEFORE ROTATION

In our approach, field renormalization was performed on the physical fields, or better said, after rotation, to the h^0, H^0, A^0, G^0, $H^\pm$, $G^\pm$ basis. We could have applied field renormalization on the components of the doublets H_1, H_2, Eq. (2.3). To make contact with some of the early papers [15,16,20] on the renormalization of the Higgs sector, we therefore introduce the most general field renormalization on the components of H_1, H_2. We define

$$\begin{pmatrix}\varphi^0_1\\ \varphi^0_2\end{pmatrix}_0 = \begin{pmatrix} Z^{1/2}_{\varphi^0_1} & Z^{1/2}_{\varphi^0_1\varphi^0_2}\\ Z^{1/2}_{\varphi^0_2\varphi^0_1} & Z^{1/2}_{\varphi^0_2}\end{pmatrix}\begin{pmatrix}\varphi^0_1\\ \varphi^0_2\end{pmatrix}, \tag{B1}$$

$$\begin{pmatrix}\phi^\pm_1\\ \phi^\pm_2\end{pmatrix}_0 = \begin{pmatrix} Z^{1/2}_{\phi^\pm_1} & Z^{1/2}_{\phi^\pm_1\phi^\pm_2}\\ Z^{1/2}_{\phi^\pm_2\phi^\pm_1} & Z^{1/2}_{\phi^\pm_2}\end{pmatrix}\begin{pmatrix}\phi^\pm_1\\ \phi^\pm_2\end{pmatrix}, \tag{B2}$$

$$\begin{pmatrix}\phi^0_1\\ \phi^0_2\end{pmatrix}_0 = \begin{pmatrix} Z^{1/2}_{\phi^0_1} & Z^{1/2}_{\phi^0_1\phi^0_2}\\ Z^{1/2}_{\phi^0_2\phi^0_1} & Z^{1/2}_{\phi^0_2}\end{pmatrix}\begin{pmatrix}\phi^0_1\\ \phi^0_2\end{pmatrix}. \tag{B3}$$

As explained in the text, these constants are immediately transformed into the set of matrices Z_P, Z_C, Z_S. Or we can go from the set Z_P, Z_C, Z_S to the set defined by Eqs. (B1)–(B3). For example,

$$\begin{aligned}\delta Z_{G^0} &= c_\beta^2 \delta Z_{\varphi_1^0} + s_\beta^2 \delta Z_{\varphi_2^0} + c_\beta s_\beta(\delta Z_{\varphi_1^0\varphi_2^0} + \delta Z_{\varphi_2^0\varphi_1^0}),\\ \delta Z_{G^0A^0} &= c_\beta s_\beta(\delta Z_{\varphi_2^0} - \delta Z_{\varphi_1^0}) + c_\beta^2 \delta Z_{\varphi_1^0\varphi_2^0} - s_\beta^2 \delta Z_{\varphi_2^0\varphi_1^0},\\ \delta Z_{A^0G^0} &= c_\beta s_\beta(\delta Z_{\varphi_2^0} - \delta Z_{\varphi_1^0}) + c_\beta^2 \delta Z_{\varphi_2^0\varphi_1^0} - s_\beta^2 \delta Z_{\varphi_1^0\varphi_2^0},\\ \delta Z_{A^0} &= s_\beta^2 \delta Z_{\varphi_1^0} + c_\beta^2 \delta Z_{\varphi_2^0} - c_\beta s_\beta(\delta Z_{\varphi_1^0\varphi_2^0} + \delta Z_{\varphi_2^0\varphi_1^0}),\end{aligned} \tag{B4}$$

$$\begin{aligned}\delta Z_{G^\pm} &= c_\beta^2 \delta Z_{\phi_1^\pm} + s_\beta^2 \delta Z_{\phi_2^\pm} + c_\beta s_\beta(\delta Z_{\phi_1^\pm\phi_2^\pm} + \delta Z_{\phi_2^\pm\phi_1^\pm}),\\ \delta Z_{G^\pm H^\pm} &= c_\beta s_\beta(\delta Z_{\phi_2^\pm} - \delta Z_{\phi_1^\pm}) + c_\beta^2 \delta Z_{\phi_1^\pm\phi_2^\pm} - s_\beta^2 \delta Z_{\phi_2^\pm\phi_1^\pm},\\ \delta Z_{H^\pm G^\pm} &= c_\beta s_\beta(\delta Z_{\phi_2^\pm} - \delta Z_{\phi_1^\pm}) + c_\beta^2 \delta Z_{\phi_2^\pm\phi_1^\pm} - s_\beta^2 \delta Z_{\phi_1^\pm\phi_2^\pm},\\ \delta Z_{H^\pm} &= s_\beta^2 \delta Z_{\phi_1^\pm} + c_\beta^2 \delta Z_{\phi_2^\pm} - c_\beta s_\beta(\delta Z_{\phi_1^\pm\phi_2^\pm} + \delta Z_{\phi_2^\pm\phi_1^\pm}),\end{aligned} \tag{B5}$$

$$\begin{aligned}\delta Z_{H^0} &= c_\alpha^2 \delta Z_{\phi_1^0} + s_\alpha^2 \delta Z_{\phi_2^0} + c_\alpha s_\alpha(\delta Z_{\phi_1^0\phi_2^0} + \delta Z_{\phi_2^0\phi_1^0}),\\ \delta Z_{H^0h^0} &= c_\alpha s_\alpha(\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}) + c_\alpha^2 \delta Z_{\phi_1^0\phi_2^0} - s_\alpha^2 \delta Z_{\phi_2^0\phi_1^0},\\ \delta Z_{h^0H^0} &= c_\alpha s_\alpha(\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}) + c_\alpha^2 \delta Z_{\phi_2^0\phi_1^0} - s_\alpha^2 \delta Z_{\phi_1^0\phi_2^0},\\ \delta Z_{h^0} &= s_\alpha^2 \delta Z_{\phi_1^0} + c_\alpha^2 \delta Z_{\phi_2^0} - c_\alpha s_\alpha(\delta Z_{\phi_1^0\phi_2^0} + \delta Z_{\phi_2^0\phi_1^0}),\end{aligned} \tag{B6}$$

$$\begin{aligned}\delta Z_{H^0h^0} + \delta Z_{h^0H^0} &= (c_\alpha^2 - s_\alpha^2)(\delta Z_{\phi_1^0\phi_2^0} + \delta Z_{\phi_2^0\phi_1^0})\\ &\quad + 2c_\alpha s_\alpha(\delta Z_{\phi_2^0} - \delta Z_{\phi_1^0}),\\ \delta Z_{H^0h^0} - \delta Z_{h^0H^0} &= \delta Z_{\phi_1^0\phi_2^0} - \delta Z_{\phi_2^0\phi_1^0}.\end{aligned} \tag{B7}$$

Our renormalization conditions in Eq. (4.36) on $\hat{\Sigma}_{ii}$ will turn into

$$\begin{aligned}\mathrm{Re}\Sigma'_{A^0A^0}(M_{A^0}^2) &= s_\beta^2 \delta Z_{\varphi_1^0} + c_\beta^2 \delta Z_{\varphi_2^0}\\ &\quad - c_\beta s_\beta(\delta Z_{\varphi_1^0\varphi_2^0} + \delta Z_{\varphi_2^0\varphi_1^0}),\end{aligned} \tag{B8}$$

$$\begin{aligned}\mathrm{Re}\Sigma'_{H^\pm H^\pm}(M_{H^\pm}^2) &= s_\beta^2 \delta Z_{\phi_1^\pm} + c_\beta^2 \delta Z_{\phi_2^\pm}\\ &\quad - c_\beta s_\beta(\delta Z_{\phi_1^\pm\phi_2^\pm} + Z_{\phi_2^\pm\phi_1^\pm}),\end{aligned} \tag{B9}$$

$$\begin{aligned}\mathrm{Re}\Sigma'_{H^0H^0}(M_{H^0}^2) &= c_\alpha^2 \delta Z_{\phi_1^0} + s_\alpha^2 \delta Z_{\phi_2^0}\\ &\quad + c_\alpha s_\alpha(\delta Z_{\phi_1^0\phi_2^0} + \delta Z_{\phi_2^0\phi_1^0}),\end{aligned} \tag{B10}$$

$$\begin{aligned}\mathrm{Re}\Sigma'_{h^0h^0}(M_{h^0}^2) &= s_\alpha^2 \delta Z_{\phi_1^0} + c_\alpha^2 \delta Z_{\phi_2^0}\\ &\quad - c_\alpha s_\alpha(\delta Z_{\phi_1^0\phi_2^0} + \delta Z_{\phi_2^0\phi_1^0}).\end{aligned} \tag{B11}$$

In fact, in [15,16,20] only two renormalization constants are introduced, one for each doublet through

$$H_i \to (1 + \tfrac{1}{2}\delta Z_{H_i})H_i, \qquad i = 1, 2. \tag{B12}$$

This means that

$$\begin{aligned}\delta Z_{\phi_i^0} &= \delta Z_{\varphi_i^0} = \delta Z_{\phi_i^\pm} = \delta Z_{H_i},\\ \delta Z_{\phi_i^0\phi_j^0} &= \delta Z_{\varphi_i^0\varphi_j^0} = \delta Z_{\phi_i^\pm\phi_j^\pm} = 0, \qquad i \neq j.\end{aligned} \tag{B13}$$

Since wave-function renormalization is applied on the doublets, it also contributes a shift to v_i. Another shift on this parameter is also applied, $v_i \to v_i - \tilde{\delta} v_i$, as to all other Lagrangian parameters. Compared to our shift δv_i, we have

$$\delta v_i = \tilde{\delta} v_i - \tfrac{1}{2}\delta Z_{H_i} v_i. \tag{B14}$$

Note that with only δZ_{H_1} and δZ_{H_2}, in view of Eqs. (B10), (B11), and (B13) we have

$$\begin{aligned}\delta Z_{H_1} - \delta Z_{H_2} &= -\frac{1}{2c_{2\alpha}}(\mathrm{Re}\Sigma'_{h^0h^0}(M_{h^0}^2)\\ &\quad - \mathrm{Re}\Sigma'_{H^0H^0}(M_{H^0}^2)).\end{aligned} \tag{B15}$$

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