

Tachyonic decay of unstable Dirichlet branesWilliam Nelson* and Mairi Sakellariadou[†]*Department of Physics, King's College, University of London, Strand WC2R 2LS, London, United Kingdom*

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We consider an unstable bound system of two supersymmetric Dirichlet branes of different dimensionality (p, p' with $p' < p$) embedded in a flat noncompactified IIA or IIB type background. We study the decay, via tachyonic condensation, of such unstable bound states leading to a pair of bound $D(p-1), Dp'$ -branes. We show that only when the gauge fields carried by the Dp -brane are localized perpendicular to the tachyon direction, then tachyon condensation will indeed take place. We perform our analysis by combining both the Hamiltonian and the Lagrangian approaches.

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I. INTRODUCTION

In the context of type II string theories, BPS Dirichlet branes with even (within type IIA string theory) or odd (within type IIB) number of tangential spatial directions can arise. Such objects, called Dp -branes, are invariant under half of the space-time supersymmetry transformations of the theory and are charged under a $(p+1)$ -form gauge field, having its origin in the Ramond-Ramond sector. Antibrane (\bar{D}) carry opposite Ramond-Ramond charge and have opposite orientation than D-branes. Starting with a $Dp\text{-}\bar{D}p$ -brane (with p even) in type IIA string theory, and modding it out by the exact symmetry of the theory $(-1)^{F_L}$ (where F_L denotes the contribution to the space-time fermion number from the left-moving sector of the string world sheet), one can define [1] a single non-BPS Dirichlet brane of type IIB. Similarly, one can construct non-BPS Dirichlet branes of type IIA. Thus, type IIB string theory contains BPS D-branes of odd dimension and non-BPS D-branes of even dimension; the vice versa holds for type IIA string theory. Alternatively, non-BPS Dirichlet branes can be constructed as tachyonic kink solutions on the brane-anti-brane system, with the energy density concentrated around a $(p-1)$ -dimensional space. One can thus claim an equivalence between tachyonic kink solution and non-BPS Dirichlet branes.

In either type IIA or type IIB string theory, non-BPS Dirichlet branes are unstable due to the appearance of a tachyonic mode. More precisely, when the tachyon condenses to its minimum, the tachyonic ground state cannot be distinguished from the vacuum, in the sense that it carries neither charge nor energy density. Even though these branes are unstable, one may obtain stable non-BPS Dirichlet branes by projecting out the tachyonic mode.

In what follows, we study the decay, via tachyonic condensation, of unstable bound states of Dirichlet (D) branes of different dimensionality, embedded in a flat non-

compactified d -dimensional space-time. The choice of the background as a flat IIA, or IIB, type will specify whether p is even, or odd, respectively. Our starting point is Sen's effective action for unstable Dirichlet branes [2], which is presented in Sec. II. The decay of the branes becomes apparent from the presence of a tachyonic mode on the brane world volume. To make our analysis more transparent we first discuss the bosonic sector of the theory in Sec. III, and then the fermionic sector in Sec. IV. We consider two complementary approaches, the Hamiltonian approach and the Lagrangian one, following Refs. [3,4], respectively. In the first approach [3], discussed in Sec. III A, we express the Hamiltonian of the system in terms of the canonical variables. Despite the fact that in the tachyon condensation limit the Lagrangian vanishes, the canonical momenta, and thus the Hamiltonian, remain well defined. In the second approach [4], discussed in Sec. III B, the action remaining after condensation is directly calculated. These two approaches have been so far considered independently in the literature. However, they can work together allowing us to understand the geometric restrictions that are required for the system to fully condense. This is indeed the novelty of our work. We discuss our findings in Sec. V.

We note that throughout this work we are using units in which the fundamental string tension g_s is equal to $1/(2\pi)$. The world-volume signature is taken to be $\eta = (- + \dots +)$.

II. TACHYON EFFECTIVE ACTION

The classical tachyon effective Lagrangian describing the dynamics of the tachyon field of a Dp -brane (i.e., unstable Dirichlet brane) of type IIA or IIB superstring theory is given by the sum of Dirac-Born-Infeld (\mathcal{L}_{DBI}) and Wess-Zumino (\mathcal{L}_{WZ}) type terms, as

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{DBI}} + \mathcal{L}_{\text{WZ}}. \quad (2.1)$$

To write down the effective Lagrangian for Dp -branes, one considers the local symmetries of such theories, which consist of a general coordinate invariance of the world

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volume, a local fermionic symmetry, and a U(1) gauge invariance. The local symmetries are then used to make a gauge choice in which the unphysical degrees of freedom can vanish.

The Dirac-Born-Infeld type term of the effective Lagrangian, which provides a good description of our system under the assumption that T is large while its second and higher derivatives are small, can be written as¹

$$\mathcal{L}_{\text{DBI}} = -V(T)\sqrt{-\det(\Pi_\mu^I \Pi_{\nu I} + \mathcal{F}_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \quad (2.2)$$

where $\mu, \nu \dots = 0, 1, \dots, p$ are the world-volume indices and $I, J, \dots = 0, 1, \dots, d-1$ are the target-space indices; T denotes the tachyon field; $V(T)$ is the tachyon potential, identified with the vanishing tension of the decaying branes. Note that we have explicitly included the kinetic term as it plays a crucial role in what follows.

The supersymmetric quantity Π_μ^I is related to another supersymmetric quantity $\partial_\mu \Theta$ through

$$\Pi_\mu^I = \partial_\mu X^I - \bar{\Theta} \Gamma^I \partial_\mu \Theta, \quad (2.3)$$

while the supersymmetric expression of the world-volume gauge field is

$$\begin{aligned} \mathcal{F}_{\mu\nu} = & F_{\mu\nu} - \bar{\Theta} \Psi \Gamma_I \partial_\mu \Theta \left(\partial_\nu X^I - \frac{1}{2} \bar{\Theta} \Gamma^I \partial_\nu \Theta \right) \\ & + \bar{\Theta} \Psi \Gamma_I \partial_\nu \Theta \left(\partial_\mu X^I - \frac{1}{2} \bar{\Theta} \Gamma^I \partial_\mu \Theta \right), \end{aligned} \quad (2.4)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the world-volume electromagnetic field, and X^I are world-volume scalars that give the transverse motion of the brane. For a 10-dimensional space-time, Θ are 32 component fermionic spinors and Γ^I are the 32×32 Dirac matrices. Finally, Ψ stands for either $\Gamma_{11} = \Gamma_0 \Gamma_1 \cdots \Gamma_9$, if p is even (flat type IIA background), or for the Pauli matrix τ_3 , if p is odd (flat type IIB background).

We impose the *static gauge* condition on the transverse scalars [5],

$$X^I = \begin{cases} \sigma^\mu & \text{for } I < p+1 \\ X^m & \text{for } I \geq p+1, \end{cases} \quad (2.5)$$

where $m, n \dots = p+1, p+2, \dots, d-1$ and σ^μ are the world-volume coordinates. In the static gauge condition, the $(p+1)$ target-space coordinates are identified with the world-volume coordinates. The remaining spatial coordinates can be seen as transverse excitations on the Dp -brane. Half of the 32 components of the Θ coordinates can be eliminated by gauge fixing the fermionic sector, so that one of the Majorana-Weyl spinors Θ_\pm equals zero, namely [5],

$$\Theta = \begin{pmatrix} \Theta_+ \\ \Theta_- \end{pmatrix} = \begin{pmatrix} \Theta_+ \\ 0 \end{pmatrix}. \quad (2.6)$$

We note that $(\mathbb{1} \pm \Gamma_{11})\Theta = 2\Theta_\pm$. Thus, Eq. (2.2) becomes [5]

$$\mathcal{L}_{\text{DBI}} = -V(T)\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \quad (2.7)$$

where

$$B_{\mu\nu} \equiv -2\bar{\Theta}_+(\Gamma_\nu + \Gamma_m \partial_\nu X^m) \partial_\mu \Theta_+ + (\bar{\Theta}_+ \Gamma_I \partial_\mu \Theta_+)(\bar{\Theta}_+ \Gamma^I \partial_\nu \Theta_+). \quad (2.8)$$

The Wess-Zumino type term in the effective Lagrangian is a $(p+1)$ -form, describing the coupling of the Ramond-Ramond background field strengths to the Dirichlet brane. Under a local fermionic symmetry, \mathcal{L}_{WZ} vanishes under contraction with Θ_- . Thus, the total effective Lagrangian is given by Eq. (2.7), namely,

$$\mathcal{L}_{\text{eff}} = -V(T)\sqrt{-\det(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T)}, \quad (2.9)$$

with $B_{\mu\nu}$ as defined in Eq. (2.8).

III. BOSONIC SECTOR

Let us first focus on the bosonic sector of the theory, i.e. $B_{\mu\nu} = 0$; we include Θ_+ in Sec. IV. The form of the tachyonic potential can contain topological obstructions that prevent the system from fully decaying. In particular,

¹This action has been basically taken from Ref. [5]; however, we have treated the tachyon similarly to the X^I . Thus, we have included the tachyon kinetic term inside the determinant, as discussed in Refs. [6,7].

for non-BPS branes the potential is such that $V(T \rightarrow \pm\infty) = 0$, with $V(0)$ a maximum [4]. In this situation, a $(p-1)$ -dimensional kink defect forms, preventing the tachyon reaching its true vacuum at all space-time coordinates. It is well known that such a kink precisely reproduces the dynamics of a $D(p-1)$ -brane in the absence of world-volume gauge fields. Here we show that this result is also true when the gauge fields are turned on, with some intuitive restrictions on the geometry of the gauge fields and the tachyon.

A Dp -brane can contain a Dp' -brane (with p' smaller than p), forming a bound state. This bound state is just a

Dp -brane with the electromagnetic charge of the Dp' -brane dissolved onto its world volume. Such a bound system turns out to be particularly relevant here, as it is precisely the system we are dealing with. We will first investigate this system using the Hamiltonian approach and we will then proceed with the Lagrangian one.

A. Hamiltonian approach

If we neglect the fermionic components, it was shown in Ref. [3] that the Hamiltonian obtained from the gauge fixed Lagrangian, Eq. (2.7), can be written as

$$\begin{aligned} \mathcal{H} = & [(P_X^m)^2 + (P_T)^2 + (\partial_i X_m P_X^m + \partial_i T P_T + F_{ij} P_A^i)^2 \\ & + (P_A^i \partial_i X_m)^2 + (P_A^i \partial_i T)^2 + (P_A^i)^2 \\ & + V^2(T) \det(h)]^{1/2}, \end{aligned} \quad (3.1)$$

where

$$h_{ij} \equiv \delta_{ij} + \partial_i X^m \partial_j X_m + F_{ij} + \partial_i T \partial_j T,$$

with $i, j, \dots = 1, 2, \dots, p$ the spatial world-volume indices and P_A^i, P_X^m, P_T the canonical momenta of A_i, X_m, T , respectively.²

If we consider the set of variables $\chi_N = (A_i, X_m, T)$ where $N = 1 \dots d$, and define $\partial_M \chi_N = 0$ for $M > p$, then the above Hamiltonian, Eq. (3.1), can be written formally as

$$\mathcal{H} = [(P_\chi^N)^2 + (F_{MN} P_\chi^N)^2 + V^2(T) \det(h)]^{1/2}, \quad (3.2)$$

where P_χ^N are the canonical momenta of χ_N and $F_{MN} = \partial_M \chi_N - \partial_N \chi_M$. Equation (3.2) turns out to be a particularly useful way of expressing the Hamiltonian for including fermionic degrees of freedom, as one can see from Sec. IV.

At this point it is worth making the following remark, comparing our analysis and our subsequently obtained results with that of Ref. [3]. The kinetic terms of the tachyon were neglected in Ref. [3]. It was stated that they could be included at any point, on the same footing as the transverse scalars. This is of course true, but only valid until the limit $T \rightarrow \pm\infty$ is taken. It is indeed the nonzero spatial tachyon derivatives that result in the localization of the kink. This is the reason for which a *string gas* solution was found in Ref. [3] as the end point of the tachyon condensation. In the following we show that accounting for the tachyon kinetic term leads to the expected localization.

The usefulness of the Hamiltonian approach is that it allows the $T \rightarrow \pm\infty$ limit to be taken while explicitly keeping track of the canonical momenta of the gauge fields, which must be conserved throughout the tachyon evolution. In the $T \rightarrow \pm\infty$ limit, $V(T) \rightarrow 0$ and the

Hamiltonian, Eq. (3.2), becomes

$$\mathcal{H} = [(P_\chi^N)^2 + (F_{MN} P_\chi^M)^2]^{1/2}; \quad (3.3)$$

we have taken the $T \rightarrow \pm\infty$ limit in the canonical momenta.

To show how this is derived, let us consider the specific case of a three-dimensional non-BPS brane.³ In this case, the tachyon is a function of only one world-volume coordinate, which we will take, without loss of generality, to be the x -direction. Following the procedure outlined in Ref. [4], we consider the field configuration

$$T(x) = f(ax), \quad (3.4)$$

where $f(w)$ is some arbitrary function, which however must satisfy

$$f(-w) = -f(w), \quad f'(w) > 0 \quad \forall w, \quad f(\pm\infty) = \pm\infty; \quad (3.5)$$

a is a constant which we will later take to infinity. Thus, $T = +\infty$ ($T = -\infty$), for $x > 0$ ($x < 0$). Expanding Eq. (3.3) into terms that depend on the tachyon and its derivatives and those that do not, we obtain

$$\mathcal{H} = [(\mathcal{H}_{\text{mod } T})^2 + (P_\chi^i \partial_i T)^2 + (P_T \partial_i T)^2 + P_T^2]^{1/2} \quad (3.6)$$

$$\begin{aligned} = & [(\mathcal{H}_{\text{mod } T})^2 + a^2 [f'(ax)]^2 (P_\chi^x)^2 \\ & + P_T^2 \{a^2 [f'(ax)]^2 + 1\}]^{1/2}; \end{aligned} \quad (3.7)$$

here χ represents the degrees of freedom *other* than the tachyon and $\mathcal{H}_{\text{mod } T}$ is the Hamiltonian of a Dp -brane (in this case $p = 3$) without a tachyon.

We are concerned with static solutions after the tachyon has fully condensed, in which case $P_T = 0$, since $\dot{T} = 0$. Thus, for the Hamiltonian to be finite in the limit $a \rightarrow \infty$, we require $P_\chi^x = 0$, i.e. the canonical momenta of all the world-volume degrees of freedom are constrained to be perpendicular to the direction of the kink. In this case, $\mathcal{H}_{\text{mod } T}$ is just the Hamiltonian of a $D(p-1)$ -brane (in the case we consider here, this is a two-dimensional Dirichlet brane) with no tachyon, as indeed expected. In particular, we note that the canonical momentum of the world-volume electromagnetic field is perpendicular to the kink formed at the end of tachyon condensation and hence it is conserved.

We extend the above analysis by considering (not necessarily small) fluctuations of bosonic fields around the kink background. We study the simple case of a translation along the x direction, which corresponds to fluctuations of the tachyon T of the form

$$T = f(a(x - t(y, z))). \quad (3.8)$$

²In the appendix we describe, this rather tedious derivation, including the fermionic degrees of freedom.

³One can generalize this argument to other dimensionality, by applying T-duality.

In this case we find

$$\begin{aligned} \mathcal{H} = & [(\mathcal{H}_{\text{mod } T})^2 + a^2[f'(a(x - t(y, z)))]^2\{P_\chi^x \\ & + (P_\chi^y \partial_2 t)^2 + (P_\chi^z \partial_3 t)^2\}^2 + a^2[f'(a(x \\ & - t(y, z))]P_T]^2\{1 + (\partial_y t)^2 + (\partial_z t)^2\} + P_T^2\}^{1/2}. \end{aligned} \quad (3.9)$$

Looking for static solutions in which $P_T = 0$, we find again that for the Hamiltonian to be finite in the $a \rightarrow \infty$ limit, the canonical momenta of the degrees of freedom, P_χ , must be perpendicular to the direction of the kink. We are thus left again with the Hamiltonian of a two-dimensional Dirichlet brane.

We have thus shown that the Dirac-Born-Infeld (DBI) action describing the dynamics of the tachyon field on a non-BPS Dp -brane has a kink solution described by the DBI action on a BPS $D(p - 1)$ -brane.

B. Lagrangian approach

Instead of working with the Hamiltonian, one can directly evaluate the Lagrangian, Eq. (2.7). To do so, we follow the method described in Ref. [4] and extend it to explicitly include nonzero world-volume electromagnetic fields, by using the fact that the canonical momenta are constrained to lie perpendicular to the tachyon. Without the previous Hamiltonian analysis this is an assumption; however, here it is a consistency requirement if the dynamics are to reach a fully condensed configuration. The action given by Eq. (2.7) for a D3-brane, in the absence of fermionic degrees of freedom, is just

$$S = - \int dt dx dy dz V(T) \sqrt{-\det(\mathcal{M})}, \quad (3.10)$$

where $\mathcal{M}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + \partial_\mu T \partial_\nu T$.

We consider that the tachyon is given from the field configuration described in Eq. (3.8) and use that for the Hamiltonian to be finite, all the degrees of freedom must evolve perpendicular to the kink (the kink is along the x direction and y, z coordinates are perpendicular to the x direction), namely,

$$\begin{aligned} A_\mu(x, t(y, z)) = & \begin{cases} 0 & \text{for } \mu = x \\ a_\alpha(t(y, z)) & \text{for } \mu = \alpha \end{cases} \quad \text{and} \\ X^I(x, t(y, z)) = & x^I(t(y, z)); \end{aligned} \quad (3.11)$$

A_μ, X^I fields are independent of x . Thus, we obtain

$$\mathcal{M}_{xx} = 1 + a^2(f')^2, \quad (3.12)$$

$$\mathcal{M}_{x\alpha} = \mathcal{M}_{\alpha x} = -a^2(f')^2 \partial_\alpha t, \quad (3.13)$$

$$\mathcal{M}_{\alpha\beta} = [a^2(f')^2 - 1] \partial_\alpha t \partial_\beta t + \mathbf{m}_{\alpha\beta}, \quad (3.14)$$

where

$$\mathbf{m}_{\alpha\beta} \equiv \eta_{\alpha\beta} + \partial_\alpha X^m \partial_\beta X_m + F_{\alpha\beta} + \partial_\alpha t \partial_\beta t \quad (3.15)$$

is just the equivalent of \mathcal{M} with terms depending of the x direction being removed.

By manipulating the determinant of the metric (as in Ref. [4]), to first order, we find

$$\det(\mathcal{M}) \sim a^2(f')^2(\det(\mathbf{m})). \quad (3.16)$$

Thus, the action, in the limit $a \rightarrow \infty$, i.e. after the tachyon condensation, is

$$\lim_{a \rightarrow \infty} S = - \int dt dx dy dz V(f) a f' \sqrt{-\det(\mathbf{m})} \quad (3.17a)$$

$$= -\mathcal{T}_2 \int dt dy dx \sqrt{-\det(\mathbf{m})}, \quad (3.17b)$$

where

$$\mathcal{T}_2 \equiv \int V(\tilde{x}) d\tilde{x} \quad (3.18)$$

is taken to be the tension of the resulting D2-brane. Equation (3.17b) is the world-volume action of a D2-brane.

Thus, the classical effective theory describing the dynamics of the tachyon field on a non-BPS Dp -brane has a kink solution of finite tension described by a codimension one BPS brane.

IV. FERMIONIC SECTOR

Including the spinors rapidly leads to rather complicated equations; however, these can be simplified by considering a change of (fermionic) coordinates

$$\partial_\mu Y_I = \bar{\Theta}_+ \Gamma_I \partial_\mu \Theta_+, \quad (4.1)$$

with the restriction,

$$\partial_\mu X_n (\partial_\nu Y_m)^\dagger = \partial_\nu X_m (\partial_\mu Y_n)^\dagger. \quad (4.2)$$

One can then explicitly check that the equations of motion derived from

$$\begin{aligned} \hat{\mathcal{L}} = & -V(T) [-\det(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m - 2\partial_\mu Y_\nu \\ & - 2\partial_\mu Y^m \partial_\nu Y_m + \partial_\mu Y^I \partial_\nu Y_I + F_{\mu\nu} + \partial_\mu T \partial_\nu T)]^{1/2} \end{aligned} \quad (4.3)$$

coincide with those obtained from Eq. (2.7).

To be explicit, the equations of motion (e.o.m.) obtained from Eq. (2.7) are

$$\partial_\nu X^I - \bar{\Theta}_+ \Gamma^I \partial_\nu \Theta_+ = 0, \quad \text{e.o.m. for } \bar{\Theta}_+ \quad (4.4)$$

$$\left(\frac{1}{2} V(T) \sqrt{-\det(\mathcal{M})} \mathcal{M}_{[\mu\nu]}^{-1} \partial^\nu X_m \right)^\mu = 0, \quad \text{e.o.m. for } X \quad (4.5)$$

$$\frac{1}{2}V(T)\sqrt{-\det(\mathcal{M})}\mathcal{M}_{[\mu\nu]}^{-1}\partial^\mu X^m(\bar{\Theta}_+\Gamma_m)^{\nu} = 0, \quad (4.6)$$

e.o.m. for Θ_+ ,

where $\mathcal{M}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T$. Equation (4.6) restricts, $\partial_\mu X^m \partial_\nu \bar{\Theta}_+\Gamma_m$ to be symmetric in μ and ν .

The relevant equations of motion from Eq. (4.3) are

$$\left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}(\partial^\nu X_m - 2\partial^\mu Y_m)\right)^{\nu} + \left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}\partial^\nu X_m\right)^{\mu} = 0, \quad \text{e.o.m. for } X \quad (4.7)$$

$$\left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}(\partial^\nu Y_m - 2\partial^\mu X_m)\right)^{\mu} + \left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}\partial^\mu Y_m\right)^{\beta} = 0, \quad \text{e.o.m. for } Y_m \quad (4.8)$$

$$\left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}(\partial^\nu Y_\gamma - 2\delta_\gamma^\mu)\right)^{\mu} + \left(\frac{1}{2}V(T)\sqrt{-\det(\mathcal{N})}\mathcal{N}_{\mu\nu}^{-1}\partial^\mu Y_\gamma\right)^{\nu} = 0, \quad \text{e.o.m. for } Y_\gamma, \quad (4.9)$$

where

$$\mathcal{N}_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} - 2\partial_\mu Y_\nu - 2\partial_\mu Y^m \partial_\nu Y_m + \partial_\mu Y^I \partial_\nu Y_I + \partial_\mu T \partial_\nu T. \quad (4.10)$$

Equations (4.7) and (4.8) imply that $\partial_\mu X_m = \partial_\nu Y_m$. Setting $\partial_\mu Y_\nu = \eta_{\mu\nu}$, to be compatible with the static gauge (this can be seen by doing this change of variables before enforcing the static gauge constraint), one can recover Eq. (4.4). Thus, Eq. (4.8) reproduces Eq. (4.5). By doing so, we have lost the equation of motion for Θ_+ , Eq. (4.6); however, the restriction we place on $\partial_\mu Y_m$, Eq. (4.2), ensures that Eq. (4.6) is always satisfied. In this sense, the Lagrangian given in Eq. (4.3) produces the *on shell* dynamics of the brane. We finally make the remark that the equations of motion for the tachyon and world-volume gauge fields are the same.

In these coordinates, the Hamiltonian becomes (see appendix)

$$\begin{aligned} \mathcal{H} = & [(P_X^m)^2 + (P_T)^2 + (P_Y^I)^2 + (I_i)^2 \\ & + (P_A^i \partial_i X^m)^2 (P_A^i \partial_i T)^2 + (P_A^i \partial_i Y_I)^2 \\ & + (P_A^i)^2 + V(T)^2 \det(h)]^{1/2}. \end{aligned} \quad (4.11)$$

This is precisely the Hamiltonian one arrives at by using the Lagrangian Eq. (4.3), and the result of the equation of motion, $\partial_\mu Y^I = \partial_\mu X^I$, since in that case the new coordi-

nates, Y_I , appear on the same footing as the transverse scalars, X_I , which is how they enter Eq. (4.11).

Finally, we note that just as in Ref. [3], this Hamiltonian can formally be written as

$$\mathcal{H} = \sqrt{(P_X^N)^2 + (P_X^M F_{MN})^2}, \quad (4.12)$$

where χ_N is the set of variables, (A_i, X_m, T, Y_I) with $N, M \dots = 1, 2, \dots, d + d$ and P_X^N are their canonical momenta, with $F_{MN} = \partial_M \chi_N - \partial_N \chi_M$, where

$$\partial_N = \begin{cases} \partial_M & \text{for } N \leq p \\ 0 & \text{for } N > p. \end{cases} \quad (4.13)$$

Thus the conclusions of the bosonic case carry over immediately for the explicit inclusion of fermionic degrees of freedom.

V. CONCLUSIONS

We have shown that a non-BPS Dp -brane carrying non-zero world-volume electromagnetic fields decays into a $D(p-1)$ -brane with the electromagnetic fields conserved and localized on that brane. The requirement that these gauge fields be conserved manifests itself as a restriction on the direction along which the tachyon can decay.

In previous literature, it was assumed that any gauge fields on the Dp -brane were perpendicular to the tachyon, enabling the $D(p-1)$ -brane to form. Here, we have used the Hamiltonian approach, to demonstrate that it is only when this condition is met that the tachyon can indeed condense.

We have considered that the tachyon depends on a particular direction and shown that only when the gauge fields are localized perpendicular to this direction, is it possible for the tachyon condensation to occur. A corollary of course is that if the gauge fields are nonzero in all the world-volume directions, then tachyon condensation cannot occur, or at least the energy produced by coupling between the tachyon's spatial derivatives and the gauge fields would prevent the true tachyon vacuum by being reached.

In particular, this shows that a bound system composed by two branes of different dimensionality (Dp, Dp' bound brane system with $p' < p$), which can be described by just a Dp -brane with world-volume gauge fields, decays exactly as a standard Dp -brane, namely, by forming a bound state of a $D(p-1)$ and a Dp' -brane.

Finally, it would be interesting to investigate whether we can recover these results by following the gauge theory approach of brane-antibrane systems discussed in Ref. [8].

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APPENDIX

We start by extracting the time derivatives of the Lagrangian Eq. (2.7) using the identity [3],

$$\begin{aligned} \det(\mathcal{M}_{\mu\nu}) &\equiv \det(\eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X_m + F_{\mu\nu} + B_{\mu\nu} \\ &\quad + \partial_\mu T \partial_\nu T) \\ &= (\dot{X}^m \dot{X}_m + \dot{T} \dot{T} + B_{00} - 1) \det(h) \\ &\quad + E_i^{(+)} D_{ij}(h) E_j^{(-)}, \end{aligned} \quad (\text{A1})$$

where

$$\begin{aligned} E_i^{(\pm)} &= F_{0i} \pm \dot{X}^m \partial_i X_m \pm B_{0i} \pm \dot{T} \partial_i T, \\ h_{ij} &= \delta_{ij} + \partial_i X^m \partial_j X_m + F_{ij} + B_{ij} + \partial_i T \partial_j T, \end{aligned}$$

and

$$D_{ij}(h) - (-1)^{i+j} \Delta_{ij}(h)$$

with $\Delta_{ij}(h)$ being the determinant of h with the i th row and j th column removed. Note that, when the inverse exists, we have $D_{ij}(h) = \det(h) h_{ij}^{-1}$. Here we have used the fact that $B_{ij} = B_{ji}$, by employing the equation of motion for $\bar{\Theta}_+$.

Using this we find that the canonical momenta are

$$\begin{aligned} P_X^m &= \frac{-V(T)}{\sqrt{-\det(\mathcal{M})}} \left[-\dot{X}^m \det(h) + \frac{1}{2} (E_i^{(+)} D_{ij}(h) \right. \\ &\quad \left. - D_{ji} R_i^{(-)}) \partial_j X^m \right], \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} P_{\Theta_+} &= \frac{-V(T)}{\sqrt{-\det(\mathcal{M})}} \left[(\bar{\Theta}_+ \Gamma_I \dot{\Theta}_+) (\bar{\Theta}_+ \Gamma^I) \det(h) \right. \\ &\quad + \frac{1}{2} (\bar{\Theta}_+ \Gamma_I) (\bar{\Theta}_+ \Gamma^I \partial_i \Theta_+) D_{ij}(h) E_j^{(-)} \\ &\quad \left. - \frac{1}{2} E_i^{(+)} D_{ij}(h) (\bar{\Theta}_+ \Gamma_I \partial_j \Theta_+) (\bar{\Theta}_+ \Gamma^I) \right], \end{aligned} \quad (\text{A3})$$

$$P_A^i = \frac{V(T)}{2\sqrt{-\det(\mathcal{M})}} [D_{ij}(h) E_j^{(-)} + E_j^{(+)} D_{ji}(h)], \quad (\text{A4})$$

$$\begin{aligned} P_T &= \frac{-V(T)}{\sqrt{-\det(\mathcal{M})}} \left[-\dot{T} \det(h) + \frac{1}{2} (E_i^{(+)} D_{ij}(h) \right. \\ &\quad \left. - D_{ij}(h) E_j^{(-)}) \partial_j T \right], \end{aligned} \quad (\text{A5})$$

where we have used the equation of motion for $\bar{\Theta}$ to write

$$B_{\mu\nu} = -(\bar{\Theta}_+ \Gamma_I \partial_\mu \Theta_+) (\bar{\Theta}_+ \Gamma^I \partial_\nu \Theta_+)$$

and assumed $V(T) \neq V(T, \dot{T})$. Thus, the Hamiltonian can be written as

$$\begin{aligned} \mathcal{H} &= P_X^I \dot{X}_I + P_{\Theta_+} \dot{\Theta}_+ + P_A^i F_{0i} + P_T \dot{T} - \mathcal{L} \\ &= \frac{V(T) \det(h)}{\sqrt{-\det(\mathcal{M})}}, \end{aligned} \quad (\text{A6})$$

where we have fixed the final gauge freedom via $\partial_i A_0 = 0$.

Let us define

$$\begin{aligned} C_I &\equiv \mathcal{H} \left[(\bar{\Theta}_+ \Gamma_I \dot{\Theta}_+) - \frac{1}{2} (E_j^{(+)} h_{ji}^{-1} - h_{ij}^{-1} E_j^{(-)}) \right. \\ &\quad \left. \times (\bar{\Theta}_+ \Gamma_I \partial_i \Theta_+) \right], \end{aligned} \quad (\text{A7})$$

and use the identities

$$\begin{aligned} \frac{\mathcal{H}}{2} (E_j^{(+)} h_{ji}^{-1} - E_j^{(-)} h_{ij}^{-1}) &= \partial_i X_m P_X^m + \partial_i T P_T + F_{ij} P_A^i \\ &\quad + \partial_i \Theta_+ P_{\Theta_+} \equiv I_i, \end{aligned} \quad (\text{A8})$$

$$\begin{aligned} \mathcal{H}^2 [\dot{X}_m \dot{X}^m + \dot{T} \dot{T} + B_{00} + E_j^{(+)} h_{ji}^{-1} E_j^{(-)}] \\ = (P_X^m)^2 + (P_T)^2 + (C_I)^2 + (I_i)^2 + (P_A^i \partial_i X^m)^2 \\ + (P_A^i \partial_i T)^2 + [P_A^i (\bar{\Theta}_+ \Gamma_I \partial_i \Theta_+)]^2 + (P_A^i)^2, \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} \mathcal{H}^2 [1 - \dot{X}_m \dot{X}^m - \dot{T} \dot{T} - B_{00} 0 E_i^{(+)} h_{ij}^{(-)} E_j^{(-)}] \\ = V(T)^2 \det(h), \end{aligned} \quad (\text{A10})$$

to find

$$\begin{aligned} \mathcal{H} &= [(P_X^m)^2 + (P_T)^2 + (C_I)^2 + (\partial_i X_m P_X^m + \partial_i T P_T \\ &\quad + F_{ij} P_A^j + \partial_i \Theta_+ P_{\Theta_+})^2 + (P_A^i \partial_i X^m)^2 + (P_A^i \partial_i T)^2 \\ &\quad + (P_A^i \bar{\Theta}_+ \Gamma_I \partial_i \Theta_+)^2 + (P_A^i)^2 + V(T)^2 \det(h)]^{1/2}. \end{aligned} \quad (\text{A11})$$

Notice that the bosonic case, $\Theta_+ = \bar{\Theta}_+ = P_{\Theta_+} = C_I = 0$, gives us Eq. (3.1), as expected.

The fermionic terms lend themselves to a simplification, by noting that

$$P_{\Theta_+} = C_I \bar{\Theta}_+ \Gamma^I. \quad (\text{A12})$$

Thus, we see that all the Θ_+ terms occur in the combination

$$\bar{\Theta}_+ \Gamma_I \partial_i \Theta_+, \quad \text{and} \quad C_I. \quad (\text{A13})$$

If we define new coordinates Y_I , such that $\partial_\mu Y_I = \bar{\Theta}_+ \Gamma_I \partial_\mu \Theta_+$, then

$$\begin{aligned} P_Y^I &= \mathcal{H} \left[(\bar{\Theta}_+ \Gamma^I \dot{\Theta}_+) + -\frac{1}{2} (E_i^{(+)} h_{ij}^{-1} - h_{ij}^{-1} E_j^{(-)}) \right. \\ &\quad \left. \times (\bar{\Theta}_+ \Gamma^I \partial_i \Theta_+) \right] = C^I. \end{aligned} \quad (\text{A14})$$

So, in terms of Y_I the Hamiltonian Eq. (A11) becomes

$$\begin{aligned} \mathcal{H} &= [(P_X^m)^2 + (P_T)^2 + (P_Y^I)^2 + (I_i)^2 \\ &\quad + (P_A^i \partial_i X^m)^2 (P_A^i \partial_i T)^2 + (P_A^i \partial_i Y_I)^2 \\ &\quad + (P_A^i)^2 + V(T)^2 \det(h)]^{1/2}. \end{aligned} \quad (\text{A15})$$

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