Effects of particle production during inflation

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The impact of particle production during inflation on the primordial curvature perturbation spectrum is investigated both analytically and numerically. We obtain an oscillatory behavior on small scales, while on large scales the spectrum is unaffected. The amplitude of the oscillations is proportional to the number of coupled fields, their mass, and the square of the coupling constant. The oscillations are due to a discontinuity in the second time derivative of the inflaton, arising from a temporary violation of the slow-roll conditions. A similar effect on the power spectrum should be produced also in other inflationary models where the slow-roll conditions are temporarily violated.

I. INTRODUCTION

Inflation is considered one of the most promising candidates to explain the statistical features of the observable Universe revealed by the detection of the cosmic microwave background (CMB) [1,2] and large scale structure surveys such as the Sloan Digital Sky Survey (SDSS) [3].

In the simplest models, inflation is an exponential expansion period of the Universe, driven by a scalar field, called inflaton, slowly rolling down its potential. One of its main predictions is a primordial curvature perturbation spectrum of the form $P_R(k) = k^{n_s-1}$, where n_s is the so-called scalar spectral index.

Various extensions of the simplest models have been proposed, and in this paper we will consider the coupling of the inflaton to another massive scalar field, which has been previously investigated by different groups, leading to apparently conflicting results [4,5]. The first group was in fact obtaining just a local peak in the power spectrum for scales which leave the horizon around the time of particle production, while the second obtained a small scale oscillatory behavior leading to a step between large and small scales. We study the problem both analytically and numerically, obtaining an intermediate result which confirms the oscillation of the power spectrum on small scales, but without any step. We also derive an analytical approximation for the curvature power spectrum, which clearly shows the dependency of the amplitude and period of the oscillations on the mass and number of the coupled fields and the coupling constant.

We mention that the presence of a temporary non-slow-roll stage during inflation and its effect on the scalar and tensor perturbation spectrum as well as the resulting CMB anisotropy have been investigated in [6,7]. Our analysis may be regarded as a special case where the perturbation spectrum can be studied analytically, and confirms other general studies [8–10] of the effects of singularities in the inflaton potential. See also a recent paper by Joy *et al.* [11] for comparizon with the WMAP data.

The paper is organized as follows. In Sec. II, we briefly describe the main features and motivations of the model we study. In Sec. III, we describe our analysis and present the numerical results for the spectrum of the curvature perturbation. In Sec. IV, we adopt an analytical approximation and derive the spectrum in both large and small k limits. In Sec. V, we summarize the results obtained and provide some ideas about possible extensions.

II. MODEL

We will consider a theory with the potential,

$$V(\phi, \varphi) = V_0 + \tfrac{1}{2}m_\phi^2\phi^2 + \tfrac{1}{2}N(m_\varphi - g\phi)^2\varphi^2, \quad (1)$$

where we assume V_0 dominates over the other terms during the period of interest, so that the evolution of H can be neglected in the calculation.

The equation for the inflaton can be written in the form [4]

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2\phi - gN(m_\varphi - g\phi)\langle\varphi^2\rangle = 0; \quad (2)$$

$$\langle\varphi^2\rangle \approx \theta(t - t_0)\frac{\mathcal{C}}{m_\varphi - g\phi}n_0\left(\frac{a}{a_0}\right)^{-3},$$
$$n_0 = g^{3/2}\frac{|\dot{\phi}_0|^{3/2}}{(2\pi)^3}, \quad (3)$$

where $\mathcal{C}$ is a constant of order unity, t_0 is the time at which $g\phi = m_\varphi$, when the effective mass of the φ field becomes zero and most of the particles are produced. Here and in the following, the suffix 0 denotes a quantity evaluated at $t = t_0$. The two point function can be approximated with the expression above after renormalizing by subtraction of its asymptotic past value, when no particles were produced [4].

It can be shown that the time scale Δt_c over which particles are produced is much smaller than the Hubble time, $H\Delta_c \sim 10^{-3} \ll 1$ [5]. This justifies our approxima-

tion of $\langle \varphi^2 \rangle$ by a step function as given by Eq. (3) for scales $k/a_0 < \Delta t_c^{-1} \sim 10^3 H$.

For later convenience, we slightly rewrite the above field equation as

$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = S(t); \tag{4}$$

$$S = \theta(t - t_0)gN\mathcal{C}n_0\left(\frac{a}{a_0}\right)^{-3}. \tag{5}$$

We solve the equation for the curvature perturbation on comoving hypersurfaces:

$$\mathcal{R}_c'' + 2\frac{z'}{z}\mathcal{R}_c + k^2\mathcal{R}_c = 0; \qquad z \equiv \frac{a\dot{\phi}}{H}, \tag{6}$$

where a prime denotes the conformal time derivative, $' = d/d\eta = a\,d/dt$. The inflaton perturbation on flat hypersurfaces $\delta\phi_f$, which is to be quantized on subhorizon scales, is related to $\mathcal{R}_c$ as

$$\delta\phi_f = -\frac{\dot{\phi}}{H}\mathcal{R}_c. \tag{7}$$

We assume that there is no isocurvature perturbation after inflation, So, the power spectrum of the curvature perturbation is given by

$$2\pi P_{\mathcal{R}}^{1/2}(k) = \sqrt{2k^3}|\mathcal{R}_k(t_f)|, \tag{8}$$

where $\mathcal{R}_k$ is a properly normalized mode function, and t_f is the time at which inflation ends.

III. CALCULATION

First we solve the inflaton background. We assume the inflaton is slow-rolling before t_0. Introduce two independent homogeneous solutions of (4),

$$U_\pm(t) = \exp[\lambda_\pm Ht]; \qquad \lambda_\pm = -\frac{3}{2}\left(1 \mp \sqrt{1 - 4\mu^2/9}\right), \tag{9}$$

where $\mu^2 \equiv m_\phi^2/H^2$, and the Wronskian,

$$W(t) \equiv \dot{U}_+U_- - \dot{U}_-U_+ = (\lambda_+ - \lambda_-)HU_+U_-$$
$$= (\lambda_+ - \lambda_-)He^{-3Ht}, \tag{10}$$

the solution of our interest is expressed as

$$\phi(t) = \phi_0 U_+(t - t_0) + \int_{-\infty}^{\infty} dt' G(t - t')S(t'), \tag{11}$$

where the Green function G is given by

$$G = \theta(t - t')\frac{U_+(t)U_-(t') - U_-(t)U_+(t')}{W(t')}. \tag{12}$$

Note that $U_+(t)$ describes the slow-roll solution, while $U_-(t)$ the rapidly decaying solution; $\lambda_+ \approx -\mu^2/3$ and $\lambda_- \approx -3 + \mu^2/3$ for $\mu^2 \ll 1$.

With the source term given by Eq. (5), we obtain the solution explicitly as

$$\phi(t) = \phi_0 U_+(t - t_0) + \frac{\theta(t - t_0)M^3}{\lambda_+\lambda_-(\lambda_+ - \lambda_-)H^2}$$
$$\times [-\lambda_+ U_+(t - t_0) + \lambda_- U_-(t - t_0)$$
$$+ (\lambda_+ - \lambda_-)e^{-3H(t-t_0)}], \tag{13}$$

where we have introduced the mass scale M by

$$M^3 \equiv gN\mathcal{C}n_0 = g^{5/2}N\mathcal{C}\frac{|\dot{\phi}_0|^{3/2}}{(2\pi)^3}. \tag{14}$$

Taking the time derivative of this equation gives

$$\dot{\phi} = \lambda_+ H\phi_0 u_+(t - t_0) + \frac{\theta(t - t_0)M^3}{\lambda_+\lambda_-(\lambda_+ - \lambda_-)H}$$
$$\times [-\lambda_+^2 U_+(t - t_0) + \lambda_-^2 U_-(t - t_0)$$
$$+ (\lambda_+^2 - \lambda_-^2)e^{-3H(t-t_0)}]. \tag{15}$$

We note that this result implies the presence of a step in $\dot{\phi}$. At late times, $H(t - t_0) \gg 1$, the above equation reduces to

$$\dot{\phi} = \lambda_+ H\phi_0\left(1 - \frac{M^3}{\lambda_-(\lambda_+ - \lambda_-)H^2\phi_0}\right)U_+(t - t_0). \tag{16}$$

Thus there is a step of the relative magnitude $\Delta\dot{\phi}/\dot{\phi} \sim M^3/(9H^2\phi_0)$ compared to the case of no particle production, and it will be reflected in the overall shape of the spectrum in general. However, as we shall see immediately below, for the values of the parameters we choose, the step turns out to be negligible.

For ease of comparison, following [4,5], we will make the following choices for the model parameters:

$$m_\phi = 10^{-6}m_{pl}, \qquad m_\varphi = 2m_{pl},$$
$$g = 1, \qquad V_0 = 5m_\phi^2 m_\varphi^2, \tag{17}$$

where $m_{pl} = G^{-1/2}$ is the Planck mass. For this choice of the parameters, we find the step relative to the case of no particle production is small, $\Delta\dot{\phi}/\dot{\phi} \sim 10^{-5}N$ unless N is extremely large. The behaviors of $\dot{\phi}$ and $\ddot{\phi}$ are plotted in Figs. 1 and 2, respectively.

Setting $u \equiv a\delta\phi_f = -z\mathcal{R}_c$, where $z = a\dot{\phi}/H$, we have

$$u'' + \left(k^2 - \frac{z''}{z}\right)u = 0. \tag{18}$$

Since we assumed that the potential is dominated by V_0, the time variation of H in z can be neglected and the scale factor a may be approximated by that of a pure de Sitter universe, $a = (-H\eta)^{-1}$. On the other hand, the time variation of $\dot{\phi}$ cannot be neglected, particularly at and after
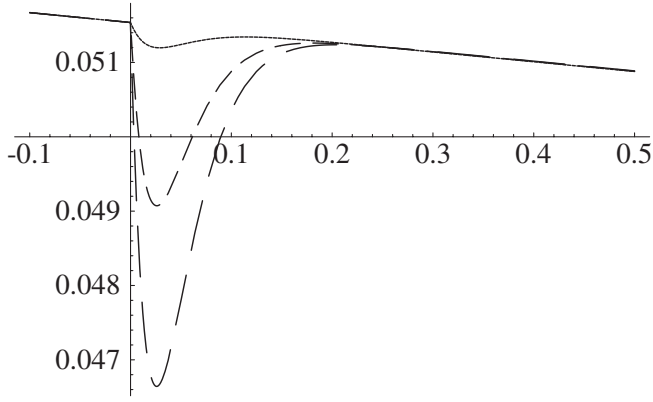
FIG. 1. $\dot{\phi}(t)/(m_p m_\phi)$ is plotted for $-0.1 < m_\phi(t - t_0) < 0.5$. The solid line corresponds to $N = 1$, the small dashed line to $N = 8$, and the long dashed line to $N = 16$.

the transition. For $\dot{\phi}$, we use the solution given by Eq. (15) with the identification $Ht = \ln(-H\eta)$.

At $\eta = \eta_0$, $\ddot{\phi}$ is discontinuous. Hence z'', which contains the third derivative $\dddot{\phi}$, contains a delta function. This implies u' is discontinuous at $\eta = \eta_0$. To evaluate this discontinuity, we calculate the contribution of the delta function in z'',

$$D_0 \equiv \int_{\eta_0 - \epsilon}^{\eta_0 + \epsilon} \frac{z''}{z} d\eta = [\ddot{\phi}_{0+} - \ddot{\phi}_{0-}] \frac{a_0}{\dot{\phi}_0}$$
$$= N g n_0 \mathcal{C} \frac{a_0}{\dot{\phi}_0} = \frac{M^3 a_0}{\dot{\phi}_0}, \qquad (19)$$

where $\ddot{\phi}_{0-}$ and $\ddot{\phi}_{0+}$ are the values of $\ddot{\phi}$ right before and after $t = t_0$, respectively. Thus the matching condition at $\eta = \eta_0$ for u is given by

$$u'_{0+} = u'_{0-}(\eta_0) + D_0 u_{0-}, \qquad u_{0+} = u_{0-}. \qquad (20)$$

Turning back to the original variable $\mathcal{R}_c$, it is noted that this matching condition implies that both $\mathcal{R}'_c$ and $\mathcal{R}_c$ are continuous at $\eta = \eta_0$. This is of course consistent with the
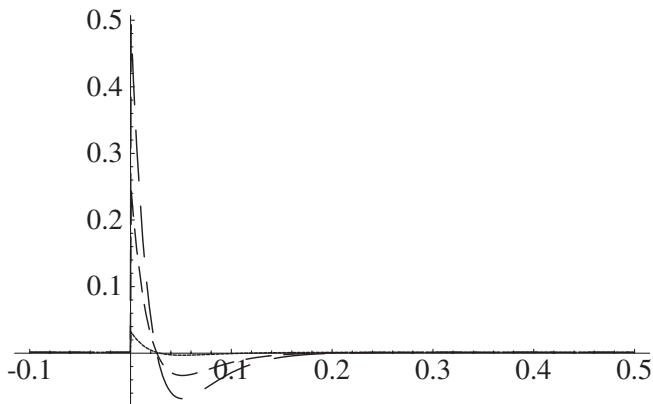
evolution equation (6) for $\mathcal{R}_c$, in which there is no delta function.

To calculate the power spectrum, we split it into two parts, corresponding to the modes greater or smaller than k_0, where $k_0 = (aH)_0$. We assume the standard Bunch-Davies vacuum for $\delta\phi_f$ at $\eta \to -\infty$, and solve for the positive frequency mode functions. Thus at sufficiently early times, $\eta \to -\infty$, the mode function can be well approximated by

$$u_< = v \equiv \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right), \qquad (21)$$

where $u_<$ denotes the mode function at $\eta < \eta_0$. For numerical analysis, we use this as the initial condition for each mode when it is inside the horizon and when $\eta < \eta_0$. Then we return to the original variable $\mathcal{R}_c$ and numerically integrate Eq. (6). As we noted in the above, there is no delta function in this equation, but only a discontinuity in z'/z. Hence it can be numerically integrated across the time $\eta = \eta_0$ without any problem.

For modes $k < k_0$, for which the sudden change in the effective potential of the inflaton happens after horizon crossing, Eq. (21) continues to be a good approximation until a mode crosses the horizon. Hence using it as the initial condition at horizon crossing, we solve the differential equation numerically.

For modes $k > k_0$, the particle production takes place before the modes leave the horizon. So, we set the initial condition at a sufficiently early time $\eta = \eta_i < \eta_0$ common to all the modes, when the mode functions are well approximated by Eq. (21). Then we integrate Eq. (6) numerically.

The numerical results for the power spectrum are given in Figs. 3–5. For the modes $k < k_0$, we do not find any appreciable evolution of modes on superhorizon scales except for modes close to $k = k_0$. For the modes $k > k_0$,

FIG. 2. $\ddot{\phi}(t)/(m_p m_\phi^2)$ is plotted for $-0.1 < m_\phi(t - t_0) < 0.5$. The parameters are the same as Fig. 1.
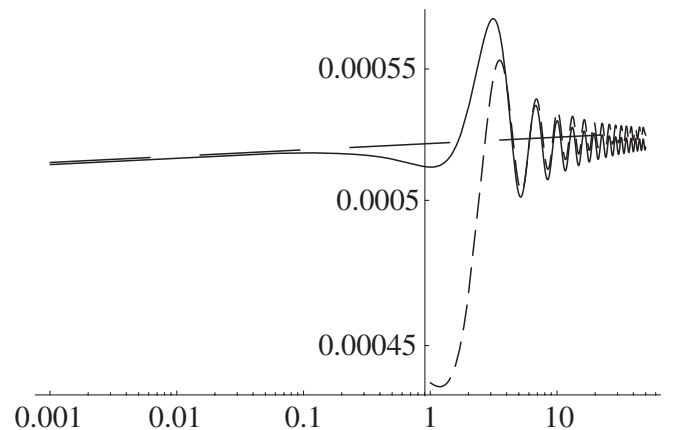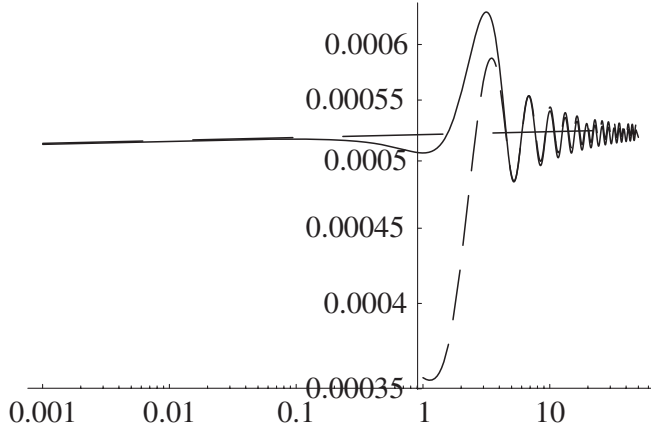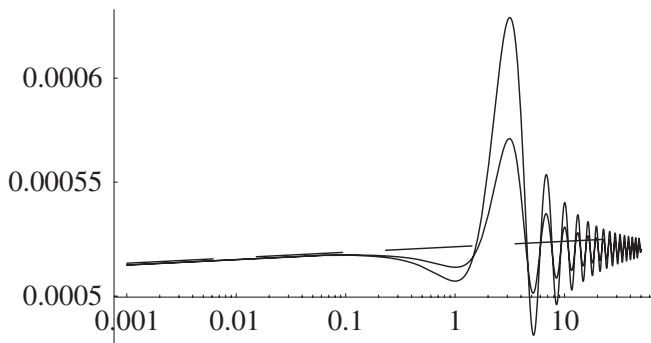

FIG. 3. $P_{\mathcal{R}}^{1/2}(k)$ is plotted for $3 \times 10^{-2} < k/(a_0 H_0) < 50$ in the case of $N = 8$. The solid line is the numerical result, the dashed line is the analytical approximation, and the long dashed line is the spectrum in the absence of particle production.

FIG. 4. The same as Fig. 3, but in the case of $N = 16$.

FIG. 5. $P_{\mathcal{R}}^{1/2}(k)$ in the cases of both $N = 8$ and $N = 16$ are plotted for $3 \times 10^{-2} < k/(a_0 H_0) < 50$. Clearly the amplitude of oscillations is larger for larger N, approximately in proportion to N. The long dashed line is the spectrum in the absence of coupling.

the oscillatory feature of the spectrum is in agreement with [5] and other studies such as [12].

Our result that there appears no step in the spectrum is in agreement with Adams *et al.* [12], while it differs from Elgaroy *et al.* [5] who claim that a step in the power spectrum is produced due to the nonconservation of entropy perturbation. We disagree with this interpretation, since a source term should be present for the superhorizon evolution of $\mathcal{R}_c$ due to entropy perturbations, which is absent in the present case.

In order to confirm our numerical results, we consider an analytical approximation to the problem in the next section. We find a good agreement between the numerical results and analytical approximations.

IV. ANALYTICAL APPROXIMATION OF THE SPECTRUM

In this section, we consider an analytical approximation for the power spectrum. As in the case of numerical analysis, we split the modes into the two, $k < k_0$ and $k > k_0$, and discuss them separately.

For the modes $k < k_0$, since the transition occurs on superhorizon scales, the only possible time variation of $\mathcal{R}_c$ is due to the discontinuity in z'/z in Eq. (6). However, since $z'/z = a\dot{z}/z$, it grows exponentially large as time goes on and the discontinuity becomes totally irrelevant, unless the magnitude of the discontinuity is exponentially large (which is apparently not the case). Therefore, for $k \ll k_0$, there can be no time evolution of $\mathcal{R}_c$ on superhorizon scales. In fact, by using the technique developed in [13], one can explicitly show that the spectrum can be modified only near $k = k_0$ and quickly approaches the standard result at $k \ll k_0$. Thus the curvature perturbation spectrum is given by the standard formula,

$$P_{\mathcal{R}}^{1/2}(k) = \frac{H^2}{2\pi|\dot{\phi}(t_k)|} \quad (k \ll k_0), \tag{22}$$

where t_k is the horizon crossing time, $k = a(t_k)H$.

As for the modes $k > k_0$, the analysis is a bit more complicated. First we note that, as we stated before, the mode functions u are well approximated by Eq. (21) when $\eta < \eta_0$. After the transition, there is a short period when the slow-rolling solution for the background inflaton does not hold. Nevertheless, for sufficiently large k, for which the mode is still deep inside the horizon at the time of transition, this small violation of the slow-roll conditions is totally negligible because $k^2 \gg z''/z$. Hence the approximate solution v defined in Eq. (21) is still a valid solution even at $\eta > \eta_0$, including the non-slow-roll period right after the transition. However, v is no longer the positive frequency mode function there. Instead the desired mode function at $\eta > \eta_0$ should be expressed as a linear combination of v and v^*. Hence we may set

$$u_> = \alpha_k v + \beta_k v^*, \tag{23}$$

where $u_>$ denotes the mode function at $\eta > \eta_0$. It is useful to note that the coefficients α_k and β_k can be expressed in terms of $u_>$, v, and $v*$ as

$$\alpha_k = -i(v^{*\prime} u_> - v^* u_>'), \qquad \beta_k = i(v' u_> - v u_>'). \tag{24}$$

This gives the formula for the spectrum,

$$P_{\mathcal{R}}^{1/2}(k) = \frac{H^2}{2\pi|\dot{\phi}(t_k)|}|\alpha_k - \beta_k|. \tag{25}$$

It should be noted that $|\alpha_k|^2 - |\beta_k|^2 = 1$. This relation can be used as a consistency check.

Now, the matching condition (20) implies

$$u_>(\eta_{0+}) = v_0, \qquad u_>'(\eta_{0+}) = v_0' + D_0 v_0. \tag{26}$$

Applying Eq. (27) to an epoch right after the transition, $\eta = \eta_{0+}$, with $u_>$ and $u_>'$ given by these equations, we find

$$\alpha_k = 1 + iD_0 v_0 v_0^* = 1 + i\frac{D_0}{2k}\left(1 + \frac{1}{(k\eta_0)^2}\right),$$

$$\beta_k = -iD_0 v_0^2 = -i\frac{D_0}{2k}\left(1 - \frac{i}{k\eta_0}\right)^2 e^{-2ik\eta_0}. \tag{27}$$

Inserting them into Eq. (25), we obtain the spectrum at $k \gg k_0$ as

$$P_{\mathcal{R}}(k)^{1/2} = \frac{H^2}{2\pi|\dot{\phi}(t_k)|}\Bigg(1 + \frac{D_0}{k}\Bigg[\sin 2k\eta_0 + O\Big(\frac{1}{k\eta_0}\Big)\Bigg]$$

$$+ \frac{D_0^2}{2k^2}\Bigg[1 + \cos 2k\eta_0 + O\Big(\frac{1}{k\eta_0}\Big)\Bigg]\Bigg)^{1/2} \quad (k \gg k_0). \tag{28}$$

As seen from Figs. 3 and 4, the above analytical approximation agrees well with the numerical results at large k. However, the analytical approximation loses its accuracy for modes which leave the horizon around the time of particle production η_0. This is because the effect of non-slow-rolling is non-negligible for these modes and the approximate solution (21) is no longer valid.

The spectrum at $k > k_0$ behaves like a dumped harmonic oscillator, where the amplitude of the oscillations is proportional to D_0, provided $D_0/k < 1$. As one can guess from the formula (25), together with Eq. (27), these oscillations are due to an interference between the positive and negative frequency mode functions, or contamination of negative frequency modes with positive frequency modes due to the transition. One might doubt if this is due to the approximation of replacing an otherwise smooth function by a step function in Eq. (3). However, as we mentioned there, our approximation is valid for $k < a_0/\Delta t = k_0(H\Delta t)^{-1} \sim 10^3 k_0$, while the oscillations are present for all $k > k_0$. This confirms that the oscillations are not an artifact of the approximation but real.

Using the expression for D_0, Eq. (19), the amplitude is evaluated as

$$\frac{D_0}{k} = \frac{a_0}{k}\frac{M^3}{\dot{\phi}_0} = \frac{a_0 H}{k}\frac{Ng^2}{\sqrt{3}(2\pi)^3}\frac{m_\phi m_\varphi^{1/2}}{H^{3/2}}. \tag{29}$$

This analytical estimate not only confirms the numerical result that the oscillations amplitude is proportional to the number of coupled fields N, but also shows that it is proportional to the square root of the mass m_φ and is proportional to the square of the coupling constant g. It also shows that the period of the oscillations is given by $k_0\pi$.

V. CONCLUSION

We have studied the impact of the coupling of the inflaton to a scalar field on the primordial curvature perturbation spectrum. We found the presence of an oscillatory behavior on small scales for the modes which leave the horizon after the time of particle production. We have also presented a very good analytical approximation for the evolution of small scale modes. This method can be applied to the general case of a sudden change in the inflaton potential, leading to a temporary violation of the slow-roll conditions.

The amplitude of the oscillations is proportional to the number of coupled fields N and the square root of their mass $m_\varphi^{1/2}$, and to the square of the coupling constant g^2. The period of the oscillations is $k_0\pi$, where k_0 is the wavelength that crosses the horizon right at the time of the particle production.

On large scales, $k < k_0$, the power spectrum is virtually unaffected by the particle production, contrary to what was claimed in a previous numerical investigation [5]. This is because the violation of the slow-roll conditions can affect only on those modes close to $k = k_0$ on superhorizon scales, and the curvature perturbation is conserved on sufficiently large superhorizon scales, $k \ll k_0$, no matter what occurs there.

Our results are quite general in the sense that in models where slow-roll conditions are temporarily violated, the spectrum will have oscillations on scales smaller than the mode which leaves the horizon at the time of transition, while it will remain unchanged on the larger scales. The presence of such features in the observed CMB spectrum could help to determine the magnitude and the lapse of periods during which the slow-roll conditions are violated, although it may be difficult in practice to distinguish such a feature of a primordial origin from a similar feature due to intermediate astrophysical processes happening before and/or after recombination.

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