Stable de Sitter vacua in four-dimensional supergravity originating from five dimensions

O. Ögetbil*

Physics Department, Pennsylvania State University, University Park, Pennsylvania 16802, USA
(Received 9 September 2008; published 4 November 2008)

The five-dimensional stable de Sitter ground states in $\mathcal{N} = 2$ supergravity obtained by gauging $SO(1, 1)$ symmetry of the real symmetric scalar manifold (in particular, a generic Jordan family manifold of the vector multiplets) simultaneously with a subgroup R_s of the R-symmetry group descend to four-dimensional de Sitter ground states under certain conditions. First, the holomorphic section in four dimensions has to be chosen carefully by using the symplectic freedom in four dimensions; second, a group contraction is necessary to bring the potential into a desired form. Under these conditions, stable de Sitter vacua can be obtained in dimensionally reduced theories (from 5D to 4D) if the semidirect product of $SO(1, 1)$ with $\mathbb{R}^{(1,1)}$ together with a simultaneous R_s is gauged. We review the stable de Sitter vacua in four dimensions found in earlier literature for $\mathcal{N} = 2$ Yang-Mills Einstein supergravity with the $SO(2, 1) \times R_s$ gauge group in a symplectic basis that comes naturally after dimensional reduction. Although this particular gauge group does not descend directly from five dimensions, we show that its contraction does. Hence, two different theories overlap in certain limits. Examples of stable de Sitter vacua are given for the cases: (i) $R_s = U(1)_R$, (ii) $R_s = SU(2)_R$, and (iii) $\mathcal{N} = 2$ Yang-Mills/Einstein supergravity theory coupled to a universal hypermultiplet. We conclude with a discussion regarding the extension of our results to supergravity theories with more general homogeneous scalar manifolds.

DOI: 10.1103/PhysRevD.78.105001 PACS numbers: 04.65.+e

I. INTRODUCTION

Supergravity theories are local gauge theories of supersymmetry and were first formulated in 1970s [1–3].[1] There are two ways of studying supergravity in a certain dimension. Either one can construct it directly from field content and symmetries (both local and global) that the action must have, or one can obtain them from higher dimensions by dimensional reduction. Supergravity theories that are obtained purely by dimensional reduction from 10- or 11-dimensional supergravity are low energy effective limits of some superstring theory/M theory. In such cases, their scalar manifold is the moduli space of the compactification. For certain extended supergravity theories, gauging a symmetry of the action may yield a potential term $V(\phi)$ of scalar fields. The ground states of the resulting theory are determined by the critical points (say, ϕ_0) of the potential term.

Scalar fields play a fundamental role in the description of cosmological models. In fact, the assumption that the energy-momentum tensor is dominated by scalar potential energy density $V(\phi)$ has been the starting point of many inflationary models[2] [6,7]. If the value of the potential at its critical point is positive $[V'|_{\phi_0} = 0, V(\phi_0) > 0]$,[3] the case with zero kinetic energy ($\dot{\phi} = 0$) corresponds to de Sitter space with a positive cosmological constant. The current accelerated expansion of the Universe [8,9] can be explained by either a positive vacuum energy $V(\phi_0)$ or a scalar field in a slow-roll regime $\dot{\phi}^2/2 \ll V(\phi)$ on a near de Sitter background (quintessence) [10–12].

There are two possible ways of explaining the positive vacuum energy in terms of scalar potentials $V(\phi)$. The observed cosmological constant may correspond to the minimum of a scalar potential, in which case the Universe will continue to accelerate forever. However, the de Sitter regime might be transient; i.e. it might correspond to a local maximum or a saddle point of the scalar potential. Models with slow-roll inflation ($|V''| \ll |V|$) and fast-roll inflation ($|V''| \sim |V|$) have been considered in [13]. In such cases either the scalar potential vanishes as the field rolls to $\phi \rightarrow \infty$ and the Universe reaches a Minkowski stage, or the scalar field rolls to the minimum of the potential with $V(\phi) < 0$ [or $V(\phi) \rightarrow -\infty$, such that the potential does not have a minimum at all] and the Universe may eventually collapse.

The evidence of a small positive cosmological constant attracted interest in finding stable de Sitter ground state solutions in supersymmetric theories. In the context of supersymmetric theories, anti-de Sitter (AdS) ground states emerge naturally in contrast to de Sitter ground states. This is due to the fact that the de Sitter superalgebras usually have noncompact R-symmetry subalgebras, which leads to the existence of ghosts if the supersymmetry is to be fully preserved. Nevertheless, exact supersymmetry is not observed in nature, and supersymmetry must be a broken symmetry. There are two main approaches to study de Sitter ground state solutions of supersymmetric theories.

*oogetbil@phys.psu.edu

[1]For a review about gauged supergravity theories of various dimensions that have been studied extensively since then, see [4].

[2]For a general review and further references on inflationary cosmology, see [5].

[3]$V' \equiv \partial V / \partial \phi$.

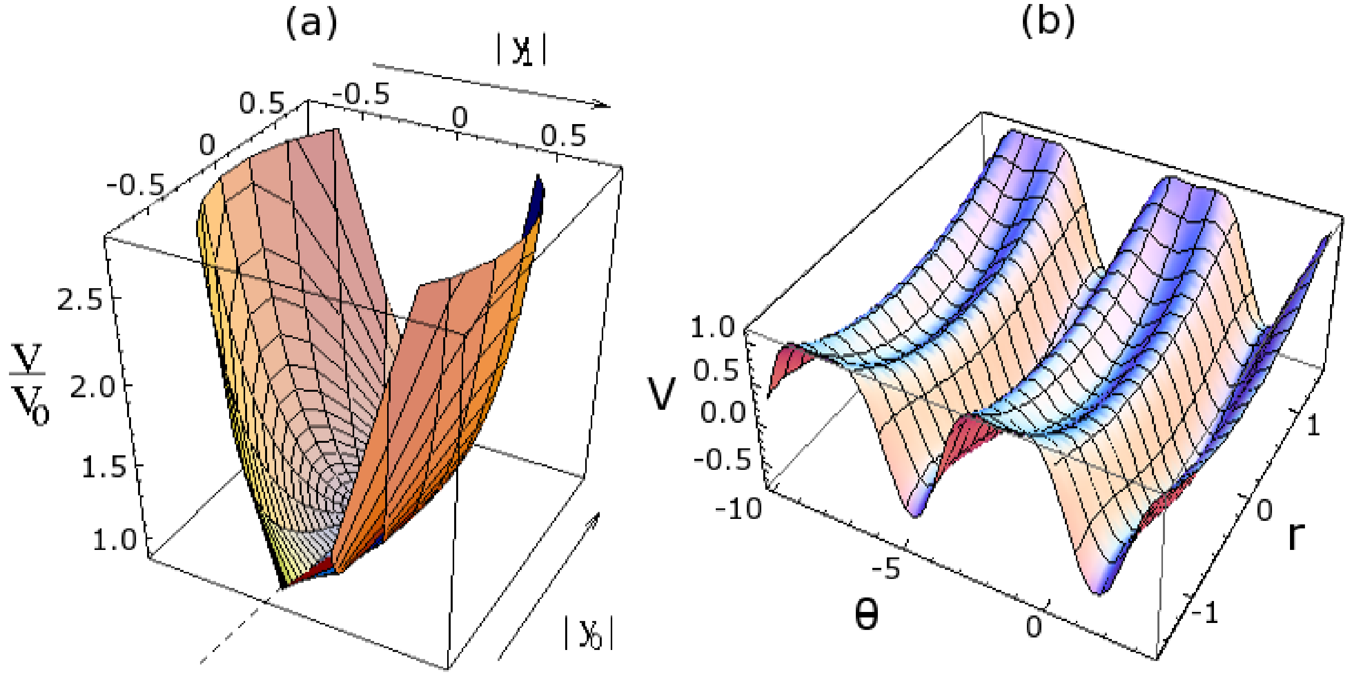

FIG. 1 (color online). Examples of de Sitter extrema in supergravity theories. (a) Stable minima with a flat direction. The potential belongs to the 4D, $\mathcal{N} = 2$ supergravity coupled to 3 vector multiplets, considered in [25]. This figure is taken from [26]. (b) Saddle point, where the scalar rolls into a Minkowski minimum on one side and anti-de Sitter minimum on the other. This is 4D, $\mathcal{N} = 2$ supergravity coupled to 1 hypermultiplet, considered in [27]. The potential includes instanton corrections.

One can start from a fundamental theory (a superstring or M theory), study compactifications on various internal manifolds, and, with the combined effects of the warped geometries of the internal manifold and tree-level corrections to the 4D Kähler potential, obtain a potential in four dimensions that admits de Sitter critical points [14–24]. On the other hand, one can search for such potentials in the extended gauged supergravity theories directly [13,25–34]. Figure 1 shows two examples obtained from 4D, $\mathcal{N} = 2$ supergravity theories [25–27]. A novel result of these studies is that the mass squared of the scalar fields is on the order of the cosmological constant, i.e. the value of the scalar potential at its extremum,

$$m_\phi^2 \sim \Lambda. \tag{1.1}$$

Any quantum corrections to the scalar masses will be related to the cosmological constant $\Lambda = 3H_0^2 \sim 10^{-120} M_{\rm Planck}^4$ and will be very small [13].

In this paper, we will take the second approach and start with studying five-dimensional gauged supergravity theories [35–38] that received renewed attention more recently due their role within the AdS/CFT correspondences in string theory [39–42], the Randall-Sundrum braneworld scenario [43–45], and M/superstring theory compactifications on Calabi-Yau manifolds with fluxes [46–49]. It is believed that the 5D, $\mathcal{N} = 8$ gauged supergravity [36–38] is a consistent nonlinear truncation of the lowest lying Kaluza-Klein modes of type IIB supergravity on $AdS_5 \times S^5$ [50–54]. Moreover, certain braneworld scenarios based on M theory compactifications have 5D, $\mathcal{N} = 2$ gauged supergravity as their effective field theories [55–59].

We adopt the convention introduced in [60] to classify the gaugings of $\mathcal{N} = 2$ supergravity theories in five and four dimensions. The ungauged $\mathcal{N} = 2$ supergravity coupled to vector and/or hypermultiplets is referred to as (ungauged) Maxwell-Einstein supergravity theories (MESGT). In the absence of hypermultiplets, these theories have a global symmetry group of the form $G \times SU(2)_R$[4] in five dimensions, where G is generally the isometry group of the scalar manifold of the vector multiplets,[5] and $SU(2)_R$ is the automorphism group of the underlying supersymmetry algebra, which is also commonly referred as the "R-symmetry group." Theories obtained by gauging a $U(1)_R$ subgroup of $SU(2)_R$ by coupling a linear combination of vector fields to the fermions [35], which are the only fields that transform nontrivially under $SU(2)_R$, are called gauged Maxwell-Einstein supergravity theories (gauged MESGT). On the other hand, if only a subgroup K of the symmetry group G of the action is being

[4]The global symmetry group is $G \times SU(2,1)$ if a universal hypermultiplet is coupled to the theory where $SU(2,1)$ is the isometry group of the hyperscalar manifold. Note that $SU(2)_R \subset SU(2,1)$.

[5]For the generic non-Jordan family, which will be defined in the next section, a parabolic subgroup of G is the symmetry of the whole Lagrangian [61].

gauged, the theory is referred to as a Yang-Mills/Einstein supergravity theory (YMESGT). Note that the theories which include tensor fields fall into this category. A theory with a gauge group $K \times U(1)_R$ is called a gauged Yang-Mills/Einstein supergravity theory (gauged YMESGT).

Pure 5D, $\mathcal{N} = 2$ supergravity was constructed in [62,63], coupling to vector multiplets was done in [35,64], and tensor fields were added to the theory in [60]. Coupling of hypers to these theories was done in [65]. Vacua of $U(1)_R$ gauged 5D, $\mathcal{N} = 2$ MESGTs and YMESGTs without hypers and tensors were studied in [35]. Vacua of the generic Jordan family models, which will be defined in the next section, with Abelian gaugings and tensors have been investigated in [66], the full R-symmetry group gauging was done in [67], and a study for vacua of some other gauged theories were carried out in [68]. We will give two examples from the literature [66,68,69] of the stable de Sitter vacua of 5D, $\mathcal{N} = 2$ supergravity theories coupled to vector, tensor multiplets and a universal hypermultiplet. Then, following the dimensional reduction process of [64,70], we will look for de Sitter ground states in four dimensions. The analysis in 5D is somewhat easier than in 4D, mainly because in 4D, the U duality is an on-shell symmetry, whereas in 5D it is a symmetry of the Lagrangian. Moreover, 5D theories have real geometry, while the geometry in 4D is complex. Therefore, whereas our study in 5D in an earlier work [69] covered all possible ground states, in 4D, motivated by experimental observations, we will concentrate only on de Sitter solutions.

The organization of this paper is as follows. In Sec. II, we start with reviewing the field content of the 5D, $\mathcal{N} = 2$ supergravity. The potential terms arising from noncompact $SO(1,1)$ real scalar manifold isometry gauging and a subgroup R_s of the R-symmtery group $SU(2)_R$ will be given. It will turn out that an $SO(1,1) \times R_s$ gauged YMESGT has stable de Sitter ground states in five dimensions. Section III takes the story down to four dimensions. The symplectic freedom related to the de Roo-Wagemans rotations will be used to find de Sitter ground states. In fact, the stable five-dimensional de Sitter ground states we will demonstrate in Sec. II and those found in [25,26] coincide in certain limits. This relation is revealed by introducing contractions on the gauge groups. Most of the calculations of this section use the symmetric generic Jordan family as the scalar manifold, although in the last subsection we discuss extending our results to the more general homogeneous scalar manifolds. Section IV collects the summary of all of our results and proposes future directions. In Appendix A, one can find the bosonic part of the four- and five-dimensional Lagrangians, the elements of very special geometry, and the derivation of the potential terms from more fundamental quantities. In Appendix B, we list the Killing vectors and their corresponding prepotentials of the hyperscalar manifold isometries that will be used to carry out the hypergaugings throughout the paper. Appendix C gives the quadratic coordinate transformations between the parametrization we use in the paper and Calabi-Vesentini coordinates that were used in [25,26]. Certain scalar potential terms are given in Appendix D in their full form due to their lengthiness. They will be referred within the text in Sec. III. The contents of this paper constitute part of the author's Ph.D. thesis [71].

II. FIVE-DIMENSIONAL $\mathcal{N} = 2$ SUPERGRAVITY THEORIES

A. The basics and the scalar potential terms

The field content of the ungauged (before tensor or hypermultiplet coupling) $\mathcal{N} = 2$ MESGT is

$$\{e_{\hat{\mu}}^{\hat{m}}, \Psi_{\hat{\mu}}^{i}, A_{\hat{\mu}}^{I}, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\}, \tag{2.1}$$

where

$$i = 1, 2, \qquad I = 1, 2, \ldots, \tilde{n} + 1,$$
$$\tilde{a} = 2, 3, \ldots, \tilde{n} + 1, \qquad \tilde{x} = 2, 3, \ldots, \tilde{n} + 1.$$

The "graviphoton" is combined with the $\tilde{n}$ vector fields of the $\tilde{n}$ vector multiplets into a single $(\tilde{n} + 1)$-plet of vector fields $A_{\hat{\mu}}^{I}$ labeled by the index I. The indices $\tilde{a}, \tilde{b}, \ldots$ and $\tilde{x}, \tilde{y}, \ldots$ are the flat and the curved indices, respectively, of the $\tilde{n}$-dimensional target manifold $\mathcal{M}_{VS}^{5}$ of the real scalar fields, which we will define below.

The bosonic part of the Lagrangian is given in Appendix A. The global symmetries of these theories are of the form $SU(2)_R \times G_{(5)}$, where $SU(2)_R$ is the R-symmetry group of the $\mathcal{N} = 2$ Poincaré superalgebra and $G_{(5)}$ is the subgroup of the group of isometries of the scalar manifold that extends to the symmetries of the full action. Gauging a subgroup $K_{(5)}$ of $G_{(5)}$ requires dualization of some of the vector fields to self-dual tensor fields if they are transforming in a nontrivial representation of $K_{(5)}$. More formally, the field content, when $2n_T$ of the vector fields are dualized to tensor fields, becomes

$$\{e_{\hat{\mu}}^{\hat{m}}, \Psi_{\hat{\mu}}^{i}, A_{\hat{\mu}}^{I}, B_{\hat{\mu}\hat{\nu}}^{M}, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\}, \tag{2.2}$$

where now

$$i = 1, 2, \qquad I = 1, 2, \ldots, n_V + 1,$$
$$M = 1, 2, \ldots, 2n_T, \qquad \tilde{I} = 1, 2, \ldots, \tilde{n} + 1,$$
$$\tilde{a} = 2, 3, \ldots, \tilde{n} + 1, \qquad \tilde{x} = 2, 3, \ldots, \tilde{n} + 1,$$

with $\tilde{n} = n_V + 2n_T$. Tensor multiplets come in pairs with four spin-1/2 fermions [i.e. two $SU(2)_R$ doublets] and two scalars. Tensor coupling generally introduces a scalar potential of the form [60]:

$$P_{(5)}^{(T)} = \frac{3\sqrt{6}}{16} h^{I} \Lambda_{I}^{MN} h_{M} h_{N}. \tag{2.3}$$

Here Λ_I^{MN} are the transformation matrices of the tensor fields and $h_{\tilde{I}}$ and $h^{\tilde{I}}$ are elements of the "very special" geometry of the scalar manifold $\mathcal{M}_{VS}^5$ that has the metric $\overset{o}{a}_{\tilde{I}\tilde{J}}$ which is used to raise and lower the indices $\tilde{I}, \tilde{J}, \ldots$.

When the full R-symmetry group $SU(2)_R$ is being gauged, the potential gets the contribution

$$P_{(5)}^{(R)} = -4C^{ij\tilde{K}}\delta_{ij}h_{\tilde{K}}, \tag{2.4}$$

where i and j are adjoint indices of $SU(2)$. If, instead, the $U(1)_R$ subgroup is being gauged, the contribution to the potential becomes

$$P_{(5)}^{(R)} = -4C^{IJ\tilde{K}}V_I V_J h_{\tilde{K}}. \tag{2.5}$$

The expressions that lead to the derivation of the above potential terms can be found in Appendix A.

We will look at the cases where the scalar manifold $\mathcal{M}_{VS}^5$ is a symmetric space. Such spaces are further divided in two categories, depending on whether they are associated with a Jordan algebra or not. The spaces that are associated with Jordan algebras are of the form $\mathcal{M}_{VS}^5 = \frac{\mathrm{Str}_0(J)}{\mathrm{Aut}(J)}$, where $\mathrm{Str}_0(J)$ and $\mathrm{Aut}(J)$ are the reduced structure group and the automorphism group, respectively, of a real, unital Jordan algebra J, of degree three [64,72], or more specifically,

(i) generic Jordan family:

$$J = \mathbb{R} \oplus \Sigma_{\tilde{n}}:\ \mathcal{M}_{VS}^5 = \frac{SO(\tilde{n}-1,1)\times SO(1,1)}{SO(\tilde{n}-1)},\quad \tilde{n} \geq 1;$$

(ii) magical Jordan family:

$$\begin{aligned} J_3^{\mathbb{R}}:&\ \mathcal{M}_{VS}^5 = \frac{SL(3,\mathbb{R})}{SO(3)}, \quad \tilde{n} = 5,\\ J_3^{\mathbb{C}}:&\ \mathcal{M}_{VS}^5 = \frac{SL(3,\mathbb{C})}{SU(3)}, \quad \tilde{n} = 8,\\ J_3^{\mathbb{H}}:&\ \mathcal{M}_{VS}^5 = \frac{SU^*(6)}{Usp(6)}, \quad \tilde{n} = 14,\\ J_3^{\mathbb{O}}:&\ \mathcal{M}_{VS}^5 = \frac{E_{6(-26)}}{F_4}, \quad \tilde{n} = 26; \end{aligned} \tag{2.6}$$

(iii) generic non-Jordan family:

$$\mathcal{M}_{VS}^5 = \frac{SO(1,\tilde{n})}{SO(\tilde{n})}, \quad \tilde{n} \geq 1.$$

In addition to the supergravity multiplet, n_V vector multiplets, and $2n_T$ tensor multiplets, one can couple hypermultiplets into the theory. A universal hypermultiplet

$$\{\zeta^a, q^X\} \tag{2.7}$$

contains a spin-$1/2$ fermion doublet $A = 1, 2$ and four real scalars $X = 1, \ldots, 4$. The total manifold of the scalars $\phi = (\varphi, q)$ then becomes

$$\mathcal{M}_{\text{scalar}}^5 = \mathcal{M}_{VS}^5 \otimes \mathcal{M}_Q,$$

with $\dim_{\mathbb{R}}\mathcal{M}_{VS}^5 = n_V + 2n_T$ and $\dim_{\mathbb{Q}}\mathcal{M}_Q = 1$. The quaternionic hyperscalar manifold $\mathcal{M}_Q$ of the scalars of a single hypermultiplet has the isometry group $SU(2,1)$. Gauging a subgroup of this group introduces an extra term in the scalar potential [65]

$$P_{(5)}^{(H)} = 2\mathcal{N}_{iA}\mathcal{N}^{iA}, \tag{2.8}$$

where $\mathcal{N}^{iA} = (\sqrt{6}/4)h^I K_I^X f_X^{iA}$, with f_X^{iA} being the quaternionic vielbeins, $f_X^{iA} f_{YiA} = g_{XY}$, g_{XY} is the metric of the quaternionic-Kähler hypermultiplet scalar manifold [73]

$$\begin{aligned} ds^2 = \frac{dV^2}{2V^2} + \frac{1}{2V^2}(d\sigma + 2\theta d\tau - 2\tau d\theta)^2 \\ + \frac{2}{V}(d\tau^2 + d\theta^2), \end{aligned} \tag{2.9}$$

and K_I^X being the Killing vectors given in Appendix B together with their corresponding prepotentials. The determinant of the metric is $1/V^6$, and it is positive definite and well behaved everywhere except $V = 0$. But, since in the Calabi-Yau derivation V corresponds to the volume of the Calabi-Yau manifold [58], we restrict ourselves to the positive branch $V > 0$.

When the R symmetry is gauged in a theory that contains hypers, the potential $P_{(5)}^{(R)}$ gets some modification due to the fact that the fermions in the hypermultiplet are doublets under the R-symmetry group $SU(2)_R$. It becomes

$$P_{(5)}^{(R)} = -4C^{IJ\tilde{K}}\vec{P}_I \cdot \vec{P}_J h_{\tilde{K}}, \tag{2.10}$$

where $\vec{P}_I$ are the prepotentials corresponding to the Killing vectors K_I^X.

The total scalar potential, which includes terms from tensor coupling, R-symmetry gauging, and hypercoupling, is given by

$$\begin{aligned} P_{(5)} &\equiv e^{-1}\mathcal{L}_{\text{pot}} = -g^2 P_{(5)}^{(T)} - g_R^2 P_{(5)}^{(R)} - g_H^2 P_{(5)}^{(H)} \\ &\equiv -g^2 \mathcal{P}_{(5)} = -g^2(P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} + \kappa P_{(5)}^{(H)}), \end{aligned} \tag{2.11}$$

where $\lambda = g_R^2/g^2$ and $\kappa = g_H^2/g^2$; g_R, g_H, and g are coupling constants, which need not be all independent. Any point on the scalar manifold where the first derivatives of the total scalar potential with respect to all scalars vanish will be a solution to the corresponding model.

Supersymmetry of the solutions.—Demanding supersymmetric variations of the fermions vanish at the critical points of the theory, the conditions that need to be satisfied are found as [66,73]

$$\langle W^{\tilde{a}}\rangle = \langle P^{\tilde{a}}\rangle = \langle \mathcal{N}_{iA}\rangle = 0, \tag{2.12}$$

where $W^{\tilde{a}}$ and $P^{\tilde{a}}$ are defined in (A4). Any ground state that does not satisfy all of these conditions is not supersymmetric. One can see that any supersymmetric solution must be of the form

$$P_{(5)}|_{\phi^c} = -4\lambda \vec{P} \cdot \vec{P}(\phi^c), \tag{2.13}$$

which is negative semidefinite. Hence we know from beginning that any de Sitter-type ground state of the theories we will consider will have broken supersymmetry. The parametrization of the Killing vectors of the hyperscalar manifold, which is outlined in Appendix B, yields $K_I^X|_{q^c} \neq 0$ for noncompact generators. Here the point $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$ is the base point of the hyperscalar manifold; i.e. the compact Killing vectors of the hyperisometry generate the isotropy group of this point. This point will be used as the hypercoordinate candidate of the critical points. As a consequence, $\langle \mathcal{N}_{iA} \rangle \neq 0$, and hence theories including noncompact hypergauging will not have supersymmetric critical points either.

B. Gauging a compact symmetry group of the hyperisometry

The total potential is of the form $P_{(5)} = P_{(5)}^{(T)} + \lambda P_{(5)}^{(R)} + \kappa P_{(5)}^{(H)}$. The most general way of doing simultaneous $U(1)_R$ gauging together with $U(1)_H$ gauging of the hypermultiplet isometry ($\lambda = \kappa$) is done by selecting a linear combination of compact Killing vectors from (B3). One can easily see that, at the base point $q^c = \{V = 1, \sigma = \theta = \tau = 0\}$ of the hyperscalar manifold, all of these compact generators vanish. Therefore one has $\mathcal{N}^{iA} = 0$ and as a consequence [68]

$$P_{(5)}^{(H)}|_{q^c} = \frac{\partial P_{(5)}^{(H)}}{\partial \varphi}\bigg|_{q^c} = \frac{\partial P_{(5)}^{(H)}}{\partial q}\bigg|_{q^c} = 0. \tag{2.14}$$

$P_{(5)}^{(T)}$ is a function of the real scalars $\varphi^{\tilde{x}}$ only. On the other hand, $P_{(5)}^{(R)}$ of (2.10) is of the form $P_{(5)}^{(R)} \sim f(\varphi)g(q)$, where $g(q) = \vec{P}_I \cdot \vec{P}_J(q)\delta^{IJ}$ for the generic family. $g(q)$ has an extremum point at the base point of the hyperscalar manifold (i.e. $\frac{dg}{dq}|_{q^c} = 0$). This leads to

$$\frac{\partial P_{(5)}^{(T)}}{\partial q}\bigg|_{q^c} = \frac{\partial P_{(5)}^{(R)}}{\partial q}\bigg|_{q^c} = \frac{\partial P_{(5)}}{\partial q}\bigg|_{q^c} = \frac{\partial^2 P_{(5)}}{\partial \varphi \partial q}\bigg|_{q^c} = 0,$$

and hence the Hessian is in block-diagonal form. The fact that $g(q) \geq 0$ makes it impossible to convert the nonminimum critical points that correspond to the upper block of the Hessian ($\frac{\partial^2 P_{(5)}}{(\partial \varphi)^2}$) to minimum points of the potential or change its sign at the critical point. Therefore a $U(1)_H$ gauging will not change the nature of an existing critical point. One can arrive at the same result by gauging a $SU(2)_H$ subgroup of the isometry group $SU(2,1)$ of the hyperscalar manifold.

Equation (2.14) does not hold for noncompact generators of the hyperisometry. Indeed gauging a noncompact hypersymmetry generally leads to stable and unstable de Sitter ground states in five dimensions as was shown in a previous work [69]. However, this topic will not be covered in this paper. Instead we will concentrate on studying de Sitter ground states that result from gauging a noncompact symmetry of the real scalar manifold of the vector multiplets.

C. Two models with stable de Sitter ground states

The real scalar manifolds of the two models we will discuss belong to the generic Jordan family.[6] These two models will play an important role in the four-dimensional stable dS vacua calculations in Sec. III.

The theory being considered is $\mathcal{N} = 2$ supergravity coupled to $\tilde{n}$ Abelian vector multiplets and with real scalar manifold $\mathcal{M}_{VS}^5 = SO(\tilde{n}-1,1) \times SO(1,1)/SO(\tilde{n}-1)$, $\tilde{n} \geq 1$. The cubic polynomial can be written in the form [66]

$$N(h) = \frac{3\sqrt{3}}{2} h^1[(h^2)^2 - (h^3)^2 - \cdots - (h^{\tilde{n}+1})^2]. \tag{2.15}$$

The nonzero $C_{\tilde{I}\tilde{J}\tilde{K}}$'s are

$$C_{122} = \frac{\sqrt{3}}{2},$$

$$C_{133} = C_{144} = \cdots = C_{1,\tilde{n}+1,\tilde{n}+1} = -\frac{\sqrt{3}}{2},$$

and their permutations. The constraint $N = 1$ can be solved by

$$h^1 = \frac{1}{\sqrt{3}||\varphi||^2}, \qquad h^a = \sqrt{\frac{2}{3}}\varphi^a, \tag{2.16}$$

with $a, b = 2, 3, \ldots, \tilde{n}+1$ and $||\varphi||^2 = \varphi^a \eta_{ab} \varphi^b$, where $\eta_{ab} = (+ - - \ldots -)$. The scalar field metric $g_{\tilde{x}\tilde{y}}$ and vector field metric $\overset{o}{a}_{\tilde{I}\tilde{J}}$ that appear in the kinetic terms in the Lagrangian are positive definite in the region $||\varphi||^2 > 0$. In order to have theories that have a physical meaning, our investigation is restricted to this region. As a consequence, one must have $\varphi^2 \neq 0$.

The isometry group of the real scalar manifold $\mathcal{M}_{VS}^5$ is $G_{(5)} = SO(\tilde{n}-1,1) \times SO(1,1)$. The gauging of an $SO(1,1)$ or an $SO(2)$ subgroup of $SO(\tilde{n}-1,1)$ will lead to dualization of vectors to tensor fields, and this gives a scalar potential term. In the generic Jordan family there are no vector fields that are nontrivially charged when the gauge group is non-Abelian, and hence gauging a non-Abelian subgroup of $G_{(5)}$ will not give a scalar potential

[6]It is possible to embed these models into magical Jordan family theories, provided that there is a sufficient number of vector fields to perform the respective gaugings [69].

term. It is also possible to gauge the R-symmetry group $SU(2)_R$ or its subgroup $U(1)_R$.

Gauging $SO(1,1)$ symmetry.—The $SO(1,1)$ subgroup of the isometry group of the scalar manifold acts nontrivially on the vector fields $A^2_{\hat{\mu}}$ and $A^3_{\hat{\mu}}$. Hence these vector fields must be dualized to antisymmetric tensor fields. The index $\tilde{I}$ is decomposed as

$$\tilde{I} = (I, M),$$

with $I, J, K = 1, 4, 5, \ldots, \tilde{n}+1$ and $M, N, P = 2, 3$. The fact that the only nonzero C_{IMN} are C_{1MN} for the theory at hand requires $A^1_{\hat{\mu}}$ to be the $SO(1,1)$ gauge field because of $\Lambda^M_{IN} \sim \Omega^{MP} C_{IPN}$ [cf. Eq. (2.3)]. All of the other $A^I_{\hat{\mu}}$ with $I \neq 1$ are spectator vector fields with respect to the $SO(1,1)$ gauging. The potential term (2.3) that comes from the tensor coupling is found to be (taking $\Omega^{23} = -\Omega^{32} = -1$)

$$P^{(T)}_{(5)} = \frac{1}{8}\frac{[(\varphi^2)^2 - (\varphi^3)^2]}{||\varphi||^6}. \tag{2.17}$$

For the function $W_{\tilde{x}}$ that enters the supersymmetry transformation laws of the fermions, one obtains

$$\begin{aligned} W_4 &= W_5 = \cdots = W_{\tilde{n}+1} = 0, \\ W_2 &= -\frac{\varphi^3}{4||\varphi||^4}, \\ W_3 &= \frac{\varphi^2}{4||\varphi||^4}. \end{aligned} \tag{2.18}$$

Since W_3 can never vanish, there can be no $\mathcal{N}=2$ supersymmetric critical point.

Taking the derivative of the total potential $P_{(5)} = P^{(T)}_{(5)}$ with respect to $\varphi^{\tilde{x}}$, one finds

$$\partial_{\varphi^2} P_{(5)} = B\varphi^2, \qquad \partial_{\varphi^3} P_{(5)} = -B\varphi^3,$$
$$\partial_{\varphi^b} P_{(5)} = -B\varphi^b + \frac{\varphi^b}{4||\varphi||^6}, \qquad b = 4, \ldots, \tilde{n}+1,$$

where

$$B = -\frac{3}{4}\frac{(\varphi^2)^2 - (\varphi^3)^2}{||\varphi||^8} + \frac{1}{4||\varphi||^6} < 0. \tag{2.19}$$

Since $\partial_{\varphi^2} P_{(5)}$ cannot be brought to zero, the potential $P_{(5)} = P^{(T)}_{(5)}$ alone does not have any critical points. However, one can gauge R symmetry to get additional potential terms.

1. $SO(1,1) \times SU(2)_R$ symmetry gauging

For such a gauging, one needs at least $\tilde{n} \geq 5$. Choosing $A^4_{\hat{\mu}}$, $A^5_{\hat{\mu}}$, and $A^6_{\hat{\mu}}$ as the $SU(2)_R$ gauge fields, one finds

$$P_{(5)} = P^{(T)}_{(5)} + \lambda P^{(R)}_{(5)},$$

with

$$P^{(R)}_{(5)} = 6||\varphi||^2 \tag{2.20}$$

and $P^{(T)}_{(5)}$ given in (2.17). Taking the derivative of the total potential with respect to $\varphi^{\tilde{x}}$, one finds

$$\begin{aligned} \partial_{\varphi^2} P_{(5)} &= (B + 12\lambda)\varphi^2, \\ \partial_{\varphi^3} P_{(5)} &= -(B + 12\lambda)\varphi^3, \\ \partial_{\varphi^b} P_{(5)} &= -(B + 12\lambda)\varphi^b + \frac{\varphi^b}{4||\varphi||^6}, \\ &\quad b = 4, \ldots, \tilde{n}+1, \end{aligned} \tag{2.21}$$

with B defined in (2.19). Setting the first equation to zero means

$$B = -12\lambda \tag{2.22}$$

since $\varphi^2 \neq 0$. The last equation then implies $\varphi^b_c = 0$. From (2.22) we find

$$\frac{1}{||\varphi_c||^6} = 24\lambda. \tag{2.23}$$

The value of $||\varphi_c||^2 = (\varphi^2_c)^2 - (\varphi^3_c)^2$ is fixed by λ but not φ^2_c and φ^3_c individually. The value of the potential at these critical points is

$$P_{(5)}|_{\varphi^c} = \frac{3}{8||\varphi_c||^4}, \tag{2.24}$$

and therefore it corresponds to a one-parameter family of de Sitter ground states. The stability of the critical points is checked by calculating the eigenvalues of the Hessian of the potential, which are easily found as

$$\Big\{0, \frac{3[(\varphi^2_c)^2 + (\varphi^3_c)^2]}{||\varphi_c||^8}, \underbrace{\frac{1}{4||\varphi_c||^6}, \ldots, \frac{1}{4||\varphi_c||^6}}_{(\tilde{n}-2)\text{ times}}\Big\}.$$

The eigenvalues are all non-negative, and thus the one-parameter family of de Sitter critical points is found to be stable [69].

2. $SO(1,1) \times U(1)_R$ symmetry gauging

The calculation in [66] for $\tilde{n} = 3$ was later generalized to arbitrary $\tilde{n} \geq 3$ in [68]. Let us briefly quote their results. A linear combination $A_{\hat{\mu}}[U(1)_R] = V_I A^I_{\hat{\mu}}$ of the vector fields is taken as the $U(1)_R$ gauge field. The scalar potential is now

$$P_{(5)} = P^{(T)}_{(5)} + \lambda P^{(R)}_{(5)},$$

where

$$P_{(5)}^{(R)} = -4\sqrt{2}V_1 V_i \varphi^i ||\varphi||^{-2} + 2|V|^2||\varphi||^2, \tag{2.25}$$

with $i = 4, \ldots, \tilde{n} + 1$ and $|V|^2 = V_i V_i$. Demanding $\partial_{\varphi^{\tilde{x}}} P_{(5)} = 0$, one obtains the following conditions:

$$\begin{aligned} \frac{\varphi_c^i}{||\varphi_c||^4} &= 16\sqrt{2}\lambda V_1 V_i, \\ \frac{1}{||\varphi_c||^6} &= -\frac{1}{2}(16\sqrt{2}\lambda V_1 |V|)^2 + 8\lambda |V|^2, \end{aligned} \tag{2.26}$$

with the constraints

$$|V|^2 > 0, \qquad 32\lambda (V_1)^2 < 1. \tag{2.27}$$

Given a set of V_I subject to (2.27), we see that $||\varphi||^2$ and φ^i [and thus $(\varphi^2)^2 - (\varphi^3)^2$] are completely determined by (2.26) but φ^2 and φ^3 are otherwise undetermined. The value of the potential at this one-parameter family of critical points becomes

$$P_{(5)}|_{\varphi^c} = 3\lambda ||\varphi||^2 |V|^2 (1 - 32\lambda (V_1)^2), \tag{2.28}$$

and this corresponds to de Sitter vacua. The stability is checked by calculating the eigenvalues of the Hessian of the potential at the critical point. We can use the $SO(1,1)$ invariance together with the $SO(\tilde{n} - 2)$ of the φ^i to take for any critical point $\varphi_c = (\varphi^2, 0, \varphi^4, 0, \ldots, 0)$. With these choices the Hessian becomes block-diagonal at the critical point. φ^3 is a zero mode, and the sector $\varphi^5, \ldots, \varphi^{\tilde{n}+1}$ consists of a unit matrix times $\frac{1}{4}||\varphi||^{-6}$. The only non-diagonal part of the Hessian is

$$\partial_{\tilde{x}} \partial_{\tilde{y}} P_{(5)}|_{\tilde{x},\tilde{y}=2,4} = \gamma \begin{pmatrix} (\varphi^2)^2[6(\varphi^2)^2 + 5(\varphi^4)^2] & -\varphi^2[8(\varphi^2)^2\varphi^4 + 3(\varphi^4)^3] \\ -\varphi^2[8(\varphi^2)^2\varphi^4 + 3(\varphi^4)^3] & \frac{1}{4}[2(\varphi^2)^4 + 37(\varphi^2)^2(\varphi^4)^2 + 5(\varphi^4)^4] \end{pmatrix},$$

with $\gamma = ||\varphi||^{-8}[2(\varphi^2)^2 - (\varphi^4)^2]^{-1}$. The determinant and the trace of this part of the Hessian are

$$\begin{aligned} \det \partial\partial P_{(5)} &= \frac{12(\varphi^2)^6 - 12(\varphi^2)^4(\varphi^4)^2 + 11(\varphi^2)^2(\varphi^4)^4}{4||\varphi||^{14}[2(\varphi^2)^2 - (\varphi^4)^2]^2}, \\ \mathrm{tr}\, \partial\partial P_{(5)} &= \frac{26(\varphi^2)^4 + 57(\varphi^2)^2(\varphi^4)^2 + 5(\varphi^4)^4}{4||\varphi||^8[2(\varphi^2)^2 - (\varphi^4)^2]}, \end{aligned}$$

respectively, which are both positive because of $(\varphi^2)^2 > (\varphi^4)^2$, and therefore the family of critical points is found to be stable. We note that, although the above quantities are both positive, they are slightly different than the ones found in [68], where the authors fixed the coupling constants with $\lambda = 1$. Figure 2 shows the plot of the potential (2.25) for the special case $\tilde{n} = 3$, $V_1 = 0$, and $\lambda = 1$.

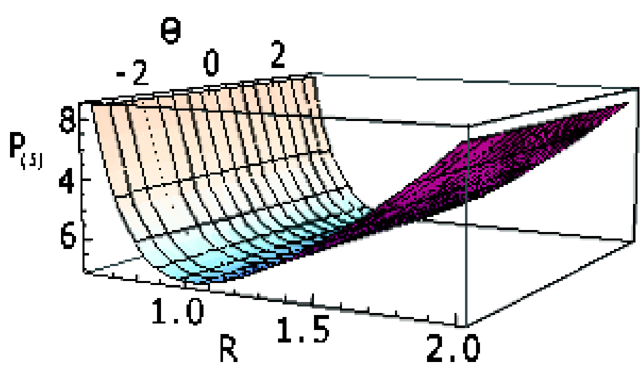

FIG. 2 (color online). The extrema of the potential $P_{(5)}(R, \theta)$ due to $SO(1,1) \times U(1)_R$ gauging, evaluated at $\varphi^4 = 0$; $V_1 = 0$ and $\lambda = 1$; with parametrization $\varphi^2 = R\cosh\theta$, $\varphi^3 = R\sinh\theta$. The zero eigenvalue of the Hessian corresponds to the flat direction of the potential at its minima.

III. FOUR-DIMENSIONAL $\mathcal{N} = 2$ SUPERGRAVITY THEORIES

Having discussed two possible gaugings that result in de Sitter ground states from the scalar potentials of $\mathcal{N} = 2$ supergravity theories with symmetric scalar manifolds in five dimensions, we now move on to de Sitter ground states of the four-dimensional $\mathcal{N} = 2$ supergravity theories obtained by dimensional reduction. The details of the dimensional reduction process can be found in [64,70]. Here we quote the necessary tools for the calculation of the scalar potentials. The bosonic sectors of the Lagrangians before and after the dimensional reduction are given in Appendix A.

Before we begin, let us see what kind of ground states we would get just by considering ordinary dimensional reduction. The dimensionally reduced potential derived from (2.11), in the absence of hypers[7] reads (A25) and (A27)

$$\begin{aligned} P_{(4)} = e^{-\sigma} P_{(5)}^{(T)} + \lambda e^{-\sigma} P_{(5)}^{(R)} + \tfrac{3}{4} e^{-3\sigma} \overset{o}{a}_{\tilde{I}\tilde{J}} (A^I M_{I\tilde{K}}^{\tilde{I}} h^{\tilde{K}}) \\ \times (A^J M_{J\tilde{L}}^{\tilde{J}} h^{\tilde{L}}), \end{aligned} \tag{3.1}$$

where $M_{I\tilde{K}}^{\tilde{I}}$ are the $K_{(5)}$-transformation matrices defined in (A28). The scalars of the above potential are $\varphi^{\tilde{x}}$, A^I, and σ. Taking the σ derivative of the potential, setting it equal to zero, and plugging the result back into the potential gives us the value of the potential at the critical point ϕ^c as

[7] Adding hypers results in an additional $P_{(4)}^{(H)}$ in the dimensionally reduced potential (3.1), which is given in (A26). The two terms of $P_{(4)}^{(H)}$ have the same powers of σ and A^I as the first and third terms above and can be absorbed in them by proper field redefinitions, and hence it will not change our result.

$$P_{(4)}|_{\phi^c} = -\tfrac{3}{2}e^{-3\sigma}\overset{o}{a}_{\tilde{I}\tilde{J}}(A^I M^{\tilde{I}}_{I\tilde{K}}h^{\tilde{K}})(A^J M^{\tilde{J}}_{J\tilde{L}}h^{\tilde{L}}). \tag{3.2}$$

The derivative of the potential with respect to any A^I must vanish at the critical point. Hence we arrive at

$$A^I \frac{\partial P_{(4)}}{\partial A^I}\bigg|_{\phi^c} = \frac{3}{2}e^{-3\sigma}A^I A^J \overset{o}{a}_{\tilde{I}\tilde{J}} M^{\tilde{I}}_{I\tilde{K}} M^{\tilde{J}}_{J\tilde{L}} h^{\tilde{K}} h^{\tilde{L}} = 0. \tag{3.3}$$

So if a critical point exists the potential vanishes there [cf. Eq. (3.2)], and there is no possibility for an (anti)de Sitter ground state. Since cosmological observations imply that the Universe has a very small positive cosmological constant, we must find a way around this problem.

It was shown in [70] that the dimensionally reduced 5D Yang-Mills-Einstein supergravity theories coupled to tensor multiplets result in 4D theories that have gauge groups of the form $K_{(4)} = K_{(5)} \ltimes \mathcal{H}^{n_T+1}$, where $\mathcal{H}^{n_T+1}$ is a Heisenberg group of dimension $n_T + 1$ and $\ltimes$ denotes the semidirect product. On the other hand, stable de Sitter vacua were found for 4D, $\mathcal{N} = 2$ theories in [25], where the authors showed that the three necessary ingredients to obtain stable de Sitter vacua are non-Abelian, noncompact gauge groups, $SO(2,1)$, in particular; Fayet-Iliopoulos (FI) terms that are possible only for $SU(2)$ or $U(1)$ factors, which can be identified by the $SU(2)_R$ or $U(1)_R$ gaugings, and the de Roo-Wagemans (dRW) rotation. The last ingredient uses additional symmetries in four dimensions, where the isometry group is larger than in five dimensions. In order to make use of these symmetries, we first need to review the structure of the complex geometry of four-dimensional $\mathcal{N} = 2$ supergravity theories. Once this is achieved, it will be easier to see the five-dimensional origins of de Sitter ground states that we will show how to obtain in four dimensions.

A. The geometry

The scalar manifold of the theory we studied in the last section, when reduced to four dimensions, is the special Kähler manifold [64,74]

$$\mathcal{M}^4_{VS} = \mathcal{ST}[2, n-1] = \frac{SU(1,1)}{U(1)} \frac{SO(2, n-1)}{SO(2) \times SO(n-1)}. \tag{3.4}$$

In four dimensions, there are $n = \tilde{n} + 1$ vector multiplets and n complex scalars. The $(n+1)$ field strengths $\mathcal{F}^{A\mu\nu}$ and their magnetic duals $\mathcal{G}_{A\mu\nu}$ transform in the $(2, n+1)$ representation of the U-duality group $\mathcal{U} = SU(1,1) \times SO(2, n-1)$. The models with stable de Sitter vacua that we will discuss in this section originate from the five-dimensional YMESGTs with gauge groups $SO(1,1) \times U(1)_R$ or $SO(1,1) \times SU(2)_R$. The $SO(1,1)$ factor, as we will show, will become a subgroup of $SO(2, n-1)$ in four dimensions. This is similar to the models with stable de Sitter vacua found in [25], where the full $SO(2,1)$ is gauged. Note that the $SU(1,1)_G$ symmetry of the pure 5D, $\mathcal{N} = 2$ supergravity reduced to four dimensions is not the $SU(1,1) = SO(2,1)$ factor in the four-dimensional U-duality group $\mathcal{U}$. It is rather a diagonal subgroup of $SU(1,1)$ times an $SO(2,1)$ subgroup of $SO(2, n-1)$ under which the following decompositions occur [75]:

$$\begin{gathered} SO(2,1) \times SO(2, n-1) \supset SO(2,1) \times SO(2,1) \times SO(n-2) \supset SO(2,1)_G \times SO(n-2), \\ (2, n+1) = (2,3,1) \oplus (2,1,n-2) = (4,1) \oplus (2,1) \oplus (2, n-2). \end{gathered}$$

Note that the four-dimensional graviphoton transforms in the spin-3/2 representation of $SO(2,1)_G$ along with some linear combination of the other vectors in the theory, and, due to the mixing, one can say that it does not descend directly from the five-dimensional graviphoton. Instead, it is a linear combination of the vector that comes from the dimensional reduction of the fünfbein and the vector that is obtained by the dimensional reduction of the five-dimensional graviphoton. We will address this issue in Sec. III C 2.

The scalars can be used to define the complex coordinates [64,70]

$$z^{\tilde{I}} = \frac{1}{\sqrt{3}}A^{\tilde{I}} + \frac{ie^{\sigma}}{\sqrt{2}}h^{\tilde{I}}. \tag{3.5}$$

These n complex coordinates can be interpreted as the inhomogeneous coordinates of the $(n+1)$-dimensional complex vector $(\tilde{I} = 1, \ldots, n)$

$$X^A = \begin{pmatrix} X^0 \\ X^{\tilde{I}} \end{pmatrix} = \begin{pmatrix} 1 \\ z^{\tilde{I}} \end{pmatrix}. \tag{3.6}$$

One can introduce the prepotential[8]

$$F(X^A) = -\frac{1}{3\sqrt{3}} C_{\tilde{I}\tilde{J}\tilde{K}} \frac{X^{\tilde{I}}X^{\tilde{J}}X^{\tilde{K}}}{X^0} \tag{3.7}$$

to write the holomorphic (symplectic) section

[8]Note that the prepotential given here differs by a factor $\sqrt{6}$ from that of [70].

$$\Omega_0 = \begin{pmatrix} X^A \\ F_B \end{pmatrix} = \begin{pmatrix} X^A \\ \partial_B F \end{pmatrix} = \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} = \begin{pmatrix} 1 \\ z^I \\ z^M \\ \frac{1}{3\sqrt{3}}[C_{IJK}z^I z^J z^K + 3C_{IMN}z^I z^M z^N] \\ -\frac{1}{\sqrt{3}}[C_{IJK}z^J z^K + C_{IMN}z^M z^N] \\ -\frac{2}{\sqrt{3}}C_{MNI}z^N z^I \end{pmatrix}, \tag{3.8}$$

with $\tilde{I} = (I, M)$. The reason the above manifold is called a *special Kähler manifold* is that one can write a Kähler potential in terms of the holomorphic section Ω:

$$\mathcal{K} = -\log(i\langle\Omega|\bar{\Omega}\rangle) = -\log[i(\bar{X}^A F_A - \bar{F}_A X^A)]. \tag{3.9}$$

The Kähler potential is used to form the Kähler metric on the scalar manifold $\mathcal{M}^4_{VS}$ of the four-dimensional theory as

$$g_{\tilde{I}\bar{\tilde{J}}} \equiv \partial_{\tilde{I}}\partial_{\bar{\tilde{J}}}\mathcal{K}. \tag{3.10}$$

It is also possible to introduce the covariantly holomorphic section [76–79]

$$V = \begin{pmatrix} L^A \\ M_B \end{pmatrix} \equiv e^{\mathcal{K}/2}\Omega = e^{\mathcal{K}/2}\begin{pmatrix} X^A \\ F_B \end{pmatrix}, \tag{3.11}$$

which obeys

$$\nabla_{\bar{\tilde{I}}} V = (\partial_{\bar{\tilde{I}}} - \tfrac{1}{2}\partial_{\bar{\tilde{I}}}\mathcal{K})V = 0. \tag{3.12}$$

By defining

$$U_{\tilde{I}} = \nabla_{\tilde{I}} V = \left(\partial_{\tilde{I}} + \frac{1}{2}\partial_{\tilde{I}}\mathcal{K}\right)V \equiv \begin{pmatrix} f^A_{\tilde{I}} \\ h_{B|\tilde{I}} \end{pmatrix}, \tag{3.13}$$

the *period matrix* is introduced via relations

$$\bar{M}_A = \bar{\mathcal{N}}_{AB}\bar{L}^B, \qquad h_{A|\tilde{I}} = \bar{\mathcal{N}}_{AB} f^B_{\tilde{I}}, \tag{3.14}$$

which can be solved by introducing two $(n+1)\times(n+1)$ vectors

$$f^A_{\tilde{C}} = \begin{pmatrix} f^A_{\tilde{I}} \\ \bar{L}^A \end{pmatrix}, \qquad h_{A|\tilde{C}} = \begin{pmatrix} h_{A|\tilde{I}} \\ \bar{M}_A \end{pmatrix} \tag{3.15}$$

and setting

$$\bar{\mathcal{N}}_{AB} = h_{A|\tilde{C}} \cdot (f^{-1})^{\tilde{C}}{}_B. \tag{3.16}$$

Whenever the prepotential F exists, the period matrix has the form [80–82]

$$\mathcal{N}_{AB} = \bar{F}_{AB} + 2i\frac{\mathrm{Im}(F_{AC})\,\mathrm{Im}(F_{BD})L^C L^D}{\mathrm{Im}(F_{CD})L^C L^D}, \tag{3.17}$$

where $F_{AB} = \partial_A\partial_B F$.

A symplectic rotation C of the holomorphic section obeys $C^T\omega C = \omega$ for

$$\omega = \begin{pmatrix} 0 & \mathbb{1}_{n+1} \\ -\mathbb{1}_{n+1} & 0 \end{pmatrix}.$$

B. Gauge group representation and dRW angles

The special Kähler manifold (3.4) of vector multiplets has the isometry group $G_{(4)} = SU(1,1)\times SO(2, n-1)$. If we are to gauge a subgroup $K_{(4)} \subset G_{(4)}$, then the symplectic representation R of $G_{(4)}$, under which the electric field strengths and their magnetic duals transform, must be decomposed as

$$G_{(4)} \supset K_{(4)}, \qquad R = \text{adj.} + \text{adj.} + \text{singlets} + \text{singlets}. \tag{3.18}$$

The electric and magnetic field strengths are in the doublet representation of $SU(1,1)$ and in the $n+1$ vector representation of $SO(2, n-1)$. The noncompact non-Abelian gauge group $K_{(4)} = SO(2,1)$ which is a necessary ingredient to obtain stable de Sitter vacua in 4D, $\mathcal{N} = 2$ supergravity is embedded in $SO(2, n-1)$. The $SO(2,1)$ generators t_A form an adjoint representation. The symplectic embedding of this representation into the fundamental representation of $Sp(2(n+1), \mathbb{R})$ is given by

$$T_A = \begin{pmatrix} t_A & 0 \\ 0 & -t_A^T \end{pmatrix} \in Sp(2(n+1), \mathbb{R}), \qquad A = 0, 2, 3, \tag{3.19}$$

and the corresponding algebra $[T_A, T_B] = f^C_{AB} T_C$ is

$$[T_0, T_2] = T_3, \qquad [T_2, T_3] = -T_0, \qquad [T_3, T_0] = -T_2. \tag{3.20}$$

Here f^C_{AB} are the structure constants of the algebra.

In addition to the $SO(2,1)$, one can gauge a $U(1)_R$ (or $SU(2)_R$) R-symmetry group for theories with $n > 2$ (or $n > 4$) vector multiplets using the remaining vectors (or a linear combination of them) as gauge fields. The dRW angles, as first introduced for $\mathcal{N} = 4$ supergravity [83,84] and later used in $\mathcal{N} = 2$ supergravity as an ingredient to obtain de Sitter vacua [25,26], parametrize the relative embedding of the R-symmetry group within $Sp(2(n+1), \mathbb{R})$. They mix the electric and magnetic components of the symplectic section prior to the gauging by a "nonperturbative" rotation. The dRW-rotation matrix has to be chosen in such a way that it commutes with $SO(2,1)$ symmetry gauging. For example, we will use the following dRW matrix for the models where we gauge a $SO(2,1)\times U(1)_R$ symmetry [25,26]:

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_n & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ 0 & 0 & \mathbb{1}_n & 0 \\ 0 & \sin\theta & 0 & \cos\theta \end{pmatrix}. \quad (3.21)$$

The holomorphic section and the covariantly holomorphic section are rotated via

$$\Omega \to \Omega_R = \mathcal{R}\Omega, \qquad V \to V_R = \mathcal{R}V. \quad (3.22)$$

C. Symplectic rotation

The symplectic section (3.8) is written in the most natural way when one comes from five down to four dimensions. But it has shortcomings. The translations $z^M \to z^M + b^M$ act on the symplectic section in such a way that the electric components mix with magnetic ones so that the transformation matrix is not block-diagonal, which is not suitable if symmetries are to be gauged in the standard way. In this section we will give two inequivalent examples of symplectic rotations that will bring Ω_0 in bases where this problem does not occur.

1. Günaydın-McReynolds-Zagermann (GMZ) rotation

We start with observing how Ω_0 varies under the infinitesimal translation $z^M \to z^M + b^M$ [70]:

$$\Omega_0 = \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} \to \begin{pmatrix} X^0 \\ X^I \\ X^M \\ F_0 \\ F_I \\ F_M \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ b^M X^0 \\ -b^M F_M \\ -\frac{2}{\sqrt{3}} b^M C_{IMN} X^N \\ -\frac{2}{\sqrt{3}} b^N C_{IMN} X^I \end{pmatrix}. \quad (3.23)$$

In the original basis a combined infinitesimal translation and infinitesimal K transformation with parameter α^I is generated by

$$\mathcal{O} = \mathbb{1}_{2n+2} + \begin{pmatrix} B & 0 \\ C & -B^T \end{pmatrix}, \quad (3.24)$$

with

$$B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \alpha^I f_{IJ}^K & 0 \\ b^M & 0 & \alpha^I \Lambda_{IN}^M \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B_{IM} \\ 0 & B_{MI} & 0 \end{pmatrix}, \quad (3.25)$$

where

$$B_{IM} := \frac{-2}{\sqrt{3}} C_{IMN} b^N. \quad (3.26)$$

By having a closer look at (3.23) we see that (X^0, F_I, X^M) transform among themselves, as do (F_0, X^I, F_M). In order to make the translations block-diagonal we exchange F_0 with X^0 and F_M with X^M. The symplectic rotation

$$\begin{pmatrix} X^A \\ F_B \end{pmatrix} \to \begin{pmatrix} \check{X}^A \\ \check{F}_B \end{pmatrix} \equiv \mathcal{S}\begin{pmatrix} X^A \\ F_B \end{pmatrix}, \quad \begin{pmatrix} F^A_{\mu\nu} \\ G_{\mu\nu B} \end{pmatrix} \to \begin{pmatrix} \check{F}^A_{\mu\nu} \\ \check{G}_{\mu\nu B} \end{pmatrix} \equiv \mathcal{S}\begin{pmatrix} F^A_{\mu\nu} \\ G_{\mu\nu B} \end{pmatrix}, \quad \begin{pmatrix} L^A \\ M_B \end{pmatrix} \to \begin{pmatrix} \check{L}^A \\ \check{M}_B \end{pmatrix} \equiv \mathcal{S}\begin{pmatrix} L^A \\ M_B \end{pmatrix} \quad (3.27)$$

that achieves this is [70]

$$\mathcal{S} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \delta_I^J & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & D^{MN} \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_I^J & 0 \\ 0 & 0 & D_{MN} & 0 & 0 & 0 \end{pmatrix}, \quad (3.28)$$

where $D_{MN} = -\sqrt{2}\Omega_{MN}$ and $D_{MN}D^{NP} = \delta_M^P$.

The potential terms.—The holomorphic Killing vectors

$$K_A^{\tilde{I}} = i g^{\tilde{I}\bar{\tilde{J}}} \partial_{\bar{\tilde{J}}} P_A, \quad (3.29)$$

that are determined in terms of the Killing prepotentials [80,82,85–87]

$$P_A = e^{\mathcal{K}}(\check{F}_B f_{AC}^B \bar{\check{X}}^C + \bar{\check{F}}_B f_{AC}^B \check{X}^C), \quad (3.30)$$

can be used to show that the potential in the canonical form

$$V = e^K (\check{X}^A \bar{K}_A^{\tilde{I}}) g_{\tilde{I}\bar{\tilde{J}}} (\bar{\check{X}}^B K_B^{\tilde{J}}) \quad (3.31)$$

is indeed equal to $P_{(4)}^{(T)}$ of (A25) [70]. Here, f_{BC}^A's are the structure constants of the gauge group.

Now we turn to the calculation of the potential $P_{(4)}^{(R)}$ rising from the R-symmetry gauging. For gauge groups with $U(1)$ or $SO(3) = SU(2)$ factors, there is a superrenormalizable term, known as an FI term [88,89] that can be added to the Lagrangian. The variation of this term under a supersymmetry transformation is a total derivative, and it yields a supersymmetric term in the action. FI terms are used in effective field theories for standard model building or cosmology quite often. It has been recently emphasized that these terms in $\mathcal{N} = 1$ or $\mathcal{N} = 2$, $D = 4$ supersymmetric models are related to R-symmetry gauging [90,91]. Here we will verify this statement by reformulating an already known $P_{(4)}^{(R)}$ potential, coming from five dimensions, in terms of complex geometry elements and comparing the expressions. The potential term we will consider is given by [85]

$$V' = (U^{(AB)} - 3\bar{\check{L}}^{(A}\check{L}^{B)})\mathcal{P}^x_A\mathcal{P}^x_B, \tag{3.32}$$

where U^{AB} is defined as

$$U^{AB} \equiv f^A_{\tilde{I}} f^B_{\bar{\tilde{J}}} g^{\tilde{I}\bar{\tilde{J}}} = -\tfrac{1}{2}(\mathrm{Im}\mathcal{N})^{-1|AB} - \bar{\check{L}}^A\check{L}^B. \tag{3.33}$$

The negative definite term in (3.32) is the gravitino mass contribution, while the U^{AB} term is the gaugino shift contribution. $\mathcal{P}^x_A$ are called the *triholomorphic moment maps* for the gauge group action on quaternionic scalars, with x being an $SU(2)$ index. When a hypermultiplet is coupled to the theory, the potential (3.32) carries contact interactions between the real and hyperscalars. In this case the triholomorphic moment maps $\mathcal{P}^x_A$ that describe the action of the R-symmetry gauge group on the quaternionic scalars are associated to the Killing prepotentials of the isometries of the hyperscalar manifold [85,92]. This is analogous to the five-dimensional theory (cf. Appendixes A and B). An FI term can be assigned to the moment maps if (and only if [91]) hypers are absent from the theory. For such models the triholomorphic moment maps satisfy the equivariance condition [25,85,92]

$$-\epsilon^{xyz}\mathcal{P}^y_A\mathcal{P}^z_B = f^C_{AB}\mathcal{P}^x_C. \tag{3.34}$$

In the $SU(2)_R$ case, one can set $f^x_{yz} = e\epsilon_{xyz}$, where e is some number, and this condition is satisfied via

$$\mathcal{P}^x_A = \begin{cases} -\delta^x_y & \text{for } A = 3 + y, \\ 0 & \text{otherwise,} \end{cases} \tag{3.35}$$

whereas, in the $U(1)_R$ case, for each generator one can set an FI term

$$\mathcal{P}^x_A = \begin{cases} e\delta^x_3 & A\text{: index for the } U(1)_R \text{ gauge vector,} \\ 0 & \text{otherwise.} \end{cases} \tag{3.36}$$

Example.—Let us now calculate the V' potential for a specific model with $n = 4$ vector multiplets where the $U(1)_R$ gauge field is a linear combination of A^1_μ and A^4_μ. This is indeed the model we discussed in Sec. II C 2 before the dimensional reduction. Using (2.25) and (A27), one can write the $U(1)_R$ potential in four dimensions as

$$\begin{aligned} P^{(R)}_{(4)} &= e^{-\sigma}P^{(R)}_{(5)} \\ &= e^{-\sigma}(-4\sqrt{2}V_1V_4\varphi^4||\varphi||^{-2} + 2(V_4)^2||\varphi||^2). \end{aligned} \tag{3.37}$$

On the other hand, the moment map for this type of gauging can be written as

$$\mathcal{P}^x_A = \delta^{x3}(e_1\delta_{A1} + e_4\delta_{A4}), \tag{3.38}$$

where e_1 and e_4 parametrize the linear combination of the gauge fields. Then the potential (3.32) becomes

$$\begin{aligned} V'_{n=4} = {}& e_1^2(U^{(11)} - 3\bar{\check{L}}^{(1}\check{L}^{1)}) + 2e_1e_4(U^{(14)} - 3\bar{\check{L}}^{(1}\check{L}^{4)}) \\ & + e_4^2(U^{(44)} - 3\bar{\check{L}}^{(4}\check{L}^{4)}), \end{aligned} \tag{3.39}$$

and after some calculation one can find

$$\begin{aligned} & U^{(11)} - 3\bar{\check{L}}^{(1}\check{L}^{1)} = 0, \\ & U^{(14)} - 3\bar{\check{L}}^{(1}\check{L}^{4)} = -\frac{\mathrm{Im}z^4}{(\mathrm{Im}z^2)^2 - (\mathrm{Im}z^3)^2 - (\mathrm{Im}z^4)^2}, \\ & U^{(44)} - 3\bar{\check{L}}^{(4}\check{L}^{4)} = \frac{1}{2\,\mathrm{Im}z^1}. \end{aligned} \tag{3.40}$$

By using (2.16) and (3.5) on (3.40), we conclude that $V'_{n=4} = P^{(R)}_{(4)}$ if we identify $e_1 = \pm(\frac{8}{3})^{1/4}V_1$ together with $e_4 = \pm(\frac{8}{3})^{1/4}V_4$.

One can arrive at a similar conclusion by gauging the full $SU(2)_R$ instead. In this case $P^{(R)}_{(4)} = 6e^{-\sigma}||\varphi||^2$ and $V' = 3/(2\,\mathrm{Im}z_1)$, which are again directly proportional to each other.

2. A new basis

The GMZ rotation we discussed in the last subsection resolves the block-diagonality problem of translational symmetries, but there are a few more steps to take in order to find a symplectic section that will allow us to find de Sitter vacua. First, it is convenient to work in a symplectic section that satisfies the constraint

$$X^A\eta_{AB}X^B = F_A\eta^{AB}F_B = 0 \tag{3.41}$$

for $\eta_{AB} = \mathrm{diag}(+ + - \ldots -)$[9] so that the $SO(2, n-1)$ invariance is evident. Note that we restrict our analysis to the generic Jordan family (3.4). Other types of scalar manifolds will be discussed in Sec. III G.

Under infinitesimal translations $z^M \to z^M + b^M$, Ω_0 transforms as in (3.23). We noted that (X^0, F_I, X^M) transform among themselves, as do (F_0, X^I, F_M). This time we are exchanging some of X^I with F_I keeping in mind that we are constrained by (3.41). Exchanging all of X^I with F_I will not leave this equation invariant. Therefore we decompose the index I as $I = (1, i)$, swap X^1 with F_1, and keep the other X^i and F_i intact. By looking at (3.23) we see that one must have

$$b^MC_{iMN}X^N = 0 \tag{3.42}$$

in order to keep the translations block-diagonal. This is indeed satisfied for all types of gaugings of the generic Jordan family isometries.

As we discussed earlier, the bare graviphoton in four dimensions is a linear combination of the vectors A^0_μ and A^1_μ which are obtained by reduction from five dimensions. By taking a linear combination of X^0 and F_1 (F_0 and X^1)

[9]In general, the order of the + and − entries depend on the type of gauging, but their numbers are fixed.

for $\tilde{X}^0$ ($\tilde{F}_0$), we isolate the bare graviphoton as $\tilde{A}^0_\mu$. The new symplectic section $\tilde{\Omega}$ is given by the rotation of Ω_0 by

$$\tilde{\mathcal{S}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & \tilde{\Lambda}^M_{1N} & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta^j{}_i & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Lambda}_{1M}{}^N \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_j{}^i & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \end{pmatrix}. \tag{3.43}$$

The rescaling $\tilde{\Lambda}^M_{IN} \equiv \sqrt{2}\Lambda^M_{IN} = \frac{2}{\sqrt{3}}\Omega^{MP}C_{INP}$ is done for future convenience. It is easy to verify that the matrix S is symplectic. More explicitly, we have

$$\tilde{\Omega} = \begin{pmatrix} \tilde{X}^0 \\ \tilde{X}^M \\ \tilde{X}^j \\ \tilde{X}^1 \\ \tilde{F}_0 \\ \tilde{F}_M \\ \tilde{F}_j \\ \tilde{F}_1 \end{pmatrix} = \tilde{\mathcal{S}}\Omega_0 = \tilde{\mathcal{S}} \begin{pmatrix} X^0 \\ X^1 \\ X^i \\ X^N \\ F_0 \\ F_1 \\ F_i \\ F_N \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{6}}(C_{1JK}z^Jz^K + C_{1MN}z^Mz^N) \\ \tilde{\Lambda}^M_{1N}z^N \\ z^i \\ \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}(C_{1JK}z^Jz^K + C_{1MN}z^Mz^N) \\ -\frac{1}{\sqrt{2}}z^1 + \frac{1}{3\sqrt{6}}(C_{IJK}z^Iz^Jz^K + 3C_{IMN}z^Iz^Mz^N) \\ -\frac{2}{\sqrt{3}}\tilde{\Lambda}_{1M}{}^P C_{PNI}z^Nz^I \\ -\frac{1}{\sqrt{3}}C_{iJK}z^Jz^K \\ \frac{1}{\sqrt{2}}z^1 + \frac{1}{3\sqrt{6}}(C_{IJK}z^Iz^Jz^K + 3C_{IMN}z^Iz^Mz^N) \end{pmatrix}. \tag{3.44}$$

Here 0 is now the graviphoton index. The combined infinitesimal $z^M \to z^M + b^M$ translation and infinitesimal $K_{(5)}$ transformation with parameter α^I is generated by the symplectic matrix

$$\tilde{\mathcal{O}} \equiv \tilde{\mathcal{S}}\mathcal{O}\tilde{\mathcal{S}}^{-1} = \mathbb{1}_{2n+2} + \begin{pmatrix} \tilde{B} & \tilde{C} \\ \tilde{C}^T & -\tilde{B}^T \end{pmatrix}, \tag{3.45}$$

with

$$\tilde{B} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}}\tilde{\Lambda}_{1M}{}^P B_{P1} & 0 & 0 \\ \frac{1}{\sqrt{2}}\tilde{\Lambda}^M_{1N}b^N & \alpha^I\Lambda^M_{IN} & 0 & \frac{1}{\sqrt{2}}\tilde{\Lambda}^M_{1N}b^N \\ 0 & 0 & \alpha^I f^k_{Ij} & 0 \\ 0 & -\frac{1}{\sqrt{2}}\tilde{\Lambda}_{1M}{}^P B_{P1} & 0 & 0 \end{pmatrix},$$

$$\tilde{C} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & -\alpha^I f^1_{Ij} & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha^I f^j_{I1} & 0 & 0 & \alpha^I f^j_{I1} \\ 0 & 0 & \alpha^I f^1_{Ij} & 0 \end{pmatrix}, \tag{3.46}$$

where $B_{IM} := -\frac{2}{\sqrt{3}}C_{IMN}b^N$. In order to represent the combined translations and $K_{(5)}$ transformations by block-diagonal matrices, one must have an algebra with $f^1_{Ij} = f^j_{I1} = 0$. Here the index 1 corresponds to the five-dimensional graviphoton, which can only be a gauge field if the gauge group is Abelian because it is a singlet under the action of five-dimensional isometry group $SO(\tilde{n}-1, 1)$. Therefore this condition is automatically satisfied, and hence $\tilde{C} = 0$. Next, by setting $\tilde{B}^C_B = \alpha^A f^C_{AB}$ one can find

$$\begin{aligned} &f^j_{ik}, \qquad f^M_{IN} = \Lambda^M_{IN}, \\ &f^0_{MN} = -f^1_{MN} = -\frac{1}{\sqrt{3}}\Lambda_{1M}{}^P C_{1PN}, \\ &f^M_{N0} = f^M_{N1} = -\Lambda^M_{1N} \end{aligned} \tag{3.47}$$

as nonvanishing components, as well as $\alpha^M = -b^M$.

D. de Sitter vacua

We will now demonstrate how to obtain stable de Sitter vacua by starting with the holomorphic section (3.44). The model to be considered is 4D, $\mathcal{N} = 2$ supergravity coupled to $n = 4$ vector multiplets with gauge group $K_{(4)} = SO(2, 1) \times U(1)_R$. This model can be trivially extended to arbitrary n as we will discuss at the end of this section. Note that this type of gauging was first used in [25,26] to obtain de Sitter vacua where the authors preferred to use Calabi-Vesentini coordinates to parametrize the complex scalars. The mapping between our notation and theirs can be found in Appendix C.

1. Potential $P^{(T)}_{(4)}$ from global isometry gauging

The global isometry group $G_{(4)}$ for the model with 4 vector multiplets is $SU(1, 1) \times SO(2, 3)$. A potential is introduced by gauging the subgroup $SO(2, 1) \subset SO(2, 3)$:

$$P^{(T)}_{(4)} = e^K(\check{X}^A \bar{K}^{\tilde{I}}_A)g_{\tilde{I}\bar{\tilde{J}}}(\bar{\check{X}}^B K^{\tilde{J}}_B). \tag{3.48}$$

The structure constants f^A_{BC} of the $SO(2, 1)$ algebra (3.20) read

$$f^3_{02} = f^2_{03} = -f^3_{20} = -f^2_{30} = 1, \qquad f^0_{32} = -f^0_{23} = 1. \tag{3.49}$$

The gauge fields are the "timelike" $\tilde{A}^0_\mu$, $\tilde{A}^2_\mu$ and the "spacelike" $\tilde{A}^3_\mu$ with respect to $SO(2,3)$ with signature $(++---)$; and the Killing vectors determined by (3.29) and (3.30) are given by

$$\vec{K}_0 = \begin{pmatrix} 0 \\ -w_3 \\ -w_2 \\ 0 \end{pmatrix}, \qquad \vec{K}_2 = \begin{pmatrix} 0 \\ -\frac{1}{\sqrt{2}} w_2 w_3 \\ \frac{1}{2\sqrt{2}}(2 - w_2^2 - w_3^2 + w_4^3) \\ -\frac{1}{\sqrt{2}} w_3 w_4 \end{pmatrix},$$

$$\vec{K}_3 = \begin{pmatrix} 0 \\ \frac{1}{2\sqrt{2}}(2 + w_2^2 + w_3^2 + w_4^3) \\ \frac{1}{\sqrt{2}} w_2 w_3 \\ \frac{1}{\sqrt{2}} w_2 w_4 \end{pmatrix}, \tag{3.50}$$

where we defined $w_{\tilde{I}} \equiv z^{\tilde{I}}$. The full potential term is given in (D1). It simplifies significantly when evaluated at $\mathrm{Re}(w_i) = 0$:

$$P^{(T)}_{(4)}|_{\mathrm{Re}(w_i)=0} = \frac{(\mathrm{Im}w_2^2 - \mathrm{Im}w_3^2)(2 + ||\mathrm{Im}w||^2)^2}{16\,\mathrm{Im}w_1 ||\mathrm{Im}w||^4}, \tag{3.51}$$

with $||\mathrm{Im}w||^2 \equiv (\mathrm{Im}w_2^2 - \mathrm{Im}w_3^2 - \mathrm{Im}w_4^2)$. Note also that this potential term satisfies

$$\frac{\partial P^{(T)}_{(4)}}{\partial \mathrm{Re}(w_i)}\bigg|_{\mathrm{Re}(w_i)=0} = 0. \tag{3.52}$$

2. $U(1)_R$ potential

We are considering a theory with $n = 4$ vector multiplets, and the vector field that gauges the $U(1)_R$ symmetry is $\tilde{A}^1_\mu$. Hence we choose the moment map to be

$$\mathcal{P}^x_A = \delta^{x3}\delta_{A1}. \tag{3.53}$$

Then the $U(1)_R$ potential term is given by

$$P^{(R)}_{(4)} = U^{11} - 3\bar{\tilde{L}}^1\tilde{L}^1 \tag{3.54}$$

with the following definitions:

$$\begin{aligned} \tilde{L}^A &\equiv SL^A \\ U^{AB} &\equiv f^A_{\tilde{I}} f^B_{\bar{\tilde{J}}} g^{\tilde{I}\bar{\tilde{J}}} = -\tfrac{1}{2}(\mathrm{Im}\mathcal{N})^{-1|AB} - \bar{\tilde{L}}^A\tilde{L}^B, \end{aligned} \tag{3.55}$$

$A, B = (0, 2, 3, 4, 1)$.

3. No dRW rotation

For simplicity let us assume no de Roo-Wagemans rotation. One can show that [85]

$$U^{AB} - 3\bar{\tilde{L}}^A\tilde{L}^B = -\frac{\eta^{AB}}{2\,\mathrm{Im}w_1}, \tag{3.56}$$

with $\eta^{AB} := \mathrm{diag}(++---)$. Then the potential (3.54) is

$$P^{(R)}_{(4)} = \frac{1}{2\,\mathrm{Im}w_1} \sim \frac{1}{e^\sigma h^1} \sim e^{-\sigma}||\varphi||^2, \tag{3.57}$$

where an overall positive multiplier is neglected. We note that this potential is proportional to the last term of (3.37), and, because of the diagonality of (3.56), one cannot get a term proportional to the first term by using a linear combination of vectors as the gauge field. One way to interpret this is as follows: Because of the symplectic rotation (3.43), the five-dimensional gauge field $A^1_{\hat{\mu}}$ is decomposed in two parts. One part contributes to the four-dimensional gauge vector $\tilde{A}^1_\mu$ and the other to the four-dimensional bare graviphoton $\tilde{A}^0_\mu$. It is this second part of $A^1_{\hat{\mu}}$ that leads to the first term of (3.37), which does not contribute to the four-dimensional gauge field in this particular choice of the holomorphic section.

4. dRW rotation

The de Roo-Wagemans matrix (3.21) rotates the symplectic section [(3.44), with $n = 4$] to

$$\begin{pmatrix} \frac{1}{2\sqrt{2}}(2 - ||w||^2) \\ w_2 \\ w_3 \\ w_4 \\ \frac{1}{2\sqrt{2}}(2 + ||w||^2)(\cos\theta + w_1\sin\theta) \\ -\frac{1}{2\sqrt{2}} w_1(2 - ||w||^2) \\ -w_1 w_2 \\ w_1 w_3 \\ w_1 w_4 \\ -\frac{1}{2\sqrt{2}}(2 + ||w||^2)(\sin\theta - w_1\cos\theta) \end{pmatrix}, \tag{3.58}$$

where $||w||^2 \equiv [w_2^2 - w_3^2 - w_4^2]$. Using MATHEMATICA we evaluated the potential as

$$P^{(R)}_{(4)} = \frac{|\cos\theta + w_1\sin\theta|^2}{2\,\mathrm{Im}w_1}. \tag{3.59}$$

This potential agrees with [25] by applying the coordinate transformations outlined in Appendix C.

5. Critical points

The total potential of the current model with $n = 4$ vector multiplets and $K_{(4)} = SO(2,1) \times U(1)_R$ gauge group evaluated at $\mathrm{Re}(w_i) = 0$ is given by

$$\begin{aligned} P_{(4)}|_{\mathrm{Re}(w_i)=0} &= (P^{(T)}_{(4)} + \lambda P^{(R)}_{(4)})|_{\mathrm{Re}(w_i)=0} \\ &= \frac{1}{2\,\mathrm{Im}w_1} \\ &\quad\times\bigg(\frac{(\mathrm{Im}w_2^2 - \mathrm{Im}w_3^2)(2 + ||\mathrm{Im}w||^2)^2}{8||\mathrm{Im}w||^4} \\ &\quad + \lambda|\cos\theta + w_1\sin\theta|^2\bigg). \end{aligned} \tag{3.60}$$

The critical points of this potential have coordinates which obey

$$w_1 = -\cot\theta + \frac{i\csc\theta}{\sqrt{\lambda}}, \qquad (\mathrm{Im}w_2)^2 - (\mathrm{Im}w_3)^2 = 2, \qquad \mathrm{Re}w_i = 0, \qquad \mathrm{Im}w_4 = 0, \tag{3.61}$$

and the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\lambda}\sin\theta = \frac{1}{\mathrm{Im}w_1^c}, \tag{3.62}$$

which is positive definite in the physically relevant region[10] ($0 < \theta < \pi$). Writing (3.61) in terms of real scalar fields, we obtain the conditions

$$A_c^i = \varphi_c^4 = 0, \qquad A_c^1 = -\sqrt{3}\cot\theta, \qquad e^{3\sigma_c} = \frac{6\sqrt{6}\csc\theta}{\sqrt{\lambda}}, \qquad [(\varphi_c^2)^2 - (\varphi_c^3)^2] = 6e^{-2\sigma_c}. \tag{3.63}$$

We see that, for a given θ, the values of all of the scalars, including the dilaton σ, at the critical point are fixed. The only exception is that the term $[(\varphi_c^2)^2 - (\varphi_c^3)^2]$ is fixed whereas φ_c^2 and φ_c^3 are not, individually. Observe that this was also the case in five dimensions when the gauge group was $K_{(5)} = SO(1,1) \times U(1)_R$ (cf. Sec. II C 2).

The stability of this family of critical points can be studied by calculating the eigenvalues of the Hessian of the potential evaluated at the extremum. When this is normalized by the inverse of the metric (3.10)

$$g^{\bar{I}\bar{J}}|_{\phi^c} = \begin{pmatrix} 4\,\mathrm{Im}w_1^2 & 0 & 0 & 0 \\ 0 & 4\,(\mathrm{Im}w_2^2 - 1) & 4\,\mathrm{Im}w_2\,\mathrm{Im}w_3 & 0 \\ 0 & 4\,\mathrm{Im}w_2\,\mathrm{Im}w_3 & 4\,(\mathrm{Im}w_2^2 - 1) & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}, \tag{3.64}$$

it gives the mass matrix of the scalar fields

$$\frac{\partial_{\bar{I}}\partial^{\bar{J}}P_{(4)}}{P_{(4)}}\bigg|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\mathrm{Im}w_2^2}{2} & \frac{1}{2}\,\mathrm{Im}w_2\,\mathrm{Im}w_3 & 0 \\ 0 & -\frac{1}{2}\,\mathrm{Im}w_2\,\mathrm{Im}w_3 & -\frac{\mathrm{Im}w_3^2}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.65}$$

with "complex" eigenvalues (2, 1, 1, 0).[11] Thus the family critical points corresponds to stable de Sitter vacua.

One can extend this result to a theory coupled to an arbitrary number $n > 2$ of vector multiplets by trivially extending the holomorphic section, and the value of the potential at the extremum will not change. The mass matrix will contain $n - 3$ diagonal entries with the value 1, and the values of the extra scalars at the extremum will be zero.

E. The five-dimensional connection

Dimensionally reducing a 5D, $\mathcal{N} = 2$ YMESGT with isometry gauging group $K_{(5)}$ yields a 4D, $\mathcal{N} = 2$ YMESGT with an isometry gauging group $K_{(4)} = K_{(5)} \ltimes \mathcal{H}^{n_T+1}$ [70], where n_T is the number of tensor multiplets coupled to the theory and $\mathcal{H}^{n_T+1}$ is the Heisenberg group generated by translations and the central charge. This Heisenberg group factor exist only if tensors are coupled to the theory.

The model discussed in the last section with gauge group $K_{(4)} = SO(2,1) \times U(1)_R$ has stable de Sitter vacua. Unfortunately, it cannot be obtained from five dimensions directly. One can immediately think of gauging a subgroup $K_{(5)} = SO(1,2)$ of the global isometry group for one of the three families (2.6) in five dimensions. For the generic Jordan family, the resulting theory after dimensional reduction still has the gauge group $K_{(4)} = SO(1,2)$. This type of gauging does not yield a scalar potential in five dimensions because tensors are absent from the theory, but it does in four dimensions due to the last term of (3.1). For the magical Jordan family, there will be tensors transforming under $SO(1,2)$, and hence the gauge group in four dimensions is $SO(1,2) \ltimes \mathcal{H}^{n_T+1}$; for the generic non-Jordan family, $SO(1,2)$ is not gaugeable because one cannot find vector fields that transform under the adjoint representation of $SO(1,2)$ to use as the gauge fields. The first two of these families allow for four-dimensional theories with an $SO(1,2)$ factor in the gauge group, but this is not the same $SO(2,1)$ gauge group factor we discussed in the last section. The former one is a subgroup of $SO(1,2) \times SO(1, n-3) \subset SO(2, n-1)$ and has one timelike and two spacelike dimensions, whereas the latter is a subgroup of $SO(2,1) \times SO(n-2) \subset SO(2, n-1)$ and has two timelike and one spacelike dimensions. Therefore the model with the $SO(2,1)$ gauge group factor we discussed in the last section does not originate from five dimensions.[12]

Nevertheless, this is not the end of the story. In five dimensions de Sitter vacua were found for the $SO(1,1) \times U(1)_R$ gauging, and in four dimensions they were found for the $SO(2,1) \times U(1)_R$ gauging. In this section, we will show that under an appropriate group contraction of $SO(2,1)$ one can find a theory which can be obtained from the five-dimensional $SO(1,1) \times U(1)_R$ theory under another appropriate group contraction and that has a potential that allows stable de Sitter ground states.

[10] The imaginary part of w_1 is proportional to $1/||\varphi||^2$, which has to be positive definite in order to have positive kinetic terms in the Lagrangian. See Sec. II C for a more thorough discussion.

[11] The reason why we called these complex eigenvalues is based on the fact that the derivatives $\partial_{\bar{I}}$ are with respect to the complex scalars $z^{\bar{I}}$. The same mass matrix can be obtained by taking the derivatives with respect to $\bar{z}^{\bar{I}}$.

[12] However, this does not rule out the possibility that the $SO(1,2)$ gauging may result in non-Minkowski ground states in four dimensions. See Sec. III F 5 for this type of gauging.

1. Contracting the algebra

A geometrical interpretation for the contraction can be given by introducing the n-dimensional inhomogeneous coordinates u_a ($a = 0, 2, 3, \ldots, n$) that parametrize a hyperboloid embedded in n-dimensional space by $u_a \eta^{ab} u_b = R^2$, where $\eta_{ab} = \mathrm{diag}(+ + - \ldots -)$ and R is the radius of curvature. The scalars $v_k (k = 2, 3, \ldots, n)$ parametrize an $(n-1)$-dimensional hypersurface. This hypersurface is mapped onto the hyperboloid embedded in n-dimensional space by the stereographical projection

$$u_0 = \frac{R^2 - ||v||^2}{R^2 + ||v||^2} R, \qquad u_k = \frac{2R^2 v_k}{R^2 + ||v||^2}, \tag{3.66}$$

where $||v||^2 = [v_2^2 - v_3^2 - \cdots - v_n^2]$. The inverse mapping is

$$v_k = \frac{R u_k}{R + u_0}. \tag{3.67}$$

For $n = 4$, the $SO(2, 1)$ symmetry on the four-dimensional hyperboloid is generated by the Killing vectors, which in terms of homogeneous hypersurface coordinates are formulated by

$$\vec{K}_0 = \begin{pmatrix} 0 \\ -w_3 \\ -w_2 \\ 0 \end{pmatrix}, \qquad \vec{K}_2 = \begin{pmatrix} 0 \\ -\frac{w_2 w_3}{R} \\ \frac{R^2 - w_2^2 - w_3^2 + w_4^2}{2R} \\ -\frac{w_3 w_4}{R} \end{pmatrix}, \qquad \vec{K}_3 = \begin{pmatrix} 0 \\ \frac{R^2 + w_2^2 + w_3^2 + w_4^2}{2R} \\ \frac{w_2 w_3}{R} \\ \frac{w_2 w_4}{R} \end{pmatrix}. \tag{3.68}$$

Note that if the real v_i are extended to the complex w_i and $R = \sqrt{2}$, these are the same Killing vectors we evaluated in (3.50). By taking the large R limit, the hyperboloid is locally flattened and the group is contracted [93,94] to $SO(1,1) \ltimes \mathbb{R}^{(1,1)}$. Let us observe this by defining the new generators as

$$\vec{K}'_0 \equiv \vec{K}_0, \qquad \vec{K}'_2 \equiv \frac{2\vec{K}_2}{R}, \qquad \vec{K}'_3 \equiv \frac{2\vec{K}_3}{R} \tag{3.69}$$

and evaluating the Lie brackets

$$[\vec{K}'_0, \vec{K}'_2] = \vec{K}'_3, \qquad [\vec{K}'_0, \vec{K}'_3] = \vec{K}'_2, \qquad [\vec{K}'_2, \vec{K}'_3] = -\frac{4}{R^2} \vec{K}'_0. \tag{3.70}$$

By taking the limit $R \to \infty$, the last of these Lie brackets vanishes, and we see that the new Killing vectors generate the Lie algebra of the Poincaré group in two dimensions which is the semidirect product of "Lorentz boosts" $SO(1, 1)$ with "translations" $\mathbb{R}^{(1,1)}$.

Meanwhile, for the five-dimensional gauge group $K_{(5)} = SO(1, 1)$, the structure constants (3.47) determine the following algebra in four dimensions:

$$\left[\frac{T_0 - T_1}{\sqrt{2}}, T_2\right] = 0, \qquad \left[\frac{T_0 + T_1}{\sqrt{2}}, T_3\right] = T_2,$$
$$\left[\frac{T_0 - T_1}{\sqrt{2}}, T_3\right] = 0, \qquad \left[\frac{T_0 + T_1}{\sqrt{2}}, T_2\right] = T_3, \tag{3.71}$$
$$[T_2, T_3] = \frac{T_0 - T_1}{\sqrt{2}}.$$

They define the Lie algebra of a central extension of the Lie algebra $SO(1, 1) \circledS \mathbb{R}^{(1,1)}$, with the central charge corresponding to the generator $\frac{1}{\sqrt{2}}(T_0 - T_1)$. Here "$\circledS$" denotes "semidirect sum." $\frac{1}{\sqrt{2}}(T_0 + T_1)$ rotates T_2 and T_3 into each other and corresponds to the bare graviphoton in five dimensions which acted as the $SO(1, 1)$ gauge field. Note that this result parallels completely the situation in Sec. III C 1 (cf. [70]).

By defining the new generators

$$\begin{pmatrix} W_0 \\ W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ -\beta & \beta & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix} \begin{pmatrix} (T_0 - T_1)/\sqrt{2} \\ (T_0 + T_1)/\sqrt{2} \\ T_2 \\ T_3 \end{pmatrix}, \tag{3.72}$$

one can rewrite the algebra as

$$[W_2, W_3] = \tfrac{1}{2}(\beta^2 W_0 - \beta W_1), \qquad [W_0, W_2] = W_3,$$
$$[W_0, W_3] = W_2, \qquad [W_1, W_2] = \beta W_3, \tag{3.73}$$
$$[W_1, W_3] = \beta W_2.$$

In the limit $\beta \to 0$ the transformation matrix above becomes noninvertible, but this is expected since information is generically lost during group contractions, and the algebra reduces to $SO(1, 1) \circledS \mathbb{R}^{(1,1)}$ without central charge. This is the same algebra as (3.70) in the large R region. Thus the two different limits of the two different theories overlap. Now, we will calculate the extrema of the scalar potential that they will generate.

2. Potential by $(SO(1,1) \ltimes \mathbb{R}^{(1,1)}) \times U(1)_R$ *gauging*

Using the Killing vectors (3.69) in the large R limit, the potential (3.48) is calculated as in (D2). When evaluated at $\mathrm{Re}(w_k) = 0$ ($k = 2, 3, \ldots, n$), it takes the form

$$P_{(4)}^{(T)}|_{\mathrm{Re}(w_k)=0} = \frac{(\mathrm{Im} w_2^2 - \mathrm{Im} w_3^2)(4 + ||\mathrm{Im} w||^2)^2}{64\, \mathrm{Im} w_1 ||\mathrm{Im} w||^4}, \tag{3.74}$$

where $||\mathrm{Im} w||^2 \equiv (\mathrm{Im} w_2^2 - \mathrm{Im} w_3^2 - \cdots - \mathrm{Im} w_n^2)$. This potential term satisfies

$$\left.\frac{\partial P_{(4)}^{(T)}}{\partial \mathrm{Re}(w_k)}\right|_{\mathrm{Re}(w_k)=0} = 0. \tag{3.75}$$

The dRW rotation is done prior to the gauging. One must choose the $U(1)_R$ gauge field $\tilde{A}^b_\mu$ among $\tilde{A}^i_\mu$ $(i = 4, \ldots, n)$[13] and dRW rotate $\tilde{X}^b$ and $\tilde{F}_b$ into each other. The dRW matrix is given by

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_{n-1} & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & 0 & \sin\theta & 0 \\ 0 & 0 & \mathbb{1}_n & 0 & 0 \\ 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.76}$$

where we chose $b = n$. Note that with this type of gauging one must have $n > 3$ vector multiplets [cf. $n > 2$ for the $SO(2, 1) \times U(1)_R$ gauging after the dRW rotation (3.21)]. The calculation of the $U(1)_R$ potential is similar to the last case, but it has the same expression

$$P^{(R)}_{(4)} = \frac{|\cos\theta + w_1 \sin\theta|^2}{2\,\mathrm{Im} w_1}. \tag{3.77}$$

The critical points of the total potential $P_{(4)} = P^{(T)}_{(4)} + \lambda P^{(R)}_{(4)}$ are given by

$$w^c_1 = -\cot\theta + \frac{i\csc\theta}{\sqrt{2\lambda}}, \qquad (\mathrm{Im} w^c_2)^2 - (\mathrm{Im} w^c_3)^2 = 4,$$
$$\mathrm{Re} w^c_k = 0, \qquad \mathrm{Im} w^c_i = 0, \tag{3.78}$$

and the value of the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\frac{\lambda}{2}}\sin\theta = \frac{1}{2\,\mathrm{Im} w^c_1}. \tag{3.79}$$

Writing these in terms of real scalars, we again see that, for a given θ, the values of all of the scalars, including the dilaton σ, at the critical point are fixed. The only exception is that the term $[(\varphi^2_c)^2 - (\varphi^3_c)^2]$ is fixed whereas φ^2_c and φ^3_c are not, individually.

The mass matrix for this potential evaluated at the family of critical points is

$$\frac{\partial_{\bar{I}}\partial^{\bar{J}} P_{(4)}}{P_{(4)}}\bigg|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\mathrm{Im} w_2^2}{4} & \frac{1}{4}\,\mathrm{Im} w_2\,\mathrm{Im} w_3 & 0 \\ 0 & -\frac{1}{4}\,\mathrm{Im} w_2\,\mathrm{Im} w_3 & -\frac{\mathrm{Im} w_3^2}{4} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{3.80}$$

which has eigenvalues (2, 1, 1, 0) and hence the extrema correspond to stable de Sitter vacua. The zero eigenvalue is due to the remaining $SO(1, 1)$ symmetry and means that there is a flat direction along the extrema.

The effect of group contraction to the potential.—Without the contraction outlined in the last subsection, i.e. using the structure constants of the algebra (3.71), the potential $P^{(T)}_{(4)}$ evaluated at $\mathrm{Re}(w_i) = 0$ is given by

$$\frac{\mathrm{Im} w_2^2 - \mathrm{Im} w_3^2}{2\,\mathrm{Im} w_1 ||\mathrm{Im} w||^4}. \tag{3.81}$$

Subtracting this expression from (3.74) will give the contribution of the group contraction to the scalar potential as

$$\frac{(\mathrm{Im} w_2^2 - \mathrm{Im} w_3^2) P_+ P_-}{64\,\mathrm{Im} w_1 ||\mathrm{Im} w||^4}, \tag{3.82}$$

where $P_\pm = ||\mathrm{Im} w||^2 + 4(1 \pm \sqrt{2})$. This term is positive definite in the neighborhood of the extrema, where $||\mathrm{Im} w||^2 \sim 4$. A quick calculation shows that the $P^{(T)}_{(4)}$ potential (3.81), together with the $P^{(R)}_{(4)}$ potential (3.77), does not have any critical points.

F. More examples

1. $(SO(1, 1) \ltimes \mathbb{R}^{(1,1)}) \times SU(2)_R$ gauging

In order to do such a gauging along with a dRW rotation, one must have $n > 5$ vector multiplets. $P^{(T)}$ is as given in (3.74).

For the $SU(2)_R$ gauging, the moment map is as defined in (3.35), and the gauge fields are chosen to be $\tilde{A}^b_\mu$ $(b = n-2, n-1, n)$. $\tilde{X}^b$ and $\tilde{F}_b$ are rotated into each other via the dRW matrix

$$\mathcal{R} = \begin{pmatrix} \mathbb{1}_{n-3} & 0 & 0 & 0 & 0 \\ 0 & \cos\theta \mathbb{1}_3 & 0 & \sin\theta \mathbb{1}_3 & 0 \\ 0 & 0 & \mathbb{1}_{n-2} & 0 & 0 \\ 0 & -\sin\theta \mathbb{1}_3 & 0 & \cos\theta \mathbb{1}_3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3.83}$$

The resulting $SU(2)_R$ potential is given by

$$P^{(R)}_{(4)} = (U^{(AB)} - 3\bar{\tilde{L}}^{(A}\tilde{L}^{B)})\mathcal{P}^x_A \mathcal{P}^x_B$$
$$= \sum_{y=n-2}^{n} (U^{(yy)} - 3\bar{\tilde{L}}^{(y}\tilde{L}^{y)}) = \frac{3|\cos\theta + w_1 \sin\theta|^2}{2\,\mathrm{Im} w_1}, \tag{3.84}$$

which differs from (3.77) only by a factor of 3. Each $SU(2)_R$ generator gives the same contribution as the $U(1)_R$ generator in the Abelian case. The total potential is

$$P_{(4)}|_{\mathrm{Re}(w_k)=0} = (P^{(T)}_{(4)} + \lambda P^{(R)}_{(4)})|_{\mathrm{Re}(w_k)=0}$$
$$= \frac{(\mathrm{Im} w_2^2 - \mathrm{Im} w_3^2)(4 + ||\mathrm{Im} w||^2)^2}{64\,\mathrm{Im} w_1 ||\mathrm{Im} w||^4} + \frac{3\lambda|\cos\theta + w_1 \sin\theta|^2}{2\,\mathrm{Im} w_1}. \tag{3.85}$$

The critical points of the total potential $P_{(4)} = P^{(T)}_{(4)} + \lambda P^{(R)}_{(4)}$ are given by

[13] Choosing $\tilde{A}^1_\mu$ as the $U(1)_R$ gauge field as we did in Sec. III D 4 would result in rotating $\tilde{X}^1$ and $\tilde{F}_1$ into each other. But in this case, the presence of tensors makes it impossible to keep the translations block-diagonal.

$$w_1^c = -\cot\theta + \frac{i\csc\theta}{\sqrt{6\lambda}}, \qquad (\mathrm{Im}w_2^c)^2 - (\mathrm{Im}w_3^c)^2 = 4, \qquad \mathrm{Re}w_k^c = 0, \qquad \mathrm{Im}w_i^c = 0, \tag{3.86}$$

and the value of the potential evaluated at these points is

$$P_{(4)}|_{\phi^c} = \sqrt{\frac{3\lambda}{2}}\sin\theta = \frac{1}{2\,\mathrm{Im}w_1^c}. \tag{3.87}$$

Writing these in terms of real scalars, we again see that, for a given θ, the values of all of the scalars, including the dilaton σ, at the critical point are fixed. The only exception is that the term $[(\varphi_c^2)^2 - (\varphi_c^3)^2]$ is fixed whereas φ_c^2 and φ_c^3 are not, individually.

The mass matrix for this potential evaluated at the family of critical points is

$$\frac{\partial_{\tilde{I}}\partial^{\tilde{J}}P_{(4)}}{P_{(4)}}\Bigg|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & \frac{\mathrm{Im}w_2^2}{4} & \frac{1}{4}\mathrm{Im}w_2\,\mathrm{Im}w_3 & 0 \\ 0 & -\frac{1}{4}\mathrm{Im}w_2\,\mathrm{Im}w_3 & -\frac{\mathrm{Im}w_3^2}{4} & 0 \\ 0 & 0 & 0 & \mathbb{1}_{n-3} \end{pmatrix}, \tag{3.88}$$

which has eigenvalues

$$(2, \underbrace{1, \ldots, 1}_{(n-2)\text{ times}}, 0),$$

and hence the extrema correspond to stable de Sitter vacua with a flat direction due to the remaining $SO(1,1)$ symmetry.

2. $(SO(1,1)\ltimes\mathbb{R}^{(1,1)}) \times U(1)_R$ *gauging with hypers*

The authors of [25] studied a model with 5 vector multiplets and 4 hypermultiplets; the scalars of the hypermultiplets spanned the hyperbolic space $\frac{SO(4,2)}{SO(4)\times SO(2)}$, and the gauge group was $SO(2,1)\times SU(2)$. Here we shall consider the coupling of a single hypermultiplet to supergravity and an arbitrary number n of vector multiplets. We use the same symmetric space $\mathcal{M}_Q = \frac{SU(2,1)}{SU(2)\times U(1)}$ formalism for the scalar manifold of a single hypermultiplet that we also studied on five dimensions in Sec. II. The scalars that span this space are $q^X = (V, \theta, \tau, \sigma)$. Gauging $(SO(1,1)\ltimes\mathbb{R}^{(1,1)}) \times U(1)_R$ gives three contributions to the scalar potential.

$P_{(4)}^{(T)}$ is not affected by the hypercoupling, so we take it as given in (3.74). Meanwhile, gauging hyperisometries introduces the potential term (A26), which is written in the canonical form as [85,95]

$$P_{(4)}^{(H)} = 4e^{\mathcal{K}}(K_A^X\bar{\tilde{X}}^A)g_{XY}(K_B^Y\tilde{X}^B), \tag{3.89}$$

with $K_A^X = V_A Y^a T_a^X$ ($a = 1, 2, 3$), where V_A determine the linear combination of vectors to use as the $U(1)_R$ gauge field. Y^a, on the other hand, determine the linear combination of the hyperisometries T_a^X that are gauged. T_a^X, the Killing vectors that generate the symmetries of the isometry group $SU(2,1)$, are given in Appendix B. At the base point $q^c = (V = 1, \theta = \tau = \sigma = 0)$ of the hyperscalar manifold, where the hyperspace metric g_{XY} (2.9) becomes diagonal, this potential satisfies

$$P_{(4)}^{(H)}|_{q^c} = \frac{\partial P_{(4)}^{(H)}}{\partial w_{\tilde{I}}}\Bigg|_{q^c} = \frac{\partial P_{(4)}^{(H)}}{\partial q}\Bigg|_{q^c} = 0 \tag{3.90}$$

because of the vanishing Killing vectors at that point. The third contribution is the $U(1)_R$ potential

$$P_{(4)}^{(R)} = (U^{(AB)} - 3\bar{\check{L}}^{(A}\check{L}^{B)})\vec{\mathcal{P}}_A\vec{\mathcal{P}}_B, \tag{3.91}$$

where the momentum map is written in terms of the Killing prepotentials as $\vec{\mathcal{P}}_A = V_A Y^a \vec{P}_a$. We choose $\tilde{A}_\mu^n$ as the $U(1)_R$ gauge field and set $V_n = 1$. The dRW-rotation matrix that mixes the electric and the magnetic components of the holomorphic section is given in (3.76).

The total potential

$$\begin{aligned} P_{(4)}|_{\mathrm{Re}(w_i)=0,q^c} &= [P_{(4)}^{(T)} + \lambda(P_{(4)}^{(R)} + P_{(4)}^{(H)})]_{\mathrm{Re}(w_k)=0,q^c} \\ &= \frac{(\mathrm{Im}w_2^2 - \mathrm{Im}w_3^2)(4 + \|\mathrm{Im}w\|^2)^2}{64\,\mathrm{Im}w_1\|\mathrm{Im}w\|^4} \\ &\quad + \frac{\lambda|\cos\theta + w_1\sin\theta|^2(Y^aY^a)}{8\,\mathrm{Im}w_1} \end{aligned}$$

has extrema at

$$\phi^c = \Bigg\{ w_1^c = -\cot\theta + \frac{i\sqrt{2}\csc\theta}{\sqrt{\lambda(Y^aY^a)}}, \quad (\mathrm{Im}w_2^c)^2 - (\mathrm{Im}w_3^c)^2 = 4, \quad \mathrm{Re}w_k^c = 0, \quad \mathrm{Im}w_i^c = 0, \quad V^c = 1, \quad \theta^c = \tau^c = \sigma^c = 0\Bigg\},$$

where it takes the value

$$P_{(4)}|_{\phi^c} = \frac{\sqrt{\lambda(Y^aY^a)}}{2\sqrt{2}}\sin\theta = \frac{1}{2\,\mathrm{Im}w_1^c}.$$

The values of all of the scalars at the critical point are fixed, except w_2 and w_3, which satisfy $(\mathrm{Im}w_2^c)^2 - (\mathrm{Im}w_3^c)^2 = 4$. This remaining $SO(1,1)$ symmetry leads to a flat direction along the extrema. Joining the scalar indices $\zeta = (\tilde{I}, X)$, the expression for the mass matrix is written as

$$\frac{\partial_\zeta \partial^\xi P_{(4)}}{P_{(4)}}\Bigg|_{\phi^c} = \begin{pmatrix} 2 & 0 & 0 & 0 & & & & \\ 0 & \frac{\mathrm{Im}w_2^2}{4} & \frac{1}{4}\mathrm{Im}w_2\,\mathrm{Im}w_3 & 0 & \dots & & & \\ 0 & -\frac{1}{4}\mathrm{Im}w_2\mathrm{Im}w_3 & -\frac{\mathrm{Im}w_3^2}{4} & 0 & & & & \\ 0 & 0 & 0 & \mathbb{1}_{n-3} & & & & \\ & \vdots & & & \frac{1}{4} & & & \\ & & & & & 1 & & \\ & & & & & & \frac{1}{2} & \\ & & & & & & & \frac{1}{8} \end{pmatrix},$$

where the last 4 entries belong to the hypers. This matrix has all non-negative eigenvalues

$$(2, \underbrace{1, \ldots, 1}_{(n-2)\ \text{times}}, 0, \tfrac{1}{4}, 1, \tfrac{1}{2}, \tfrac{1}{8}),$$

where again the last 4 entries are the masses of the hyperscalars, and hence the extrema correspond to stable de Sitter vacua.

3. Yet another holomorphic section?

Applying a symplectic transformation

$$\tilde{X}^A \rightarrow \tilde{F}_A, \qquad \tilde{F}_A \rightarrow -\tilde{X}^A \tag{3.92}$$

on the holomorphic section [(3.44), with $n = 4$], acting on it with the dRW matrix (3.76), and gauging $K_{(4)} = (SO(1,1)\ltimes\mathbb{R}^{(1,1)}) \times U(1)_R$ lead to the scalar potential

$$\begin{aligned} P_{(4)}|_{\mathrm{Re}(w_k)=0} &= [P_{(4)}^{(T)} + \lambda P_{(4)}^{(R)}]_{\mathrm{Re}(w_k)=0} \\ &= \frac{(\mathrm{Im}w_2^2 - \mathrm{Im}w_3^2)(4 + \|\mathrm{Im}w\|^2)^2(\mathrm{Re}w_1^2 + \mathrm{Im}w_1^2)}{64\,\mathrm{Im}w_1\|\mathrm{Im}w\|^4} \\ &\quad + \lambda\frac{|\sin\theta - w_1\cos\theta|^2}{2\,\mathrm{Im}w_1}, \end{aligned} \tag{3.93}$$

which has critical points at

$$w_1^c = \frac{\sqrt{2\lambda}\sin\theta}{1 + 2\lambda\cos^2\theta}(\sqrt{2\lambda}\cos\theta + i),$$
$$(\mathrm{Im}w_2^c)^2 - (\mathrm{Im}w_3^c)^2 = 4, \qquad \mathrm{Re}w_k^c = \mathrm{Im}w_i^c = 0.$$

At this family of critical points, the potential takes the value of

$$P_{(4)}|_{\phi^c} = \sqrt{\frac{\lambda}{2}}\sin\theta, \tag{3.94}$$

and the mass matrix $\frac{\partial_{\bar{I}}\partial^{\bar{J}}P_{(4)}}{P_{(4)}}$ has eigenvalues (2, 1, 1, 0); hence, this corresponds to stable dS vacua with a flat direction for $0 < \theta < \pi$.

Observe that, when $\theta = \pi/2$, apart from the $X^0\&F_1$ and $X^1\&F_0$ mixing in the transformation (3.43), this corresponds to the GMZ holomorphic section that we introduced in Sec. III C 1. Although this seems like just a specific case, it will play an important role in finding stable de Sitter vacua when we study general homogeneous scalar manifolds below.

4. de Sitter vacua from GMZ holomorphic section

The procedure of obtaining de Sitter ground states using the GMZ holomorphic section, which is obtained by acting on Ω_0 by the transformation matrix (3.28), involves a similar group contraction. Consider a five-dimensional $SO(1,1)$ gauged YMESGT coupled to $\tilde{n} = 3$ vector multiplets that has $A^1_{\hat{\mu}}$ as the gauge field. The vectors $A^2_{\hat{\mu}}$ and $A^3_{\hat{\mu}}$ are charged under the gauge group and need to be dualized to tensors. After the dimensional reduction, this becomes a four-dimensional theory coupled to $n = 4$ vector multiplets with a gauge group $K_{(4)} = SO(1,1)\ltimes\mathbb{R}^{(1,1)}$ with central charge [70]. The structure constants are

$$f^0_{23} = -\sqrt{2}, \qquad f^2_{13} = f^3_{12} = \frac{1}{\sqrt{2}}, \tag{3.95}$$

which are antisymmetric in the lower indices. With these structure constants, one can calculate the Killing vectors (3.29) that generate $K_{(4)}$ as

$$\begin{aligned} &\vec{K}_0 = 0, \qquad \vec{K}_1 = \begin{pmatrix} 0 \\ w_3/\sqrt{2} \\ w_2/\sqrt{2} \\ 0 \end{pmatrix}, \\ &\vec{K}_2 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}, \qquad \vec{K}_1 = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}. \end{aligned} \tag{3.96}$$

The contraction will be done by going to a basis with the following rotation of the Killing vectors:

$$\vec{K}'_0 = \vec{K}_0 - \vec{K}_1, \qquad \vec{K}'_1 = \vec{K}_0 + \vec{K}_1,$$
$$\vec{K}'_2 = \vec{K}_2, \qquad \vec{K}'_3 = \vec{K}_3. \tag{3.97}$$

It is straightforward to show that the new Killing vectors generate $SO(1,1)\ltimes\mathbb{R}^{(1,1)}$ without central charge. After some calculation we found that the potential $P^{(T)}$, defined in (3.48), is indeed equal to the $P^{(T)}$ given in (3.93).

In addition to $K_{(4)}$, one can also gauge the $U(1)_R$ symmetry. Choosing the gauge field as A^4_μ, this will result in a potential term $P^{(R)}_{(4)} = 1/(2\,\mathrm{Im}w_1)$ that is the $P^{(R)}_{(4)}$ given in (3.93) when $\theta = \pi/2$. The calculation in the last subsection shows that the total potential $P_{(4)} = P^{(T)}_{(4)} + \lambda P^{(R)}_{(4)}$ has de Sitter minima with a flat direction.

5. $SO(1,2)$ *gauging from five dimensions*

One can start with a gauged YMESGT in five dimensions with an isometry gauging group $K_{(5)} = SO(1,2)$. For the generic Jordan family, the only charged vector fields are the gauge fields $A^2_{\hat\mu}$, $A^3_{\hat\mu}$, and $A^4_{\hat\mu}$, which transform under the adjoint representation of this $SO(1,2)$. There are no tensor fields, and no scalar potential is introduced.

After the dimensional reduction, the gauge group is still $SO(1,2)$, but this is a different subgroup of the global isometry group in four dimensions than what we gauged in Sec. III D. The former one is a subgroup of $SO(1,1)\times SO(1,n-2)\subset SO(2,n-1)$ and has one timelike and two spacelike dimensions, whereas the latter is a subgroup of $SO(2,1)\times SO(n-2)\subset SO(2,n-1)$ and has two timelike and one spacelike dimensions. In contrast to the case before the dimensional reduction, gauging $SO(1,2)$ results in a scalar potential in four dimensions due to the second term in (A25). Taking the structure constants as $f^2_{34} = -f^3_{42} = -f^4_{23} = 1$, this potential is evaluated to be

$$P^{(T)}_{(4)} = \frac{Q_{23} + Q_{24} - Q_{34}}{2\,\mathrm{Im}w_1 ||\mathrm{Im}w||^4}, \tag{3.98}$$

with $Q_{kl} = (w_k\bar{w}_l - w_l\bar{w}_k)^2$. Unfortunately, this potential does not admit any ground states other than Minkowskian. One can gauge $U(1)_R$ (in four dimensions) in addition to the $SO(1,2)$ symmetry which adds the term (3.77) to the potential. But it is easy to verify that the total potential does not have any critical points in this case.

At this point, perhaps it is worth mentioning again that the four-dimensional theories that have different holomorphic sections as their starting points, which are related by just a symplectic transformation, describe different physics. For the generic Jordan family, an $K_{(5)} = SO(1,2)\times U(1)_R$ gauged YMESGT has Minkowski and anti-de Sitter ground states in five dimensions. The Minkowski ground states survive in four dimensions if one works with the GMZ holomorphic section, due to a term in the $U(1)_R$ potential that does not exist in the potential that is derived from our holomorphic section. We stressed this issue below Eq. (3.57).

G. Beyond generic Jordan family

For the holomorphic section we obtained by the rotation (3.43), satisfying Eq. (3.42) was crucial to keep the translations block-diagonal. This equation is trivially satisfied for the generic Jordan family ($C_{iMN} = 0$), but for other types of scalar manifolds, such as the magical Jordan family, it does not hold, in general. This problem can be evaded by dRW rotating all $\tilde{X}^i$ and $\tilde{F}_i$ by $\pi/2$ radians. The entire symplectic transformation matrix, including the dRW rotation with $\theta = \pi/2$,

$$\check{\mathcal{S}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D^{MN} \\ 0 & 0 & \delta^j_i & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 & D_{MN} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta^i_j & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix} \tag{3.99}$$

acts on Ω_0 in the following way:

$$\check{\Omega} = \begin{pmatrix} \check{X}^0 \\ \check{X}^M \\ \check{X}^j \\ \check{X}^1 \\ \check{F}_0 \\ \check{F}_M \\ \check{F}_j \\ \check{F}_1 \end{pmatrix} = \check{\mathcal{S}}\Omega_0 = \check{\mathcal{S}} \begin{pmatrix} X^0 \\ X^1 \\ X^i \\ X^N \\ F_0 \\ F_1 \\ F_i \\ F_N \end{pmatrix}.$$

Here $D_{MN} = -\sqrt{2}\Omega_{MN}$ and $D_{MN}D^{NP} = \delta^P_M$, and, again, we decomposed the index I as $I = (1, i)$.

Furthermore, in order to gauge $\check{K} \equiv K_{(4)} \times U(1)_R = (SO(1,1)\ltimes\mathbb{R}^{(1,1)})\times U(1)_R$ which was the four-dimensional gauge group for the theories with de Sitter solutions that originate from five dimensions, the isometry group needs to contain a subgroup $SO(2, r-1)$, with $r \geq 3$.

So far, we have studied *symmetric* space scalar manifolds only. Now we relax this restriction and look for *homogeneous* (but not necessarily symmetric) space scalar manifolds that have $SO(2, r-1)$, $r\geq 3$, as a subsector. We have to reanalyze how the holomorphic section transforms under the translations $z^M \to z^M + b^M$ because C_{IJM} does not necessarily vanish in homogeneous spaces:

$$\check{\Omega} \to \check{\Omega} + \begin{pmatrix} \frac{1}{\sqrt{2}} b^M D_{MN} \check{X}^N \\ \sqrt{\frac{2}{3}} D^{MN} b^P \{C_{1NP}(\check{X}^1 - \check{X}^0) + \sqrt{2} C_{iNP} \check{X}^i\} \\ 0 \\ \frac{1}{\sqrt{2}} b^M D_{MN} \check{X}^N \\ -\sqrt{\frac{2}{3}} b^M \{C_{1MN} D^{NP} \check{F}_P + C_{11M} \frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{1jM} \check{X}^j\} \\ \frac{1}{\sqrt{2}} D_{MN} b^N (\check{F}_0 + \check{F}_1) \\ -\sqrt{\frac{2}{3}} b^M \{C_{iMN} D^{NP} \check{F}_P + C_{i1M} \frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{ijM} \check{X}^j\} \\ -\sqrt{\frac{2}{3}} b^M \{C_{1MN} D^{NP} \check{F}_P + C_{11M} \frac{\check{X}^0 - \check{X}^1}{\sqrt{2}} - C_{1jM} \check{X}^j\} \end{pmatrix}.$$

Observe that, in order to keep the translations block-diagonal, i.e. to have $\check{X}^A$ and $\check{F}_A$ transform among themselves,

$$C_{IJM} \stackrel{!}{=} 0 \tag{3.100}$$

must hold.

de Wit and Van Proeyen classified homogeneous very special manifolds and gave their corresponding cubic polynomials in [96,97]. These spaces are of the form G/H, where G is the isometry group and H is its isotropy subgroup. G is not necessarily semisimple; thus not all of the homogeneous spaces have a clear name. In their classification, the homogeneous spaces are denoted as $L(q, P)$. Here q characterizes the real Clifford algebras [$\mathcal{C}(q+1, 0)$] that are in one-to-one correspondence with homogeneous special manifolds. These have signatures $(q+1, 1)$ for real (in five dimensions) and $(q+2, 2)$ for Kähler (in four dimensions) manifolds, which are related to each other with what is called the **r** map. The non-negative integer P denotes the multiplicity of the representation of the Clifford algebra. For $q \neq 4m$ (m is a non-negative integer), P is unique. When $q = 4m$, there are two inequivalent representations. In this case the homogeneous space is denoted by $L(4m, P, \dot{P})$. Note that $L(4m, P, 0) = L(4m, 0, P) \equiv L(4m, P)$. Table I lists the special cases where $L(q, P)$ are symmetric manifolds. The cubic polynomial that has an invariance group that acts transitively on the special real manifolds can be specified in the general form

$$\begin{aligned} N(h) &= C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} \\ &= 3\{\hat{h}^1 (\hat{h}^2)^2 - \hat{h}^1 (\hat{h}^\beta)^2 - \hat{h}^2 (\hat{h}^m)^2 + \gamma_{\beta mn} \hat{h}^\beta \hat{h}^m \hat{h}^n\}, \end{aligned} \tag{3.101}$$

where the index $\tilde{I} = 1, \ldots, n$ is decomposed into $I = 1, 2, \beta, m$, with $\beta = 3, \ldots, (q+3)$ and $m = (q+4), \ldots, (q+3+(P+\dot{P})\mathcal{D}_{q+1} = n)$. The dimension $\mathcal{D}_{q+1}$ of the irreducible representation of the Clifford algebra with positive signature in $q+1$ dimensions is given by

TABLE I. *Symmetric very special manifolds.* $L(-1, P)$, which correspond to the generic non-Jordan family, are symmetric in five dimensions but not their images under the **r** map. $L(0, P)$ is the generic Jordan family, and the last 4 entries are the magical Jordan family manifolds. The number n is the complex dimension of the Kähler space, which also is the number of vector multiplets in four dimensions. This table is adapted from [96].

$L(q, P)$	n	Real	Kähler
$L(-1, 0)$	2	$SO(1, 1)$	$[\frac{SU(1,1)}{U(1)}]^2$
$L(-1, P)$	$2 + P$	$\frac{SO(P+1,1)}{SO(P+1)}$	
$L(0, P)$	$3 + P$	$SO(1, 1) \times \frac{SO(P+1,1)}{SO(P+1)}$	$\frac{SU(1,1)}{U(1)} \times \frac{SO(P+2,2)}{SO(P+2)\times SO(2)}$
$L(1, 1)$	6	$\frac{S\ell(3,R)}{SO(3)}$	$\frac{Sp(6)}{U(3)}$
$L(2, 1)$	9	$\frac{S\ell(3,C)}{SU(3)}$	$\frac{SU(3,3)}{SU(3)\times SU(3)\times U(1)}$
$L(4, 1)$	15	$\frac{SU^*(6)}{Sp(3)}$	$\frac{SO^*(12)}{SU(6)\times U(1)}$
$L(8, 1)$	27	$\frac{E_6}{F_4}$	$\frac{E_7}{E_6\times U(1)}$

$$\mathcal{D}_{q+1} = 1 \quad \text{for } q = -1, 0, \qquad \mathcal{D}_{q+1} = 2 \quad \text{for } q = 1,$$
$$\mathcal{D}_{q+1} = 4 \quad \text{for } q = 2, \qquad \mathcal{D}_{q+1} = 8 \quad \text{for } q = 3, 4,$$
$$\mathcal{D}_{q+1} = 16 \quad \text{for } q = 5, 6, 7, 8, \qquad \mathcal{D}_{q+8} = 16\mathcal{D}_q.$$

The constraint $r \geq 3$ translates into $q \geq 0$. Hence we immediately see that $\check{K}$ is not gaugeable for generic non-Jordan family $L(-1, P)$. Let us investigate the cases ($q = 0$) and ($q > 0$) separately.

Case 1 ($q = 0$).—If either of P or $\dot{P}$ vanishes, the homogeneous space corresponds to the symmetric generic Jordan family, which we have studied already. For non-vanishing P and $\dot{P}$, one can write the cubic polynomial as

$$N(h) = 3\{h^1[(h^2)^2 - (h^3)^2 - (h^x)^2] - (h^2 - h^3)(h^{\dot{x}})^2\} \tag{3.102}$$

after the reparametrization

$$\hat{h}^1 = h^2 + h^3, \qquad \hat{h}^2 = \frac{h^1 + h^2 - h^3}{2},$$
$$\hat{h}^3 = \frac{-h^1 + h^2 - h^3}{2}, \qquad \hat{h}^x = h^x, \qquad \hat{h}^{\dot{x}} = h^{\dot{x}},$$

where the index m is decomposed into P indices x and $\dot{P}$ indices $\dot{x}$. The fields h^2 and h^3 are charged under the gauge group $K_{(4)}$, and the corresponding vector fields A^2_μ and A^3_μ need to be dualized to tensor fields. Hence the index $\tilde{I} = (I, M)$ is split as follows: $I = 1, x, \dot{x}$; $M = 2, 3$. But then $C_{\dot{x}\dot{y}M} \neq 0$, and hence the translations will not remain block-diagonal; i.e. $K_{(4)}$ is not gaugeable in the standard way.

Case 2 ($q > 0$).—All of these spaces $L(q > 0, P, \dot{P})$, which also include the symmetric magical Jordan family for ($q = 1, 2, 4, 8$; $P = 1$), contain $K_{(4)} = SO(1,1) \ltimes \mathbb{R}^{(1,1)}$ subsectors. Consider the cubic form in the most general form as given in (3.101). Choosing A^1_μ as the gauge field, one can find a $K_{(4)}$ generator such that $\hat{h}^2$ and $\hat{h}^3$ rotate into each other keeping $(\hat{h}^2)^2 - (\hat{h}^3)^2$ fixed. Because they are charged under the gauge group, the corresponding vector fields need to be dualized to tensor fields. The rest of $\hat{h}^\beta$ are $K_{(4)}$ singlets, and a linear combination of the corresponding vector fields can be used as the $U(1)_R$ [or even $SU(2)_R$ if $q \geq 3$] gauge field(s). h^m form $(P + \dot{P})\mathcal{D}_{q+1}/2$ doublets under $K_{(4)}$, and their corresponding vector fields are dualized to tensor fields. All of the conditions are satisfied, and we conclude that the homogeneous spaces of the type $L(q > 0, P)$ admit stable de Sitter vacua when $\check{K}$ is gauged.

IV. DISCUSSIONS

Stable de Sitter vacua of 4D, $\mathcal{N} = 2$ YMESGTs were found in [25,26]. The main goal of this paper was to relate these four-dimensional theories to the theories in five dimensions with various gaugings. The authors of these papers asserted that three ingredients are necessary to obtain de Sitter vacua:

(i) noncompact gauge groups,
(ii) FI terms, and
(iii) dRW rotation.

The noncompact gauge group they used in the three models they studied is $SO(2, 1)$. We showed that this is *not* the only gauge group that admits a potential that one needs to obtain de Sitter vacua. One can indeed contract this group to $SO(1,1) \ltimes \mathbb{R}^{(1,1)}$, and de Sitter vacua is preserved under this contraction. We need to emphasize that whereas the $SO(2, 1)$ gauged theories do not directly descend from five dimensions, their contracted counterparts do. FI terms are available for gauge groups that have $U(1)$ or $SU(2)$ factors. The variation of such terms in the Lagrangian is a total derivative, and they yield supersymmetric terms in the action. References [90,91] point out that adding FI terms to the Lagrangian is indeed equivalent to gauging R symmetry. In the three models they studied, Fre, Trigiante and Van Proeyen considered $\mathcal{N} = 2$ supergravity with a complex scalar manifold of the form $\mathcal{M}^4_{VS} = \mathcal{ST}[2, n-1] = \frac{SU(1,1)}{U(1)} \times \frac{SO(2,n-1)}{SO(2)\times SO(n-1)}$, parametrized by Calabi-Vesentini coordinates. These correspond to the symmetric generic Jordan family, which describes the geometry of a real manifold of the form $\mathcal{M}^5_{VS} = \frac{SO(\tilde{n}-1,1)\times SO(1,1)}{SO(\tilde{n}-1)}$ in five dimensions. Here n and $\tilde{n}$ denote the number of vector multiplets coupled to supergravity in four and five dimensions, respectively, and they are related by $n = \tilde{n} + 1$. For such theories one has a certain amount of freedom to choose a holomorphic (symplectic) section upon dimensional reduction. This freedom is parametrized by dRW angles θ. Different choices of θ yield different gauged models with different physics. We use the notation of [70,74] to parametrize the complex manifold instead of Calabi-Vesentini coordinates for two main reasons. First, in the former parametrization the five-dimensional connection is as clear as it could be, as the complex scalar fields are obtained directly from dimensional reduction, and, second, generalizing the results to homogeneous manifolds is significantly easier. The mapping between two parametrizations can be found in Appendix C.

As we stressed earlier, stable de Sitter vacua exist in five-dimensional $SO(1, 1) \times R_s$ gauged YMESGTs, where the R_s denotes a subgroup of the full R-symmetry group $SU(2)_R$ [67–69]. These theories descend to four-dimensional theories that have the gauge group $(SO(1,1) \ltimes \mathbb{R}^{(1,1)}) \times R_s$ with a central charge [70]. The procedure in establishing de Sitter ground states in four dimensions from these theories includes finding an appropriate holomorphic section by means of a dRW rotation and contracting the gauge group. The contraction rotates some of the group generators into each other, eliminates the central charge, and gives a positive definite contribution to the potential. Without this contribution, the potential does not have any ground states, and that makes the group contraction essential. We showed that these theories can also be obtained from four-dimensional $SO(2, 1) \times R_s$ gauged YMESGTs, which were considered in [25,26] to have stable de Sitter vacua, by means of a different contraction.

In analogy to five dimensions, the theories with generic Jordan family scalar manifolds have stable de Sitter vacua for $(SO(1,1) \ltimes \mathbb{R}^{(1,1)}) \times R_s$ gaugings. R_s can be either $U(1)_R$ or $SU(2)_R$. In either case, the de Sitter minima in four dimensions that we found has a flat direction. Recall that this was also the case in five dimensions before the dimensional reduction. In addition to vector/tensor multiplets, one can couple a universal hypermultiplet and simultaneously gauge $U(1)$ or $SU(2)$ symmetry of its quaternionic hyperscalar manifold. We showed that, again in analogy to five dimensions, this type of extra gauging preserves the nature of the de Sitter ground states. The theories with noncompact hyperisometry gauging, which lead to stable de Sitter ground states in five dimensions, still need to be checked in four dimensions to complete the analogy. This topic is not covered in this paper, and we leave it for future investigation.

The same results can be achieved by starting either with the GMZ symplectic section [70] or with the symplectic section that we introduced in (3.44), which has a closer connection to the Calabi-Vesentini basis used in [25,26]. While in either case a gauge group contraction that rotates some of the generators into each other and eliminates the central charge is essential, it should be noted that one does not need an extra dRW rotation for the GMZ symplectic section, because it is already "dRW rotated" by $\theta = \pi/2$ radians with respect to our symplectic section.

In four dimensions, general homogeneous (but not necessarily symmetric) scalar manifolds $L(q, P)$ admit de Sitter vacua provided that they contain a $(SO(1,1) \ltimes \mathbb{R}^{(1,1)}) \times R_s$ subsector. These spaces are limited to $L(q \geq 0, P)$. For the symmetric generic Jordan family spaces $L(0, P)$, one has the freedom to choose the dRW angle from $0 < \theta < \pi$. This choice affects the values of the scalar fields and the value of the potential at the de Sitter minima. For the spaces of type $L(q > 0, P)$, on the other hand, the value of the dRW angle has to be fixed to $\theta = \pi/2$; otherwise the translational variations of the holomorphic section do not become block-diagonal, and one cannot gauge the theory in the standard way. Observe that the GMZ symplectic section carries this rotation to begin with.

The spaces of the type $L(q \geq 0, P)$ have de Sitter minima, but one can analyze them for other ground states. However, the analysis of extrema of the homogeneous spaces in their full generality is involved and requires a separate study.

Having found the recipe that starts with a five-dimensional $SO(1,1) \times U(1)_R$ gauged $\mathcal{N} = 2$ YMESGT and ends with stable de Sitter vacua in four dimensions, one can ask the question: Is it possible to embed the theory into a fundamental superstring theory or M theory? There are several directions one can take to answer this question. Compactifications of type IIA and type IIB superstring theories on Calabi-Yau threefolds yield ungauged supergravity theories in four dimensions. Using the same method, it was shown in [46] that a 5D, $\mathcal{N} = 2$ MESGT coupled to hypers can be obtained by compactifying 11-dimensional supergravity. In particular, the Hodge number $h_{(1,1)}$ of the threefold corresponds to the number of vector fields (including graviphoton) in the resulting 5D, $\mathcal{N} = 2$ MESGT, whereas $h_{(2,1)} + 1$ corresponds to the number of hypermultiplets. Type IIA or type IIB supergravity in ten dimensions compactified on T^6 results in $N = 8$ supergravity in four dimensions. Similarly, 5D, $\mathcal{N} = 8$ supergravity can be obtained by compactifying 11-dimensional supergravity on T^6. By orbifolding (modding out by discrete groups) the four-dimensional theory, Sen and Vafa considered examples of models with broken supersymmetries [98]. In one of the several models, the scalar manifold belongs to the generic Jordan family. As was pointed out in [99], another model they considered is the $J_3^{\mathbb{H}}$ of the magical Jordan family. These results can be extended to the 11D-to-5D compactifications. However, these types of compactifications result in ungauged theories. Whether one can obtain gauged versions of these theories is an open problem to be investigated.

One should note that the no-go theorem of [100] asserts that a Minkowskian ($\mathbb{R}^d$) or de Sitter (dS^d) theory in d dimensions cannot be obtained from a higher dimensional supergravity theory by a nonsingular compactification if the higher (for our consideration, 10- or 11-) dimensional supergravity theory

(i) does not have an action with higher curvature corrections,
(ii) admits a nonpositive potential, or
(iii) contains massless fields with positive kinetic energy terms.

As the authors point out, there are possible ways to evade this no-go theorem, namely,

(i) including higher curvature stringy corrections in the supergravity equations,
(ii) starting from a theory that already has a positive cosmological constant,
(iii) starting from an alternative supergravity theory that has negative kinetic energy terms for some scalar fields (e.g. type IIa* supergravity theories considered by [101]), or
(iv) choosing the internal space to be noncompact, i.e. doing a "noncompact compactification."

In fact, the last case is known to produce lower dimensional theories with noncompact gaugings [30,102], which, as was found in the literature and demonstrated in this paper, is a necessary ingredient to obtain five- and four-dimensional de Sitter vacua.

Meanwhile, after solving the stabilization problem of compactification of internal dimensions [14,103] by incorporating instanton corrections to the superpotential [cf. (i)], it was possible to find de Sitter vacua from string theory. This moduli stabilization fixes the runaway behavior of the axion-dilaton fields, which was also a problem we encountered upon dimensional reduction in the beginning of Sec. III. In the original Kachru-Kallosh-Linde-Trivedi (KKLT) scenario, the moduli stabilization brings the minimum of the scalar potential to a finite negative value. Then the addition of an anti-$D3$ brane lifts this minimum to a state with positive vacuum energy. In our construction, on the other hand, a similar effect was established through dRW rotation and gauging the noncompact $SO(1,1)$ subgroup of the global isometry group of the scalar manifold simultaneously with a subgroup of the R-symmetry group. Finding a relation between our and a KKLT-like scenario is an interesting problem, and we leave this for a future study.

ACKNOWLEDGMENTS

I am grateful to my academic advisor Murat Günaydin for his suggestions and guidance throughout this work. I thank Mario Trigiante for his help with clearing out confusions regarding FI-term computations in Sec. III. I also thank the referee for constructive comments related to the discussions in Sec. IV. This work was supported in part by the National Science Foundation under Grant No. PHY-0555605. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.

APPENDIX A: "VERY SPECIAL GEOMETRY," THE LAGRANGIANS IN FIVE AND FOUR DIMENSIONS, AND THE DERIVATION OF THE POTENTIAL TERMS

The bosonic sector of the 5D, $\mathcal{N}=2$ gauged Yang-Mills-Einstein supergravity[14] coupled to tensor and hypermultiplets is described by the Lagrangian [with metric signature $(-++++)$] [60,65,104]

$$\begin{aligned}\hat{e}^{-1}\mathcal{L}^{(5)} = &-\frac{1}{2}\hat{R} - \frac{1}{4}\overset{o}{a}_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{\hat{\mu}\hat{\nu}}\mathcal{H}^{\tilde{J}\hat{\mu}\hat{\nu}} \\ &-\frac{1}{2}g_{XY}\mathcal{D}_{\hat{\mu}}q^X\mathcal{D}^{\hat{\mu}}q^Y - \frac{1}{2}g_{\tilde{x}\tilde{y}}\mathcal{D}_{\hat{\mu}}\varphi^{\tilde{x}}\mathcal{D}^{\hat{\mu}}\varphi^{\tilde{y}} \\ &+\frac{\hat{e}^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}F^I_{\hat{\mu}\hat{\nu}}F^J_{\hat{\rho}\hat{\sigma}}A^K_{\hat{\tau}} \\ &+\frac{\hat{e}^{-1}}{4g}\epsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}\hat{\tau}}\Omega_{MN}B^M_{\hat{\mu}\hat{\nu}}\mathcal{D}_{\hat{\rho}}B^N_{\hat{\sigma}\hat{\tau}} - P_{(5)}(\varphi,q).\end{aligned} \tag{A1}$$

Here, non-Abelian field strengths $\mathcal{F}^I_{\hat{\mu}\hat{\nu}} \equiv F^I_{\hat{\mu}\hat{\nu}} + gf^I_{JK}A^J_{\hat{\mu}}A^K_{\hat{\nu}}$ $(I = 1, 2, \ldots, n_V + 1)$ of the gauge group $K_{(5)}$ and the self-dual tensor fields $B^M_{\hat{\mu}\hat{\nu}}$ $(M = 1, 2, \ldots, 2n_T)$ are grouped together to define the tensorial quantity $\mathcal{H}^{\tilde{I}}_{\hat{\mu}\hat{\nu}} \equiv (\mathcal{F}^I_{\hat{\mu}\hat{\nu}}, B^M_{\hat{\mu}\hat{\nu}})$, with $\tilde{I} = 1, 2, \ldots, n_V + 2n_T + 1$. The potential term $P_{(5)}(\varphi, q)$ is given by

$$P_{(5)}(\varphi,q) = g^2(P^{(T)}_{(5)}(\varphi) + \lambda P^{(R)}_{(5)}(\varphi,q) + \kappa P^{(H)}_{(5)}(q)), \tag{A2}$$

where

$$P^{(T)}_{(5)} = 2W_{\tilde{x}}W^{\tilde{x}}, \qquad P^{(R)}_{(5)} = -4\vec{P}\cdot\vec{P} + 2\vec{P}^{\tilde{x}}\cdot\vec{P}_{\tilde{x}}, \qquad P^{(H)}_{(5)} = 2\mathcal{N}_X\mathcal{N}^X, \tag{A3}$$

and $\lambda = g_R^2/g^2$, $\kappa = g_H^2/g^2$. The quantities given in the above expression are defined as

$$W_{\tilde{x}} \equiv -\frac{\sqrt{6}}{8}\Omega^{MN}h_{M\tilde{x}}h_N = \frac{\sqrt{6}}{4}h^IK^{\tilde{x}}_I, \qquad \vec{P} \equiv h^I\vec{P}_I, \qquad \vec{P}_{\tilde{x}} \equiv h^I_{\tilde{x}}\vec{P}_I, \qquad \mathcal{N}^X \equiv \frac{\sqrt{6}}{4}h^IK^X_I, \tag{A4}$$

where $K^{\tilde{x}}_I$ and K^X_I are Killing vectors acting on the scalar and the hyperscalar parts, respectively, of the total scalar manifold $\mathcal{M}^5_{\rm scalar} = \mathcal{M}^5_{VS}\otimes\mathcal{M}_Q$; $\vec{P}_I$ are the Killing prepotentials which will be defined below; Ω^{MN} is the inverse of Ω_{MN}, which is the constant invariant antisymmetric tensor of the gauge group $K_{(5)}$; and h^I and $h^I_{\tilde{x}}$ are elements of the very special manifold $\mathcal{M}^5_{VS}$ described by the hypersurface

$$N(h) = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} = 1, \qquad \tilde{I},\tilde{J},\tilde{K} = 1,\ldots,\tilde{n}+1, \tag{A5}$$

of the $\tilde{n}+1$-dimensional space $M = \{h^{\tilde{I}} \in \mathbb{R}^{\tilde{n}+1} | N(h) = C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}h^{\tilde{K}} > 0\}$ with metric

$$a_{IJ} = -\tfrac{1}{3}\partial_I\partial_J\ln N(h). \tag{A6}$$

The terms $P^{(T)}_{(5)}$ and $P^{(H)}_{(5)}$ are semipositive definite in the physically relevant region, whereas $P^{(R)}_{(5)}$ can have both signs. $\mathcal{M}^5_{VS}$ is determined completely by the totally symmetric tensor $C_{\tilde{I}\tilde{J}\tilde{K}}$. The scalar field metric on this hypersurface is the induced metric from the embedding space, which is given by

$$g_{\tilde{x}\tilde{y}} = \tfrac{3}{2}a_{\tilde{I}\tilde{J}}h^{\tilde{I}}_{,\tilde{x}}h^{\tilde{J}}_{,\tilde{y}}|_{N=1} = -3C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}h^{\tilde{J}}_{,\tilde{x}}h^{\tilde{K}}_{,\tilde{y}}|_{N=1}, \tag{A7}$$

where "$,\tilde{x}$" denotes a derivative with respect to $\varphi^{\tilde{x}}$. The definitions

$$\begin{aligned}\overset{o}{a}_{\tilde{I}\tilde{J}} &\equiv a_{\tilde{I}\tilde{J}}|_{N=1} = -2C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}} + 3h_{\tilde{I}}h_{\tilde{J}}, \\ h_{\tilde{I}} &\equiv C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{J}}h^{\tilde{K}} = \overset{o}{a}_{\tilde{I}\tilde{J}}h^{\tilde{J}}, \\ h^{\tilde{I}}_{\tilde{x}} &\equiv -\sqrt{\frac{3}{2}}h^{\tilde{I}}_{,\tilde{x}}, \\ h_{\tilde{I}\tilde{x}} &\equiv \overset{o}{a}_{\tilde{I}\tilde{J}}h^{\tilde{J}}_{\tilde{x}} = \sqrt{\frac{3}{2}}h_{\tilde{I},\tilde{x}}\end{aligned} \tag{A8}$$

help us write the algebraic constraints of the very special geometry

$$h^{\tilde{I}}h_{\tilde{I}} = 1, \qquad h^{\tilde{I}}_{\tilde{x}}h_{\tilde{I}} = h_{\tilde{I}\tilde{x}}h^{\tilde{I}} = 0, \qquad h^{\tilde{I}}_{\tilde{x}}h^{\tilde{J}}_{\tilde{y}}\overset{o}{a}_{\tilde{I}\tilde{J}} = g_{\tilde{x}\tilde{y}}. \tag{A9}$$

There are also differential constraints to be satisfied:

$$\begin{aligned}h_{\tilde{I}\tilde{x};\tilde{y}} &= \sqrt{\frac{2}{3}}(g_{\tilde{x}\tilde{y}}h_{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}}h^{\tilde{z}}_{\tilde{I}}), \\ h^{\tilde{I}}_{\tilde{x};\tilde{y}} &= -\sqrt{\frac{2}{3}}(g_{\tilde{x}\tilde{y}}h^{\tilde{I}} + T_{\tilde{x}\tilde{y}\tilde{z}}h^{\tilde{I}\tilde{z}}),\end{aligned} \tag{A10}$$

where ";" is the covariant derivative using the Christoffel connection calculated from the metric $g_{\tilde{x}\tilde{y}}$ and

$$T_{\tilde{x}\tilde{y}\tilde{z}} \equiv C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{I}}_{\tilde{x}}h^{\tilde{J}}_{\tilde{y}}h^{\tilde{K}}_{\tilde{z}}. \tag{A11}$$

Using (A7)–(A9) one can derive

$$\overset{o}{a}_{\tilde{I}\tilde{J}} = h_{\tilde{I}}h_{\tilde{J}} + h^{\tilde{x}}_{\tilde{I}}h_{\tilde{J}\tilde{x}}, \tag{A12}$$

$$h^{\tilde{x}}_{\tilde{I}}h_{\tilde{J}\tilde{x}} = -2C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}} + 2h_{\tilde{I}}h_{\tilde{J}}. \tag{A13}$$

The indices $\tilde{I}$, $\tilde{J}$, and $\tilde{K}$ are raised and lowered by $\overset{o}{a}_{\tilde{I}\tilde{J}}$, and its inverse $\overset{o}{a}{}^{\tilde{I}\tilde{J}}$. $P^{(T)}_{(5)}$ can now be written in a more compact form

[14] For the full Lagrangian, see [65,66]

$$P^{(T)}_{(5)} = \frac{3}{8}\Omega^{MN}\Omega^{PR}C_{MRI}h_N h_P h^I = \frac{3\sqrt{6}}{16}\Lambda_I^{MN}h_M h_N h^I, \tag{A14}$$

with Λ^M_{IN} being the transformation matrices of the tensor fields under the gauge group $K_{(5)}$:

$$\Lambda_I^{MN} = \Lambda^M_{IP}\Omega^{PN} = \frac{2}{\sqrt{6}}\Omega^{MR}C_{IRP}\Omega^{PN}. \tag{A15}$$

Gauging the R symmetry introduces the potential term $P^{(R)}_{(5)} = -4\vec{P}\cdot\vec{P} + 2\vec{P}^{\tilde{x}}\cdot\vec{P}_{\tilde{x}}$, where $\vec{P} = h^I\vec{P}_I$ and $\vec{P}_{\tilde{x}} = h^I_{\tilde{x}}\vec{P}_I$ are vectors that transform under the R-symmetry group that is being gauged. For the $SU(2)_R$ gauging one can take

$$\vec{P}_I = \vec{e}_I,$$

where $\vec{e}_I$ satisfy $\vec{e}_i\times\vec{e}_j = d_{ij}{}^k\vec{e}_k$ and $\vec{e}_i\cdot\vec{e}_j = \delta_{ij}$ when i, j, and k are the $SU(2)_R$ adjoint indices [d^k_{ij} are the $SU(2)$ structure constants], and $\vec{e}_I = 0$ otherwise. With this convention and the use of (A8) and (A9), the potential term simplifies to

$$P^{(R)}_{(5)} = -4C^{ij\tilde{K}}\delta_{ij}h_{\tilde{K}}. \tag{A16}$$

If the $U(1)_R$ subgroup of $SU(2)_R$ is being gauged, one can take

$$\vec{P}_I = V_I\vec{e},$$

where $\vec{e}$ is an arbitrary vector in the $SU(2)$ space and V_I are some constants that define the linear combination of the vector fields $A^I_{\hat{\mu}}$ that is used as the $U(1)_R$ gauge field:

$$A_{\hat{\mu}}[U(1)_R] = V_I A^I_{\hat{\mu}}.$$

The potential term then can be written as

$$P^{(R)}_{(5)} = -4C^{IJ\tilde{K}}V_I V_J h_{\tilde{K}}. \tag{A17}$$

If tensors are coupled to the theory, the V_I have to be constrained by

$$V_I f^I_{JK} = 0,$$

with f^I_{JK} being the structure constants of $K_{(5)}$. When the target manifold $\mathcal{M}_{VS}$ is associated with a Jordan algebra, the following equality holds componentwise:

$$C^{\tilde{I}\tilde{J}\tilde{K}} = C_{\tilde{I}\tilde{J}\tilde{K}} = \text{const}.$$

After the dimensional reduction from five to four, the Lagrangian (A1) becomes [70]

$$\begin{aligned} e^{-1}\mathcal{L}^{(4)} = {}& -\frac{1}{2}R - \frac{3}{4}\overset{\circ}{a}_{\tilde{I}\tilde{J}}(\mathcal{D}_\mu\tilde{h}^{\tilde{I}})(\mathcal{D}^\mu\tilde{h}^{\tilde{J}}) - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{IJ}(\mathcal{D}_\mu A^I)(\mathcal{D}^\mu A^J) - \frac{1}{2}e^{-2\sigma}g_{XY}(\mathcal{D}_\mu q^X)(\mathcal{D}^\mu q^Y) \\ & - e^{-2\sigma}\overset{\circ}{a}_{IM}(\mathcal{D}_\mu A^I)B^{\mu M} - \frac{1}{2}e^{-2\sigma}\overset{\circ}{a}_{MN}B^M_\mu B^{\mu N} + \frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}B^M_{\mu\nu}(\partial_\rho B^N_\sigma + gA^I_\rho\Lambda^N_{IP}B^P_\sigma) \\ & + \frac{e^{-1}}{g}\epsilon^{\mu\nu\rho\sigma}\Omega_{MN}W_{\mu\nu}B^M_\rho B^N_\sigma + \frac{e^{-1}}{2\sqrt{6}}C_{MNI}\epsilon^{\mu\nu\rho\sigma}B^M_{\mu\nu}B^N_{\rho\sigma}A^I - \frac{1}{4}e^\sigma\overset{\circ}{a}_{MN}B^M_{\mu\nu}B^{N\mu\nu} - \frac{1}{2}e^\sigma\overset{\circ}{a}_{IM}(\mathcal{F}^I_{\mu\nu} \\ & + 2W_{\mu\nu}A^I)B^{M\mu\nu} - \frac{1}{4}e^\sigma\overset{\circ}{a}_{IJ}(\mathcal{F}^I_{\mu\nu} + 2W_{\mu\nu}A^I)(\mathcal{F}^{J\mu\nu} + 2W^{\mu\nu}A^J) - \frac{1}{2}e^{3\sigma}W_{\mu\nu}W^{\mu\nu} \\ & + \frac{e^{-1}}{2\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma}\{\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma}A^K + 2\mathcal{F}^I_{\mu\nu}W_{\rho\sigma}A^J A^K + \frac{4}{3}W_{\mu\nu}W_{\rho\sigma}A^I A^J A^K\} - g^2 P_{(4)}, \end{aligned} \tag{A18}$$

where

$$\tilde{h}^{\tilde{I}} \equiv e^\sigma h^{\tilde{I}}, \tag{A19}$$

$$\mathcal{D}_\mu A^I \equiv \partial_\mu A^I + gA^J_\mu f^I_{JK}A^K, \tag{A20}$$

$$\mathcal{F}^I_{\mu\nu} \equiv 2\partial_{[\mu}A^I_{\nu]} + gf^I_{JK}A^J_\mu A^K_\nu, \tag{A21}$$

$$\mathcal{D}_\mu\tilde{h}^{\tilde{I}} \equiv \partial_\mu\tilde{h}^{\tilde{I}} + gA^I_\mu M^{\tilde{I}}_{I\tilde{K}}\tilde{h}^{\tilde{K}}, \tag{A22}$$

$$\mathcal{D}_\mu q^X \equiv \partial_\mu q^X + g_H A^I_\mu K^X_I, \tag{A23}$$

and the total scalar potential $P_{(4)}$ is given by

$$P_{(4)} = P^{(T)}_{(4)} + \frac{g_H^2}{g^2}P^{(H)}_{(4)}, \tag{A24}$$

where

$$P^{(T)}_{(4)} \equiv e^{-\sigma}P^{(T)}_{(5)} + \tfrac{3}{4}e^{-3\sigma}\overset{\circ}{a}_{\tilde{I}\tilde{J}}(A^I M^{\tilde{I}}_{(I)\tilde{K}}h^{\tilde{K}})(A^J M^{\tilde{J}}_{(J)\tilde{L}}h^{\tilde{L}}) \tag{A25}$$

and

$$P^{(H)}_{(4)} \equiv e^{-\sigma}P^{(H)}_{(5)} + \tfrac{1}{2}e^{-3\sigma}(A^I K^X_I)g_{XY}(A^J K^Y_J), \tag{A26}$$

which would get an additional term of the form

$$\frac{g_R^2}{g^2}P^{(R)}_{(4)} \equiv \frac{g_R^2}{g^2}e^{-\sigma}P^{(R)}_{(5)}(h^{\tilde{I}}) \tag{A27}$$

if the R symmetry is being gauged. The transformation matrices $M_{(I)\tilde{K}}^{\tilde{J}}$ that correspond to the gauge group $K_{(5)}$ are decomposed as follows:

$$M_{(I)\tilde{K}}^{\tilde{J}} = \begin{pmatrix} f_{IK}^{J} & 0 \\ 0 & \Lambda_{IM}^{N} \end{pmatrix}. \tag{A28}$$

f_{IK}^{J} are always antisymmetric in the lower two indices.

APPENDIX B: KILLING VECTORS OF THE HYPERISOMETRY

The eight Killing vectors k_α^X that generate isometry group $SU(2,1)$ of the hyperscalar manifold are given by [73]

$$\vec{k}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \quad \vec{k}_2 = \begin{pmatrix} 0 \\ 2\theta \\ 0 \\ 1 \end{pmatrix}, \quad \vec{k}_3 = \begin{pmatrix} 0 \\ -2\tau \\ 1 \\ 0 \end{pmatrix}, \quad \vec{k}_4 = \begin{pmatrix} 0 \\ 0 \\ -\tau \\ \theta \end{pmatrix}, \quad \vec{k}_5 = \begin{pmatrix} V \\ \sigma \\ \theta/2 \\ \tau/2 \end{pmatrix},$$
$$\vec{k}_6 = \begin{pmatrix} 2V\sigma \\ \sigma^2 - (V + \theta^2 + \tau^2)^2 \\ \sigma\theta - \tau(V + \theta^2 + \tau^2) \\ \sigma\tau + \theta(V + \theta^2 + \tau^2) \end{pmatrix}, \quad \vec{k}_7 = \begin{pmatrix} -2V\theta \\ -\sigma\theta + V\tau + \tau(\theta^2 + \tau^2) \\ \frac{1}{2}(V - \theta^2 + 3\tau^2) \\ -2\theta\tau - \sigma/2 \end{pmatrix}, \quad \vec{k}_8 = \begin{pmatrix} -2V\tau \\ -\sigma\tau - V\theta - \theta(\theta^2 + \tau^2) \\ -2\theta\tau + \sigma/2 \\ \frac{1}{2}(V + 3\theta^2 - \tau^2) \end{pmatrix}. \tag{B1}$$

The corresponding prepotentials are

$$\vec{p}_1 = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{4V} \end{pmatrix}, \quad \vec{p}_2 = \begin{pmatrix} -\frac{1}{\sqrt{V}} \\ 0 \\ -\frac{\theta}{V} \end{pmatrix}, \quad \vec{p}_3 = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{V}} \\ \frac{\tau}{V} \end{pmatrix}, \quad \vec{p}_4 = \begin{pmatrix} -\frac{\theta}{\sqrt{V}} \\ -\frac{\tau}{\sqrt{V}} \\ \frac{1}{2} - \frac{\theta^2+\tau^2}{2V} \end{pmatrix}, \quad \vec{p}_5 = \begin{pmatrix} -\frac{\tau}{2\sqrt{V}} \\ \frac{\theta}{2\sqrt{V}} \\ -\frac{\sigma}{4V} \end{pmatrix},$$
$$\vec{p}_6 = \begin{pmatrix} -\frac{1}{\sqrt{V}}[\sigma\tau + \theta(-V + \theta^2 + \tau^2)] \\ \frac{1}{\sqrt{V}}[\sigma\theta - \tau(-V + \theta^2 + \tau^2)] \\ -\frac{V}{4} - \frac{1}{4V}[\sigma^2 + (\theta^2 + \tau^2)^2] + \frac{3}{2}(\theta^2 + \tau^2) \end{pmatrix}, \quad \vec{p}_7 = \begin{pmatrix} \frac{4\theta\tau+\sigma}{2\sqrt{V}} \\ \frac{3\tau^2-\theta^2}{2\sqrt{V}} - \frac{\sqrt{V}}{2} \\ -\frac{3}{2}\tau + \frac{1}{2V}[\sigma\theta + \tau(\theta^2 + \tau^2)] \end{pmatrix},$$
$$\vec{p}_8 = \begin{pmatrix} -\frac{3\theta^2-\tau^2}{2\sqrt{V}} + \frac{\sqrt{V}}{2} \\ \frac{\sigma-4\theta\tau}{2\sqrt{V}} \\ \frac{3}{2}\theta + \frac{1}{2V}[\sigma\tau - \theta(\theta^2 + \tau^2)] \end{pmatrix}. \tag{B2}$$

It is easier to see that the Killing vectors close to the $SU(2,1)$ algebra if they are recast in the following combinations:

$$SU(2) \quad \begin{cases} T_1 = \frac{1}{4}(k_2 - 2k_8), \\ T_2 = \frac{1}{4}(k_3 - 2k_7), \\ T_3 = \frac{1}{4}(k_1 + k_6 - 3k_4), \end{cases}$$
$$U(1) \quad \left\{ T_8 = \frac{\sqrt{3}}{4}(k_4 + k_1 + k_6), \right. \tag{B3}$$
$$\frac{SU(2,1)}{U(2)} \quad \begin{cases} T_4 = k_5, \\ T_5 = -\frac{1}{2}(k_1 - k_6), \\ T_6 = -\frac{1}{4}(k_3 + 2k_7), \\ T_7 = -\frac{1}{4}(k_2 + 2k_8). \end{cases}$$

This basis is chosen for convenience such that the generators T_1, T_2, T_3, and T_8 are the isotropy group of the point $(V, \sigma, \theta, \tau) = (1, 0, 0, 0)$. The metric hyperscalar manifold becomes diagonal at this point. In all of the theories that have hypercoupling, we will take this basis point q^C for a possible candidate of the hypercoordinates of a critical point. The Killing vectors K_I^X are then given by $V_I^\alpha k_\alpha^X$, and the corresponding prepotentials $\vec{P}_I$ are $V_I^\alpha \vec{p}_\alpha$, where V_I^α are constants that determine which isometries are being gauged and what linear combination of vector fields is being used. In particular,

$$K_I^X = \begin{cases} T_1^X, T_2^X, T_3^X & \text{for } SU(2) \text{ gauging}, \\ V_I W^k T_k^X, \quad k = 1, 2, 3, 8 & \text{for } U(1) \text{ gauging}, \\ V_I W^k T_k^X, \quad k = 4, 5, 6, 7 & \text{for } SO(1,1) \text{ gauging}, \end{cases}$$

where V_I and W^k are constants depending on the model.

APPENDIX C: TRANSFORMATIONS BETWEEN TWO PARAMETRIZATIONS

For $\mathcal{N}=2$ supergravity coupled to n vector multiplets and no tensors, the symplectic section (3.8) takes the following form [70]:

$$\Omega_0 = \begin{pmatrix} 1 \\ z_1 \\ z_2 \\ z_a \\ \frac{1}{2}z_1\|z\|^2 \\ -\frac{1}{2}\|z\|^2 \\ -z_1 z_2 \\ z_1 z_a \end{pmatrix}. \tag{C1}$$

Here $\|z\|^2 = [(z_2)^2 - (z_3)^2 - \cdots - (z_n)^2]$ and $a = 3, \ldots, n$. Fre, Trigiante, and Van Proeyen [25] use Calabi-Vesentini coordinates for which $(X^\Lambda, F_\Lambda = \eta_{\Lambda\Sigma} S X^\Sigma; X^\Lambda X^\Sigma \eta_{\Lambda\Sigma} = 0, \eta_{\Lambda\Sigma} = \mathrm{diag}(+,+,-,\ldots,-))$ holds. More explicitly [25],

$$\Omega_{CV} = \begin{pmatrix} X^\Lambda \\ F_\Sigma \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(1+\|y\|^2) \\ \frac{1}{2}i(1-\|y\|^2) \\ y_1 \\ y_{a-1} \\ \frac{1}{2}S(1+\|y\|^2) \\ \frac{1}{2}iS(1-\|y\|^2) \\ -Sy_1 \\ -Sy_{a-1} \end{pmatrix}, \tag{C2}$$

where $\|y\|^2 = y_1^2 + \cdots + y_{n-1}^2$. The transformations between the two notations are given by

$$\frac{1}{2}(1+\|y\|^2) = \frac{1}{2\sqrt{2}}(2-\|z\|^2), \qquad \frac{1}{2}i(1-\|y\|^2) = z_2,$$
$$y_{a-2} = z_a, \qquad y_{n-1} = \frac{1}{2\sqrt{2}}(2+\|z\|^2), \qquad S = -z_1. \tag{C3}$$

The matrix for the symplectic rotation $C\Omega_{CV} = \Omega_0$ is given by

$$C = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & \mathbb{1}_{n-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathbb{1}_{n-1} & 0 \end{pmatrix}. \tag{C4}$$

It is easy to see that the symplectic section (C2) together with the coordinate transformations (C3) is a particular case of (3.44) and also that $C = S^{-1}$.

APPENDIX D: VARIOUS POTENTIAL TERMS

The $P^{(T)}$ potential terms given here are calculated for a $\mathcal{N}=2$, 4D YMESGT coupled to $n=4$ vector/tensor multiplets.

Gauging $K_{(4)} = SO(2,1)$ symmetry results in the following potential:

$$\begin{aligned} &-i[(\bar{w}_2^2 - \bar{w}_3^2)w_2^4 + 2\bar{w}_2(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2^3 + (\bar{w}_2^4 - 2(w_3^2 - \bar{w}_3 w_3 + w_4^2 + \bar{w}_3^2 + \bar{w}_4^2 + 4)\bar{w}_2^2 + 2w_3^2\bar{w}_3^2 + 2w_4^2\bar{w}_3^2 \\ &+ (\bar{w}_3^2 + \bar{w}_4^2)^2 + 4(\bar{w}_4^2 + 1) - 2w_3\bar{w}_3(\bar{w}_3^2 + \bar{w}_4^2 + 2))w_2^2 + 2\bar{w}_2((\bar{w}_2^2 - \bar{w}_3^2 - \bar{w}_4^2 - 2)w_3^2 + 8\bar{w}_3 w_3 + (w_4^2 + 2)(\bar{w}_2^2 - \bar{w}_3^2 \\ &- \bar{w}_4^2 - 2))w_2 - w_3^2\bar{w}_2^4 - 4w_3^2 - (\bar{w}_3(w_3^2 - \bar{w}_3 w_3 + w_4^2 + 2) - w_3\bar{w}_4^2)(\bar{w}_3 w_3^2 - (\bar{w}_3^2 + \bar{w}_4^2 + 4)w_3 + (w_4^2 + 2)\bar{w}_3) \\ &+ \bar{w}_2^2(w_3^4 - 2\bar{w}_3 w_3^3 + 2(w_4^2 + \bar{w}_3^2 + \bar{w}_4^2)w_3^2 - 2(w_4^2 + 2)\bar{w}_3 w_3 + (w_4^2 + 2)^2)]/[2(w_1 - \bar{w}_1)((w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 \\ &- (w_4 - \bar{w}_4)^2)^2]. \end{aligned} \tag{D1}$$

Gauging $K_{(4)} = SO(1,1) \ltimes \mathbb{R}^{(1,1)}$ (no central charge) symmetry, on the other hand, results in the following potential:

$$\begin{aligned} &-i[-2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2^2 + 8(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)\bar{w}_3 w_2 \\ &+ 4\bar{w}_2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)w_2 - 8w_3(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 \\ &+ 2(-w_2^2 + w_3^2 + w_4^2 + 2)(w_2 - \bar{w}_2)(w_3 - \bar{w}_3)\bar{w}_3(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 + (-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_2(-(w_2 - \bar{w}_2)^2 \\ &- (w_3 - \bar{w}_3)^2 + (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)w_2 + 8w_3(w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3) - 8(-w_2^2 + w_3^2 + w_4^2 + 2) \\ &\times (w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3)\bar{w}_3 - 2(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_3^2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2) \\ &+ 4w_3\bar{w}_3(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2) - 2(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_2^2(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 \\ &+ (w_4 - \bar{w}_4)^2) + 2w_3(-w_2^2 + w_3^2 + w_4^2 + 2)(w_2 - \bar{w}_2)\bar{w}_2(w_3 - \bar{w}_3)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2) - 2w_3^2(-(w_2 - \bar{w}_2)^2 \\ &- (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2) + w_3(-w_2^2 + w_3^2 + w_4^2 + 2)\bar{w}_3(-(w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 \\ &- (w_4 - \bar{w}_4)^2)(-\bar{w}_2^2 + \bar{w}_3^2 + \bar{w}_4^2 + 2)]/[2(w_1 - \bar{w}_1)((w_2 - \bar{w}_2)^2 - (w_3 - \bar{w}_3)^2 - (w_4 - \bar{w}_4)^2)^3]. \end{aligned} \tag{D2}$$

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