

Thermal behavior of π - π scattering lengths in the Nambu–Jona-Lasinio model

M. Loewe* and Jorge Ruiz A.†

Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago 22, Chile

J. C. Rojas‡

Departamento de Física, Universidad Católica del Norte, Angamos 0610, Antofagasta, Chile

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We calculate the thermal evolution of π - π scattering lengths, in the framework of the Nambu–Jona-Lasinio model. The thermal corrections were calculated at the one-loop level using thermofield dynamics. We present also results for the pion thermal mass. Our procedure implies the modeling of a propagating scalar meson as a resummation of chains of quark bubbles, which is presented explicitly. We compare our results with previous analysis of this problem in the framework of different theoretical approaches.

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I. INTRODUCTION

The Nambu–Jona-Lasinio model (NJL) is an effective, nonrenormalizable, low-energy description of hadron dynamics [1,2]. The main motivation for this model is the natural way in which the phenomenon of chiral symmetry breaking appears. The model has the same global symmetries of QCD, being therefore appropriate for describing low-energy hadronic phenomenology. Of special interest are processes involving pions, since they play a quite special role among hadrons. They are the lightest hadrons, being also pseudoscalar Goldstone bosons.

In heavy ion relativistic collisions, it turns out that the pion multiplicity dominates by far any other hadronic signature, being dominant in the particle content of the central rapidity region, where we expect the quark gluon plasma to be created. This suggests that pion interactions, i.e. scattering processes, will be important. Since those scattering processes occur in the presence of a thermal environment, our discussion should be presented in the frame of finite temperature quantum field theory. The main parameters associated to low energy π - π scattering are the so-called scattering lengths, which depend on the isospin channel. The first measurements of π - π scattering lengths was done by Rosselet *et al.* [3]. A review about the present experimental status of π - π scattering lengths can be found in [4].

In this paper we address once again the problem of determining the thermal behavior of the π - π scattering lengths, in the frame of the Nambu–Jona-Lasinio (NJL) model, using the thermofield dynamics (TFD) formalism. For details about this formalism see [5,6]. The novelty of our discussion is mainly technical, including the resummation in TFD of bubble diagrams associated to a propagating sigma meson, how to regularize products of distributions,

as well as other nontrivial intermediate steps that appear in the calculation of the diagrams.

It should be noticed that the thermal behavior of the π - π scattering lengths in the NJL model has been discussed long time ago in the imaginary time formalism, or Matsubara approach [7]. Our results, specially for $a_2(T)$, do not agree with this paper. We do agree qualitatively, however, with the discussion of the thermal behavior of π - π scattering lengths in the frame of chiral perturbation theory [8], and with a recent analysis based on the linear sigma model [9]. Our results indicate that the variation of the scattering lengths as function of temperature, in particular, for $a_2(T)$, begin at much lower values of temperatures than the behavior found in [8,9].

The plan of this article is the following: after presenting the general formalism, we classify the relevant diagrams for getting the π - π scattering lengths according to the NJL model. Then, we proceed to extend our calculation to the finite temperature scenario. For this purpose, a resummation procedure of quark bubbles is presented in order to describe the thermal sigma meson. As a by-product of our analysis we found also the thermal evolution of the pion mass. Finally, we compare our results with previous articles about this subject.

II. SCATTERING LENGTHS AND THE RELEVANT DIAGRAMS AT ZERO TEMPERATURE

In general, the π - π scattering amplitude can be parametrized according to

$$T_{\alpha\beta;\delta\gamma} = A(s, t, u)\delta_{\alpha\beta}\delta_{\delta\gamma} + A(t, s, u)\delta_{\alpha\gamma}\delta_{\beta\delta} + A(u, t, s)\delta_{\alpha\delta}\delta_{\beta\gamma}, \quad (1)$$

where the α , β , γ , and δ denote isospin components.

By using appropriate projection operators it is possible to get the following isospin-dependent scattering amplitudes

$$T^0 = 3A(s, t, u) + A(t, s, u) + A(u, t, s), \quad (2)$$

*mloewe@fis.puc.cl

†jruiza@uc.cl

‡jurojas@ucn.cl

$$T^1 = A(t, s, u) - A(u, t, s), \quad (3)$$

$$T^2 = A(t, s, u) + A(u, t, s), \quad (4)$$

where T^I denotes a scattering amplitude in a given isospin channel.

Below the inelastic threshold the partial scattering amplitudes can be parametrized as [10]

$$T_\ell^I = \left(\frac{s}{s - 4m\pi^2} \right)^{1/2} \frac{1}{2i} (e^{2i\delta_\ell^I(s)} - 1), \quad (5)$$

where δ_ℓ is a phase shift in the ℓ channel. In fact our last expression can be expanded according to

$$\Re(T_\ell^I) = \left(\frac{p^2}{m_\pi^2} \right)^\ell \left(a_\ell^I + \frac{p^2}{m_\pi^2} b_\ell^I + \dots \right). \quad (6)$$

The parameters a_ℓ^I and b_ℓ^I are the scattering lengths and scattering slopes, respectively. In general the scattering lengths satisfy $|a_0| > |a_1| > |a_2| > \dots$. If we are only interested in the scattering lengths a_0^I , it is enough to calculate the scattering amplitude T^I in the static limit or at threshold i.e. when $s \rightarrow 4m_\pi^2$, $t \rightarrow 0$, and $u \rightarrow 0$

$$a_0^I = \frac{1}{32\pi} T^I(s \rightarrow 4m_\pi^2, t \rightarrow 0, u \rightarrow 0). \quad (7)$$

Our Lagrangian is given by

$$\mathcal{L}_{NJL} = \bar{\psi}(i\not{\partial} - m_0)\psi + G((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2), \quad (8)$$

where ψ denotes the quark fields, G is a coupling constant and m_0 denotes the current quark mass. We will also consider an effective coupling $g_{\pi qq}$ between the internal quark lines and the external pions. Notice that the model does not include a field associated to the scalar sigma meson. The propagator of the sigma meson is represented in the random phase approximation (RPA) trough a geometrical sum of quark bubbles's chains. See Fig. 1, where we show the effective exchange of a sigma meson in the s channel.

The lowest-order diagrams that contribute to the π - π scattering lengths in the NJL model are shown in Fig. 2, where the lines with an arrow inside the loops denote the quarks, and the double lines the sigma meson. The external legs are of course the physical pions with momentum p which couple to the quarks trough the effective coupling $g_{\pi qq}$.

At zero temperature these diagrams have been calculated by Schulze [11].

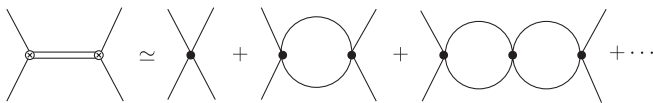


FIG. 1. Effective meson exchange modeled as a chain of quark bubbles.

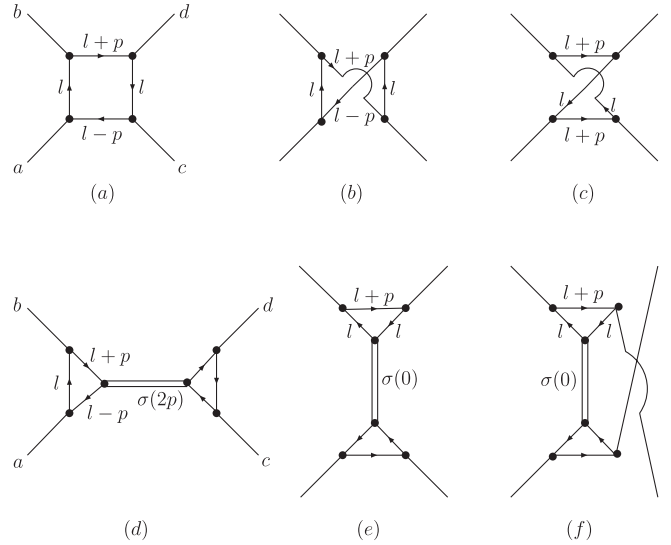


FIG. 2. Relevant Feynman diagrams for the calculation of the π - π scattering lengths.

It is important to remark that the fermion propagators include the constituent quark mass m , instead of the current quark mass m_0 , because our calculations are done in the chiral broken phase. The quarks acquire their constituent mass due to a condensation of quark-antiquark pairs that appear in a completely analogous way to the Cooper pairs in the BCS theory of superconductivity. This quark-antiquark pairing mechanism implies the appearance of the constituent mass trough the so-called gap equation

$$1 = \frac{m_0}{m} + 8iGN_cN_f \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - m^2 + i\epsilon}, \quad (9)$$

where $N = N_cN_f$, being $N_c = 3$ the numbers of colors and N_f the numbers of flavors. In our case we will take $N_f = 2$.

III. THERMOFIELD DYNAMICS AND THE EFFECTIVE SIGMA MESON PROPAGATOR

In TFD we have to double the number of degrees of freedom in order to express any thermal average in the ensemble as a vacuum expectation value in a populated vacuum. The extra fields are called thermal ghosts and, therefore, the propagators in general will be given by a two by two matrix. In the case of fermions, the matrix propagator has the form

$$S_F(l) = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix}, \quad (10)$$

where

$$\begin{aligned}
 S_{11} &= (\mathcal{I} + m) \left(\frac{i}{l^2 - m^2 + i\varepsilon} - \frac{2\pi\delta(l^2 - m^2)}{e^{l_0/T} + 1} \right), \\
 S_{12} &= (\mathcal{I} + m) \frac{\varepsilon(l_0) 2\pi\delta(l^2 - m^2) e^{l_0/2T}}{e^{l_0/T} + 1} = S_{21}, \quad (11) \\
 S_{22} &= (\mathcal{I} + m) \left(\frac{i}{l^2 - m^2 - i\varepsilon} + \frac{2\pi\delta(l^2 - m^2)}{e^{l_0/T} + 1} \right),
 \end{aligned}$$

where ε is the sign function. Note that the S_{11} element is exactly the fermion Dolan–Jackiw propagator [12]. We may define $S_{11}(p) = S_F^0(p) + S_F^\beta(p)$, separating explicitly the zero temperature from the finite temperature contribution. The other components of the matrix propagator appear only in internal lines which do not couple to external physical particles. Therefore, in our case, we will have to deal with the full-matrix propagator structure, at the one-loop level, only for the diagrams where a sigma meson is exchanged. As we said, it is given as a sum of chains of quark bubbles. Notice also that in general, there are two different coupling constants for the four quark vertices, depending on the type of quark lines that come together [5,6].

We will now construct the effective thermal sigma meson propagator. In general, the chain of bubbles will imply a resummation of the type $\sum_{n=0}^{\infty} \mathbf{D}^n$, where

$$\mathbf{D} = \begin{pmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{pmatrix} \quad (12)$$

is a matrix whose elements D_{ij} represent a bubble with an ‘‘open vertex’’ of type i at the left side of the bubble, and with a vertex of type j at the right side ($i, j = 1, 2$). In order to avoid double counting of vertices when powers of \mathbf{D} are considered, we have to introduce quark bubbles with an open vertex only at the left side. It is clear that each element D_{ij}^2 involves the sum of the two possibilities to build a chain with two bubbles, in such a way that we have an open vertex of type i at the left side and a vertex of type j at the right side. This means that the two possible values for the vertex in between are included. The same happens for every matrix element of some power n of \mathbf{D} .

It is convenient to define the following matrix

$$\begin{aligned}
 \mathbf{Z} &\equiv \frac{1}{1 - D_{11} - D_{22} + D_{11}D_{22} - D_{12}D_{21}} \\
 &\times \begin{pmatrix} v_1(1 - D_{22}) & v_1D_{12} \\ v_2D_{21} & v_2(1 - D_{11}) \end{pmatrix} \quad (13)
 \end{aligned}$$

that essentially corresponds to $\sum_{n=0}^{\infty} \mathbf{D}^n$, where, we have explicitly introduced the vertices v_i associated to the left side of the sum of the quark bubbles. A similar treatment was introduced previously in the discussion of thermal renormalons in the $\lambda\phi^4$ theory [13].

In order to appreciate the role of the Z matrix let us consider diagrams (d), (e), and (f) of Fig. 2. There we have quark triangles that couple to the effective sigma meson through open vertices. To be more explicit, let us denote by

Γ_i the triangle diagram with an internal open vertex of type i . Then we introduce the vectors

$$\mathbf{Y}^\top = (\Gamma_1 \quad \Gamma_2), \quad \mathbf{Y} = \begin{pmatrix} \Gamma_1 \\ \Gamma_2 \end{pmatrix}. \quad (14)$$

In this way, the complete amplitude for these kind of diagrams, including both the zero as well as the finite temperature parts, is given in this matrix language by

$$\mathcal{T} = \mathbf{Y}^\top \mathbf{Z} \mathbf{Y}. \quad (15)$$

In the case of the triangle diagrams (d), (e), and (f) of Fig. 2, we need the following elements

$$D_{11} = \left(\frac{1}{i} \Pi^0 + 2 \frac{1}{i} \Pi^\beta \right) v_1 \quad (16)$$

$$D_{12} = 0 \quad (17)$$

$$D_{21} = 0 \quad (18)$$

$$D_{22} = \left(\frac{1}{i} \Pi^0 - 2 \frac{1}{i} \Pi^\beta \right) v_2 \quad (19)$$

to build the \mathbf{Z} matrix. In the above expression $-i\Pi^0(-i\Pi^\beta)$ denotes a zero (finite) temperature single bubble contribution. Notice that $D_{12} = D_{21} = 0$ because there is no support for the corresponding integrals, as can be checked, since we are using constituent quark masses m , being $m > 2m_\pi$. In order to handle other singular contributions, like products of delta functions with the same argument, it was necessary to take different external pion momenta, i.e. a kinematical configuration away from the threshold. In this way, singularities disappear and we are able to compute the corresponding integrals, finding that they vanish identically when the limit to the threshold configuration is taken. Because of the same reasons, the triangle diagram Γ_2 which couples to the sigma meson through a vertex v_2 , vanishes. In this way, for the triangle diagrams we found

$$\Gamma_1 = \Gamma_d^0 + 4\Gamma_d^\beta, \quad (20)$$

$$\Gamma_2 = 0, \quad (21)$$

where

$$\Gamma_d^\beta = \frac{2m}{\pi^2} N(g_{\pi qq}^\beta)^2 \int_0^\Lambda dl \frac{l^2 n_F(E_l)}{E_l(4E_l^2 - m_\pi^2)}, \quad (22)$$

where $E_l = \sqrt{l^2 + m^2}$ and n_F is the usual Fermi–Dirac distribution

$$n_F(z) \equiv \frac{1}{e^{z_0/T} + 1}. \quad (23)$$

We have introduced a temperature dependent coupling $g_{\pi qq}^\beta$ between pions and quarks. This coupling can be obtained as

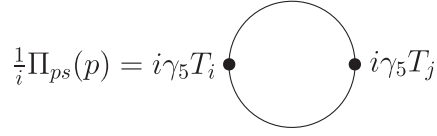


FIG. 3. The single quark bubble that model a pion in the NJL approach.

$$(g_{\pi qq}^\beta)^{-2} = \left(\frac{\partial \Pi_{ps}}{\partial p^2} + 2 \frac{\partial \Pi_{ps}^\beta}{\partial p^2} \right) \Big|_{p^2=(m_\pi^\beta)^2}, \quad (24)$$

where $-i\Pi_{ps}$ is the single quark bubble (pseudoscalar case) shown in Fig. 3. At this point it is worthwhile to remember that such a quark bubble is used to model a pion in the NJL approach. T selects the isospin channel: $T_i = T_j = \tau_3$ corresponds to the π^0 and/or $T_i = \tau^{(\pm)}$ and $T_j = \tau^{(\mp)}$ are associated to the $\pi^{(\pm)}$. In the previous equation, m_π^β is the thermal pion mass which is obtained in the next section.

IV. TEMPERATURE CORRECTIONS TO THE π - π SCATTERING LENGTHS

Following the notation introduced by Schulze [11], all diagrams at zero temperature are given in terms of the three following integrals

$$\begin{aligned} I(p) &= \int^\Lambda \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2 + i\varepsilon)((l+p)^2 - m^2 + i\varepsilon)}, \\ K(p) &= \int^\Lambda \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2 + i\varepsilon)^2((l+p)^2 - m^2 + i\varepsilon)}, \\ L(p) &= \int^\Lambda \frac{d^4 l}{(2\pi)^4} \frac{1}{(l^2 - m^2 + i\varepsilon)^2((l+p)^2 - m^2 + i\varepsilon)^2}, \end{aligned} \quad (25)$$

where Λ is a 3-momentum cutoff. After performing the angular and the l_0 integrals, we found

$$I(p) = \frac{i}{2\pi^2} \int_0^\Lambda dl \frac{l^2}{E_l(4E_l^2 - p^2)}, \quad (26)$$

$$K(p) = \frac{i}{8\pi^2} \int_0^\Lambda dl \frac{l^2(p^2 - 12E_l^2)}{E_l^3(4E_l^2 - p^2)^2}, \quad (27)$$

$$L(p) = \frac{i}{4\pi^2} \int_0^\Lambda dl \frac{l^2(20E_l^2 - p^2)}{E_l^3(4E_l^2 - p^2)^3}. \quad (28)$$

In the NJL approach, the gap equation plays a fundamental role. At finite temperature [2] it acquires the form

$$\begin{aligned} 1 &= \frac{m_0}{m} + 8iGN \int^\Lambda \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m^2 + i\varepsilon} \\ &+ 8iGN \int^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{in_F(E_l)}{E_l}. \end{aligned} \quad (29)$$

The thermal corrections to the effective sigma meson propagator, at the level of a single quark bubble are shown in the Fig. 3, but where the $i\gamma_5 T$ has been replaced by the identity matrix according to the NJL Lagrangian. This loop is given by

$$\frac{1}{i} \Pi^\beta(p) = 2iN \left(\int^\Lambda \frac{d^3 l}{(2\pi)^3} \frac{n_F(E_l)}{E_l} + (4m^2 - p^2)J(p) \right), \quad (30)$$

where $J(p)$ is

$$J(p) = \frac{1}{2\pi^2} \int_0^\Lambda dl \frac{l^2 n_F(E_l)}{E_l(p^2 - 4E_l^2)}. \quad (31)$$

Notice that at this level the Dolan–Jackiw propagators are enough. In particular, the thermal contributions arise from the insertion of only one pure thermal propagator S_F^β in the loop. The other propagator has to be the normal one at zero temperature. It is easy to see that two thermal propagators do not contribute. The denominator $D_Z(p)$ that appears in the \mathbf{Z} matrix (13) is given by

$$D_Z(p) = 1 - 2G\Pi_{ps}^0(p) - 4G\Pi_{ps}^\beta(p). \quad (32)$$

Using the thermal mass gap Eq. (29) this expression can be written as

$$D_Z(p) = \frac{m_0}{m} + 4iGNp^2(I(p) + 2iJ(p)). \quad (33)$$

Following the same procedure, we could also construct an effective pion propagator, which is not necessary for our purpose of getting the π - π scattering lengths. From this analysis, however, we get the thermal pion mass determined from the pole of the effective propagator

$$(m_\pi^\beta)^2 = -\frac{m_0}{m} \frac{1}{4iGN(I(m_\pi) + 2iJ(m_\pi))}. \quad (34)$$

The behavior of m_π^β is shown in Fig. 4. This allows us to express the quotient m_0/m in terms of the other quantities that appear in the previous equations. Replacing m_0/m in (33) we get

$$\begin{aligned} D_Z(p) &= 4iGN(p^2 - 4m^2)(I(p) + 2iJ(p)) \\ &- 4iGNm_\pi^2(I(m_\pi) + 2iJ(m_\pi)). \end{aligned} \quad (35)$$

An expression for m_π^β was also found through the mass gap equation in the Matsubara or imaginary time formalism by previous authors [14]. We agree qualitatively with them.

It is important to remark that in Eq. (35), m_π is the usual pion mass at zero temperature because we are implementing a calculation at the one-loop level. The inclusion of m_π^β in the last expression goes beyond this approximation.

Finally, we get the following expression, consistent at the one-loop perturbation theory, for the effective sigma meson propagator

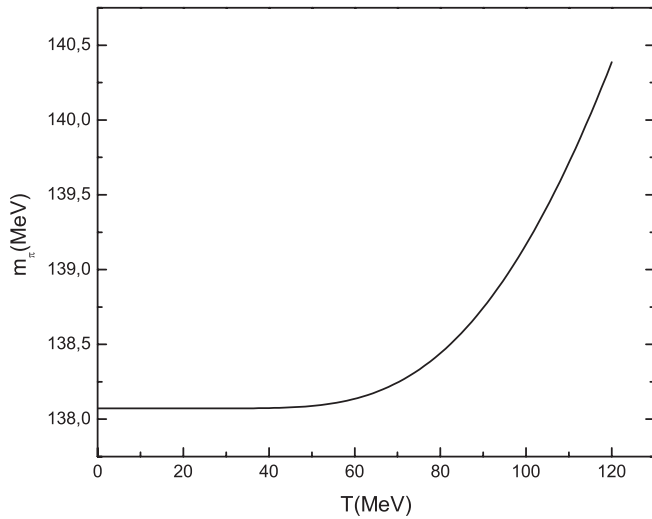


FIG. 4. Thermal evolution of the pion mass.

$$D_{\sigma}^{0+\beta}(p) = \frac{1}{2N} \frac{4iGN}{D_Z(p)}. \quad (36)$$

In addition to the triangle diagram given in Eq. (22), we need, also associated to the diagram 2(e), the following quantities

$$\begin{aligned} \Gamma_{e_-}^{\beta} &= \frac{4m}{\pi^2} N(g_{\pi qq}^{\beta})^2 \int_0^{\Lambda} \frac{l^2 n_F(E_l)(4E_l^2 + m_{\pi}^2)}{E_l(4E_l^2 - m_{\pi}^2)^2} dl, \\ \Gamma_{e_0}^{\beta} &= \frac{8}{\pi^2} N(g_{\pi qq}^{\beta})^2 \int_0^{\Lambda} \frac{\partial}{\partial m^2} \left(\frac{m l^2 n_F(E_l) E_l}{(4E_l^2 - m_{\pi}^2)} \right) dl \\ &\quad - \frac{2}{\pi^2 m} N(g_{\pi qq}^{\beta})^2 M_e, \end{aligned} \quad (37)$$

where

$$M_e = \int_0^{\Lambda} \frac{l^2 n_F(E_l)(m_{\pi}^2 m^2 + (4m^2 + 2m_{\pi}^2)E_l^2 - 8E_l^4)}{E_l(4E_l^2 - m_{\pi}^2)^2} dl. \quad (38)$$

For the diagrams (d), (e), and (f) of Fig. 2, we get

$$\begin{aligned} \mathcal{T}_d^{0+\beta} &= D_{\sigma}^{0+\beta}(2m_{\pi})(\Gamma_d^0 + 4\Gamma_d^{\beta})^2, \\ \mathcal{T}_e^{0+\beta} &= D_{\sigma}^{0+\beta}(0)(\Gamma_e^0 + \Gamma_{e_-}^{\beta} + 2\Gamma_{e_0}^{\beta})^2, \\ \mathcal{T}_f^{0+\beta} &= \mathcal{T}_e^{0+\beta}, \end{aligned} \quad (39)$$

where [11]

$$\begin{aligned} \Gamma_d^0 &= -8NmI(m_{\pi})(g_{\pi qq}^0)^2, \\ \Gamma_e^0 &= -8Nm(I(0) - m_{\pi}^2 K(m_{\pi}))(g_{\pi qq}^0)^2. \end{aligned} \quad (40)$$

$g_{\pi qq}^0$ can be read from the first term in Eq. (24). In terms of our integrals $I(p)$ and $K(p)$, it can be rewritten as

$$g_{\pi qq}^0 = (-Ni(I(0) + I(m_{\pi}) - m_{\pi}^2 K(m_{\pi}))^{-1/2}. \quad (41)$$

We find $g_{\pi qq}^0 = 3.56$.

Concerning the temperature corrections to the box diagrams (a), (b), and (c) in Fig. 2, we only need the Dolan–Jackiw fermion propagators. For the same reasons we mentioned previously, in each box diagram will survive only the insertion of one pure thermal fermion propagator S^{β} , which can be any of the four propagators inside the box. Two, three, or four thermal insertions vanish identically. The results are

$$\begin{aligned} \mathcal{T}_a^{0+\beta} &= \mathcal{T}_a^0 + 2\mathcal{T}_{a_{\pm}}^{\beta} + 2\mathcal{T}_{a_0}^{\beta}, & \mathcal{T}_b^{0+\beta} &= \mathcal{T}_a^{0+\beta}, \\ \mathcal{T}_c^{0+\beta} &= \mathcal{T}_c^0 + 4\mathcal{T}_{c_{\pm}}^{\beta}, \end{aligned} \quad (42)$$

where

$$\begin{aligned} \mathcal{T}_{a_{\pm}}^{\beta} &= \frac{8i}{\pi^2} N(g_{\pi qq}^{\beta})^4 \int_0^{\Lambda} \frac{l^2 n_F(E_l) E_l}{(4E_l^2 - m_{\pi}^2)^2} dl, \\ \mathcal{T}_{a_0}^{\beta} &= -\frac{4i}{\pi^2} N(g_{\pi qq}^{\beta})^4 \int_0^{\Lambda} \frac{\partial}{\partial m^2} \left(\frac{l^2 n_F(E_l) E_l}{(4E_l^2 - m_{\pi}^2)} \right) dl \\ &\quad - \frac{2im_{\pi}^2}{\pi^2} N(g_{\pi qq}^{\beta})^4 \int_0^{\Lambda} \frac{l^2 n_F(E_l)}{E_l(4E_l^2 - m_{\pi}^2)^2} dl, \\ \mathcal{T}_{c_{\pm}}^{\beta} &= \frac{4N(g_{\pi qq}^{\beta})^4}{i\pi^2} \int_0^{\Lambda} \frac{\partial}{\partial m^2} \left(\frac{l^2 n_F(E_l) E_l (4E_l^2 + m_{\pi}^2)}{(4E_l^2 - m_{\pi}^2)^2} \right) dl \\ &\quad - \frac{2m_{\pi}^2 i}{\pi^2} N(g_{\pi qq}^{\beta})^4 \int_0^{\Lambda} \frac{l^2 n_F(E_l) (m_{\pi}^2 + 12E_l^2)}{E_l(4E_l^2 - m_{\pi}^2)^3} dl. \end{aligned} \quad (43)$$

\mathcal{T}_a^0 and \mathcal{T}_c^0 were also obtained in [11], and are given by

$$\begin{aligned} \mathcal{T}_a^0 &= 4N(g_{\pi qq}^{\beta})^4 (m_{\pi}^2 K(m_{\pi}) - I(0) - I(m_{\pi})), \\ \mathcal{T}_c^0 &= 8N(g_{\pi qq}^{\beta})^4 \left(2m_{\pi}^2 K(m_{\pi}) - I(0) - \frac{1}{2} m_{\pi}^4 L(m_{\pi}) \right). \end{aligned} \quad (44)$$

Using these results, we are able to find the thermal π - π scattering lengths. For each diagram in Fig. 2, the total amplitude is given as the sum of the zero temperature and the finite temperature parts. In this way we get

$$\begin{aligned} \mathcal{T}_a^{0+\beta} &= \mathcal{T}_a^0 + 2\mathcal{T}_{a_{\pm}}^{\beta} + 2\mathcal{T}_{a_0}^{\beta}, \\ \mathcal{T}_b^{0+\beta} &= \mathcal{T}_a^{0+\beta}, \\ \mathcal{T}_c^{0+\beta} &= \mathcal{T}_c^0 + 4\mathcal{T}_{c_{\pm}}^{\beta}, \\ \mathcal{T}_d^{0+\beta} &= D_{\sigma}^{0+\beta}(2m_{\pi})(\Gamma_d^0 + 4\Gamma_d^{\beta})^2, \\ \mathcal{T}_e^{0+\beta} &= D_{\sigma}^{0+\beta}(0)(\Gamma_e^0 + \Gamma_{e_-}^{\beta} + 2\Gamma_{e_0}^{\beta})^2, \\ \mathcal{T}_f^{0+\beta} &= \mathcal{T}_e^{0+\beta}, \end{aligned} \quad (45)$$

where the superscript 0 refers to zero temperature terms, which are given in [11]. To express them, we have used our functions I , K , and L . Finally, the scattering lengths are given by

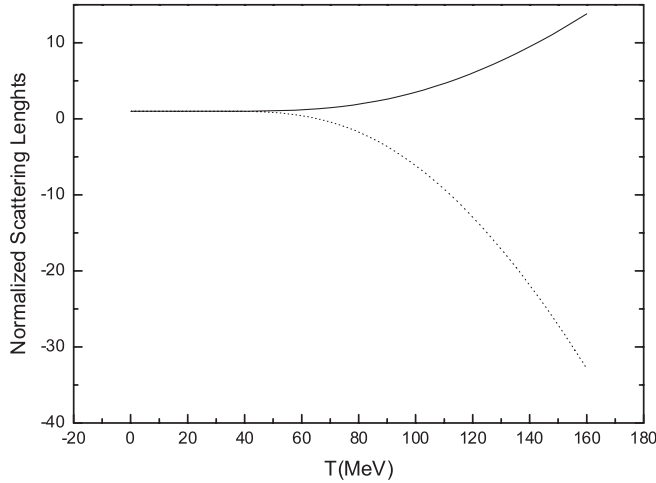


FIG. 5. Thermal dependence of the π - π scattering lengths, normalized to its zero-temperature value. The upper curve correspond to $a_0^{0+\beta}/a_0^0$, and the lower one to $a_2^{0+\beta}/a_2^0$. Temperatures are in MeV.

$$a_0^{0+\beta} = \frac{1}{32\pi} (6\mathcal{T}_a^{0+\beta} - \mathcal{T}_c^{0+\beta} + 3\mathcal{T}_d^{0+\beta} + 2\mathcal{T}_e^{0+\beta}),$$

$$a_2^{0+\beta} = \frac{1}{32\pi} (2\mathcal{T}_c^{0+\beta} + 2\mathcal{T}_e^{0+\beta}). \quad (46)$$

Their thermal dependence is shown in Fig. 5.

The parameters we used in the calculation are $\Lambda = 631$ MeV, $m = 339$ MeV, $m_\pi (= m_\pi^\beta(T=0)) \simeq 138$ MeV. This set of parameters implies also $m_0 = 5.5$ MeV (the average between the lightest quark masses, $m_u \simeq 4$ MeV, $m_d \simeq 7$ MeV), $f_\pi = 93$ MeV, and $G \simeq 5.51$ GeV⁻². These values for the parameters are standard in the literature for the case with a 3-dimensional cutoff [7].

At zero temperature we found $a_0 \simeq 0.161$, $a_2 \simeq -0.043$. These numbers are in agreement with Weinberg's results [15] $a_0^W = 7m_\pi^2/32\pi f_\pi^2 = 0.16$, and $a_2^W = -2m_\pi^2/32\pi f_\pi^2 = -0.044$. However, they disagree with the experimental results [3], $a_0 = 0.26 \pm 0.05$ and $a_2 = -0.028 \pm 0.012$. Nevertheless, our goal here was to find the thermal evolution of the scattering lengths normalized by the zero temperature values.

Our results agree qualitatively well with two previous calculations. The first one [8], was done in the frame of the chiral perturbation theory at the one-loop level, whereas the linear sigma model was used in the second analysis [9]. In our case, the scattering lengths vary faster as function of temperature compared with the other two papers. However, we disagree qualitatively with the results obtained in [7]. This calculation was done also in the frame of the NJL model, but using the imaginary time formalism to compute the thermal corrections.

The reason behind our disagreement with the results in [7] is because the authors go beyond the one-loop order perturbation theory when they introduced a temperature dependent pion mass in their expression for the sigma meson propagator which already emerged from a one-loop calculation (after resummation).

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