Matter power spectrum for the generalized Chaplygin gas model: The Newtonian approach

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(Received 24 October 2008; published 21 November 2008)

We model the cosmic medium as the mixture of a generalized Chaplygin gas and a pressureless matter component. Within a neo-Newtonian approach (in which, different from standard Newtonian cosmology, the pressure enters the homogeneous and isotropic background dynamics) we compute the matter power spectrum. The 2dFGRS data are used to discriminate between unified models of the dark sector (a purely baryonic matter component of roughly 5% of the total energy content and roughly 95% generalized Chaplygin gas) and different models, for which there is separate dark matter, in addition to that accounted for by the generalized Chaplygin gas. Leaving the corresponding density parameters free, we find that the unified models are strongly disfavored. On the other hand, using unified model priors, the observational data are also well described, in particular, for small and large values of the generalized Chaplygin gas parameter α. The latter result is in agreement with a recent, more qualitative but fully relativistic, perturbation analysis in [V. Gorini, A. Y. Kamenshchik, U. Moschella, O. F. Piatella, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 02 (2008) 016.].

DOI: 10.1103/PhysRevD.78.103523 PACS numbers: 98.80.−k, 95.35.+d, 95.36.+x

I. INTRODUCTION

The crossing of different cosmological observations, in particular, the anisotropy spectrum of the cosmic microwave background radiation (CMBR), the luminosity distance of supernovae of type Ia, gravitational lensing and baryonic acoustic oscillations, indicates that around 95% of the cosmic substratum is not directly detectable through electromagnetic emission [1–4]. As long as one accepts General Relativity (GR) to be valid, the now widely accepted conclusion is that most of the substance in the Universe must be of nonbaryonic origin. This dynamically dominating nonbaryonic substratum is usually divided into two components: dark matter, a pressureless, agglomerating component, being present in local structures like galaxies and clusters of galaxies, and smoothly distributed dark energy, an exotic fluid with negative pressure. Dark matter is required in order to explain the observed anomalies in the dynamics of galaxies and cluster of galaxies, as well as to generate the large-scale structures in the Universe; dark energy is required in order to account for the present stage of accelerated expansion of the Universe and for the position of the first acoustic peak in the CMBR spectrum. The nature of these dark components remains a mystery. For reviews on the subject see [5,6].

Among the host of models that have been proposed for dark matter and dark energy over the last years, there are unified models of the dark sector according to which there is just one dark component that simultaneously plays the role of dark matter and dark energy. The most popular proposal along this line is the Chaplygin gas, an exotic fluid with negative pressure that scales as the inverse of the energy density [7]. This phenomenologically introduced equation of state can be given a string theory based motivation [8]. It has also been generalized in different phenomenological ways [9]. Another example for a unification scenario for the dark sector is a bulk viscous model of the cosmic substratum [10]. While the Chaplygin gas model (in its traditional and generalized forms) has been very successful in explaining the supernovae type Ia data [11], there are claims that it does not pass the tests connected with structure formation because of predicted but not observed strong oscillations of the matter power spectrum [12]. It should be mentioned, however, that oscillations in the Chaplygin gas component do not necessarily imply corresponding oscillations in the observed baryonic power spectrum [13]. For previous studies of inhomogeneities in Chaplygin gases see also [14].

The generalized Chaplygin gas is characterized by the equation of state

$$p = -\frac{A}{\rho^{\alpha}}. \tag{1}$$

For $A > 0$ the pressure p is negative, hence it may induce an accelerated expansion of the Universe. The corresponding sound speed is positive as long as $\alpha > 0$. Recently, a gauge-invariant analysis of the baryonic matter power spectrum for generalized Chaplygin gas cosmologies was shown to be compatible with the data for parameter values $\alpha \approx 0$ and $\alpha \geq 3$ [15]. This result seems to strengthen the role of Chaplygin gas type models as competitive candi-

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dates for the dark sector. The present work provides a further investigation along these lines. While we shall rediscover the mentioned results of [15], albeit in a different framework, we also extend the scope of the analysis in the following sense. The authors of [15] have shown that Chaplygin gas cosmologies are consistent with the data from structure formation for certain parameter configurations. Here we ask additionally whether or not the data really favor generalized Chaplygin gases as unified models of the dark sector. By leaving the density parameters of the Chaplygin gas and the nonrelativistic matter component, respectively, free, we allow for a matter fraction that can be different from the pure baryonic part. This is equivalent to hypothetically admit the existence of an additional dark matter component. In other words, we do not prescribe the unified model from the start. Moreover, our study is not restricted to the spatially flat case. The 2dFGRS data are then used to test whether or not the unified model, requiring that the matter component describes baryonic matter with a density parameter of the order of 5% only, is favored.

Now, a precise estimation of the cosmological parameters using the matter power spectrum is very involved, since a detailed discussion of many physical processes like free streaming of neutrinos, electron diffusion, etc. is necessary. We shall avoid such more complex analysis by using conveniently the BBKS transfer function [16–18], which connects the primordial spectrum with the spectrum observed today, to impose the initial conditions. We believe that this type of analysis retains the essential features of the process and leads to quantitatively relevant results.

Our study relies on a *neo-Newtonian* approach which represents a major simplification of the problem. In some sense, the neo-Newtonian equations can be seen as the introduction of a first-order relativistic correction to the usual Newtonian equations [19]. The neo-Newtonian equations for cosmology [19–22] modify the Newtonian equations in a way that makes the pressure dynamically relevant already for the homogeneous and isotropic background. This allows us to describe an accelerated expansion of the Universe as the consequence of a sufficiently large effective negative pressure in a Newtonian framework. While the neo-Newtonian approach reproduces the GR background dynamics exactly, differences occur at the perturbative level. However, the GR first-order perturbation dynamics and its neo-Newtonian counterpart coincide exactly in the case of a vanishing sound speed [22]. One may therefore expect that the neo-Newtonian perturbation dynamics reproduces the correct GR results on all perturbation scales at least for small values of the sound speed. For constant equations of state it has been demonstrated that the correct large-scale behavior in the synchronous gauge is reproduced [21]. On small scales one expects the spatial pressure gradient term to be relevant and the difference to the GR dynamics should be of minor importance. Since the observational data correspond to modes that are well inside the Hubble radius, the use of a Newtonian type approach seems therefore adequate.

On this basis our analysis extends previous neo-Newtonian studies to the two-component case. One of the components is a generalized Chaplygin gas, the other one represents pressureless matter. The advantage of employing a neo-Newtonian approach is a gain in simplicity and transparency. While in future work all results will have to be confirmed within GR, we shall ensure already here that in the region of overlap between GR and neo-Newtonian dynamics our results coincide with the corresponding GR results. Our neo-Newtonian approach reproduces the parameter estimations for the unified dark matter/dark energy in [15] also numerically. We mention, that the coincidence of neo-Newtonian and GR results on the scales of interest here was also demonstrated in [23]. Backed up by this success of the neo-Newtonian approach we then enlarge the scope of our analysis and test the validity of the unified model itself by relaxing the unified model priors used in [15]. Denoting the present value of the Chaplygin gas density parameter by Ω_{c0}, we admit the total present matter density parameter Ω_{m0} to be the sum of an additional dark matter component with density parameter Ω_{dm0} and the baryon contribution Ω_{b0}, i.e., $\Omega_{m0} = \Omega_{dm0} + \Omega_{b0}$. Leaving the density parameters free, we investigate whether or not the unified model with $\Omega_{c0} \approx 0.96$, $\Omega_{b0} \approx 0.04$ and $\Omega_{dm0} \approx 0$ is favored by the large-scale structure data. We mention that a similar investigation using supernova type Ia data reveals that the unification scenario is the most favored one [11].

Our Chaplygin gas cosmology has four free parameters: the value of α, the present Chaplygin gas and dark matter density parameters Ω_{c0} and Ω_{dm0}, respectively, and the present Chaplygin gas sound speed v_0^2. There are two main observational sources concerning the matter power spectrum today: the 2dFGRS and the SDSS data sets [24,25]. For reasons to be discussed later, we will mainly use the 2dFGRS data. For $\Omega_{dm0} \approx 0$, $\Omega_{b0} \approx 0.04$ and $\Omega_{c0} \approx 0.96$, equivalent to the unified model, we obtain a very good fit of the date where very small or very large values of α are preferred. This reproduces the GR results of [15] in a Newtonian context. On the other hand, when Ω_{dm0}, Ω_{c0} and α are left free, large values of Ω_{dm0} and small values of Ω_{c0} are preferred, thus disfavoring the unification model. The same result is obtained when all four parameters (Ω_{dm0}, Ω_{c0}, α and $\bar{A}$) are left free. If the curvature is fixed to zero, as indicated by the Wilkinson Microwave Anisotropy Probe (WMAP) results [26], implying $\Omega_{c0} \approx 1 - \Omega_{m0}$, the predictions do not change substantially and a scenario with almost no dark energy is again preferred. In all cases, including those for which the unification scenario is imposed from the beginning, the minimum values of the χ^2 statistical parameter are very similar. This does not seem to allow definite predictions of the model. Any conclusion seems to depend on the chosen priors. We compare our results with those obtained from the ΛCDM model, for which the power spectrum test indicates $\Omega_{dm0} \approx 0.25$. However, almost no restrictions on the value of the cosmological constant are obtained. In fact, the matter power spectrum seems to be a good indicator for dark matter but not for dark energy.

The paper is organized as follows. In Sec. II we recall the generalized Chaplygin gas model and the basic equations of standard Newtonian cosmology. In Sec. III we introduce the neo-Newtonian framework for the two-component model of a generalized Chaplygin gas and pressureless matter and establish the perturbation equations for this system. In Sec. IV the power spectrum is determined, from which the probability distribution functions for each parameter are obtained. Our results are discussed in Sec. IV.

II. THE GENERALIZED CHAPLYGIN GAS MODEL

The generalized Chaplygin gas is characterized by the equation of state (1), implying a negative pressure and a positive sound speed as long as $A > 0$ and $\alpha > 0$. The observational constraints from supernova type Ia data indicate that negative values for α are favored, but the dispersion is high enough to allow for a large range of positive values for this parameter [11]. Negative values for α imply an imaginary sound velocity, leading to small-scale instabilities at the perturbative level. Rigorously, the general situation is more complex: such instabilities for fluids with negative pressure may disappear if the hydrodynamical approach is replaced by a more fundamental description using, e,g., scalar fields. However, this is not true for the Chaplygin gas: even in a fundamental approach, using, for example, the Born-Infeld action, the sound speed square may be negative if $\alpha < 0$. For this reason we shall not allow α to be negative.

The traditional Chaplygin gas model is characterized by $\alpha = 1$. It is a consequence of the Nambu-Goto action parametrized in light-cone coordinates. Through some suitable transformations, the light-cone parametrized Nambu-Goto action reduces to the action of a Newtonian fluid that obeys the equation of state (1) with $\alpha = 1$ [8]. In this sense, it is somehow natural to construct a cosmological Chaplygin gas scenario within a Newtonian framework. To be precise, the symmetries of the Lagrangian are broken when gravity is included. But this drawback cannot even be cured by using a relativistic version: in order to preserve the symmetries of the original Nambu-Goto action a full string model must be implemented. But the Newtonian approach remains a reasonable approximation because of the mentioned relation between the Chaplygin gas and the Nambu-Goto action.

Now, establishing a Newtonian model for a universe in accelerated expansion seems to be impossible. In traditional Newtonian cosmology the pressure does not play any role in an isotropic and homogeneous universe: the universe evolves always with the scale factor $a(t) \propto t^{2/3}$, implying a decelerated expansion. This coincides with the relativistic cosmology for a pressureless fluid. The pressure becomes relevant only at perturbative level: there the nature of the fluid is essential for the evolution of the density contrast. For the evolution of density perturbations in Newtonian cosmology see Ref. [27]. The specific application to the Chaplygin gas model has been discussed in [28]. Let us sketch its main lines. The Newtonian cosmology is defined through the continuity equation, the Euler equation and the Poisson equation [27]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \tag{2}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho} - \nabla \phi, \tag{3}$$

$$\nabla^2 \phi = 4\pi G \rho. \tag{4}$$

In Eqs. (2)–(4) the pressure appears only in the form of a gradient. Hence, the pressure itself does not enter the dynamics of a spatially homogeneous background, i.e., the equations do not depend on the nature of the fluid. However, at perturbative level the relevant equation is

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left\{\frac{k^2 v_s^2}{a^2} - 4\pi G \rho\right\}\delta = 0, \tag{5}$$

where $\delta = \frac{\delta\rho}{\rho}$ is the density contrast, $\delta\rho$ being a first-order fluctuation around the background solution, $v_s^2 = \partial p / \partial \rho$ is the sound velocity and k is the wave number of the perturbation. If we consider a fluid whose equation of state is given by $p = \kappa \rho^{\varepsilon}$, the solution is

$$\delta = t^{-1/6}\left\{c_1 J_{5/(6\nu)}\left(\frac{\Lambda t^{-\nu}}{\nu}\right) + c_2 J_{-5/(6\nu)}\left(\frac{\Lambda t^{-\nu}}{\nu}\right)\right\}, \tag{6}$$

with $\nu = \varepsilon - \frac{4}{3}$.

For the Chaplygin gas model the perturbations initially grow as in the matter-dominated universe, later they decrease, finally approaching zero, which is the value for the cosmological constant model [28].

III. NEO-NEWTONIAN APPROACH

The drawback of standard Newtonian cosmology, the absence of a pressure term in the background dynamics, has been cured in a simple way [21]: in the conservation Eq. (2) one takes into account the work done by the pressure during the expansion of the Universe. At the same time, the equation for the gravitational potential must be modified in order to render the equations compatible. This has been done in Refs. [19–21]. The final equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) + p \nabla \cdot \vec{v} = 0, \tag{7}$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\nabla p}{\rho + p} - \nabla \phi, \tag{8}$$

$$\nabla^2 \phi = 4\pi G(\rho + 3p). \tag{9}$$

For the case of two noninteracting fluids with energy densities ρ_c and ρ_m and pressures p_c and $p_m = 0$, respectively, the equations are

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot (\rho_c \vec{v}_c) + p_c \nabla \cdot \vec{v}_c = 0, \tag{10}$$

$$\frac{\partial \vec{v}_c}{\partial t} + \vec{v}_c \cdot \nabla \vec{v}_c = -\frac{\nabla p_c}{\rho_c + p_c} - \nabla \phi, \tag{11}$$

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot (\rho_m \vec{u}_m) = 0, \tag{12}$$

$$\frac{\partial \vec{v}_m}{\partial t} + \vec{v}_m \cdot \nabla \vec{v}_m = -\nabla \phi, \tag{13}$$

$$\nabla^2 \phi = 4\pi G(\rho_m + \rho_c + 3p_c). \tag{14}$$

The subscript m stands for pressureless matter and the subscript c for the (generalized) Chaplygin gas component. Considering now an isotropic and homogeneous universe with $\rho = \rho(t)$, $p = p(t)$ and $\vec{v} = \frac{\dot{a}}{a}\vec{r}$, we find

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}(\rho_m + \rho_c), \tag{15}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_c + \rho_m + 3p_c). \tag{16}$$

These equations are identical to the corresponding equations for a homogeneous and isotropic universe in GR. In a sense, the neo-Newtonian formulation intends to reproduce the equations of GR, but in a Newtonian conceptual framework.

While there is a complete equivalence between the general relativistic and the neo-Newtonian equations in the homogeneous and isotropic background, this is no longer the case at the perturbative level. As already mentioned, the GR first-order perturbation dynamics and its neo-Newtonian counterpart coincide exactly only in the case of a vanishing sound speed [22]. But also for small values of the sound speed the neo-Newtonian perturbation dynamics will very likely be a reasonable approximation. Since the observational data correspond to modes that are well inside the Hubble radius, the use of a Newtonian type approach seems adequate, at least as a first and transparent step towards a full relativistic treatment.

Defining the fractional density contrasts

$$\delta_c = \frac{\delta \rho_c}{\rho_c} \quad \text{and} \quad \delta_m = \frac{\delta \rho_m}{\rho_m} \tag{17}$$

for the Chaplygin gas and matter components, respectively, the first-order perturbation equations for the system (10)–(14) are

$$\begin{aligned} &\ddot{\delta}_c + \left\{2\frac{\dot{a}}{a} - \frac{\dot{\omega}_c}{1+\omega_c} + 3\frac{\dot{a}}{a}(v_c^2 - \omega_c)\right\}\dot{\delta}_c \\ &\quad + \left\{3\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)(v_c^2 - \omega_c) + 3\frac{\dot{a}}{a}\left[\dot{v}_c^2 - \dot{\omega}_c\frac{(1+v_c^2)}{1+\omega_c}\right]\right. \\ &\quad \left. + \frac{v_c^2 k^2}{a^2} - 4\pi G\rho_c(1+\omega_c)(1+3v_c^2)\right\}\delta_c \\ &= 4\pi G\rho_m(1+\omega_c)\delta_m \end{aligned} \tag{18}$$

and

$$\ddot{\delta}_m + 2\frac{\dot{a}}{a}\dot{\delta}_m - 4\pi G\rho_m\delta_m = 4\pi G\rho_m(1+3v_c^2)\delta_c, \tag{19}$$

where $v_c^2 = \frac{\partial p_c}{\partial \rho_c}$ and $\omega_c = \frac{p_c}{\rho_c}$. The quantity k^2 denotes the square of the comoving wave vector. Dividing the Eqs. (18) and (19) by H_0^2 and redefining the time as $tH_0 \to t$, these equations become dimensionless. In terms of the scale factor a as dynamical variable, the system (18) and (19) takes the form

$$\begin{aligned} &\delta_c'' + \left\{\frac{2}{a} + g(a) - \frac{\omega_c'(a)}{1+\omega_c(a)} - 3\frac{1+\alpha}{a}\omega_c(a)\right\}\delta_c' \\ &\quad - \left\{3\left[\frac{g(a)}{a} + \frac{1}{a^2}\right](1+\alpha)\omega_c(a) + \frac{3}{a}\left(\frac{1+\alpha}{1+\omega_c(a)}\right)\omega_c'(a)\right. \\ &\quad + \frac{\alpha\omega_c(a)k^2 l_H^2}{a^2 f(a)} + \frac{3}{2}\frac{\Omega_{c0}}{f(a)}h(a)[1+\omega_c(a)] \\ &\quad \left.\times[1-3\alpha\omega_c(a)]\right\}\delta_c = \frac{3}{2}\frac{\Omega_{m0}}{a^3 f(a)}[1+\omega_c(a)]\delta_m \end{aligned} \tag{20}$$

and

$$\begin{aligned} &\delta_m'' + \left[\frac{2}{a} + g(a)\right]\delta_m' - \frac{3}{2}\frac{\Omega_{m0}}{a^3 f(a)}\delta_m \\ &\quad = \frac{3}{2}\frac{\Omega_{c0}}{f(a)}h(a)[1-3\alpha\omega_c(a)]\delta_c, \end{aligned} \tag{21}$$

where $l_H = cH_0^{-1}$ is the present Hubble radius and c is the velocity of light. The prime denotes a derivative with respect to a and the definitions

$$f(a) = \dot{a}^2 = \left[\frac{\Omega_{m0} + \Omega_{c0}a^3 h(a)}{a} + \Omega_{k0}\right], \tag{22}$$

$$g(a) = \frac{\ddot{a}}{\dot{a}^2} = -\frac{\Omega_{m0} + \Omega_{c0}[h(a) - 3\bar{A}h^{-\alpha}]a^3}{2a[\Omega_{m0} + \Omega_{c0}a^3 h(a) + \Omega_{k0}a]}, \tag{23}$$

$$h(a) = [\bar{A} + (1-\bar{A})a^{-3(1+\alpha)}]^{1/(1+\alpha)}, \tag{24}$$

$$\omega_c(a) = -\frac{\bar{A}}{h(a)^{1+\alpha}}, \tag{25}$$

with

$$\bar{A} = \frac{A}{\rho_{c0}^{1+\alpha}}, \qquad v_{s0}^2 = \alpha \bar{A} \tag{26}$$

have been used. Recall that $\Omega_{m0} = \Omega_{dm0} + \Omega_{b0}$. For the unified model to be an adequate description one expects $\Omega_{m0} \approx \Omega_{b0}$. In case the data indicate a substantial fraction of Ω_{dm0}, the unified model will be disfavored.

IV. THE POWER SPECTRUM: COMPARING THE THEORY WITH OBSERVATIONS

The power spectrum is defined by

$$\mathcal{P} = \delta_k^2, \tag{27}$$

where δ_k is the Fourier transform of the dimensionless density contrast δ_m. We will constrain the free parameters using the quantity

$$\chi^2 = \sum_i \left(\frac{\mathcal{P}_i^o - \mathcal{P}_i^t}{\sigma_i} \right)^2, \tag{28}$$

where $\mathcal{P}_i^o$ is the observational value for the power spectrum, $\mathcal{P}_i^t$ is the corresponding theoretical result and σ_i denotes the error bar. The index i refers to a measurement corresponding to given wave number. The quantity (28) qualifies the fitting of the observational data for a given theoretical model with specific values of the free parameters. Hence, χ^2 is a function of the free parameters of the model. The probability distribution function is then defined as

$$F(x_n) = F_0 e^{-\chi^2(x_n)/2}, \tag{29}$$

where the x_n denote the ensemble of free parameters and F_0 is a normalization constant. In order to obtain an estimation for a given parameter one has to integrate (marginalize) over all the other ones. For a more detailed description of this statistical analysis see Ref. [11].

The 2dFGRS [24] and the SDSS [25] are the main surveys to obtain matter power spectrum data. The last one covers a larger range of scales but the error bars are more narrow for the former one. There are some discussions in the literature concerning the relation between the different data [29]. In fact, the use of one or the other or the combination of both may result in different parameter estimations. For our model, however, the difference in using one or the other set of data is not significant (we have verified this!). Hence, from now on we focus on the 2dFGRS observational data for the power spectrum. We use the data that are related with the linear approximation, that is, those for which $kh^{-1} \leq 0.185\ \mathrm{Mpc}^{-1}$, where h is defined by $H_0 \equiv 100 \cdot h\ \mathrm{km/s \cdot Mpc}$. This definition should not be confused with the preceding definition of the function $h(a)$.

To fix the initial conditions we use the following procedure. The ΛCDM power spectrum is well fitted using the BBKS transfer function [16]. Then, employing the perturbed equations for the ΛCDM model and integrating back from today to a distant past, say $z = 1.000$, we fix the shape of the transfer function at that moment. The spectrum determined in this way is then used as the initial condition for our Chaplygin gas model. This procedure is described in more detail in Refs. [17,18].

To "gauge" our approach, let us first consider the ΛCDM model. In the general (nonflat) case there are two parameters: Ω_{dm0} and $\Omega_{\Lambda 0}$. In Fig. 1 we show the two-dimensional probability distribution function (PDF) as well the one-dimensional PDFs for the dark matter parameter Ω_{dm0} and for the cosmological constant parameter $\Omega_{\Lambda 0}$, respectively. From the two-dimensional graphic it is clear that there is a large degeneracy for the parameter $\Omega_{\Lambda 0}$, while the region of allowable values for Ω_{dm0} is quite narrow. The degeneracy for the cosmological constant density is less visible in the one-dimensional PDF graphic, but it is still considerable. Incidentally, the minimum value for the χ^2 parameter is 0.3822 for $\Omega_{dm0} = 0.2387$ and $\Omega_{\Lambda 0} = 0.5937$, corresponding to an open universe.

The four free parameters to be constrained in our Chaplygin gas model are Ω_{dm0}, Ω_{c0}, $\bar{A}$ and α. An analysis with four free parameters is computationally hard. For this reason we shall start working with sets of three or two free parameters, fixing the remaining one or two, respectively. Only afterwards we consider the most general case in

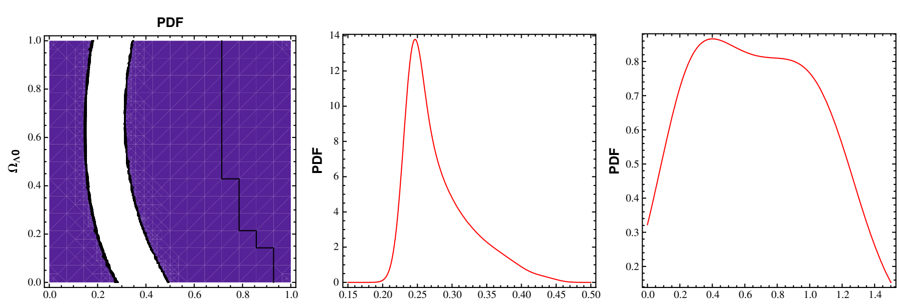

FIG. 1 (color online). The two-dimensional probability distribution function (PDF) for Ω_{dm0} and $\Omega_{\Lambda 0}$ (left) and the corresponding one-dimensional probability distribution functions for the nonflat ΛCDM model. In the left panel: the darker the color, the smaller the probability.

which all parameters are left free. This strategy will allow us to check the consistency of the final results. The baryonic component Ω_{b0} is kept fixed in agreement with the nucleosynthesis results. We use the value obtained by the recent five-year WMAP results, $\Omega_{b0} = 0.043$ (with $h = 0.72$). We will consider the following cases: (i) a spatially flat universe with no separate dark matter component, i.e. $\Omega_{dm0} = 0$, a baryonic component given by $\Omega_{b0} = 0.043$ and a dark sector component $\Omega_{c0} = 0.957$—there are two free parameters, α and $\bar{A}$; (ii) a flat universe with the density parameters free, except for the condition $\Omega_{c0} = 1 - \Omega_{dm0} - \Omega_{b0}$, with the parameter $\bar{A}$ fixed ($\bar{A} = 0.15$ and $\bar{A} = 0.95$); (iii) a flat universe with α, $\bar{A}$ and one density parameter free; (iv) the parameter $\bar{A}$ fixed (with values 0.15 and 0.95), while α and the two density parameters are free; (v) all four parameters free. Case (i) is the configuration studied in Ref. [15]. Our results for the PDF essentially confirm what was obtained in [15]: the one-dimensional PDF, after marginalizing over $\bar{A}$, is higher near $\alpha = 0$ and for $\alpha > 2$. For $\bar{A}$, the one-dimensional PDF, after marginalizing over α, is initially high, then it decreases until $\bar{A} \sim 0.7$, subsequently it is increasing again. This behavior is shown in Fig. 2. Considering the two-dimensional distribution, the minimum value for χ^2 is obtained for $\alpha = 3.57$ and $\bar{A} = 0$, with $\chi^2_{\rm min} = 0.378$, which is a value slightly better than that for the ΛCDM model, $\chi^2_{\rm min} = 0.382$. However, the superluminal sound speed renders this model unphysical. (For a possible modification that could preserve causality, see [15].)

In case (ii) we relax the restriction that the pressureless matter component is entirely given by baryons. It will turn out that this leads to curious results. For vanishing spatial curvature $\Omega_{c0} = 1 - \Omega_{dm0} - \Omega_{b0}$ is valid. As before, Ω_{b0} is fixed and we fix also $\bar{A} = 0.95$. Now, varying Ω_{dm0}, we span a two-dimensional PDF which depends on α and Ω_{dm0}. This two-dimensional PDF and the corresponding one-dimensional PDFs for α and Ω_{dm0}, respectively, are shown in Fig. 3. Again, values near $\alpha = 0$ and for $\alpha > 2$ are favored. On the other hand, the PDF for Ω_{dm0} decreases as Ω_{dm0} increases. This seems to favor the unification scenario which requires a small Ω_{dm0}. However, if we vary Ω_{c0} instead of Ω_{dm0}, we find that the PDF for Ω_{c0} also decreases as Ω_{c0} increases as is shown in Fig. 4. This seems to lead to the opposite conclusion than in the previous case: now the unified scenario which requires a large Ω_{c0} seems to be disfavored. Such a contradiction seems to be an artifact of the marginalization procedure as can be seen in the corresponding two-dimensional PDFs in Figs. 3 and 4: in the first case the probabilities are high near $\alpha = 0$ and for low values of Ω_{dm0}, but at the same time the minimum value for χ^2 is obtained for $\alpha = 0$ and $\Omega_{dm0} = 1$; on the other hand, in the second case, the probabilities

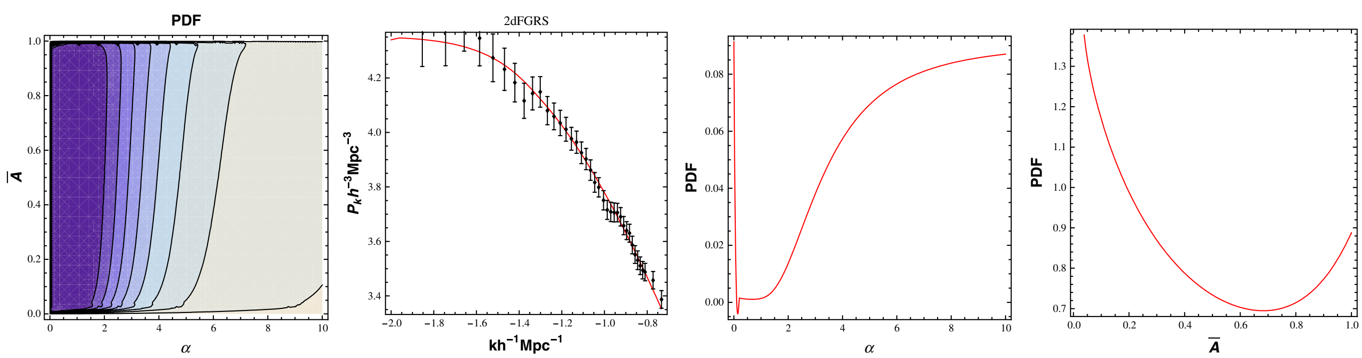

FIG. 2 (color online). The results for the flat case with $\Omega_{b0} = 0.043$, $\Omega_{dm0} = 0$ and $\Omega_{c0} = 0.957$, corresponding to the unification scenario (case (i)). From left to right: the two-dimensional PDF for α and $\bar{A}$ (with the same color convention as before), the best fitting curve for the power spectrum, the one-dimensional PDFs for α and $\bar{A}$.

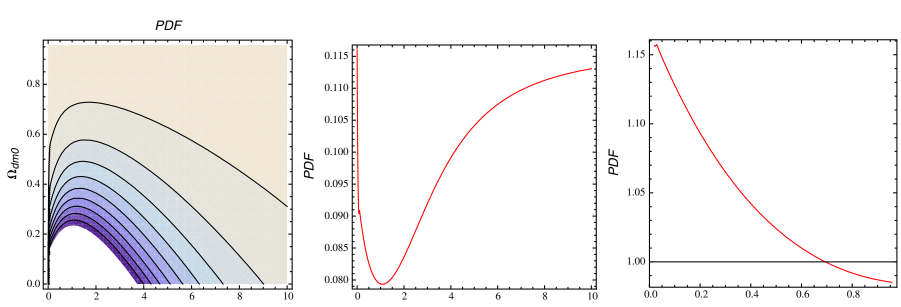

FIG. 3 (color online). The results for case (ii) with $\Omega_{b0} = 0.043$, $\Omega_{dm0} = 1 - \Omega_{c0} - \Omega_{b0}$. From left to right: the two-dimensional PDF for α and Ω_{dm0}, the one-dimensional PDFs for α and Ω_{dm0}.

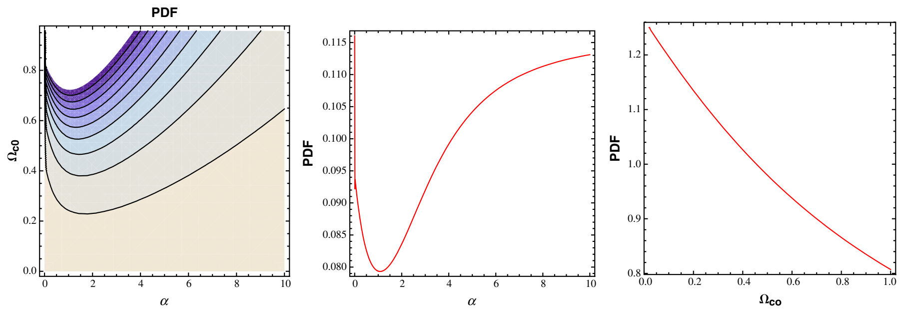

FIG. 4 (color online). The results for case (ii) with $\Omega_{b0} = 0.043$, $\Omega_{c0} = 1 - \Omega_{md0} - \Omega_{b0}$. From left to right: the two-dimensional PDF for α and Ω_{c0}, the one-dimensional PDFs for α and Ω_{c0}.

are high near $\alpha = 0$ and $\Omega_{c0} = 0$, which are also the values for which the minimum of χ^2 is obtained. We conclude that under the given conditions the unification scenario is disfavored. The same results are obtained for $\bar{A} = 0.15$. In all these cases the minimum value for χ^2 is essentially the same as before.

To test the previous result, we construct again a three-dimensional parameter space with zero spatial curvature but leaving α and $\bar{A}$ free (case (iii)). Again, the PDF for α is initially decreasing but increasing later for $\alpha > 2$, while the PDF for Ω_{md0} is an increasing function of Ω_{md0}, as well as the PDF for $\bar{A}$ increases with $\bar{A}$. This result is shown in Fig. 5. If we now vary Ω_{c0}, its PDF is a decreasing function of Ω_{c0} as shown in Fig. 6.

The curious fact that the *antiunification* scenario seems to be favored is confirmed by another three-dimensional parameter space study for which α, Ω_{dm0} and Ω_{c0} are left free, while $\bar{A} = 0.95$ (case iv). Notice that the curvature now is arbitrary. As can be seen in Fig. 7, the PDF for α shows the same behavior as in the previous cases, while the PDF for Ω_{dm0} now increases with Ω_{dm0}. On the other hand, the PDF for Ω_{c0} decreases with Ω_{c0}. The *antiunification* scenario is clearly favored in this case, and there is no contradiction as for configuration (ii). As in the previous cases, the minimum value for χ^2 is around 0.378.

Varying all four parameters, all the preceding results are confirmed. The one-dimensional PDFs for α, $\bar{A}$, Ω_{dm0} and Ω_{c0} are displayed in Fig. 8. It can be seen that the preferred values are either $\alpha \ll 1$ or $\alpha \gtrsim 2$, while the probability is higher for large values of Ω_{dm0} and small values of Ω_{c0}.

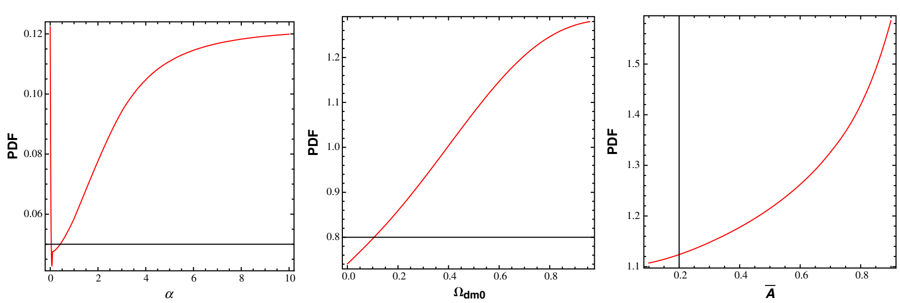

FIG. 5 (color online). The results for case (iii) with $\Omega_{dm0} = 1 - \Omega_{c0} - \Omega_{b0}$. From left to right: the one-dimensional PDFs for α, Ω_{dm0} and $\bar{A}$.

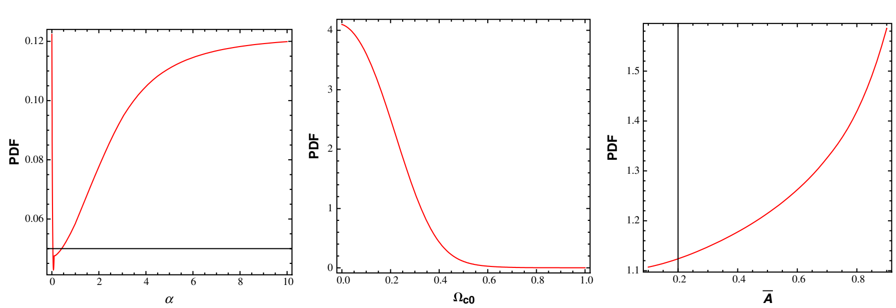

FIG. 6 (color online). The results for case (iii) with $\Omega_{c0} = 1 - \Omega_{dm0} - \Omega_{b0}$. From left to right: the one-dimensional PDFs for α, Ω_{c0} and $\bar{A}$.

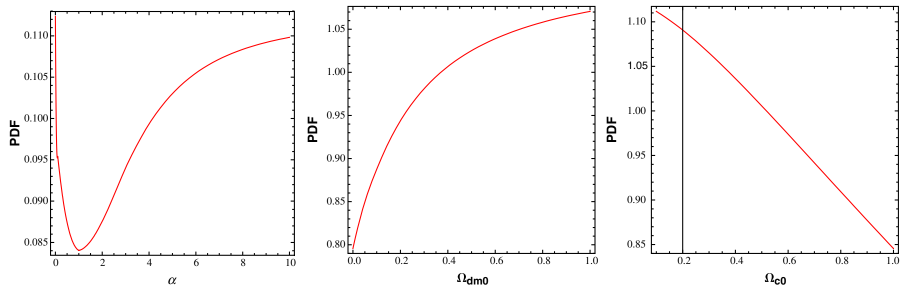

FIG. 7 (color online). The results for case (iv). From left to right: the one-dimensional PDFs for α, Ω_{dm0} and Ω_{c0}.

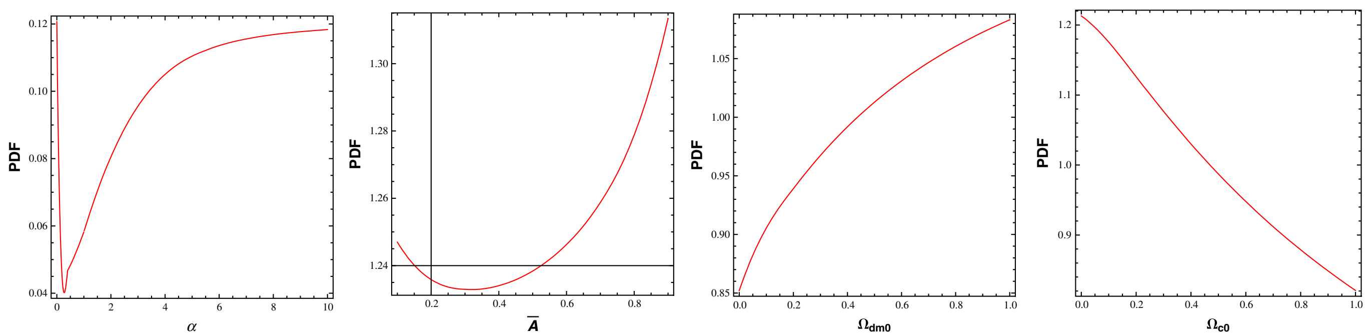

FIG. 8 (color online). The results for the general case with four free parameters (case (v)). From left to right: the one-dimensional PDFs for α, $\bar{A}$, Ω_{c0} and Ω_{dm0}.

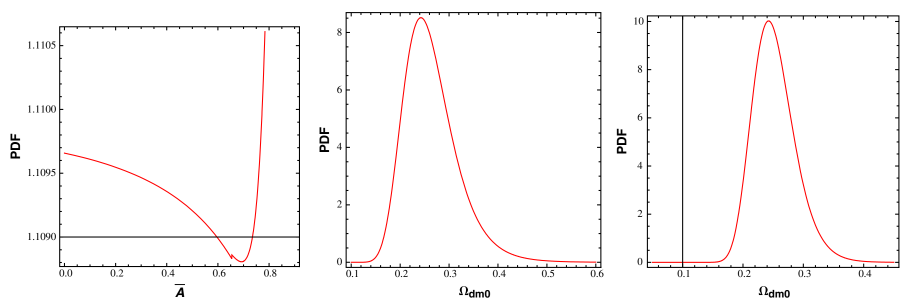

FIG. 9 (color online). The results for the flat case with $\alpha = 0$. From left to right: the one-dimensional PDFs for $\bar{A}$, for Ω_{dm0} with $\bar{A} \neq 1$ and for Ω_{dm0} with $\bar{A} = 1$.

Finally, let us consider the particular case $\alpha = 0$. In this situation, the neo-Newtonian perturbation dynamics exactly coincides with that of GR. The ΛCDM model is recovered for $\bar{A} = 1$. The results for $\alpha = 0$ and $\bar{A} \neq 1$ as well as for $\alpha = 0$ and $\bar{A} = 1$, both for a flat universe, are displayed in Fig. 9: the predictions for Ω_{dm0} are essentially the same as for the ΛCDM model. For $\bar{A} \neq 1$, the probability for $\bar{A}$ grows as $\bar{A}$ approaches unity. This shows that the method employed is consistent.

V. ANALYSIS OF THE RESULTS AND CONCLUSIONS

In the present work we obtained statistical information about the matter power spectrum by comparing the theoretical results for the generalized Chaplygin gas model with the 2dFGRS observational data. The free parameters of the model are the equation of states parameters α and $\bar{A}$ and the density parameters Ω_{dm0} and Ω_{c0}. The complete four-dimensional analysis is computationally hard but still feasible. We have complemented it by a detailed study of the cases for which only two or three parameters are free. This allows us to verify the consistency and the correctness of the method employed here.

If the unification scenario with dark matter and dark energy as a single fluid in a spatially flat universe is imposed from the beginning (case (i)), the results of Ref. [15] are essentially confirmed: there are parameter ranges for which the data are well described by the generalized Chaplygin gas model. The probability distribution function for α is high for very small (near zero) or very large (greater than 2) values of α. Allowing the parameter $\bar{A}$ to vary, we find that its one-dimensional PDF initially decreases with $\bar{A}$, but increases as $\bar{A} = 1$ is approached. Notice that values $\alpha > 1$ imply a superluminal sound speed and are therefore unphysical (see, however, [15]).

A different picture emerges for different priors. Leaving the density parameters for the Chaplygin gas and the pressureless matter components free allows us to test the unified models (pressureless matter is entirely baryonic) against models in which there is separate dark matter, not accounted for by the Chaplygin gas (cases (ii) and (iii)). We find that the unification scenario is clearly disfavored. The PDF is highest in regions with very small values for Ω_{c0} and large values for Ω_{dm0}. The behavior for α remains essentially the same as in the previous case. This result is confirmed when the condition of a spatially flat universe is relaxed and both density parameters are allowed to vary freely (case iv): the one-dimensional PDFs for α and Ω_{c0} are decreasing functions of α and Ω_{c0}, respectively, while the PDF for Ω_{dm0} increases with Ω_{dm0}. Finally, the full four-dimensional analysis of the phase space (case (v)) reproduces the results for the lower-dimensional cases (ii)–(iv).

What is the origin of these apparently contradictory results? The first aspect to be mentioned is that the matter power spectrum data only poorly constrain the dark energy component. Even for the ΛCDM model the matter power spectrum gives information mainly on the dark matter component, the dark energy component remaining largely imprecise. It is not by chance that the dark energy concept emerged from the supernova data. Our results for the Chaplygin gas model show that a large amount of dark matter, different from those described by the Chaplygin gas, is necessary to fit the data. However, the dispersion is quite high. For the flat case with a three-dimensional parameter space we find at 2σ, that $\Omega_{dm0} = 1^{+0.00}_{-0.91}$. Another point is the use of the neo-Newtonian formalism. However, for small values of the parameter α, the main case of interest here, the differences to the full general relativistic treatment are not expected to be substantial. Moreover, in the cases of overlap the results of the full theory are reproduced. Finally, possible statistical subtleties may influence the outcome of the investigation. But as far as we could test the statistical analysis (precision, crossing different information, etc.), the results seem to be robust. If this is really the case, we must perhaps live with the fact that, while the SNe type Ia data favor a unified model of the dark sector [11], this scenario is disfavored if large-scale structure data are taken into account, unless specific priors are imposed.

ACKNOWLEDGMENTS

We thank FAPES and CNPq (Brazil) for partial financial support (Grants 093/2007 (CNPq and FAPES) and EDITAL FAPES No. 001/2007). J. C. F. thanks also the French-Brazilian scientific cooperation CAPES/COFECUB and the Institut of Astrophysique de Paris (IAP) for their kind hospitality during the elaboration of this work.

[1] S. Hannestad, Int. J. Mod. Phys. A **21**, 1938 (2006).
[2] T. Padmanabhan, arXiv:asto-ph/0510492.
[3] J. A. Friedman, M. S. Turner, and D. Huterer, arXiv:0803.0982.
[4] V. Sahni, Lect. Notes Phys. **653**, 141 (2004).
[5] L. Bergström, Rep. Prog. Phys. **63**, 793 (2000).
[6] G. Bertone, D. Hooper, and J. Silk, Phys. Rep. **405**, 279 (2005).
[7] A. Y. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001).
[8] R. Jackiw, arXiv:physics/0010042.
[9] M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **66**, 043507 (2002).
[10] R. Colistete Jr., J. C. Fabris, J. Tossa, and W. Zimdahl, Phys. Rev. D **76**, 103516 (2007).
[11] R. Colistete Jr, J. C. Fabris, S. V. B. Gonçalves, and P. E. de Souza, Int. J. Mod. Phys. D **13**, 669 (2004); R. Colistete Jr., J. C. Fabris, and S. V. B. Gonçalves, Int. J. Mod. Phys. D **14**, 775 (2005); R. Colistete Jr. and J. C. Fabris, Classical Quantum Gravity **22**, 2813 (2005).
[12] H. Sandvik, M. Tegmark, M. Zaldarriaga, and I. Waga, Phys. Rev. D **69**, 123524 (2004).
[13] L. M. G. Beça, P. P. Avelino, J. P. M. de Carvalho, and C. J. A. P. Martins, Phys. Rev. D **67**, 101301 (2003).
[14] N. Bilic, G. P. Tupper, and R. D. Viollier, Phys. Lett. B**535**, 17 (2002); N. Bilic, R. J. Lindebaum, G. P. Tupper, and R. D. Viollier, J. Cosmol. Astropart. Phys. 11 (2004) 008; M. C. Bento, O. Bertolami, and A. A. Sen, Phys. Rev. D **70**, 083519 (2004); R. R. R. Reis, M. Makler, and I. Waga, Phys. Rev. D **69**, 101301 (2004); R. A. Sussman, arXiv:0801.3324; N. Bilic, G. P. Tupper, and R. D. Viollier, arXiv: 0809.0375.
[15] V. Gorini, A. Y. Kamenshchik, U. Moschella, O. F. Piatella, and A. A. Starobinsky, J. Cosmol. Astropart. Phys. 02 (2008) 016.
[16] J. M. Bardeen, J. R. Bond, N. Kaiser, and A. S. Szalay, Astrophys. J. **304**, 15 (1986); J. Martin, A. Riazuelo, and M. Sakellariadou, Phys. Rev. D **61**, 083518 (2000).
[17] J. C. Fabris, I. L. Shapiro, and J. Solà, J. Cosmol. Astropart. Phys. 02 (2007) 016.

[18] H. A. Borges, S. Carneiro, J. C. Fabris, and C. Pigozzo, Phys. Rev. D **77**, 043513 (2008).
[19] E. R. Harrison, Ann. Phys. (N.Y.) **35**, 437 (1965).
[20] W. H. McCrea, Proc. R. Soc. London **206**, 562 (1951).
[21] J. A. S. Lima, V. Zanchin, and R. Brandenberger, Mon. Not. R. Astron. Soc. **291**, L1 (1997).
[22] R. R. R. Reis, Phys. Rev. D **67**, 087301 (2003); **68**, 089901 (2003).
[23] L. R. Abramo, R. C. Batista, L. Liberato, and R. Rosenfeld, arXix:0806.3461.
[24] S. Cole *et al.*, Mon. Not. R. Astron. Soc. **362**, 505 (2005).
[25] M. Tegmark *et al.*, Phys. Rev. D **69**, 103501 (2004).
[26] J. Dunkley *et al.*, arXiv: 0803.0586.
[27] S. Weinberg, *Gravitation and Cosmology* (Wiley, New York, 1972).
[28] J. C. Fabris, S. V. B. Gonçalves, and P. E. de Souza, Gen. Relativ. Gravit. **34**, 53 (2002).
[29] S. Cole, A. G. Sanchez, and S. Wilkins, arXiv:astro-ph/0611178.