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Consistent scenario for $B \to PS$ decays

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We consider $B \to PS$ decays where P stands for pseudoscalar and S for a heavy (1500 MeV) scalar meson. We achieve agreement with available experimental data, which includes two orders of magnitude hierarchy, assuming the scalars mesons are two quark states. The contribution of the dipolar penguin operator $\mathcal{O}_{11}$ is quantified.

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I. INTRODUCTION

The scalar sector below 2 GeV is poorly understood, nevertheless several features—like the presence of two multiplets and several of their properties—naturally arise in the analysis of a number of authors. A first set of scalars with masses around 1.5 GeV [1] is grouped in a heavy multiplet, including the $K_0^*(1430)$, $a_0(1450)$, $f_0(1500)$ for the octet, $f_0(1370)$ which is identified with the singlet and the $f_0(1710)$ which seems to be mainly glueball. The octet is nearly degenerate, like similar pseudoscalar, vector, axial vector and tensor multiplets, their widths are small (≤ 100 MeV). The mixing angles seems to be small except by the singlet-glueball which is around $-20°$, according to H. Y. Cheng in Ref. [2]. It has been more difficult to establish the lighter multiplet, even the existence and nature of some of their members is in doubt. The light multiplet should include the $a_0(980)$, $f_0(980)$ and the $\kappa = K_0^*(800)$ in the octet; while the singlet could be identified with the $\sigma = f_0(600)$. The mixing is not clear and their widths are very large. Ideally, the former multiplet can be identified as the ground state of quark-antiquark bound states with angular momenta one while the latter with the ground state of four quarks systems with zero angular momenta. In the real world an undetermined mixing between the two multiplets is expected. Alternatively both multiplets could be identified as quark-antiquark states with angular momenta one, the lighter being the ground state while the heavier the first excited state.

A full understanding of the scalar multiplets previously described remains a challenge, both from the experimental perspective as well as from the theoretical point of view [1]. To start with, there is not enough conclusive experimental information regarding the existence and properties of the scalars. Notice that the information is poor not because of the lack of sources of scalar mesons, for example, many of the decays of particles containing c or b quarks involve the production of scalar mesons. The information on the scalars is scarce because of the large width they have since that produces a large overlap with nearby resonances and with the background. In spite of those problems, precise experimental results are available [1,3,4] for the mass and width of the f_0 and K_0^*, for the β angle [5] of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and for several partial widths. It has been speculated that the α angle can be extracted in processes involving scalars [6] and new projects like the LHCB [7] will improve the old measurements and obtain new results. Relevant to our work are the branching ratios for the $B \to PS$ decays measured by different groups, which show a nontrivial hierarchy. The experimental data collected in Table I suggest that, for $B \to PS$ decays including members of the heavy scalar multiplet, the order of magnitude of the branching ratios involving the $K_0^*(1430)$, the $f_0(1370)$, $f_0(1500)$ and the $a_0(1450)$ are different.

On the theoretical side the situation is not better. The origin of the difficulties are the nonperturbative regime of QCD and the limited computer capacity for the lattice approach. The nature of the observed scalars has been discussed at length, and proposals exist to identify them as 2 or 4 quark states, glueballs, molecules, etc., and several theoretical formalisms have been developed to calculate nonleptonic decays. The simplest one is the so-called "Naive Factorization Approach" (NFA) [9], which in general produces the correct order of magnitude and its predictions are in rough agreement with the experimental results. Discrepancies are known to occur in two cases, for "color suppressed" processes and when important rescattering effects are involved, for example, processes where direct CP violation is relevant [10,11]. The advantage of

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TABLE I. Branching ratios for the $B \to PS$ decays (in units of 10^{-6}), for the heavier scalar multiplet. The values reported for the widths marked with $*$ include the corresponding branching of the scalar decaying channel. To obtain the NFA predictions we used $B(f_0(1370) \to 2\pi) = 0.26(1)$, $B(f_0(1500) \to 2\pi) = 0.35(2)$ and for the $a_0(1450) \to \pi\eta$ no reliable value exists [3].

Decay	BELLE	*BABAR*	HFAG [3]	B_{exp}	NFA	NFA + $\mathcal{O}_{11}$	QCDF [8]	pQCD [8]
$\pi^- a_0^+(1450)(\pi\eta)$		$<2.3^*$	$<2.3^*$		8		3.1	
$\pi^+ a_0^-(1450)$					2		0.5	
$\pi^- a_0^0(1450)$					4		2.5	
$\pi^- f_0(1370)$		<3	<3					
$\pi^- f_0(1500)$								
$\pi^0 a_0^-(1450)$					0.01		1.1	
$K^+ a_0^-(1450)$		$<3.1^*$	$<3.1^*$		1		0.3	
$K^+ a_0^0(1450)$					0.5		0.2	
$K^- f_0(1370)(\pi\pi)$		$<10.7^*$	$<10.7^*$	<41	8	7		
$K^- f_0(1500)(\pi\pi)$		$0.73 \pm 0.21 \pm 0.47^*$	$0.7(5)^*$	2(1)	23	21		55
$\bar{K}^0 a_0^-(1450)$							0.1	
$K^0 a_0^0(1450)$							0.1	
$K^0 f_0(1370)$					7	7		
$K^0 f_0(1500)$					22	21		42
$\pi^- K_0^{*+}(K^+\pi^0)$	$49.7 \pm 3.8 \pm 3.8^{+1.2}_{-4.8}$	$25.4^{+3.0+6.1}_{-3.7-5.6}$	34(5)	34(5)	45	45	11	43
$\pi^+ K_0^{*0}(K^+\pi^-)$	$51.6 \pm 1.7 \pm 6.8^{+1.8}_{-3.1}$	$32.0 \pm 1.2^{10.8}_{-6}$	45(6)	45(6)	45 (in)	45 (in)	11	48
$\pi^0 K_0^{*+}$					25	25	5.3	29
ηK_0^{*+}		$15.8 \pm 2.2 \pm 1.4 \pm 1.7$	16(3)	16(3)	7	7		
$\pi^0 K_0^{*0}$		$11.7^{+1.4+4}_{-1.3-3.6}$	12(4)	12(4)	17	17	6.4	18
ηK_0^{*0}		$9.6 \pm 1.4 \pm 0.7 \pm 1.1$	10(2)	10(2)	7	7		

formalism where a systematic expansion is implemented and where higher-order correction can be organized and controlled is of great importance (QCDF, SCET, pQCD, LCSR, etc. [12–15]), in particular, when high accuracy predictions are required.

Additional reasons to study the $B \to PS$ decays are that they offer a window to study the spectroscopy and the dynamics of the scalar sector and that the $B \to 3P$ decays get a contribution from the $B \to PS$, PV, PT, so that in order to achieve an appropriated estimate for the former decay the latter must be well known [16]. In a similar way one can argue that in order to extract signals of possible new physics, the contribution of low-lying conventional physics has to be known in detail, including the contributions of the scalar mesons [17]. We believe that the understanding of the physical origin of the hierarchy of scales appearing in the $B \to PS$ decays can shed some light on the nature of the scalars [8,18]. Complementary information on the nature of the scalars may be obtained from $D \to PS$ physics [19]: in the first case through the decay constants, $\bar{f}^s$ while in the latter through the F^{DS} form factors. The purpose of the present work is to consider the $B \to PS$ decays with S a member of the heavy scalar multiplet. We assume that the leading contribution to these processes is given by the NFA and that, in first approximation, contributions other than the leading one can be safely neglected. In these conditions the dominant contribution can be clearly identified and the existence of the scales in the branching ratios naturally arises. Besides the NFA our approach can be summarized along the following lines: we include ten dimension-six four-quark operators and the dimension-five chromomagnetic operator O_{11} [20]; annihilation contributions are included and the form factors required are obtained by using sum rules, so infrared divergences are absent. This approach, together with $SU(3)$ symmetry, allows us to reproduce the pattern observed experimentally.

II. BRANCHING RATIOS AND MIXING

Our results are summarized in Table I. It is worth remarking that both the experimental data and our results point to the existence of branching ratios that range from 45 to 0.5 (in units of 10^{-6}). In the following paragraphs we introduce the notation conventions and explain the procedure we follow to obtain these branching ratios. Within the NFA the hadronic matrix elements can be reduced to products of decay constants and form factors. In order to achieve this, one uses the "vacuum saturation" approximation and neglects other intermediate states. This seems to be a reasonable assumption since the hadronic resonances have masses in the 1–2 GeV range, far from the m_b region. For the invariant amplitude we write $\mathcal{M}_{f\to i} = \langle f|H|i\rangle = G_F A_{f\to i}/\sqrt{2}$ while the branching ratios are given by $B = \tau_B G_F^2 |A|^2 p/16\pi m_B^2 = \tau_B G_F^2 |A|^2/32\pi m_B$, with τ_B the B lifetime. The decay constants and form factors are defined as [8,9,18,21]

$$
\begin{aligned}
\langle P(p)|A_\mu|0\rangle &= -if_P p_\mu; \\
\langle S(p)|V_\mu|0\rangle &= f_S p_\mu = \frac{m_2 - m_1}{m_S}\bar{f}_S p_\mu \\
\langle f_0|q\bar{q}|0\rangle &= m_{f_0}\bar{f}_{f_0}, \\
\langle S(p_2)|L_\mu|P(p_1)\rangle &= -i\bigg[\bigg(p_1 + p_2 - \frac{m_1^2 - m_2^2}{q^2}q\bigg)_\mu F_+^{M_1M_2} \\
&\quad + \frac{m_1^2 - m_2^2}{q^2}q_\mu F_0^{M_1M_2}(q^2)\bigg] \qquad (1)
\end{aligned}
$$

with $q = p_1 - p_2$.

We have left to the Appendix details regarding the effective Hamiltonian we use—which includes ten dimension-six operators and the so-called $\mathcal{O}_{11}$ operator—and the matrix elements' evaluation. The most interesting decays are those involving the $S = K_0^*(1430)$ both because they have the largest branching ratio (around 40, in units of 10^{-6}) and because the theoretical predictions are the cleanest. The a_6 term is by far the dominant one. The amplitudes are proportional to $\lambda_{ts} f_{K_0^*} a_6 m_{K_0^*}^2/m_s m_b \sim \lambda_{ts} a_6 m_b m_{K_0^*}\bar{f}_{K_0^*}$ times $SU(3)$ factors. The origin of the enhancement is a combination of large CKM matrix elements, the nonvanishing decay constant and a large $m_{K_0^*}$ (chiral enhancement) mass. The $SU(3)$ symmetry allows us to relate different decays involving the K_0^* and so, by measuring one of them, one can predict the others, a fact that is not distorted by the $\mathcal{O}_{11}$ contributions. For the numerical analysis we used the following input parameters: $F^{B\pi} = 0.27(4)$, $F^{BK} = 0.33(4)$, $m_s(2.1) = 90$ MeV, $F^{Ba_0(1450)} = F^{BK_0^*(1430)} = 0.26$ and, when required, $SU(3)$ relations are invoked. Although predictions for $f_{K_0^*}$ are available [8], we preferred to include the $B^+ \to \pi^+ K_0^{*0}$ experimental value as an input, obtaining thus $f_{K_0^*}^{\rm eff.} \simeq 58$ MeV ($f_{K_0^*}^{\rm eff.} \simeq 56$ MeV when the $\mathcal{O}_{11}$ is taken into account). The branching ratios we obtain for other channels involving the K_0^* are reported in Table I. Notice that the value obtained for $f_{K_0^*}$ is not far from the theoretical predictions (see Table II).

We now consider the decays involving $S = f_0(1370)$, $f_0(1500)$ and $f_0(1700)$. Their relevance stems from the large branching ratios predicted for them [8]—of the same order as the K_0^*—and also due to the possible glueball nature of the $f_0(1700)$. Their amplitudes are proportional to $\lambda_{ts} a_6 m_b m_{K_0^*}\bar{f}_{f_0}^s$ times $SU(3)$ factors and mixing angles (s–quark content). Our predictions for these processes are included in Table I; unfortunately the experimental results are still inconclusive. Note that except for the $f_0(1500)$ decay channel, the NFA plus $SU(3)$ symmetry for the heavy scalar multiplet leads to predictions for the branching ratios in rough agreement with the experimental values. However, even if the experimental data is poor the discrepancy between our results and experimental data is evident—there is a 1 order of magnitude difference. In this sense it is important to remark that in order to obtain the results of Table I we assumed, following H. Y. Cheng [2], a mixing between the glueball, singlet and octet components given by

$$
\begin{aligned}
\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1700) \end{pmatrix} &= \begin{pmatrix} 0.78 & 0.51 & -0.36 \\ -0.54 & 0.84 & 0.03 \\ 0.32 & 0.18 & 0.93 \end{pmatrix} \begin{pmatrix} N \\ S \\ G \end{pmatrix} \\
&= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -s_{23}c_{12} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{2}{3}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} N \\ S \\ G \end{pmatrix} \qquad (2)
\end{aligned}
$$

TABLE II. Decay constants for scalars (in MeV). The heavy scalars are assumed to be two-quark states. Notice that the constants computed by Cheng were obtained by using sum rules, OPE and Renormalization Group equations that render $\bar{f}$ scale dependant.

Ref.	$(f/\bar{f})_{K_0^*(1430)}, \bar{f}$	$\bar{f}_{a_0(1450)}$	$\bar{f}^s_{f_0(1500)}$	m_s [GeV]
Meurice-87 [22]	27	...	...	
Narison-89 [22]	40(6)	...	...	
Maltman [22]	42(2)	390(159)	...	
Chernyak-01 [22]	70(10)	...	...	
Shakin-01 [22]	30	207	...	
Pennington-01 [22]	...	...	...	
Du-04 [22]	42(8), 427(85)		...	0.14
Cheng-05 [8] at $\mu = 1$ GeV	445(50)	460(50)	490(50)	0.119
Cheng-05 [8] at $\mu = 2.1$ GeV	550(60)	570(60)	605(60)	0.09
Lu-07 [23] at $\mu = 1$ GeV	349–375	325–350	381–426	

where $s_i = \sin\theta_i$ and so on. The angles $\theta_{12} \simeq 2^o$, $\theta_{13} \simeq -21^o$ and $\theta_{23} \simeq 2^o$ are the mixing between singlet-octet, singlet-glueball and octet-glueball, respectively. The singlet and the octet are $f_0(1370) \sim f_{\text{sing.}} = \sqrt{2/3}N + S/\sqrt{3}$, $f_0(1500) \sim f_{\text{oct.}} = N/\sqrt{3} - S\sqrt{2/3}$, $S = \bar{s}s$, $N = (\bar{u}u + \bar{d}d)/\sqrt{2}$ and $G = gg$ the glueball. Thus, in this approach [2], there is only a small mixing between the singlet and the glueball. Using these values the prediction for $B \to f_0(1500)K$ is in conflict with the experimental data. One way to avoid this problem is to leave θ_{12} as a free parameter, keeping the others fixed. Using the experimental data we obtain the following inequality for the mixing between the singlet and the octet:

$$\left| -s_{12}\sqrt{\frac{1}{3}} + c_{12}\sqrt{\frac{2}{3}} \right| \leq 0.34. \tag{3}$$

These constraints lead two possible values:

$$35° \leq \theta_{12} \leq 74° \tag{4}$$

$$215° \leq \theta_{12} \leq 254°. \tag{5}$$

It is worth noticing that these values for the mixing are close to those mentioned by several groups [1].

Finally for the decays involving the $a_0(1450)$, $B \to a_0(1450)\pi, a_0(1450)K$, the terms proportional to $a_4 - a_6 \sim 0$ almost vanish and the branching ratios are smaller. Two different cases must be considered. The first when the amplitude is dominated by the tree-level contribution a_1 (the amplitudes are proportional to $\lambda_{ud} a_1 m_B^2 f_\pi$), then the theoretical prediction is reliable, and the branchings are predicted to be around 10 (in units of 10^{-6}). The second case arises when no tree-level contribution exists and terms like annihilation are dominant. In this case the branchings are of order 0.1–1 (in units of 10^{-6}), but the theoretical uncertainties are larger since other contributions (Final States Interactions, for example [11]) may be important. Unfortunately little is known about these corrections.

III. SUMMARY

In this work we studied the $B \to PS$ decay where S stands for a member of the heavy scalar multiplet. The computations have been done assuming the heavy scalar multiplet is a two-quark state, using $SU(3)$ symmetry and the naive factorization approach. Our conclusions can be summarized as follows:

(i) Within the error bars, it is possible to reproduce the hierarchy of branching ratios experimentally observed in the $B \to PS$ decays, whether or not the operator $\mathcal{O}_{11}$ is included.

(ii) When the singlet-octet mixing given by [1] is used, we obtain a prediction for the $f_0(1500)$ which is 1 order of magnitude above the experimental limit. A solution to this problem can be obtained by modifying the mixing matrix. In such a case one obtains a constraint on the singlet-octet mixing and its s-quark content.

(iii) The contribution of the $\mathcal{O}_{11}$ operator is around 30% in decay channels involving the K_0^*. The $\mathcal{O}_{11}$ contributions approximately keep the $SU(3)$ relations between different decay channels.

(iv) Our approach is based upon the assumption of the vacuum saturation dominance. Nonfactorizable effects, final state interactions and other physical mechanisms are expected to lead corrections to the NFA, and although a failure of the NFA cannot be excluded, the order of magnitude of the branching ratios is not expected to change. As far as we can see *strong* deviations from our predictions could hint not to discrepancy on the mechanisms involved in the processes but to the structure assumed for the scalar mesons, for example, the two-quark structure of the mesons or their glue content.

(v) The approach used in this work can be improved in a number of ways by using QCD-based formalisms. In these schemes experimental data is required as input; unfortunately the large error bars in existing experimental data induce large theoretical errors preventing thus any interpretation of the results [21].

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APPENDIX: SOMES AMPLITUDES

Some of amplitudes, including O_{11} contribution, in the NFA are given by

$$
\begin{aligned}
A_{B^-\to\pi^- S^0} \simeq{}& \lambda_{ud} a_1 (X^{\pi^-}_{B^- S^0} + X^{B^-}_{S^0\pi^-}) - \lambda_{td}\bigg[\bigg(a_4 + a_{10} - \frac{(a_6+a_8)m_\pi^2}{\hat{m}(m_b+m_u)}\bigg) X^{\pi^-}_{B^- S^0} + \bigg(a_4 + a_{10} - \frac{(a_6+a_8)m_B^2}{\hat{m}(m_b+m_u)}\bigg) X^{B^-}_{S^0\pi^-} \\
&+ (a_8 - 2a_6)\tilde{X}^{S^0_d}_{B^-\pi^-}\bigg] \\
A_{B^-\to\pi^0 a_0^-} \simeq{}& \lambda_{ud}[a_1(X^{a_0^-}_{B^-\pi^0} + X^{B^-}_{a_0^-\pi^0}) + a_2 X^{\pi^0_u}_{B^- a_0^-}] - \lambda_{td}\bigg[(a_4 + a_{10}) X^{a_0^-}_{B^-\pi^0} - 2(a_6+a_8)\tilde{X}^{a_0^-}_{B^-\pi^0} \\
&- \bigg(a_4 - \frac{3}{2}(a_9 - a_7) - \frac{1}{2}a_{10} - \frac{(a_6+a_8)m_\pi^2}{m_u(m_b+m_d)}\bigg) X^{\pi^0_u}_{B^- a_0^-} + \bigg(a_4 + a_{10} - \frac{(a_6+a_8)m_B^2}{\hat{m}(m_b+m_u)}\bigg) X^{B^-}_{a_0^-\pi^0}\bigg] \\
A_{\bar{B}^0\to\pi^0 S^0} \simeq{}& \lambda_{ud} a_2 (X^{\pi^0_u}_{\bar{B}^0 a_0^0} + X^{\bar{B}^0}_{(a_0^0\pi^0)_u}) - \lambda_{td}\bigg[\bigg(-\frac{3}{2}a_7 + \frac{(2a_6 - a_8)m_\pi^2}{2m_d(m_b - m_d)}\bigg) X^{\pi^0_u}_{\bar{B}^0 a_0^0} + \bigg(2a_5 + \frac{3}{2}a_7 + \frac{(2a_6 - a_8)m_B^2}{2m_d(m_b+m_d)}\bigg) X^{\bar{B}^0}_{(a_0^0\pi^0)_u}\bigg] \\
A_{\bar{B}^0\to\bar{K}^0 S^0} \simeq{}& -\lambda_{ts}\bigg[\bigg(a_4 - \frac{a_{10}}{2} - (a_6^{\rm eff.} - a_8/2) r^K_\chi\bigg) X^{\bar{K}^0}_{\bar{B}^0 S^0} - \bigg(2a_6 - a_8 - \frac{30}{32}a_{11}\bigg)\tilde{X}^{S^0_s}_{\bar{B}^0\bar{K}^0} \\
&+ \bigg(a_4 - \frac{a_{10}}{2} - \frac{(2a_6 - a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\bigg) X^{\bar{B}^0}_{S^0\bar{K}^0}\bigg] \\
A_{\bar{B}^0\to\pi^0\bar{K}^{*0}_0} \simeq{}& \lambda_{us} a_2 X^{\pi^0}_{\bar{B}^0\bar{K}^{*0}_0} - \lambda_{ts}\bigg[\bigg(a_4 - \frac{a_{10}}{2} - (a_6^{\rm eff.} - a_8/8) r^*_\chi\bigg) X^{\bar{K}^{*0}_0}_{\bar{B}^0\pi^0} + \frac{3}{2}(a_9 - a_7) X^{\pi^0}_{\bar{B}^0\bar{K}^{*0}_0} \\
&+ \bigg(a_4 - \frac{a_{10}}{2} - \frac{(2a_6 - a_8)m_B^2}{(m_b+m_d)(m_s+m_d)}\bigg) X^{\bar{B}^0}_{\bar{K}^{*0}_0\pi^0}\bigg]
\end{aligned}
\tag{A1}
$$

and $A_{\bar{B}^0\to K^{*+}_0\pi^-} = 0$, $S^0 = a_0^0$, σ and f_0, $r^*_\chi \simeq 2m^2_{K^*_0}/m_b m_s$ and $a_6^{\rm eff.} r^M_\chi = a_6 r^M_\chi - a_{11}[12(1 - r^M_\chi) - 1]/32$.

$$
\begin{aligned}
X^{K^-}_{B^- f_0} &= \langle K^-|(\bar{u}s)_L|0\rangle\langle f_0|(\bar{u}b)_L|B^-\rangle = f_K(m_B^2 - m_{f_0}^2)F_0^{B^- f_0}(m_\pi^2) = f_K \frac{F_0^{\bar{B}^0 a_0^+}(m_K^2)}{\sqrt{2}}(m_B^2 - m_{f_0}^2)\sin\phi_S \\
X^{\bar{K}^0}_{\bar{B}^0 f_0} &= \langle \bar{K}^0|(\bar{s}d)_L|0\rangle\langle f_0|(\bar{d}b)_L|\bar{B}^0\rangle = f_{\bar{K}^0}(m_B^2 - m_{f_0}^2)F_0^{\bar{B}^0 f_0}(m_K^2) = f_K \frac{F_0^{\bar{B}^0 a_0^+}(m_K^2)}{\sqrt{2}}(m_B^2 - m_{f_0}^2)\sin\phi_S \\
X^{K^{*-}_0}_{\bar{B}^0\pi^+} &= \langle K^{*-}_0|(\bar{s}u)_L|0\rangle\langle \pi^+|(\bar{u}b)_L|\bar{B}^0\rangle = f_{K^{*-}_0}(m_B^2 - m_\pi^2)F_0^{\bar{B}^0\pi^+}(m^2_{K^*_0}) = -f_{K^{*-}_0}(m_B^2 - m_\pi^2)F_0^{B^0\pi^-}(m^2_{K^*_0}) \\
X^{\bar{K}^{*0}_0}_{B^-\pi^-} &= \langle \bar{K}^{*0}_0|(\bar{s}d)_L|0\rangle\langle \pi^-|(\bar{d}b)_L|B^-\rangle = f_{\bar{K}^{*0}_0}(m_B^2 - m_\pi^2)F_0^{\bar{B}^-\pi^-}(m^2_{K^*_0}) = -f_{K^{*0}_0}(m_B^2 - m_\pi^2)F_0^{B^0\pi^-}(m^2_{K^*_0}) \\
X^{K^{*-}_0}_{B^-\pi^0} &= \langle K^{*-}_0|(\bar{s}u)_L|0\rangle\langle \pi_0|(\bar{u}b)_L|B^-\rangle = f_{K^{*-}_0}(m_B^2 - m_{\pi^0}^2)F_0^{B^-\pi_0}(m^2_{K^*_0}) = -f_{K^*_0}\frac{F_0^{B^0\pi^-}(m_*^2)}{\sqrt{2}} \\
X^{\pi^0_u}_{\bar{B}^0\bar{K}^{*0}_0} &= \langle \pi^0|(\bar{u}u)_L|0\rangle\langle \bar{K}^{*0}_0|(\bar{s}b)_L|\bar{B}^0\rangle = \frac{f_\pi}{\sqrt{2}}(m_B^2 - m^2_{K^*_0})F_0^{\bar{B}^0\bar{K}^{*0}_0}(m_\pi^2) = \frac{f_\pi}{\sqrt{2}}(m_B^2 - m^2_{K^*_0}) r_{K\pi} F_0^{\bar{B}^0 a_0^+}(m_\pi^2) \\
X^{\bar{K}^{*0}_0}_{\bar{B}^0\pi^0} &= \langle K^{*0}_0|(\bar{s}d)_L|0\rangle\langle \pi^0|(\bar{d}b)_L|\bar{B}^0\rangle = f_{K^{*0}_0}(m_B^2 - m_{\pi^0}^2)F_0^{\bar{B}^0\pi^0}(m^2_{K^*_0}) = f_{K^*_0}(m_B^2 - m_\pi^2)\frac{F_0^{B^0\pi^-}(m_*^2)}{\sqrt{2}} \\
\tilde{X}^{f_{0d}}_{B^-\pi^-} &= \langle f_0|\bar{d}d|0\rangle\langle \pi^-|\bar{d}b|B^-\rangle = m_S \bar{f}^d_{f_0}\frac{m_B^2 - m_\pi^2}{m_b - m_d}F_0^{B^-\pi^-}(m_{f_0}^2) = -\frac{1}{\sqrt{2}}\frac{m_B^2 - m_\pi^2}{m_b - m_d} m_{f_0}\bar{f}^n_{f_0}F_0^{B^0\pi^-}(m_{f_0}^2) \\
\tilde{X}^{f_{0s}}_{B^- K^-} &= \langle f_0|\bar{s}s|0\rangle\langle K^-|\bar{s}b|B^-\rangle = m_{f_0}\bar{f}^s_{f_0}\frac{m_B^2 - m_K^2}{m_b - m_s}F_0^{B^- K^-}(m_{f_0}^2) = \frac{m_B^2 - m_K^2}{m_b - m_s} r_{K\pi} m_{f_0}\bar{f}^s_{f_0}F_0^{B^0\bar{\pi}^-}(m_{f_0}^2) \\
\tilde{X}^{f_{0s}}_{\bar{B}^0\bar{K}^0} &= \langle f_0|\bar{s}s|0\rangle\langle \bar{K}^0|\bar{s}b|\bar{B}^0\rangle = m_{f_0}\bar{f}^s_{f_0}\frac{m_B^2 - m_K^2}{m_b - m_s}F_0^{\bar{B}^0\bar{K}^0}(m_{f_0}^2) = \tilde{X}^{f_{0s}}_{B^- K^-}
\end{aligned}
\tag{A2}
$$

with $r_{K\pi} = F^{BK}/F^{B\pi} \simeq f_K/f_\pi \simeq 1.21(9)$.

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