Cosmological *CPT* violating effect on CMB polarization

Mingzhe Li[1,2,3] and Xinmin Zhang[4]

[1]*Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany*
[2]*Department of Physics, Nanjing University, Nanjing 210093, People's Republic of China*
[3]*Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University–Purple Mountain Observatory, Nanjing 210093, People's Republic of China*
[4]*Institute of High Energy Physics, Chinese Academy of Sciences, P.O. Box 918-4, Beijing 100049, People's Republic of China*

(Received 15 October 2008; published 14 November 2008)

A dark energy scalar (or a function of the Ricci scalar) coupled with the derivative to the matter fields will violate the *CPT* symmetry during the expansion of the Universe. This type of cosmological *CPT* violation helps to generate the baryon number asymmetry and gives rise to the rotation of the photon polarization which can be measured in the astrophysical and cosmological observations, especially the experiments of the cosmic microwave background radiation. In this paper, we derive the rotation angle in a fully general relativistic way and present the rotation formulas used for the cosmic microwave background data analysis. Our formulas include the corrections from the spatial fluctuations of the scalar field. We also estimate the magnitude of these corrections in a class of dynamical dark energy models for quintessential baryo/leptogenesis.

DOI: 10.1103/PhysRevD.78.103516 PACS numbers: 98.80.Cq, 98.70.Vc, 98.80.Es

I. INTRODUCTION

In the standard model of particle physics, *CPT* is a fundamental symmetry. Probing its violation is an important way to search for new physics beyond the standard model. Up to now, *CPT* symmetry has passed a number of high precision experimental tests in the ground-based laboratory, and no definite signal of its violation has been observed [1]. So the present *CPT* violating effects, if they exist, should be very small to be amenable to the experimental limits.

The *CPT* symmetry could be violated dynamically in the expanding Universe. To show it, consider a scalar boson ϕ which effectively couples to a fermion current J^μ, with the Lagrangian given by

$$\mathcal{L}_{\rm int} = \frac{c}{M}\nabla_\mu \phi J^\mu, \tag{1}$$

where c is a dimensionless constant and M is the cutoff scale. The interaction in (1) is *CPT* conserved; however, during the expansion of the Universe, the background value $\dot{\phi}$ does not vanish and *CPT* is broken spontaneously. This type of *CPT* violation occurs naturally in theories of dynamical dark energy and has interesting implications in particle physics and cosmology. In models of quintessential baryo/leptogenesis [2–4], the scalar field ϕ in (1) is the dark energy scalar (quintessence [5–7], k-essence [8], phantom [9], quintom [10,11], etc.). In the early Universe, the field ϕ with the interaction in (1) generates the baryon number asymmetry, and at late times it drives the accelerating expansion of the Universe. One of the features of these models is a unified description of the present accelerating expansion and the generation of the matter and antimatter asymmetry of our Universe. Furthermore, differing from the original proposal for spontaneous baryogenesis by Cohen and Kaplan [12], since the dark energy scalar has been existing up to the present epoch, the corresponding *CPT* violation could be tested in laboratory experiments and cosmology. In Refs. [2,3], we have pointed out that, to produce the enough baryon number asymmetry, the dark energy should be significant in the radiation-dominated epoch. This is the case if the dark energy has the tracking behavior, i.e., its density decays almost at the same rate with that of radiation as the Universe expanding. Along this line, the gravitational baryo/leptogenesis [13,14] has been proposed in which a function of curvature scalar R replaces the ϕ field in (1). There are other motivations in the literature, e.g., Refs. [15–20].

The current J^μ in Eq. (1) is not necessary to be the baryon current for baryogenesis. It could be other currents which are not orthogonal to J^μ_B or J^μ_{B-L}. In [21], we have proposed, for example, that J^μ is the left-handed part of the $B-L$ current $J^\mu_{(B-L)_L}$. Besides the generation of baryon number asymmetry, this kind of coupling will bring a new effect to the photon sector. This is because $J^\mu_{(B-L)_L}$ is anomalous under the electromagnetic interaction

$$\nabla_\mu J^\mu_{(B-L)_L} \sim -\frac{\alpha_{\rm em}}{3\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}. \tag{2}$$

Hence the interaction in Eq. (1) would induce the following effective coupling through the anomaly equation:

$$\mathcal{L}_{\rm int} = -\frac{2c\alpha_{\rm em}}{3\pi M}\nabla_\mu \phi A_\nu \tilde{F}^{\mu\nu} \equiv p_\mu A_\nu \tilde{F}^{\mu\nu}, \tag{3}$$

where A_ν is the electromagnetic vector potential, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$ is the strength tensor, and $\tilde{F}^{\mu\nu} = 1/2\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual. This Chern-Simons term leads to the rotations of the polarization vectors of photons when propagating over the cosmological distance [22].[1] The change in the position angle of the polarization plane $\Delta\chi$, characterizing CPT violation in this scenario, can be obtained by observing polarized radiation from distant sources such as radio galaxies, quasars [22,27], and the cosmic microwave background (CMB) [28,29]. Assuming the rotation angle is homogeneous and isotropic $\Delta\chi = \Delta\bar{\chi}$, the CMB power spectra would be rotated as [29,30]

$$\begin{aligned} C_l^{TT,\text{obs}} &= C_l^{TT}, \qquad C_l^{TE,\text{obs}} = C_l^{TE}\cos(2\Delta\bar{\chi}),\\ C_l^{TB,\text{obs}} &= C_l^{TE}\sin(2\Delta\bar{\chi}),\\ C_l^{EE,\text{obs}} &= C_l^{EE}\cos^2(2\Delta\bar{\chi}) + C_l^{BB}\sin^2(2\Delta\bar{\chi}),\\ C_l^{BB,\text{obs}} &= C_l^{EE}\sin^2(2\Delta\bar{\chi}) + C_l^{BB}\cos^2(2\Delta\bar{\chi}),\\ C_l^{EB,\text{obs}} &= \tfrac{1}{2}\sin(4\Delta\bar{\chi})(C_l^{EE} - C_l^{BB}). \end{aligned} \tag{4}$$

In the formulas above, the quantities with the superscript obs are those observed after the rotation. T, E, and B represent the temperature, the electriclike and magneticlike polarization modes, respectively.

In Ref. [29], with Feng and Li, we did the simulations on the measurement of $\Delta\bar{\chi}$ with the future high precision CMB experiments CMBPol [31] and PLANCK [32] using the rotation formulas (4). We pointed out that in such experiments the EB spectrum will be the most sensitive probe of such CPT violation; this is because the EB power spectrum is generated by the rotation of the EE power spectrum, which is a more sensitive probe of the primordial fluctuations than the TT and TE spectra. In [30], with Feng, Xia, and Chen, we first found that a nonzero rotation angle $\Delta\bar{\chi} = -6.0 \pm 4.0$ deg (1σ) is mildly favored by the CMB polarization data from the three-year Wilkinson Microwave Anisotropy Probe (WMAP3) observations [33–37] and the January 2003 Antarctic flight of BOOMERanG (hereafter B03) [38–40] (see also Refs. [41–44]). This is a signal in some sense of the cosmological CPT violation mentioned above. Later on, Cabella, Natoli, and Silk [45] performed a wavelet analysis of the temperature and polarization maps of the CMB delivered by WMAP3. They set a limit on the rotation angle $\Delta\bar{\chi} = -2.5 \pm 3.0$ deg (1σ). This is consistent with our result because they considered WMAP3 data only. Using the full data of B03 and the WMAP3 angular power spectra, one of the authors (X. Z.) with Xia *et al.* [46] has found that $\Delta\bar{\chi} = -6.2 \pm 3.8$ deg (1σ). This result improved the measurement given by our previous paper [30]. Recently, the WMAP experiment has published the five-year results for the CMB angular power spectra which include the TB and EB information [47,48]. They used the polarization power spectra of WMAP5 TE/TB $(2 \le l \le 450)$ and $EE/BB/EB$ $(2 \le l \le 23)$ to determine this rotation angle [49] and found that $\Delta\bar{\chi} = -1.7 \pm 2.1$ deg (1σ). However, when B03 data are included, one of the authors (X. Z.) with Xia *et al.* [50] found that $\Delta\bar{\chi} = -2.6 \pm 1.9$ deg (1σ). Again, a small CPT-violating effect is mildly detected by current data.

We note that the rotation formulas given in (4) are valid only for a homogeneous and isotropic rotation angle and are obtained in the Minkowski spacetime or the spatially flat Friedmann-Robertson-Walker spacetime which is conformally equivalent to the former. This is expected to be a good approximation when the coupled scalar field ϕ is the dark energy or the function of the curvature scalar, because in these cases ϕ is very homogeneous in the observed Universe, while the accompanied perturbations are much smaller. Usually, its background part makes the dominant contributions. One of the aims of this paper is to study the secondary effects due to the perturbations of ϕ, which leads to the anisotropies of the rotation angle. For this purpose, we first study the Maxwell theory modified by the Chern-Simons term in the general curved spacetime and investigate the possibility of obtaining the rotation angle in a fully general relativistic framework. The spatial fluctuations of the scalar field make the rotation angle inhomogeneous and anisotropic and bring higher order corrections to the rotation formulas Eq. (4). Specifically, we evaluate the magnitude of these corrections in the models of tracking dark energy, as required by the quintessential baryo/leptogenesis, and found the corrections are negligible. However, for some other models, the corrections could be sizable. This paper is organized as follows. In Sec. II, we present the relevant equations of the modified electromagnetic theory under the geometric optics approximation. In Sec. III, we study the generalized Stokes parameters and the changes in CMB power spectra in Sec. IV. In Sect. V, we evaluate the corrections due to quintessence fluctuations in the quintessential baryogenesis model. Section VI is the conclusion.

II. BASIC EQUATIONS

The full Lagrangian of the Maxwell theory modified by the Chern-Simons term (3) (without other sources) is

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + p_\mu A_\nu \tilde{F}^{\mu\nu}. \tag{5}$$

This Lagrangian is not gauge-invariant, but the action integral $S = \int \mathcal{L} d^4x$ is gauge-independent because p_μ is defined in (3) as the derivative of the scalar field. The equation of motion can be obtained through varying this Lagrangian with respect to A_ν:

$$\nabla_\mu F^{\mu\nu} = -2p_\mu \tilde{F}^{\mu\nu}. \tag{6}$$

The right-hand side of the above equation is brought by the Chern-Simons term. But the identity is unchanged:

$$\nabla_\mu F_{\rho\sigma} + \nabla_\rho F_{\sigma\mu} + \nabla_\sigma F_{\mu\rho} = 0. \tag{7}$$

[1] Such a term also breaks the Einstein equivalence principle [23,24] in the short wavelength limit and breaks causality in the long wavelength limit [25,26].

We will study these equations in a gauge-independent way though it is easier to do it by choosing the Lorenz gauge [21]. For this purpose we make a differentiation to Eq. (7) and get

$$\Box F_{\rho\sigma} + 2\nabla_\rho(p_\mu \tilde{F}^\mu_\sigma) - 2\nabla_\sigma(p_\mu \tilde{F}^\mu_\rho) - [F^\alpha_\rho R_{\alpha\sigma} - F^\alpha_\sigma R_{\alpha\rho} - F^{\mu\alpha} R_{\alpha\mu\rho\sigma}] = 0, \tag{8}$$

where $R_{\alpha\sigma}$ and $R_{\alpha\mu\rho\sigma}$ are Ricci and Riemann tensors, respectively.

Since we are studying the light which propagates in the cosmological scales, the geometric optics approximation (GOA) applies very well. With this approximation, the solution to the equation of motion is supposed to be

$$F^{\mu\nu} = (a^{\mu\nu} + \epsilon b^{\mu\nu} + \epsilon^2 c^{\mu\nu} + \cdots)e^{iS/\epsilon}, \tag{9}$$

where we made a complexification to the electromagnetic field, but ϵ is a small real parameter and S is a real function. This ansatz means that the phase of the wave varies much faster than the amplitude. We define the wave vector as

$$k_\mu \equiv \nabla_\mu S, \tag{10}$$

which represents the travel direction of the photon.

Substituting the ansatz (9) into Eqs. (8) and (7) and dropping out the terms containing Ricci and Riemann tensors, we have

$$\begin{aligned}\Box(a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots) &+ \frac{2i}{\epsilon} k^\mu \nabla_\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots)\\ &+ \frac{i}{\epsilon}(\nabla_\mu k^\mu)(a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots)\\ &- \frac{1}{\epsilon^2} k_\mu k^\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots)\\ = -2[(\nabla_\rho p^\mu)(\tilde{a}_{\mu\sigma} &+ \epsilon \tilde{b}_{\mu\sigma} + \cdots)\\ &+ p^\mu(\nabla_\rho \tilde{a}_{\mu\sigma} + \epsilon \nabla_\rho \tilde{b}_{\mu\sigma} + \cdots)\\ &+ \frac{ik_\rho}{\epsilon} p^\mu (\tilde{a}_{\mu\sigma} + \epsilon \tilde{b}_{\mu\sigma} + \cdots)] + 2[\rho \to \sigma]\end{aligned} \tag{11}$$

and

$$[\nabla_\mu(a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots) + \frac{i}{\epsilon} k_\mu (a_{\rho\sigma} + \epsilon b_{\rho\sigma} + \cdots)] + [\rho\sigma\mu] + [\sigma\mu\rho] = 0. \tag{12}$$

At the leading order of the GOA, Eq. (12) gives

$$k_\mu a_{\rho\sigma} + k_\rho a_{\sigma\mu} + k_\sigma a_{\mu\rho} = 0, \tag{13}$$

which implies that $a_{\rho\sigma}$ should have the following antisymmetric form:

$$a_{\rho\sigma} = k_\rho a_\sigma - k_\sigma a_\rho. \tag{14}$$

Then we collect the terms of Eq. (11) at the orders of $1/\epsilon^2$ and $1/\epsilon$, respectively. At the order of $1/\epsilon^2$, we have

$$k_\mu k^\mu = 0. \tag{15}$$

The propagation equation of k^μ can be obtained by differentiating the above equation again:

$$\begin{aligned} 0 &= \nabla_\nu(k_\mu k^\mu) = 2\nabla^\mu S \nabla_\nu \nabla_\mu S = 2\nabla^\mu S \nabla_\mu \nabla_\nu S\\ &= 2k^\mu \nabla_\mu k_\nu. \end{aligned} \tag{16}$$

This is a geodesic equation. The vector k^μ defines an affine parameter λ which measures the distance along the light ray:

$$k^\mu \equiv \frac{dx^\mu}{d\lambda}. \tag{17}$$

We can see from (16) that k^μ is parallelly transported along the light curve $x^\mu(\lambda)$. In other words, photons travel along null geodesics. These results are the same as those of the standard Maxwell theory. The modification due to the Chern-Simons term appears at the order of $1/\epsilon$:

$$\mathcal{D} a^\nu + \frac{\theta}{2} a^\nu = -p_\mu \epsilon^{\mu\nu\rho\sigma} k_\rho a_\sigma, \tag{18}$$

where we have considered Eq. (14) and defined the operator $\mathcal{D} \equiv k^\mu \nabla_\mu$. The quantity $\theta = \nabla_\mu k^\mu$ describes the expansion of the bundle of the light. Without the modification, the right-hand side of the above equation would vanish. Its physical meaning is that the polarization vector of the photon is not parallelly transported along the light ray as we will see in the next section. In addition, by applying the GOA to the original equation

$$\nabla_\mu F^{\mu\nu} = -2p_\mu \tilde{F}^{\mu\nu}, \tag{19}$$

we have

$$k_\mu a^\mu = 0. \tag{20}$$

The basic results we got above are Eqs. (18) and (16) with two orthogonality relations (15) and (20).

III. STOKES PARAMETERS

It is convenient to use the Stokes parameters to study the polarization of radiation. The four Stokes parameters are well defined in Minkowski spacetime (the inertial frame). Considering a monochromatic electromagnetic wave of frequency ω_0 propagating in the $+z$ direction

$$\begin{aligned} E_x &= a_x(t)\exp[i(\omega_0 t - \theta_x(t))],\\ E_y &= a_y(t)\exp[i(\omega_0 t - \theta_y(t))], \end{aligned} \tag{21}$$

the Stokes parameters are defined as the time averages

$$\begin{aligned} I &\equiv \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle, \qquad Q \equiv \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle,\\ U &\equiv \langle E_x E_y^* \rangle + \langle E_x^* E_y \rangle, \qquad V \equiv i[\langle E_x E_y^* \rangle - \langle E_x^* E_y \rangle]. \end{aligned} \tag{22}$$

In general relativity, these definitions should be generalized. This can be done by using the tetrad formalism. A tetrad is a set of four orthogonal unit basis vectors $e^\mu_{(a)}$, with

$a = 0, 1, 2, 3$. At each point x, we can attach a tetrad which transforms between the coordinate frame and the local inertial frame at x. For a vector field $B_\mu(x)$, its components in the local inertial frame are

$$\bar{B}_a = e^\mu_{(a)} B_\mu. \tag{23}$$

The Latin indices are lowered and raised by the Minkowski metric η^{ab}, the Greek indices, however, by the coordinate metric $g^{\mu\nu}$. The tetrad has the following properties:

$$g_{\mu\nu} e^\mu_{(a)} e^\nu_{(b)} = \eta_{ab}, \qquad \eta^{ab} e^\mu_{(a)} e^\nu_{(b)} = g^{\mu\nu}. \tag{24}$$

We can set the tetrad frame at each point as follows. Consider the rest frame of the free fall observer, in which the four-velocity is $\bar{u}^a = \delta^a_0$. Furthermore, we require the observer to see the light traveling along the $+z$ direction, and hence $\bar{k}^a = \omega(\delta^a_0 + \delta^a_3)$. So, after transforming to the coordinate frame,

$$u^\mu = e^\mu_{(a)} \bar{u}^a = e^\mu_{(0)} \tag{25}$$

and

$$k^\mu = e^\mu_{(a)} \bar{k}^a = \omega(u^\mu + e^\mu_{(3)}). \tag{26}$$

Hence,

$$e^\mu_{(0)} = u^\mu, \qquad e^\mu_{(3)} = \frac{1}{\omega}(k^\mu - \omega u^\mu), \tag{27}$$

where $\omega \equiv k_\mu u^\mu$ is the frequency measured by the observer. The other tetrad vectors $e^\mu_{(1)}$ and $e^\mu_{(2)}$ are unit spacelike, orthogonal to each other and to $e^\mu_{(0)}$, $e^\mu_{(3)}$, and therefore orthogonal to k^μ.

The electric vector in general spacetime for an observer with four-velocity u^μ is defined as

$$E^\mu \equiv F^{\mu\nu} u_\nu. \tag{28}$$

At the leading order of the GOA mentioned at the last section, it is

$$E^\mu = a^{\mu\nu} u_\nu e^{iS/\epsilon} = (k^\mu a^\nu - k^\nu a^\mu) u_\nu e^{iS/\epsilon}. \tag{29}$$

Transforming it to the local inertial frame, we get the x and y components of the electric field in this frame easily:

$$E_x = \bar{E}_1 = E_\mu e^\mu_{(1)}, \qquad E_y = \bar{E}_2 = E_\mu e^\mu_{(2)}. \tag{30}$$

In the local inertial frame, the definitions of the Stokes parameters (22) are applicable. By applying the above equations to (22), we get the general expressions of the Stokes parameters in curved spacetime [51,52]:

$$\begin{aligned} I &= \omega^2 L_{\mu\nu}(e^\mu_{(1)} e^\nu_{(1)} + e^\mu_{(2)} e^\nu_{(2)}), \\ Q &= \omega^2 L_{\mu\nu}(e^\mu_{(1)} e^\nu_{(1)} - e^\mu_{(2)} e^\nu_{(2)}), \\ U &= \omega^2 L_{\mu\nu}(e^\mu_{(1)} e^\nu_{(2)} + e^\mu_{(2)} e^\nu_{(1)}), \\ V &= i\omega^2 L_{\mu\nu}(e^\mu_{(1)} e^\nu_{(2)} - e^\mu_{(2)} e^\nu_{(1)}), \end{aligned} \tag{31}$$

where $L_{\mu\nu} \equiv \langle a_\mu a^*_\nu \rangle$ satisfies the following equation making use of Eq. (18):

$$\mathcal{D} L_{\mu\nu} + \theta L_{\mu\nu} = -p_\alpha k_\beta (\epsilon^{\alpha\beta\gamma}{}_\mu L_{\gamma\nu} + \epsilon^{\alpha\beta\gamma}{}_\nu L_{\mu\gamma}). \tag{32}$$

We can see that the Stokes parameters are coordinate scalars but not Lorentz scalars. We require the tetrad frames to be not physically rotating. In order to do that, we set the tetrad vectors at each point so that $e^\mu_{(1)}$ and $e^\mu_{(2)}$ are parallelly transported along the light curve. So it is straightforward to get the propagation equations of the four parameters along the light curve:

$$\mathcal{D} F_0 + \theta F_0 = 0, \tag{33}$$

$$\mathcal{D} F_1 + \theta F_1 = 2 p_\mu k^\mu F_2, \tag{34}$$

$$\mathcal{D} F_2 + \theta F_2 = -2 p_\mu k^\mu F_1, \tag{35}$$

$$\mathcal{D} F_3 + \theta F_3 = 0, \tag{36}$$

where $F_a \equiv (I, Q, U, V)/\omega^2$. Equation (33) means the conservation of the light flux. Equation (36) indicates that the Stokes V, which describes the net circular polarization, vanishes if it is zero at the beginning. This is the case for CMB where the polarization is produced at the last scattering. Since the Stokes V cannot be produced by Thomson scattering, it remains zero afterwards. In short, the net circular polarization remains vanishing even in the presence of the Chern-Simons term. The terms in the right-hand sides of Eqs. (34) and (35) are the effects of the Chern-Simons term which rotates the polarization angle of the light. The polarization angle defined by $\chi \equiv 1/2 \arctan(U/Q) = 1/2 \arctan(F_2/F_1)$ satisfies

$$\mathcal{D}\chi + p_\mu k^\mu = 0. \tag{37}$$

This angle when measured at the point f is rotated by

$$\Delta\chi = \chi_f - \chi_i = -\int_i^f p_\mu k^\mu d\lambda = -\int_i^f p_\mu dx^\mu(\lambda), \tag{38}$$

compared with that at the point i when the photon was emitted. From (3), $p_\mu = -(2c\alpha_{\rm em})/(3\pi M)\partial_\mu \phi$, the rotation angle is given by

$$\Delta\chi = \frac{2c\alpha_{\rm em}}{3\pi M}(\phi_f - \phi_i). \tag{39}$$

Defining

$$F_\pm \equiv F_1 \pm i F_2, \tag{40}$$

it satisfies from Eqs. (34) and (35) that

$$F^f_\pm = F^i_\pm \exp\left(-\int_i^f \theta d\lambda\right) \exp(\pm i 2\Delta\chi). \tag{41}$$

The Chern-Simons term modifies the result by merely adding the rotation factor $\exp(\pm i2\Delta\chi)$. Hence the observed Stokes parameters should be

$$(Q \pm iU)^{\text{obs}} = \exp(\pm i2\Delta\chi)(Q \pm iU). \tag{42}$$

This is the basic result obtained in this section. It describes the rotation of the polarization of a single bundle of light. It is the starting point to study the rotated CMB power spectra in the next section.

IV. CMB POWER SPECTRA

In order to analyze the CMB map, we usually make multipole expansion. In the flat universe,[2] we can expand the temperature and polarization anisotropies in terms of appropriate spin-weighted harmonic functions on the sky [54]:

$$\begin{aligned} T(\hat{\boldsymbol{n}}) &= \sum_{lm} a_{T,lm} Y_{lm}(\hat{\boldsymbol{n}}), \\ (Q \pm iU)(\hat{\boldsymbol{n}}) &= \sum_{lm} a_{\pm 2,lm\pm 2} Y_{lm}(\hat{\boldsymbol{n}}). \end{aligned} \tag{43}$$

The expressions for the expansion coefficients are

$$\begin{aligned} a_{T,lm} &= \int d\Omega Y^*_{lm}(\hat{\boldsymbol{n}}) T(\hat{\boldsymbol{n}}), \\ a_{\pm 2,lm} &= \int d\Omega_{\pm 2} Y^*_{lm}(\hat{\boldsymbol{n}})(Q \pm iU)(\hat{\boldsymbol{n}}). \end{aligned} \tag{44}$$

Instead of $a_{2,lm}$ and $a_{-2,lm}$, it is convenient to introduce their linear combinations

$$\begin{aligned} a_{E,lm} &= -(a_{2,lm} + a_{-2,lm})/2, \\ a_{B,lm} &= i(a_{2,lm} - a_{-2,lm})/2. \end{aligned} \tag{45}$$

The power spectra are defined as

$$\langle a^*_{X',l'm'} a_{X,lm} \rangle = C_l^{X'X} \delta_{l'l} \delta_{m'm} \tag{46}$$

with the assumption of statistical isotropy. In the equation above, X' and X denote the temperature T and the E and B modes of the polarization field, respectively. For Gaussian theories, the statistical properties of the CMB temperature/polarization map are specified fully by these six spectra. In the standard case, $C_l^{TB} = C_l^{EB} = 0$.

Considering the rotation in Eq. (42), the expressions for the expansion coefficients become

$$\begin{aligned} a^{\text{obs}}_{\pm 2,lm} &= \int d\Omega_{\pm 2} Y^*_{lm}(\hat{\boldsymbol{n}})(Q \pm iU)^{\text{obs}}(\hat{\boldsymbol{n}}) \\ &= \int d\Omega_{\pm 2} Y^*_{lm}(\hat{\boldsymbol{n}}) \exp(\pm i2\Delta\chi)(Q \pm iU)(\hat{\boldsymbol{n}}), \end{aligned} \tag{47}$$

and $a_{T,lm}$ remains unchanged. The rotation angle in (39) depends on time as well as space generally. It can be separated as the background part, which is homogeneous and isotropic, and the perturbation, which is randomly distributed on the sky:

$$\Delta\chi = \Delta\bar{\chi} + \Delta\delta\chi, \tag{48}$$

where

$$\Delta\bar{\chi} = \frac{2c\alpha_{\text{em}}}{3\pi M}[\bar{\phi}(\eta_0) - \bar{\phi}(\eta_{\text{dec}})], \tag{49}$$

$$\Delta\delta\chi = -\frac{2c\alpha_{\text{em}}}{3\pi M}\delta\phi(\boldsymbol{x}_{\text{dec}}, \eta_{\text{dec}}). \tag{50}$$

In the above equations the subscript 0 indicates the present values and dec means the values at the time of matter-radiation decoupling. The homogeneous part $\Delta\bar{\chi}$ is the same one that appeared in the previous rotation formulas (4). The final value of the fluctuation $\delta\phi(\boldsymbol{x}_0, \eta_0)$ is neglected because it only gives rise to a dipole contribution due to our motion with respect to the CMB frame. In the flat universe, $\boldsymbol{x}_{\text{dec}} = (\eta_0 - \eta_{\text{dec}})\hat{\boldsymbol{n}}$ when putting the observer at the origin of the coordinate system. Similar to the studies on Faraday rotation of the CMB polarization by a stochastic magnetic field [55], we expand $\Delta\delta\chi$ on the sky:

$$\Delta\delta\chi = \sum_{lm} b_{lm} Y_{lm}(\hat{\boldsymbol{n}}), \tag{51}$$

and define its angular power spectrum as

$$\langle b^*_{l'm'} b_{lm} \rangle = C_l^{\chi} \delta_{l'l} \delta_{m'm}, \tag{52}$$

where we have also assumed statistical isotropy of b_{lm}. This angular power spectrum is related to the power spectrum of $\delta\phi$ at time η_{dec}, which can be seen from the following discussions. Expanding $\delta\phi(\boldsymbol{x}_{\text{dec}}, \eta_{\text{dec}})$ in terms of Fourier functions, we have

$$\begin{aligned} \delta\phi(\boldsymbol{x}_{\text{dec}}, \eta_{\text{dec}}) &= \int \frac{d^3k}{(2\pi)^{3/2}} \phi_{\boldsymbol{k}}(\eta_{\text{dec}}) e^{i\boldsymbol{k}\cdot\hat{\boldsymbol{n}}\Delta\eta} \\ &= \int \frac{d^3k}{(2\pi)^{3/2}} \phi_{\boldsymbol{k}}(\eta_{\text{dec}}) \sum_l (2l+1) i^l j_l(k\Delta\eta) \\ &\quad \times P_l(\hat{\boldsymbol{k}}\cdot\hat{\boldsymbol{n}}) \\ &= \int \frac{d^3k}{(2\pi)^{3/2}} \phi_{\boldsymbol{k}}(\eta_{\text{dec}}) \sum_{lm} 4\pi i^l j_l(k\Delta\eta) \\ &\quad \times Y^*_{lm}(\hat{\boldsymbol{k}}) Y_{lm}(\hat{\boldsymbol{n}}), \end{aligned} \tag{53}$$

where $\Delta\eta \equiv \eta_0 - \eta_{\text{dec}}$, j_l is the spherical Bessel function, and P_l is the Legendre polynomial. Comparing it with Eqs. (50) and (51), we get

$$b_{lm} = -\frac{8c\alpha_{\text{em}}}{3M} i^l \int \frac{d^3k}{(2\pi)^{3/2}} \phi_{\boldsymbol{k}}(\eta_{\text{dec}}) j_l(k\Delta\eta) Y^*_{lm}(\hat{\boldsymbol{k}}). \tag{54}$$

With the help of the definition of the power spectrum of

[2]For the treatment of CMB anisotropies in open and closed universes, please see [53].

$\delta\phi$,

$$\langle \phi^*_{\boldsymbol{k}'}(\eta_{\rm dec})\phi_{\boldsymbol{k}}(\eta_{\rm dec})\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_\phi(k,\eta_{\rm dec})\delta^3(\boldsymbol{k}-\boldsymbol{k}'), \quad (55)$$

we can find that C_l^χ in Eq. (52) is

$$C_l^\chi = \frac{16c^2\alpha_{\rm em}^2}{9\pi M^2}\int\frac{dk}{k}\mathcal{P}_\phi(k,\eta_{\rm dec})j_l^2(k\Delta\eta), \quad (56)$$

and

$$\sum_l (2l+1)C_l^\chi = 4\pi\langle\Delta\delta\chi^2\rangle = \frac{16c^2\alpha_{\rm em}^2}{9\pi M^2}\langle\delta\phi^2\rangle, \quad (57)$$

where $\delta\phi = \delta\phi(\boldsymbol{x}_{\rm dec},\eta_{\rm dec})$ and the arguments $\boldsymbol{x}_{\rm dec}$, $\eta_{\rm dec}$ are suppressed in the following.

With these formulas, we can calculate the coefficients after the rotation:

$$\begin{aligned} a^{\rm obs}_{\pm2,lm} &= \int d\Omega\, {}_{\pm2}Y^*_{lm}(\hat{\boldsymbol{n}})(Q\pm iU)^{\rm obs}(\hat{\boldsymbol{n}}) \\ &= \exp(\pm i2\Delta\bar\chi)\sum_{l_1m_1}a_{\pm2,l_1m_1}\int d\Omega\, {}_{\pm2}Y^*_{lm}(\hat{\boldsymbol{n}}) \\ &\quad\times\exp(\pm2i\Delta\delta\chi)\,{}_{\pm2}Y_{l_1m_1}(\hat{\boldsymbol{n}}) \\ &= \exp(\pm i2\Delta\bar\chi)\sum_{l_1m_1}a_{\pm2,l_1m_1}F^{\pm}_{lml_1m_1}. \end{aligned} \quad (58)$$

In the last equality, we have defined

$$F^{\pm}_{lml_1m_1} \equiv \int d\Omega\, {}_{\pm2}Y^*_{lm}(\hat{\boldsymbol{n}})\exp(\pm2i\Delta\delta\chi)\,{}_{\pm2}Y_{l_1m_1}(\hat{\boldsymbol{n}}). \quad (59)$$

So

$$\begin{aligned} a^{\rm obs}_{E,lm} &= \frac{1}{2}\sum_{l_1m_1}[(e^{i2\Delta\bar\chi}F^+_{lml_1m_1}+e^{-i2\Delta\bar\chi}F^-_{lml_1m_1})a_{E,l_1m_1} \\ &\quad + i(e^{i2\Delta\bar\chi}F^+_{lml_1m_1}-e^{-i2\Delta\bar\chi}F^-_{lml_1m_1})a_{B,l_1m_1}], \\ a^{\rm obs}_{B,lm} &= \frac{1}{2}\sum_{l_1m_1}[(-i)(e^{i2\Delta\bar\chi}F^+_{lml_1m_1}-e^{-i2\Delta\bar\chi}F^-_{lml_1m_1})a_{E,l_1m_1} \\ &\quad + (e^{i2\Delta\bar\chi}F^+_{lml_1m_1}+e^{-i2\Delta\bar\chi}F^-_{lml_1m_1})a_{B,l_1m_1}]. \end{aligned} \quad (60)$$

To calculate the observed correlations of T, E, and B, we make the following assumptions: (i) the rotation field χ or ϕ is uncorrelated with the primordial T, E, and B modes; (ii) the rotation angle is small everywhere. In addition, we have $C_l^{TB} = C_l^{EB} = 0$ for primordial modes. Hence, we need only to calculate the following six correlations: $\langle F^{\pm}_{lml'm'}\rangle$, $\sum_{l_1m_1}C_{l_1}^{XX'}\langle F^{+*}_{l'm'l_1m_1}F^+_{lml_1m_1}\rangle$, $\sum_{l_1m_1}C_{l_1}^{XX'}\times\langle F^{-*}_{l'm'l_1m_1}F^-_{lml_1m_1}\rangle$, $\sum_{l_1m_1}C_{l_1}^{XX'}\langle F^{-*}_{l'm'l_1m_1}F^+_{lml_1m_1}\rangle$, and $\sum_{l_1m_1}C_{l_1}^{XX'}\langle F^{+*}_{l'm'l_1m_1}F^-_{lml_1m_1}\rangle$. Up to the quadratic order of $\Delta\delta\chi$, we have

$$\begin{aligned} \langle F^{\pm}_{lml'm'}\rangle &\simeq \langle 1\pm2i\Delta\delta\chi-2\Delta\delta\chi^2\rangle\delta_{ll'}\delta_{mm'} \\ &= (1-2\langle\Delta\delta\chi^2\rangle)\delta_{ll'}\delta_{mm'} \end{aligned} \quad (61)$$

and

$$\begin{aligned} \sum_{l_1m_1}C_{l_1}^{XX'}\langle F^{+*}_{l'm'l_1m_1}F^+_{lml_1m_1}\rangle &\simeq \sum_{l_1m_1}C_{l_1}^{XX'}\int d\Omega' d\Omega[1-4\langle\Delta\delta\chi^2\rangle+4\langle\Delta\delta\chi(\hat{\boldsymbol{n}}')\Delta\delta\chi(\hat{\boldsymbol{n}})\rangle]\,{}_2Y_{l'm'}(\hat{\boldsymbol{n}}')\,{}_2Y^*_{l_1m_1}(\hat{\boldsymbol{n}}')\,{}_2Y^*_{lm}(\hat{\boldsymbol{n}})\,{}_2Y_{l_1m_1}(\hat{\boldsymbol{n}}) \\ &= (1-4\langle\Delta\delta\chi^2\rangle)C_l^{XX'}\delta_{ll'}\delta_{mm'} + 4\sum_{l_1m_1l_2m_2}C_{l_1}^{XX'}C_{l_2}^\chi\int d\Omega' d\Omega\,{}_2Y_{l'm'}(\hat{\boldsymbol{n}}')\,{}_2Y^*_{l_1m_1}(\hat{\boldsymbol{n}}')Y^*_{l_2m_2}(\hat{\boldsymbol{n}}')\,{}_2 \\ &\quad\times Y^*_{lm}(\hat{\boldsymbol{n}})\,{}_2Y_{l_1m_1}(\hat{\boldsymbol{n}})Y_{l_2m_2}(\hat{\boldsymbol{n}}). \end{aligned} \quad (62)$$

The remaining integrals in the above equation may be expressed in terms of the Wigner-3j symbol through the general relation [56]:

$$\int d\Omega\, {}_sY^*_{lms_1}Y_{l_1m_1s_2}Y_{l_2m_2} = (-1)^{m+s}\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}}\begin{pmatrix} l & l_1 & l_2 \\ s & -s_1 & -s_2\end{pmatrix}\begin{pmatrix} l & l_1 & l_2 \\ -m & m_1 & m_2\end{pmatrix}. \quad (63)$$

So

$$\sum_{l_1m_1}C_{l_1}^{XX'}\langle F^{+*}_{l'm'l_1m_1}F^+_{lml_1m_1}\rangle \simeq (1-4\langle\Delta\delta\chi^2\rangle)C_l^{XX'}\delta_{ll'}\delta_{mm'} + \sum_{l_1l_2}C_{l_1}^{XX'}C_{l_2}^\chi\frac{(2l_1+1)(2l_2+1)}{\pi}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0\end{pmatrix}^2\delta_{ll'}\delta_{mm'}, \quad (64)$$

where we have used the orthogonality relation of the 3j symbol

$$\sum_{m_1m_2}(2l+1)\begin{pmatrix} l & l_1 & l_2 \\ m & m_1 & m_2\end{pmatrix}\begin{pmatrix} l' & l_1 & l_2 \\ m' & m_1 & m_2\end{pmatrix} = \delta_{ll'}\delta_{mm'}. \quad (65)$$

Similarly, we can find that

$$\sum_{l_1 m_1} C_{l_1}^{XX'} \langle F_{l'm'l_1m_1}^{-*} F_{lml_1m_1}^{-} \rangle = \sum_{l_1 m_1} C_{l_1}^{XX'} \langle F_{l'm'l_1m_1}^{+*} F_{lml_1m_1}^{+} \rangle \tag{66}$$

and

$$\begin{aligned} \sum_{l_1 m_1} C_{l_1}^{XX'} \langle F_{l'm'l_1m_1}^{-*} F_{lml_1m_1}^{+} \rangle &= \sum_{l_1 m_1} C_{l_1}^{XX'} \langle F_{l'm'l_1m_1}^{+*} F_{lml_1m_1}^{-} \rangle \\ &\simeq (1 - 4\langle \Delta \delta \chi^2 \rangle) C_l^{XX'} \delta_{ll'} \delta_{mm'} \\ &\quad + \sum_{l_1 l_2} (-1)^{L+1} C_{l_1}^{XX'} C_{l_2}^{\chi} \frac{(2l_1+1)(2l_2+1)}{\pi} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \delta_{ll'} \delta_{mm'}, \end{aligned} \tag{67}$$

where $L = l + l_1 + l_2$ and we have used the permutation property of the $3j$ symbol

$$\begin{pmatrix} l & l_1 & l_2 \\ -m & -m_1 & -m_2 \end{pmatrix} = (-1)^L \begin{pmatrix} l & l_1 & l_2 \\ m & m_1 & m_2 \end{pmatrix}. \tag{68}$$

Consequently, we obtained the rotation formulas of the power spectra

$$\begin{aligned} C_l^{TT,\mathrm{obs}} &= C_l^{TT}, \qquad C_l^{TE,\mathrm{obs}} = C_l^{TE} \cos(2\Delta\bar{\chi})(1 - 2\langle \Delta\delta\chi^2 \rangle), \qquad C_l^{TB,\mathrm{obs}} = C_l^{TE} \sin(2\Delta\bar{\chi})(1 - 2\langle \Delta\delta\chi^2 \rangle), \\ C_l^{EE,\mathrm{obs}} &= [C_l^{EE} \cos^2(2\Delta\bar{\chi}) + C_l^{BB} \sin^2(2\Delta\bar{\chi})](1 - 4\langle \Delta\delta\chi^2 \rangle) \\ &\quad + \sum_{l_1 l_2} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi} \{[1 + (-1)^{L+1} \cos(4\Delta\bar{\chi})] C_{l_1}^{EE} + [1 + (-1)^L \cos(4\Delta\bar{\chi})] C_{l_1}^{BB}\}, \\ C_l^{BB,\mathrm{obs}} &= [C_l^{EE} \sin^2(2\Delta\bar{\chi}) + C_l^{BB} \cos^2(2\Delta\bar{\chi})](1 - 4\langle \Delta\delta\chi^2 \rangle) \\ &\quad + \sum_{l_1 l_2} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi} \{[1 + (-1)^{L} \cos(4\Delta\bar{\chi})] C_{l_1}^{EE} + [1 + (-1)^{L+1} \cos(4\Delta\bar{\chi})] C_{l_1}^{BB}\}, \\ C_l^{EB,\mathrm{obs}} &= \frac{1}{2} \sin(4\Delta\bar{\chi})(C_l^{EE} - C_l^{BB})(1 - 4\langle \Delta\delta\chi^2 \rangle) \\ &\quad + \sin(4\Delta\bar{\chi}) \sum_{l_1 l_2} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l_1+1)(2l_2+1)}{2\pi} C_{l_2}^{\chi} (-1)^{L+1} (C_{l_1}^{EE} - C_{l_1}^{BB}). \end{aligned} \tag{69}$$

In comparisons with those in Eq. (4), Eq. (69) included the corrections from spatial fluctuations.

From Eq. (69), we can see first that $C_l^{TB,\mathrm{obs}}$ and $C_l^{EB,\mathrm{obs}}$ are proportional to $\sin(\Delta\bar{\chi})$, which vanish when $\Delta\bar{\chi} = 0$. This is understandable because *CPT* is violated only by the background field. Second, we find that

$$\begin{aligned} \sum_l (2l+1)(C_l^{EE,\mathrm{obs}} + C_l^{BB,\mathrm{obs}}) &= \sum_l (2l+1)(C_l^{EE} + C_l^{BB})(1 - 4\langle \Delta\delta\chi^2 \rangle) \\ &\quad + \sum_{ll_1l_2} \begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}^2 \frac{(2l+1)(2l_1+1)(2l_2+1)}{\pi} C_{l_2}^{\chi} (C_{l_1}^{EE} + C_{l_1}^{BB}) \\ &= \sum_l (2l+1)(C_l^{EE} + C_l^{BB})(1 - 4\langle \Delta\delta\chi^2 \rangle) + 4\langle \Delta\delta\chi^2 \rangle \sum_{l_1} (2l_1+1)(C_{l_1}^{EE} + C_{l_1}^{BB}) \\ &= \sum_l (2l+1)(C_l^{EE} + C_l^{BB}), \end{aligned} \tag{70}$$

where we have used another orthogonality relation of the $3j$ symbol

$$\sum_l (2l+1) \begin{pmatrix} l & l_1 & l_2 \\ -m_1 - m_2 & m_1 & m_2 \end{pmatrix}^2 = 1. \tag{71}$$

The equality in Eq. (70) is the direct consequence of invariance of $Q^2 + U^2$ under the rotation (42).

V. THE EVALUATION ON THE MAGNITUDE OF CORRECTIONS IN THE TRACKING DARK ENERGY MODEL

Equations (69) indicated that the most important corrections appear at the order of $\langle \Delta\delta\chi^2\rangle$. In this section, we consider a model for quantitative estimation on the corrections. Specifically, we take the quintessential baryo/leptogenesis model as we mentioned in the introduction. For such a model, we have

$$\langle \Delta\delta\chi^2\rangle = \frac{4c^2\alpha_{\rm em}^2}{9\pi^2 M^2}\langle\delta\phi^2\rangle \sim \frac{10^{-5}}{M^2}\langle\delta\phi^2\rangle. \tag{72}$$

As was pointed out in [2], to generate enough baryon number asymmetry, the quintessence field ϕ should have tracking behavior, which happens, for example, in the Albrecht and Skordis model [57]. In the following, we will evaluate $\langle \Delta\delta\chi^2\rangle$ in this model. We consider the perturbed metric in the Newtonian gauge:

$$ds^2 = (1+2\Phi)dt^2 - a^2(1-2\Phi)dx^i dx^i, \tag{73}$$

where $t = \int a d\eta$ is the cosmic time and Φ is the gravitational potential. The linear perturbation equation of the quintessence is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\nabla^2}{a^2}\delta\phi + V''(\phi)\delta\phi = 4\dot{\phi}\,\dot{\Phi} - 2V'\Phi. \tag{74}$$

In the above equation, the dot denotes the derivative with respect to t. The general solution to this equation is decomposed into two parts: the adiabatic mode and the isocurvature one. For the model of quintessence with a tracking solution, the isocurvature perturbation decays away quickly [2,58]. We need only calculate the adiabatic perturbation, which satisfies the adiabatic condition

$$\frac{\delta p}{\dot{p}} = \frac{\delta\rho}{\dot{\rho}}, \tag{75}$$

i.e.,

$$\Phi = \frac{d}{dt}\left(\frac{\delta\phi}{\dot{\phi}}\right). \tag{76}$$

With the equation above, we can find that at the time of matter-radiation decoupling (in the matter-dominated epoch) the adiabatic perturbation of quintessence on large scales is

$$\delta\phi = \frac{2\dot{\phi}}{3H}\Phi = \frac{2}{\sqrt{3}}\sqrt{\Omega_\phi}M_{\rm pl}\Phi, \tag{77}$$

where we have considered the exact tracking behavior of quintessence, $w_\phi = w_m = 0$, and the well-known result $\Phi =$ constant in the matter-dominated era. The parameter $\Omega_\phi \le 10^{-2}$ [59,60] is the density of quintessence at this time, and $M_{\rm pl} = 1/\sqrt{8\pi G} \sim 10^{18}$ GeV is the reduced Planck mass. So, with $\langle\Phi^2\rangle \sim 10^{-10}$, we have

$$\langle \Delta\delta\chi^2\rangle \sim 10^{-7}\frac{M_{\rm pl}^2}{M^2}\langle\Phi^2\rangle \sim 10^{-17}\frac{M_{\rm pl}^2}{M^2}. \tag{78}$$

If $\langle \Delta\delta\chi^2\rangle \ll 1$, the corrections in Eqs. (69) can be neglected safely, which happens for the cutoff scale $M \gg 10^{-8}M_{\rm pl} \sim 10^{10}$ GeV. In the quintessential baryo/leptogenesis model, $M \sim 10^8 T_D$ [2], where T_D is the decoupling temperature of lepton number violating interaction and is around 10^{11} GeV [14].

VI. CONCLUSION

In this paper, we have studied the effects of the interaction with the derivative coupling of the scalar field to photons given by the Chern-Simons term in the general curved spacetime. Under the geometric optics approximation, we have obtained the general form of the rotation angle in a gauge-invariant method. We have calculated the corrections brought by the spatial fluctuations of the scalar field to the rotation formulas. These corrections exist due to the dynamics of the scalar field[3]; however, they have not been considered in the literature on the CMB data analysis. We have estimated the magnitude of the corrections in a model of scalar field for the quintessential baryo/leptogenesis scenario and fortunately found that the corrections are very small and can be neglected safely in the fit to the CMB data. The same techniques can be applied to the case of gravitational leptogenesis in which the coupled scalar is the function of the gravitational field. Similar techniques can be developed to other cases in which the Chern-Simons parameter has other origins. For example, in a more complicated case, where the parameter is not statistically isotropic or even has no power spectrum, the space components of p_μ in Eq. (3) will bring the correlations between $a_{T,l'm'}$ and $a_{E,lm}$ of different $l'm'$ and lm and so on. These complications are beyond the scope of this paper, and we leave them in the future work.

ACKNOWLEDGMENTS

M. L. is grateful to Sergei Kopeikin for useful correspondences and to Taotao Qiu and Jun-Qing Xia for helpful discussions. This work is supported in part by National Science Foundation of China under Grants No. 10533010 and No. 10675136 and the 973 program No. 2007CB815401 and by the Chinese Academy of Sciences under Grant No. KJCX3-SYW-N2.

[3]During the writing of this paper, the paper [61] appeared, which has some similarities with the calculations of corrections from the fluctuations of the scalar field in this paper.

[1] R. Lehnert, arXiv:hep-ph/0611177.
[2] M. Li, X. Wang, B. Feng, and X. Zhang, Phys. Rev. D **65**, 103511 (2002).
[3] M. Li and X. Zhang, Phys. Lett. B **573**, 20 (2003).
[4] A. De Felice, S. Nasri, and M. Trodden, Phys. Rev. D **67**, 043509 (2003).
[5] C. Wetterich, Nucl. Phys. **B302**, 668 (1988).
[6] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988).
[7] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[8] C. Armendariz-Picon, V. F. Mukhanov, and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000).
[9] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002).
[10] B. Feng, X. L. Wang, and X. M. Zhang, Phys. Lett. B **607**, 35 (2005).
[11] M. Li, B. Feng, and X. Zhang, J. Cosmol. Astropart. Phys. 12 (2005) 002.
[12] A. Cohen and D. Kaplan, Phys. Lett. B **199**, 251 (1987).
[13] H. Davoudiasl, R. Kitano, G. D. Kribs, H. Murayama, and P. J. Steinhardt, Phys. Rev. Lett. **93**, 201301 (2004).
[14] H. Li, M. Li, and X. Zhang, Phys. Rev. D **70**, 047302 (2004).
[15] O. Bertolami, D. Colladay, V. A. Kostelecky, and R. Potting, Phys. Lett. B **395**, 178 (1997).
[16] V. A. Kostelecky, R. Lehnert, and M. J. Perry, Phys. Rev. D **68**, 123511 (2003).
[17] O. Bertolami, R. Lehnert, R. Potting, and A. Ribeiro, Phys. Rev. D **69**, 083513 (2004).
[18] G. L. Alberghi, R. Casadio, and A. Tronconi, Mod. Phys. Lett. A **22**, 339 (2007).
[19] B. Mukhopadhyay, Classical Quantum Gravity **24**, 1433 (2007).
[20] M. Sinha and B. Mukhopadhyay, Phys. Rev. D **77**, 025003 (2008).
[21] M. Li, J. Q. Xia, H. Li, and X. Zhang, Phys. Lett. B **651**, 357 (2007).
[22] S. M. Carroll, G. B. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
[23] W. T. Ni, Phys. Rev. Lett. **38**, 301 (1977).
[24] W. T. Ni, Prog. Theor. Phys. Suppl. **172**, 49 (2008).
[25] R. Lehnert and R. Potting, Phys. Rev. Lett. **93**, 110402 (2004).
[26] R. Lehnert and R. Potting, Phys. Rev. D **70**, 125010 (2004); **70**, 129906(E) (2004).
[27] S. M. Carroll, Phys. Rev. Lett. **81**, 3067 (1998).
[28] A. Lue, L. M. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
[29] B. Feng, H. Li, M. Li, and X. Zhang, Phys. Lett. B **620**, 27 (2005).
[30] B. Feng, M. Li, J.-Q. Xia, X.-L. Chen, and X. Zhang, Phys. Rev. Lett. **96**, 221302 (2006).
[31] http://cmbpol.uchicago.edu.
[32] http://www.rssd.esa.int/index.php?project=PLANCK.
[33] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 377 (2007).
[34] L. Page *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 335 (2007).
[35] G. Hinshaw *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 288 (2007).
[36] N. Jarosik *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **170**, 263 (2007).
[37] Available from http://lambda.gsfc.nasa.gov/product/map/current/.
[38] W. C. Jones *et al.*, Astrophys. J. **647**, 823 (2006).
[39] T. E. Montroy *et al.*, Astrophys. J. **647**, 813 (2006).
[40] Available from http://cmb.phys.cwru.edu/boomerang/.
[41] G. C. Liu, S. Lee, and K. W. Ng, Phys. Rev. Lett. **97**, 161303 (2006).
[42] V. A. Kostelecky and M. Mewes, Phys. Rev. Lett. **99**, 011601 (2007).
[43] A. Kostelecky and M. Mewes, arXiv:0809.2846.
[44] C. Q. Geng, S. H. Ho, and J. N. Ng, Can. J. Phys. **86**, 587 (2008).
[45] P. Cabella, P. Natoli, and J. Silk, Phys. Rev. D **76**, 123014 (2007).
[46] J.-Q. Xia, H. Li, X. Wang, and X. Zhang, Astron. Astrophys. **483**, 715 (2008).
[47] G. Hinshaw *et al.*, arXiv:0803.0732.
[48] M. R. Nolta *et al.*, arXiv:0803.0593.
[49] E. Komastu *et al.*, arXiv:0803.0547.
[50] J.-Q. Xia, H. Li, G.-B. Zhao, and X. Zhang, Astrophys. J. **679**, L61 (2008).
[51] A. M. Anile and R. A. Breuer, Astrophys. J. **189**, 39 (1974).
[52] S. Kopeikin and P. Korobkov, arXiv:gr-qc/0510084.
[53] W. Hu, U. Seljak, M. J. White, and M. Zaldarriaga, Phys. Rev. D **57**, 3290 (1998).
[54] M. Zaldarriaga and U. Seljak, Phys. Rev. D **55**, 1830 (1997).
[55] A. Kosowsky, T. Kahniashvili, G. Lavrelashvili, and B. Ratra, Phys. Rev. D **71**, 043006 (2005).
[56] W. Hu, Phys. Rev. D **62**, 043007 (2000).
[57] A. Albrecht and C. Skordis, Phys. Rev. Lett. **84**, 2076 (2000).
[58] L. R. W. Abramo and F. Finelli, Phys. Rev. D **64**, 083513 (2001).
[59] R. Bean, S. H. Hansen, and A. Melchiorri, Phys. Rev. D **64**, 103508 (2001).
[60] M. Doran and G. Robbers, J. Cosmol. Astropart. Phys. 06 (2006) 026.
[61] M. Pospelov, A. Ritz, and C. Skordis, arXiv:0808.0673.