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Hyperon polarization in unpolarized scattering processes

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Transverse polarization in the hyperon (Λ) production in the unpolarized deep inelastic scattering and pp collisions is studied in the twist-three approach, considering the contribution from the quark-gluon-antiquark correlation distribution in nucleon. We further compare our results for deep inelastic scattering to a transverse momentum dependent factorization approach, and find consistency between the two approaches in the intermediate transverse momentum region. We also find that in pp collisions, there are only derivative terms contributions, and the nonderivative terms vanish.

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I. INTRODUCTION

Single transverse spin asymmetry (SSA) phenomena have a long history, starting with the observation of the large transverse polarization of hyperon (Λ) in unpolarized nucleon-nucleon scattering [1,2]. It has imposed theoretical challenges to understand these phenomena in quantum chromodynamics (QCD) [2,3]. In recent years, this physics has attracted strong interest from both experiment and theory sides. For example, the experimental observation of SSAs in semi-inclusive hadron production in deep inelastic scattering (SIDIS), in inclusive hadron production in pp scattering at collider energy at RHIC, and the relevant azimuthal asymmetric distribution of hadron production in e^+e^- annihilation have motivated theoretical developments in the last few years [4–9]. Among these developments, two mechanisms in the QCD framework have been most explored to study the large SSAs observed in the experiments. One is the so-called twist-three quark-gluon correlation approach [3,10–15], and the other is the transverse momentum dependent (TMD) approach where the intrinsic transverse momentum of partons inside the nucleon plays an important role [4–7,16,17]. Recent studies have shown that these two approaches are consistent with each other in the intermediate transverse momentum region where both apply [18].

In particular, theoretical developments have been made to understand the Λ polarization in unpolarized hadronic reactions [19–22]. Similar to the SSA in inclusive hadron production in $p^\uparrow p \to \pi X$, the hyperon polarization in unpolarized pp scattering $pp \to \Lambda_\uparrow X$ receives the contributions from (naive)-time-reversal-odd effects in the distribution and fragmentation parts [21,22]. In order to understand the experimental observations of $pp \to \Lambda_\uparrow X$, one has to take into account both contributions.

On the other hand, in the deep inelastic scattering process, one can separate these two contributions because they have different azimuthal angle dependence [23]. To describe these effects, one can calculate the Λ polarization in the twist-three approach [22] or use the TMD mechanism [23]. For example, the contribution from the T-odd effects in the distribution part is associated with the so-called Boer-Mulders TMD quark distribution $h_1^\perp$ [23] multiplied by the TMD transversity fragmentation function H_{1T} when the transverse momentum of the produced Λ is much smaller than the hard scale $P_{\Lambda\perp} \ll Q$, where Q is the virtuality of the virtual photon in DIS process. On the other hand, we can also calculate this contribution from the twist-three mechanism when the transverse momentum is much larger than the nonperturbative scale $\Lambda_{\rm QCD}$: $P_{\Lambda\perp} \gg \Lambda_{\rm QCD}$. From our calculations, we find that these two approaches indeed provide a unique description for Λ polarization at the intermediate transverse momentum region in the semi-inclusive DIS. The large transverse momentum Boer-Mulders function calculated in this context can also be used in other processes, like Drell-Yan lepton pair production in pp scattering [24].

The rest of the paper is organized as follows. In Sec. II, we study the Λ polarization in semi-inclusive deep inelastic scattering $e + p \to e + \Lambda_\uparrow + X$ by calculating the contribution from the twist-three quark-gluon-antiquark correlation function from nucleon. We will then take the limit of the small transverse momentum $P_{\Lambda\perp} \ll Q$, and

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compare it to the prediction from the TMD factorization approach. In the TMD picture, the polarization of Λ comes from the Beor-Mulders function $h_1^\perp$. We briefly discuss the extension to pp collisions and conclude in Sec III.

II. Λ TRANSVERSE POLARIZATION IN SEMI-INCLUSIVE DIS

In the SIDIS process $ep \to e'\Lambda_\uparrow X$, the differential cross section can be formulated as

$$\frac{d\sigma(S_{\Lambda\perp})}{dx_B dy dz_h d^2\vec{P}_\Lambda} = \frac{2\pi\alpha_{\rm em}^2}{Q^2} L^{\mu\nu}(l,q) W_{\mu\nu}(P,q,P_\Lambda,S_{\Lambda\perp}), \tag{1}$$

where $\alpha_{\rm em}$ is the electromagnetic coupling, l and P are incoming momenta for the lepton and nucleon, q is the momentum for the exchanged virtual photon with $Q^2 = -q.q$, P_Λ and $S_{\Lambda\perp}$ are the momentum and transverse polarization vector for the final state Λ, respectively, and we have $S_{\Lambda\perp}\cdot P_\Lambda = 0$. The kinematic variables are defined as $x_B = \frac{Q^2}{2P\cdot q}$, $z_h = \frac{P\cdot P_\Lambda}{P\cdot q}$, $y = \frac{P\cdot q}{P\cdot l}$. In the above equation, $L^{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic tensors, respectively. They are given by

$$L^{\mu\nu}(l,q) = 2\left(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu}\frac{Q^2}{2}\right), \tag{2}$$

$$\begin{aligned} W^{\mu\nu}(P,q,P_\Lambda,S_{\Lambda\perp}) = \frac{1}{4z_h}\sum_X \int \frac{d^4\zeta}{(2\pi)^4} \\ \times e^{iq\cdot\zeta}\langle P|J_\mu(\zeta)|XP_\Lambda S_{\Lambda\perp}\rangle \\ \times \langle XP_\Lambda S_{\Lambda\perp}|J_\nu(0)|P\rangle, \end{aligned} \tag{3}$$

where l' is the momentum for the final-state lepton, J^μ is the quark electromagnetic current, and X represents all other final-state hadrons other than the observed hyperon Λ.

It is convenient to write the momentum of the virtual photon in terms of the incoming and outgoing hadron momenta,

$$q^\mu = q_t^\mu + \frac{q\cdot P_\Lambda}{P\cdot P_\Lambda}P^\mu + \frac{q\cdot P}{P\cdot P_\Lambda}P_\Lambda^\mu, \tag{4}$$

where q_t^μ is transverse to the momentum of the initial and final hadrons: $q_t^\mu P_\mu = q_t^\mu P_{\Lambda\mu} = 0$. Here q_t^μ is a spacelike vector, and we define $\vec{q}_\perp^2 \equiv -q_t^2$. In the hadron frame, the final-state hadron will have the momentum

$$P_\Lambda^\mu = \frac{x_B\vec{P}_{\Lambda\perp}^2}{z_h Q^2}P^+ p^\mu + z_h\frac{Q^2}{2x_B P^+}n^\mu + P_{\Lambda\perp}^\mu, \tag{5}$$

where $P_{\Lambda\perp}$ is the hyperon transverse momentum in the hadron frame $P^+ = 1/\sqrt{2}(P^0 + P^z)$, and we use the conventional definition for light-cone vector p^μ, n^μ: $p = (1^+, 0^-, 0_\perp)$, $n = (0^+, 1^-, 0_\perp)$. From the above definitions, we will find $\vec{q}_\perp^2 = \vec{P}_{\Lambda\perp}^2/z_h^2$.

We will calculate the hadronic tensor $W^{\mu\nu}$ at large transverse momentum in perturbative QCD, by radiating a hard gluon in the final state. They are expressed in terms of integrated parton distribution and fragmentation functions or the quark-gluon-antiquark correlations, according to a collinear factorization [25]. In the calculations, it is convenient to decompose the hadronic tensor $W^{\mu\nu}$ in terms of individual tensors [26,27],

$$W^{\mu\nu} = \sum_{i=1}^{9} V_i^{\mu\nu} W_i, \tag{6}$$

where the W_i are structure functions, and can be projected out from $W^{\mu\nu}$ by $W_i = W_{\alpha\beta}\tilde{V}_i^{\alpha\beta}$ with the corresponding inverse tensors $\tilde{V}_i$. Both V_i, $\tilde{V}_i$ can be constructed from four orthonormal basis vectors [26]: T^μ, X^μ, Y^μ, Z^μ with normalization $T^\mu T_\mu = 1$, $X^\mu X_\mu = Y^\mu Y_\mu = Z^\mu Z_\mu = -1$. These four vectors can be further constructed by P^μ, q^μ, $S_{\Lambda\perp}^\mu$, q_t^μ. In this paper, we choose a special frame, where the q_t^μ is parallel to X^μ, and the target proton and final state Λ have spatial components only in the Z direction. In the small q_t ($P_{\Lambda\perp}$) region, we have checked that this frame leads to the same result as that in the normal hadron frame.

As mentioned in the introduction, in this paper, we are interested in calculating the differential cross section in the intermediate transverse momentum region, $\Lambda_{\rm QCD} \ll P_{\Lambda\perp} \ll Q$. In the calculations, we will utilize the power counting method, and only keep the leading power contributions and neglect all higher order corrections in terms of $P_{\Lambda\perp}/Q$. For the spin-average Λ production in SIDIS, the differential cross section will be identical to any other hadron production process except we have to change the associated fragmentation function for the hyperon. This cross section in the above limit will be consistent with the TMD factorization approach in the intermediate transverse momentum region as has been shown before, for example, in the pion production in SIDIS [18].

For the Λ polarization dependent cross section, we have two separate contributions from the twist-three quark-gluon-antiquark correlations in the parton distribution or fragmentation. In this paper, we will only focus on the parton distribution part, whereas that from the fragmentation part can follow accordingly. We also note that these two contributions will have different azimuthal angular dependence in SIDIS in the small transverse momentum limit. For the contribution from the parton distribution part, following the Qiu-Sterman formalism, the corresponding spin-dependent hadronic tensor can be written as [3]

$$\begin{aligned} W^{\mu\nu} = \int \frac{d^4k_1}{(2\pi)^4} \\ \times \frac{d^4k_2}{(2\pi)^4} T_\rho(k_1,k_2) H^{\mu\nu,\rho}(k_1,k_2,P_{\Lambda\perp},S_{\Lambda\perp}) H_{1T}(z), \end{aligned} \tag{7}$$

where T and H represent the twist-three function and the partonic hard-scattering amplitude, respectively. These two parts are connected by the two independent integrals over the momentum k_1 and k_2 that they share. In the above expansion, spinor and color indices connecting the hard part and long-distance parts have already been separated, which leads to the hard part $H(k_1, k_2, P_{\Lambda\perp}, S_{\Lambda\perp})$ being contracted with $(1/2)\not{p}\gamma_\perp^\rho/(2\pi)$. The transversity fragmentation H_{1T} for Λ production is defined as

$$H_{1T}(z) = \frac{1}{2z}\sum_X \int \frac{dy^+}{4\pi} e^{-iP_\Lambda^- y^+/z}\langle 0|\psi(0^+)|P_\Lambda S_{\Lambda\perp}, X\rangle \times \langle P_\Lambda S_{\Lambda\perp}, X|\bar{\psi}(y^+)S_{\perp\mu} i\sigma^{\mu -}\gamma^5|0\rangle, \quad (8)$$

where X represents all other particles in the final state except for Λ, $S_\perp^\mu$ is the transverse polarization vector of the final-state hadron. The next step is to perform a collinear expansion of the expression:

$$k_i^\mu = x_i P^\mu + k_{i,\perp}^\mu, \quad (9)$$

where minus component has been neglected since it is beyond the order in $k_{i,\perp}$ that we consider. The collinear expansion enables us to reduce the four-dimensional integral to a integral convolution in the light-cone momentum fractions of the initial partons. Expanding $H^{\mu\nu}$ in the partonic momentum at $k_1 = x_1 P$ and $k_2 = x_2 P$, we have

$$H^{\mu\nu,\rho}(k_1, k_2, P_{\Lambda\perp}, S_{\Lambda\perp}) = H^{\mu\nu,\rho}(x_1, x_2, P_{\Lambda\perp}, S_{\Lambda\perp}) + \frac{\partial H^{\mu\nu,\rho}}{\partial k_1^\alpha}(x_1, x_2)(k_1 - x_1 P)^\alpha + \frac{\partial H^{\mu\nu,\rho}}{\partial k_2^\alpha}(x_1, x_2)(k_2 - x_2 P)^\alpha + \ldots. \quad (10)$$

The above expansion allows us to integrate over three of the four components of each of the loop momenta k_i, and the hadronic tensor $W^{\mu\nu}$ will depend on the chiral-odd spin-independent twist-three quark-gluon-antiquark correlation function [3,22]

$$T_F^{(\sigma)}(x_1, x_2) = \int \frac{dy_1^- dy_2^-}{4\pi} e^{iy_2^-(x_2-x_1)P^+ - iy_1^- x_1 P^+} \times \langle P|\bar{\psi}(y_1^-)\sigma^{+\mu} g F_{+\mu}(y_2^-)\psi(0)|P\rangle, \quad (11)$$

where the sums over color and spin indices are implicit, $|P\rangle$ denotes the unpolarized proton state, ψ is the quark field, $F_{+\mu}$ is the gluon field tensor, and the gauge link has been suppressed. Because of parity and time-reversal invariance, we have the relation $T_F^{(\sigma)}(x_1, x_2) = T_F^{(\sigma)}(x_2, x_1)$.

Similar to the SSA in π production in SIDIS, the strong interaction phase necessary for having a nonvanishing Λ transverse polarization arises from the interference between an imaginary part of the partonic scattering amplitude with the extra gluon and the real scattering amplitude without a gluon. The imaginary part is due to the pole of the parton propagator associated with the integration over the gluon momentum fraction x_g. Depending on which propagator's pole contributes, the amplitude may get contributions from $x_g = 0$ ("soft-pole") and $x_g \neq 0$ ("hard-pole" or "soft-fermion-pole") [18]. The diagrams contributing to the Λ polarization in SIDIS will be the same as those calculated for the SSA in π production. The only difference is that we have to replace the Qiu-Sterman matrix element T_F with the above unpolarized quark-gluon-antiquark correlation function $T_F^{(\sigma)}$, and the unpolarized fragmentation function $D(z)$ for π with the transversity fragmentation function for Λ. We further notice that the soft-fermion-pole contribution is power suppressed in the limit of $P_{\Lambda\perp} \ll Q$, similar to the SSA in π production. In Fig. 1, we show some examples of the soft-pole and hard-pole diagrams. The calculations will be similar to those in [3,12,18]. We perform the calculation in the covariant gauge. There are a total of eight diagrams contributing to the soft-pole and 12 diagrams for the hard-pole contributions. Since the calculation formalism has been well established, we only give the final result and refer the reader to the references for details.

We are interested in obtaining the differential cross section in the limit of $P_{\Lambda\perp} \ll Q$. In this limit, we further find that only V_4 and V_9 in Eq. (6) contribute in the leading power of $P_{\Lambda\perp}/Q$. V_4 and V_9 are defined as

$$V_4^{\mu\nu} = X^\mu X^\nu - Y^\mu Y^\nu, \quad (12)$$

$$V_9^{\mu\nu} = X^\mu Y^\nu + X^\mu Y^\nu, \quad (13)$$

and the associated $\tilde{V}_4$ and $\tilde{V}_9$ are given by,

$$\tilde{V}_4^{\mu\nu} = \frac{1}{2}(X^\mu X^\nu - Y^\mu Y^\nu), \quad (14)$$

$$\tilde{V}_9^{\mu\nu} = \frac{1}{2}(X^\mu Y^\nu + Y^\mu X^\nu). \quad (15)$$

These two terms contribute the same to the differential

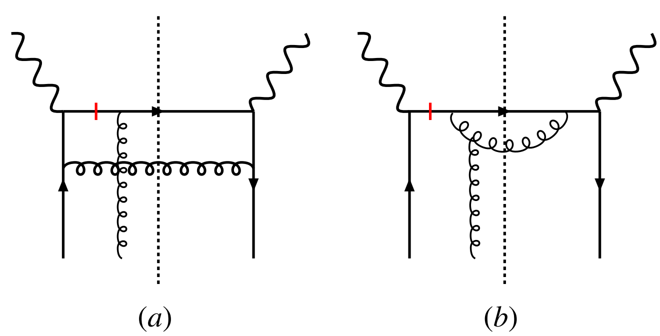

FIG. 1 (color online). The example diagrams for the soft-pole (a) and hard-pole (b) contributions to Λ transverse polarization in the semi-inclusive DIS process. The short bars indicate the pole contribution from the propagators.

cross sections except the azimuthal angular dependence. The contribution from V_4 is proportional to $\cos(2\phi^l_\Lambda)\times\sin(\phi^l_s-\phi^l_\Lambda)$, where ϕ^l_Λ and ϕ^l_s are the azimuthal angles of the transverse momentum $P_{\Lambda\perp}$ and the polarization vector $S_{\Lambda\perp}$ of Λ relative to the lepton scattering plane. On the other hand, the V_9 contribution is proportional to $\sin(2\phi^l_\Lambda)\cos(\phi^l_s-\phi^l_\Lambda)$. The total contributions from these two terms will be proportional to $\sin(\phi^l_\Lambda+\phi^l_s)$.

Summing up both soft-pole and hard-pole contributions from all the diagrams similar to those in Fig. 1, we obtain the following Λ polarization dependent differential cross section from the contributions from V_4 and V_9:

$$\frac{d\sigma(S_{\Lambda\perp})}{dx_B dy dz_h d^2\vec{P}_{\Lambda\perp}}\bigg|_{V_4+V_9} = -\frac{4\pi\alpha_{\rm em}^2 s}{Q^4}x_B(1-y)\sin(\phi^l_\Lambda+\phi^l_s)\frac{1}{z_h^2}\frac{\alpha_s}{2\pi^2}\frac{2}{|\vec{q}_\perp|}\int\frac{dxdz}{xz}\delta\left(\vec{q}_\perp^2-\frac{Q^2(1-\xi)(1-\hat{\xi})}{\xi\hat{\xi}}\right) \times\left\{x\frac{\partial}{\partial x}T_F^{(\sigma)}(x,x)\frac{1}{2N_c}\left[\frac{(\xi+\hat{\xi}-1)}{\hat{\xi}(1-\hat{\xi})}\right]-T_F^{(\sigma)}(x,x)\frac{1}{2N_c}\left[\frac{(\hat{\xi}-\xi^2+2\xi-1)}{(1-\xi)(1-\hat{\xi})\hat{\xi}}\right]\right. \left.+T_F^{(\sigma)}(x,x_B)\left(\frac{1}{2N_c}+\hat{\xi}C_F\right)\left[\frac{1}{(1-\xi)(1-\hat{\xi})}\right]\right\}H_{1T}(z), \tag{16}$$

where $\xi=x_B/x$ and $\hat{\xi}=z_h/z$. In the above result, the first term in the bracket is the derivative term coming from the soft-gluon pole, the second term is the nonderivation contribution from soft-gluon pole, and the third is the hard-pole contribution whose derivative term vanishes.

In order to compare to the TMD factorization formalism, we will extrapolate our results into the region of $\Lambda_{\rm QCD}\ll P_{h\perp}\ll Q$. In doing the expansion, we only keep the terms leading in $P_{h\perp}/Q$, and neglect all higher powers. For small $P_{h\perp}$, the delta function can be expanded as

$$\delta\left(\vec{q}_\perp^2-\frac{Q^2(1-\xi)(1-\hat{\xi})}{\xi\hat{\xi}}\right)=\frac{\xi\hat{\xi}}{Q^2}\left\{\frac{\delta(\xi-1)}{(1-\hat{\xi})_+}+\frac{\delta(\hat{\xi}-1)}{(1-\xi)_+}+\delta(\xi-1)\delta(\hat{\xi}-1)\ln\frac{Q^2}{\vec{q}_\perp^2}\right\}. \tag{17}$$

With this expansion, the spin-dependent cross section in the small $P_{h\perp}$ limit can be written as

$$\frac{d\sigma(S_{\Lambda\perp})}{dx_B dy dz_h d^2\vec{P}_{\Lambda\perp}}\bigg|_{P_{\Lambda\perp}\ll Q} = -\frac{4\pi\alpha_{\rm em}^2 s}{Q^4}x_B(1-y)\times\sin(\phi^l_h+\phi^l_{st})\frac{1}{z_h^2}\frac{\alpha_s}{2\pi^2}\frac{1}{|\vec{q}_\perp|^3}\times\int\frac{dxdz}{xz}H_{1T}(z)\{A\delta(\hat{\xi}-1)+B\delta(\xi-1)\}, \tag{18}$$

where

$$A=\frac{1}{2N_c}\left\{\left[x\frac{\partial}{\partial x}T_F^{(\sigma)}(x,x)\right]2\xi+T_F^{(\sigma)}(x,x)\frac{2\xi(\xi-2)}{(1-\xi)_+}\right\}+\frac{C_A}{2}T_F^{(\sigma)}(x,x_B)\frac{2}{(1-\xi)_+},$$
$$B=C_F T_F^{(\sigma)}(x,x)\left[\frac{2\hat{\xi}}{(1-\hat{\xi})_+}+2\delta(\hat{\xi}-1)\ln\frac{Q^2}{\vec{q}_\perp^2}\right]. \tag{19}$$

On the other side, the transverse momentum dependent factorization can be applied in the small $P_{h\perp}\ll Q$. Therefore one expects the above result can be reproduced in this approach. The Λ polarization dependent cross section can be factorized as the following form [8,9]:

$$\frac{d\sigma(S_{\Lambda\perp})}{dx_B dy dz_h d^2\vec{P}_{\Lambda\perp}}=\frac{4\pi\alpha_{\rm em}^2 s}{Q^4}x_B(1-y)\sin(\phi^l_\Lambda+\phi^l_s)\times\int\frac{k_\perp\cdot\hat{\vec{P}}_{\Lambda\perp}}{M}h_{1,{\rm DIS}}^\perp(x,k_\perp)H_{1T}(z,p_\perp)\times(S(\lambda_\perp))^{-1}H_{UUT}(Q^2), \tag{20}$$

where $\hat{\vec{P}}_{\Lambda\perp}$ is the unit vector in direction of $\vec{P}_{\Lambda\perp}$, $h_{1,{\rm DIS}}^\perp$ is the TMD Boer-Mulders function for DIS process, and H_{1T} the TMD transversity fragmentation for Λ. $S(\lambda_\perp)$ and $H_{UUT}(Q^2)$ are the soft factor and hard factor, respectively. The simple integral symbol represents a complicated integral: $\int=\int d^2\vec{k}_\perp d^2\vec{p}_\perp d^2\vec{\lambda}_\perp\delta^2(z\vec{k}_\perp+\vec{p}_\perp+\vec{\lambda}_\perp-\vec{P}_{\Lambda\perp})$. We have suppressed the sum over all flavors and factorization scale dependence in the parton distribution function and fragmentation function.

When the $k_\perp$ is of the order of $\Lambda_{\rm QCD}$, the TMD dependent parton distribution functions are entirely nonperturbative objects. But in the region $\Lambda_{\rm QCD}\ll k_\perp\ll Q$, the TMD factorization still holds and at the same time $k_\perp$ dependent parton distribution function $h_1^\perp$ can be calculated in terms of the twist-three parton correlation function within the perturbative QCD. This provides us a chance to make contact with the result from the collinear factorization formalism. The perturbative calculation follows the similar procedure as that in [18]. Finally, one obtains

$$h_{1,{\rm DIS}}^\perp(x_B,k_\perp)=-\frac{\alpha_s}{2\pi^2}\frac{M_p}{(\vec{k}_\perp^2)^2}\int\frac{dx}{x}\left\{A+C_F T_F^{(\sigma)}(x,x)\times\delta(\xi-1)\left(\ln\frac{x_B^2\zeta^2}{\vec{k}_\perp^2}-1\right)\right\}, \tag{21}$$

where A is given in Eq. (19), and $\xi=x_B/x$. We note that the native-time-reversal-odd TMD Boer-Mulders function is process dependent. The above result is for SIDIS. When we apply the above to the Drell-Yan process, the Boer-Mulders function shall change the sign. Similarly, for the TMD transversity fragmentation function, we have

$$H_{1T}(z_h, p_\perp) = \frac{\alpha_s}{2\pi^2}\frac{1}{\vec{p}_\perp^2}C_F\int\frac{dz}{z}H_{1T}(z)\left[\frac{2\hat{\xi}}{(1-\hat{\xi})_+} + \delta(1-\hat{\xi})\left(\ln\frac{\hat{\xi}^2}{\vec{p}_\perp^2}-1\right)\right], \tag{22}$$

where $H_{1T}(z)$ is the integrated transversity quark fragmentation function defined in Eq. (8), and $\hat{\xi} = z_h/z$.

To obtain the final result, we let one of the transverse momentum $\vec{k}_\perp$, $\vec{p}_\perp$, $\vec{l}_\perp$ be of the order of $\vec{P}_\perp$ and the others are much smaller. After integrating the delta function, one has

$$\frac{d\sigma(S_{\Lambda\perp})}{dx_B dy dz_h d^2\vec{P}_{\Lambda\perp}} = -\frac{4\pi\alpha_{\rm em}^2 s}{Q^4}x_B(1-y)\sin(\phi_\Lambda^l+\phi_s^l) \times\frac{z_h}{|\vec{P}_{\Lambda\perp}|^3}\frac{\alpha_s}{2\pi^2}\int\frac{dxdz}{xz}H_{1T}(z) \times\{A\delta(\hat{\xi}-1)+B\delta(\xi-1)\}, \tag{23}$$

where we have used the relation $T_F^{(\sigma)}(x,x) = -\int d^2k_\perp\frac{|k_\perp|^2}{M_p}h_{1,{\rm DIS}}^\perp(x,k_\perp^2)$ [7,28]. Obviously, we reproduce the differential cross sections from the collinear factorization calculation.

This clearly demonstrates that in the intermediate transverse momentum region, the twist-three collinear factorization approach and the TMD factorization approach provide a unique picture for the Λ polarization in the unpolarized semi-inclusive DIS process. This is because the observable we calculated above is the leading contribution in the limit of $P_{\Lambda\perp}/Q$, and the TMD factorization is valid [8,9].

III. CONCLUSION

In summary, in this paper, we studied the Λ polarization in the unpolarized semi-inclusive DIS and pp collisions. In the SIDIS process, we compared the twist-three approach with the TMD factorization approach and found that they are consistent with each other at the intermediate transverse momentum region. We have also calculated the large transverse momentum behavior for the naive time-reversal-odd Boer-Mulders quark distribution in the twist-three approach from the quark-gluon-antiquark correlation function in an unpolarized nucleon. This distribution has a number of important applications in the Drell-Yan lepton azimuthal distribution in pp scattering. For example, the $\cos 2\phi$ angular distribution has a contribution from two Boer-Mulders functions from the incoming nucleons [24]. From our calculations above, we shall be able to study the large transverse momentum behavior for this $\cos 2\phi$ angular distribution.

The extension to the Λ polarization in hadronic scattering is straightforward. The diagrams will be similar to what have been calculated for the SSA in inclusive hadron production in $pp_\uparrow \to \pi X$ collisions [12]. Similar to what we have calculated in the last section, we will have both derivative and nonderivative contributions. The derivative terms have been calculated in [22]. Using the same method as that in [12], we calculated the nonderivative terms, and found that the nonderivative terms vanish for Λ polarization in hard partonic scattering processes. This indicates that the compact formula [12] containing both derivative and nonderivative contributions may not work in general, and shows a counterexample of the derivation of the compact formula in [15]. It will be interesting to further investigate the reason for this observation. These extensions will be presented elsewhere.

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