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Hard Pomeron-odderon interference effects in the production of $\pi^+\pi^-$ pairs in high energy $\gamma\gamma$ collisions at the LHC

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We estimate the production of two meson pairs in high energy photon-photon collisions produced in ultraperipheral collisions at LHC. We show that the study of charge asymmetries may reveal the existence of the perturbative Odderon and discuss the concrete event rates expected at the LHC. Sizable rates and asymmetries are expected in the case of proton-proton collisions and medium values of $\gamma\gamma$ energies $\sqrt{s_{\gamma\gamma}} \approx 20$ GeV. Proton-proton collisions will benefit from a high rate due to a large effective $\gamma\gamma$ luminosity and ion-ion collisions with a somewhat lower rate from the possibility to trigger on ultra-peripheral collisions and a reduced background from strong interactions.

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I. INTRODUCTION

Hadronic reactions at low momentum transfer and high energies are described in the framework of QCD in terms of the dominance of color singlet exchanges corresponding to a few Reggeized gluons. The charge conjugation even sector of the t-channel exchanges is understood as the QCD Pomeron. The charge-odd exchange is less well understood, although the need for the Odderon contribution [1,2], in particular to understand the different behaviors of pp and $\bar{p}p$ elastic cross sections [3], is quite generally accepted. Studies of specific channels where the Odderon contribution is expected to be singled out have turned out to be very disappointing [4] but new channels have recently been proposed [5–8]. Another strategy to reveal the Odderon has been first initiated in Ref. [9], where it was stressed that the study of observables where Odderon effects are present at the amplitude level—and not at the squared amplitude level—is mandatory to get a convenient sensitivity to a rather small normalization of this contribution. This approach has been extended to the production of pion pairs [10–15] since the $\pi^+\pi^-$ state does not have any definite charge parity and therefore both Pomeron and Odderon exchanges may contribute to the production amplitude.

In this paper, we study within perturbative QCD (pQCD) the charge asymmetries in the production of two pion pairs in photon-photon collisions,

$$\gamma(q, \varepsilon)\gamma(q', \varepsilon') \rightarrow \pi^+(p_+)\pi^-(p_-)\pi^+(p'_+)\pi^-(p'_-), \quad (1)$$

where ε and ε' are the initial photon polarization vectors, see Fig. 1. We have in mind the ultraperipheral collisions of protons or nuclei of high energies, which constitute a promising new way to study QCD processes initiated by two quasireal photons [16–18]. At high energy the application of pQCD for the calculation of a part of this process is justified by the presence of a hard scale: the momentum transfer $t = (q - p_+ - p_-)^2$ from an initial photon to a final pion pair, $-t$ being of the order of a few GeV^2. The amplitude of this process may be calculated within k_T factorization, as the convolution of two impact factors representing the photon to pion pair transitions and a two or three gluon exchange. The impact factors themselves include the convolution of a perturbatively calculable hard part with a nonperturbative input, the two pion generalized distribution amplitudes (GDA) which parametrize the quark-antiquark to hadron transition. Since the $\pi^+\pi^-$ system is not a charge parity eigenstate, the GDA includes two charge parity components and allows for a study of the corresponding interference term. The relevant GDA is here just given by the light cone wave function of the two pion system [19].

II. KINEMATICS

Let us first specify the kinematics of the process under study, namely the photon-photon scattering as stated in Eq. (1). We introduce a Sudakov representation of all

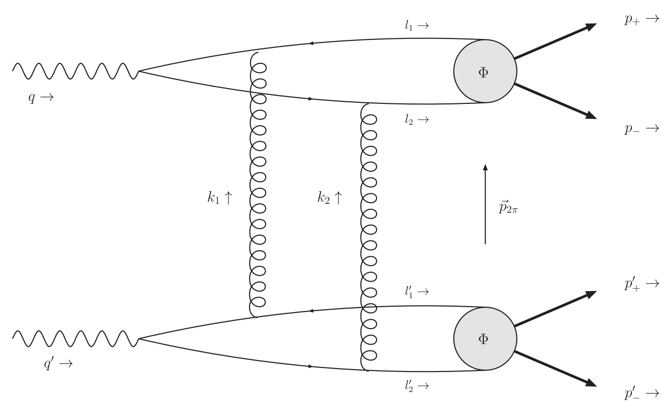

FIG. 1. Kinematics of the reaction $\gamma\gamma \rightarrow \pi^+\pi^-\pi^+\pi^-$ in a sample Feynman diagram of the two gluon exchange process.

particle momenta using the Sudakov lightlike momenta p_1, p_2. The initial photon momenta are written as

$$q^\mu = p_1^\mu, \qquad q'^\mu = p_2^\mu, \tag{2}$$

where $s = 2p_1 \cdot p_2$. The momenta of the two pion systems are given by

$$\begin{aligned} p_{2\pi}^\mu &= \left(1 - \frac{\vec{p}_{2\pi}^{\,2}}{s}\right) p_1^\mu + \frac{m_{2\pi}^2 + \vec{p}_{2\pi}^{\,2}}{s} p_2^\mu + p_{2\pi\perp}^\mu, \\ p_{2\pi\perp}^2 &= -\vec{p}_{2\pi}^{\,2}, \\ p_{2\pi}'^\mu &= \left(1 - \frac{\vec{p}_{2\pi}'^{\,2}}{s}\right) p_2^\mu + \frac{m_{2\pi}'^2 + \vec{p}_{2\pi}'^{\,2}}{s} p_1^\mu + p_{2\pi\perp}'^\mu, \\ p_{2\pi\perp}'^2 &= -\vec{p}_{2\pi}'^{\,2}. \end{aligned} \tag{3}$$

The (massless) quark momentum l_1 and antiquark momentum l_2 inside the upper loop before the formation of the two pion system are parametrized as

$$l_1^\mu = z p_1^\mu + \frac{(\vec{l} + z\vec{p}_{2\pi})^2}{zs} p_2^\mu + (l_\perp + z p_{2\pi\perp})^\mu, \tag{4}$$

$$l_2^\mu = \bar{z} p_1^\mu + \frac{(-\vec{l} + \bar{z}\vec{p}_{2\pi})^2}{\bar{z}s} p_2^\mu + (-l_\perp + \bar{z} p_{2\pi\perp})^\mu, \tag{5}$$

where $2\vec{l}$ is the relative transverse momentum of the quarks forming the two pion system and $\bar{z} = 1 - z$, up to small corrections of the order $\vec{p}_{2\pi}^{\,2}/s$. Following the collinear approximation of the factorization procedure in the description of the two pion formation through the generalized distribution amplitude, we put $\vec{l} = \vec{0}$ in the hard amplitude.

In a similar way as in (4) and (5) we parametrize the momenta of the produced pions as

$$p_+^\mu = \zeta p_1^\mu + \frac{m_\pi^2 + (\vec{p} + \zeta\vec{p}_{2\pi})^2}{\zeta s} p_2^\mu + (p_\perp + \zeta p_{2\pi\perp})^\mu, \tag{6}$$

$$p_-^\mu = \bar{\zeta} p_1^\mu + \frac{m_\pi^2 + (-\vec{p} + \bar{\zeta}\vec{p}_{2\pi})^2}{\bar{\zeta}s} p_2^\mu + (-p_\perp + \bar{\zeta} p_{2\pi\perp})^\mu, \tag{7}$$

where $2\vec{p}$ is now their relative transverse momentum, $\zeta = \frac{p_2 \cdot p_+}{p_2 \cdot p_{2\pi}}$ is the fraction of the longitudinal momentum $p_{2\pi}$ carried by the produced π^+, and $\bar{\zeta} = 1 - \zeta$. The variable ζ is related to the polar angle θ which is defined in the rest frame of the pion pair by

$$\beta \cos\theta = 2\zeta - 1, \qquad \beta \equiv \sqrt{1 - \frac{4m_\pi^2}{m_{2\pi}^2}}. \tag{8}$$

Since the "longitudinal part" of the two pion wave function depends only on the angle θ and does not depend on the azimuthal decay angle ϕ (in the same rest frame of the pair), we focus on the calculation of charge asymmetries expressed in terms of θ (see below). Similar expressions as Eqs. (4)–(8) are used for the lower quark loop. Since we are interested in photon interactions in ultraperipheral collisions of hadrons, the photons are quasireal and hence predominantly transversely polarized. The polarization vectors of the photons are written as

$$\vec{\varepsilon}(+) = -\frac{1}{\sqrt{2}}(1, i), \qquad \vec{\varepsilon}(-) = \frac{1}{\sqrt{2}}(1, -i). \tag{9}$$

We will consider spin averaged cross sections since hadron colliders do not produce polarized photon beams.

III. SCATTERING AMPLITUDES

It is well known that at high energies ($s \gg |t|$) the amplitude factorizes into impact factors convoluted over the two-dimensional transverse momenta of the t-channel gluons.

For the Pomeron exchange, which corresponds in the Born approximation of QCD to the exchange of two gluons in a color singlet state, see Fig. 1, the impact representation has the form

$$\mathcal{M}_{\mathbb{P}} = -is \int \frac{d^2\vec{k}_1 d^2\vec{k}_2 \delta^{(2)}(\vec{k}_1 + \vec{k}_2 - \vec{p}_{2\pi})}{(2\pi)^2 \vec{k}_1^{\,2} \vec{k}_2^{\,2}} J_{\mathbb{P}}^{\gamma_T}(\vec{k}_1, \vec{k}_2) \times J_{\mathbb{P}}^{\gamma_T}(\vec{k}_1, \vec{k}_2), \tag{10}$$

where $J_{\mathbb{P}}^{\gamma_T}(\vec{k}_1, \vec{k}_2)$ is the impact factor for the transition $\gamma_T \to \pi^+\pi^-$ via Pomeron exchange.

The corresponding representation for the Odderon exchange, i.e. the exchange of three gluons in a color singlet state, is given by the formula

$$\mathcal{M}_{\mathbb{O}} = -\frac{8\pi^2 s}{3!} \int \frac{d^2\vec{k}_1 d^2\vec{k}_2 d^2\vec{k}_3 \delta^{(2)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 - \vec{p}_{2\pi})}{(2\pi)^6 \vec{k}_1^{\,2} \vec{k}_2^{\,2} \vec{k}_3^{\,2}} \times J_{\mathbb{O}}^{\gamma_T}(\vec{k}_1, \vec{k}_2, \vec{k}_3) J_{\mathbb{O}}^{\gamma_T}(\vec{k}_1, \vec{k}_2, \vec{k}_3), \tag{11}$$

where $J_{\mathbb{O}}^{\gamma_T}(\vec{k}_1, \vec{k}_2, \vec{k}_3)$ is the impact factor for the transition $\gamma_T \to \pi^+\pi^-$ via Odderon exchange.

The impact factors are calculated by the use of standard methods, see e.g. Ref. [20] and references therein.

A. Impact factors for $\gamma \to \pi^+\pi^-$

The leading order calculation in pQCD of the impact factors gives

$$J_{\mathbb{P}}^{\gamma_T}(\vec{k}_1, \vec{k}_2) = -\frac{ieg^2\delta^{ab}}{4N_C} \int_0^1 dz (z - \bar{z}) \times \vec{\varepsilon}(T) \vec{Q}_{\mathbb{P}}(\vec{k}_1, \vec{k}_2) \Phi^{I=1}(z, \zeta, m_{2\pi}^2), \tag{12}$$

where the vector $\vec{Q}_{\mathbb{P}}(\vec{k}_1, \vec{k}_2)$ is defined by

$$\vec{Q}_{\mathbb{P}}(\vec{k}_1, \vec{k}_2) = \frac{z\vec{p}_{2\pi}}{z^2\vec{p}_{2\pi}^2 + \mu^2} - \frac{\bar{z}\vec{p}_{2\pi}}{\bar{z}^2\vec{p}_{2\pi}^2 + \mu^2} + \frac{\vec{k}_1 - z\vec{p}_{2\pi}}{(\vec{k}_1 - z\vec{p}_{2\pi})^2 + \mu^2} - \frac{\vec{k}_1 - \bar{z}\vec{p}_{2\pi}}{(k_1 - \bar{z}\vec{p}_{2\pi})^2 + \mu^2}. \tag{13}$$

The calculation of the Odderon exchange contribution gives

$$J_{\mathbb{O}}^{\gamma_T}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = -\frac{ieg^3 d^{abc}}{8N_C}\int_0^1 dz(z-\bar{z}) \times \vec{\varepsilon}(T)\vec{Q}_{\mathbb{O}}(\vec{k}_1, \vec{k}_2, \vec{k}_3)\frac{1}{3}\Phi^{I=0}(z, \zeta, m_{2\pi}^2), \tag{14}$$

where we have used the definition

$$\vec{Q}_{\mathbb{O}}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{z\vec{p}_{2\pi}}{z^2\vec{p}_{2\pi}^2 + \mu^2} + \frac{\bar{z}\vec{p}_{2\pi}}{\bar{z}^2\vec{p}_{2\pi}^2 + \mu^2} + \sum_{i=1}^{3}\left(\frac{\vec{k}_i - z\vec{p}_{2\pi}}{(\vec{k}_i - z\vec{p}_{2\pi})^2 + \mu^2} + \frac{\vec{k}_i - \bar{z}\vec{p}_{2\pi}}{(\vec{k}_i - \bar{z}\vec{p}_{2\pi})^2 + \mu^2}\right). \tag{15}$$

B. Generalized two pion distribution amplitudes

A crucial point of the present study is the choice of an appropriate two pion distribution amplitude (GDA) [12–15,19,21–23] which includes the full strong interaction related to the production of the two pion system.

Based on an expansion of the GDA in Gegenbauer polynomials $C_n^m(2z-1)$ and in Legendre polynomials $P_l(2\zeta - 1)$ [24,25], it is believed that only the first terms give a significant contribution:

$$\Phi^{I=1}(z, \zeta, m_{2\pi}) = 6z\bar{z}\beta f_1(m_{2\pi})\cos\theta, \tag{16}$$

$$\Phi^{I=0}(z, \zeta, m_{2\pi}) = 5z\bar{z}(z-\bar{z})\left[-\frac{3-\beta^2}{2}f_0(m_{2\pi}) + \beta^2 f_2(m_{2\pi})P_2(\cos\theta)\right], \tag{17}$$

where $f_1(m_{2\pi})$ can be identified with the electromagnetic pion form factor $F_\pi(m_{2\pi})$. For our calculation we use the following F_π parametrization inspired by Ref. [26]:

$$f_1(m_{2\pi}) = F_\pi(m_{2\pi}) = \frac{1}{1+b}BW_\rho(m_{2\pi}^2)\frac{1 + aBW_\omega(m_{2\pi}^2)}{1+a}, \tag{18}$$

with

$$BW_\rho(m_{2\pi}^2) = \frac{m_\rho^2}{m_\rho^2 - m_{2\pi}^2 - im_{2\pi}\Gamma_\rho(m_{2\pi}^2)} \tag{19}$$

$$\Gamma_\rho(m_{2\pi}^2) = \Gamma_\rho\frac{m_\rho^2}{m_{2\pi}^2}\left(\frac{m_{2\pi}^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2} \tag{20}$$

$$BW_\omega(m_{2\pi}^2) = \frac{m_\omega^2}{m_\omega^2 - m_{2\pi}^2 - im_\omega\Gamma_\omega}. \tag{21}$$

As masses and widths we use $m_\rho = 775.49$ MeV, $\Gamma_\rho = 146.2$ MeV, $m_\omega = 782.65$ MeV, and $\Gamma_\omega = 8.49$ MeV [27]. We fit the remaining free parameters to the data compiled in Ref. [28] obtaining $a = 1.78 \times 10^{-3}$ and $b = -0.154$. Including a hypothetical ρ' resonance as originally used in Ref. [26] gives a significant better fit to the data at large $m_{2\pi}$ but has only a small effect on the asymmetry which is the main object of our studies.

For the $I = 0$ component we use different models. The first model follows Ref. [12] and reads

$$f_{0/2}(m_{2\pi}) = e^{i\delta_{0/2}(m_{2\pi})}|BW_{f_{0/2}}(m_{2\pi}^2)|. \tag{22}$$

The phase shifts $\delta_{0/2}$ are those from the elastic $\pi^+\pi^-$ scattering, for which we use the parametrization of Ref. [29] below and that of Ref. [30] above the $K\bar{K}$ threshold.

$|BW_{f_{0/2}}(m_{2\pi}^2)|$ is the modulus of the Breit-Wigner amplitudes,

$$BW_{f_{0/2}}(m_{2\pi}^2) = \frac{m_{f_{0/2}}^2}{m_{f_{0/2}}^2 - m_{2\pi}^2 - im_{f_{0/2}}\Gamma_{f_{0/2}}}, \tag{23}$$

with $m_{f_0} = 980$ MeV, $\Gamma_{f_0} = 40$–100 MeV, $m_{f_2} = 1275.1$ MeV, and $\Gamma_{f_2} = 185$ MeV [27].

In the second model—elaborated in Ref. [23]—the functions $f_{0/2}$ are the corresponding Omnès functions for S and D waves constructed by dispersion relations from the phase shifts of the elastic pion scattering:

$$f_l(m_{2\pi}) = \exp\left(\pi I_l + \frac{m_{2\pi}^2}{\pi}\int_{4m_\pi^2}^\infty ds\frac{\delta_l(s)}{s^2(s - m_{2\pi}^2 - i\varepsilon)}\right), \quad \text{with } I_l = \frac{1}{\pi}\int_{4m_\pi^2}^\infty ds\frac{\delta_l(s)}{s^2}. \tag{24}$$

The assumption that the phases of the GDA equal those of the elastic scattering loses its solid base beyond the $K\bar{K}$ threshold. As discussed in Refs. [23,31] it might well be that the actual phases of the GDA are closer to the phases of the corresponding $\mathcal{T}$ matrix elements $(\eta_l e^{2i\delta_l} - 1)/(2i)$. The third model for the $I = 0$ component of the GDA takes this into account by using the technique of model 2 with these phases $\delta_{\mathcal{T},l}$ of the $\mathcal{T}$ matrix elements.

While the first and the second model give quite compatible results, model 3 differs from them significantly. The most striking difference is the absence of pronounced f_0

resonance effects in model 3. In fact, measurements at HERMES [32] do not observe a resonance effect at the f_0 mass even though a confirmation by an independent experiment would be desirable. From the same measurements at HERMES, one can draw the conclusion that using $\delta_{T,2}$ and δ_2 for the f_2 region are both compatible with data [23]. Having this in mind, we consider also a fourth model—a mixed description with the f_0 contribution from model 3 and the f_2 contribution from model 2.

C. Photon exchange amplitude

The photon has the same C parity as the Odderon. Therefore, its exchange between the two quark-antiquark systems can mimic an Odderon exchange. The according amplitude is straightforward to calculate and reads

$$\mathcal{M}_\gamma = \frac{s}{2t} J_\gamma^{\gamma_T} J_\gamma^{\gamma_T}, \tag{25}$$

with

$$J_\gamma^{\gamma_T} = \frac{e^2}{2}\int_0^1 dz(z-\bar{z})\vec{\varepsilon}(T)\vec{p}_{2\pi}\left(\frac{z}{\mu^2 + z^2\vec{p}_{2\pi}^2} + \frac{\bar{z}}{\mu^2+\bar{z}^2\vec{p}_{2\pi}^2}\right)\Phi^{I=0}(z,\zeta,m_{2\pi}^2). \tag{26}$$

In our concrete process the photon exchange amplitude amounts to the order of 10% of the Odderon exchange amplitude. Although we do not neglect this contribution, it is clear that the asymmetry described in the following section is driven by the Odderon/Pomeron interference.

IV. CHARGE ASYMMETRIES AND RATES

In the following we make use of the fact that the GDAs for C-even and for C-odd pion pairs are orthogonal to each other in the space of Legendre polynomials in $\cos\theta$. As a consequence, in the total cross section the interference term completely vanishes. In contrast only the interference term survives, when the amplitude squared is multiplied by $\cos\theta$ before the angular integration which corresponds to selecting the charge asymmetric contribution. The asymmetry we are interested in is defined as

$$A(t, m_{2\pi}^2, m_{2\pi}'^2) = \frac{\int \cos\theta\cos\theta' d\sigma(t, m_{2\pi}^2, m_{2\pi}'^2, \theta, \theta')}{\int d\sigma(t, m_{2\pi}^2, m_{2\pi}'^2, \theta, \theta')} = \frac{\int_{-1}^1 d\cos\theta \int_{-1}^1 d\cos\theta' 2\cos\theta\cos\theta' \mathrm{Re}[\mathcal{M}_{\mathbb{P}}(\mathcal{M}_{\mathbb{O}} + \mathcal{M}_\gamma)^*]}{\int_{-1}^1 d\cos\theta \int_{-1}^1 d\cos\theta' [|\mathcal{M}_{\mathbb{P}}|^2 + |\mathcal{M}_{\mathbb{O}} + \mathcal{M}_\gamma|^2]}. \tag{27}$$

The obtained landscape as a function of the two invariant masses is not particularly illuminating and would be difficult to measure. To reduce the complexity, we integrate over the invariant mass of one of the two pion systems to obtain

$$\begin{aligned}\hat{A}(t, m_{2\pi}^2; m_{\min}^2, m_{\max}^2) &= \frac{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int \cos\theta\cos\theta' d\sigma(t, m_{2\pi}^2, m_{2\pi}'^2, \theta, \theta')}{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int d\sigma(t, m_{2\pi}^2, m_{2\pi}'^2, \theta, \theta')} \\ &= \frac{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int_{-1}^1 d\cos\theta \int_{-1}^1 d\cos\theta' 2\cos\theta\cos\theta' \mathrm{Re}[\mathcal{M}_{\mathbb{P}}(\mathcal{M}_{\mathbb{O}} + \mathcal{M}_\gamma)^*]}{\int_{m_{\min}^2}^{m_{\max}^2} dm_{2\pi}'^2 \int_{-1}^1 d\cos\theta \int_{-1}^1 d\cos\theta' [|\mathcal{M}_{\mathbb{P}}|^2 + |\mathcal{M}_{\mathbb{O}} + \mathcal{M}_\gamma|^2]}.\end{aligned} \tag{28}$$

Let us note that, since two pion pairs are always in the same C-parity state, because of $C = +$ parity of the initial $\gamma\gamma$ state, it is necessary to keep in Eq. (28) the integration weight $\cos\theta'$. Without this weight the single charge asymmetry would vanish. The deviations from this vanishing of the asymmetry may serve as a measure of experimental uncertainties.

Considering an analytic calculation of the matrix element in Eq. (11), it turns out that it would require the calculation of two-dimensional two-loop box diagrams, whose off-shellness for all external legs is different. The techniques developed in Refs. [33–35] cannot be applied here due to the very elaborated topology of the most complicated diagrams involved (square box with one additional diagonal line). Instead we rely on a numerical evaluation by Monte Carlo methods. In particular, we make use of a modified version of VEGAS as it is provided by the CUBA library [36]. The result for the asymmetry $\hat{A}$ at $t = -1$ GeV^2 (respectively $t = -2$ GeV^2) is shown in Fig. 2 (respectively Fig. 3). Since our framework is only justified for $m_{2\pi}^2 < -t$ (in fact strictly speaking, one even needs $m_{2\pi}^2 \ll -t$), we keep $m_{2\pi}$ below 1 GeV (respectively 1.4 GeV). In each case, we present the expected asymmetry with the GDAs parametrized as discussed above.

In order to evaluate the feasibility of the Odderon search, we need to supplement the calculation of the asymmetry with rate estimates in ultraperipheral collisions at hadron colliders. This rate depends on the Pomeron dominated photon-photon cross section and on the equivalent photon flux. The total photon-photon cross section falls off rapidly with increasing $|t|$ (see Fig. 4). Therefore, the integration mainly depends on the lower limit of $|t|$ inte-

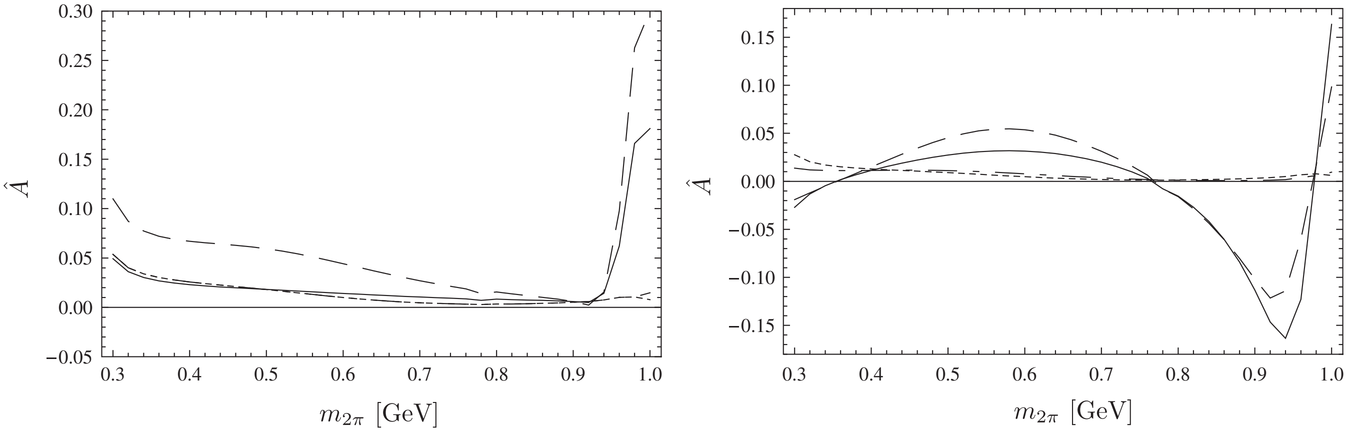

FIG. 2. Asymmetry $\hat{A}$ at $t = -1\ \mathrm{GeV}^2$ for model 1 (solid), 2 (dashed), 3 (dotted), and 4 (dash-dotted)—model 3 and 4 are nearly on top of each other. The left part has $m_{\min} = 0.3$ GeV and $m_{\max} = m_\rho$, while the right part has $m_{\min} = m_\rho$ and $m_{\max} = 1$ GeV.

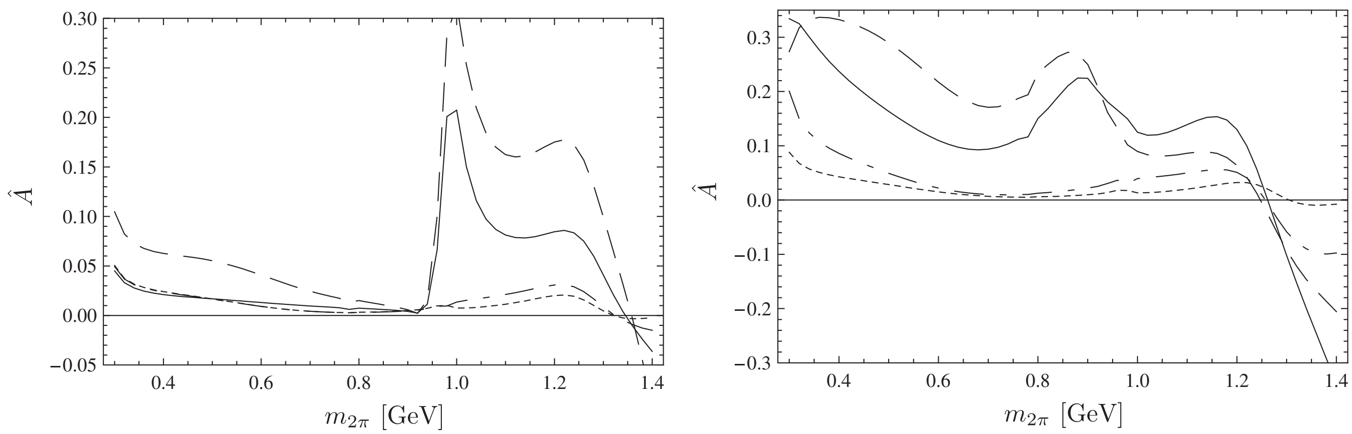

FIG. 3. Asymmetry $\hat{A}$ at $t = -2\ \mathrm{GeV}^2$ for model 1 (solid), 2 (dashed), 3 (dotted), and 4 (dash-dotted). The left part has $m_{\min} = 0.3$ GeV and $m_{\max} = m_\rho$, while the right part has $m_{\min} = m_{f_0}$ and $m_{\max} = 1.4$ GeV.

gration ($t_{\min}$). Already for $t_{\min} = -1\ \mathrm{GeV}^2$ we find $\sigma_{\gamma\gamma} = 1.1$ pb.

Although in Ref. [16] it is claimed that the photon flux is best for medium-weight ions, and especially superior to that of protons, this is in fact not true. To obtain Fig. 6 of Ref. [16], the effective $\gamma\gamma$ luminosity has been calculated for protons and ions by the Monte Carlo program TPHIC [37] which is based on the Weizsäcker-Williams method [38,39] with the additional condition of nonoverlapping ions [40,41], but the authors used a quite small luminosity for proton-proton collisions at the LHC ($14\,000\ \mathrm{mb}^{-1}\,\mathrm{s}^{-1}$ instead of the official design luminosity $10^7\ \mathrm{mb}^{-1}\,\mathrm{s}^{-1}$ [42,43]). Very unfortunately, the identical figure is reprinted in Ref. [17] while a nonconsistent p-p luminosity of $10^7\ \mathrm{mb}^{-1}\,\mathrm{s}^{-1}$ is cited.

As already was shown by Cahn and Jackson [41], the $\gamma\gamma$ luminosity in the case of ions can be expressed in terms of a universal function $\xi(z)$, where $z = MR \approx 5.665A^{4/3}$ with M being the mass and R the radius of the ion, and a prefactor proportional to Z^4. Since the luminosity for ions at the LHC decreases roughly as Z^{-4}, the prefactor's Z dependence is more or less compensated and only the universal function ξ remains which is exponentially decreasing with z. Hence, lighter projectiles provide a larger

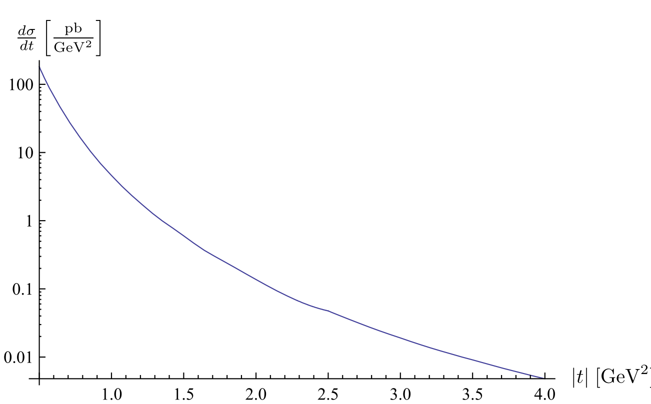

FIG. 4 (color online). t dependence of photon-photon cross section.

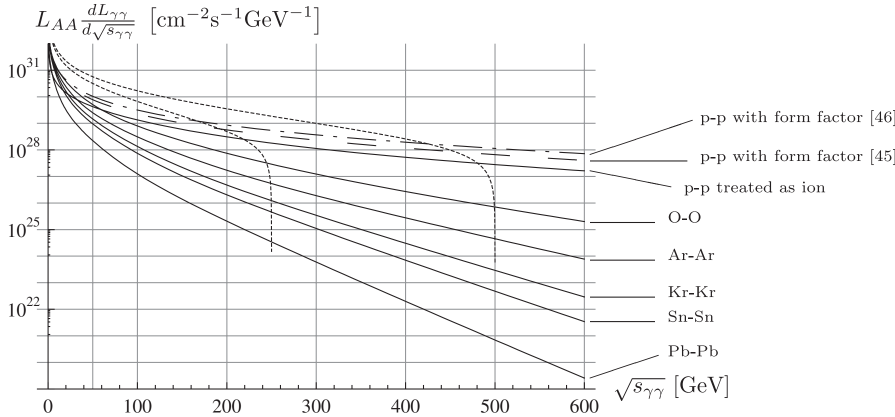

FIG. 5. Effective $\gamma\gamma$ luminosities for the collision of p-p based on Ref. [45] (dash-dotted) and Ref. [46] (dashed). The results using the parametrization of Ref. [41] for ions are given by solid lines for p-p, O^8_{16}-O^8_{16}, Ar^{18}_{40}-Ar^{18}_{40}, Kr^{36}_{84}-Kr^{36}_{84}, Sn^{50}_{120}-Sn^{50}_{120}, and Pb^{82}_{208}-Pb^{82}_{208} from top to bottom. For ions we used the average luminosities as given in Ref. [44], for proton we used $L_{pp} = 10^{34}$ cm^{-2} s^{-1}. For comparison also effective $\gamma\gamma$ luminosities at the ILC are given for $\sqrt{s_{e^+e^-}} = 250$ GeV and $\sqrt{s_{e^+e^-}} = 500$ GeV (both as dotted lines).

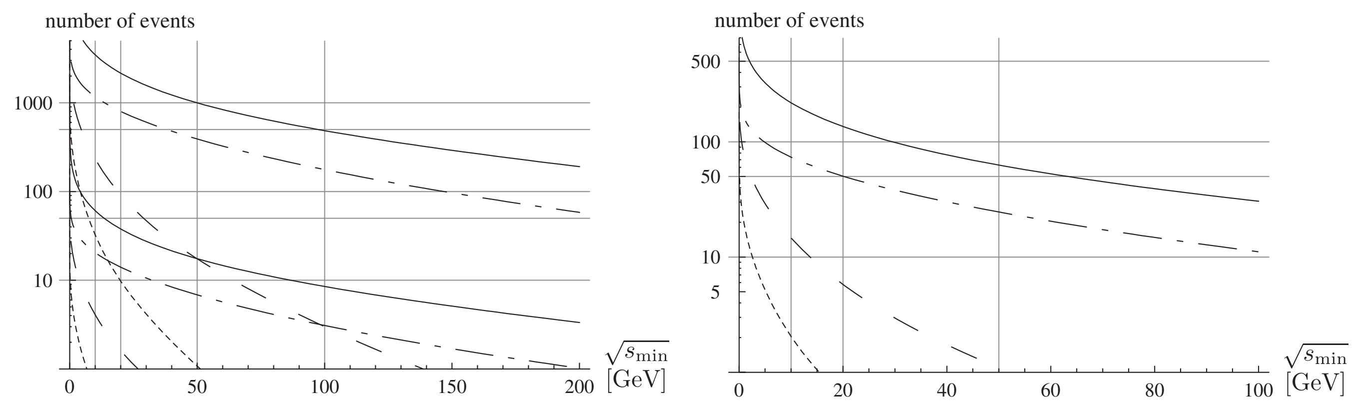

FIG. 6. Rate of production of two pion pairs in ultraperipheral collisions in dependence on the lower cut $s_{\min}$ given in "events per month" in the case of ions, and "events per six months" in the case of protons which in both cases correspond to one year of running of LHC. The left-hand side plot shows the rate for $t_{\min} = -1$ GeV^2, the right-hand side that for $t_{\min} = -2$ GeV^2. The solid line displays the result for p-p collision using Ref. [45], the dash-dotted that for protons treated as heavy ions, the dashed one that for Ar-Ar collisions, and the dotted line that for Pb-Pb collisions. On the left figure, also the much smaller rates coming from the Odderon exchange are shown (with the same dashing).

effective $\gamma\gamma$ luminosity, with the protons offering the highest luminosity.

Therefore, we provide a corrected overview over the various effective $\gamma\gamma$ luminosities in Fig. 5. For the different ion scenarios that are discussed in Ref. [44], we use the parametrization of Ref. [41] which relies on the Weizsäcker-Williams method [38,39] with the additional condition of nonoverlapping ions [40,41]. For protons usually a calculation based on the proton electric dipole form factor $F_E(Q^2) = 1/(1 + \frac{Q^2}{0.71\ \mathrm{GeV}^2})$ in combination with the Weizsäcker-Williams method is used, as it can be found in Ref. [45]. In Ref. [46] a slightly improved version is given, which lowers the photon flux. An inclusion of the corresponding magnetic dipole moment [47] would lead to a flux between those both. For this reason, we use the formulas given in Refs. [45,46] for the case of proton-proton collisions. We also provide the results for the proton treated as a heavy ion because it might be that the nonoverlap condition—reducing the flux—is of importance [46], even if such a procedure does not include the proton form factor. We consider such a lower result as a conservative lower estimate. Since the luminosity factors will cancel in the asymmetry, this uncertainty does not affect our conclusions on the Odderon effects. For comparison, we provide also the effective $\gamma\gamma$ luminosities at the intended ILC where the design luminosity for e^+e^- collisions is $2 \cdot 10^{34}$ cm^{-2} s^{-1} [48].[1]

As shown in Fig. 5, the effective $\gamma\gamma$ luminosity decreases rapidly with increasing energy. Since our inter-

[1]In the calculation of the equivalent photon spectrum [49] we used $q_{\max} = 100$ GeV.

mediate hard scale t is quite small, we are not forced to consider extremely large photon-photon energies. The only condition on the minimal photon-photon energy ($s_{\min}$) is $s_{\min} \gg |t|$ to ensure the validity of high energy factorization into two separated pion systems. Hence, a $s_{\min}$ of 400 GeV^2 could be already enough. The effect of varying $s_{\min}$ is displayed in Fig. 6.

For p-p collisions, the rates are high and even for $t_{\min} = -2\ \mathrm{GeV}^2$ sizable. Although heavy ions would offer the possibility to trigger on ultraperipheral collisions by detecting neutrons from giant dipole resonances in the zero degree calorimeters, the rates that can be read off from Fig. 6 are rather low. Only for medium-weight ions there might be the possibility to measure the process.

In hadron-hadron collisions the process of interest could as well be connected by Pomerons to the colliding hadrons. In that case one would have to deal with the unknown hadron-Pomeron and Pomeron-Pomeron-two-pion couplings. Therefore, we consider that process as background. This circumstance would make heavy ions preferable because of the different scaling of Pomeron and photon coupling when changing from proton to nuclei scattering. However, experimentally such a background can be suppressed by refusing events with a total p_T larger than some small cutoff. Indeed, the $\gamma\gamma$ events dominate, due to the photon propagator singularities, when each of the transverse momentum is small, which on the average is satisfied when the *total* transverse momentum is imposed to be small.

Inclusion of high energy evolution of the two gluon exchange *à la* Balitksy-Fadin-Kuraev-Lipatov [50–55] would increase the total cross section but at the same time diminish the asymmetry since the Odderon including Bartels-Kwieciński-Praszałowicz evolution [56,57] has a smaller intercept. The study of these effects goes beyond the goal of this paper. Since we demonstrated that the rate and asymmetry were sizable for moderate values of the $\gamma\gamma$ energy, we do not expect evolution effects to play a major role in the proposed strategy to uncover Odderon effects.

V. CONCLUSION

We have investigated in real photon-photon collision the production of two $\pi^+\pi^-$ pairs well separated in rapidity. Because of the nonfixed C parity of these pairs, beside a Pomeron exchange an Odderon exchange is possible as well. We have calculated both contributions in a perturbative approach—justified by t providing the hard scale—where the only soft building blocks needed are the GDAs of the pion pairs. We have shown that a charge asymmetry in the polar angle θ (defined in the rest frame of the pion pairs) is linearly dependent on the Odderon amplitude and moreover is sizable but GDA-model dependent. In fact, the predicted asymmetry depends much of the two pion GDAs, which are still not really known although HERMES measurements of two pion electroproduction [32] disfavor models with a strong f_0 coupling to the $\pi^+\pi^-$ state. We however think that higher statistics data, which may come from a JLab experiment at 6 or 12 GeV, are definitely needed before one can trust a definite model of the GDAs. Because two pion deep electroproduction in the low energy domain is dominated by quark exchanges, this test of the GDA models is independent of any Odderon search. This looks like a prerequisite to a trustable extraction of the Odderon signal—or of an upper bound on Odderon exchange amplitudes—from ultraperipheral collisions.

We have discussed the possibility to measure this process in ultraperipheral collisions at LHC. Although in ion-ion collision it is easier to trigger on ultraperipheral collision with a lower background from diffractive events from strong interactions, the event rates are too low. In proton-proton collision the rates are high enough to measure the asymmetry even though isolating ultraperipheral collisions in proton-proton collisions may be a challenge for experimentalists.

Let us finally note that the background from strong interactions would be completely absent in an e^+e^- collider which via Compton-backscattering could work as a very effective $\gamma\gamma$ collider. At the ILC [48] for a nominal electron beam energy of 250 GeV, the luminosity for photon-photon collisions would be $3.5 \cdot \times 10^{33}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ [58] for the high energy photons (energy fraction at least 80% of the maximal possible energy fraction) or even higher, if optimized for photon-photon collisions [59]. As Fig. 5 reveals, even running in the electron-positron mode the effective $\gamma\gamma$ luminosity is slightly larger than at the LHC. For these reasons, the ILC would provide an ideal environment to study the process of interest.

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[1] L. Łukaszuk and B. Nicolescu, Nuovo Cimento Lett. **8**, 405 (1973).
[2] H. G. Dosch, C. Ewerz, and V. Schatz, Eur. Phys. J. C **24**, 561 (2002).
[3] A. Breakstone *et al.*, Phys. Rev. Lett. **54**, 2180 (1985).
[4] J. Olsson (H1 Collaboration), in *Yalta 2001, New Trends in High-Energy Physics*, edited by G. V. Bugrij (Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine, 2001), p. 79.
[5] A. Bzdak, L. Motyka, L. Szymanowski, and J. R. Cudell, Phys. Rev. D **75**, 094023 (2007).
[6] L. Szymanowski (2007), in *Hamburg 2007, Blois07, Forward Physics and QCD*, edited by J. Bartels *et al.* (DESY and Univ. Hamburg, Hamburg, Germany, 2007), p. 254.
[7] L. Szymanowski, in Yalta 2001, New Trends in High-Energy Physics, Ref. [4], p. 123.
[8] S. Braunewell and C. Ewerz, Phys. Rev. D **70**, 014021 (2004).
[9] S. J. Brodsky, J. Rathsman, and C. Merino, Phys. Lett. B **461**, 114 (1999).
[10] I. F. Ginzburg, I. P. Ivanov, and N. N. Nikolaev, Eur. Phys. J. direct C **5**, 02 (2003).
[11] I. F. Ginzburg, I. P. Ivanov, and N. N. Nikolaev, Eur. Phys. J. C **32S1**, 23 (2003).
[12] P. Hägler, B. Pire, L. Szymanowski, and O. V. Teryaev, Phys. Lett. B **535**, 117 (2002).
[13] P. Hägler, B. Pire, L. Szymanowski, and O. V. Teryaev, Nucl. Phys. **A711**, 232 (2002).
[14] P. Hägler, B. Pire, L. Szymanowski, and O. V. Teryaev, Eur. Phys. J. C **26**, 261 (2002).
[15] P. Hägler, B. Pire, L. Szymanowski, and O. V. Teryaev, Nucl. Phys. B, Proc. Suppl. **121**, 155 (2003).
[16] G. Baur, K. Hencken, D. Trautmann, S. Sadovsky, and Y. Kharlov, Phys. Rep. **364**, 359 (2002).
[17] K. Hencken *et al.*, Phys. Rep. **458**, 1 (2008). With regard to $\gamma\gamma$ luminosities in p-p collisions, Fig. 3 should be modified to account for a realistic p-p luminosity, as we discuss in Sec. IV.
[18] R. Schicker (ALICE Collaboration), Nucl. Phys. B, Proc. Suppl. **179–180**, 196 (2008).
[19] M. Diehl, T. Gousset, and B. Pire, Phys. Rev. D **62**, 073014 (2000).
[20] I. F. Ginzburg and D. Y. Ivanov, Nucl. Phys. **B388**, 376 (1992).
[21] M. Diehl, T. Gousset, B. Pire, and O. Teryaev, Phys. Rev. Lett. **81**, 1782 (1998).
[22] D. Y. Ivanov, M. Diehl, A. Schaefer, and N. Warkentin, Proc. Sci. DIFF2006 (2006) 032 [arXiv:hep-ph/0611138].
[23] N. Warkentin, M. Diehl, D. Y. Ivanov, and A. Schafer, Eur. Phys. J. A **32**, 273 (2007).
[24] M. V. Polyakov, Nucl. Phys. **B555**, 231 (1999).
[25] N. Kivel, L. Mankiewicz, and M. V. Polyakov, Phys. Lett. B **467**, 263 (1999).
[26] J. H. Kuhn and A. Santamaria, Z. Phys. C **48**, 445 (1990).
[27] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).
[28] L. M. Barkov *et al.*, Nucl. Phys. **B256**, 365 (1985).
[29] J. R. Peláez and F. J. Ynduráin, Phys. Rev. D **71**, 074016 (2005).
[30] R. Kamiński, J. R. Peláez, and F. J. Ynduráin, Phys. Rev. D **74**, 014001 (2006).
[31] B. Ananthanarayan, I. Caprini, G. Colangelo, J. Gasser, and H. Leutwyler, Phys. Lett. B **602**, 218 (2004).
[32] A. Airapetian *et al.* (HERMES Collaboration), Phys. Lett. B **599**, 212 (2004).
[33] D. J. Broadhurst, Phys. Lett. B **307**, 132 (1993).
[34] B. Pire, L. Szymanowski, and S. Wallon, Eur. Phys. J. C **44**, 545 (2005).
[35] M. Segond, L. Szymanowski, and S. Wallon, Eur. Phys. J. C **52**, 93 (2007).
[36] T. Hahn, Comput. Phys. Commun. **168**, 78 (2005).
[37] K. Hencken, Y. V. Kharlov, G. V. Khaustov, S. A. Sadovsky, and V. D. Samoylenko, Report No. IFVE-96-38, 1996.
[38] C. F. von Weizsäcker, Z. Phys. **88**, 612 (1934).
[39] E. J. Williams, Phys. Rev. **45**, 729 (1934).
[40] G. Baur and L. G. Ferreira Filho, Nucl. Phys. **A518**, 786 (1990).
[41] R. N. Cahn and J. D. Jackson, Phys. Rev. D **42**, 3690 (1990).
[42] J. Bosser *et al.*, CERN Report No. CERN-LHC-PROJECT-REPORT-370, 2000.
[43] O. Brüning *et al.*, CERN Report No. CERN-2004-003-V-1, 2004.
[44] D. Brandt, CERN Report No. CERN-LHC-PROJECT-REPORT-450, 2000.
[45] M. Drees and D. Zeppenfeld, Phys. Rev. D **39**, 2536 (1989).
[46] J. Nystrand, Nucl. Phys. **A752**, 470 (2005).
[47] B. A. Kniehl, Phys. Lett. B **254**, 267 (1991).
[48] J. Brau *et al.*, Report No. ILC-REPORT-2007-001, 2007.
[49] V. M. Budnev, I. F. Ginzburg, G. V. Meledin, and V. G. Serbo, Phys. Rep. **15**, 181 (1975).
[50] V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. **60B**, 50 (1975).
[51] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **44**, 443 (1976).
[52] E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977).
[53] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).
[54] V. S. Fadin and L. N. Lipatov, Phys. Lett. B **429**, 127 (1998).
[55] M. Ciafaloni and G. Camici, Phys. Lett. B **430**, 349 (1998).
[56] J. Bartels, Nucl. Phys. **B175**, 365 (1980).
[57] J. Kwieciński and M. Praszałowicz, Phys. Lett. **94B**, 413 (1980).
[58] V. I. Telnov, Physics Options at the ILC. GG6 Summary at Snowmass2005, econf C0508141, PLEN0020 (2005).
[59] V. I. Telnov, Acta Phys. Pol. B **37**, 1049 (2006).