PHYSICAL REVIEW D **78,** 111103(R) (2008)

Determination of the neutrino mass hierarchy at an intermediate baseline

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(Received 21 July 2008; revised manuscript received 24 October 2008; published 10 December 2008)

It is generally believed that neutrino mass hierarchy can be determined at a long baseline experiment, often using accelerator neutrino beams. Reactor neutrino experiments at an intermediate baseline have the capability to distinguish normal or inverted hierarchy. Recently, it has been demonstrated that the mass hierarchy could possibly be identified using Fourier transform to the L/E spectrum if the mixing angle $\sin^2(2\theta_{13}) > 0.02$. In this study, a more sensitive Fourier analysis is introduced. We found that an ideal detector at an intermediate baseline (~ 60 km) could identify the mass hierarchy for a mixing angle $\sin^2(2\theta_{13}) > 0.005$, without requirements on accurate information of reactor neutrino spectra and the value of Δm^2_{32}.

DOI: 10.1103/PhysRevD.78.111103 PACS numbers: 13.15.+g, 14.60.Lm, 14.60.Pq

Recent results from solar, atmospheric, reactor, and accelerator neutrino experiments all show that neutrinos are massive and can oscillate from one type to another. Among all the six mixing parameters, three of them are known, two unknown, and one of them, the mass-squared difference Δm^2_{32}, defined as $m^2_3 - m^2_2$, is only known to be $|\Delta m^2_{32}| = (2.43 \pm 0.13) \times 10^{-3}$ eV^2 (68% C.L.) from accelerator neutrino experiments [1]. The question, if the mass hierarchy is normal ($\Delta m^2_{32} > 0$) or inverted ($\Delta m^2_{32} < 0$), is not known now but is fundamental to particle physics.

For normal hierarchy (NH) or inverted hierarchy (IH), the neutrino mass-squared difference has the following relations:

$$\begin{aligned} \Delta m^2_{31} &= \Delta m^2_{32} + \Delta m^2_{21} \\ \text{NH: } |\Delta m^2_{31}| &= |\Delta m^2_{32}| + |\Delta m^2_{21}| \\ \text{IH: } |\Delta m^2_{31}| &= |\Delta m^2_{32}| - |\Delta m^2_{21}| \end{aligned} \quad (1)$$

In principle, the mass hierarchy can be determined by precision measurements of $|\Delta m^2_{31}|$ and $|\Delta m^2_{32}|$. In fact, it is extremely difficult since Δm^2_{21} is only $\sim$3% of $|\Delta m^2_{32}|$, hence $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$ have to be measured with a precision much better than 3%.

Effects of mass hierarchy can be amplified by matter effects if the baseline is large enough, say several hundreds to thousands of kilometers. Such experiments often need accelerator-based neutrino beams and huge detectors. Proposals such as T2K [2,3], Nova [3–5], and T2KK [6] have mass hierarchy sensitivity in the $\nu_\mu \to \nu_e$ channel if θ_{13} is large enough (i.e. $\sin^2(2\theta_{13}) \geq 0.03$). In addition, they are affected by the (δ_{CP}, sign(Δm^2_{32})) degeneracy [7,8]. At a magic baseline [9,10], $L \sim 7000$ km, the degeneracy can be canceled, but it requires a very intensive source such as a neutrino factory or a beta beam, which will not be available in the near future. A method using atmospheric neutrinos [11,12] with a baseline of $L \sim 10^4$ km and the neutrino energy of $E \sim 1$ GeV is sensitive to mass hierarchy for very small or even the null value of θ_{13}, if the measurement precision of $|\Delta m^2_{32}|$ is better than 2%.

A method using reactor neutrino-based intermediate baseline (40–65 km) experiments has been explored based on precision measurement of distortions of the energy spectrum due to nonzero θ_{13} [13,14]. Recently, a study [15] shows a new method to distinguish normal or inverted hierarchy after a Fourier transform of the L/E spectrum of reactor neutrinos. It is observed that the Fourier power spectrum has a small shoulder next to the main peak, and their relative position can be used to determine the mass hierarchy. A filter method is used to improve the sensitivity to the mass hierarchy up to $\sin^2(2\theta_{13}) > 0.02$, if Δm^2_{32} is known *a priori*. Comparing to a normal L/E analysis, the Fourier analysis naturally separates the mass hierarchy information from uncertainties of the reactor neutrino spectra and other mixing parameters, which is critical for very small $\sin^2(2\theta_{13})$ oscillations.

In this paper, we report that if a proper Fourier transform is applied and if all information is fully utilized, the capability of an intermediate baseline reactor experiment to determine the neutrino mass hierarchy can be improved for a smaller mixing angle θ_{13} without knowing Δm^2_{32} *a priori*. In the following, we will use a reactor neutrino spectrum to illustrate the method, but such a method can be generalized to other experiments.

For a reactor neutrino experiment, the observed neutrino spectrum at a baseline L, $F(L/E)$, can be written as

$$F(L/E) = \phi(E)\sigma(E)P_{ee}(L/E),$$

where E is the electron antineutrino ($\bar{\nu}_e$) energy, $\phi(E)$ is the flux of $\bar{\nu}_e$ from the reactor, $\sigma(E)$ is the interaction cross section of $\bar{\nu}_e$ with matter, and $P_{ee}(L/E)$ is the $\bar{\nu}_e$ survival probability.

The $\bar{\nu}_e$ flux $\phi(E)$ from the reactor can be parameterized as [16],

$$\begin{aligned}\phi(E) = {} & 0.58\exp(0.870 - 0.160E - 0.091E^2) \\ & + 0.30\exp(0.896 - 0.239E - 0.0981E^2) \\ & + 0.07\exp(0.976 - 0.162E - 0.0790E^2) \\ & + 0.05\exp(0.793 - 0.080E - 0.1085E^2), \end{aligned} \quad (2)$$

where four exponential terms are contributions from isotopes ^{235}U, ^{239}Pu, ^{238}U, and ^{241}Pu in the reactor fuel, respectively.

The leading-order expression for the cross section [17] of inverse-β decay ($\bar{\nu}_e + p \to e^+ + n$) is

$$\sigma^{(0)} = 0.0952 \times 10^{-42}\ \mathrm{cm}^2 (E_e^{(0)} p_e^{(0)}/1\ \mathrm{MeV}^2), \quad (3)$$

where $E_e^{(0)} = E_\nu - (M_n - M_p)$ is the positron energy when neutron recoil energy is neglected, and $p_e^{(0)}$ is the positron momentum. The survival probability of $\bar{\nu}_e$ can be expressed as [18]

$$\begin{aligned} P_{ee}(L/E) &= 1 - P_{21} - P_{31} - P_{32} \\ P_{21} &= \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2(\Delta_{21}) \\ P_{31} &= \cos^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{31}) \\ P_{32} &= \sin^2(\theta_{12})\sin^2(2\theta_{13})\sin^2(\Delta_{32}), \end{aligned} \quad (4)$$

where $\Delta_{ij} = 1.27\Delta m_{ij}^2 L/E$, Δm_{ij}^2 is the neutrino mass-squared difference ($m_i^2 - m_j^2$) in eV^2, θ_{ij} is the neutrino mixing angle, L is the baseline from reactor to $\bar{\nu}_e$ detector in meters, and E is the $\bar{\nu}_e$ energy in MeV.

$P_{ee}(L/E)$ has three oscillation components, P_{21}, P_{31}, and P_{32}, corresponding to three oscillation frequencies in L/E space, which are proportional to $|\Delta m_{ij}^2|$, respectively. Their relative amplitude (oscillation intensity), is about 40:2:1 from a global fit [19] of mixing parameters as listed in Table I. The oscillation component $1 - P_{21}$ dominates the P_{ee} oscillation, while P_{31} and P_{32}, which are sensitive to the neutrino mass hierarchy, are suppressed by the small value of $\sin^2(2\theta_{13})$.

The observed neutrino spectrum in L/E space, taking the baseline L to be 60 km and all the other parameters from Table I except $\sin^2(2\theta_{13})$, is shown in Fig. 1, together with that of no oscillation. For comparison, the oscillation spectrum without P_{31} and P_{32} are also shown. For a very small $\sin^2(2\theta_{13})$, a normal χ^2 analysis on the L/E spectrum with binned data, which requires accurate knowledge on the neutrino energy spectra and much smaller binning than the energy resolution, is difficult for the mass hierarchy study.

TABLE I. Neutrino mixing parameters from a global fit, updated in 2007, as the inputs to this study.

pParameter	Best fit	2σ	3σ
$\Delta m_{21}^2[10^{-5}\ \mathrm{eV}^2]$	7.6	7.3–8.1	7.1–8.3
$\lvert\Delta m_{32}^2\rvert[10^{-3}\ \mathrm{eV}^2]$	2.4	2.1–2.7	2.0–2.8
$\sin^2\theta_{12}$	0.32	0.28–0.37	0.26–0.40
$\sin^2\theta_{23}$	0.50	0.38–0.63	0.34–0.67
$\sin^2\theta_{13}$	0.007	≤ 0.033	≤ 0.050

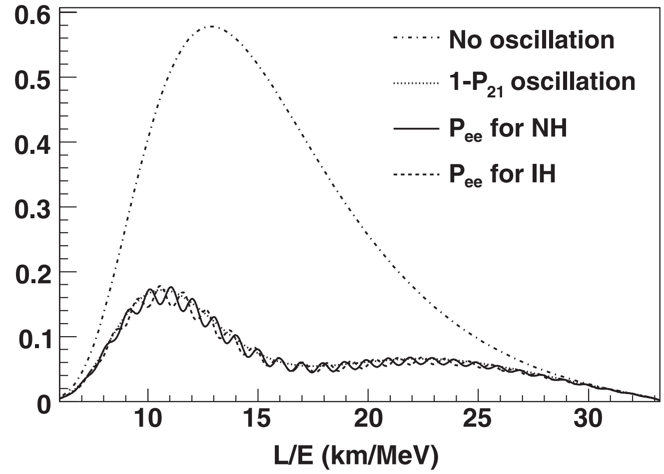

FIG. 1. Reactor neutrino spectra at a baseline of 60 km in L/E space for no oscillation (dashed-dotted line), $1 - P_{21}$ oscillation (dotted line) and P_{ee} oscillation in the cases of NH and IH, assuming $\sin^2(2\theta_{13}) = 0.1$.

Since neutrino masses all appear in the frequency domain as shown in Eq. (4), a Fourier transform of $F(L/E)$ shall enhance the sensitivity to the mass hierarchy. The frequency spectrum can be obtained by the following Fourier sine transform (FST) and Fourier cosine transform (FCT):

$$\begin{aligned} \mathrm{FST}(\omega) &= \int_{t_{\min}}^{t_{\max}} F(t)\sin(\omega t)dt \\ \mathrm{FCT}(\omega) &= \int_{t_{\min}}^{t_{\max}} F(t)\cos(\omega t)dt, \end{aligned} \quad (5)$$

where ω is the frequency, $\omega = 2.54\Delta m_{ij}^2$; $t = \frac{L}{E}$ is the variable in L/E space, varying from $t_{\min} = \frac{L}{E_{\max}}$ to $t_{\max} = \frac{L}{E_{\min}}$.

Since P_{ee} is a linear combination of $1 - P_{21}$, P_{31} and P_{32}, FST and FCT spectra can be divided into three components corresponding to $1 - P_{21}$, P_{31}, and P_{32}, respectively. Figure 2 shows the three components of the FST and FCT spectra together with full P_{ee} oscillation for both the NH and IH cases. The oscillation frequency is proportional to Δm_{ij}^2, so we can scale the frequency to be δm^2 and plot the spectra in axis of δm^2 in the interested frequency range of $1.8 \times 10^{-3}\ \mathrm{eV}^2 < \delta m^2 < 3.0 \times 10^{-3}\ \mathrm{eV}^2$. From Fig. 2, we know that

(1) P_{31} and P_{32} components dominate the FCT and FST spectra in the interested frequency range of $1.8 \times 10^{-3}\ \mathrm{eV}^2 < \delta m^2 < 3.0 \times 10^{-3}\ \mathrm{eV}^2$, since $|\Delta m_{31}^2|$ and $|\Delta m_{32}^2|$ are in this range, while $1 - P_{21}$ is very weak, since its oscillation frequency is in a much lower range. The FST and FCT spectra of P_{ee} are

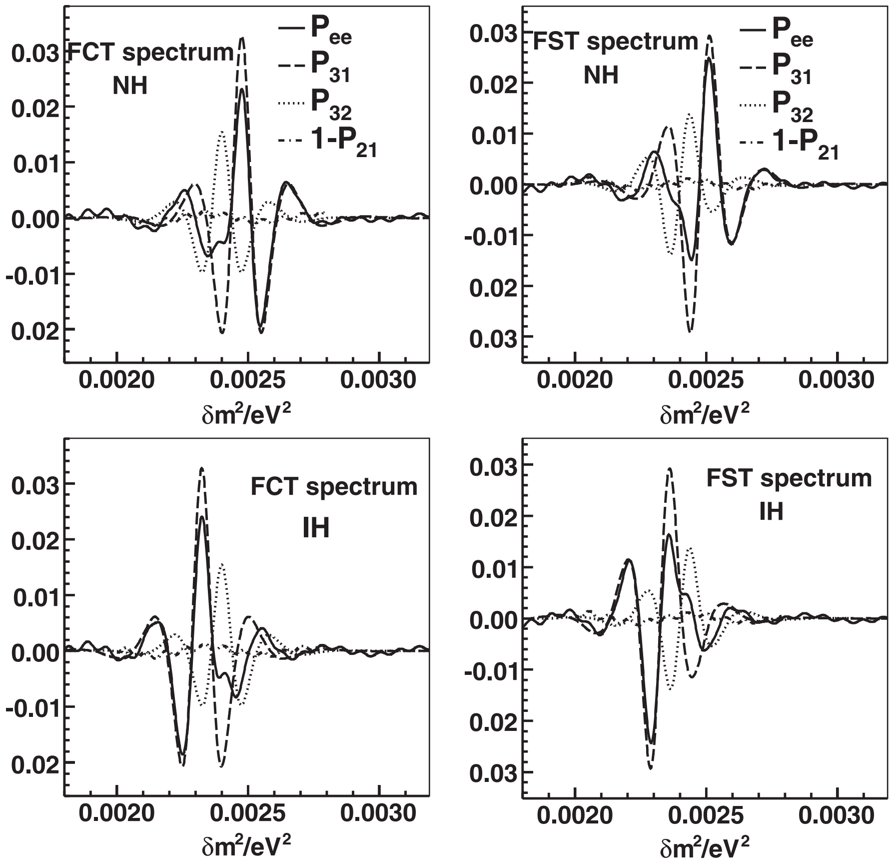

FIG. 2. FST and FCT transform spectra for $1 - P_{21}$ component (dotted line), P_{32} component (dashed line), P_{31} component (dotted-dashed line) and all the components of P_{ee} (solid line) in the cases of NH and IH.

approximately the sum of P_{31} and P_{32} components, which are sensitive to mass hierarchy.

(2) For NH, the P_{32} FCT and FST spectra are left shifted with respect to the P_{31} spectra because $|\Delta m^2_{32}| < |\Delta m^2_{31}|$; while for IN, the P_{32} spectra are right shifted because $|\Delta m^2_{32}| > |\Delta m^2_{31}|$.

(3) The peak of FCT spectrum corresponds to the zero point of FST spectrum. This feature is helpful to identify the position of $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$, without knowing their accurate values *a priori*.

(4) For FCT spectrum, P_{32} and P_{31} components have similar shapes with the peak around $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$, respectively, and two valleys on each side of the peak. The amplitude of P_{32} to that of P_{31} has a ratio of about 1:2 determined by $\tan^2(\theta_{12})$. The shapes of P_{32} and P_{31} are left-right symmetric with respect to their peaks (mirror symmetric). This symmetry is broken for P_{ee} as an approximate sum of P_{32} and P_{31} in different ways for NH and IH. For NH, the peak of P_{32} is at the left of the valley of P_{31}, while for IH, the peak of P_{32} is at the right of the valley of P_{31}. This feature can be used to distinguish NH and IH.

(5) For FST spectrum, the shapes of P_{32} and P_{31} are positive-negative symmetric with respect to zero (rotation symmetric) around $|\Delta m^2_{32}|$ and $|\Delta m^2_{31}|$, respectively. This symmetry is broken for P_{ee} in different ways for NH and IH. For NH, the peak of P_{32} is at the valley position of P_{31}, while for IH, the valley of P_{32} is at the peak position of P_{31}. This feature can be also used to distinguish NH and IH.

As discussed above and shown in Fig. 2, the normal or inverted mass hierarchy can be distinguished by the symmetry breaking features of the FCT and FST spectra. To quantify these features, two parameters, RL and PV, are introduced as the following:

$$RL = \frac{RV - LV}{RV + LV}, \qquad PV = \frac{P - V}{P + V}, \tag{6}$$

where RV is the amplitude of the right valley and LV is the amplitude of the left valley in the FCT spectrum. P is the amplitude of the peak and V is the amplitude of the valley in the FST spectrum. From the above discussion, we know that

$$\begin{aligned} RL > 0 \quad \text{and} \quad PV > 0 \Rightarrow \text{NH} \\ RL < 0 \quad \text{and} \quad PV < 0 \Rightarrow \text{IH}. \end{aligned} \tag{7}$$

The values of RL and PV as well as the shapes of FCT and FST spectra depend on the baseline and neutrino mixing parameters. Parameters such as $\sin^2\theta_{12}$, Δm^2_{21}, and Δm^2_{32} are relatively well known; hence, only small uncertainties are introduced. The baseline and $\sin^2(2\theta_{13})$ are more important and are discussed below.

(1) Baseline determines the oscillation cycles. To maximize the symmetry breaking of FCT and FST spectra, we scan the baseline length and find that the peak (valley)of P_{32} spectrum lays on the valley (peak) of P_{31} spectrum around 60 km. The widths of peaks and valleys of the Fourier spectra, which are proportional to $1/L$, are also determined by baseline. In an extreme case, the peaks and valleys of P_{31} and P_{32} spectra all become δ functions at infinite baseline, and hence are well separated from each other. In fact, this is already the case at 200 km and the mass hierarchy can be determined by looking at the position of the smaller peak (P_{32} component). If it is on the left side of the main peak (P_{31} component), it is NH. Otherwise, it is IH. However, since the neutrino flux from reactors is proportional to $1/L^2$, shorter baseline, say at 60 km, is the best from an experimental point of view. The actual optimum baseline can be determined by taking into account both statistical and systematical errors.

(2) $\sin^2(2\theta_{13})$ determines the amplitude of the Fourier spectra of P_{31} and P_{32}. At $\sin^2(2\theta_{13}) = 0$, P_{31}, and P_{32} components will vanish, and no features can be used to discriminate the mass hierarchy. A minimum value of $\sin^2(2\theta_{13})$ to distinguish NH and IH experimentally will be analyzed by taking into account possible experimental errors [20].

In order to understand the robustness of the discrimination method using FCT and FST spectra, values of baseline are scanned from 46 to 72 km; $\sin^2(2\theta_{13})$ from 0.005 to 0.05. The resultant RL and PV values are well separated

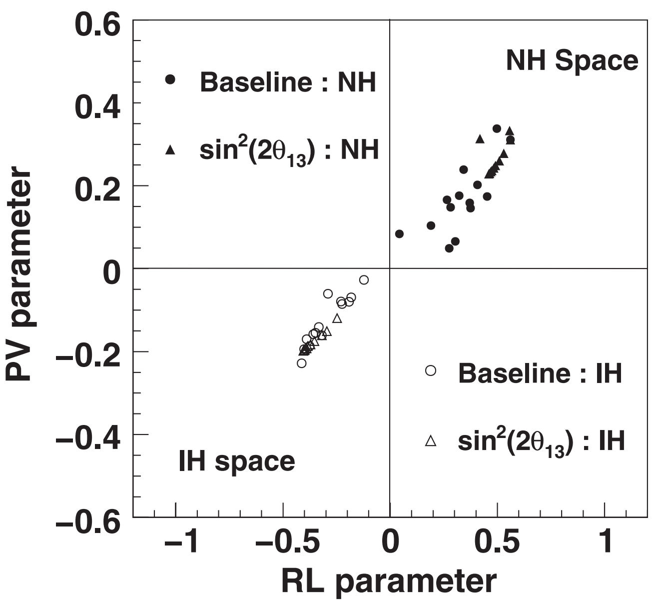

FIG. 3. Distribution of RL and PV values for different parameters of baseline and $\sin^2(2\theta_{13})$. For each parameter to be scanned, the default baseline is 60 km and all the other parameters are the values as in Table I. Two clusters of RL and PV values are clearly seen in the NH and IH cases.

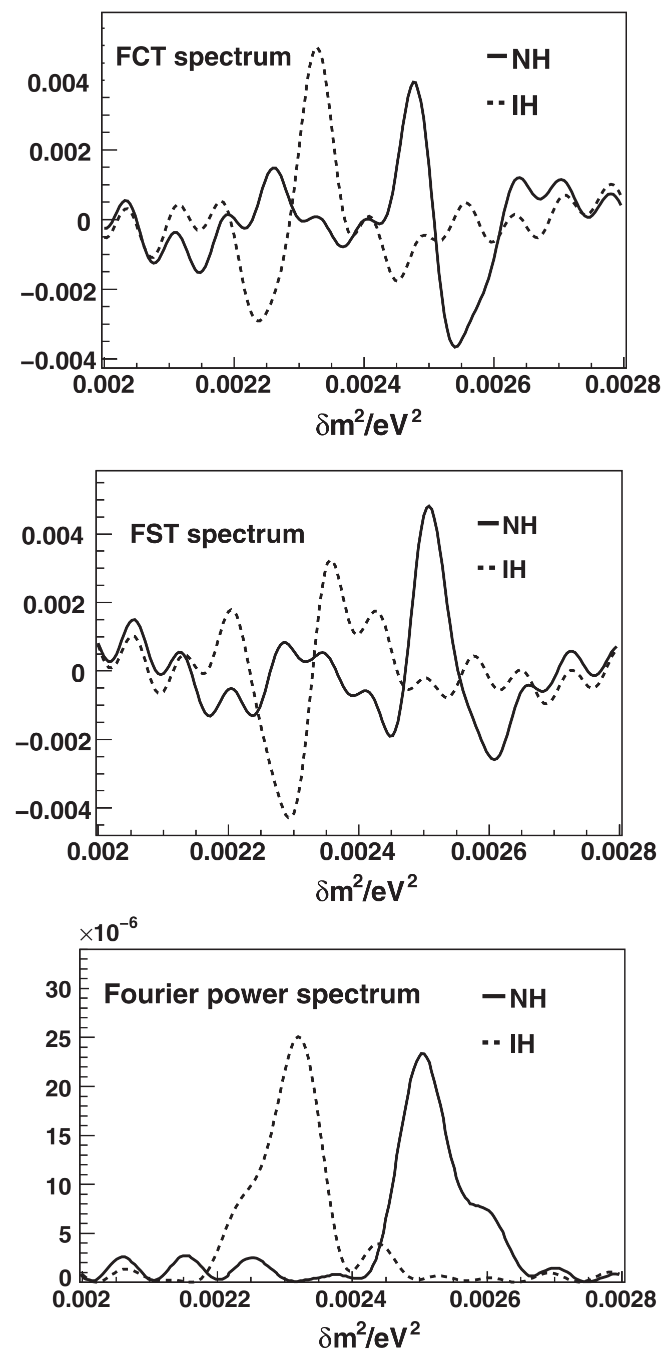

FIG. 4. The FCT and FST spectra and Fourier power spectrum for $\sin^2(2\theta_{13}) = 0.005$. The solid line is for NH and the dashed line is for IH. The FCT and FST spectra have distinctive features to identify the mass hierarchy, which looks more sensitive than the Fourier power spectrum method.

into two clusters, corresponding to the case of NH and IH, respectively, as shown in Fig. 3.

The FCT and FST spectra for $\sin^2(2\theta_{13}) = 0.005$ are shown in Fig. 4. Although a detailed experimental analysis of error contour is to be completed [20], the features of NH and IH are still very distinctive. On the FCT spectrum, a valley appears at the left of the prominent peak for IH, and a peak appears at the left of the valley for NH. On the FST spectrum, there is a clear valley for IH, while for NH it is a peak. In comparison, the Fourier power spectrum used in Ref. [15] is also shown in Fig. 4. The FCT and FST method is more sensitive than the Fourier power spectrum method for a very small $\sin^2(2\theta_{13})$.

For even smaller $\sin^2(2\theta_{13})$, the main peak becomes less significant. For example, if the main peak is required to be twice higher than that of noise, $\sin^2(2\theta_{13})$ must be greater than 0.005 in order to clearly identify the main peak, for a variety of neutrino energy spectra in a reasonable range.

For a realistic experiment in the near future, the energy resolution and statistics are of the most concern. At 60 km, θ_{12} has the least impact to the mass hierarchy determination. The energy resolution must be good enough not to smear the difference between P_{31} and P_{32}, which requires the energy resolution be better than $3\%/\sqrt{E}$. A detector with a mass at 10 kton level may be necessary, depending on the size of θ_{13}. If shortening the baseline, the noise in the Fourier spectra from θ_{12} oscillation increases, thus degrade the sensitivity. In the mean time, requirements to the energy resolution and the detector size are relaxed. The optimization of the baseline as well as the energy resolution and detector size for different θ_{13} assumptions are undergoing.

In summary, the method to discriminate the mass hierarchy has been studied by using a FST and FCT transform to the observed reactor neutrino L/E spectra. The FCT and FST spectra can separate P_{31} and P_{32} oscillation components from the large $1 - P_{21}$ component in a specific δm^2 range. Features of mass hierarchy are enhanced in this representation and more sensitive than that of the Fourier power spectrum at very small $\sin^2(2\theta_{13})$. We found that an ideal detector at an intermediate baseline (~ 60 km) could identify the mass hierarchy for a mixing angle $\sin^2(2\theta_{13}) >$ 0.005, without requirements on accurate information of reactor neutrino spectra and the value of Δm^2_{32}. A paper of a detailed analysis of experimental errors will be released soon [20]. Similar methods can be applied to other experiments using different neutrino sources, such as accelerator-based neutrino beams or atmospheric neutrinos.

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