TeV scale Lee-Wick fields out of large extra dimensional gravity

Feng Wu[1,*] and Ming Zhong[2,3,+]

[1]*Department of Physics, Nanchang University, 330031, China*
[2]*Department of Physics, National University of Defense Technology, Hunan 410073, China*
[3]*Kavli Institute for Theoretical Physics China, CAS, Beijing 100190, China*

(Received 12 July 2008; published 13 October 2008)

We study the gravitational corrections to the Maxwell, Dirac, and Klein-Gordon theories in the large extra dimension model in which the gravitons propagate in the $(4 + n)$-dimensional bulk, while the gauge and matter fields are confined to the four-dimensional world. The corrections to the two-point Green's functions of the gauge and matter fields from the exchanges of virtual Kaluza-Klein gravitons are calculated in the gauge independent background field method. In the framework of effective field theory, we show that the modified one-loop renormalizable Lagrangian due to quantum gravitational effects contains a TeV scale Lee-Wick partner of every gauge and matter field as extra degrees of freedom in the theory. Thus the large extra dimension model of gravity provides a natural mechanism for the emergence of these exotic particles which were recently used to construct an extension of the standard model.

DOI: 10.1103/PhysRevD.78.085010 PACS numbers: 12.60.Cn, 04.50.Cd, 04.62.+v, 11.15.Bt

I. INTRODUCTION

In constructing a finite version of quantum electrodynamics [1,2], Lee and Wick introduced a physical massive vector field to play the role of the regulator in Pauli-Villars regularization, and made the mass, charge, and wave function renormalizations all finite. The Lee-Wick QED (LW QED) is Poincaré invariant, gauge invariant, and unitary. Recently, Grinstein, O'Connell, and Wise [3] extended this idea to the standard model (SM) and proposed that a higher derivative term is added for each field in the SM. In this scenario each propagator includes an extra degree of freedom corresponding to a massive LW particle. The extended SM is shown to be free of quadratic divergences. Therefore the mass of the Higgs particle is stable and the hierarchy problem is solved. The Lee-Wick SM (LWSM) does not provide any information on the masses of Lee-Wick particles. However, they are expected to have masses at the TeV scale in various phenomenological studies. Extensive discussions of the phenomenology of the LWSM, including its implications for LHC and linear collider physics [4–6], neutrino physics [7], the flavor changing neutral currents [8], the electroweak precision constraints [9,10], and another Lee-Wick extension of the SM [11], have been made. Some theoretical works on the perturbative unitarity, the one-loop renormalization, and the causality problem of the LWSM have been discussed in [12–14], respectively. Chiral symmetry breaking and fermion mass generation triggered by a higher derivative term were shown in [15].

We studied the Maxwell-Einstein theory in [16]. We considered the effect of gravity by expanding the metric around the flat background spacetime and calculated the photon self-energy in the framework of the gauge independent background field method. We showed that the one-loop gravitational corrections induce a new higher derivative term with mass dimension six, which is the term needed in the LWSM. In a $(3 + 1)$-dimensional renormalizable theory, quantum corrections will generate possible UV divergences only to the relevant and marginal operators whose mass dimensions are less than 5. This assures the predictiveness of the theory. General relativity, the theory of gravitational interactions, on the other hand, is not renormalizable after quantization [17,18]. This is one of the reasons that general relativity was considered to be incompatible with quantum mechanics. In the quantized version of general relativity, one would not be able to reabsorb all the UV divergences into the coupling constant in the original Lagrangian. That is, new counterterms are needed at each order of perturbative calculations when trying to renormalize the theory. Nevertheless, a modern point of view is that a nonrenormalizable theory might be sensible and the reliable predictions could still be made from it within the framework of effective field theories [19,20]. From the value of the only dimensional coupling constant G, Newton's constant, in the Hilbert-Einstein Lagrangian, one can see that gravitational effects are tiny at energies $E \ll M_{\rm pl} \sim 10^{19}\ \mathrm{GeV}/c^2$. It makes sense to treat general relativity as a low energy effective field theory of some unknown fundamental theory and consider its quantum effects [21]. The effects due to nonrenormalizable terms are suppressed by inverse powers of the Planck scale $M_{\rm pl}$, the mass scale of new physics. In this sense, the higher derivative term permitted by symmetry should be added to the Lagrangian at the beginning so that the theory is one-loop renormalizable. The modified Maxwell-Einstein theory contains a Lee-Wick vector field as an extra degree of freedom. Since gravity does exist in nature, it provides a natural mechanism for the emergence of this exotic particle. Different from the other scenarios, all the Lee-Wick

*fengwu@ncu.edu.cn
+zhongm@nudt.edu.cn

fields generated in this way have their masses of order $M_{\rm pl}$, and thus presently escape from any experimental measurement.

In this work we will investigate the gauge-matter-gravity system where the gravitons propagate in $(4+n)$-dimensional spacetime while the gauge and matter fields live in the normal four-dimensional world. Because of phenomenological implications, we take the compactification scale $1/R$ of the n extra dimensions to be as low as 10^{-4} eV. The Planck scale $M_{{\rm pl}(4+n)}$ of the $(4+n)$-dimensional theory is taken to be of order 1 TeV, as is the case in the ADD model [22]. We will show, by explicit calculations in the background field method, that up to one-loop order, quantum corrections to each sector of the gauge, fermion, and scalar coming from the exchange of virtual Kaluza-Klein gravitons will generate a new type of divergence which corresponds to a higher derivative operator with mass dimension six. Summation of the Kaluza-Klein towers greatly improves the coefficients of these operators up to $\sim 1\ {\rm TeV}^{-2}$. Therefore, one needs to modify the Lagrangian of the system and include the higher derivative operators at the beginning to absorb the divergent quantum corrections. This modification is natural and consistent with the framework of effective field theories. The modified theory now contains a Lee-Wick partner whose mass is at the TeV scale for every gauge, fermion, and scalar particle. On the one hand, these Lee-Wick particles are the necessary components of the extension of the SM in [3] and turn out to be interesting and significant in describing the physics at the TeV scale. On the other hand, to our knowledge, they were ignored in the relevant phenomenological studies of the ADD model in the literature.

The rest of the paper is organized into three parts. In Sec. II, we describe the Maxwell, Dirac, and Klein-Gordon theories in the context of the ADD gravity. We compactify the gravity on an n-dimensional torus T^n and perform a Kaluza-Klein decomposition. In Sec. III, we compute the one-loop gravitational corrections to the gauge, fermion, and scalar self-energy in the framework of the background field method and discuss the implications of the higher derivative operators. Similar calculations on the various self-energy corrections have been made in the conventional gauge dependent method [23,24], where a cutoff procedure has been used in the loop momentum integrations and the summations of the Kaluza-Klein states. In this work we use dimensional regularization so that one can easily track out the momenta of the background fields. We present our discussions and conclusions in the final section.

II. THE FORMALISM AND KALUZA-KLEIN DECOMPOSITION OF GRAVITY

Our starting point is the Hilbert-Einstein, Maxwell, Dirac, and Klein-Gordon theory in which the gravity propagates in the $(4+n)$-dimensional bulk while the gauge and matter fields live in four-dimensional spacetime. The extra dimensions are compactified on a torus T^n. The action of the theory has the form

$$S = -\int d^4x d^n y (\mathcal{L}_{\rm HE} + \mathcal{L}_{\rm M} + \mathcal{L}_{\rm D} + \mathcal{L}_{\rm KG}), \qquad (1)$$

where the Lagrangians for gravity and gravity-gauge-matter couplings are

$$\begin{aligned}
\mathcal{L}_{\rm HE} &= \frac{1}{\hat{\kappa}^2}\sqrt{(-1)^{3+n}|\hat{g}^{(4+n)}|}\hat{R},\\
\mathcal{L}_{\rm M} &= \frac{1}{4}\sqrt{-|\hat{g}^{(4)}|}\hat{g}^{\mu\lambda}\hat{g}^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}\delta^{(n)}(y),\\
\mathcal{L}_{\rm D} &= -\sqrt{-|\hat{g}^{(4)}|}\bar{\psi}\left[i\hat{e}^{\mu}_{\alpha}\gamma^{\alpha}\left(D_\mu + \frac{1}{2}\hat{\omega}_{\mu ab}\sigma^{ab}\right)\right]\psi\,\delta^{(n)}(y),\\
\mathcal{L}_{\rm KG} &= -\frac{1}{2}\sqrt{-|\hat{g}^{(4)}|}(\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_s^2\varphi^2)\delta^{(n)}(y), \qquad (2)
\end{aligned}$$

with $\hat{\kappa}^2 \equiv 16\pi\hat{G}_N \sim \frac{1}{M^{2+n}_{{\rm pl}(4+n)}}$. The hatted symbols $\hat{R}$, $\hat{g}^{\hat{\mu}\hat{\nu}}$, and $\hat{G}$ are the Ricci scalar, the metric tensor, and the Newton constant in the $(4+n)$ bulk. The index $\hat{\mu}$ extends over the full $(4+n)$ dimensions, and μ over $(3+1)$ dimensions. The vierbein $\hat{e}^{\nu}_{\mu}$ is defined by $\hat{e}^{\alpha}_{\mu}\hat{e}^{\beta}_{\nu}\eta_{\alpha\beta} = \hat{g}_{\mu\nu}$. The spin connection $\hat{\omega}_{\mu ab}$ can be solved in terms of the vierbein and $\sigma^{ab} = \frac{1}{4}[\gamma^a, \gamma^b]$. The derivative D_μ is internal gauge symmetry covariant. The fermion is massless because we are interested in the case of high energies when electroweak symmetry is unbroken. The scalar field is taken to be real and thus it does not carry any nontrivial charges. We ignore the cosmological constant term since it is irrelevant to our discussions. The action (1) is invariant under general coordinate and $U(1)$ gauge transformations. For simplicity, we assume that the compactification scales of the extra n-dimensional spaces y_i are all roughly equal to R.

In the following we use the background field method [25] and choose the background spacetime to be Minkowski space. The basic idea of the method is to expand the fields appearing in the classical action to the background fields and the quantum fields. The quantum fields are the variables of integration in the functional integral. A gauge choice is made in such a way that it breaks the gauge invariance of the quantum gauge field, but retains gauge invariance in terms of the background gauge field. Therefore, one is able to quantize a gauge field theory without losing the explicit gauge invariance. The Green's functions obey the naive Ward identities of gauge invariance, and even the unphysical quantities like divergent counterterms take a gauge invariant form, which makes the following discussion unambiguous. The background field method is used extensively in gravity. The first relevant papers calculating the one-loop quantum gravitational effects on the gauge and matter fields are [18,26].

The graviton field $\hat{g}_{\hat{\mu}\hat{\nu}}(x, y)$, gauge field $A(x)$, and matter fields $\psi(x)$ and $\varphi(x)$ can be written as sums of background fields $(\hat{\eta}_{\hat{\mu}\hat{\nu}}, \bar{A}(x), \Psi(x), \Phi(x))$ and quantum fluctuations

$(\hat{h}_{\hat{\mu}\,\hat{\nu}}, a(x), \tilde{\psi}(x), \tilde{\varphi}(x))$:

$$\begin{aligned} \hat{g}_{\hat{\mu}\,\hat{\nu}}(x, y) &= \hat{\eta}_{\hat{\mu}\,\hat{\nu}} + \hat{\kappa}\hat{h}_{\hat{\mu}\,\hat{\nu}}(x, y), \\ A(x) &= \bar{A}(x) + a(x), \\ \psi(x) &= \Psi(x) + \tilde{\psi}(x), \\ \varphi(x) &= \Phi(x) + \tilde{\varphi}(x). \end{aligned} \tag{3}$$

Here $\hat{\eta}_{\hat{\mu}\,\hat{\nu}}$ is the Minkowski metric of the bulk.

Now we consider the Hilbert-Einstein term in action (1). Expanding the gravity field, one can get the linearized Fierz-Pauli Lagrangian

$$\begin{aligned} \frac{-\sqrt{(-1)^{3+n}|\hat{g}^{(4+n)}|}\hat{R}(x, y)}{\hat{\kappa}^2} &= \frac{1}{4}(\partial^{\hat{\mu}}\hat{h}\partial_{\hat{\mu}}\hat{h} - \partial^{\hat{\mu}}\hat{h}^{\hat{\nu}\,\hat{\rho}}\partial_{\hat{\mu}}\hat{h}_{\hat{\nu}\,\hat{\rho}} \\ &\quad + 2\partial_{\hat{\nu}}\hat{h}^{\hat{\nu}\,\hat{\mu}}\partial^{\hat{\rho}}\hat{h}_{\hat{\rho}\,\hat{\mu}} \\ &\quad - 2\partial_{\hat{\nu}}\hat{h}^{\hat{\nu}\,\hat{\mu}}\partial_{\hat{\mu}}\hat{h}) + \mathcal{O}(\hat{\kappa}), \end{aligned} \tag{4}$$

where $\hat{h} \equiv \hat{h}^{\hat{\mu}}_{\hat{\mu}}$. It is the kinematic of the graviton, and we have ignored the self-interaction terms of gravitons since they are irrelevant to our present discussions.

To perform the Kaluza-Klein reduction of Eq. (4) to four-dimensional spacetime, we parametrize the field $\hat{h}_{\hat{\mu}\,\hat{\nu}}$ as

$$\hat{h}_{\hat{\mu}\,\hat{\nu}} = V_n^{-(1/2)}\begin{pmatrix} h_{\mu\nu} + \eta_{\mu\nu}\phi & A_{\mu j} \\ A_{i\nu} & 2\phi_{ij} \end{pmatrix}, \tag{5}$$

where $V_n = R^n$ is the volume of the n-dimensional compactified torus T^n, $\phi = \sum_i \phi_{ii}$, the subscripts $\mu, \nu = 0, 1, 2, 3$, and $i, j = 4, 5, \ldots, 3 + n$. The fields $h_{\mu\nu}$, $A_{\mu i}$, and ϕ_{ij} are the Lorentz tensor, vector, and scalar, respectively. They have the following mode expansions:

$$h_{\mu\nu}(x, y) = \sum_{\vec{n}} h^{\vec{n}}_{\mu\nu}(x)\exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R}\right), \tag{6}$$

$$A_{\mu i}(x, y) = \sum_{\vec{n}} A^{\vec{n}}_{\mu i}(x)\exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R}\right), \tag{7}$$

$$\phi_{ij}(x, y) = \sum_{\vec{n}} \phi^{\vec{n}}_{ij}(x)\exp\left(i\frac{2\pi\vec{n}\cdot\vec{y}}{R}\right), \tag{8}$$

with $\vec{n} = (n_1, n_2, \ldots, n_n)$. After a straightforward calculation, we find that Eq. (4) reduces to the expression in terms of the massive Kaluza-Klein modes

$$\begin{aligned} &-\frac{1}{4}\sum_{\vec{n}}(2\partial^\mu A^{\vec{n},i\nu}\partial_\mu A^{-\vec{n}}_{i\nu} - 2\partial_\mu A^{\vec{n},i\mu}\partial_\nu A^{-\vec{n},i\nu} - 2m^2_{\vec{n}}A^{\vec{n},i\mu}A^{-\vec{n}}_{i\mu} - 2m_{n_i}m_{n^j}A^{\vec{n},i\mu}A^{-\vec{n}}_{j\mu} + \partial^\mu h^{\vec{n},\nu\rho}\partial_\mu h^{-\vec{n}}_{\nu\rho} - \partial^\mu h^{\vec{n}}\partial_\mu h^{-\vec{n}} \\ &\quad - 2\partial_\nu h^{\vec{n},\nu\mu}\partial^\rho h^{-\vec{n}}_{\rho\mu} + 2\partial_\mu h^{\vec{n}}\partial_\nu h^{-\vec{n},\mu\nu} - m^2_{\vec{n}}h^{\vec{n},\mu\nu}h^{-\vec{n}}_{\mu\nu} + m^2_{\vec{n}}h^{\vec{n}}h^{-\vec{n}} + 2\partial^\mu\phi^{\vec{n}}\partial_\mu\phi^{-\vec{n}} + 4\partial^\mu\phi^{\vec{n},ij}\partial_\mu\phi^{-\vec{n}}_{ij} \\ &\quad - 4m^2_{\vec{n}}\phi^{\vec{n},ij}\phi^{-\vec{n}}_{ij} - 8m_{n_i}m_{n^j}\phi^{\vec{n},ik}\phi^{-\vec{n}}_{jk} + 8m_{n_i}m_{n_j}\phi^{\vec{n}}\phi^{-\vec{n},ij} + 2m^2_{\vec{n}}h^{\vec{n}}\phi^{-\vec{n}} + 4m_{n_i}m_{n_j}h^{\vec{n}}\phi^{-\vec{n},ij} + i4m_{n^i}\partial_\mu A^{\vec{n},j\mu}\phi^{-\vec{n}}_{ij} \\ &\quad + i4m_{n_i}\phi^{\vec{n}}\partial_\mu A^{-\vec{n},i\mu} + i4m_{n^i}\partial_\mu h^{\vec{n},\mu\nu}A^{-\vec{n}}_{i\nu} + i2m_{n_i}h^{\vec{n}}\partial_\mu A^{-\vec{n},i\mu} - i2m_{n_i}\partial_\mu h^{\vec{n}}A^{-\vec{n},i\mu}), \end{aligned} \tag{9}$$

where $m^2_{\vec{n}} \equiv -m_{n_i}m_{n^i} = -\frac{4\pi^2 n_i n^i}{R^2} = \frac{4\pi^2 n_i n_i}{R^2}$ since we have used the flat spacetime metric tensor $\mathrm{diag}(\hat{\eta}) = (1, -1, \ldots, -1)$. In deriving Eq. (9), we have used the relation between the four-dimensional and the $(4 + n)$-dimensional Newton constants $\kappa^2 R^n = \hat{\kappa}^2$. The last two lines contain the mixing terms $h\phi$, $A\phi$, and hA. All of them result from the last two terms in Eq. (4). It is more convenient to cancel such terms in practical calculations. For this purpose, we add a special de Donder gauge fixing term

$$-\tfrac{1}{2}(\partial_{\hat{\rho}}\hat{h}^{\hat{\rho}\,\hat{\mu}}\partial^{\hat{\sigma}}\hat{h}_{\hat{\sigma}\,\hat{\mu}} - \partial_{\hat{\rho}}\hat{h}^{\hat{\rho}\,\hat{\mu}}\partial_{\hat{\mu}}\hat{h} + \tfrac{1}{4}\partial_{\hat{\mu}}\hat{h}\partial^{\hat{\mu}}\hat{h}) \tag{10}$$

to the Fierz-Pauli Lagrangian (4), rather than redefine the fields as in [23]. The first two terms will cancel the last two terms in Eq. (4) so that the mixing terms in Eq. (9) do not appear again. The Lagrangian of the gravity can then be written as a simple form,

$$\begin{aligned} \mathcal{L}_{\rm HE} = &-\frac{1}{4}\sum_{\vec{n}}(\partial^\mu h^{\vec{n},\nu\rho}\partial_\mu h^{-\vec{n}}_{\nu\rho} - \frac{1}{2}\partial^\mu h^{\vec{n}}\partial_\mu h^{-\vec{n}} - m^2_{\vec{n}}h^{\vec{n},\nu\rho}h^{-\vec{n}}_{\nu\rho} + \frac{1}{2}m^2_{\vec{n}}h^{\vec{n}}h^{-\vec{n}} + 2\partial^\mu A^{\vec{n},i\nu}\partial_\mu A^{-\vec{n}}_{i\nu} - 2m^2_{\vec{n}}A^{\vec{n},i\nu}A^{-\vec{n}}_{i\nu} \\ &+ 4\partial^\mu\phi^{\vec{n},ij}\partial_\mu\phi^{-\vec{n}}_{ij} + 2\partial^\mu\phi^{\vec{n}}\partial_\mu\phi^{-\vec{n}} - 4m^2_{\vec{n}}\phi^{\vec{n},ij}\phi^{-\vec{n}}_{ij} - 2m^2_{\vec{n}}\phi^{\vec{n}}\phi^{-\vec{n}}) + \mathcal{O}(\kappa). \end{aligned} \tag{11}$$

In this gauge, the propagators for the massive Kaluza-Klein states $h^{\vec{n}}_{\mu\nu}$, $A^{\vec{n}}_{i\mu}$, and $\phi^{\vec{n}}_{ij}$ are

$$\triangle^h_{\vec{n}\mu\nu,\vec{m}\rho\sigma}(k) = -i\frac{\delta_{\vec{n},-\vec{m}}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})}{k^2 - m^2_{\vec{n}} + i\epsilon}, \tag{12}$$

$$\triangle^{A}_{\vec{n}i\mu,\vec{m}j\nu}(k) = -i\frac{\delta_{\vec{n},-\vec{m}}\delta_{ij}\eta_{\mu\nu}}{k^2 - m^2_{\vec{n}} + i\epsilon}, \tag{13}$$

$$\triangle^{\phi}_{\vec{n}ij,\vec{m}kl}(k) = -i\frac{\delta_{\vec{n},-\vec{m}}[\frac{1}{4}(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) - \frac{1}{4+2n}\delta_{ij}\delta_{kl}]}{k^2 - m^2_{\vec{n}} + i\epsilon}. \tag{14}$$

Obviously the spin-2 state $h^{\vec{n}}_{\mu\nu}$ is not a physical state. This can be seen explicitly from the numerator of its propagator. The physical massive spin-2 state with the right polarization tensor is constructed from $h^{\vec{n}}_{\mu\nu}$, $A^{\vec{n}}_{i\mu}$, and $\phi^{\vec{n}}_{ij}$. Its explicit form is shown in [23]. However, the rearrangement of the physical spin-2, $(n-1)$ spin-1, and $n(n-1)/2$ spin-0 states into $h^{\vec{n}}_{\mu\nu}$, $A^{\vec{n}}_{i\mu}$, and $\phi^{\vec{n}}_{ij}$ states shown above greatly simplifies our calculations.

Note that we do not include ghost parts in the Lagrangian since they do not contribute to the one-loop order.

III. LEE-WICK PARTICLES FROM GRAVITATIONAL CORRECTIONS

Now we turn to the interactions of gravity with the gauge and matter fields. What we are concerned with are the higher derivative operators of mass dimension six from the one-loop gravitational corrections to the two-point Green's functions for the gauge and matter wave operators. The coefficients of the quantum induced higher derivative operators are divergent. In terms of the effective theory, one should include each higher derivative operator with an arbitrary parameter $a_i (i = 1, 2, 3)$ in the Lagrangian, and then introduce counterterms to cancel the divergences and renormalize the parameters a_i. The tadpole diagrams, though they do not vanish because of the appearance of the massive Kaluza-Klein states in the loops, are irrelevant to the higher derivative operators. Thus, in what follows, we only need to calculate the rainbow diagrams and write down the divergent terms explicitly. We will present a rather detailed description on the gauge field in Sec. III A, and leave out similar ones on the fermion and scalar field sectors. We would like to emphasize that since gravitons do not carry gauge charges, the results of the one-loop corrections to the two-point Green's functions can be applied directly to the non-Abelian case.

A. Lee-Wick gauge bosons

The second term in action (1) specifies the interaction of the gauge field with the graviton. Adding the Lorentz gauge fixing term $\frac{-1}{2\xi}(\partial_\mu a_\nu)^2$ for the photon field to the action, we find the propagator for the photon a_μ,

$$\triangle^{a}_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon}\left[\eta_{\mu\nu} - (1-\xi)\frac{k_\mu k_\nu}{k^2}\right]. \tag{15}$$

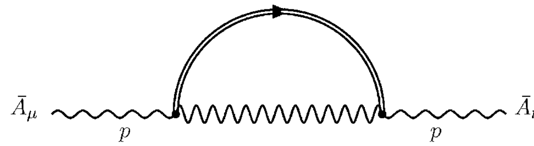

FIG. 1. The rainbow diagram generating the higher derivative term of the gauge field. The internal wavy and double lines represent a_μ and $h^{\vec{n}}_{\mu\nu}$ fields, respectively. The external wavy lines are background photon fields.

Up to order κ^2, the relevant interaction terms contain $(h^{\vec{n}} a\bar{A})$, $(h^{\vec{n}} h^{\vec{m}}\bar{A}\bar{A})$, and $(h^{\vec{n}}\phi^{\vec{m}}\bar{A}\bar{A})$ vertices, from which only the first one is of interest in our present consideration since it is the only vertex to compose the rainbow diagram shown in Fig. 1.

$$\mathcal{L}^{\rm int}_{\rm M} \sim \kappa\sum_{\vec{n}}\Big[h^{\vec{n}}_{\mu\nu}\partial_\lambda\bar{A}_\rho\partial_\tau a_\sigma\Big(\eta^{\lambda\tau}\eta^{\nu\sigma}\eta^{\mu\rho} + \eta^{\mu\lambda}\eta^{\rho\sigma}\eta^{\nu\tau} + \frac{1}{2}\eta^{\mu\nu}\eta^{\rho\tau}\eta^{\lambda\sigma} - \eta^{\lambda\sigma}\eta^{\mu\rho}\eta^{\nu\tau} - \eta^{\mu\lambda}\eta^{\rho\tau}\eta^{\nu\sigma} - \frac{1}{2}\eta^{\mu\nu}\eta^{\lambda\tau}\eta^{\rho\sigma}\Big)\Big] + \mathcal{O}(\kappa^2). \tag{16}$$

The diagram is of order κ^2 of the gravitational coupling. We evaluate the loop integral in the dimensional regularization (DR) scheme to extract the singularity, rather than introduce a hard cutoff as in [23,24]. By a straightforward calculation, it can be shown that the correction obtained in the framework of the background field method is independent of the gauge parameter ξ, as it should be. The result of the diagram is

$$\Pi^{R}_{\mu\nu}(p^2) = i\frac{\kappa^2}{24\pi^2}\frac{1}{\epsilon}p^2(p^2\eta_{\mu\nu} - p_\mu p_\nu)\sum_{\vec{n}}\mathbf{1} + i\frac{3\kappa^2}{8\pi^2}\frac{1}{\epsilon}\times(p^2\eta_{\mu\nu} - p_\mu p_\nu)\sum_{\vec{n}}m^2_{\vec{n}} + [\text{finite part}]. \tag{17}$$

The contributions of the extra dimensions are embodied in the summation over the Kaluza-Klein states in a tower. In the limit of four-dimensional spacetime, the second term vanishes because the graviton is massless. The summation can be written as an integration in terms of the mass $m^2_{\vec{n}}$ when the Kaluza-Klein states are nearly degenerate [23],

$$\sum_{\vec{n}} f(m_{\vec{n}}) = \int_0^{\Lambda^2} dm^2_{\vec{n}}\rho(m_{\vec{n}})f(m_{\vec{n}}), \tag{18}$$

where

$$\rho(m_{\vec{n}}) = \frac{R^n m^{n-2}_{\vec{n}}}{(4\pi)^{n/2}\Gamma(n/2)} \tag{19}$$

is the Kaluza-Klein state density. As is well known, using the DR scheme, which is a mass independent scheme, heavy states do not decouple. Thus we have introduced an explicit cutoff Λ to regularize the mass integration. That is, we include only a finite number of low-lying Kaluza-Klein states and assume all other states decouple from the low energy physics we are interested in. This cutoff does not break any gauge symmetry and our calculation is gauge independent. The nearly degenerate condition is satisfied when the energy scale R^{-1} characterizing the Kaluza-Klein excitations is much less than the physical scale Λ. This is indeed the case in the large extra dimension model. In general, we have $\Lambda \leq M_{\mathrm{pl}(4+n)}$, since the effective theory is only expected to be valid below the fundamental scale $M_{\mathrm{pl}(4+n)}$. Various phenomenological studies have suggested that the cutoff should be $\Lambda \sim M_{\mathrm{pl}(4+n)} \sim 1$ TeV [23,24,27]. Our final result for the rainbow diagram is

$$\begin{aligned}\Pi^R_{\mu\nu}(p^2) = {} & i\frac{\hat{\kappa}^2}{12\pi^2}\left[\frac{\Lambda^n}{(4\pi)^{n/2}\Gamma(n/2)n}\right] \\ & \times \frac{1}{\epsilon}p^2(p^2\eta_{\mu\nu} - p_\mu p_\nu) \\ & + i\frac{3\hat{\kappa}^2}{4\pi^2}\left[\frac{\Lambda^{n+2}}{(4\pi)^{n/2}\Gamma(n/2)(n+2)}\right] \\ & \times \frac{1}{\epsilon}(p^2\eta_{\mu\nu} - p_\mu p_\nu) + [\text{finite part}]. \end{aligned} \tag{20}$$

The first term of Eq. (20) shows that, different from renormalizable theories, the correction due to gravitons generates a new type of divergence which cannot be absorbed in the Maxwell-Einstein action. In flat spacetime, the term $ip^2(p_\mu p_\nu - p^2\eta_{\mu\nu})$ in the truncated photon-photon correlation function corresponds to the dimension six operator $-\frac{1}{2}\partial_\mu \bar{F}^{\mu\nu}\partial^\rho \bar{F}_{\rho\nu}$. This is the leading higher derivative term allowed by symmetries. Without renormalizability as an axiom, it should be included in the Lagrangian. If one unnaturally neglects it, the theory will lack its predictiveness at one-loop order. The second term which vanishes in the four-dimensional theory results from the summation of the Kaluza-Klein states. It contributes to the one-loop β function of the gauge coupling and leads to the power-law running of the gauge coupling.

Note that the one-loop correction to the gauge couplings from a tower of gauge boson Kaluza-Klein states is of order Λ^n [28,29]. Here we show explicitly that the correction coming from graviton Kaluza-Klein states is of order Λ^{n+2}. The effect on the unification of gauge coupling constants caused by this term is itself an interesting subject and will be reported elsewhere [30].

When we take $n = 0$ in Eq. (20), we will come to the case of gravity in four-dimensional spacetime that we have discussed in [16]. But since we start from different definitions of the action (1) and κ in this work, the result here differs by a factor of $\frac{1}{8}$ from that in [16].

Now let us consider the modified Maxwell theory in curved spacetime with the required higher derivative term. The action has the form

$$\begin{aligned} & -\int d^4x d^n y\Big[\sqrt{-|\hat{g}^{(4)}|}\Big(\frac{1}{4}\hat{g}^{\mu\lambda}\hat{g}^{\nu\rho}F_{\mu\nu}F_{\lambda\rho} \\ & - \frac{a_1}{2M^2}\hat{g}^{\mu\rho}\hat{g}^{\nu\lambda}\hat{g}^{\sigma\tau}\mathcal{D}_\mu F_{\rho\sigma}\mathcal{D}_\nu F_{\lambda\tau}\Big)\delta^{(n)}(y)\Big] \end{aligned} \tag{21}$$

where a_1 is a dimensionless parameter and will be renormalized by introducing a counterterm to cancel the divergence of the first term in Eq. (20) at a certain renormalization condition. The operator $\mathcal{D}_\mu$ is the spacetime covariant derivative. For convenience, we have defined a dimension-one parameter $M \equiv \sqrt{1/\hat{\kappa}^2\Lambda^n} \sim 1$ TeV. Based on the previous discussion, the origin of the last term is clear. The existence of gravity naturally provides a mechanism to generate this nonrenormalizable term. With this term, the gauge sector in the theory is one-loop renormalizable.

Let us consider again the four-dimensional background spacetime to be a Minkowski one and focus our discussion on the gauge sector. The part of the action we are interested in is the quadratic terms of the gauge field,

$$-\int d^4x\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{a_1}{2M^2}\partial_\mu F^{\mu\nu}\partial^\rho F_{\rho\nu}\right), \tag{22}$$

from which we can find the propagator in the background spacetime:

$$\frac{-i}{p^2 - \frac{a_1}{M^2}p^4 + i\epsilon}\left[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{q^2} + \xi\left(1 - a_1\frac{p^2}{M^2}\right)\frac{p_\mu p_\nu}{p^2}\right], \tag{23}$$

where we have used the same Lorentz gauge fixing term as before. The propagator contains two poles: One corresponds to the massless photon, and the other one at $p^2 = \frac{M^2}{a_1}$ corresponds to the Lee-Wick particle with mass $\frac{M}{\sqrt{a_1}}$ for positive a_1.

Remember the electric field $E^i = F^{i0}$ and magnetic fields $B^i = -\epsilon^{ijk}F_{jk}$. "Maxwell's equations" derived from Eq. (22) read

$$\vec{\nabla}\cdot\vec{B} = 0, \tag{24}$$

$$\frac{\partial\vec{B}}{\partial t} + \vec{\nabla}\times\vec{E} = 0, \tag{25}$$

$$\vec{\nabla}\cdot\vec{D} = 0, \tag{26}$$

$$\frac{\partial\vec{D}}{\partial t} - \vec{\nabla}\times\vec{H} = 0, \tag{27}$$

where

$$\vec{D} \equiv \left(1 + \frac{a_1}{M^2}\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\right)\vec{E}, \tag{28}$$

$$\vec{H} \equiv \left(1 + \frac{a_1}{M^2}\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right)\right)\vec{B}. \tag{29}$$

The first set comes from the gauge invariance of the system. The second set are the equations of motion derived from Eq. (22). One can easily see that the solution $\vec{D} = \vec{E}$ and $\vec{H} = \vec{B}$ corresponds to the well-known massless photon. The other independent solution, $\vec{D} = 0$ and $\vec{H} = 0$, implies

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + \frac{M^2}{a_1}\right)\vec{E} = 0, \tag{30}$$

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 + \frac{M^2}{a_1}\right)\vec{B} = 0. \tag{31}$$

This corresponds to the Lee-Wick particle with mass $\frac{M}{\sqrt{a_1}}$.

B. Lee-Wick fermions

We now switch to the third term in the action (1) which contains the interaction terms of the fermion field with the gravitons. Since only the rainbow diagrams plotted in Fig. 2 can contribute to the higher derivative operator, we are interested in the following interaction terms exclusively:

$$\begin{aligned}\frac{\kappa}{2}\sum_{\vec n}\{\bar{\tilde{\psi}}[&i\gamma^\mu(\partial_b h^{\vec n}_{a\mu} + \eta_{a\mu}\partial_b\phi^{\vec n})\sigma^{ab} \\ &- i(h^{\vec n,\mu}_\alpha + \delta^\mu_\alpha\phi^{\vec n})\gamma^\alpha\partial_\mu + (h^{\vec n} + 4\phi^{\vec n})i\gamma^\mu\partial_\mu]\Psi \\ &+ \bar\Psi[i\gamma^\mu(\partial_b h^{\vec n}_{a\mu} + \eta_{a\mu}\partial_b\phi^{\vec n})\sigma^{ab} \\ &- i(h^{\vec n,\mu}_\alpha + \delta^\mu_\alpha\phi^{\vec n})\gamma^\alpha\partial_\mu + (h^{\vec n} + 4\phi^{\vec n})i\gamma^\mu\partial_\mu]\tilde\psi\},\end{aligned} \tag{32}$$

where Ψ and $\tilde\psi$ are the background field and fluctuation of the fermion. Our final result for these two diagrams is

$$\begin{aligned}\Sigma^R(\not p) = &-i\frac{\hat\kappa^2}{128\pi^2}\left[\frac{\Lambda^n}{(4\pi)^{n/2}\Gamma(n/2)n(n+2)}\right] \\ &\times\frac{1}{\epsilon}(n-16)p^2\not p \\ &+ i\frac{\hat\kappa^2}{64\pi^2}\left[\frac{\Lambda^{n+2}}{(4\pi)^{n/2}\Gamma(n/2)(n+2)^2}\right] \\ &\times\frac{1}{\epsilon}(30-12n)\not p + [\text{finite part}].\end{aligned} \tag{33}$$

After a Fourier transform to the configuration space, the first term corresponds to a dimension-six operator $\frac{1}{M^2}\bar\Psi\not\partial\not\partial\not\partial\Psi$. The second term contributes to the renormalization of the fermion field.

In the effective theory, we should add the higher derivative term permitted by symmetry at the beginning. Thus the modified curved spacetime Dirac action becomes

$$\begin{aligned}\int d^4x d^n y\Big[\sqrt{-|\hat g^{(4)}|}\,\bar\psi\Big(&i\hat e^\mu_\alpha\gamma^\alpha\mathcal{D}_\mu \\ &+ i\frac{a_2}{M^2}\hat e^\mu_\alpha\hat e^\nu_\beta\hat e^\rho_\delta\gamma^\alpha\gamma^\beta\gamma^\delta\mathcal{D}_\mu\mathcal{D}_\nu\mathcal{D}_\rho\Big)\psi\,\delta^{(n)}(y)\Big],\end{aligned} \tag{34}$$

where the derivative is defined as $\mathcal{D}_\mu = D_\mu + \frac{1}{2}\hat\omega_{\mu ab}\sigma^{ab}$. The effective action in four-dimensional Minkowski spacetime is of the form

$$\int d^4x\bar\psi(i\not D + i\frac{a_2}{M^2}\not D\not D\not D)\psi. \tag{35}$$

The propagator of the higher derivative theory is

$$\frac{i}{\not p - \frac{a_2}{M^2}p^2\not p} = \frac{-i}{\frac{a_2}{M^2}(p^2 - \frac{M^2}{a_2})\not p}, \tag{36}$$

which implies two freedoms of the fermion: One is the massless fermion, and the other one is its Lee-Wick partner with mass $\frac{M}{\sqrt{a_2}}$.

C. Lee-Wick scalars

The fourth term in the action (1) is the Klein-Gordon theory in curved spacetime. In terms of the Kaluza-Klein states of the graviton, the relevant interaction terms can be reduced to

$$\begin{aligned}-\frac{\kappa}{2}\sum_{\vec n}&h^{\vec n}_{\mu\nu}[2\partial^\mu\tilde\varphi\partial^\nu\Phi - \eta^{\mu\nu}(\partial_\lambda\tilde\varphi\partial^\lambda\Phi - m_s^2\tilde\varphi\Phi)] \\ &+ \kappa\sum_{\vec n}\phi^{\vec n}(\partial_\lambda\tilde\varphi\partial^\lambda\Phi - 2m_s^2\tilde\varphi\Phi).\end{aligned} \tag{37}$$

Similar to the gauge and fermion field sectors, only the rainbow diagrams plotted in Fig. 3 contribute to the higher derivative operator. Summing up these two diagrams, we find the final result,

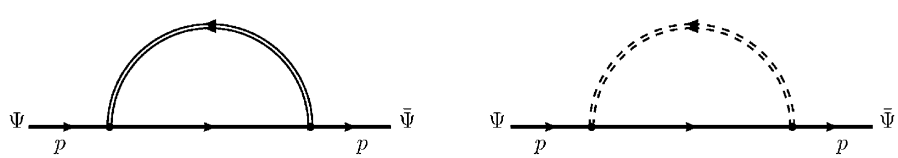

FIG. 2. The rainbow diagrams generating the higher derivative term of the fermion. The internal line, double lines, and dashed double lines represent $\tilde\psi$, $h^{\vec n}_{\mu\nu}$, and $\phi^{\vec n}_{ij}$ fields, respectively. The external lines are background fermion fields.

FIG. 3. The rainbow diagrams generating the higher derivative term of the scalar. The internal dashed line, double lines, and dashed double lines represent $\tilde{\varphi}$, $h^{\vec{n}}_{\mu\nu}$, and $\phi^{\vec{n}}_{ij}$ fields, respectively. The external dashed lines are background scalar fields.

$$\Omega^R(p^2) = -i\frac{\hat{\kappa}^2}{16\pi^2}\frac{\Lambda^n}{(4\pi)^{n/2}\Gamma(n/2)}\frac{1}{\epsilon}\left(\frac{1}{n+2}p^4 + \frac{n+16}{n(n+2)}m_s^2 p^2 + \frac{5n+8}{(n+2)^2}\Lambda^2 p^2 + \frac{8n-16}{n(n+2)}m_s^4\right) + [\text{finite part}]. \tag{38}$$

It is easy to transform the first term to configuration space. It is a higher derivative operator $\frac{1}{M^2}(\partial^2\Phi)^2$. The other three terms work on the renormalizations of the field and the mass of the scalar. Note that when there are not any extra dimensions, i.e. $n = 0$, the higher derivative operator does not appear.

To write down a one-loop renormalizable theory, we should include the higher derivative operator allowed by symmetry in action (1). Then the modified action has the form

$$\frac{1}{2}\int d^4x d^n y\sqrt{-|\hat{g}^{(4)}|}[\hat{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{a_3}{M^2}(\hat{g}^{\mu\nu}\mathcal{D}_\mu\mathcal{D}_\nu\varphi)^2 - m_s^2\varphi^2]\delta^{(n)}(y). \tag{39}$$

By a straightforward reduction, the quadratic terms in the flat four-dimensional world can be obtained,

$$\frac{1}{2}\int d^4x[\partial_\mu\varphi\partial^\mu\varphi - \frac{a_3}{M^2}(\partial^2\varphi)^2 - m_s^2\varphi^2]. \tag{40}$$

One can easily find the propagator to be

$$\frac{i}{p^2 - \frac{a_3}{M^2}p^4 - m_s^2}. \tag{41}$$

For $\frac{M}{\sqrt{a_3}} \gg m_s$, it has poles at $p^2 \simeq m_s^2$ and at $p^2 \simeq M^2/a_3$, indicating the description of 2 degrees of freedom: the scalar and its Lee-Wick partner.

IV. CONCLUSIONS

We have studied Maxwell, Dirac, and Klein-Gordon theories in the model of large extra dimensional gravity in which the gravitons propagate in the $(4+n)$-dimensional bulk, while the gauge and matter fields are confined to the four-dimensional world. The one-loop corrections to the two-point Green's functions of the gauge, fermion, and scalar fields from the exchange of virtual Kaluza-Klein gravitons have been calculated with the gauge independent background field method. We show that the gravitational corrections generate a new type of divergence which corresponds to a higher derivative operator with mass dimension six for every field. Besides, the one-loop corrections to the β function of the gauge coupling from a tower of graviton Kaluza-Klein states is found to be of order Λ^{n+2}.

In the framework of effective field theories, the higher derivative operators permitted by the symmetry should be added to the Lagrangian at the beginning so that one can subtract the divergences and renormalize the theory. We show that the modified one-loop renormalizable Lagrangian contains a TeV scale Lee-Wick partner of every gauge and matter field as an extra degree of freedom. Since gravitons do not carry gauge charges, the same results can be applied directly to the non-Abelian case. Thus the large extra dimension model of gravity provides a natural mechanism for the emergence of these exotic particles which were introduced in [3] to construct an extension of the standard model.

ACKNOWLEDGMENTS

The research of F. W. is supported in part by the Chinese Ministry of Education Project No. 208072. M. Z. is supported in part by the research fund of National University of Defense Technology.

[1] T. D. Lee and G. C. Wick, Nucl. Phys. **B9**, 209 (1969).
[2] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).
[3] B. Grinstein, D. O'Connell, and M. B. Wise, Phys. Rev. D **77**, 025012 (2008).
[4] T. G. Rizzo, J. High Energy Phys. 06 (2007) 070.
[5] T. G. Rizzo, J. High Energy Phys. 01 (2008) 042.
[6] F. Krauss, T. E. J. Underwood, and R. Zwicky, Phys. Rev. D **77**, 015012 (2008).
[7] J. R. Espinosa, B. Grinstein, D. O'Connell, and M. B. Wise, Phys. Rev. D **77**, 085002 (2008).
[8] T. R. Dulaney and M. B. Wise, Phys. Lett. B **658**, 230 (2008).

[9] E. Álvarez, L. D. Rold, C. Schat, and A. Szynkman, J. High Energy Phys. 04 (2008) 026.
[10] T. E. J. Underwood and R. Zwicky, arXiv:0805.3296.
[11] C. D. Carone and R. F. Lebed, arXiv:0806.4555.
[12] B. Grinstein, D. O'Connell, and M. B. Wise, Phys. Rev. D **77**, 065010 (2008).
[13] B. Grinstein and D. O'Connell, arXiv:0801.4034.
[14] B. Grinstein, D. O'Connell, and M. B. Wise, arXiv:0805.2156.
[15] E. Gabrielli, Phys. Rev. D **77**, 055020 (2008).
[16] F. Wu and M. Zhong, Phys. Lett. B **659**, 694 (2008).
[17] B. S. DeWitt, Phys. Rev. **160**, 1113 (1967); **162**, 1195 (1967); **162**, 1239 (1967); **171**, 1834(E) (1968).
[18] G. 't Hooft, Nucl. Phys. **B62**, 444 (1973); G. 't Hooft and M. Veltman, Ann. Inst. Henri Poincaré, A **20**, 69 (1974).
[19] J. Polchinski, arXiv:hep-th/9210046.
[20] D. B. Kaplan, arXiv:nucl-th/9506035.
[21] J. F. Donoghue, Phys. Rev. D **50**, 3874 (1994).
[22] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).
[23] T. Han, J. D. Lykken, and R. J. Zhang, Phys. Rev. D **59**, 105006 (1999).
[24] T. Han, D. Marfatia, and R. J. Zhang, Phys. Rev. D **62**, 125018 (2000).
[25] L. F. Abbott, Acta Phys. Pol. B **13**, 33 (1982).
[26] S. Deser and P. van Nieuwenhuizen, Phys. Rev. D **10**, 401 (1974); **10**, 411 (1974); S. Deser, H. S. Tsao, and P. van Nieuwenhuizen, *ibid.* **10**, 3337 (1974).
[27] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. **B544**, 3 (1999); S. Nussinov and R. Shrock, Phys. Rev. D **59**, 105002 (1999); E. A. Mirabelli, M. Perelstein, and M. E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J. L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999); K. Agashe and N. G. Deshpande, Phys. Lett. B **456**, 60 (1999); M. L. Graesser, Phys. Rev. D **61**, 074019 (2000); T. G. Rizzo, Phys. Rev. D **59**, 115010 (1999); C. Balazs, D. A. Dicus, H. J. He, W. W. Repko, and C. P. Yuan, Phys. Rev. Lett. **83**, 2112 (1999); K. M. Cheung and W. Y. Keung, Phys. Rev. D **60**, 112003 (1999); K. M. Cheung, Phys. Lett. B **460**, 383 (1999); K. Y. Lee, H. S. Song, J. H. Song, and C. Yu, Phys. Rev. D **60**, 093002 (1999); H. Davoudiasl, Phys. Rev. D **60**, 084022 (1999); T. G. Rizzo, Phys. Rev. D **60**, 115010 (1999); K. M. Cheung, Phys. Rev. D **61**, 015005 (1999); X. G. He, Phys. Rev. D **61**, 036007 (2000).
[28] K. R. Dienes, E. Dudas, and T. Gherghetta, Phys. Lett. B **436**, 55 (1998).
[29] K. R. Dienes, E. Dudas, and T. Gherghetta, Nucl. Phys. **B537**, 47 (1999).
[30] F. Wu and M. Zhong, arXiv:0809.1913.