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Exclusive semileptonic decays of $\Lambda_b \to \Lambda l^+ l^-$ in supersymmetric theories

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The weak decays of $\Lambda_b \to \Lambda l^+ l^-$ ($l = e$, μ) are investigated in the minimal supersymmetric standard model (MSSM) and also in supersymmetric (SUSY) SO(10) grand unified models. In the MSSM special attention is paid to the neutral Higgs bosons (NHBs) as they make quite a large contribution in exclusive $B \to X_s l^+ l^-$ decays at large $\tan\beta$ regions of parameter space of SUSY models, since part of SUSY contributions is proportional to $\tan^3\beta$. The analysis of decay rate, forward-backward asymmetries, lepton polarization asymmetries, and the polarization asymmetries of the Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ show that the values of these physical observables are greatly modified by the effects of NHBs. In the SUSY SO(10) grand unified theory model, the new physics contribution comes from the operators which are induced by the NHBs' penguins and also from the operators having chirality opposite to that of the corresponding standard model (SM) operators. SUSY SO(10) effects show up only in the decay $\Lambda_b \to \Lambda + \tau^+ \tau^-$ where the longitudinal and transverse lepton polarization asymmetries deviate significantly from the SM value while the effects in the decay rate, forward-backward asymmetries, and polarization asymmetries of final state Λ baryon are very mild. The transverse lepton polarization asymmetry in $\Lambda_b \to \Lambda + \tau^+ \tau^-$ is almost zero in the SM and in the MSSM model. However, it can reach to -0.1 in the SUSY SO(10) grand unified theory model and could be seen at the future colliders; hence this asymmetry observable will provide us useful information to probe new physics and discriminate between different models.

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I. INTRODUCTION

From the last decade, rare decays induced by flavor-changing neutral currents (FCNCs) $b \to s l^+ l^-$ have become the main focus of the studies due to the CLEO measurement of the radiative decay $b \to s\gamma$ [1]. In the standard model (SM) these decays are forbidden at tree level and can only be induced by the Glashow-Iliopoulos-Maiani mechanism [2] via loop diagrams. Hence, such decays will provide helpful information about the parameters of the Cabbibo-Kobayashi-Maskawa (CKM) matrix [3,4] elements as well as various hadronic form factors. In the literature there have been intensive studies on the exclusive decays $B \to P(V, A) l^+ l^-$ [5–11] both in the SM and beyond, where the notions P, V, and A denote the pseudoscalar, vector, and axial vector mesons, respectively.

It is generally believed that supersymmetry (SUSY) is not only one of the strongest competitors of the SM but is also the most promising candidate of new physics. The reason is that it offers a unique scheme to embed the SM in a more fundamental theory where many theoretical problems such as gauge hierarchy, origin of mass, and Yukawa couplings can be resolved. One direct way to search for SUSY is to discover SUSY particles at high-energy colliders, but unfortunately, so far no SUSY particles have been found. Another way is to search for its effects through indirect methods. The measurement of invariant mass spectrum, forward-backward asymmetry, and polarization asymmetries are the suitable tools to probe new physics effects. For most of the SUSY models, the SUSY contributions to an observable appear at loop level due to the R-parity conservation. Therefore, it has been realized for a long time that rare processes can be used as a good probe for the searches of SUSY, since in these processes the contributions of SUSY and SM arise at the same order in perturbation theory [12].

Motivated from the fact that in the two Higgs doublet model and in other SUSY models, neutral Higgs bosons (NHBs) could contribute largely to the inclusive processes $B \to X_s l^+ l^-$, as part of supersymmetric contributions is proportional to the $\tan^3\beta$ [13]. Subsequently, the physical observables, like branching ratio and forward-backward asymmetry, in the large $\tan\beta$ region of parameter space in SUSY models can be quite different from that in the SM. In addition, similar effects in exclusive $B \to K(K^*) l^+ l^-$ decay modes are also investigated [12], where the analysis of decay rates, forward-backward asymmetries, and polarization asymmetries of the final state lepton indicate the significant role of NHBs. It is believed that physics beyond the SM is essential to explain the problem of neutrino oscillation. To this purpose, a number of SUSY SO(10) models have been proposed in the literature [14–17]. One such model is the SUSY SO(10) grand unified models (GUT), in which there is a complex flavor nondiagonal down-type squark mass matrix element of 2nd and 3rd generations of order one at the GUT scale [16]. This can induce large flavor off-diagonal coupling such as the coupling of the gluino to the quark and squark which belong to different generations. These couplings are in general complex and may contribute to the process of flavor-changing neutral currents (FCNCs). The above analysis of physical observables in $B \to K(K^*)l^+l^-$ decay is extended in the SUSY SO(10) GUT model in Ref. [18]. It is believed that the effects of the counterparts of usual chromomagnetic and electromagnetic dipole moment operators as well as semileptonic operators with opposite chirality are suppressed by m_s/m_b in the SM, but in SUSY SO(10) GUTs their effect can be significant, since δ_{23}^{dRR} can be as large as 0.5 [16,18]. Apart from this, δ_{23}^{dRR} can induce new operators as the counterparts of usual scalar operators in SUSY models due to NHB penguins with gluino-down-type squark propagator in the loop. It has been shown [18] that the forward-backward asymmetries as well as the longitudinal and transverse decay widths of $B \to K(K^*)l^+l^-$ decay are sensitive to these NHBs' effect in the SUSY SO(10) GUT model which can be detected in the future B factories. Apart from these decays, there are some studies in the literature on the rare B decays in some of these SUSY models [19–21].

Compared to the B meson decays, the investigations of FCNC $b \to s$ transition for bottom baryon decays $\Lambda_b \to \Lambda l^+l^-$ are much behind because more degrees of freedom are involved in the bound state of the baryon system at the quark level. From the experimental point of view, the only drawback of bottom baryon decays is that the production rate of the Λ_b baryon in b quark hadronization is about 4 times less than that of the B meson. Theoretically, the major interest in baryonic decays can be attributed to the fact that they can offer a unique ground to extract the helicity structure of the effective Hamiltonian for $b \to s$ transition in the SM and beyond, which is lost in the hadronization of the mesonic case. The key issue in the study of exclusive baryonic decays is to properly evaluate the hadronic matrix elements for $\Lambda_b \to \Lambda$, namely, the transition form factors which are obviously governed by nonperturbative QCD dynamics. Currently, there have been some studies in the literature on $\Lambda_b \to \Lambda$ transition form factors in different models including pole model (PM) [22], covariant oscillator quark model (COQM) [23], MIT bag model (BM)[24], and the nonrelativistic quark model [25], QCD sum rule approach (QCDSR) [26], perturbative QCD (pQCD) approach [27], and also in the light-cone sum rules approach (LCSR) [28]. Using these form factors, the physical observables like decay rates, forward-backward asymmetries, and polarization asymmetries of the Λ baryon as well as of the final state leptons in $\Lambda_b \to \Lambda l^+l^-$ were studied in great detail in the literature [29–37]. It is pointed out that these observables are very sensitive to the new physics, for instance, the polarization asymmetries of the Λ baryon in $\Lambda_b \to \Lambda l^+l^-$ decays heavily depend on the right-handed current, which is much suppressed in the SM [32].

In this paper, we will investigate the exclusive decay $\Lambda_b \to \Lambda l^+l^-$ ($l = \mu, \tau$) both in the minimal supersymmetric standard model (MSSM) as well as in the SUSY SO (10) GUT model [16]. We evaluate the branching ratios, forward-backward asymmetries, lepton polarization asymmetries and polarization asymmetries of the Λ baryon with special emphasis on the effects of NHBs in the MSSM. It is pointed out that different sources of the vector current could manifest themselves in different regions of phase space. For a low value of momentum transfer, the photonic penguin dominates, while the Z penguin and W box become important towards a high value of momentum transfer [12]. In order to search the region of momentum transfer with large contributions from NHBs, the above decay in a certain large $\tan\beta$ region of parameter space has been analyzed in SuperGravity (SUGRA) and M-theory inspired models [38]. We extend this analysis to the SUSY SO(10) GUT model [12], where there are some primed counterparts of the usual SM operators. For instance, the counterparts of usual operators in $B \to X_s\gamma$ decay are suppressed by m_s/m_b and consequently negligible in the SM because they have opposite chiralities. These operators are also suppressed in minimal flavor violating (MFV) models [39,40], however, in the SUSY SO(10) GUT model their effects can be significant. The reason is that the flavor nondiagonal squark mass matrix elements are the free parameters and some of them have significant effects in rare decays of B mesons [41]. In our numerical analysis for $\Lambda_b \to \Lambda l^+l^-$ decays, we shall use the results of the form factors calculated by the LCSR approach in Ref. [28], and the values of the relevant Wilson coefficient for MSSM and SUSY SO(10) GUT models are borrowed from Refs. [12,18]. The effects of SUSY contributions to the decay rate and zero position of forward-backward asymmetry are also explored in this work. Our results show that not only the decay rates are sensitive to the NHBs' contribution but the zero-point of the forward-backward asymmetry also shifts remarkably. It is known that the hadronic uncertainties associated with the form factors and other input parameters have negligible effects on the lepton polarization asymmetries and polarization asymmetries of the Λ baryon in $\Lambda_b \to \Lambda l^+l^-$ decays. We have also studied these asymmetries in the SUSY models mentioned above and found that the effects of NHBs are quite significant in some regions of parameter space of SUSY.

The paper is organized as follows. In Sec. II, we present the effective Hamiltonian for the dilepton decay $\Lambda_b \to \Lambda l^+l^-$. Section III contains the definitions and numbers of the form factors for the said decay using the LCSR approach. In Sec. IV we present the basic formulas of physical observables like decays rate, forward-backward asymmetries (FBAs), and polarization asymmetries of the lepton and that of the Λ baryon in $\Lambda_b \to \Lambda l^+l^-$. Section V is devoted to the numerical analysis of these observables, and the brief summary and concluding remarks are given in Sec. VI.

II. EFFECTIVE HAMILTONIAN

After integrating out the heavy degrees of freedom in the full theory, the general effective Hamiltonian for $b \to sl^+l^-$ in the SUSY SO(10) GUT model can be written as [18]

$$H_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \bigg[\sum_{i=1}^{2} C_i(\mu)O_i(\mu) + \sum_{i=3}^{10} (C_i(\mu)O_i(\mu) + C_i'(\mu)O_i'(\mu)) + \sum_{i=1}^{8} (C_{Q_i}(\mu)Q_i(\mu) + C_{Q_i}'(\mu)Q_i'(\mu)) \bigg], \tag{1}$$

where $O_i(\mu)$ $(i = 1, \ldots, 10)$ are the four-quark operators and $C_i(\mu)$ are the corresponding Wilson coefficients at the energy scale μ [42]. Using renormalization group equations to resum the QCD corrections, Wilson coefficients are evaluated at the energy scale $\mu = m_b$. The theoretical uncertainties associated with the renormalization scale can be substantially reduced when the next-to-leading-logarithm corrections are included [43]. The new operators $Q_i(\mu)$ $(i = 1, \ldots, 8)$ come from the NHBs' exchange diagrams, whose manifest forms and corresponding Wilson coefficients can be found in [44,45]. The primed operators are the counterparts of the unprimed operators, which can be obtained by flipping the chiralities in the corresponding unprimed operators. It needs to be pointed out that these primed operators will appear only in the SUSY SO(10) GUT model and are absent in the SM and MSSM [12].

The explicit expressions of the operators responsible for $\Lambda_b \to \Lambda l^+l^-$ transition are given by

$$\begin{aligned}
O_7 &= \frac{e^2}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu}P_R b)F^{\mu\nu}, \quad O_7' = \frac{e^2}{16\pi^2} m_b(\bar{s}\sigma_{\mu\nu}P_L b)F^{\mu\nu} \quad O_9 = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l),\\
O_9' &= \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l) \quad O_{10} = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma_5 l), \quad O_{10}' = \frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu\gamma_5 l)\\
Q_1 &= \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{l}l), \qquad Q_1' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{l}l) \quad Q_2 = \frac{e^2}{16\pi^2}(\bar{s}P_R b)(\bar{l}\gamma_5 l), \quad Q_2' = \frac{e^2}{16\pi^2}(\bar{s}P_L b)(\bar{l}\gamma_5 l)
\end{aligned} \tag{2}$$

with $P_{L,R} = (1 \pm \gamma_5)/2$. In terms of the above Hamiltonian, the free quark decay amplitude for $b \to sl^+l^-$ can be derived as [13]:

$$\begin{aligned}
\mathcal{M}(b \to sl^+l^-) = -\frac{G_F\alpha}{\sqrt{2}\pi} V_{tb}V_{ts}^* \bigg\{ & C_9^{\rm eff}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l) + C_{10}(\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu\gamma_5 l) - 2m_b C_7^{\rm eff}\\
& \times \left(\bar{s}i\sigma_{\mu\nu}\frac{q^\nu}{s}P_R b\right)(\bar{l}\gamma^\mu l) + C_{Q_1}(\bar{s}P_R b) \times (\bar{l}l) + C_{Q_2}(\bar{s}P_R b)(\bar{l}\gamma_5 l)\\
& + (C_i(m_b) \leftrightarrow C_i'(m_b)) \bigg\}
\end{aligned} \tag{3}$$

where $s = q^2$ and $q = p_{\Lambda_b} - p_\Lambda$ is the momentum transfer. Because of the absence of the Z boson in the effective theory, the operator O_{10} can not be induced by the insertion of four-quark operators. Therefore, the Wilson coefficient C_{10} does not renormalize under QCD corrections and hence it is independent on the energy scale. Moreover, the above quark level decay amplitude can receive additional contributions from the matrix element of four-quark operators, $\sum_{i=1}^{6}\langle l^+l^-s|O_i|b\rangle$, which are usually absorbed into the effective Wilson coefficient $C_9^{\rm eff}(\mu)$. To be more specific, we can decompose $C_9^{\rm eff}(\mu)$ into the following three parts [46–52]

$$C_9^{\rm eff}(\mu) = C_9(\mu) + Y_{\rm SD}(z, s') + Y_{\rm LD}(z, s'),$$

where the parameters z and s' are defined as $z = m_c/m_b$, $s' = q^2/m_b^2$. $Y_{\rm SD}(z, s')$ describes the short-distance contributions from four-quark operators far away from the $c\bar{c}$ resonance regions, which can be calculated reliably in the perturbative theory. The long-distance contributions $Y_{\rm LD}(z, s')$ from four-quark operators near the $c\bar{c}$ resonance cannot be calculated from first principles of QCD and are usually parametrized in the form of a phenomenological Breit-Wigner formula making use of the vacuum saturation approximation and quark-hadron duality. The manifest expressions for $Y_{\rm SD}(z, s')$ and $Y_{\rm LD}(z, s')$ can be written as [28–32]

$$\begin{aligned}
Y_{\rm SD}(z, s') = {} & h(z, s')(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) - \frac{1}{2}h(1, s')(4C_3(\mu)\\
& + 4C_4(\mu) + 3C_5(\mu) + C_6(\mu)) - \frac{1}{2}h(0, s') \times (C_3(\mu) + 3C_4(\mu)) + \frac{2}{9}(3C_3(\mu) + C_4(\mu)\\
& + 3C_5(\mu) + C_6(\mu)),
\end{aligned} \tag{4}$$

$$\begin{aligned}
Y_{\rm LD}(z, s') = {} & \frac{3}{\alpha_{\rm em}^2}(3C_1(\mu) + C_2(\mu) + 3C_3(\mu) + C_4(\mu) + 3C_5(\mu) + C_6(\mu)) \sum_{j=\psi,\psi'} \omega_j(q^2)k_j\\
& \times \frac{\pi\Gamma(j \to l^+l^-)M_j}{q^2 - M_j^2 + iM_j\Gamma_j^{\rm tot}},
\end{aligned} \tag{5}$$

with

$$h(z, s') = -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2} \times \begin{cases} \ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi & \text{for } x \equiv 4z^2/s' < 1 \\ 2\arctan\frac{1}{\sqrt{x-1}} & \text{for } x \equiv 4z^2/s' > 1 \end{cases},$$

$$h(0, s') = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}i\pi. \quad (6)$$

The nonfactorizable effects [53–56] from the charm loop can bring about further corrections to the radiative $b \to s\gamma$ transition, which can be absorbed into the effective Wilson coefficient $C_7^{\rm eff}$. Specifically, the Wilson coefficient $C_7^{\rm eff}$ is given by [32]

$$C_7^{\rm eff}(\mu) = C_7(\mu) + C_{b\to s\gamma}(\mu),$$

with

$$C_{b\to s\gamma}(\mu) = i\alpha_s\left[\frac{2}{9}\eta^{14/23}(G_1(x_t) - 0.1687) - 0.03C_2(\mu)\right], \quad (7)$$

$$G_1(x) = \frac{x(x^2-5x-2)}{8(x-1)^3} + \frac{3x^2\ln^2 x}{4(x-1)^4}, \quad (8)$$

where $\eta = \alpha_s(m_W)/\alpha_s(\mu)$, $x_t = m_t^2/m_W^2$, $C_{b\to s\gamma}$ is the absorptive part for the $b \to sc\bar{c} \to s\gamma$ rescattering and we have dropped out the tiny contributions proportional to the CKM sector $V_{ub}V_{us}^*$. In addition, $C_7'^{\rm eff}(\mu)$ and $C_9'^{\rm eff}(\mu)$ can be obtained by replacing the unprimed Wilson coefficients with the corresponding prime ones in the above formula.

III. MATRIX ELEMENTS AND FORM FACTORS IN LIGHT-CONE SUM RULES

With the free quark decay amplitude available, we can proceed to calculate the decay amplitudes for $\Lambda_b \to \Lambda\gamma$ and $\Lambda_b \to \Lambda l^+ l^-$ at hadron level, which can be obtained by sandwiching the free quark amplitudes between the initial and final baryon states. Consequently, the following four hadronic matrix elements,

$$\begin{aligned} &\langle\Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P+q)\rangle, \\ &\langle\Lambda(P)|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P+q)\rangle, \\ &\langle\Lambda(P)|\bar{s}\sigma_{\mu\nu} b|\Lambda_b(P+q)\rangle, \\ &\langle\Lambda(P)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\Lambda_b(P+q)\rangle, \end{aligned} \quad (9)$$

need to be computed. Generally, the above matrix elements can be parametrized in terms of the form factors as [32–36]:

$$\langle\Lambda(P)|\bar{s}\gamma_\mu b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)(g_1\gamma_\mu + g_2 i\sigma_{\mu\nu}q^\nu + g_3 q_\mu)\Lambda_b(P+q), \quad (10)$$

$$\langle\Lambda(P)|\bar{s}\gamma_\mu\gamma_5 b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)(G_1\gamma_\mu + G_2 i\sigma_{\mu\nu}q^\nu + G_3 q_\mu)\gamma_5\Lambda_b(P+q), \quad (11)$$

$$\langle\Lambda(P)|\bar{s}\sigma_{\mu\nu} b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)[h_1\sigma_{\mu\nu} - ih_2(\gamma_\mu q_\nu - \gamma_\nu q_\mu) - ih_3(\gamma_\mu P_\nu - \gamma_\nu P_\mu) - ih_4(P_\mu q_\nu - P_\nu q_\mu)]\Lambda_b(P+q), \quad (12)$$

$$\langle\Lambda(P)|\bar{s}\sigma_{\mu\nu}\gamma_5 b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)[H_1\sigma_{\mu\nu} - iH_2(\gamma_\mu q_\nu - \gamma_\nu q_\mu) - iH_3(\gamma_\mu P_\nu - \gamma_\nu P_\mu) - iH_4(P_\mu q_\nu - P_\nu q_\mu)]\gamma_5\Lambda_b(P+q), \quad (13)$$

where all the form factors g_i, G_i, h_i and H_i are functions of the square of momentum transfer q^2. Contracting Eqs. (12) and (13) with the four momentum q^μ on both sides and making use of the equations of motion

$$q^\mu(\bar{\psi}_1\gamma_\mu\psi_2) = (m_1 - m_2)\bar{\psi}_1\psi_2 \quad (14)$$

$$q^\mu(\bar{\psi}_1\gamma_\mu\gamma_5\psi_2) = -(m_1 + m_2)\bar{\psi}_1\gamma_5\psi_2 \quad (15)$$

we have

$$\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}q^\nu b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)(f_1\gamma_\mu + f_2 i\sigma_{\mu\nu}q^\nu + f_3 q_\mu)\Lambda_b(P+q), \quad (16)$$

$$\langle\Lambda(P)|\bar{s}i\sigma_{\mu\nu}\gamma_5 q^\nu b|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)(F_1\gamma_\mu + F_2 i\sigma_{\mu\nu}q^\nu + F_3 q_\mu)\gamma_5\Lambda_b(P+q), \quad (17)$$

with

$$f_1 = \frac{2h_2 - h_3 + h_4(m_{\Lambda_b} + m_\Lambda)}{2}q^2, \quad (18)$$

$$f_2 = \frac{2h_1 + h_3(m_\Lambda - m_{\Lambda_b}) + h_4 q^2}{2}, \quad (19)$$

$$f_3 = \frac{m_\Lambda - m_{\Lambda_b}}{q^2}f_1, \quad (20)$$

$$F_1 = \frac{2H_2 - H_3 + H_4(m_{\Lambda_b} - m_\Lambda)}{2}q^2, \quad (21)$$

$$F_2 = \frac{2H_1 + H_3(m_\Lambda + m_{\Lambda_b}) + H_4 q^2}{2}, \tag{22}$$

$$F_3 = \frac{m_\Lambda + m_{\Lambda_b}}{q^2} F_1. \tag{23}$$

Because of the conservation of vector current, the form factors f_3 and g_3 do not contribute to the decay amplitude of $\Lambda_b \to \Lambda l^+ l^-$. To incorporate the NHBs' effect one needs to calculate the matrix elements involving the scalar $\bar{s}b$ and the pseudoscalar $\bar{s}\gamma_5 b$ currents, which can be parametrized as

$$\langle \Lambda(P)|\bar{s}b|\Lambda_b(P+q)\rangle = \frac{1}{m_b + m_s}\bar{\Lambda}(P)[g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2]\Lambda_b(P+q), \tag{24}$$

$$\langle \Lambda(P)|\bar{s}\gamma_5 b|\Lambda_b(P+q)\rangle = \frac{1}{m_b - m_s}\bar{\Lambda}(P)[G_1(m_{\Lambda_b} + m_\Lambda) - G_3 q^2]\gamma_5\Lambda_b(P+q). \tag{25}$$

The various form factors f_i and g_i appearing in the above equations are not independent in the heavy quark limit and one can express them in terms of two independent form factors ξ_1 and ξ_2 in HQET defined by [28]

$$\langle \Lambda(P)|\bar{b}\Gamma s|\Lambda_b(P+q)\rangle = \bar{\Lambda}(P)[\xi_1(q^2) + \not{v}\xi_2(q^2)] \times \Gamma\Lambda_b(P+q), \tag{26}$$

with Γ being an arbitrary Lorentz structure and v_μ being the four-velocity of the Λ_b baryon. Comparing Eqs. (10), (11), (16), and (17) and Eq. (26), one can arrive at [32–36]

$$f_1 = F_1 = \frac{q^2}{m_{\Lambda_b}}\xi_2, \tag{27}$$

$$f_2 = F_2 = g_1 = G_1 = \xi_1 + \frac{m_\Lambda}{m_{\Lambda_b}}\xi_2, \tag{28}$$

$$f_3 = \frac{m_\Lambda - m_{\Lambda_b}}{m_{\Lambda_b}}\xi_2, \tag{29}$$

$$F_3 = \frac{m_\Lambda + m_{\Lambda_b}}{m_{\Lambda_b}}\xi_2, \tag{30}$$

$$g_2 = G_2 = g_3 = G_3 = \frac{\xi_2}{m_{\Lambda_b}}. \tag{31}$$

It is known that Eq. (26) is successful at the zero recoil region (with large q^2) in the heavy quark limit. As for the large recoil region, one might think that these relations would be broken since light degrees of freedom could receive large excitations. However, as pointed out in Ref. [57], the effective theory of soft-collinear interactions and HQET are independent on both the energy of the light hadron (E) and heavy quark mass (m_b) in the limit of $E \to \infty$, which indicates that Eq. (26) is still well defined owing to the tiny effects from $1/m_b$, $1/E$, and α_s corrections [57] under the assumption of the Feynman mechanism.

TABLE I. Numerical results for the form factors $f_2(0)$, $g_2(0)$, and parameters a_1 and a_2 involved in the double-pole fit of Eq. (32) for both twist-3 and twist-6 sum rules with $M_B^2 \in [3.0, 6.0]$ GeV^2, $s_0 = 39 \pm 1$ GeV^2.

Parameter	Twist-3	Up to twist-6
$f_2(0)$	$0.14^{+0.02}_{-0.01}$	$0.15^{+0.02}_{-0.02}$
a_1	$2.91^{+0.10}_{-0.07}$	$2.94^{+0.11}_{-0.06}$
a_2	$2.26^{+0.13}_{-0.08}$	$2.31^{+0.14}_{-0.10}$
$g_2(0)(10^{-2}$ GeV$^{-1})$	$-0.47^{+0.06}_{-0.06}$	$1.3^{+0.2}_{-0.4}$
a_1	$3.40^{+0.06}_{-0.05}$	$2.91^{+0.12}_{-0.09}$
a_2	$2.98^{+0.09}_{-0.08}$	$2.24^{+0.17}_{-0.13}$

Because of our poor understanding of nonperturbative QCD dynamics, one has to rely on some approaches to calculate the form factors answering for $\Lambda_b \to \Lambda$ transition. It is suggested that the soft nonperturbative contribution to the transition form factor can be calculated quantitatively in the framework of the LCSR approach [58–62], which is a fully relativistic approach and well rooted in quantum field theory, in a systematic and almost model-independent way. As a marriage of standard QCDSR technique [63–65] and theory of hard exclusive process [66–73], LCSR cures the problem of QCDSR applying to the large momentum transfer by performing the operator product expansion (OPE) in terms of twist of the relevant operators rather than their dimension [74]. Therefore, the principal discrepancy between QCDSR and LCSR consists in that nonperturbative vacuum condensates representing the long-distance quark and gluon interactions in the short-distance expansion are substituted by the light-cone distribution amplitudes (LCDAs) describing the distribution of longitudinal momentum carried by the valence quarks of the hadronic bound system in the expansion of transverse-distance between partons in the infinite momentum frame.

Considering the distribution amplitude up to twist-6, the form factors for $\Lambda_b \to \Lambda l^+ l^-$ have been calculated in [28] to the accuracy of leading conformal spin, where the pole model was also employed to extend the results to the whole kinematical region. Specifically, the dependence of form factors on transfer momentum are parametrized as

$$\xi_i(q^2) = \frac{\xi_i(0)}{1 - a_1 q^2/m_{\Lambda_b}^2 + a_2 q^4/m_{\Lambda_b}^4}, \tag{32}$$

where ξ_i denotes the form factors f_2 and g_2. The numbers of parameters $\xi_i(0)$, a_1, a_2 have been collected in Table I.

IV. FORMULA FOR OBSERVABLES

In this section, we proceed to perform the calculations of some interesting observables in phenomenology including decay rates, forward-backward asymmetry, and polarization asymmetries of the final state lepton and of the Λ baryon. From Eq. (3), it is straightforward to obtain the decay amplitude for $\Lambda_b \to \Lambda l^+ l^-$ as

$$\mathcal{M}_{\Lambda_b \to \Lambda l^+ l^-} = -\frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* [T_\mu^1 (\bar{l}\gamma^\mu l) + T_\mu^2 (\bar{l}\gamma^\mu \gamma_5 l) + T^3 (\bar{l} l)], \tag{33}$$

where the auxiliary functions T_μ^1, T_μ^2, and T^3 are given by

$$\begin{aligned} T_\mu^1 = \bar{\Lambda}(P)\Big[&\{\gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu\} C_9^{\rm eff} + \{\gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) \\ &+ (g_3 + G_3\gamma_5) q_\mu\} C_9^{\prime\rm eff} - \frac{2m_b}{s}\{\gamma_\mu (f_1 + F_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (f_2 + F_2\gamma_5) + (f_3 + F_3\gamma_5) q_\mu\} C_7^{\rm eff} \\ &- \frac{2m_b}{s}\{\gamma_\mu (f_1 - F_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (f_2 - F_2\gamma_5) + (f_3 - F_3\gamma_5) q_\mu\} C_7^{\prime\rm eff} \Big] \Lambda_b(P+q), \end{aligned} \tag{34}$$

$$\begin{aligned} T_\mu^2 = \bar{\Lambda}(P)\Big[&\{\gamma_\mu (g_1 - G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 - G_2\gamma_5) + (g_3 - G_3\gamma_5) q_\mu\} C_{10} + \{\gamma_\mu (g_1 + G_1\gamma_5) + i\sigma_{\mu\nu} q^\nu (g_2 + G_2\gamma_5) \\ &+ (g_3 + G_3\gamma_5) q_\mu\} C_{10}^\prime - \frac{q_\mu}{2m_l m_b}\{[g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2] + [G_1(m_{\Lambda_b} + m_\Lambda) - G_3 q^2]\gamma_5\} C_{Q_2} \\ &- \frac{q_\mu}{2m_l m_b}\{[g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2] - [G_1(m_{\Lambda_b} + m_\Lambda) - G_3 q^2]\gamma_5\} C_{Q_2}^\prime \Big] \Lambda_b(P+q), \end{aligned} \tag{35}$$

and

$$\begin{aligned} T^3 = \frac{1}{m_b}\bar{\Lambda}(P)[&\{[g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2] + [G_1(m_{\Lambda_b} + m_\Lambda) \\ &- G_3 q^2]\gamma_5\} C_{Q_1} + \{[g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2] \\ &- [G_1(m_{\Lambda_b} + m_\Lambda) - G_3 q^2]\gamma_5\} C_{Q_1}^\prime] \Lambda_b(P+q). \end{aligned} \tag{36}$$

For convenience, we can also rewrite the decay amplitude in the following form:

$$\mathcal{M}_{\Lambda_b \to \Lambda l^+ l^-} = -\frac{G_F \alpha}{2\sqrt{2}\pi} V_{tb} V_{ts}^* [T_\mu^1 (\bar{l}\gamma^\mu l) + T_\mu^2 (\bar{l}\gamma^\mu \gamma_5 l) + T^3 (\bar{l} l)], \tag{37}$$

with

$$\begin{aligned} T_\mu^1 &= \bar{\Lambda}(P)[\gamma_\mu (A_1 + A_2\gamma_5) + i\sigma_{\mu\nu} q^\nu (B_1 + B_2\gamma_5) \\ &\quad + q_\mu (D_1 + D_2\gamma_5)] \Lambda_b(P+q), \\ T_\mu^2 &= \bar{\Lambda}(P)[\gamma_\mu (A_3 + A_4\gamma_5) + i\sigma_{\mu\nu} q^\nu (B_3 + B_4\gamma_5) \\ &\quad + q_\mu (D_3 + D_4\gamma_5)] \Lambda_b(P+q), \\ T_\mu^3 &= \bar{\Lambda}(P)(E_1 + E_2\gamma_5) \Lambda_b(P+q). \end{aligned} \tag{38}$$

The functions A_i, B_i, D_i, and E_i are defined as

$$\begin{aligned} &A_1 = g_1(C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} f_1(C_7^{\rm eff} + C_7^{\prime\rm eff}), \quad A_2 = G_1(-C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} f_1(C_7^{\rm eff} - C_7^{\prime\rm eff}), \\ &B_1 = g_2(C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} f_2(C_7^{\rm eff} + C_7^{\prime\rm eff}), \quad B_2 = G_2(-C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} F_2(C_7^{\rm eff} - C_7^{\prime\rm eff}), \\ &D_1 = g_3(C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} f_3(C_7^{\rm eff} + C_7^{\prime\rm eff}), \quad D_2 = G_3(-C_9^{\rm eff} + C_9^{\prime\rm eff}) - \frac{2m_b}{s} F_3(C_7^{\rm eff} - C_7^{\prime\rm eff}), \\ &A_3 = g_1(C_{10} + C_{10}^\prime), \quad A_4 = G_1(-C_{10} + C_{10}^\prime), \quad B_3 = g_2(C_{10} + C_{10}^\prime), \quad B_4 = G_2(-C_{10} + C_{10}^\prime), \\ &D_3 = g_3(C_{10} + C_{10}^\prime) - \frac{g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2}{2m_l m_b}(C_{Q_2} + C_{Q_2}^\prime), \quad D_4 = G_3(-C_{10} + C_{10}^\prime) - \frac{G_1(m_{\Lambda_b} - m_\Lambda) + G_3 q^2}{2m_l m_b}(C_{Q_2} - C_{Q_2}^\prime), \\ &E_1 = \frac{g_1(m_{\Lambda_b} - m_\Lambda) + g_3 q^2}{m_b}(C_{Q_1} + C_{Q_1}^\prime), \quad E_2 = \frac{G_1(m_{\Lambda_b} - m_\Lambda) - G_3 q^2}{m_b}(C_{Q_1} - C_{Q_1}^\prime). \end{aligned} \tag{39}$$

It needs to be pointed out that the terms proportional to q_μ in T_μ^1, namely D_1 and D_2, do not contribute to the decay amplitude with the help of the equation of motion for lepton fields. Besides, one can also find that the above results can indeed reproduce those obtained in the SM with $C_i^\prime = 0$ and $T^3 = 0$.

A. The differential decay rates of $\Lambda_b \to \Lambda l^+ l^-$

The differential decay width of $\Lambda_b \to \Lambda l^+ l^-$ in the rest frame of the Λ_b baryon can be written as [75],

$$\frac{d\Gamma(\Lambda_b \to \Lambda l^+ l^-)}{ds} = \frac{1}{(2\pi)^3}\frac{1}{32 m_{\Lambda_b}^3} \times \int_{u_{\min}}^{u_{\max}} |\tilde{M}_{\Lambda_b \to \Lambda l^+ l^-}|^2 du, \tag{40}$$

where $u = (p_\Lambda + p_{l^-})^2$ and $s = (p_{l^+} + p_{l^-})^2$; p_Λ, p_{l^+}, and p_{l^-} are the four-momenta vectors of Λ, l^+, and l^-, respectively. $\tilde{M}_{\Lambda_b \to \Lambda l^+ l^-}$ denotes the decay amplitude after performing the integration over the angle between the l^- and Λ baryon. The upper and lower limits of u are given by

$$u_{\max} = (E_\Lambda^* + E_l^*)^2 - \left(\sqrt{E_\Lambda^{*2} - m_\Lambda^2} - \sqrt{E_l^{*2} - m_l^2}\right)^2,$$
$$u_{\min} = (E_\Lambda^* + E_l^*)^2 - \left(\sqrt{E_\Lambda^{*2} - m_\Lambda^2} + \sqrt{E_l^{*2} - m_l^2}\right)^2, \tag{41}$$

where E_Λ^* and E_l^* are the energies of Λ and l^- in the rest frame of lepton pair

$$E_\Lambda^* = \frac{m_{\Lambda_b}^2 - m_\Lambda^2 - s}{2\sqrt{s}}, \qquad E_l^* = \frac{\sqrt{s}}{2}. \tag{42}$$

Putting everything together, we can achieve the decay rates and invariant mass distributions of $\Lambda_b \to \Lambda l^+ l^-$ with and without long-distance contributions as

$$\begin{aligned}
\frac{d\Gamma}{ds} = {} & \frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2}{128 m_{\Lambda_b}^3 \pi^4}\sqrt{1 - \frac{4m_l^2}{s}}\sqrt{\lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}\bigg\{\frac{m_{\Lambda_b} m_l}{2m_b} f_2^2 (m_{\Lambda_b} - m_\Lambda)(s - (m_\Lambda + m_{\Lambda_b})^2)(C_{Q_1}C_{10}^* + C_{Q_1}^* C_{10}) \\
& + m_{\Lambda_b} m_b (1 + 2m_l^2/s)((m_{\Lambda_b}^2 - m_\Lambda^2 - t)(f_2^2 + g_2^2 s) - 4 f_2 g_2 s m_{\Lambda_b})(C_9^{*\mathrm{eff}} C_7^{\mathrm{eff}} + C_7^{*\mathrm{eff}} C_9^{\mathrm{eff}}) + s\frac{m_{\Lambda_b}^2}{2m_b^2}((m_\Lambda + m_{\Lambda_b})^2 \\
& - s) f_2^2 |C_{Q_2}|^2 + \frac{2}{3s} m_b^2 (1 + 2m_l^2/s)(\lambda(2f_2^2 + g_2^2 s) + 3s(m_{\Lambda_b}^2 + m_\Lambda^2 - s)(f_2^2 + g_2^2 s) + 6 f_2 g_2 s m_\Lambda (m_{\Lambda_b}^2 - m_\Lambda^2 + s))|C_7^{\mathrm{eff}}|^2 \\
& + \frac{1}{6}(1 + 2m_l^2/s)(\lambda(f_2^2 + 2g_2^2 s) + 3s(m_{\Lambda_b}^2 + m_\Lambda^2 - s)(f_2^2 + g_2^2 s) + 6 f_2 g_2 s m_\Lambda (m_{\Lambda_b}^2 - m_\Lambda^2 + s))|C_9^{\mathrm{eff}}|^2 \\
& + \frac{1}{6}(((1 + 2m_l^2/s)\lambda + 3(1 - 2m_l^2/s)(m_{\Lambda_b}^2 + m_\Lambda^2 - s)) f_2^2 - g_2^2 s (1 - 4m_l^2/s)(\lambda - 3((m_{\Lambda_b}^2 - m_\Lambda^2)^2 + (m_{\Lambda_b}^2 + m_\Lambda^2) s)) \\
& + 6 f_2 g_2 s m_\Lambda (1 - 4m_l^2/s)(m_{\Lambda_b}^2 - m_\Lambda^2 + s))|C_{10}|^2\bigg\},
\end{aligned} \tag{43}$$

where

$$\lambda = \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s) = m_{\Lambda_b}^4 + m_\Lambda^4 + s^2 - 2m_{\Lambda_b}^2 m_\Lambda^2 - 2m_\Lambda^2 s - 2 s m_{\Lambda_b}^2. \tag{44}$$

In Eq. (43) we have given the result in the MSSM with NHBs and ignored the contribution from the primed operators which appear in the SUSY SO(10) GUT model as the results are very tiny.

B. FBAs of $\Lambda_b \to \Lambda l^+ l^-$

Now we are in a position to explore the FBAs of $\Lambda_b \to \Lambda l^+ l^-$, which is an essential observable sensitive to the new physics effects. To calculate the forward-backward asymmetry, we consider the following double differential decay rate formula for the process $\Lambda_b \to \Lambda l^+ l^-$

$$\frac{d^2\Gamma(s, \cos\theta)}{ds\, d\cos\theta} = \frac{1}{(2\pi)^3}\frac{1}{64 m_{\Lambda_b}^3}\lambda^{1/2}(m_{\Lambda_b}^2, m_\Lambda^2, s) \times \sqrt{1 - \frac{4m_l^2}{s}}|\tilde{M}_{\Lambda_b \to \Lambda l^+ l^-}|^2, \tag{45}$$

where θ is the angle between the momentum of the Λ_b baryon and l^- in the dilepton rest frame. Following Refs. [32,52], the differential and normalized FBAs for the semileptonic decay $\Lambda_b \to \Lambda l^+ l^-$ are defined as

$$\frac{dA_{FB}(q^2)}{ds} = \int_0^1 d\cos\theta \frac{d^2\Gamma(s, \cos\theta)}{ds\, d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma(s, \cos\theta)}{ds\, d\cos\theta} \tag{46}$$

and

$$A_{FB}(q^2) = \frac{\int_0^1 d\cos\theta \frac{d^2\Gamma(s,\cos\theta)}{ds\, d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d^2\Gamma(s,\cos\theta)}{ds\, d\cos\theta}}{\int_0^1 d\cos\theta \frac{d^2\Gamma(s,\cos\theta)}{ds\, d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d^2\Gamma(s,\cos\theta)}{ds\, d\cos\theta}}. \tag{47}$$

Following the same procedure as we did for the differential decay rate, one can easily get the expression for the forward-backward asymmetry.

C. Lepton polarization asymmetries of $\Lambda_b \to \Lambda l^+ l^-$

In the rest frame of the lepton l^-, the unit vectors along the longitudinal, normal, and transversal component of the l^- can be defined as [76]:

$$s_L^{-\mu} = (0, \vec{e}_L) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right),$$
$$s_N^{-\mu} = (0, \vec{e}_N) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_-}{|\vec{p}_\Lambda \times \vec{p}_-|}\right), \tag{48}$$
$$s_T^{-\mu} = (0, \vec{e}_T) = (0, \vec{e}_N \times \vec{e}_L),$$

where $\vec{p}_-$ and $\vec{p}_\Lambda$ are the three-momenta of the lepton l^- and Λ baryon, respectively, in the center mass (CM) frame of the l^+l^- system. Lorentz transformation is used to boost the longitudinal component of the lepton polarization to the CM frame of the lepton pair as

$$(s_L^{-\mu})_{\mathrm{CM}} = \left(\frac{|\vec{p}_-|}{m_l}, \frac{E_l \vec{p}_-}{m_l|\vec{p}_-|}\right) \tag{49}$$

where E_l and m_l are the energy and mass of the lepton in the CM frame. The normal and transverse components remain unchanged under the Lorentz boost.

The longitudinal (P_L), normal (P_N) and transverse (P_T) polarizations of lepton can be defined as:

$$P_i^{(\mp)}(s) = \frac{\frac{d\Gamma}{ds}(\vec{\xi}^{\mp} = \vec{e}^{\mp}) - \frac{d\Gamma}{ds}(\vec{\xi}^{\mp} = -\vec{e}^{\mp})}{\frac{d\Gamma}{ds}(\vec{\xi}^{\mp} = \vec{e}^{\mp}) + \frac{d\Gamma}{ds}(\vec{\xi}^{\mp} = -\vec{e}^{\mp})} \tag{50}$$

where $i = L, N, T$, and $\vec{\xi}^{\mp}$ is the spin direction along the leptons $l^{\mp}$. The differential decay rate for the polarized lepton $l^{\mp}$ in $\Lambda_b \to \Lambda l^+ l^-$ decay along any spin direction $\vec{\xi}^{\mp}$ is related to the unpolarized decay rate (40) with the following relation:

$$\frac{d\Gamma(\vec{\xi}^{\mp})}{ds} = \frac{1}{2}\left(\frac{d\Gamma}{ds}\right)[1 + (P_L^{\mp}\vec{e}_L^{\mp} + P_N^{\mp}\vec{e}_N^{\mp} + P_T^{\mp}\vec{e}_T^{\mp}) \cdot \vec{\xi}^{\mp}]. \tag{51}$$

We can achieve the expressions of longitudinal, normal, and transverse polarizations for $\Lambda_b \to \Lambda l^+ l^-$ decays as collected below, where only the results in MSSM models with NHB are given for the conciseness of this paper. Thus the longitudinal lepton polarization can be written as

$$\begin{aligned} P_L(s) = {}& \left(1/\frac{d\Gamma}{ds}\right)\frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}{768 m_b^2 m_{\Lambda_b}^3 \pi^5}\left(1 - \frac{4m_l^2}{s}\right)\{-6m_l m_b m_{\Lambda_b} f_2^2 (m_\Lambda - m_{\Lambda_b})(s - (m_\Lambda + m_{\Lambda_b})^2)(C_{Q_1}C_{10}^* \\ & + C_{Q_1}^* C_{10}) - 6m_{\Lambda_b}^2 f_2^2 s(s - (m_\Lambda + m_{\Lambda_b})^2)(C_{Q_1}^* C_{Q_2} + C_{Q_1}C_{Q_2}^*) + m_b^2(3(m_\Lambda^4 + m_{\Lambda_b}^4 - s^2)(f_2^2 + g_2^2 s) \\ & - 2m_{\Lambda_b}^2 m_\Lambda^2 (f_2^2 + g_2^2 s) - \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)(f_2^2 - g_2^2 s))(C_9^{*\mathrm{eff}} C_{10} + C_9^{\mathrm{eff}} C_{10}^*) - 12 m_b^3 m_{\Lambda_b}((m_\Lambda^2 - m_{\Lambda_b}^2 + s)(f_2^2 \\ & - g_2^2 s) - 4f_2 g_2 m_\Lambda s)(C_{10}^* C_7^{\mathrm{eff}} + C_7^{*\mathrm{eff}} C_{10})\}. \end{aligned} \tag{52}$$

Similarly, the normal lepton polarization is

$$\begin{aligned} P_N(s) = {}& \left(1/\frac{d\Gamma}{ds}\right)\frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}{1024 m_b m_{\Lambda_b}^3 \pi^4 \sqrt{s}}\sqrt{1 - \frac{4m_l^2}{s}}\bigg\{(s - 4m_l^2)(g_2 s - f_2(m_\Lambda + m_{\Lambda_b}))f_2 m_{\Lambda_b}(C_{Q_1}C_{10}^* + C_{Q_1}^* C_{10}) \\ & - 2sm_{\Lambda_b} m_b (g_2(m_\Lambda + m_{\Lambda_b}) - f_2) f_2 (C_{Q_2}^* C_7^{\mathrm{eff}} + C_7^{*\mathrm{eff}} C_{Q_2}) + sm_{\Lambda_b}(f_2(m_\Lambda + m_{\Lambda_b}) - g_2 s) f_2 (C_{Q_2}^* C_9^{\mathrm{eff}} + C_9^{*\mathrm{eff}} C_{Q_2}) \\ & - 2f_2 m_l m_b (f_2(m_{\Lambda_b}^2 - m_\Lambda^2) + g_2 m_\Lambda s)(C_9^{\mathrm{eff}} C_{10}^* + C_9^{*\mathrm{eff}} C_{10}) + 8m_l m_b^2 m_{\Lambda_b}(f_2^2 - g_2^2 s)(C_9^{\mathrm{eff}} C_7^{*\mathrm{eff}} + C_9^{*\mathrm{eff}} C_7^{\mathrm{eff}}) \\ & - 4m_l m_b^2 m_{\Lambda_b} f_2^2 (C_7^{*\mathrm{eff}} C_{10} + C_7^{\mathrm{eff}} C_{10}^*) + \frac{16 m_l m_b^3}{s}(f_2^2 (m_{\Lambda_b} - m_\Lambda)^2 - g_2^2 s^2)|C_7^{\mathrm{eff}}|^2 + 4m_l m_b s((f_2 - g_2 m_\Lambda)^2 \\ & - g_2^2 m_{\Lambda_b}^2)|C_9^{\mathrm{eff}}|^2\bigg\}, \end{aligned} \tag{53}$$

and the transverse one is given by

$$\begin{aligned} P_T(s) = {}& \left(1/\frac{d\Gamma}{ds}\right)\frac{i\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}{512 m_{\Lambda_b}^2 \pi^4 \sqrt{s}}\left(1 - \frac{4m_l^2}{s}\right)(m_{\Lambda_b}^2 - m_\Lambda^2 + s)\bigg\{\frac{1}{2}(g_2(m_\Lambda + m_{\Lambda_b}) - f_2) f_2 (C_{Q_1} C_7^{*\mathrm{eff}} \\ & - C_{Q_1}^* C_7^{\mathrm{eff}}) + \frac{1}{4m_b} f_2 (f_2(m_\Lambda + m_{\Lambda_b}) - g_2 s)(C_{Q_1}(C_9^{*\mathrm{eff}} + C_{10}^*) - C_{Q_1}^*(C_9^{\mathrm{eff}} + C_{10})) + \frac{m_l}{2m_{\Lambda_b}}((f_2 - g_2 m_\Lambda)^2 \\ & - g_2^2 m_{\Lambda_b}^2)(C_9^{\mathrm{eff}} C_{10}^* - C_9^{*\mathrm{eff}} C_{10}) + \frac{m_l m_b}{s}(f_2^2 - g_2^2 s)(C_7^{*\mathrm{eff}} C_{10} - C_{10}^* C_7^{\mathrm{eff}})\bigg\}. \end{aligned} \tag{54}$$

The $\frac{d\Gamma}{ds}$ appearing in the above equation is the one given in Eq. (43) and $\lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)$ is the same as defined in Eq. (44).

D. Λ polarization in $\Lambda_b \to \Lambda l^+ l^-$

To study the Λ spin polarization, one needs to express the Λ four spin vector in terms of a unit vector $\hat{\xi}$ along the Λ spin in its rest frame as [30]

$$s_0 = \frac{\vec{p}_\Lambda \cdot \vec{\xi}}{m_\Lambda}, \qquad \vec{s} = \vec{\xi} + \frac{s_0}{E_\Lambda + m_\Lambda}\vec{p}_\Lambda, \tag{55}$$

where the unit vectors along the longitudinal, normal, and transverse components of the Λ polarization are chosen to be

$$\hat{e}_L = \frac{\vec{p}_\Lambda}{|\vec{p}_\Lambda|}, \quad \hat{e}_N = \frac{\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)}{|\vec{p}_\Lambda \times (\vec{p}_- \times \vec{p}_\Lambda)|}, \quad \hat{e}_T = \frac{\vec{p}_- \times \vec{p}_\Lambda}{|\vec{p}_- \times \vec{p}_\Lambda|}.$$

Similar to the lepton polarization, the polarization asymmetries for the Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ can be defined as

$$P_i^{(\mp)}(s) = \frac{\frac{d\Gamma}{ds}(\vec{\xi} = \hat{e}) - \frac{d\Gamma}{ds}(\vec{\xi} = -\hat{e})}{\frac{d\Gamma}{ds}(\vec{\xi} = \hat{e}) + \frac{d\Gamma}{ds}(\vec{\xi} = -\hat{e})} \tag{56}$$

where $i = L, N, T$, and $\vec{\xi}$ is the spin direction along the Λ baryon. The differential decay rate for the polarized Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ decay along any spin direction $\vec{\xi}$ is related to the unpolarized decay rate (40) through the following relation:

$$\frac{d\Gamma(\vec{\xi})}{ds} = \frac{1}{2}\left(\frac{d\Gamma}{ds}\right)[1 + (P_L \vec{e}_L + P_N \vec{e}_N + P_T \vec{e}_T) \cdot \vec{\xi}]. \tag{57}$$

Following the same procedure as we did for the lepton polarizations, we can derive the formulae for the longitudinal, normal, and transverse polarizations of Λ baryon in the MSSM as

$$\begin{aligned}
P_L(s) ={}& \left(1/\frac{d\Gamma}{ds}\right)\frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}{64 m_\Lambda m_{\Lambda_b}^3 \pi^5 s^{3/2}}\sqrt{1 - \frac{4m_l^2}{s}}\bigg\{\frac{m_\Lambda m_{\Lambda_b} m_l}{2m_b}(m_\Lambda + m_{\Lambda_b})s^{3/2} f_2^2 (C_{Q_2}C_{10}^* + C_{10}^* C_{Q_2}) \\
&+ m_\Lambda m_{\Lambda_b} m_l (2m_l^2 + s)(g_2^2 s - f_2^2)\sqrt{s}(C_7^{*\mathrm{eff}} C_9^{\mathrm{eff}} + C_7^{\mathrm{eff}} C_9^{*\mathrm{eff}}) + \frac{m_b^2}{3}\bigg[\frac{m_\Lambda}{\sqrt{s}}((12 m_l^2((m_\Lambda^2 - m_{\Lambda_b}^2) f_2^2 + g_2^2 s^2)) \\
&- 3s(s - m_\Lambda^2 + m_{\Lambda_b}^2)(f_2^2 - g_2^2 s) + \left(1 - \frac{4m_l^2}{s}\right)(s + m_\Lambda^2 - m_{\Lambda_b}^2)(f_2^2 + g_2^2 s))\bigg]|C_7^{\mathrm{eff}}|^2 \\
&+ \frac{s}{12}\bigg[12 m_l^2 m_\Lambda \sqrt{s}((|C_{10}|^2 - |C_9^{\mathrm{eff}}|^2) f_2^2 + g_2^2((m_{\Lambda_b}^2 - m_\Lambda^2)|C_{10}|^2 - (s - m_\Lambda^2 + m_{\Lambda_b}^2)|C_9^{\mathrm{eff}}|^2)) \\
&+ m_\Lambda \sqrt{s}(f_2^2 + g_2^2 t)(3(s - m_\Lambda^2 + m_{\Lambda_b}^2) - \left(1 - \frac{4m_l^2}{s}\right)(m_\Lambda^2 - m_{\Lambda_b}^2 + s))(|C_9^{\mathrm{eff}}|^2 + |C_{10}|^2)\bigg]\bigg\}, \\
P_N(s) ={}& \left(1/\frac{d\Gamma}{ds}\right)\frac{\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \sqrt{\lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}}{512 m_b m_{\Lambda_b}^3 \pi^4 \sqrt{s}}\left(1 - \frac{4m_l^2}{s}\right)\{2 m_b m_l m_{\Lambda_b}(s - (m_\Lambda + m_{\Lambda_b})^2)(f_2(m_{\Lambda_b} - m_\Lambda) + g_2 s) \\
&\times (C_{Q_1} C_7^{*\mathrm{eff}} + C_{Q_1}^* C_7^{\mathrm{eff}}) + m_l m_{\Lambda_b} s(s - (m_\Lambda + m_{\Lambda_b})^2)(f_2 + g_2(m_{\Lambda_b} - m_\Lambda))(C_{Q_1} C_9^{*\mathrm{eff}} + C_{Q_1}^* C_9^{\mathrm{eff}}) \\
&- m_b s(m_{\Lambda_b}^2 - m_\Lambda^2 + s)(m_\Lambda f_2^2 - g_2(m_\Lambda^2 - m_{\Lambda_b}^2 + s) f_2 + g_2^2 m_\Lambda t)(C_{10} C_9^{*\mathrm{eff}} + C_{10}^* C_9^{\mathrm{eff}}) \\
&+ 4 m_b m_{\Lambda_b} s(-m_\Lambda f_2^2 + g_2(m_\Lambda^2 - m_{\Lambda_b}^2 + s) f_2 - g_2^2 m_\Lambda t)(C_{10} C_7^{*\mathrm{eff}} + C_{10}^* C_7^{\mathrm{eff}})\}, \\
P_T(s) ={}& \left(1/\frac{d\Gamma}{ds}\right)\frac{i\alpha^2 G_F^2 |V_{tb}V_{ts}^*|^2 \lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)}{512 m_b m_{\Lambda_b}^2 \pi^4 \sqrt{s}} m_l \left(1 - \frac{4m_l^2}{s}\right)\{2 m_b(f_2(m_\Lambda + m_{\Lambda_b}) - g_2 s)(C_{Q_1} C_7^{*\mathrm{eff}} - C_{Q_1}^* C_7^{\mathrm{eff}}) \\
&+ s(g_2(m_\Lambda + m_{\Lambda_b}) - f_2)(C_{Q_1} C_9^{*\mathrm{eff}} - C_{Q_1}^* C_9^{\mathrm{eff}})\},
\end{aligned} \tag{58}$$

where $\lambda(m_{\Lambda_b}^2, m_\Lambda^2, s)$ is the same as that defined in Eq. (44), and the mass of the strange quark is neglected to make the expressions more compact.

V. NUMERICAL ANALYSIS

In this section, we would like to present the numerical analysis of decay rates, FBAs, and polarization asymmetries of the lepton and Λ baryon. The numerical values of Wilson coefficients and other input parameters used in our analysis are borrowed from Refs. [12,18,28] and collected in Tables I, II, III, and IV. In the subsequent analysis, we will focus on the parameter space of large $\tan\beta$, where the NHBs' effects are significant owing to the fact that the Wilson coefficients corresponding to NHBs are proportional to $(m_b m_l / m_h)\tan^3\beta$ ($h = h^0, A^0$). Here, one $\tan\beta$ comes from the chargino-up-type squark loop and $\tan^2\beta$ comes from the exchange of the NHBs. At large value of $\tan\beta$ the $C_{Q_i}^{(\prime)}$ compete with $C_i^{(\prime)}$ and can overwhelm $C_i^{(\prime)}$ in some region as can be seen from Tables III and IV [13]. Apart from the large $\tan\beta$ limit, the other two conditions responsible for the large contributions from NHBs are: (i) the mass values of the lighter chargino and lighter

TABLE II. Values of input parameters used in our numerical analysis.

$G_F = 1.166 \times 10^{-5}$ GeV^{-2}	$\lvert V_{ts}\rvert = 41.61^{+0.10}_{-0.80} \times 10^{-3}$
$\lvert V_{tb}\rvert = 0.9991$	$m_b = (4.68 \pm 0.03)$ GeV
$m_c(m_c) = 1.275^{+0.015}_{-0.015}$ GeV	$m_s(1\text{ GeV}) = (142 \pm 28)$ MeV
$m_{\Lambda_b} = 5.62$ GeV	$m_\Lambda = 1.12$ GeV
$f_{\Lambda_b} = 3.9^{+0.4}_{-0.2} \times 10^{-3}$ GeV^2	$f_\Lambda = 6.0^{+0.4}_{-0.4} \times 10^{-3}$ GeV^2

stop should not be too large; (ii) the mass splitting of charginos and stops should be large, which also indicate large mixing between the stop sector and chargino sector [12]. Once these conditions are satisfied, the process $B \to X_s\gamma$ will not only impose constraints on C_7, but it also puts a very stringent constraint on the possible new physics. It is well known that the SUSY contribution is sensitive to the sign of the Higgs mass term μ and SUSY contributes destructively when the sign of this term becomes minus. It is pointed out in literature [12] that there exist considerable regions of SUSY parameter space in which NHBs can largely contribute to the process $b \to sl^+l^-$ due to change of the sign of C_7 from positive to negative, while the constraint on $b \to s\gamma$ is respected. Also, when the masses of SUSY particles are relatively large, say about 450 GeV, there exist significant regions in the parameter space of SUSY models in which NHBs could contribute largely. However, in these cases C_7 does not change its sign, because contributions of charged Higgs and charginos cancel each other. Hopefully, we can distinguish between these two regions of SUSY by observing $\Lambda_b \to \Lambda l^+l^-$ with ($l = \mu, \tau$).

The numerical results for the decay rates, FBAs, and polarization asymmetries of the lepton and Λ baryon are presented in Figs. 1–8. Figure 1 describes the differential decay rate of $\Lambda_b \to \Lambda l^+l^-$, from which one can see that the supersymmetric effects are quite significant for the SUSY I and SUSY II model in the high momentum transfer regions for the muon case, whereas these effects are extremely small for SUSY III and SUSY SO(10) GUT models in this case. The reason for the increase of differential decay width in the SUSY I model is the relative change in the sign of $C_7^{\rm eff}$; while the large change in the SUSY II model is due to the contribution of the NHBs. As for the SUSY III and SUSY SO (10) models, the value of the Wilson coefficients corresponding to NHBs is small and hence one expects small deviations from SM. For the tauon case, the values of Wilson coefficients corresponding to NHBs in SUSY III are larger than those for the muon case and therefore their effects are quite significant as shown in

TABLE III. Wilson Coefficients in the SM and different SUSY models but without neutral Higgs boson contributions. The primed Wilson coefficients correspond to the operators, which are opposite in helicities from those of the SM operators and these come only in the SUSY SO(10) GUT model.

Wilson coefficients	$C_7^{\rm eff}$	$C_7^{\prime\rm eff}$	C_9	C_9'	C_{10}	C_{10}'
SM	-0.313	0	4.334	0	-4.669	0
SUSYI	$+0.3756$	0	4.7674	0	-3.7354	0
SUSYII	$+0.3756$	0	4.7674	0	-3.7354	0
SUSYIII	-0.3756	0	4.7674	0	-3.7354	0
SUSY SO(10) ($A_0 = -1000$)	$-0.219 + 0i$	$0.039 - 0.038i$	$4.275 + 0i$	$0.011 + 0.0721i$	$-4.732 - 0i$	$-0.075 - 0.670i$

TABLE IV. Wilson coefficient corresponding to NHBs' contributions. SUSYI corresponds to the regions where SUSY can destructively contribute and can change the sign of C_7, but contribution of NHBs are neglected; SUSYII refers to the region where $\tan\beta$ is large and the masses of the superpartners are relatively small. SUSY III corresponds to the regions where $\tan\beta$ is large and the masses of the superpartners are relatively large. The primed Wilson coefficients are the contribution of NHBs in the SUSY SO(10) GUT model. As the neutral Higgs bosons are proportional to the lepton mass, the values shown in the table are for the μ and τ case. The values in the bracket are for the τ.

Wilson coefficients	C_{Q_1}	C_{Q_1}'	C_{Q_2}	C_{Q_2}'
SM	0	0	0	0
SUSYI	0	0	0	0
SUSYII	6.5 (16.5)	0	$-6.5(-16.5)$	0
SUSYIII	1.2 (4.5)	0	$-1.2(-4.5)$	0
SUSY SO(10) ($A_0 = -1000$)	$0.106 + 0i$	$-0.247 + 0.242i$	$-0.107 + 0i$	$-0.250 + 0.246i$
	$(1.775 + 0.002i)$	$(-4.148 + 4.074i)$	$(-1.797 - 0.002i)$	$(-4.202 + 4.128i)$

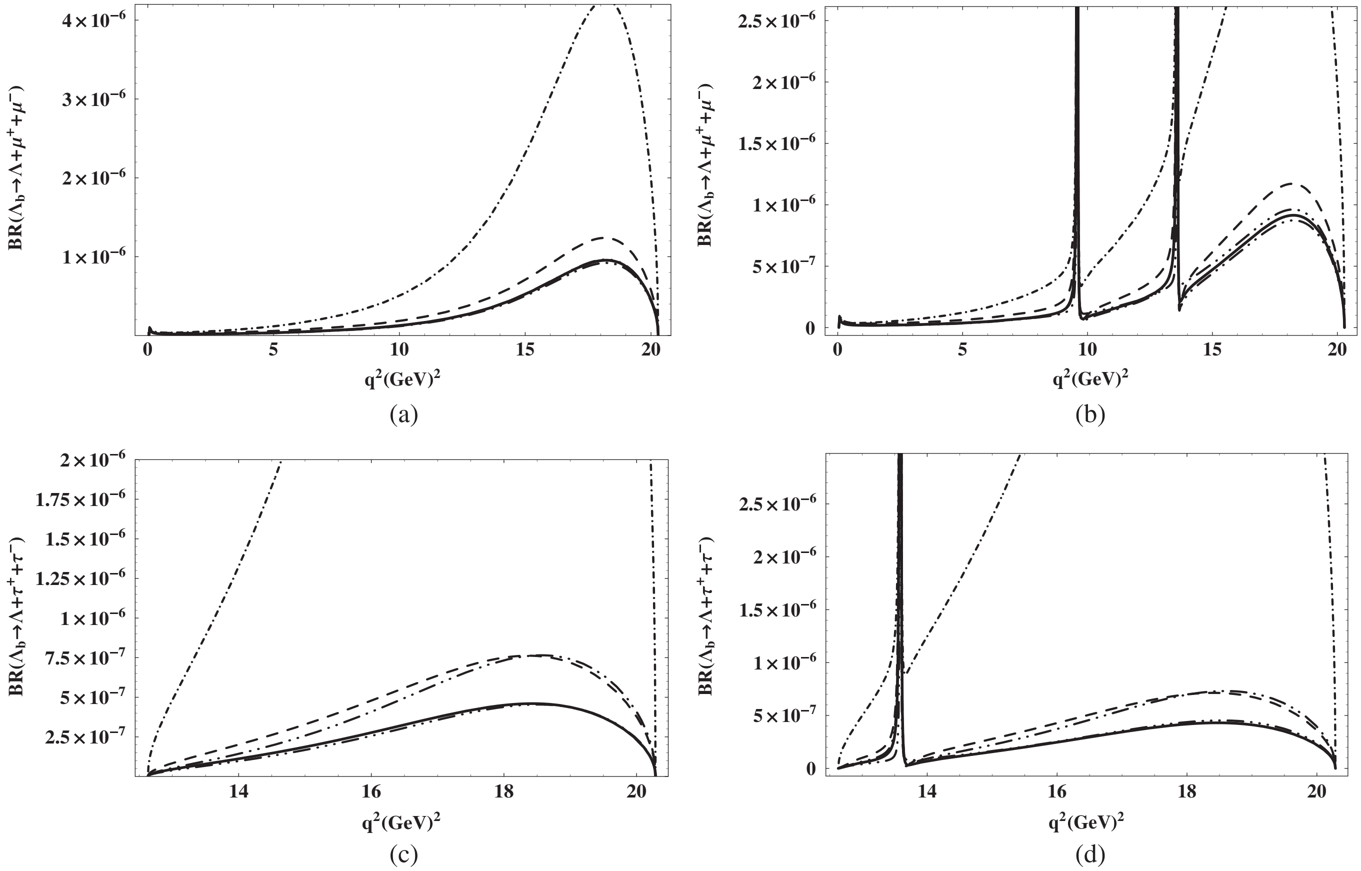


FIG. 1. The differential width for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2 without long-distance contributions (a, c) and with long-distance contributions (b, d). The solid, dashed, dashed-dot, dashed-double dot, and dashed-triple dot line represent the SM, SUSY I, SUSY II, SUSY III, and SUSY SO(10) GUT model.

Figs. 1(c) and 1(d). The numerical values of the branching fractions for $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) with and without long-distance contribution in the SM and different SUSY models are given in Table V.

In Fig. 2, the FBAs for $\Lambda_b \to \Lambda l^+ l^-$ are presented. Figures 2(a) and 2(b) describe the FBAs for $\Lambda_b \to \Lambda \mu^+ \mu^-$ with and without long-distance contributions, from which one can easily distinguish different SUSY models. It is known that in the SM the zero position of FBAs is due to the opposite sign of C_7^{eff} and C_9^{eff}. In SUSY I and SUSY II models, the sign of C_7^{eff} and C_9^{eff} are the same and hence the zero point of the FBAs disappears. Whereas, in the SUSY III model, due to the opposite sign of C_7^{eff} and C_9^{eff}, forward-backward asymmetry passes from the zero but this zero position shifts to the right from that of the SM value due to the contribution from the NHBs. Similar behavior is expected in the SUSY SO(10) GUT model but in this case the shifting is very mild as the contribution from the NHBs is very small. For $\Lambda_b \to \Lambda \tau^+ \tau^-$ the FBAs with and without long-distance contributions are represented in Figs. 2(c) and 2(d). Again, one can easily distinguish between the contributions from different SUSY models. Here, the most interesting point is that the FBAs pass through the zero point in the SUSY SO(10) GUT

TABLE V. Branching ratio for $\Lambda_b \to \Lambda l^+ l^- (l = \mu, \tau)$ in units of 10^{-6} in the SM and different SUSY models.

Branching ratio	$\Lambda_b \to \Lambda \mu^+ \mu^-$ without LD	$\Lambda_b \to \Lambda \mu^+ \mu^-$ with LD	$\Lambda_b \to \Lambda \tau^+ \tau^-$ without LD	$\Lambda_b \to \Lambda \tau^+ \tau^-$ with LD
SM	5.9	39	2.1	4
SUSYI	7.9	47	3.5	5.7
SUSYII	25	65	31	33
SUSYIII	5.6	45	3.2	5.6
SUSY SO(10) ($A_0 = -1000$)	5.92	23	2	2.6

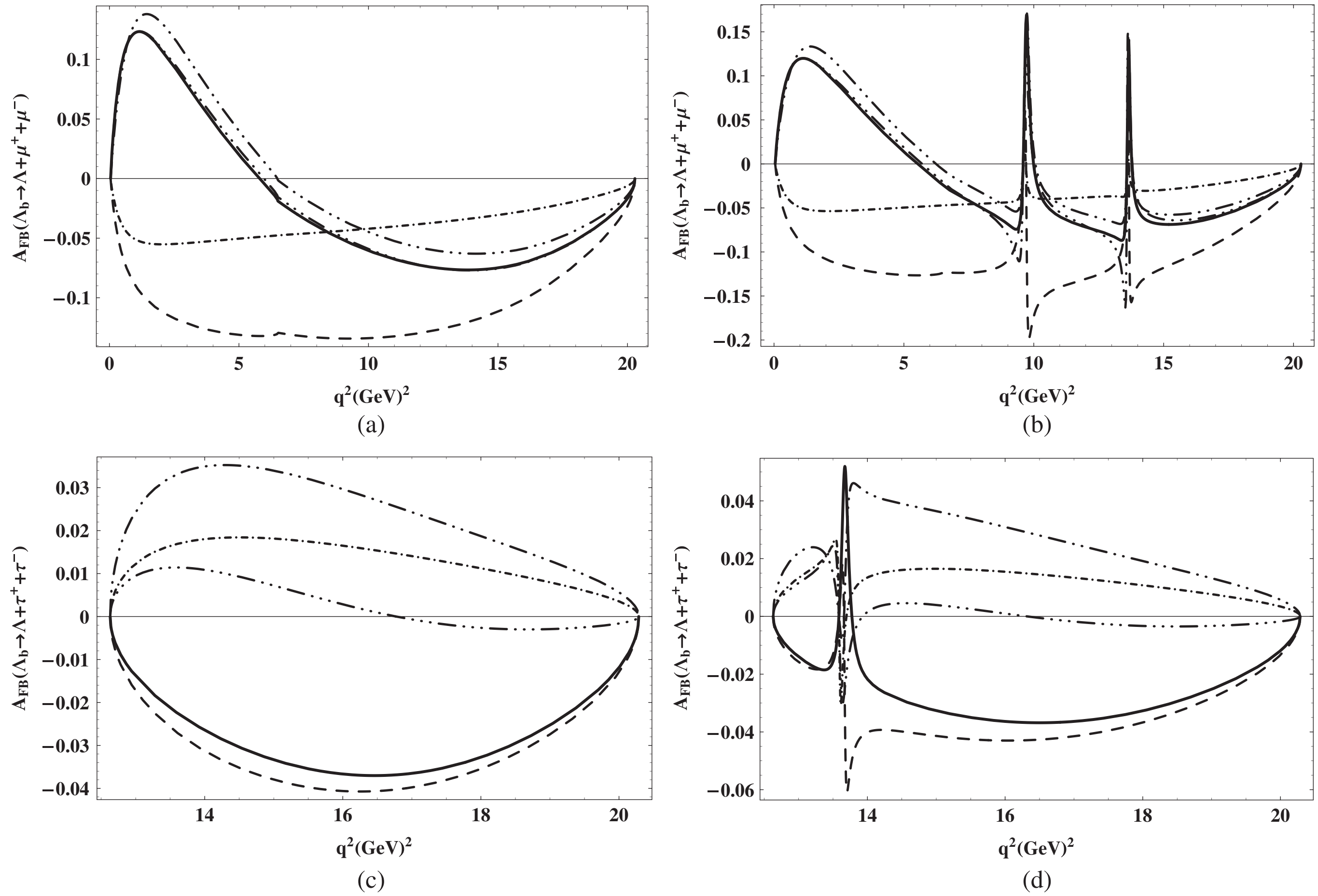


FIG. 2. Forward-backward asymmetry for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2 without long-distance contributions (a, c) and with long-distance contributions (b, d). The line conventions are same as given in the legend of Fig. 1.

model. This is due to the same sign of the C'_{Q_1} and C'_{Q_2} which suppress the large contribution coming from the C_7^{eff} and C_9^{eff} in this model. Though SUSY effects are more distinguishable in FBAs in this case, however, it is too difficult to measure it experimentally due to its small value.

Figures 3–5 describe the lepton polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$. Before, we try to explain the behavior of different polarization asymmetries with the help the formulas (52)–(54) given above. Equation (52) shows the dependence of the longitudinal lepton polarization on different Wilson coefficients, from which one can expect that the value of lepton polarization asymmetries in the SUSY I model should be greatly modified from that of the SM due to the change of sign for the term proportional to $C_7^{*\mathrm{eff}} C_{10}$. Because of this change in sign, the large positive contribution comes and the magnitude of the longitudinal polarization asymmetry decreases from that of the SM value. However, this value is expected to increase in the SUSY II model because of the NHBs' contribution, which lies in the first and second term of Eq. (52). In the SUSY III model, this asymmetry lies close to that of the SM value due to the same sign of $C_7^{*\mathrm{eff}} C_{10}$ and small contribution from the NHBs. As we have considered all the primed Wilson coefficients to be zero therefore the effect of SUSY SO(10) on longitudinal lepton polarization asymmetry will be explained by plotting it with the square of the momentum transfer.

Now, Figs. 3(a) and 3(b) show the dependence of longitudinal polarization asymmetry for the $\Lambda_b \to \Lambda \mu^+ \mu^-$ on the square of momentum transfer. The value in the SUSY I model is significantly different from that of the SM, however, this value is close to that in the SM for SUSY II and SUSY III models. Furthermore, the absolute value of longitudinal polarization asymmetry in the SUSY SO(10) is small compared to the SM model due to the complex part of the Wilson coefficients and also due to small contributions of the NHBs in this model.

Figures 3(c) and 3(d) are for the longitudinal lepton polarization asymmetries of $\Lambda_b \to \Lambda \tau^+ \tau^-$ with and without long-distance contributions, where the different SUSY models are easily distinguishable. Contrary to the muon case, the values of this asymmetry in the SUSY II and SUSY III models are even larger in magnitude than those obtained in the SM owing to the large contributions from NHBs, which are attributed to the first and second term of

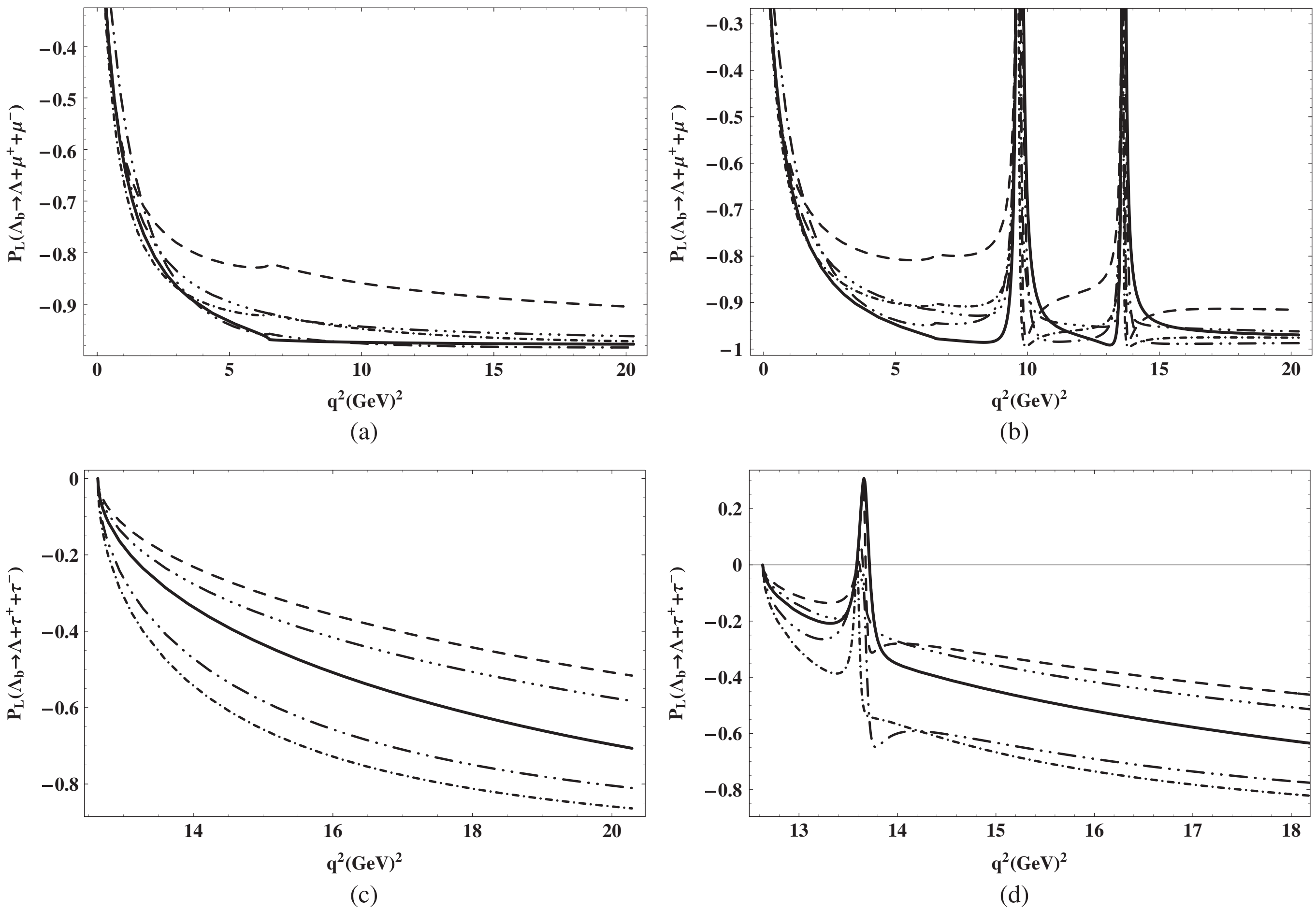


FIG. 3. Longitudinal lepton polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2 without long-distance contributions (a, c) and with long-distance contributions (b, d). The line conventions are same as given in the legend of Fig. 1.

Eq. (52). Though the large contributions come from the first term which is proportional to m_l in Eq. (52), this is overshadowed by the much larger term proportional to $m_{\Lambda_b}(C^*_{Q_1} C_{Q_2} + C^*_{Q_2} C_{Q_1})$.

The dependence of lepton normal polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ on the momentum transfer square are presented in Fig. 4. In terms of Eq. (53), one can observe that this asymmetry is sensitive to the contribu-

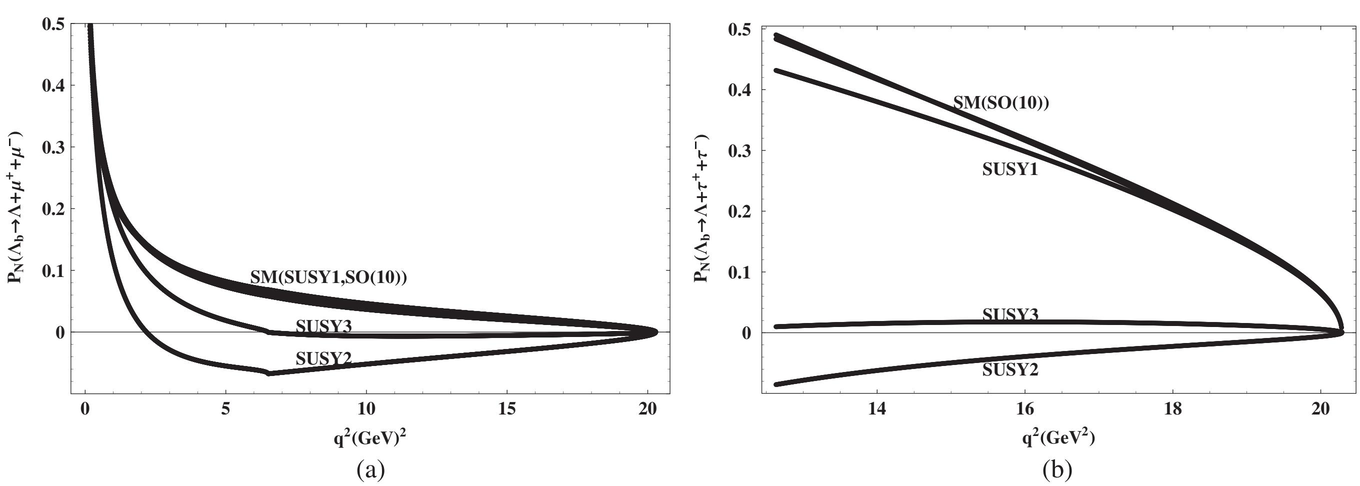
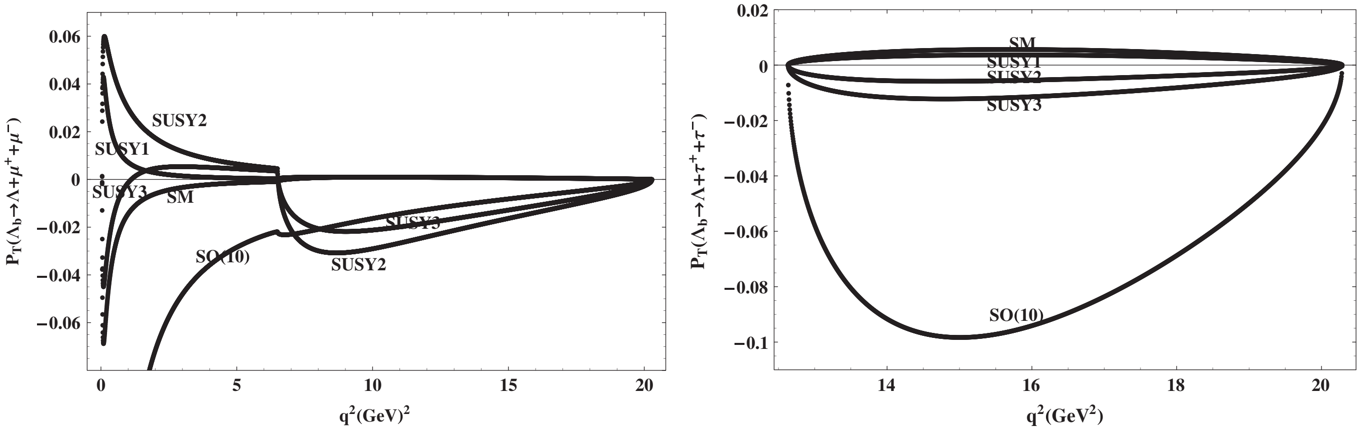


FIG. 4. Normal lepton polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2.



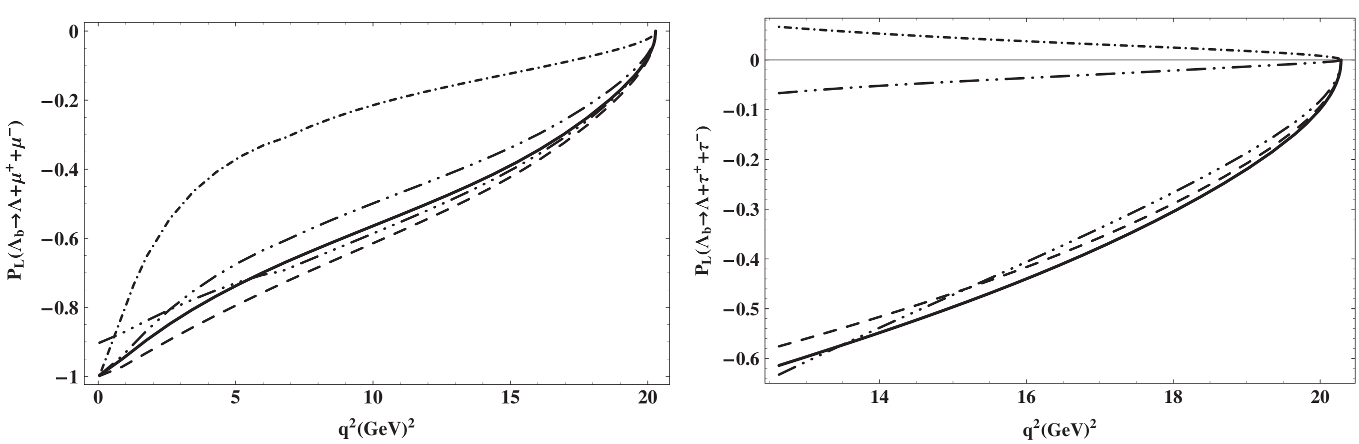
(a) (b)

FIG. 5. Transverse lepton polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2.

tion of NHBs in the SUSY II and SUSY III models, while it is insensitive to the contributions from the SUSY I and SUSY SO(10) model. It can be seen that P_N changes its sign in the case of large contributions from NHBs as indicated in Fig. 4, and this is also clear from the first three terms of Eq. (53). As expected, the contribution of NHBs from the $\tau^+\tau^-$ channel is much more significant than that from the $\mu^+\mu^-$ channel. Now, the normal polarization is proportional to the λ which approaches to zero at the large momentum transfer region and hence the normal polarization is suppressed by λ in this region.

Figure 5 shows the dependence of transverse polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ on the square of momentum transfer. From Eq. (54) we can see that it is proportional to the imaginary part of the Wilson coefficient which is negligibly small in the SM as well as in the SUSY I, SUSY II and SUSY III models. However, complex flavor nondiagonal down-type squark mass matrix elements of 2nd and 3rd generations are of order one at GUT scale in the SUSY SO(10) model, which induces complex couplings and Wilson coefficients. As a result, nonzero transverse polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ exist in this model. Even though in this case, the asymmetry effects are quite small in the $\tau^+\tau^-$ channel, the value of transverse polarization asymmetry can reach to -0.1 when the momentum transfer is around 15 GeV^2. Experimentally, to measure $\langle P_T \rangle$ of a particular decay branching ratio $\mathcal{B}$ at the $n\sigma$ level, the required number of events are $N = n^2/(\mathcal{B}\langle P_T \rangle^2)$ and if $\langle P_T \rangle \sim 0.1$, then the required number of events are almost 10^8 for Λ_b decays. Since at LHC and BTeV machines, the expected number of $b\bar{b}$ production events is around 10^{12} per year, so the measurement of transverse polarization asymmetries in the $\Lambda_b \to \Lambda l^+ l^-$ decays could discriminate the SUSY SO(10) model from the SM and other SUSY models.

Figure 6 shows the dependence of longitudinal polarization of the Λ baryon on the square of momentum transfer. One can see that the effects of NHBs are quite distinguishable in SUSY II and SUSY III models both for the $\mu^+\mu^-$ and $\tau^+\tau^-$ channels; but the values for SUSY I and SUSY II are almost close to that of the SM. As observed from



(a) (b)

FIG. 6. Longitudinal Λ polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2.

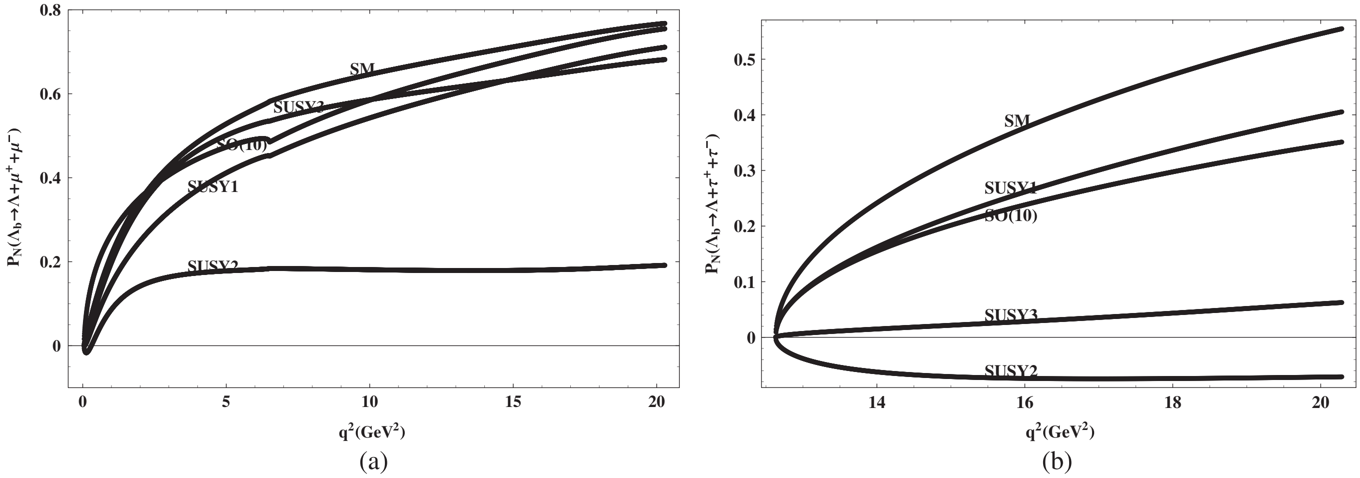


FIG. 7. Normal Λ polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2.

Eq. (58), the effects of NHBs are proportional to the mass of leptons, therefore the large deviations from the SM are expected for the tauon as presented in Fig. 6. In the SUSY II model, the value of the longitudinal polarization even changes its sign for the $\tau^+\tau^-$ channel.

With the help of Eq. (58), one can see that the normal polarization asymmetry of the Λ baryon is sensitive to the C_{Q_1} and the sign of the C_7^{eff}. It is shown that the sign of C_7^{eff} is negative in the SUSY I and II models. In particular, this asymmetry in the SUSY II model differs from that in the SM remarkably due to the large value of C_{Q_1}. Moreover, the contributions from the SUSY III and SUSY SO(10) models are also quite distinguishable from the SM as shown in Fig. 7. Therefore, the measurements of normal polarization asymmetries for $\Lambda_b \to \Lambda l^+ l^-$ in future experiments will help to distinguish different scenarios beyond the SM.

Similar to the transverse polarization asymmetry of the lepton, the transverse polarization asymmetry of the Λ baryon is also proportional to the imaginary part of $C_{Q_1} C_7^{*\mathrm{eff}}$ and $C_{Q_1} C_9^{*\mathrm{eff}}$ (c.f. Eq. (58)). It is known that these imaginary parts are quite small in the SM, SUSY I, SUSY II, and SUSY III model, and hence the values of the transverse polarization asymmetries of the Λ baryon are almost zero in these models. However, the imaginary part of the Wilson coefficient in the SUSY SO(10) model is large, and hence its effects are quite different from the other models discussed above as collected in Fig. 8. For the muon case, the transverse polarization asymmetry can reach the number -0.1 in the SUSY SO(10) model; whereas the value is too small to measure experimentally for the tauon case.

VI. CONCLUSION

We have carried out the study of invariant mass spectrum, FBAs, polarization asymmetries of the lepton and Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays in SUSY theories including the SUSY SO(10) GUT model. In particular, we analyze the effects of NHBs to this process and our main outcomes can be summarized as follows:

(i) The differential decay rates deviate sizably from that of the SM especially in the large momentum transfer

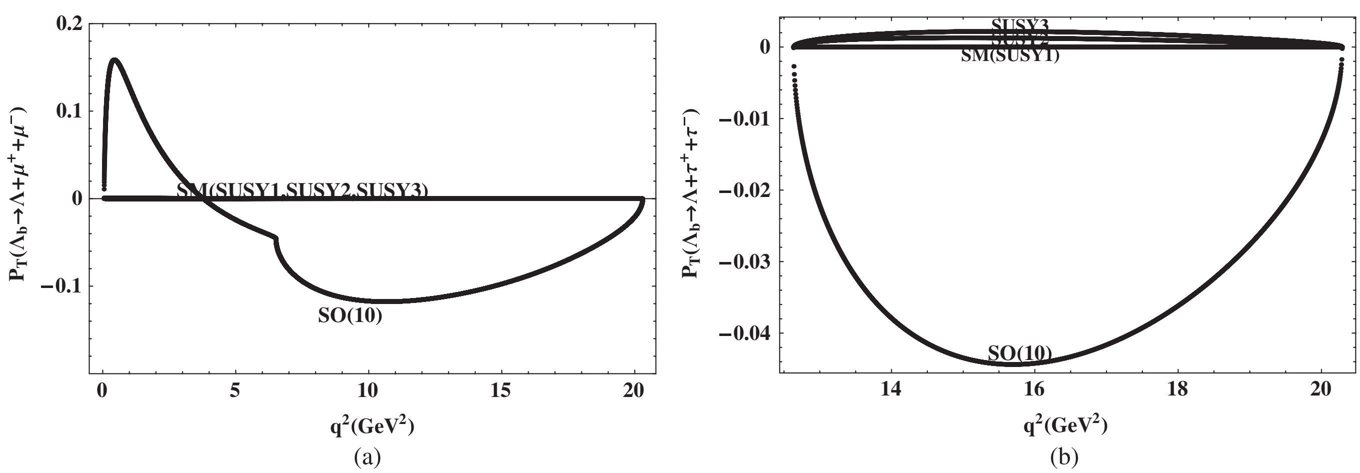


FIG. 8. Transverse Λ polarization asymmetries for the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays as functions of q^2.

region. These effects are significant in the SUSY II model where the value of the Wilson coefficients corresponding to the NHBs is large. However, the SUSY SO(10) effects in the differential decay rate of $\Lambda_b \to \Lambda l^+ l^-$ are negligibly small.

(ii) The SUSY effects show up for the FBA of the $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays and the deviations from the SM are very large especially in the SUSY I and SUSY II model where the FBAs did not pass from zero. The reason is the same sign of $C_7^{\rm eff}$ and $C_9^{\rm eff}$ in these two models, but in the SUSY III scenario it passes from the zero-point different from that of the SM. The effects of the SUSY SO(10) are quite distinguishable when the final state leptons are the tauon pair, but these are too small to be measured experimentally.

(iii) The longitudinal, normal, and transverse polarizations of leptons are calculated in different SUSY models. It is found that the SUSY effects are very promising which could be measured at future experiments and shed light on the new physics beyond the SM.

(iv) Following the same line, the longitudinal, normal, and transverse polarizations of the Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decays are calculated at length. The different SUSY effects are clearly distinguishable from each other and also from those of the SM. The transverse polarization asymmetries are proportional to the imaginary part of the Wilson coefficients. Hence it is almost zero in the SM as well as in the MSSM model, however, the Wilson coefficients in the SUSY SO(10) GUT model have a large imaginary part, and hence the value of the transverse polarization is expected to be nonzero. The maximum value of transverse polarization of $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay reaches to -0.1 for the square of momentum transfer around 10 GeV^2 and hence can be measurable at future experiments like LHC and BTeV machines where a large number of $b\bar{b}$ pairs are expected to be produced.

In short, the experimental investigation of observables, like decay rates, FBAs, lepton polarization asymmetries, and the polarization asymmetries of the Λ baryon in $\Lambda_b \to \Lambda l^+ l^-$ ($l = \mu, \tau$) decay will be used to search for the SUSY effects, in particular, the NHBs' effect, encoded in the MSSM as well as the SUSY SO(10) models.

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APPENDIX A: HELICITY AMPLITUDE OF $\Lambda_b \to \Lambda l^+ l^-$ DECAY

In this appendix, we would like to present the helicity amplitude of $\Lambda_b \to \Lambda l^+ l^-$ decay. It is convenient to regard this decay as a quasi-two-body decay $\Lambda_b \to \Lambda V^*$ followed by the leptonic decay $V^* \to l^+ l^-$, where V^* is the off-shell photon, Z boson or neutral Higgs boson. The matrix element of $\Lambda_b \to \Lambda l^+ l^-$ can be written in the following form:

$$M_{\lambda_\Lambda}^{\lambda_{l^-}\lambda_{l^+}} = g^{\mu\nu}\langle l^-(p_{l^-}, \lambda_{l^-}) l^+(p_{l^+}, \lambda_{l^+})| j_\mu^{(l)} |0\rangle \times \langle \Lambda(p_\Lambda, \lambda_\Lambda)| j_\nu^{(h)} |\Lambda_b(p_{\Lambda_b})\rangle + \ldots. \tag{A1}$$

Notice that the dots represent the contributions from the scalar leptonic current $\bar{\Lambda}(p_\Lambda)(E_1 + E_2\gamma_5)\Lambda_b(p_{\Lambda_b}) \times \bar{l}(p_{l^-}) l(p_{l^+})$, which can be decomposed as the helicity amplitudes directly. Below, we can concentrate on the amplitudes contributed from the first part. It is known that the polarization vector of V^* satisfies the completeness relation

$$\sum_{\lambda=\pm 1,0} \epsilon^\mu(q, \lambda)\epsilon^{\nu *}(q, \lambda) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}. \tag{A2}$$

Introducing a timelike polarization vector

$$\epsilon^\mu(q, t) = \frac{1}{\sqrt{q^2}} q^\mu, \tag{A3}$$

we can express the metric tensor $g^{\mu\nu}$ in terms of the polarization vector of the virtual vector particle as

$$-g^{\mu\nu} = \sum_{\lambda=\pm 1,0,t} \eta_\lambda \epsilon^\mu(q, \lambda)\epsilon^{\nu *}(q, \lambda), \tag{A4}$$

where the metric is given by $\eta_\pm = \eta_0 = -\eta_t = +1$. Now we can rewrite the matrix element of $\Lambda_b \to \Lambda l^+ l^-$ as

$$M_{\lambda_\Lambda}^{\lambda_{l^-}\lambda_{l^+}} = -\eta_{\lambda_{V^*}} H_{\lambda_\Lambda, \lambda_{V^*}} L_{\lambda_{V^*}}^{\lambda_{l^-}, \lambda_{l^+}}, \tag{A5}$$

where

$$\begin{aligned} L_{\lambda_{V^*}}^{\lambda_{l^-}, \lambda_{l^+}} &= \epsilon_{\lambda_{V^*}}^\mu \langle l^-(p_{l^-}, \lambda_{l^-}) l^+(p_{l^+}, \lambda_{l^+})| j_\mu^{(l)} |0\rangle, \\ H_{\lambda_\Lambda, \lambda_{V^*}} &= (\epsilon_{\lambda_{V^*}}^\mu)^* \langle \Lambda(p_\Lambda, \lambda_\Lambda)| j_\mu^{(h)} |\Lambda_b(p_{\Lambda_b})\rangle, \end{aligned} \tag{A6}$$

with λ_{V^*} being the polarization vector of the virtual intermediate vector boson. Since both $L_{\lambda_{V^*}}^{\lambda_{l^-}, \lambda_{l^+}}$ and $H_{\lambda_{V^*}}^{\lambda_\Lambda}$ are Lorentz invariant, we can calculate these two amplitudes in the rest frame of the lepton pair and Λ_b baryon, respectively. The kinematics in the rest frame of Λ_b can be specified as follows:

$$\begin{aligned} p^\mu_{\Lambda_b} &= (m_{\Lambda_b}, 0, 0, 0), \qquad p^\mu_\Lambda = (E_\Lambda, 0, 0, |\mathbf{p}_\Lambda|), \\ q^\mu &= (E_{V^*}, 0, 0, -|\mathbf{p}_\Lambda|), \\ \epsilon^\mu_{V^*}(0) &= \frac{1}{\sqrt{q^2}}(|\mathbf{p}_\Lambda|, 0, 0, -E_{V^*}), \\ \epsilon^\mu_{V^*}(\pm 1) &= \frac{1}{\sqrt{2}}(0, \mp 1, i, 0), \qquad \epsilon^\mu_{V^*}(t) = \frac{1}{\sqrt{q^2}} q^\mu, \end{aligned} \tag{A7}$$

where the variables E_Λ, $|\mathbf{p}_\Lambda|$, and E_{V^*} are given by

$$\begin{aligned} E_\Lambda &= \frac{m^2_{\Lambda_b} + m^2_\Lambda - q^2}{2m_{\Lambda_b}}, \qquad |\mathbf{p}_\Lambda| = \sqrt{E^2_\Lambda - m^2_\Lambda}, \\ E_{V^*} &= m_{\Lambda_b} - E_\Lambda. \end{aligned} \tag{A8}$$

It needs to be pointed out that we choose the z axes as the direction of Λ momentum. The kinematics in the rest frame of lepton pair can be specified as

$$\begin{aligned} p^\mu_{l^-} &= (E_l, |\mathbf{p}_l|\sin\theta\cos\phi, |\mathbf{p}_l|\sin\theta\sin\phi, |\mathbf{p}_l|\cos\theta), \\ p^\mu_{l^+} &= (E_l, -|\mathbf{p}_l|\sin\theta\cos\phi, -|\mathbf{p}_l|\sin\theta\sin\phi, -|\mathbf{p}_l|\cos\theta), \\ q^\mu &= \sqrt{q^2}(1,0,0,0), \quad \epsilon^\mu_{V^*}(0) = (0,0,0,1), \\ \epsilon^\mu_{V^*}(\pm 1) &= \frac{1}{\sqrt{2}}(0, \pm 1, i, 0), \quad \epsilon^\mu_{V^*}(t) = \frac{1}{\sqrt{q^2}} q^\mu, \end{aligned} \tag{A9}$$

where the variable E_l and $|\mathbf{p}_l|$ are

$$E_l = \frac{\sqrt{q^2}}{2}, \qquad |\mathbf{p}_l| = \sqrt{E^2_l - m^2_l}. \tag{A10}$$

It should be emphasized that we choose the direction of the lepton's momentum as (θ, ϕ) in the spherical coordinate, hence the direction of the antilepton's momentum is $(\pi - \theta, \pi + \phi)$. For convenience, we usually choose the $x - z$ plane as the virtual vector boson V^* decay plane, that is to say $\phi = 0$.

Now we can calculate the helicity amplitude of $\Lambda_b \to \Lambda l^+ l^-$ explicitly. To this purpose, we first give the expressions of the Dirac spinor in terms of the helicity operator's eigenstate

$$\begin{aligned} u(p, s) &= \frac{\not{p} + m}{\sqrt{p_0 + m}} \begin{pmatrix} \chi^{(s)} \\ 0 \end{pmatrix}, \\ v(p, s) &= \frac{-\not{p} + m}{\sqrt{p_0 + m}} \begin{pmatrix} 0 \\ \eta^{(s)} \end{pmatrix}, \end{aligned} \tag{A11}$$

with

$$\begin{aligned} \chi^{(1/2)} &= \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} e^{i\varphi} \end{pmatrix}, \qquad \chi^{(-1/2)} = \begin{pmatrix} -\sin\frac{\theta}{2} e^{i\varphi} \\ \cos\frac{\theta}{2} \end{pmatrix}, \\ \eta^{(1/2)} &= \chi^{(-1/2)}, \qquad \eta^{(-1/2)} = -\chi^{(1/2)}. \end{aligned} \tag{A12}$$

After some lengthy computations, we can finally derive the following helicity amplitudes:

$$\begin{aligned} M^{++}_{+1/2} &= -\sqrt{2} m_l \sin\theta H^V_{+1/2,+1} - 2m_l \cos\theta H^V_{+1/2,0} - 2m_l H^A_{+1/2,t} + \sqrt{q^2 - 4m^2_l} H^S_{1/2} \\ M^{+-}_{+1/2} &= -\sqrt{2q^2}\cos^2\frac{\theta}{2}(H^V_{+1/2,+1} + vH^A_{+1/2,+1}) + \sqrt{q^2}\sin\theta(H^V_{+1/2,0} + vH^A_{+1/2,0}), \\ M^{-+}_{+1/2} &= \sqrt{2q^2}\sin^2\frac{\theta}{2}(H^V_{+1/2,+1} - vH^A_{+1/2,+1}) + \sqrt{q^2}\sin\theta(H^V_{+1/2,0} - vH^A_{+1/2,0}), \\ M^{--}_{+1/2} &= \sqrt{2} m_l \sin\theta H^V_{+1/2,+1} + 2m_l \cos\theta H^V_{+1/2,0} - 2m_l H^A_{+1/2,t} - \sqrt{q^2 - 4m^2_l} H^S_{1/2}, \\ M^{++}_{-1/2} &= -2m_l \cos\theta H^V_{-1/2,0} + \sqrt{2} m_l \sin\theta H^V_{-1/2,-1} - 2m_l H^A_{-1/2,t} + \sqrt{q^2 - 4m^2_l} H^S_{-1/2}, \\ M^{+-}_{-1/2} &= \sqrt{q^2}\sin\theta(H^V_{-1/2,0} + vH^A_{-1/2,0}) - \sqrt{2q^2}\sin^2\frac{\theta}{2}(H^V_{-1/2,-1} + vH^A_{-1/2,-1}), \\ M^{-+}_{-1/2} &= \sqrt{q^2}\sin\theta(H^V_{-1/2,0} - vH^A_{-1/2,0}) + \sqrt{2q^2}\cos^2\frac{\theta}{2}(H^V_{-1/2,-1} - vH^A_{-1/2,-1}), \\ M^{--}_{-1/2} &= 2m_l \cos\theta H^V_{-1/2,0} - \sqrt{2} m_l \sin\theta H^V_{-1/2,-1} - 2m_l H^A_{-1/2,t} - \sqrt{q^2 - 4m^2_l} H^S_{-1/2}, \end{aligned} \tag{A13}$$

where the hadronic amplitude $H_{\lambda_\Lambda, \lambda_{V^*}}$ is given by

$$
\begin{aligned}
H^V_{+1/2,+1} &= \sqrt{2}[\sqrt{Q_-}(A_1 - B_1(m_{\Lambda_b} + m_\Lambda)) + \sqrt{Q_+}(A_2 + B_2(m_{\Lambda_b} - m_\Lambda))],\\
H^V_{+1/2,0} &= \frac{1}{\sqrt{q^2}}[\sqrt{Q_-}(A_1(m_{\Lambda_b} + m_\Lambda) - B_1 t) + \sqrt{Q_+}(A_2(m_{\Lambda_b} - m_\Lambda) + B_2 t)]\\
H^A_{+1/2,t} &= \frac{1}{\sqrt{q^2}}[\sqrt{Q_-}(A_4(m_{\Lambda_b} + m_\Lambda) - D_4 t) + \sqrt{Q_+}(A_3(m_{\Lambda_b} - m_\Lambda) + D_3 t)],\\
H^S_{+1/2} &= E_1\sqrt{Q_+} - E_2\sqrt{Q_-},\\
H^A_{+1/2,+1} &= \sqrt{2}[\sqrt{Q_-}(-A_3 + B_3(m_{\Lambda_b} + m_\Lambda)) - \sqrt{Q_+}(A_4 + B_4(m_{\Lambda_b} - m_\Lambda))],\\
H^A_{+1/2,0} &= \frac{1}{\sqrt{q^2}}[\sqrt{Q_-}(A_3(m_{\Lambda_b} + m_\Lambda) - B_3 t) + \sqrt{Q_+}(A_4(m_{\Lambda_b} - m_\Lambda) + B_4 t)],\\
H^V_{-1/2,0} &= \frac{1}{\sqrt{q^2}}[\sqrt{Q_-}(A_1(m_{\Lambda_b} + m_\Lambda) - B_1 t) - \sqrt{Q_+}(A_2(m_{\Lambda_b} - m_\Lambda) + B_2 t)],\\
H^V_{-1/2,-1} &= \sqrt{2}[\sqrt{Q_-}(A_1 - B_1(m_{\Lambda_b} + m_\Lambda)) - \sqrt{Q_+}(A_2 + B_2(m_{\Lambda_b} - m_\Lambda))],\\
H^A_{-1/2,t} &= \frac{1}{\sqrt{q^2}}[-\sqrt{Q_-}(A_4(m_{\Lambda_b} + m_\Lambda) - D_4 t) + \sqrt{Q_+}(A_3(m_{\Lambda_b} - m_\Lambda) + D_3 t)],\\
H^S_{-1/2} &= E_1\sqrt{Q_+} + E_2\sqrt{Q_-}.\\
H^A_{-1/2,0} &= \frac{1}{\sqrt{q^2}}[\sqrt{Q_-}(A_3(m_{\Lambda_b} + m_\Lambda) - B_3 t) - \sqrt{Q_+}(A_4(m_{\Lambda_b} - m_\Lambda) + B_4 t)],\\
H^A_{-1/2,-1} &= \sqrt{2}[\sqrt{Q_-}(A_3 - B_3(m_{\Lambda_b} + m_\Lambda)) - \sqrt{Q_+}(A_4 + B_4(m_{\Lambda_b} - m_\Lambda))],
\end{aligned}
\tag{A14}
$$

with

$$
\begin{aligned}
Q_+ &= (m_{\Lambda_b} + m_\Lambda)^2 - q^2,\\
Q_- &= (m_{\Lambda_b} - m_\Lambda)^2 - q^2, \qquad v = \sqrt{1 - \frac{4m_l^2}{q^2}}.
\end{aligned}
\tag{A15}
$$

The square of the matrix element for the decay of $\Lambda_b \to \Lambda l^+ l^-$ after averaging over the spin of Λ_b baryon can be written as

$$
\begin{aligned}
2|\tilde{M}|^2_{\Lambda_b\to\Lambda l^+ l^-} &= |M^{++}_{+1/2}|^2 + |M^{+-}_{+1/2}|^2 + |M^{-+}_{+1/2}|^2\\
&\quad + |M^{--}_{+1/2}|^2 + M^{++}_{-1/2}|^2 + |M^{+-}_{-1/2}|^2\\
&\quad + |M^{-+}_{-1/2}|^2 + |M^{--}_{-1/2}|^2.
\end{aligned}
\tag{A16}
$$

The differential decay rate formula for the process $\Lambda_b \to \Lambda l^+ l^-$ can be written as

$$
\frac{d^2\Gamma(s,\cos\theta)}{ds d\cos\theta} = \frac{1}{(2\pi)^3}\frac{1}{64 m^3_{\Lambda_b}}\lambda^{1/2}(m^2_{\Lambda_b}, m^2_\Lambda, s) \times\sqrt{1 - \frac{4m_l^2}{s}}|\tilde{M}_{\Lambda_b\to\Lambda l^+ l^-}|^2,
\tag{A17}
$$

where θ is the angle between the momentum of the Λ baryon and l^- in the dilepton rest frame.

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