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CP violation in $\tau \to K\pi\pi\nu_\tau$

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We consider CP-violating effects in $\tau \to K\pi\pi\nu_\tau$, assuming that a charged Higgs boson provides a new amplitude that can interfere with the usual standard model amplitude. We consider four CP-odd observables—the regular rate asymmetry, two modified rate asymmetries and a triple-product asymmetry. The regular rate asymmetry is expected to be small because it requires the interference of the new physics amplitude with the standard model amplitude containing the hadronic scalar form factor. The other CP asymmetries may be more promising in terms of their new physics reach. Numerical estimates indicate that the maximum obtainable values for the modified and triple-product asymmetries are on the order of a percent.

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I. INTRODUCTION

In the standard model (SM) of particle physics, CP violation is due to a complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. But is this the only source of CP violation? In order to answer this question, it is important to look for CP-violating effects in as many systems as possible.

One such system is τ decays. In the SM, CP violation in the τ system is essentially zero [1]; we consider, instead, a search for physics beyond the SM. In Ref. [2], we examined CP violation in strangeness-conserving τ decays. It is only natural next to turn to those processes with $\Delta S = 1$. The simplest such decay is $\tau \to K\pi\nu_\tau$. However, CP violation in this process has been analyzed in detail in Ref. [3], and we have nothing to add here. The next decay is $\tau \to K\pi\pi\nu_\tau$. This has been examined theoretically in the past in Refs. [4,5]. In this paper we update these analyses [6].

One has to assume the presence of new physics in order to get nonzero CP-violating effects when comparing $\tau^- \to K^-\pi^-\pi^+\nu_\tau$ to its CP-conjugate decay. In Ref. [4], the left-right (LR) model is assumed when the authors consider $\tau \to K\pi\pi\nu_\tau$. However, as shown in Ref. [2], if there is no LR mixing, CP violation is proportional to the mass of the neutrino, and is negligible. Thus, in the LR model, CP violation in τ decays is proportional to W_L-W_R mixing. However, we know this is small [7]. We therefore conclude that sizeable CP violation in the τ system cannot arise in the LR model.

For this reason, in this paper, we assume that the τ decay includes the exchange of a new-physics (NP) charged Higgs. Note that many NP models have two Higgs doublets, so that a charged Higgs is present. However, if the Higgs doublets give mass to the fermions, the coupling of the charged Higgs boson is generally proportional to the masses of the first- and second-generation quarks. Since these are small, CP violation in the τ system will also be small. To avoid this, if CP violation is to be observed in τ decays, the charged-Higgs coupling must be large. In other words, $\tau \to K\pi\pi\nu_\tau$ probes non-"standard" NP CP violation.

It is worth noting at this point that CLEO has searched for CP violation in $\tau \to K\pi\nu_\tau$ [8] and has set a bound on a coupling constant related to the scalar coupling of a charged Higgs (or other scalar boson) to the up and strange quarks. The experimental investigation suggested in this work would be complementary to that carried out in Ref. [8] in that it would probe the pseudoscalar coupling of the Higgs to the up and strange quarks. In the notation introduced below, the CLEO experiment probed η_S, while a CP analysis of $\tau \to K\pi\pi\nu_\tau$ would probe η_P [see Eq. (7) below].

In the presence of one NP contribution, the amplitude for the decay $\tau \to K\pi\pi\nu_\tau$ can be written

$$\mathcal{A} = A_1 + A_2 e^{i\phi} e^{i\delta}, \tag{1}$$

where ϕ and δ are the relative weak (CP-odd) and strong (CP-even) phases, respectively. The amplitude for the antiprocess, $\bar{\mathcal{A}}$, is given by the same expression, but with $\phi \to -\phi$.

In general, CP violation is obtained by comparing $|\mathcal{A}|^2$ to $|\bar{\mathcal{A}}|^2$. There are three types of signals:

(1) The full rate for a particular process involves $\sum_{\rm spins}|\mathcal{A}|^2$, integrated over the final-state momenta in the usual way. The *rate asymmetry* is given by the rate difference of the process and antiprocess.

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(2) The rate asymmetry can be altered in two ways. First, if some spins are measured, one does not sum over them. Alternatively, one can integrate asymmetrically in order to isolate certain terms in the differential width. In either case the process–antiprocess difference leads to a *modified rate asymmetry.*

(3) One can also construct *CP* asymmetries based on the quantity $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each v_i is a spin or momentum. This is a *triple product* (TP), and its value can be different for process and antiprocess, signaling *CP* violation.

The rate asymmetry or modified rate asymmetry is proportional to

$$\sin\delta \sin\phi \tag{2}$$

(integrated over phase space). Thus, this category of *CP* violation requires that the two decay amplitudes have a nonzero relative weak *and* strong phase. The TP asymmetry is proportional to

$$\cos\delta \sin\phi, \tag{3}$$

so that one does not require a strong-phase difference to get a TP asymmetry. In this paper we consider all three types of *CP* violation in $\tau \to K\pi\pi\nu_\tau$. (Ref. [4] considers only TP's.)

The remainder of this paper is organized as follows. In Sec. II, we write down the expression for the differential width for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$ in terms of various form factors, and including the NP contribution. We perform weighted integrations of the differential width over phase space to isolate certain cross terms. In Sec. III, we consider four *CP* asymmetries: the regular rate asymmetry, two modified rate asymmetries, and a triple-product asymmetry. The modified rate asymmetries and triple-product asymmetry are constructed using the weighted differential widths from Sec. II. Section IV contains a numerical analysis of the modified rate asymmetries and the triple-product asymmetry. We conclude with a few closing remarks in Sec. V.

II. DIFFERENTIAL WIDTH FOR $\tau^- \to K^-\pi^-\pi^+\nu_\tau$

We start by determining an expression for the differential width for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$, including possible NP effects due to a new charged Higgs boson H^-.

A. General expression for the differential width

Let us begin by considering the SM contribution to $\tau^- \to K^-\pi^-\pi^+\nu_\tau$. Within the SM, the relevant effective Hamiltonian is given by

$$\mathcal{H}_{\rm eff}^{\rm SM} = \frac{G_F}{\sqrt{2}} \sin\theta_c \bar{\nu}_\tau \gamma_\mu (1-\gamma_5)\tau \bar{s}\gamma^\mu(1-\gamma_5)u + \text{H.c.}, \tag{4}$$

where θ_c is the Cabibbo angle. The hadronic matrix element for the decay may be conveniently parametrized in terms of four form factors as follows [9],

$$\begin{aligned} J^\mu &\equiv \langle K^-(p_1)\pi^-(p_2)\pi^+(p_3)|\bar{s}\gamma^\mu(1-\gamma^5)u|0\rangle \\ &= [F_1(s_1, s_2, Q^2)(p_1-p_3)_\nu + F_2(s_1, s_2, Q^2) \\ &\quad \times (p_2-p_3)_\nu]T^{\mu\nu} + iF_3(s_1, s_2, Q^2)\epsilon^{\mu\nu\rho\sigma}p_{1\nu}p_{2\rho}p_{3\sigma} \\ &\quad + F_4(s_1, s_2, Q^2)Q^\mu, \end{aligned} \tag{5}$$

where $Q^\mu = (p_1+p_2+p_3)^\mu$, $T^{\mu\nu} = g^{\mu\nu} - Q^\mu Q^\nu/Q^2$, $s_1 = (p_2+p_3)^2$, and $s_2 = (p_1+p_3)^2$; also, we adopt the convention $\epsilon_{0123} = +1$ as in Ref. [9].[1] The form factors F_1-F_4 have been considered, for example, in Ref. [10]. As noted there, F_1 can arise due to the decay chain $\tau \to K_1\nu_\tau$, with $K_1 \to K^*\pi \to K\pi\pi$, while F_2 comes from $\tau \to K_1\nu_\tau$, with $K_1 \to K\rho \to K\pi\pi$. It is now known that both the $K_1(1270)$ and the $K_1(1400)$ contribute (see Sec. IVA for further details). The form factor F_3 is related to the Wess-Zumino anomaly term in the chiral Lagrangian. F_3 can be estimated by considering the decay chain $\tau \to K^*\nu_\tau$, with the intermediate K^* going to ρK or $K^*\pi$ [10]. The scalar term, F_4, is generally assumed to be negligible for this decay, since there is no suitable pseudoscalar resonance through which the decay can proceed. The authors of Ref. [11] performed a calculation of F_4 within the context of chiral perturbation theory and found that F_4 is nonzero if one includes chiral-symmetry-breaking mass terms for the quarks. The resulting expression for F_4 was found to contain both a nonresonant term (proportional to $m_\pi^2 + m_K^2$) and a resonant term. A numerical study indicated that the SM scalar contribution to the width was quite small [11]. We will consider the form factors further in Sec. IVA. At this point we simply note that F_1 and F_2 give the dominant contributions to the rate for $\tau \to K\pi\pi\nu_\tau$ [12], while F_3 is expected to give a subdominant contribution. In fact, in their experimental analysis, CLEO discards the F_3 term altogether and considers only the contributions due to F_1 and F_2 [12].

Starting from Eq. (5), the amplitude squared for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$ within the context of the SM is given by

$$|\mathcal{A}_{\rm SM}|^2 = \frac{G_F^2}{2}\sin^2\theta_c L_{\mu\nu}H^{\mu\nu}, \tag{6}$$

where $L_{\mu\nu} = M_\mu(M_\nu)^\dagger$ and $H^{\mu\nu} = J^\mu(J^\nu)^\dagger$, with $M_\mu = \bar{u}_{\nu_\tau}\gamma_\mu(1-\gamma^5)u_\tau$.

Effects due to a charged Higgs modify the effective Hamiltonian relevant for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$, adding the following terms,[2]

[1] The authors of Ref. [9] adopt the convention $\epsilon_{0123} = +1$, but do not state the precise functional form for F_3. Subsequent authors state F_3, but the sign of ϵ_{0123} is not obvious. We make a particular choice for the sign of F_3 below; changing this sign would change the sign of the related asymmetry.

[2] These expressions are similar to those in Ref. [4], although our notation differs slightly from that found there.

$$\mathcal{H}_{\rm eff}^{\rm NP} = \frac{G_F}{\sqrt{2}} \sin\theta_c [\eta_S \bar{\nu}_\tau (1+\gamma_5)\tau \bar{s} u + \eta_P \bar{\nu}_\tau (1+\gamma_5)\tau \bar{s}\gamma_5 u] + \text{H.c.} \tag{7}$$

The total effective Hamiltonian is then $\mathcal{H}_{\rm eff} = \mathcal{H}_{\rm eff}^{\rm SM} + \mathcal{H}_{\rm eff}^{\rm NP}$. In writing down Eq. (7) we have neglected terms that would involve a right-handed projection of the neutrino field. The interference of such terms with the SM amplitude would be suppressed by m_{ν_i} (assuming that the neutrino spin states are summed over).

The NP effects can be incorporated into the amplitude in a straightforward manner. We first define a new current $\tilde{J}^\mu$, which is obtained from J^μ by the replacement

$$F_4 \to \tilde{F}_4 = F_4 + \frac{f_H}{m_\tau}\eta_P, \tag{8}$$

where the pseudoscalar form factor has been defined as follows

$$\langle K^-(p_1)\pi^-(p_2)\pi^+(p_3)|\bar{s}\gamma^5 u|0\rangle = f_H. \tag{9}$$

Defining $\tilde{H}^{\mu\nu} \equiv \tilde{J}^\mu(\tilde{J}^\nu)^\dagger$, we then find the following expression for the square of the matrix element,

$$|\mathcal{A}|^2 = \frac{G_F^2}{2}\sin^2\theta_c L_{\mu\nu}\tilde{H}^{\mu\nu}. \tag{10}$$

Note that we have used the τ^- equation of motion in order to arrive at our definition of $\tilde{F}_4$. We have also neglected the mass of the neutrino.

The decays of τ leptons to final states containing two and three pseudoscalar mesons have been thoroughly analyzed in Ref. [9]. The notation described there has been adopted widely in the field and is quite standard. First, let us define several useful angles. Our definitions are identical to those in Ref. [9]. We review the various definitions here for convenience (more details may be found in Ref. [9]). The angle θ is defined in the τ rest frame; in that frame it is the angle between the direction of the hadrons ("$\vec{Q}$") and the direction of the tau in the laboratory frame. All other angles are defined in the hadronic rest frame (i.e., the frame in which $\vec{Q} \equiv \vec{p}_1 + \vec{p}_2 + \vec{p}_3 = 0$). In the hadronic rest frame we define two different coordinate systems, S and S'. These two coordinate systems are related by a Euler rotation using the Euler angles α, β and γ, as indicated in Fig. 1. The z' axis in S' is chosen as the direction of the laboratory in the hadronic rest frame ($\hat{n}_L$). The x' axis is chosen such that the τ direction ($\hat{n}_\tau$) is in the x'-z' plane, making an angle ψ with respect to the z' axis (see Fig. 1). The z axis in S is perpendicular to the plane defined by the momenta of the hadrons: $\hat{z} = \hat{n}_\perp \equiv \vec{p}_1 \times \vec{p}_2/|\vec{p}_1 \times \vec{p}_2|$. The x axis is taken to be the direction of $\vec{p}_3$; i.e., $\hat{x} = \vec{p}_3/|\vec{p}_3|$. The three Euler angles are defined as follows: α is the angle between the $(\hat{n}_L, \hat{n}_\tau)$ plane and the $(\hat{n}_L, \hat{n}_\perp)$ plane, β is the angle between $\hat{n}_L$ and $\hat{n}_\perp$, and γ is the angle between the $(\hat{n}_L, \hat{n}_\perp)$ plane and the $(\hat{n}_\perp, \hat{x})$ plane.

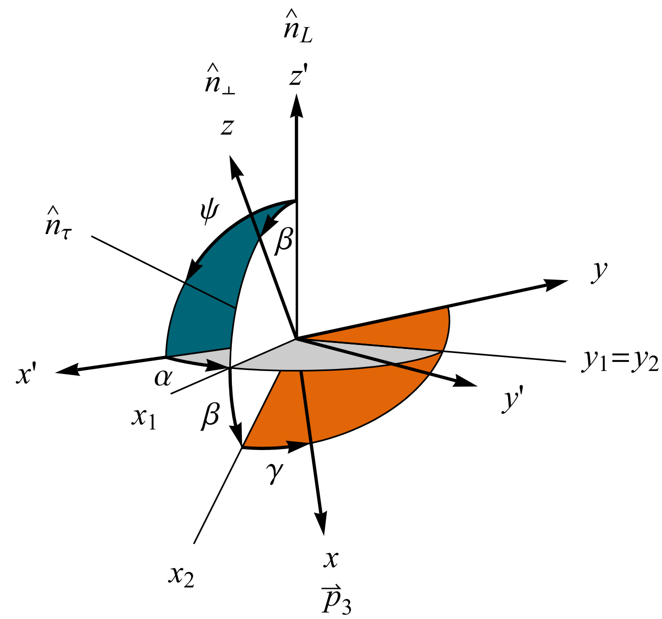

FIG. 1 (color online). Definitions of the angles α, β, γ and ψ. The Euler rotations corresponding to α, β and γ are about the z', $y_1 = y_2$ and z axes, respectively. This figure is very similar to a figure found in Ref. [9].

Having defined the various angles, we may write the differential width for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$ as follows [9],

$$d\Gamma = \frac{G_F^2 \sin^2\theta_c}{256(2\pi)^5 m_\tau}\frac{m_\tau^2 - Q^2}{m_\tau^2}\frac{dQ^2}{Q^2} ds_1 ds_2 \frac{d\alpha}{2\pi}\frac{d\gamma}{2\pi}\frac{d\cos\beta}{2} \times \frac{d\cos\theta}{2} L_{\mu\nu}\tilde{H}^{\mu\nu}, \tag{11}$$

where Q^2, s_1, and s_2 were defined below Eq. (5).

The coordinate system S is convenient for expressing the momenta of the three pseudoscalar mesons and for computing the various components of the tensor $\tilde{H}^{\mu\nu}$. In this coordinate system we have [9],

$$p_1^\mu = (E_1, p_1^x, p_1^y, 0), \tag{12}$$

$$p_2^\mu = (E_2, p_2^x, p_2^y, 0), \tag{13}$$

$$p_3^\mu = (E_3, p_3^x, 0, 0), \tag{14}$$

where

$$E_i = (Q^2 - s_i + m_i^2)/(2\sqrt{Q^2}), \tag{15}$$

$$p_3^x = \sqrt{E_3^2 - m_3^2}, \tag{16}$$

$$p_1^x = (2E_1E_3 - s_2 + m_1^2 + m_3^2)/(2p_3^x), \tag{17}$$

$$p_2^x = (2E_2E_3 - s_1 + m_2^2 + m_3^2)/(2p_3^x), \tag{18}$$

$$p_1^y = \sqrt{E_1^2 - (p_1^x)^2 - m_1^2}, \tag{19}$$

$$p_2^y = -\sqrt{E_2^2 - (p_2^x)^2 - m_2^2} = -p_1^y. \tag{20}$$

s_3 is defined analogously to s_1 and s_2 [i.e., $s_3 = (p_1 + p_2)^2$] and may be expressed in terms of s_1, s_2, and Q^2. Note that the angle between $\vec{p}_1$ and $\vec{p}_3$ is fixed for a given choice of s_1, s_2 and Q^2.

The above definitions for the various hadron momentum vectors allow us to determine simple expressions for $\tilde{H}^{\mu\nu}$ in S. We will not write out all 16 elements of the tensor. Rather, we define new quantities B_i ($i = 1, \ldots, 4$) that are related to the components of $\tilde{J}^\mu$ as follows,

$$B_1 = \tilde{J}^1 = [F_1(p_1 - p_3)^x + F_2(p_2 - p_3)^x], \tag{21}$$

$$B_2 = \tilde{J}^2 = (F_1 - F_2)p_1^y, \tag{22}$$

$$B_3 = -i\tilde{J}^3 = F_3\sqrt{Q^2}\,p_1^y p_3^x, \tag{23}$$

$$B_4 = \tilde{J}^0 = \sqrt{Q^2}\left[F_4 + \frac{f_H}{m_\tau}\eta_P\right]. \tag{24}$$

Then the components of $\tilde{H}^{\mu\nu}$ consist of various combinations $B_iB_j^*$ (in some cases multiplied by $\pm i$). Inserting these expressions into Eq. (11) and integrating over α we find an expression very similar to that given in Ref. [4],[3]

$$\begin{aligned}
\frac{d\Gamma}{dQ^2 ds_1 ds_2 d\gamma\, d\cos\beta\, d\cos\theta} = {} & \frac{G_F^2\sin^2\theta_c}{512(2\pi)^6}\frac{(m_\tau^2 - Q^2)^2}{m_\tau^3 Q^2}\bigg\{\left[\frac{2}{3}K_1 + K_2 + \frac{1}{3}\bar{K}_1(3\cos^2\beta - 1)/2\right](|B_1|^2 + |B_2|^2) \\
& + \left[\frac{2}{3}K_1 + K_2 - \frac{2}{3}\bar{K}_1(3\cos^2\beta - 1)/2\right]|B_3|^2 + K_2|B_4|^2 - \frac{1}{2}\bar{K}_1\sin^2\beta\cos2\gamma(|B_1|^2 - |B_2|^2) \\
& + \bar{K}_1\sin^2\beta\sin2\gamma\,\mathrm{Re}(B_1B_2^*) + 2\bar{K}_3\sin\beta\sin\gamma\,\mathrm{Re}(B_1B_3^*) + 2\bar{K}_2\sin\beta\cos\gamma\,\mathrm{Re}(B_1B_4^*) \\
& + 2\bar{K}_3\sin\beta\cos\gamma\,\mathrm{Re}(B_2B_3^*) - 2\bar{K}_2\sin\beta\sin\gamma\,\mathrm{Re}(B_2B_4^*) + 2\bar{K}_3\cos\beta\,\mathrm{Im}(B_1B_2^*) \\
& + \bar{K}_1\sin2\beta\cos\gamma\,\mathrm{Im}(B_1B_3^*) - \bar{K}_1\sin2\beta\sin\gamma\,\mathrm{Im}(B_2B_3^*) + 2\bar{K}_2\cos\beta\,\mathrm{Im}(B_3B_4^*)\bigg\},
\end{aligned} \tag{25}$$

Note that, of the four parameters B_i defined in Eqs. (21)–(24), only B_4 contains a non-SM weak phase. Thus, the only terms in Eq. (25) that can lead to nonzero CP asymmetries are those containing one or more powers of B_4. The parameters K_i and $\bar{K}_i$ in the above expression are defined as follows [9],

$$K_1 = 1 - P\cos\theta - (m_\tau^2/Q^2)(1 + P\cos\theta), \tag{26}$$

$$K_2 = (m_\tau^2/Q^2)(1 + P\cos\theta), \tag{27}$$

$$K_3 = 1 - P\cos\theta, \tag{28}$$

$$\bar{K}_1 = K_1(3\cos^2\psi - 1)/2 - (3/2)K_4\sin2\psi, \tag{29}$$

$$\bar{K}_2 = K_2\cos\psi + K_4\sin\psi, \tag{30}$$

$$\bar{K}_3 = K_3\cos\psi - K_4\sin\psi, \tag{31}$$

$$K_4 = \sqrt{m_\tau^2/Q^2}\,P\sin\theta, \tag{32}$$

where the parameter P denotes the polarization of the τ^-, $s_\tau^2 = -P^2$. In the numerical work in Ref. [4], a value of P was used that was relevant for LEP. In our numerical work we will take $P = 0$, which is appropriate for lower-energy experiments [9]. Note that ψ is a function of $\cos\theta$ and Q^2.

If the τ's are pair-produced at a symmetric collider,

$$\cos\theta = \frac{(2xm_\tau^2 - m_\tau^2 - Q^2)}{(m_\tau^2 - Q^2)\sqrt{1 - 4m_\tau^2/s}}, \tag{33}$$

$$\cos\psi = \frac{x(m_\tau^2 + Q^2) - 2Q^2}{(m_\tau^2 - Q^2)\sqrt{x^2 - 4Q^2/s}}, \tag{34}$$

where $x = 2E_h/\sqrt{s}$ and $s = 4E_{\mathrm{beam}}^2$, with E_h being the hadron energy in the lab [9] (see also Ref. [13]). Thus, given s (we take $s = (10.58\ \mathrm{GeV})^2$ below), Q^2 and $\cos\theta$, one can solve for x and substitute this expression into the expression for $\cos\psi$. Finally, note that if the direction of the τ^- could be determined, it would not be necessary to integrate over α. In this case it might be possible to extract other useful information for the construction of CP asymmetries. We do not consider this possibility in this work.

The differential width in Eq. (25) may now be integrated to compute the partial width for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$. Comparison with the analogous quantity for the τ^+ decay yields the regular rate asymmetry. One can also integrate over the angular variables in an asymmetric manner in such a way that certain cross terms are selected from Eq. (25). These "weighted differential widths" can then be compared to the analogous expressions for the τ^+ decay to

[3]Because of some ambiguities, it is difficult to tell if the expressions agree exactly.

yield CP-odd quantities. We consider two types of asymmetries formed in this manner—modified rate asymmetries [whose dependence on the strong and weak phases is given in Eq. (2)] and a triple product asymmetry [see Eq. (3)].

B. Weighted differential widths

The authors of Ref. [4] derived an expression for the differential width that is very similar to Eq. (25). Since they assumed that $f_H = 0$ for these decays, they only considered LR effects. We consider the complementary point of view. Assuming that f_H could be nonzero and noting that there are strong constraints on LR mixing, we consider only effects due to the exchange of a charged scalar. The analysis in Ref. [4] focused exclusively on triple products in the differential width. In our notation, these TP's correspond to the cross-terms containing the factors $\mathrm{Im}(B_i B_j^*)$. Recall that CP asymmetries formed from triple products do not require the presence of strong phases [see Eq. (3)].

In this work, we reconsider CP asymmetries formed from triple products and also consider CP asymmetries that can be formed from T-even[4] cross terms in the differential width. Both types of terms may be isolated by employing suitable weighting functions when performing the angular integrations.

We begin by defining various regions in terms of γ and β, as in Ref. [4],

$$\begin{aligned} &\mathrm{I}: 0 \le \gamma < \pi/2, \qquad \mathrm{II}: \pi/2 \le \gamma < \pi, \\ &\mathrm{III}: \pi \le \gamma < 3\pi/2, \qquad \mathrm{IV}: 3\pi/2 \le \gamma < 2\pi; \\ &\mathrm{A}: 0 \le \beta < \pi/2, \qquad \mathrm{B}: \pi/2 \le \beta < \pi; \end{aligned} \tag{35}$$

As noted above, in order for a particular term in the differential width [Eq. (25)] to contribute to a nonzero CP asymmetry, it must contain one or more powers of B_4. This is because B_4 contains the possible CP-violating phase coming from NP. Inspection of Eq. (25) leads one to the conclusion that there are four terms of interest. One is proportional to $|B_4|^2$. As we shall see below, this term arises in the regular rate asymmetry. The remaining three terms are proportional to the angular functions $\sin\beta\cos\gamma$, $\sin\beta\sin\gamma$, and $\cos\beta$. These three terms can be isolated by using appropriate weighting functions, as indicated in Table I. Thus, for example, to isolate the term in Eq. (25) proportional to $\sin\beta\sin\gamma$, the differential width is multiplied by $g_1(\gamma, \beta)$ (which is $+1$ in regions IA, IIA, IB, and IIB and -1 in the other regions) and the angular integration is carried out. This eliminates all other terms since the weighting functions are such that[5]

TABLE I. Angular weighting factors. The regions I–IV, A and B are defined in Eq. (35) in the text. The second column gives the angular functions of interest, $f_i(\gamma, \beta)$. The third column gives the weighting function $g_i(\gamma, \beta)$ that can be used to isolate $f_i(\gamma, \beta)$. The functions $g_i(\gamma, \beta)$ are simply ± 1 depending on which region γ and β fall in.

i	$f_i(\gamma, \beta)$	$g_i(\gamma, \beta)$
1	$\sin\beta\sin\gamma$	I + II − III − IV; A + B
2	$\sin\beta\cos\gamma$	I − II − III + IV; A + B
3	$\cos\beta$	I + II + III + IV; A − B

$$\iint f_i(\gamma, \beta) g_j(\gamma, \beta) \sin\beta d\gamma d\beta = 2\pi\delta_{ij} \qquad (i = 1, 2, 3). \tag{36}$$

Using the weighting functions $g_i(\gamma, \beta)$ in Table I, we define weighted differential widths as follows,

$$\frac{d\Gamma_i}{dQ^2 ds_1 ds_2} \equiv \int \frac{d\Gamma}{dQ^2 ds_1 ds_2 d\gamma d\cos\beta\, d\cos\theta} g_i(\gamma, \beta) \times \sin\beta d\beta d\gamma\, d\cos\theta. \tag{37}$$

The results for the three weighting functions are as follows,

$$\frac{d\Gamma_1}{dQ^2 ds_1 ds_2} = A(Q^2)[\langle \bar{K}_3 \rangle \mathrm{Re}(B_1 B_3^*) - \langle \bar{K}_2 \rangle \mathrm{Re}(B_2 B_4^*)], \tag{38}$$

$$\frac{d\Gamma_2}{dQ^2 ds_1 ds_2} = A(Q^2)[\langle \bar{K}_3 \rangle \mathrm{Re}(B_2 B_3^*) + \langle \bar{K}_2 \rangle \mathrm{Re}(B_1 B_4^*)], \tag{39}$$

$$\frac{d\Gamma_3}{dQ^2 ds_1 ds_2} = A(Q^2)[\langle \bar{K}_3 \rangle \mathrm{Im}(B_1 B_2^*) + \langle \bar{K}_2 \rangle \mathrm{Im}(B_3 B_4^*)], \tag{40}$$

where

$$A(Q^2) = \frac{G_F^2 \sin^2\theta_c}{128(2\pi)^5} \frac{(m_\tau^2 - Q^2)^2}{m_\tau^3 Q^2}, \tag{41}$$

$$\langle \bar{K}_i \rangle \equiv \frac{1}{2} \int_0^\pi \bar{K}_i \sin\theta d\theta. \tag{42}$$

The three weighted differential widths defined in Eqs. (38)–(40) can now be compared to their CP-conjugates in order to construct CP asymmetries. Recalling that the Higgs contribution resides in B_4 [see Eq. (24)] and noting that each of the three expressions above contains a term linear in B_4, we see that each of the resulting CP asymmetries has the possibility of being nonzero.

In the following sections we construct the CP asymmetries and then study them numerically to see if they might provide useful probes of non-SM CP violation.

[4] "T-even" here refers to the naive time-reversal operation.

[5] The weighting functions are also orthogonal to 1, $(3\cos^2\beta - 1)/2$, $\sin^2\beta\cos 2\gamma$, etc., so that only the intended cross terms are isolated. Also note that, experimentally, a more statistically significant weighting procedure might be to weight the differential width by the various functional forms f_i themselves. See Ref. [14] and also the moment analysis discussion in Ref. [9].

III. CP-ODD OBSERVABLES

Before analyzing the various CP asymmetries, let us consider the coefficients B_i defined in Eqs. (21)–(24) a bit more carefully. As noted above, the sole non-SM weak phase resides in B_4. The form factors F_i and f_H are potential sources of strong phases. We may thus parametrize the four coefficients as follows,

$$B_1 = |B_1|e^{i\delta_1}, \tag{43}$$

$$B_2 = |B_2|e^{i\delta_2}, \tag{44}$$

$$B_3 = |B_3|e^{i\delta_3}, \tag{45}$$

$$B_4 = |B_4^{(1)}|e^{i\delta_4} + |B_4^{(2)}|e^{i\delta_H + i\phi_H}, \tag{46}$$

where

$$B_4^{(1)} = \sqrt{Q^2}F_4, \qquad B_4^{(2)} = \sqrt{Q^2}\frac{f_H}{m_\tau}\eta_P, \tag{47}$$

and where δ_i and ϕ_H represent strong and weak phases, respectively. An explicit expression for the weak phase ϕ_H is as follows,

$$\phi_H = \arg(\eta_P). \tag{48}$$

This phase could in principle be of order unity.

As was the case in Ref. [2], we can consider three types of CP asymmetries. The first is the regular rate asymmetry. This asymmetry is likely to be small in $\tau \to K\pi\pi\nu_\tau$ and is therefore unlikely to be measurable in the near future. The second and third types of asymmetries are the modified rate asymmetry and the triple-product asymmetry. We consider two different modified rate asymmetries, and one triple-product asymmetry. The triple-product asymmetry is similar to one considered for the decay $\tau \to K\pi K\nu_\tau$ in Ref. [4]. The modified rate asymmetries, to our knowledge, are new relative to this decay mode. Both types of asymmetries are constructed by first performing an asymmetrical integration over the kinematical angles β and γ, as noted in Eq. (37) and Table I. Since the procedures for extracting these two types of asymmetries are similar, we consider them together in the following.

A. Rate asymmetry

Let us first consider the regular rate asymmetry. In this case the angular integrations are performed symmetrically $[g(\gamma, \beta) = 1]$ and the width for the process is compared to that for the antiprocess. The differential width for the τ^- decay in this case is given by

$$\frac{d\Gamma}{dQ^2 ds_1 ds_2} = A(Q^2)\bigg[\bigg(\frac{2}{3} + \frac{1}{3}\frac{m_\tau^2}{Q^2}\bigg)(|B_1|^2 + |B_2|^2 + |B_3|^2) + \frac{m_\tau^2}{Q^2}|B_4|^2\bigg]. \tag{49}$$

The width for the τ^+ process will have the same strong phases, but the weak phases will have their signs reversed. It is immediately evident from Eq. (49) and Eqs. (43)–(45) that the coefficients B_1, B_2, and B_3 will not give any contribution to the rate asymmetry, since they do not contain weak phases. Thus, the rate asymmetry is proportional to[6]

$$|B_4|^2 - |\bar{B}_4|^2 = 4|B_4^{(1)}||B_4^{(2)}|\sin(\delta_4 - \delta_H)\sin(\phi_H). \tag{50}$$

This expression is proportional to $|F_4 f_H \eta_P|$. The SM scalar form factor F_4 is generally thought to be small. If the NP factor η_P is also small, then the regular rate asymmetry is doubly suppressed. Given the expected smallness of the regular rate asymmetry, we do not consider it further here. As we shall see, however, other CP asymmetries can be constructed that depend on $F_i f_H \eta_P$, with $i = 1, 2, 3$. Such asymmetries may be more promising in terms of their NP reach.

B. Modified and triple-product CP asymmetries

We define CP asymmetries corresponding to the weighted differential widths [Eqs. (38)–(40)] as follows,

$$A_{CP}^{(i)} = \frac{1}{\Gamma + \bar{\Gamma}}\int\bigg(\frac{d\Gamma_i}{dQ^2 ds_1 ds_2} - \frac{d\bar{\Gamma}_i}{dQ^2 ds_1 ds_2}\bigg)dQ^2 ds_1 ds_2. \tag{51}$$

The quantities with the bars correspond to the decay $\tau^+ \to K^+\pi^+\pi^-\bar{\nu}_\tau$ and are obtained from those without the bars by changing the signs of all weak phases while leaving strong phases unchanged.[7] $A_{CP}^{(1)}$ and $A_{CP}^{(2)}$ descend from the terms containing $\mathrm{Re}(B_2 B_4^*)$ and $\mathrm{Re}(B_1 B_4^*)$ in Eqs. (38) and (39), respectively. These are both modified rate asymmetries. The third asymmetry, $A_{CP}^{(3)}$, descends from the term containing $\mathrm{Im}(B_3 B_4^*)$ in Eq. (40). This a triple-product asymmetry. Γ and $\bar{\Gamma}$ in Eq. (51) represent the partial widths for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$ and $\tau^+ \to K^+\pi^+\pi^-\bar{\nu}_\tau$, respectively. In our numerical work below we make the approximation that $\Gamma \simeq \bar{\Gamma}$, so that $\Gamma + \bar{\Gamma} \simeq 2\Gamma$. The experimental value for Γ is used.

1. Modified rate asymmetries ($i = 1, 2$)

The modified rate asymmetries, $A_{CP}^{(1)}$ and $A_{CP}^{(2)}$, require a strong phase in order to be nonzero. These asymmetries are

[6] This expression is part of an integral over phase space. Note that one or both of the strong phases could depend on Q^2, s_1, and s_2.

[7] Note that we *subtract* the width for the antiprocess from that for the process, both for the modified rate asymmetries and for the triple-product asymmetry. The authors of Ref. [4] consider only triple-product asymmetries. Their expressions for the antiprocess contain an extra overall sign; thus they add the widths for the process and antiprocess to obtain CP asymmetries. This is a notational difference. Both approaches lead (correctly) to a TP CP asymmetry that is of the form of Eq. (3).

analogous to the "polarization-dependent asymmetry" defined in Ref. [2]. In order to obtain numerical estimates for these asymmetries, let us make the following simplifying assumptions. First of all, we will assume that f_H has no Q^2, s_1, or s_2 dependence. We will also assume that f_H has no strong phase associated with it (it will be taken to be real and positive). Under these assumptions, these two asymmetries are given by

$$A_{CP}^{(1)} \simeq -\frac{m_\tau}{\Gamma+\bar{\Gamma}}\bigg[\int \frac{A(Q^2)}{\sqrt{Q^2}}\cos\psi\, p_1^y\, \mathrm{Im}(F_1 - F_2) \times dQ^2 ds_1 ds_2 d\cos\theta\bigg] f_H\, \mathrm{Im}(\eta_P), \tag{52}$$

$$A_{CP}^{(2)} \simeq \frac{m_\tau}{\Gamma+\bar{\Gamma}}\bigg[\int \frac{A(Q^2)}{\sqrt{Q^2}}\cos\psi\, \mathrm{Im}[F_1(p_1-p_3)^x + F_2(p_2-p_3)^x] dQ^2 ds_1 ds_2 d\cos\theta\bigg] f_H\, \mathrm{Im}(\eta_P), \tag{53}$$

in which we have taken the τ's to be unpolarized ($P = 0$). Recall that ψ depends on θ through Eqs. (33) and (34).

As noted above, $A_{CP}^{(1)}$ and $A_{CP}^{(2)}$ both have the generic form $\sin\phi\sin\delta$, since $\mathrm{Im}(\eta_P) \propto \sin\phi_H$ and $\mathrm{Im}(F_1 - F_2)$ and $\mathrm{Im}[F_1(p_1-p_3)^x + F_2(p_2-p_3)^x]$ are both proportional to $\sin\delta$, with δ being a strong phase. In Sec. IV B we will examine the sensitivity of these asymmetries in a particular model for the form factors.

B. Triple-product asymmetry ($i = 3$)

The third CP asymmetry, $A_{CP}^{(3)}$, is a triple-product asymmetry and is similar in some respects to the asymmetries constructed for $\tau \to K\pi\pi\nu_\tau$ in Ref. [4]. Recall, however, that in that case the authors assumed that the NP effects were due to a new right-handed gauge boson. To obtain a numerical estimate for $A_{CP}^{(3)}$, we make the same simplifying assumptions as above; i.e., we assume that f_H is real and positive (no strong phase) and that it has no Q^2, s_1 or s_2 dependence. Under these assumptions,

$$A_{CP}^{(3)} \simeq -\frac{m_\tau}{\Gamma+\bar{\Gamma}}\bigg[\int A(Q^2) \times \cos\psi\, p_1^y p_3^x\, \mathrm{Re}(F_3) dQ^2 ds_1 ds_2 d\cos\theta\bigg] f_H\, \mathrm{Im}(\eta_P). \tag{54}$$

Like the modified rate asymmetries considered above, the triple-product asymmetry $A_{CP}^{(3)}$ is proportional to $\mathrm{Im}(\eta_P)$. In contrast to $A_{CP}^{(1)}$ and $A_{CP}^{(2)}$, however, this asymmetry does not require a strong phase, since $\mathrm{Re}(F_3) \propto \cos\delta$ (where δ represents a strong phase). Having said this, there is a potential drawback with $A_{CP}^{(3)}$ in that it depends on the form factor F_3 (which is expected to be subdominant), whereas $A_{CP}^{(1)}$ and $A_{CP}^{(2)}$ depend on combinations of the dominant form factors F_1 and F_2. In the next section we perform a numerical study to examine these various factors quantitatively.

IV. NUMERICAL RESULTS

The modified and triple-product asymmetries defined above all have the form

$$A_{CP}^{(i)} = a_{CP}^{(i)} f_H\, \mathrm{Im}(\eta_P), \qquad i = 1\text{–}3, \tag{55}$$

where the $a_{CP}^{(i)}$ are constants determined by integrating over $\cos\theta$, s_1, s_2 and Q^2. In this section we assume particular functional forms for the form factors and use these to estimate the $a_{CP}^{(i)}$. It turns out that there are significant cancellations that occur as one performs the integrations over phase space. To help illustrate this cancellation, we define four differential quantities as follows,

$$\frac{da_{CP}^{(i)}}{dX}, \tag{56}$$

with X given by $M_{K\pi\pi} = \sqrt{Q^2}$, $M_{\pi\pi} = \sqrt{s_1}$, $M_{K\pi} = \sqrt{s_2}$, and $\cos\theta$. Given the cancellations that occur upon integration, experimentalists may wish to study differential CP asymmetries in addition to, or in place of, the integrated asymmetries.

A. Model for the form factors

There have been several models for the form factors F_1-F_3 over the past number of years. One model, which simply took the intermediate K_1 to be the $K_1(1400)$, may be found in Ref. [10] (see also Ref. [15]). A subsequent analysis by Finkemeier and Mirkes [16] took into account both the $K_1(1400)$ and the $K_1(1270)$ resonances and also incorporated other K^* resonances ($K^{*\prime}$ and $K^{*\prime\prime}$; see also Refs. [17,18]). Finally, an experimental analysis of the form factors was performed by the CLEO collaboration in Ref. [12].

The form factor F_3 is related to the Wess-Zumino anomaly term in the chiral Lagrangian, although the anomaly calculation does not predict the full momentum dependence of the form factor. The various models that have been proposed make different assumptions regarding F_3. The authors of Ref. [10] found that the F_3 term contributed approximately 1% to the overall width for $\tau^- \to K^-\pi^-\pi^+\nu_\tau$. The parametrization in Ref. [16] led to an anomalous contribution of order 10%. The CLEO collaboration noted that the contribution would be of order 5.5% based on a particular model (found in Ref. [15]). Since the contribution was expected to be small, they set F_3 to zero in their analysis and focused on determining the resonance structures of F_1 and F_2. The uncertainty resulting from the neglect of F_3 was incorporated into their systematic error [12].

It is not obvious which form factor parametrization one ought to employ in a numerical study such as we carry out here. One option is to be motivated almost purely by theoretical considerations. Past authors have employed such an approach. They have first used low-energy QCD to fix the overall normalizations of the form factors (or some combination of contributions). The momentum dependence of the various contributions has then been incorporated in a phenomenological manner. There are several issues that one encounters when attempting to adopt such an approach:

(1) According to Ref. [12], there is a disagreement between chiral perturbation theory and isospin regarding the relative normalizations of the form factors.

(2) Whether or not the disagreement noted in the preceding statement exists, there is a more serious problem in that theoretical predictions based on phenomenological models tend to overestimate the $\tau \to K\pi\pi\nu_\tau$ branching ratio, typically by a factor of 2 or more (see, e.g., Refs. [10,16]), although the agreement between theory and experiment improves if the K_1 states are taken to have larger widths [19].

(3) The F_3 contribution to the decay width (ignored in the CLEO analysis) can vary widely, depending on the specific phenomenological form chosen for the form factor—even if the normalization (as Q^2, s_1, $s_2 \to 0$) is held fixed. A relatively recent parameterization (which took the intermediate ρ and K^* contributions to contribute "equally" at the form factor level) led to an anomalous contribution of order 10% when the F_3 part was compared to the theoretical branching ratio [16]. The theoretical branching ratio, however, was too large by more than a factor of 2 compared to the current experimental value. When the F_3 contribution from Ref. [16] (using their preferred parameter choices) is compared to the *experimental* branching ratio, the relative F_3 contribution is over 25%. This number is much larger than the "5.5%" figure that the CLEO analysis assumed when it dropped the F_3 contribution.

Given the various issues associated with a purely "theoretical" approach to the form factors F_1-F_3, we adopt instead a more pragmatic approach based on the experimental analysis performed by CLEO. This analysis provides information regarding the relative contributions of various decay chains to F_1 and F_2, but does not provide the over-all normalization of the form factors. We fix the normalization by requiring that our numerical branching ratio agree with the experimental number. One difference between our analysis and that of CLEO is that we allow F_3 to be nonzero. Guided by Ref. [12] for F_1 and F_2 and by Ref. [10] for F_3, we write the form factors in terms of various Breit-Wigner functions as follows,

$$F_1(s_1, s_2, Q^2) = -\frac{2N}{3F_\pi}[C \cdot BW_{1270}(Q^2) + D \cdot BW_{1400}(Q^2)]BW_{K^*}(s_2), \tag{57}$$

$$F_2(s_1, s_2, Q^2) = -\frac{N}{\sqrt{3}F_\pi}[A \cdot BW_{1270}(Q^2) + B \cdot BW_{1400}(Q^2)]T_\rho^{(1)}(s_1), \tag{58}$$

$$F_3(s_1, s_2, Q^2) = \frac{N_3}{2\sqrt{2}\pi^2 F_\pi^3} BW_{K^*}(Q^2) \times \left[\frac{T_\rho^{(1)}(s_1) + \alpha BW_{K^*}(s_2)}{1+\alpha}\right], \tag{59}$$

with $\alpha = -0.2$ and $F_\pi = 93.3$ MeV. Also, we set F_4 to zero and only take f_H into account when computing the numerators of the asymmetry expressions. The constants N, N_3 and A-D will be discussed further below. The normalized Breit-Wigner propagators for the $K_1(1270)$ and the $K_1(1400)$ are assumed to be given by [12],

$$BW_{K_1}(Q^2) = \frac{-m_{K_1}^2 + i m_{K_1}\Gamma_{K_1}}{Q^2 - m_{K_1}^2 + i m_{K_1}\Gamma_{K_1}}, \tag{60}$$

with m_{K_1} and Γ_{K_1} being the mass and width for the appropriate K_1 state. As noted in the CLEO analysis, a fit to the $\tau \to K\pi\pi\nu_\tau$ data indicates that the effective $K_1(1270)$ and $K_1(1400)$ widths are larger in this decay than the respective values reported by the Particle Data Group (see also Refs. [19,20]). Following CLEO, we take the following values for our numerical analysis [12]:

$$\begin{aligned} m_{1270(1400)} &= 1.254(1.463)\ \text{GeV}, \\ \Gamma_{1270(1400)} &= 0.26(0.30)\ \text{GeV}. \end{aligned} \tag{61}$$

The Breit-Wigner propagators for the K^* and ρ are taken to have energy-dependent widths (see, for example, Refs. [10,12]),

$$BW_R(s) = \frac{-m_R^2}{s - m_R^2 + i\sqrt{s}\Gamma_R(s)}. \tag{62}$$

with

$$\Gamma_R(s) = \Gamma_R \frac{m_R^2}{s}\left(\frac{p}{p_R}\right)^3, \tag{63}$$

where

$$p = \frac{1}{2\sqrt{s}}\sqrt{[s-(m_1+m_2)^2][s-(m_1-m_2)^2]}, \tag{64}$$

$$p_R = \frac{1}{2m_R}\sqrt{[m_R^2-(m_1+m_2)^2][m_R^2-(m_1-m_2)^2]}. \tag{65}$$

When using the above expressions it is assumed that the

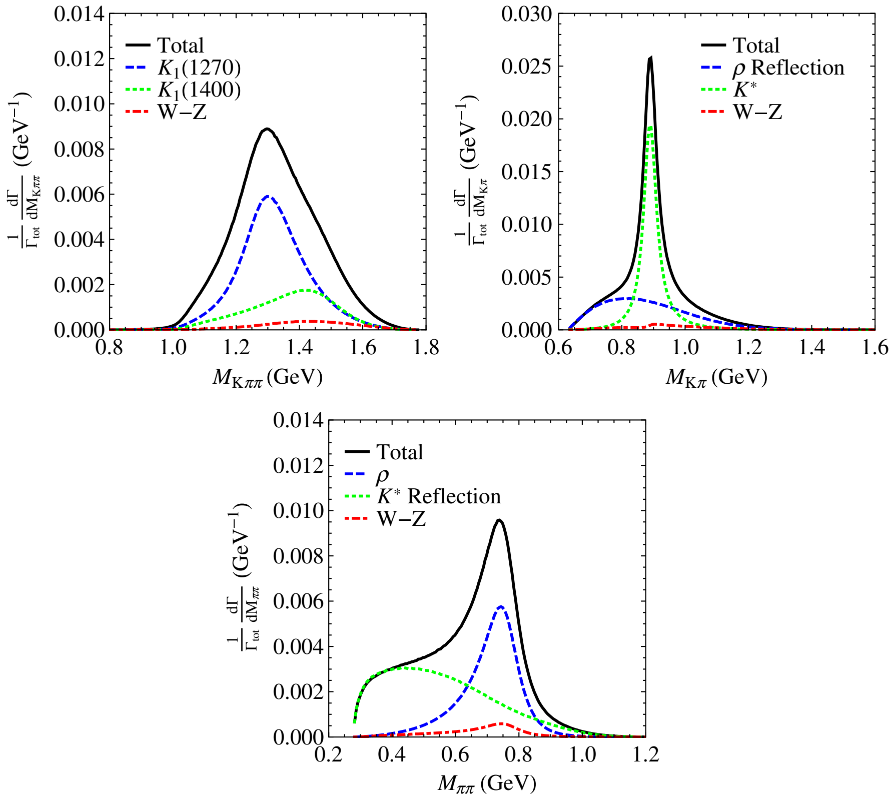

FIG. 2 (color online). Plots of the differential widths $d\Gamma/dM$, including the contributions from the various decay chains. The ρ, K^*, $K_1(1270)$, and $K_1(1400)$ curves include contributions from only the F_1 and F_2 terms. The "W-Z" curves represent the contribution from the F_3 term. (As noted in the text, F_3 is related to the Wess-Zumino anomaly.)

resonance R decays to two particles with masses m_1 and m_2. [Equation (63) also assumes that $\sqrt{s} \geq m_1 + m_2$—otherwise $\Gamma_R(s)$ should be set to zero. This condition is satisfied in all regions of phase space for the decay chains that we consider.] For the K^*, a single resonance (with an energy-dependent width) is assumed; we take $m_{K^*} = 0.892$ GeV and $\Gamma_{K^*} = 0.050$ GeV.[8] The expression for the ρ incorporates two different resonances (the ρ and the ρ'),

$$T_\rho^{(1)}(s_1) = \frac{BW_\rho(s_1) + \beta BW_{\rho'}(s_1)}{1 + \beta}, \tag{66}$$

with $\beta = -0.145$, $m_\rho = 0.773$ GeV, $m_{\rho'} = 1.370$ GeV, $\Gamma_\rho = 0.145$ GeV and $\Gamma_{\rho'} = 0.510$ GeV [16,21].

Let us now consider the constants N, N_3 and A-D in Eqs. (57)–(59). As noted above, low-energy QCD can be used to fix certain combinations of these constants. Given the issues noted above, however, our choices for these parameters are guided by experimental data. The CLEO collaboration effectively set $N_3 = 0$ in their analysis and then determined A-D [12]. The overall normalization N was not stated. We choose values that are similar to those reported in Table I of Ref. [12],

$$\begin{aligned} &A = 0.944, \qquad B = 0, \\ &C = A \times \sqrt{\tfrac{16}{42}} \times \sqrt{\tfrac{6917}{61\,636}} \simeq 0.195, \\ &D = \sqrt{1 - A^2 - C^2} \simeq 0.266. \end{aligned} \tag{67}$$

An apparent typo in Eq. (2) of Ref. [12] renders the relative signs of the constants a bit uncertain. The signs we have chosen for A-D are consistent with the signs used in Ref. [16]. Our parameter choice gives results for the differential width (see Fig. 2) that are visually similar to the results obtained in Ref. [12], although the agreement between our numerical results and those of CLEO is not perfect. Since we wish, in part, to study effects due to the inclusion of the F_3 term, we retain a nonzero value for N_3. As was noted above, there have been various estimates regarding the F_3 contribution to the width, with estimates varying from 1% to 10% in papers that we have noted. For the purpose of our numerical study, we fix N_3 such that the F_3 term contributes 5% to the $\tau \to K\pi\pi\nu_\tau$ width. This value is similar to the value assumed (and subsequently neglected) in the the CLEO analysis. The F_1 and F_2 terms

[8]Note that the intermediate K^* represents a K^{*0} in the expression for the form factor F_1, while both K^{*0} and K^{*-} appear in F_3. For simplicity we use the same mass and width for both the charged and neutral versions of this particle.

TABLE II. Calculated values for $a_{CP}^{(i)}$, $a_{CP,\mathrm{mod}}^{(i)}$, and $a_{CP,\mathrm{max}}^{(i)}$. $a_{CP,\mathrm{mod}}^{(i)}$ is computed by making the replacement $\cos\psi \to |\cos\psi|$ in Eqs. (52)–(54). This procedure helps to eliminate some of the cancellations that occur upon integration. $a_{CP,\mathrm{max}}^{(i)}$ is determined by taking the absolute values of the integrands in Eqs. (52)–(54).

i	$a_{CP}^{(i)}$	$a_{CP,\mathrm{mod}}^{(i)}$	$a_{CP,\mathrm{max}}^{(i)}$	CP asymmetry type
1	-2.2×10^{-5}	-5.2×10^{-5}	9.8×10^{-4}	Modified rate asymmetry
2	7.0×10^{-4}	1.0×10^{-3}	2.9×10^{-3}	Modified rate asymmetry
3	2.5×10^{-4}	6.2×10^{-4}	8.3×10^{-4}	Triple-product asymmetry

are taken to contribute the remaining 95%. Taking $\mathcal{B}(\tau \to K\pi\pi\nu_\tau) = 0.002\,73$ [22] and $c\tau = 87.11 \times 10^{-6}$ m [23], we find that $N \simeq 1.4088$ and $N_3 \simeq 1.4696$.

B. Numerical estimates of the *CP* asymmetries

Using the constants for N, N_3, and A-D noted above, we integrate Eq. (25) over phase space to obtain $d\Gamma/dM_{K\pi\pi}$, $d\Gamma/dM_{K\pi}$, and $d\Gamma/dM_{\pi\pi}$. The results (normalized to Γ_{tot}) are displayed in Fig. 2. The plots are similar to those in Fig. 2 of Ref. [12], although the agreement is not perfect. We also include a contribution due to F_3 (not included in the CLEO plots).

Having chosen the various coefficients, we can also perform the integrations in Eqs. (52)–(54) to obtain the numerical coefficients $a_{CP}^{(i)}$. Numerical values for these coefficients are listed in the second column of Table II. Recall that the actual CP asymmetries are obtained by multiplying the $a_{CP}^{(i)}$ by $f_H\,\mathrm{Im}(\eta_P)$ [see Eq. (55)].

Figure 3 shows plots of the differential asymmetries $da_{CP}^{(i)}/dX$, with $X = M_{K\pi\pi}$, $M_{\pi\pi}$, $M_{K\pi}$, and $\cos\theta$. In each case, integration over X yields the corresponding coefficient $a_{CP}^{(i)}$. As is apparent from the figure, each of the asymmetry coefficients undergoes considerable cancellation upon integration. Given these cancellations, experimentalists may find it advantageous to perform fits to the differential CP asymmetries instead of simply measuring the integrated asymmetries. Alternatively, it may be possible to achieve larger integrated asymmetries by employing extra weighting functions in the integration over one or more of the integration variables. As an example, we have recomputed the asymmetries with the change $\cos\psi \to |\cos\psi|$ in Eqs. (52)–(54) (as noted above, $\cos\psi$ should be an experimental observable). The third column of

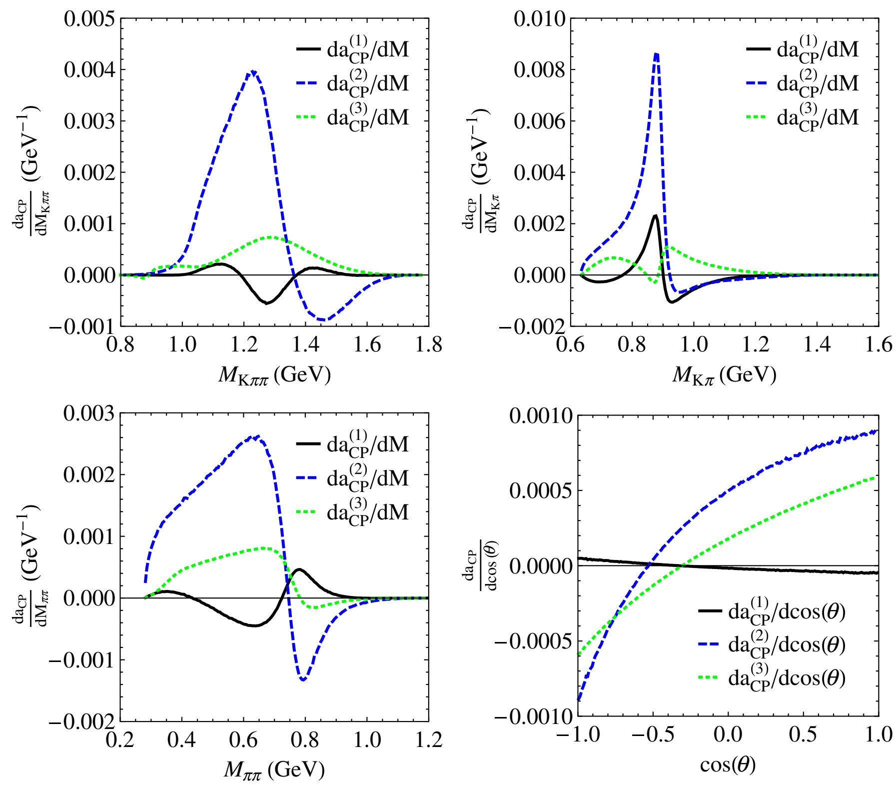

FIG. 3 (color online). Differential asymmetries showing each asymmetry's dependence on the respective integration variables.

Table II shows the resulting values for the asymmetry coefficients. As can be seen, this modification leads to modest increases in the sizes of the coefficients. Other weighting functions could also be considered. If a weighting function is chosen such that it takes on only the values ± 1 over the integration range, the largest possible asymmetry coefficients would be obtained by simply taking the absolute value of the integrand. We have computed the asymmetry coefficients under this assumption as well. The results may be found in the fourth column of Table II. The values in this column represent the maximum values obtainable for the magnitudes of the asymmetry coefficients, given the choices we have made for the form factors. Comparison of the second and fourth columns in the table illustrates the level of cancellation that the integrated asymmetry coefficients have each undergone. A considerable gain in the magnitude of each asymmetry is possible if an appropriate weighting function is adopted.

A few comments are in order. First of all, we note that the values obtained for the asymmetry coefficients, as well as the shapes of the curves in Fig. 3, depend sensitively on the coefficients A-D, N, and N_3. We have chosen particular values for illustration, but it is assumed that experimentalists would perform more accurate measurements of the coefficients A-D in tandem with performing any CP analysis. Also, recall that we have assumed that f_H is a constant and have thus pulled it outside of the various integrations. This may well be a poor approximation, in which case the expressions for the differential CP asymmetries would need to be modified to include the dependence that f_H has on the various variables. Finally, we note that more recent analyses use an expression for F_3 that differs from the expression we use [Eq. (59)]. References [16,24] use an expression that is similar to Eq. (59), except that it sets $\alpha = 1$ and $N_3 = 1$, and that it replaces $BW_{K^*}(Q^2)$ and $BW_{K^*}(s_2)$ by expressions that take into account one or both of the $K^{*\prime}/K^{*\prime\prime}$ resonances. We have performed an analysis using this modified expression for F_3; the change affects the asymmetry $A_{CP}^{(3)}$. Retaining an overall normalization constant and tuning it so that the F_3 contribution still accounts for approximately 5% of the experimental branching ratio ($N_3 \simeq 0.4206$), we find $a_{CP}^{(3)} \simeq 6.8 \times 10^{-5}$ and $a_{CP,\max}^{(3)} \simeq 7.1 \times 10^{-4}$. Since $a_{CP}^{(3)}$ depends linearly on N_3, it is straightforward to scale these numbers, should a different value for N_3 be favored. (Note that we do not quote a revised number for $a_{CP,\mathrm{mod}}^{(3)}$, since the replacement $\cos\psi \to |\cos\psi|$ actually makes the magnitude of the asymmetry smaller in this case.) Comparison with Table II shows that the asymmetries are smaller in magnitude in this case. The differential plots are also affected. We do not consider results following from this revised expression for F_3 further here, but our estimates below could easily be adapted to take this change into account.

To determine actual CP asymmetries ($A_{CP}^{(i)}$) from the asymmetry coefficients ($a_{CP}^{(i)}$), we need to know or be able to estimate the quantity $f_H \,\mathrm{Im}(\eta_P)$ [see Eq. (55)]. Let us begin with a crude estimate by assuming that the NP contribution to the width is "hiding" in the experimental uncertainty of the branching ratio. The experimental branching ratio determined by *BABAR* is $\mathcal{B}(\tau^- \to K^-\pi^-\pi^+\nu_\tau) = (0.273 \pm 0.002 \pm 0.009)\%$ [22]; i.e., the experimental measurement has a relative uncertainty of approximately 3.4%. A numerical integration of Eq. (49), performed under the assumption that only the NP part contributes [i.e., setting $B_1 = B_2 = B_3 = 0$ and $B_4 = B_4^{(2)} = \sqrt{Q^2} f_H \eta_P / m_\tau$—see Eq. (47)], shows that the experimental uncertainty is saturated when $|f_H \eta_P| \simeq 17.9$. Assuming that η_P is purely imaginary yields upper bounds on the magnitudes of the $A_{CP}^{(i)}$ in the range 3.9×10^{-4} to 0.012. Under the same assumptions regarding $f_H \eta_P$, we also find that the $A_{CP,\max}^{(i)}$ range from 0.015 to 0.052.

The above estimates may be a bit optimistic, although it is difficult to say without direct bounds on f_H and η_P. As noted in the Introduction, the CLEO collaboration has searched for CP violation in $\tau \to K\pi\nu_\tau$; they have set the following bound on the scalar coupling that they denote Λ [8],

$$-0.172 < \mathrm{Im}(\Lambda) < 0.067, \quad \text{at 90\% C.L.} \tag{68}$$

The coupling Λ is related to η_S in Eq. (7); η_P, however, does not receive a direct constraint from this experiment. η_P should scale like m_W^2/m_H^2 due to the Higgs propagator (where m_W and m_H are the W and charged Higgs masses, respectively). If the Higgs has electroweak couplings, then it would be reasonable to assume that η_P has a magnitude not exceeding unity. At this point we do not have a reliable way to estimate f_H. One possibility is to infer f_H from F_4 using the quark equations of motion, although this procedure may well have a large error. As was noted above, F_4 for this decay has been computed from the perspective of chiral perturbation theory in Ref. [11]. Using the quark equations of motion, one finds $|f_H| \sim Q^2 |F_4|/m_s$, leading to an enhancement of f_H because of the small strange quark mass. (This enhancement would be lost to some degree if the quark mass were replaced by a meson mass.) An approximate numerical examination of $|F_4|$ derived from Ref. [11][9] shows that it can be of order $1\ \mathrm{GeV}^{-1}$ for some values of Q^2, s_1, and s_2 (it is also much smaller than this for other values of the kinematical variables). A crude estimate of the maximum size of $|f_H|$ would be $|f_H| \sim m_\tau^2 \times (1\ \mathrm{GeV}^{-1})/m_s \sim (1.777\ \mathrm{GeV})^2 \times (1\ \mathrm{GeV}^{-1})/(0.095\ \mathrm{GeV}) \sim 30$. A more realistic estimate for $|f_H|$ might be in the range 1–10. Combining these estimates, we see that $|f_H \,\mathrm{Im}(\eta_P)|$ could be of order 1–10, leading to a reduction of the possible magnitudes of the

[9]We have not updated the expression to account for the possibility of contributions from both $K_1(1270)$ and $K_1(1400)$.

CP asymmetries compared with our estimates above (for which we assumed $|f_H \,\mathrm{Im}(\eta_P)| \simeq 17.9$).

V. DISCUSSION AND CONCLUDING REMARKS

We have analyzed CP violation in $\tau \to K\pi\pi\nu_\tau$ due to NP in the form of a charged Higgs boson. Noting that the couplings of a charged Higgs boson to the light quarks are suppressed in many models due to the smallness of the light quarks' masses, we have observed that CP-odd observables in $\tau \to K\pi\pi\nu_\tau$ probe non-standard NP CP violation. An experimental search for CP violation in $\tau \to K\pi\pi\nu_\tau$ would complement the search for CP violation that has already taken place in $\tau \to K\pi\nu_\tau$ [8]. In our notation, $\tau \to K\pi\pi\nu_\tau$ is sensitive to the coupling η_P, while $\tau \to K\pi\nu_\tau$ is sensitive to η_S.

We have analyzed four CP-odd observables in $\tau \to K\pi\pi\nu_\tau$—the rate asymmetry, two modified rate asymmetries and a triple-product asymmetry. The rate asymmetry is likely to be quite small because it relies on the interference of the SM scalar form factor with the NP contribution; thus, we did not make any numerical estimates for this asymmetry. The modified rate asymmetries and the triple-product asymmetry result from the interference of the NP amplitude with the SM contributions containing the form factors $F_1 - F_3$. Adopting a particular model for the form factors and making various assumptions, we have estimated the possible sizes of the CP asymmetries numerically. In our calculation it was found that each of the asymmetries underwent a substantial cancellation upon integration over the various phase space variables. Experimentalists may wish to consider differential CP asymmetries in order to avoid some of this cancellation. The maximal sizes of the three asymmetries (assuming that the cancellations could be avoided by using appropriately chosen weighting functions) were found to be in the range 0.015 to 0.052. These numbers were derived under the assumption that the only constraint on the NP contribution is that it is "hidden" in the uncertainty of the branching ratio for $\tau \to K\pi\pi\nu_\tau$. The maximal magnitudes of the asymmetries decrease if one makes more realistic assumptions regarding the hadronic form factor f_H and the NP parameter η_P.

We encourage experimentalists at the B factories to analyze their τ data sets in the manner that we have described. Future experiments, such as the Super B factories, could provide even greater sensitivity to these observables.

We close with a short comment regarding CP violation in $\tau^\mp \to K^\mp\pi^\mp K^\pm\nu_\tau$. In principle, this decay mode could be analyzed in a similar manner to what we have described. (See Ref. [4], for example.) One advantage of $\tau \to K\pi K\nu_\tau$ is that there is an intermediate pseudoscalar resonance [the $\pi'(1300)$] that could potentially enhance the hadronic current associated with the NP charged scalar exchange. We wish to point out what appears to be an error, or an oversimplification, in the literature regarding this point. The scalar form factors associated with the π' resonance in the $\tau \to 3\pi\nu_\tau$ and $\tau \to K\pi K\nu_\tau$ decays have been written down in Ref. [10]. The expression for the 3π case seems to be sensible, but the one for the $K\pi K$ case appears to make an unphysical assumption regarding the contributing decay chains. In particular, judging from the expression, one of the decay chains would seem to have an intermediate ρ decaying to a K and a π. If this is remedied by replacing the ρ by a K^*, one finds that none of the decay chains can quite proceed on shell (although there is a large uncertainty in the π' mass; furthermore, the π' does have a large width and the decay $\pi' \to K^*K$ is actually right near threshold).

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