Bounds on new light particles from high-energy and very small momentum transfer $n p$ elastic scattering data

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We found that spin-one new light particle exchanges are strongly bounded by high-energy and small momentum transfer np elastic scattering data; the analogous bound for a scalar particle is considerably weaker, while for a pseudoscalar particle no bounds can be set. These bounds are compared with the bounds extracted from low-energy *n*-Pb scattering experiments and from the bounds of π^0 and K⁺ meson decays.

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I. INTRODUCTION

The standard model of three fundamental forces describes interactions of elementary particles very well. While the electromagnetic force has a long interaction range, the short radius of the ''weak force'' $($\sim 1/1000$ fm) is determined by the heavy masses of mediating W and Z hosons (~ 100 GeV). The OCD forces$ mediating W and Z bosons (~ 100 GeV). The QCD forces
are typically contained within a confinement radius of are typically contained within a confinement radius of \sim 1 fm. The effects of the long-range gravitational force
can usually be neglected in elementary particle scattering can usually be neglected in elementary particle scattering experiments.

The search for new forces of nature is a major goal of experiments at high-energy colliders. Rare transitions and decays of fundamental particles can also shed light on new interactions. These experiments are probing for new forces at distances shorter than $1/1000$ fm. However, the existence of new forces at distances larger than the confinement radius of a nucleon $(\sim 1 \text{ fm})$ can also be probed and constrained by sensitive experiments constrained by sensitive experiments.

Experimental searches for and limits on these new forces of nature can be pursued in two directions: (a) as deviations from the Newtonian law of gravity where the new force is expressed as a modification of the $1/r^2$ law, usually by an additional Yukawa term that can be parametrized with two parameters α , the relative strength of new interaction, and λ , the characteristic radius of the interaction. Applied in the analysis of experimental data at macroscopic distances down to \sim a micrometer, this ansatz describes the possible deviations from classical gravity. (b) The second direction is a quantum field theory description of the interactions (excluding gravity) in a covariant form, which can be expressed in the lowest perturbation order through the coupling constant g and the mass of the exchanged particle

mediating the interaction μ . Covariants that can be consistently considered in this description are scalar (S), pseudoscalar (P), vector (V), and axial vector (A). We can argue that higher spins of the intermediate particle should not be considered since they lead to a nonrenormalizable theory. The particles that mediate the new force could be absent from the spectrum of known particles [1] due to their small mass and coupling constant or due to some other reason that is helping them avoid detection. In any case, if these particles are not observed, direct experimental limits on their existence in terms of g and μ are required.

In Sec. II of the present paper, we reanalyze the experimental small-angle np elastic scattering data at high energy [2] in terms of bounds on the existence of new forces expressed as S, P, V, or A covariant interactions. In Sec. III, we examine bounds that can be obtained from lower energy data.

II. BOUNDS FROM HIGH-ENERGY np SCATTERING

The data for small-angle np elastic scattering at high energy were obtained in the NA-6 experiment [2] performed at the CERN Super Proton Synchrotron a quarter of a century ago. Incident neutron energy in the experiment was 100–400 GeV, while the square of the 4-momentum transfer |t| was varied in the range 6×10^{-6} to $5 \times$ 10^{-1} GeV². The data of this experiment are consistent with extrapolation of the hadronic amplitude from higher |t| values, while at $|t| < 10^{-4}$ GeV² the differential cross section rises due to Schwinger scattering, which is the interaction of the neutron's magnetic moment with the Coulomb field of the proton or electron. The purpose of NA-6 [2] was to measure hadronic interactions at high s in the region of momentum transfer $(\sim |t| < 10^{-2} \text{ GeV}^2)$
that was usually inaccessible in the scattering of charged that was usually inaccessible in the scattering of charged hadrons due to Coulomb interactions. This is the region where the effect of a new force mediated by a light particle may be present.

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Figure 1 (similar to Fig. 16 from [2]) demonstrates that np elastic scattering data in this experiment are well described by the following formula:

$$
\frac{d\sigma}{dt} = A \exp[bt] - 2\left(\frac{\alpha k_n}{m_n}\right)^2 \frac{\pi}{t},\tag{1}
$$

where $A = (79.78 \pm 0.26) \text{ mb/GeV}^2$ and $b = (11.63 \pm 0.26) \text{ mb/GeV}^2$ 0.08) GeV^{-2} were determined from the fit to the data (data are taken from Table 7 in [2]), m_n is the neutron mass, and $k_n = -1.91$ is the neutron magnetic moment in nuclear magnetons. The factor of 2 in the Schwinger term, as will be discussed later, accounts for the scattering of the neutron's magnetic moment on the proton plus an incoherent contribution of scattering on electrons (gaseous hydrogen was used in [2] as a target). Smaller effects due to neutron polarizability are not included in the description of the data. This description [\(1](#page-1-0)) works rather satisfactorily with χ^2 = 41.5 for 31 degrees of freedom. We will refer to this description as the ''zero model'' since no new force contributions are included here.

Although quantum chromodynamics does not provide a detailed theoretical description of the hadronic elastic scattering at small $|t|$, i.e. at large impact parameters, hadronic scattering has been studied experimentally in great detail in the past and was phenomenologically well understood, e.g. in the framework of Regge models. The description of elastic data by a single exponent was a general universal feature of hadronic scattering observed at low $|t|$ in the

FIG. 1. Elastic differential neutron-proton cross sections measured in experiment [2]. For comparison, the $|t|$ region measurable in pp scattering is shown with the effect of the Coulomb interaction indicated by the dashed line.

region where it was not obstructed by Coulomb scattering (for example, see [3] and also the comparison with other experiments in [2]). This justifies, at a phenomenological level, our choice of the hadronic scattering description with a single exponent. However, in an attempt to improve the description of the data [2], we have tried several alternative modifications of the exponential term in the ''zero model'' [\(1\)](#page-1-0) involving additional parameters, including a quadratic term in the exponent and the sum of two exponents. In all of these cases, χ^2 per degrees of freedom was slightly increased demonstrating that more complicated modifications of the ''zero model'' are not statistically justifiable.

We describe the contribution of a new interaction in the following way: Let us suppose that a new light particle with mass μ exists which interacts with the neutron and proton with couplings g_n and g_p , correspondingly. Assuming scalar, pseudoscalar, vector, and axial vector couplings of this particle with nucleons, we obtain the following addition to expression [\(1\)](#page-1-0):

$$
\frac{d\sigma_i}{dt}(g,\mu)|_{\text{new}} = \frac{|A_i|^2 \cdot FF}{16\pi s(s-4m^2)},\tag{2}
$$

where $s = (p_n + p_p)^2$ is the invariant energy square and m is the nucleon mass. We parametrize the hadronic form factor's contribution as

$$
FF = \frac{1}{(1 - t/\Lambda^2)^8},\tag{3}
$$

which comes from a $1/q⁴$ decrease of the nucleon form factor, and we set Λ equal to the mass of the lightest meson resonance with appropriate quantum numbers (η' in the case of the pseudoscalar).¹ Finally, we use the following amplitude squares for different couplings:

$$
|A_S|^2 = \frac{g_S^4}{(t - \mu^2)^2} (4m^2 - t)^2,
$$
 (4)

$$
|A_P|^2 = \frac{g_P^4 t^2}{(t - \mu^2)^2},\tag{5}
$$

$$
|A_V|^2 = \frac{4g_V^4}{(t - \mu^2)^2} \bigg[s^2 - 4m^2s + 4m^4 + st + \frac{1}{2}t^2 \bigg], \tag{6}
$$

$$
|A_A|^2 = \frac{4g_A^4}{(t - \mu^2)^2} \left[s^2 + 4m^2s + 4m^4 + st + \frac{1}{2}t^2 + \frac{4m^4t^2}{\mu^4} + \frac{8m^4t}{\mu^2} \right],
$$
 (7)

where coupling constants $g_i^2 \equiv g_p^i g_n^i$.

¹We checked that our bounds for the cases of S, V, and A exchanges do not change within 1% accuracy if we substitute $FF = 1$ into Eq. [\(2](#page-1-1)); for the P exchange, we do not get a reliable bound (see below).

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It is quite natural to suppose that a new light particle's couplings with nucleons originates from its couplings with quarks. In this case, [\(6\)](#page-1-2) and [\(7](#page-1-3)) are modified. For the vector exchange, the induced magnetic moment's interaction term should be added to the scattering amplitude. Since its numerator contains a momentum transfer divided by m_N , which in the considered kinematics gives a factor much smaller than 1, we can safely neglect it and use (6) (6) in what follows. The case of the axial vector exchange is more delicate and discussed in detail in the Appendix.

We can now turn to the discussion of other features of the np elastic scattering amplitude. Though the strong interaction amplitude cannot be determined theoretically from the first principles, our confidence that $1/t$ dependence is absent in strong interactions for $|t| < m_{\pi}^2$ opens the road to bounding the light particle exchange if its mass is smaller than that of the π meson. Experimental data at $|t| < m_{\pi}^2$ matter for our bounds, which makes the precise value of Λ in the expression for FF unimportant, since in the relevant domain of $|t|$ the form factor is close to 1. For the same reason, no form factor is introduced for the Schwinger term in ([1\)](#page-1-0).

For each fixed set of parameters g_i^2 and μ describing the possible contribution of a ''new force,'' we fit the experimental distribution with a combined function ([1\)](#page-1-0) and ([2\)](#page-1-1), where parameters A and b describing the standard hadronic contribution are free. Then the maps of A, b, and the minimum values of χ^2 are composed as functions of g_i^2 and μ . Analyzing the χ^2 map, we determined the level of χ^2 [4] above which parameters of the "new force" become incompatible with experimental data at a confidence level (C.L.) greater than 90%. At this level we also examined and ensured that parameters A and b remain within the 90% C.L. close to those in the ''zero model.'' In this way, we can ensure that the new force contribution does not substitute for the standard hadronic plus electromagnetic contributions in the description of the data.

Figure 2 shows, for comparison, fits to the data for the ''zero model'' and for several excluded models for the new vector particle exchange with parameters slightly beyond the excluded limits for μ_V and g_V^2 .

Two comments need to be made on formulas (4) – (7) : First, the amplitudes with the exchange in the t-channel of a pointlike particle with spin α depend on s as s^{α} . This results in an amplitude behavior of s^0 for scalar and pseudoscalar particles and of s for vector and axial vector particles. This property of high-energy scattering amplitudes would allow us to determine the value of the spin of the new physics mediator. Second, the pseudoscalar exchange vanishes at $t = 0$. These comments explain why we will get the strongest bounds on g_A and g_V , a weaker bound on g_S , and no bound on g_P .

Before presenting fit results, let us explain why we neglect the interference of a new particle exchange amplitude with the strong amplitude and with the photon ex-

FIG. 2. Several fits to the experimental data of [2]: longdashed line—single exponent without the Schwinger contribution; solid line—the zero model description of [\(1\)](#page-1-0); dotted line the zero model plus the new vector particle contribution with $\mu = 1$ MeV and $g^2 = 0.0015$; short-dashed line–the zero model plus the new vector particle contribution with $\mu = 10$ MeV and $g² = 0.005$; dot-dashed line—the zero model plus the new vector particle contribution with $\mu = 40$ MeV and $g^2 = 0.025$.

change (Schwinger) amplitude. The strong amplitude is almost entirely imaginary in the energy domain studied in [2] ($|Re/Im|$ < 0.1 [1]). That is why it does not interfere with the real amplitude of a pointlike new particle exchange. Interference of the strong amplitude (as well as the new force amplitude) with the Schwinger term is negligible since the interference term is constant at $t = 0$,² unlike the square of the Schwinger amplitude,
which contributes significantly at small |t| because of 1/t which contributes significantly at small |t| because of $1/t$ behavior.

The mass of a light particle μ was bounded in our fits to be below 100 MeV, and the range of the coupling constants varied depending on the particular model. Compilation of the bounds obtained from the χ^2 limit for P, S, V, and A models in coordinates g^2 versus μ is presented in Fig. 3.

In the next step of our analysis, we checked that, for each model, fitted parameters A and b corresponding to the boundary of the excluded domain of g_i^2 and μ must not deviate from their zero model values by more than 1.28σ , where σ is the corresponding error of parameters A and b from the zero model. We found that these conditions are

²The Schwinger part of the interference term contains q_{μ}/q^2 , $q^2 \equiv t$, which may either multiply $q_\mu \equiv (p_1 - p_2)_\mu$, canceling
the $1/a^2$ enhancement or multiply $(p_1 + p_2)$ giving zero the $1/q^2$ enhancement, or multiply $(p_1 + p_2)$ _{μ} giving zero.

FIG. 3 (color online). Compilation of the upper bounds obtained in the current analysis in terms of g^2 and μ at a 90% C.L. No limit can be set for the pseudoscalar exchange. Brackets indicate the interval of mass μ where the analysis was validated (see text).

satisfied if μ < 40 MeV for the S, V, and A models, and cannot be satisfied for any value of μ for the P model. Thus, our bounds on the coupling strength g^2 shown in Fig. 3 should only be referred to in these validated domains (indicated in Fig. 3 by brackets). No consistent limit can be set for the P model. In addition, we should notice that the very high value of g_P obtained from the χ^2 analysis for the P model makes our perturbative approach of formula [\(5\)](#page-1-4) invalid. We should conclude, therefore, that the experimental data [2] do not provide any limit for the pseudoscalar exchange.

The factor of 2 in the Schwinger term of the zero model in [\(1](#page-1-0)) is coming from both $n-p$ and $n-e$ scattering and is an estimate of equal contribution from both. However, $n-e$ scattering occurs in a different kinematical range, and the event selection criteria in [2] could suppress the detection of electrons. Consequently, we varied the factor in the Schwinger term of the zero model ([1](#page-1-0)) from 1 (no $n - e$ contribution) to 3 (double $n - e$ contribution) and found in our analysis that this variation was not very significant, changing our limiting value for g^2 by $\pm 8\%$ for a fixed value of μ .

One can notice that, since the average $s \approx 540 \text{ GeV}^2$ in experiment [2], vector (A_V) and axial vector (A_A) amplitudes, as follows from Eqs. [\(6\)](#page-1-2) and ([7\)](#page-1-3), are practically the same (see Fig. 3), except when $\mu \le 1$ MeV. In this case, the last two terms of Eq. ([7](#page-1-3)) arising from the $q_{\mu}q_{\nu}/\mu^2$ part of the propagator of the axial vector particle start to dominate the amplitude.

Our bounds on the parameters g_V^2 and g_A^2 (Fig. 3) are rather strong; say, for $\mu = 10$ MeV, $g_{V,A}^2 < 5 \times 10^{-3}$ at 90% CT, which corresponds to 90% C.L., which corresponds to

$$
g_N^{V,A} < 0.071,\tag{8}
$$

4 times smaller than the QED coupling constant $\sqrt{4\pi\alpha} \approx$ 0.3. For the scalar exchange taking $\mu = 10$ MeV, we get a 0.3. For the scalar exchange, taking $\mu = 10$ MeV, we get a much weaker bound, $g_S^2 < 1.4$.

III. BOUNDS FROM LOWER ENERGY DATA

We will now compare our results from the previous section with other searches for new interactions [5–14] in which new light particles participate. In the literature, the effect of new forces is usually parametrized as a deviation from the Newtonian gravitational potential:

$$
V(r) = -G_N \frac{m_1 m_2}{r} [1 + \alpha_G \exp(-r/\lambda)],\qquad(9)
$$

which is an adequate approximation for the description of the effect of a new particle exchange between nonrelativistic constituents. The following relationship exists between the coupling constant α_G and characteristic length λ and our parameters g_i^2 and μ in cases of vector and scalar exchanges:

$$
\alpha_G = \frac{g_{V,S}^2}{4\pi G_N m_p m_n} = 1.35 \times 10^{37} g_{V,S}^2,\tag{10}
$$

$$
lg \alpha = lg g_{V,S}^2 + 37.13,
$$

$$
\lambda (cm) = \frac{1}{\mu (MeV) 5.05 \times 10^{10}}, \qquad lg \lambda = -lg \mu - 10.7.
$$
 (11)

The pseudoscalar exchange would not modify the potential in a nonrelativistic approximation, while axial coupling leads to an interaction among spins of constituents.

References [5–7] show e.g. that values of α_G larger than 1 are excluded for λ larger than 0.1 mm, while for smaller λ the upper bounds on α_G rapidly grow, reaching 10⁹ at the micron scale micron scale.

In Refs. [8–14], analogous bounds for shorter distances are presented. We see e.g. that for $\lambda = 10^{-13}$ m, α_G should
be less than 10^{30} or g^2 . less than 10^{-7} The corresponding be less than 10^{30} , or $g_{V,S}^2$ less than 10^{-7} . The corresponding
value of u is 2 MeV In Fig. 4, our limits for the V and S value of μ is 2 MeV. In Fig. 4, our limits for the V and S particle exchanges in terms of α_G and λ parameters are compared with the limits obtained in Refs. [5–14].

Additionally, the data on low-energy (1 keV $\leq E_n$ \leq 10 keV) neutron scattering on ^{208}Pb [15] were applied in Ref. [16] to obtain bounds on the possible contributions of a light scalar particle exchange to a neutron-nucleus potential. The upper bound on the coupling constant of a 10 MeV boson to a nucleon obtained in [16] corresponds to $g_{V,S}^2$ < 4 × 10⁻⁶. This rather restrictive bound was ob-
tained from the analysis of the shape of the differential tained from the analysis of the shape of the differential cross section of low-energy $n^{-208}Pb$ scattering, where the

FIG. 4 (color online). Experimental limits on α_G and λ from [5–14] parametrizing deviations from Newton's law. Our limits transformed into coordinates α_G and λ are also shown for comparison.

additional term, originating from the light scalar boson exchange, leads to a modification of angular dependence not observed in the experimental data. Let us stress that the same bound is valid for g_V .

According to [15,16], experimental data in the keV energy range are well described by the following expression:

$$
\frac{d\sigma}{d\Omega} = \frac{\sigma_0}{4\pi} [1 + \omega E \cos \theta],\tag{12}
$$

where $\sqrt{\sigma_0/4\pi} \approx 10$ fm $\sqrt{\sigma_0/4\pi} \approx 10$ fm and $\omega = (1.91 \pm 1)$
reV⁻¹ These numerical values are very reason- $0.42)10^{-3}$ keV⁻¹. These numerical values are very reason-
able from the nuclear scattering point of view, and from the able from the nuclear scattering point of view, and from the demand that these values are not spoiled by a Yukawa potential contribution originating from a light boson exchange, bounds on g and μ were obtained in [16]. The point is that the Yukawa amplitude, interfering with the strong interaction amplitude, will show up in the following contribution to ω for $E \rightarrow 0$:

$$
|\Delta\omega| = \frac{16m_n^2}{\sqrt{\sigma_0/4\pi}} \frac{\lambda_n^2}{4\pi} \frac{A}{\mu^4},\tag{13}
$$

and from the demand that $\Delta \omega \leq \omega$, the above mentioned bound was extracted. For an update of results obtained in [16], see [17].

It is quite natural to assume that the coupling of a new light boson with nucleons originates from its coupling with u and d quarks. In this case, bounds from pion and kaon decays [18] are applicable. Let us start with vector coupling. According to the conservation of vector current, couplings to nucleons are equal to the sum of the couplings to quarks: $2f_{uv} + f_{dv}$ for a proton and $f_{uv} + 2f_{dv}$ for a neutron (f_i are analogous to our g_N^i). The $\pi^0 \to VV$ decay contributes to $\pi^0 \rightarrow$ invisible decays and, using the experimental bound $Br(\pi^0 \rightarrow \nu \nu)$ < 2.7 \times 10⁻⁷ in [18], the following bound was obtained:

$$
\sqrt{|f_{uV}^2 - f_{dV}^2|} \le 4 \times 10^{-3},\tag{14}
$$

which is automatically satisfied for an isoscalar coupling, $f_{uv} = f_{dv}$. However, the bound on the $\pi^0 \rightarrow \gamma V$ decay, with the latter contributing into the $\pi^0 \rightarrow \gamma +$ invisible mode, will translate into bounding of isoscalar couplings as well $[18]$:

$$
\frac{2f_{uV} + f_{dV}}{3} < 1.6 \times 10^{-3}.\tag{15}
$$

Here, the experimental bound $Br(\pi^0 \to \gamma \nu \nu) < 6 \times$
⁻⁴ was used These numbers should be compared with 10^{-4} was used. These numbers should be compared with our bound on g_N^V ([8](#page-3-0)).

Since $\pi^0 \to SS$ and $\pi^0 \to S\gamma$ decays violate the corresponding P and C parities, we do not obtain bounds on f_S from these decays. C-parity conservation forbids the $\pi^0 \rightarrow$ γA decay as well, while from the bound on $\pi^0 \rightarrow$ invisible decays, we get the coupling constant bound [\(14\)](#page-4-0) for the axial vector boson.

More stringent upper bounds on the coupling constants follow from very strong experimental limits on the branching ratio Br($K^+ \rightarrow \pi^+ + \nu \nu$) < 2 × 10⁻¹⁰. The longitudinal component of the axial vector boson contributes to the decay amplitude proportionally as $(2m_q/\mu)f_{qA}$ [18], and even if the axial vector boson couples only with light quarks, we obtain

$$
f_{u,dA} \lesssim 10^{-6} \mu \text{(MeV)}.
$$
 (16)

The factor of $2m_q/\mu$ is absent when the axial vector interaction is substituted by the scalar interaction, and thus we obtain

$$
f_{u,dS} \lesssim 10^{-5}.\tag{17}
$$

Fortunately, the conservation of vector current forbids $K \to \pi V$ decays for $\mu^2 = 0$, so that is why the bound on the vector coupling for light μ is not very strong:

$$
f_{u,dV}\left(\frac{\mu}{m_K}\right)^2 \lesssim 10^{-5}.\tag{18}
$$

IV. CONCLUSIONS

Our bounds obtained from high-energy and very small momentum transfer np elastic scattering data [2] provide exclusions of new forces at distances above 5 fm, which corresponds to exchanged particle masses lighter than 40 MeV. These bounds are extracted in a covariant approach, as an alternative to the bounds on couplings at larger distances, extracted from the absence of deviations from the Newtonian gravitational law.

Both low-energy n^{-208} Pb and high-energy np scattering
ta lead to similar upper bounds on the coupling constants data lead to similar upper bounds on the coupling constants for ≈ 10 MeV vector bosons, though upper bounds from *n*-Pb scattering on the coupling constant g_V^V are \sim 30 times
lower and close to the bounds from $\pi^0 \rightarrow$ invisible and lower and close to the bounds from $\pi^0 \rightarrow$ invisible and $\pi^0 \rightarrow \gamma$ + invisible decays on the vector coupling constants with quarks.

Strong upper bounds on the new physics contribution into the $K^+ \rightarrow \pi^+$ + invisible decay allow us to get very strong bounds for scalar and axial vector bosons: $g_N^{A,S}$ 10^{-5} for a 10 MeV boson mass.

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APPENDIX

For vanishing light quark masses, their isotriplet axial current is conserved. The same should hold for the nucleon currents and is achieved by accounting for pion exchanges:

$$
\tilde{A}_A = g_A^2 \bar{n} \gamma_\beta \gamma_5 n \Big(g_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2 - m_\pi^2} \Big) \frac{(g_{\alpha\mu} - \frac{k_\alpha k_\mu}{\mu^2})}{k^2 - \mu^2} \times \Big(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2 - m_\pi^2} \Big) \bar{p} \gamma_\nu \gamma_5 p
$$
\n
$$
= \frac{g_A^2}{k^2 - \mu^2} \Big[g_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mu^2} \frac{(m_\pi^4 - 2\mu^2 m_\pi^2 + k^2 \mu^2)}{(k^2 - m_\pi^2)^2} \Big] \times \bar{n} \gamma_\alpha \gamma_5 n \bar{p} \gamma_\beta \gamma_5 p. \tag{A1}
$$

For a massless pion, the $1/\mu^2$ singularity cancels out and the expression in square brackets contains $k_{\alpha}k_{\beta}/k^2$, which, acting upon fermionic axial currents, becomes $\left(\frac{(2m_N)^2}{k^2}\right)$. The numerator of the expression for the differ-
ential cross section is regular at $k^2 \equiv t = 0$ since the ential cross section is regular at $k^2 \equiv t = 0$ since the square of the pseudoscalar exchange amplitude contains t^2 in the numerator [see [\(5](#page-1-4))], while the interference of the axial vector and pseudoscalar exchanges is proportional to t [the denominator equals $(t - \mu^2)^2$ independently of the spin of the exchanged boson].

In real life, light quarks, as well as pions, have nonzero masses, and to obtain an amplitude square for the axial vector boson exchange, we should substitute μ by $\tilde{\mu}$ in the square brackets of ([7\)](#page-1-3), where

$$
\frac{1}{\tilde{\mu}^2} = \frac{1}{\mu^2} \frac{m_{\pi}^4 - 2\mu^2 m_{\pi}^2 + t\mu^2}{(t - m_{\pi}^2)^2},
$$

and for t, $\mu^2 \ll m_{\pi}^2$ we get $\tilde{\mu} = \mu$.
The numerator of the expression

The numerator of the expression for the differential cross section is singular for $\mu \rightarrow 0$. However, in renormalizable theory, the mass of the axial vector boson equals its gauge coupling constant (g_A in our case) times the vacuum average of the corresponding Higgs field. As an example, one can have in mind the expansion of the standard model with two Higgs doublets with opposite hypercharges, where Peccei-Quinn $U(1)$ symmetry is spontaneously broken, producing an axion. In order to suppress axion couplings to quarks and leptons, the additional singlet neutral Higgs field N is usually added, which makes the axion invisible. Gauging of Peccei-Quinn $U(1)$ leads to the axial coupling of the corresponding vector boson to matter. Such a light axial vector boson is discussed, in particular, in [18], where it is light due to the smallness of the gauge coupling constant, while the vacuum average $\langle N \rangle \gg 100$ GeV, making it superweakly coupled to matter $(g_A/\mu \sim 1/\langle N \rangle)$.

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