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M2 signatures in $\psi(2S)$ radiative decays

Jonathan L. Rosner*

Enrico Fermi Institute and Department of Physics, University of Chicago, 5640 S. Ellis Avenue, Chicago, Illinois 60637, USA

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The sensitivity of observables in radiative decays $\psi(2S) \to \gamma\chi_{c1,2} \to \gamma\gamma J/\psi$ to quadrupole (M2) admixtures in the dominant electric dipole (E1) transitions is explored. Emphasis is placed on distributions in a single angle, and several examples are given.

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Radiative decays of charmonium ($c\bar{c}$) states are expected to be dominated by electric dipole (E1) transitions, with higher multipoles suppressed by powers of photon energy divided by quark mass [1]. (An early discussion of charmonium radiative transitions may be found in Ref. [2].) The search for contributions of higher multipoles is of interest as a source of information on the charmed quark's magnetic moment, which is difficult to measure directly because of the extremely short lifetime of charmed hadrons. The possibility of anomalous magnetic moments of heavy quarks being larger than those of light ones, as a result of light-quark loops, was raised in Ref. [3].

Expectations for the magnitude of magnetic quadrupole (M2) transitions in charmonium were calculated in Ref. [4], and detailed discussions of various ways to extract the M2/E1 ratio were presented in Ref. [5]. Many of these methods involved the use of distributions in several decay angles. The present note is devoted to several sources of information on the M2/E1 ratio based on distributions with respect to a single angle. These distributions can be examined in new data on $\psi(2S)$ decays from the CLEO Detector at the Cornell Electron-Positron Storage Ring [6].

The distributions which will be examined are with respect to the following angles: (1) the angle θ' between the initial positron e^+ and the photon γ' in the $\psi(2S)$ center-of-mass system (c.m.s.), where $\psi(2S) \to \gamma'\chi_c$; (2) the angle θ between the photon γ in $\chi_c \to \gamma J/\psi$ and the final ℓ^+ direction in $J/\psi \to \ell^+\ell^-$, in the J/ψ c.m.s.; (3) the angle $\theta_{\gamma'\gamma}$ between γ' and γ in the cascade $\psi(2S) \to \gamma'\chi_c \to \gamma'\gamma J/\psi$, evaluated in the χ_c c.m.s., and (4) the angle $\theta_{\gamma' P}$ in the cascade $\psi(2S) \to \gamma'\chi_c \to \gamma' P^+P^-$, where $P = \pi$ or K, between γ' and P^+, evaluated in the χ_c c.m.s. In cases (1) through (3), one obtains information on the M2/E1 admixture from χ_{c1} and χ_{c2}, while in case (4) only χ_{c2} provides information, as χ_{c1} does not decay to P^+P^-. The transitions $\psi(2S) \to \gamma'\chi_{c0}$ and $\chi_{c0} \to \gamma J/\psi$ are purely electric dipole (E1) and do not provide information on the M2 amplitude.

It is conventional to define normalized amplitudes a_i^J for the transitions $\chi_{cJ} \to \gamma J/\psi$ and b_i^J for the transitions $\psi(2S) \to \gamma'\chi_{cJ}$ such that $\sum_i |a_i^J|^2 = \sum_i |b_i^J|^2 = 1$, with $i = 1, 2, 3$ corresponding to E1, M2, and E3 amplitudes, respectively. We shall neglect possible E3 contributions, which could only enter in the case of χ_{c2} intermediate states. The current state of experimental and theoretical information on M2 admixtures is summarized in Table I. Energies for the transitions $\psi(2S) \to \gamma'\chi_{cJ}$ and $\chi_{cJ} \to \gamma J/\psi$ are summarized in Table II.

Helicity amplitudes $A^J_{|\nu|}$ for $\chi_{cJ} \to \gamma J/\psi$ decays and $B^J_{|\nu|}$ for $\psi(2S) \to \gamma'\chi_{cJ}$ are related to respective multipole amplitudes $a^J_{J_\gamma}$ and $b^J_{J'_\gamma}$ [5] as

$$A^J_{|\nu|} = \sum_{J_\gamma} a^J_{J_\gamma} \left(\frac{2J_\gamma + 1}{2J + 1}\right)^{1/2} (J_\gamma, 1, 1, |\nu| - 1 | J, |\nu|), \quad (1)$$

$$B^J_{|\nu|} = \sum_{J'_\gamma} b^J_{J'_\gamma} \left(\frac{2J'_\gamma + 1}{2J + 1}\right)^{1/2} (J'_\gamma, 1, 1, |\nu'| - 1 | J, |\nu'|). \quad (2)$$

Specifically, the transformations for the A amplitudes are

$$J = 1:\ A_1^{J=1} = \sqrt{\frac{1}{2}} a_1^{J=1} - \sqrt{\frac{1}{2}} a_2^{J=1}, \quad (3)$$

$$A_0^{J=1} = \sqrt{\frac{1}{2}} a_1^{J=1} + \sqrt{\frac{1}{2}} a_2^{J=1}. \quad (4)$$

$$J = 2:\ A_2^{J=2} = \sqrt{\frac{3}{5}} a_1^{J=2} - \sqrt{\frac{1}{3}} a_2^{J=2} + \sqrt{\frac{1}{15}} a_3^{J=2}, \quad (5)$$

$$A_1^{J=2} = \sqrt{\frac{3}{10}} a_1^{J=2} + \sqrt{\frac{1}{6}} a_2^{J=2} - \sqrt{\frac{8}{15}} a_3^{J=2}, \quad (6)$$

$$A_0^{J=2} = \sqrt{\frac{1}{10}} a_1^{J=2} + \sqrt{\frac{1}{2}} a_2^{J=2} + \sqrt{\frac{2}{5}} a_3^{J=2}, \quad (7)$$

with a similar set of transformations for the B amplitudes.

Taking the charmed quark to have a mass $m_c = 1.5\ \text{GeV}/c^2$ and an anomalous magnetic moment κ, noting the photon energies in Table II, and approximating $b_1^J \simeq a_1^J \simeq 1$, one predicts [4]

*rosner@hep.uchicago.edu

TABLE I. Magnetic quadrupole admixtures b_2^J in $\psi(2S) \to \gamma' \chi_{cJ}$ and a_2^J in $\chi_{cJ} \to \gamma J/\psi$ from Crystal Ball [7], Fermilab E-760 [8], Fermilab E-835 [9], BES-II [10], and theory with $m_c = 1.5$ GeV/c^2 [4].

Experiment	$b_2^{J=1}$	$a_2^{J=1}$	$b_2^{J=2}$	$a_2^{J=2}$
Crystal Ball	$0.077^{+0.050}_{-0.045}$	$-0.002^{+0.008}_{-0.020}$	$0.132^{+0.098}_{-0.075}$	$-0.333^{+0.116}_{-0.292}$
E-760	...	...	...	-0.14 ± 0.06
E-835	...	$0.002 \pm 0.032 \pm 0.004$	...	$-0.093^{+0.039}_{-0.041} \pm 0.046$
BES-II	...	...	$-0.051^{+0.054\text{a}}_{-0.036}$	...
Theory[b]	$0.029(1+\kappa)$	$-0.065(1+\kappa)$	$0.029(1+\kappa)$	$-0.096(1+\kappa)$

[a]Result of fit with octupole moment $b_3 = -0.027^{+0.043}_{-0.029}$.
[b]κ denotes a possible anomalous magnetic moment of the charmed quark.

TABLE II. Energies of photons in $\psi(2S) \to \gamma' \chi_{cJ}$ and $\chi_{cJ} \to \gamma J/\psi$, evaluated in rest frame of decaying particle, in MeV [11].

J	$E_{\gamma'}$	E_γ
0	261.35 ± 0.33	303.05 ± 0.32
1	171.26 ± 0.07	389.36 ± 0.07
2	127.60 ± 0.09	429.63 ± 0.08

$$b_2^{J=1} = \frac{E_{\gamma'}[\psi(2S) \to \gamma' \chi_{c1}]}{4m_c}(1+\kappa) = 0.029(1+\kappa), \quad (8)$$

$$a_2^{J=1} = -\frac{E_\gamma[\chi_{c1} \to \gamma J/\psi]}{4m_c}(1+\kappa) = -0.065(1+\kappa), \quad (9)$$

$$b_2^{J=2} = \frac{3}{\sqrt{5}} \frac{E_{\gamma'}[\psi(2S) \to \gamma' \chi_{c2}]}{4m_c}(1+\kappa) = 0.029(1+\kappa), \quad (10)$$

$$a_2^{J=2} = -\frac{3}{\sqrt{5}} \frac{E_\gamma[\chi_{c2} \to \gamma J/\psi]}{4m_c}(1+\kappa) = -0.096(1+\kappa). \quad (11)$$

These predictions are summarized in Table I.

Although the most recent measurements, those of Fermilab E-835, are consistent with predictions, no convincing signal for any M2 admixture has yet been obtained. Prediction of the *ratios* $a_2^{J=2}/a_1^{J=1}$ and $b_2^{J=2}/b_1^{J=1}$ are likely to be more reliable than individual predictions because the charmed quark mass and anomalous moment cancel in these ratios:

$$a_2^{J=2}/a_1^{J=1} = \frac{3}{\sqrt{5}} \frac{429.63}{389.36} = 1.48, \qquad b_2^{J=2}/b_1^{J=1} = \frac{3}{\sqrt{5}} \frac{127.60}{171.26} = 1.00. \quad (12)$$

We introduce a shorthand notation

$$x_J = E_\gamma[\chi_{cJ} \to J/\psi](1+\kappa)/(4m_c), \quad (13)$$

$$x'_J = -E_{\gamma'}[\psi(2S) \to \gamma' \chi_{cJ}](1+\kappa)/(4m_c), \quad (14)$$

in terms of which the helicity amplitudes for $\chi_{cJ} \to \gamma J/\psi$ have the relative values (with arbitrary normalization for overall rates) [4]

$$A_0^{J=0} = \sqrt{2}, \quad (15)$$

$$A_1^{J=1} = \sqrt{3}(1 + x_1), \qquad A_0^{J=1} = \sqrt{3}(1 - x_1), \quad (16)$$

$$A_2^{J=2} = \sqrt{6}(1 + x_2), \qquad A_1^{J=2} = \sqrt{3}(1 - x_2), \qquad A_0^{J=2} = 1 - 3x_2, \quad (17)$$

$$B_0^{J=0} = \sqrt{2}, \quad (18)$$

$$B_1^{J=1} = \sqrt{3}(1 + x'_1), \qquad B_0^{J=1} = \sqrt{3}(1 - x'_1), \quad (19)$$

$$B_2^{J=2} = \sqrt{6}(1 + x'_2), \qquad B_1^{J=2} = \sqrt{3}(1 - x'_2), \qquad B_0^{J=2} = 1 - 3x'_2. \quad (20)$$

Many observables are most simply expressed in terms of the As and Bs [5].

We begin with the distributions with respect to the angles θ' and θ defined above. They are

$$J = 0\colon\ W(\cos\theta) \propto 1 + \cos^2\theta, \quad (21)$$

$$J = 1\colon\ W(\cos\theta) \propto \frac{1+\cos^2\theta}{2}(A_0)^2 + \sin^2\theta (A_1)^2, \quad (22)$$

$$J = 2\colon\ W(\cos\theta) \propto \frac{1+\cos^2\theta}{2}[(A_2)^2 + (A_0)^2] + \sin^2\theta (A_1)^2, \quad (23)$$

with similar expressions for $\theta \to \theta'$, $A_\nu \to B_\nu$. We use the expressions for helicity amplitudes, keep terms to first order in x_J, define $z = \cos\theta$, and normalize $\int_{-1}^{1} W(z)dz = 1$. The results are

$$J = 1\colon\ W(z) = \tfrac{3}{16}[3 - z^2 + 2x(1 - 3z^2)], \quad (24)$$

$$J = 2\colon\ W(z) = \tfrac{3}{80}[13 + z^2 - 6x(1 - 3z^2)], \quad (25)$$

and similarly for primed quantities. The terms linear in x give zero when integrated over the full range of z, as expected because the M2–E1 interference term should not contribute to the decay rate.

For the transitions $\psi(2S) \to \gamma' \chi_{c1,2}$, one expects

$$x_1' = -b_1^{J=1} = -0.029, \qquad x_2' = -\frac{\sqrt{5}}{3} b_1^{J=1} = -0.021. \tag{26}$$

The expected angular distributions for the above x_J' values are compared with those for $x_J' = 0$ in Fig. 1. The expected M2 admixtures lead to distributions which are very slightly flatter in θ' than the pure E1 transitions.

For the transitions $\chi_{c1,2} \to \gamma J/\psi$ the photon energies are larger and the signs of the M2 contributions are such that the distributions in θ are slightly more pronounced than for pure E1 transitions. These distributions are shown in Fig. 2.

We next discuss the effect of an M2 admixture on the photon-photon correlation angle $\theta_{\gamma'\gamma}$. Defining $z_c \equiv \cos\theta_{\gamma'\gamma}$, the distributions in terms of helicity amplitudes are [5]:

$$J = 1\colon\ W(z_c) \propto (A_1B_1)^2 + 2(A_1B_0)^2 + 2(A_0B_1)^2 + z_c^2(2A_0^2 - A_1^2)(2B_0^2 - B_0^2), \tag{27}$$

$$\begin{aligned} J = 2\colon\ W(z_c) \propto{}& (A_2B_2)^2 + 4(A_2B_1)^2 + 4(A_1B_2)^2 \\ &+ 6(A_2B_0)^2 + 6(A_0B_2)^2 + 4(A_1B_1)^2 \\ &+ 4(A_0B_0)^2 + 6z_c^2[(A_2^2 - 2A_0^2)(B_2^2 - 2B_0^2) \\ &- 2(A_1^2 - 2A_0^2)(B_1^2 - 2B_0^2)] \\ &+ z_c^4(A_2^2 - 4A_1^2 + 6A_0^2)(B_2^2 - 4B_1^2 + 6B_0^2). \end{aligned} \tag{28}$$

Keeping terms to first order in x_J and x_J', one finds that the z_c^4 terms cancel and

$$J = 1\colon\ W(z_c) = \tfrac{3}{32}[5 + z_c^2 + 2(x_1 + x_1')(1 - 3z_c^2)], \tag{29}$$

$$J = 2\colon\ W(z_c) = \tfrac{1}{160}[73 + 21z_c^2 + 42(x_1 + x_1')(3z_c^2 - 1)]. \tag{30}$$

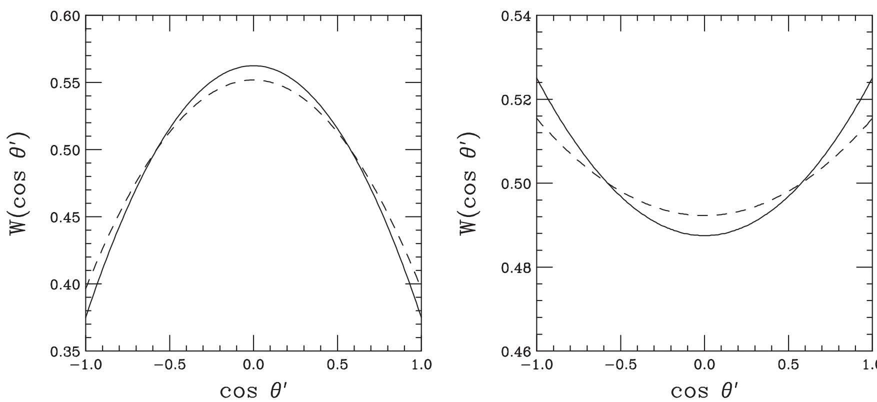

FIG. 1. Distributions in $\cos\theta'$, angle between photon γ' and beam axis in $e^+e^- \to \psi(2S) \to \gamma'\chi_{cJ}$. Solid lines: pure E1; dashed lines: including expected M2 admixture. Left: $J = 1$, $x_1' = -0.029$; right: $J = 2$, $x_2' = -0.021$.

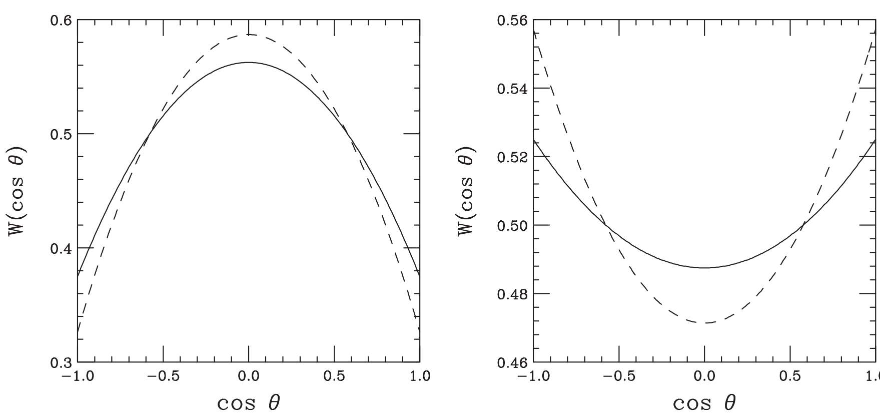

FIG. 2. Distributions in $\cos\theta$, angle between photon γ and lepton pair axis in $\chi_{cJ} \to \gamma J/\psi \to \gamma e^+e^-$, evaluated in J/ψ rest frame. Solid lines: pure E1; dashed lines: including expected M2 admixture. Left: $J = 1$, $x_1 = 0.065$; right: $J = 2$, $x_2 = 0.072$.

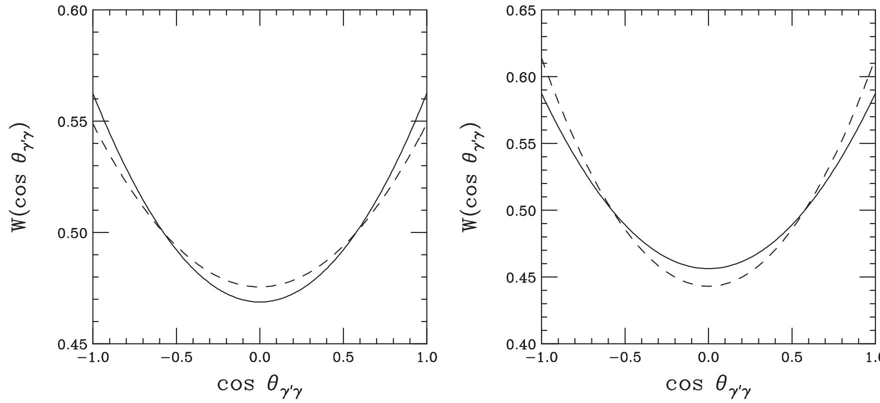

FIG. 3. Distributions in $\cos\theta_{\gamma'\gamma}$, angle between photon γ in $\chi_{cJ} \to \gamma J/\psi$ and γ' in $\psi(2S) \to \gamma' \chi_{cJ}$, evaluated in χ_{cJ} rest frame. Solid lines: pure E1; dashed lines: including expected M2 admixture. Left: $J = 1$, $x_1 + x_1' = 0.036$; right: $J = 2$, $x_2 + x_2' = 0.050$.

Here we have normalized $\int_{-1}^{1} dz_c W(z_c) = 1$. The angular distributions for $J = 1$ and $J = 2$ are shown in Fig. 3.

As for the single-angle distributions, the deviation from pure E1 behavior is proportional to a Legendre polynomial $P_2(z_c)$. The deviations are of opposite signs for $J = 1$ and $J = 2$, and slightly larger for $J = 2$ as a result of the larger value of $x_J + x_J'$.

We conclude with a discussion of the effect of an M2 admixture on the angle $\theta_{\gamma' P}$ between the photon γ' in $\psi(2S) \to \gamma \chi_{c2}$ and the pseudoscalar meson P^+ in the decay $\chi_{c2} \to P^+ P^-$, where, for example, $P = \pi$ or K. This distribution is given in terms of $z_1 \equiv \cos\theta_{\gamma' P}$ and the helicity amplitudes B by [5]

$$W(z_1) \propto 3(B_2)^2(1 - z_1^2)^2 + 12(B_1)^2(1 - z_1^2)z_1^2 + 2(B_0)^2(3z_1^2 - 1)^2. \tag{31}$$

Substituting for the helicity amplitudes, keeping terms of order x_2', and normalizing $\int_{-1}^{1} dz_1 W(z_1) = 1$, one finds

$$W(z_1) = \frac{5 - 3z_1^2}{8} + \frac{3}{4} x_2' (1 - 3z_1^2). \tag{32}$$

This expression is plotted for a pure E1 transition and for the expected M2 admixture in Fig. 4. An analysis by the BES-II Collaboration [10] based on 14 million $\psi(2S)$ decays finds no evidence for higher multipoles (see Table I).

The effects we have displayed all are rather small. Based on the previous experimental signatures summarized in Table I, the most promising distributions in which to see

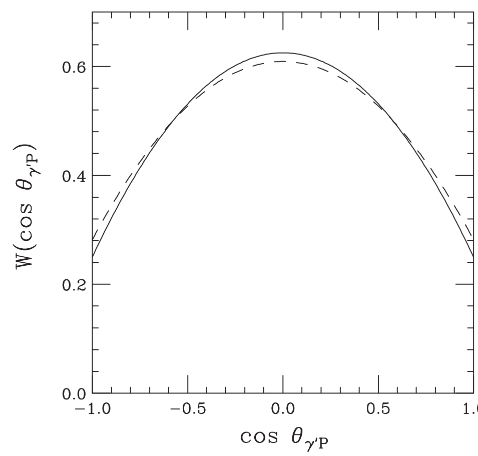

FIG. 4. Distribution in $\cos_{\theta' P}$, where $\theta_{\gamma' P}$ is the angle between the photon γ' in $\psi(2S) \to \gamma \chi_{c2}$ and the pseudoscalar meson P^+ in the decay $\chi_{c2} \to P^+ P^-$. Solid curve: pure E1; dashed curve: with expected M2 admixture corresponding to $x_2' = -0.021$.

significant effects are probably Eqs. (24) and (25) as applied to the transition $\chi_{c1,2} \to \gamma J/\psi$ (see Fig. 2). A simulation will be necessary to tell whether the present CLEO data sample of some 27 million $\psi(2S)$ decays [6] is adequate to see such effects.

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