Mass deformation of Bagger-Lambert theory in 3D with reduced $N = 1$ supersymmetry

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We introduce $SO(7)_{\rm global}$ symmetric mass terms into Bagger-Lambert theory in three dimensions. The scalar field $X_a{}^I$ and its fermionic partner ψ_a are provided with the mass m, while a quartic interaction term is induced. These new terms explicitly break the original $SO(8)_{\rm global}$ symmetry down to $SO(7)_{\rm global}$ in terms of octonion structure constants. The original supersymmetry parameter in the $\mathbf{8}_S$ is reduced to a singlet, implying that the original $N = 8$ supersymmetry is reduced to $N = 1$ supersymmetry. As illustrations, we present some nontrivial vacuum configurations with the breaking $SO(7)_{\rm global} \to SO(4)_{\rm global}$ or smaller symmetry groups. Interestingly, we also find that after a nontrivial vacuum expectation value $\langle X_a{}^I \rangle$ is developed, the vector field $A_\mu{}^{ab}$ satisfies a "self-duality" condition, and starts propagating with a mass. These results are due to the special nature of the octonion structure constants.

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I. INTRODUCTION

Coincident M2-branes have been recently attracting much attention for understanding the different aspects of M theory[1]. In particular, Bagger and Lambert (BL) [2,3] have presented an explicit lagrangian with the maximal $N = 8$ supersymmetry in three dimensions (3D) with global conformal symmetry, based on totally antisymmetric triple brackets or 3-Lie algebras (3D) with global conformal symmetry, based on totally antisymmetric triple brackets or 3-Lie algebras [4]

$$[X^I, X^J, X^K] = \frac{1}{3!}[[X^I, X^J], X^K] \pm (\text{cyclic perms.}) \quad (1.1)$$

for the element X^I of nonassociative algebra. The explicit model presented in [3] has $SO(4)_{\rm local} \times SO(8)_{\rm global}$ symmetry with a Chern-Simons (CS) term.

Afterwards, there have been considerable developments. For example, the existence of $OSp(8|4)$ superconformal symmetry in BL theory [2,3] has been confirmed [5] with possible generalizations to more general algebra including octonions. The algebraic structure [6] of BL theory [2,3] has been studied such as from the viewpoint of 3 algebra [7], or from that of embedding tensor [8], or the viewpoint of $SU(2) \times SU(2)$ instead of $SO(4)$ [9], and further generalized to arbitrary Lie algebras [10], or Janus field theory [11]. Recently, the "no-go" theorem about gauge groups has been circumvented in [12] by using nonpositive definite metric. Relationships between the M2-branes and D2-branes [13,14], as well as relationships with multiple membranes and a holographic dual [15], and also with M folds [16] have been also studied. Furthermore, mass deformations of BL theory has been considered in [17] with the original global $SO(8)_{\rm global}$ symmetry broken down to $SO(4)_{\rm global} \times SO(4)_{\rm global}$.

In the light of these activities in the new direction for superconformal CS theory, we present a mass-deformed BL theory using octonion structure constants as a combination of the ideas in [5,17]. We consider a new mass deformation of the $SO(8)_{\rm global} \times SO(4)_{\rm local}$ BL theory [3], where the original global $SO(8)_{\rm global}$ symmetry is explicitly broken down into $SO(7)_{\rm global}$, while the original $N = 8$ supersymmetry is reduced into $N = 1$. These result are accomplished through the mass terms based on octonion structure constants. Explicit $N = 1$ supersymmetric vacuum configurations with symmetry $SO(4)_{\rm global}$ or smaller are also presented.

II. OCTONION STRUCTURE CONSTANT PRELIMINARIES

Before presenting our total action, we review the basic ingredients for $SO(7)$-invariant constants[1] ϕ^{IJKL} that are defined by the original octonion structure constants $\psi^{I'J'K'}$ [18]. In this context, we use the indices $I', J', \ldots = 1, 2, \ldots, 7$. Since $SO(7)$ has a positive definite metric, there is no need to differentiate between their superscripts and subscripts. The nonzero components of the totally antisymmetric ψ's are [18]

$$\psi^{123} = \psi^{516} = \psi^{624} = \psi^{435} = \psi^{471} = \psi^{673} = \psi^{572} = +1 \quad (2.1)$$

together with other dependent components, e.g., $\psi^{213} = -1$. The "dual" components in 7D are[2]

[1]We call these ϕ's "octonion structure constants" in this paper. We use the symbol ϕ^{IJKL} instead of f^{IJKL} in order to avoid the confusion with the structure constants $f^{abc}{}_d$ in [2,3].

[2]Despite no difference between superscripts and subscripts of the indices $I', J', \ldots$, both of them are sometimes used to make contractions transparent. The same applies to $I, J, \ldots, a, b, \ldots, A, B, \ldots$, and $\dot{A}, \dot{B}, \ldots$.

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$$\varphi^{4567} = \varphi^{2374} = \varphi^{1357} = \varphi^{1276} = \varphi^{2356} = \varphi^{1245} = \varphi^{1346} = +1,$$
$$\varphi^{I'J'K'L'} \equiv +\frac{1}{3!}\epsilon^{I'J'K'L'M'N'P'}\psi_{M'N'P'}. \quad (2.2)$$

Based on these, the $SO(7)$ invariant totally antisymmetric constants ϕ^{IJKL} are defined now in terms of $SO(8)$ indices $I, J, \ldots = 1, 2, \ldots, 8$ for $\mathbf{8}_V$ by

$$\phi^{IJKL} = \begin{cases} \phi^{I'J'K'8} \equiv \psi^{I'J'K'} & (I', J', K' = 1, 2, \ldots, 7), \\ \phi^{I'J'K'L'} = \varphi^{I'J'K'L'} & (I', J', K', L' = 1, 2, \ldots, 7). \end{cases} \quad (2.3)$$

Their nonzero components are

$$\begin{aligned} &\phi^{1238} = \phi^{5168} = \phi^{6248} = \phi^{4358} = \phi^{4718} = \phi^{6738} = \phi^{5728} = +1, \\ &\phi^{4567} = \phi^{2374} = \phi^{1357} = \phi^{1276} = \phi^{2356} = \phi^{1245} = \phi^{1346} = +1, \end{aligned} \quad (2.4)$$

and any exchange of two indices cause a sign flip. Accordingly, they also satisfy "self duality" in 8D[3]

$$\phi^{IJKL} = +\frac{1}{4!}\epsilon^{IJKLMNPQ}\phi_{MNPQ} \quad \text{i.e.,} \quad \phi^{[4]} = +\frac{1}{4!}\epsilon^{[4][4]'}\phi_{[4]'}. \quad (2.5)$$

Relevantly, other important identities are

$$\begin{aligned} \phi_{IJKL}\phi^{LMNP} &= -6\delta_{[I}{}^{M}\delta_{J}{}^{N}\delta_{K]}{}^{P} + 9\phi_{[IJ}{}^{[MN}\delta_{K]}{}^{P]}, \\ \phi_{MN[2]}\phi^{[2]RS} &= +12\delta_{[M}{}^{R}\delta_{N]}{}^{S} - 4\phi_{MN}{}^{RS}, \\ \phi_{I[3]}\phi^{J[3]} &= +42\delta_{I}{}^{J}, \\ (\phi_{[4]})^2 &= +336. \end{aligned} \quad (2.6)$$

As in the formulation of reduced $SO(7)$ holonomy in $SO(8)$, in 8D [18–20] we use the projectors for the **28** of $SO(8)$

$$\begin{aligned} P_{IJ}{}^{KL} &\equiv \frac{3}{4}\left(\delta_{[I}{}^{K}\delta_{J]}{}^{L} + \frac{1}{6}\phi_{IJ}{}^{KL}\right), \\ N_{IJ}{}^{KL} &\equiv \frac{1}{4}\left(\delta_{[I}{}^{K}\delta_{J]}{}^{L} - \frac{1}{2}\phi_{IJ}{}^{KL}\right), \end{aligned} \quad (2.7)$$

respectively, into the **21** and the **7** of $SO(7)$. They satisfy the usual projector relationships

$$\begin{aligned} &P_{IJ}{}^{KL}P_{KL}{}^{MN} = P_{IJ}{}^{MN}, \\ &N_{IJ}{}^{KL}N_{KL}{}^{MN} = N_{IJ}{}^{MN}, \\ &P_{IJ}{}^{KL}N_{KL}{}^{MN} = N_{IJ}{}^{KL}P_{KL}{}^{MN} = 0. \end{aligned} \quad (2.8)$$

We need the Γ matrices for the $SO(8)$ Clifford algebra

$$\{\Gamma^I, \Gamma^J\} = +2\delta^{IJ}I. \quad (2.9)$$

Relevantly, we have the chirality projectors $P \equiv (I + \Gamma_9)/2$ for $\mathbf{8}_S$ and $N \equiv (I - \Gamma_9)/2$ for $\mathbf{8}_C$ of $SO(8)$ with $\Gamma_9 \equiv \Gamma_1\Gamma_2\cdots\Gamma_8$. We frequently use the Γ-matrix combination

$$\phi \equiv \frac{1}{4!}\phi^{IJKL}\Gamma_{IJKL} \equiv \frac{1}{4!}\phi^{[4]}\Gamma_{[4]}. \quad (2.10)$$

Also needed are the projectors $\mathcal{P}$ and $\mathcal{Q}$ satisfying

$$\begin{aligned} &\mathcal{P} \equiv \tfrac{1}{8}(P + \tfrac{1}{2}\phi), \qquad \mathcal{Q} \equiv \tfrac{7}{8}(P - \tfrac{1}{14}\phi), \\ &\mathcal{P} + \mathcal{Q} = P, \qquad \mathcal{P}^2 = \mathcal{P}, \qquad \mathcal{Q}^2 = \mathcal{Q}, \\ &\mathcal{P}\mathcal{Q} = \mathcal{Q}\mathcal{P} = 0. \end{aligned} \quad (2.11)$$

Note that $\mathcal{P}$ and $\mathcal{Q}$, respectively, project the $\mathbf{8}_S$ of $SO(8)$ into the **1** and **7** of $SO(7)$ [19,20].

The following useful relationships are easily confirmed [18–20]

$$\phi\Gamma^I = -\tfrac{1}{3}\phi^{I[3]}P\Gamma_{[3]}, \qquad \Gamma^I\phi = +\tfrac{1}{3}\phi^{I[3]}N\Gamma_{[3]},$$
$$\phi^{[I|[3]}\Gamma_{[3]}{}^{|JK]} = +3\phi^{[IJ|[2]}\Gamma_9\Gamma^{|K]}{}_{[2]}, \quad (2.12a)$$
$$\mathcal{P}\phi^{[IJ|[2]}\Gamma^{|K]}{}_{[2]} = -6\mathcal{P}\Gamma^{IJK} + 2\mathcal{P}\phi^{IJKL}\Gamma_L, \quad (2.12b)$$
$$[\phi, \Gamma^{IJK}] = +2\phi^{IJKL}\Gamma_L - 3\Gamma_9\phi^{[IJ|[2]}\Gamma^{|K]}{}_{[2]}, \quad (2.12c)$$
$$\mathcal{P}\Gamma^I\mathcal{P} = 0, \qquad \mathcal{P}\Gamma^{IJ}\mathcal{P} = 0, \qquad \mathcal{P}\Gamma^{IJK}\mathcal{P} = 0, \quad (2.12d)$$
$$\begin{aligned} &\phi^{IJKL}\Gamma_L\mathcal{P}\Gamma_9 = -\Gamma^{IJK}\mathcal{P}\Gamma_9, \\ &\phi^{IJKL}\mathcal{P}\Gamma_9\Gamma_L = +\mathcal{P}\Gamma_9\Gamma^{IJK}. \end{aligned} \quad (2.12e)$$

III. TOTAL ACTION AND SYMMETRIES

Our field content is the same as the original $SO(4)_{\text{local}} \times SO(8)_{\text{global}}$ symmetric BL theory [3], namely, $(X_a{}^I, \psi_{\alpha\dot{A}a}, A_\mu{}^{ab})$. The indices $a, b, \ldots = 1, 2, \ldots, 4$ are for the vector representation of $SO(4)$, while $I, J, \ldots = 1, 2, \ldots, 8$ are for the $\mathbf{8}_V$ of $SO(8)$.

The indices $\alpha, \beta, \ldots = 1, 2$ are for the Majorana spinors in 3D, while $\dot{A}, \dot{B}, \ldots = \dot{1}, \dot{2}, \ldots, \dot{8}$ are for the $\mathbf{8}_C$ of $SO(8)$. These two sorts of indices are frequently omitted. Since all of these indices are contracted by positive definite metrics, there is no difference between their superscript and subscripts. Relevantly, we have $\Gamma_9\psi_a = -\psi_a$.

[3] The symbol $[n]$ means the totally antisymmetric n indices to save space.

Our total action $I \equiv \int d^3x \mathcal{L}$ has the lagrangian

$$\mathcal{L} = -\frac{1}{2}(D_\mu X_a{}^I)^2 + \frac{i}{2}(\bar{\psi}_a \gamma^\mu D_\mu \psi^a) - ic\epsilon^{abcd}(\bar{\psi}_a \Gamma_{IJ}\psi_b)X_c{}^I X_d{}^J - \frac{4}{3}c^2(\epsilon^{abcd}X_b{}^I X_c{}^J X_d{}^K)^2 + \frac{1}{64}c^{-1}\epsilon^{\mu\nu\rho}\epsilon_{abcd}\left(F_{\mu\nu}{}^{ab}A_{\mu\nu}{}^{cd} - \frac{2}{3}A_\mu{}^{ab}A_\nu{}^{ce}A_{\rho e}{}^d\right) - \frac{1}{2}m^2(X_a{}^I)^2 + \frac{i}{2}m(\bar{\psi}_a\psi^a) - \frac{2}{3}cm\epsilon^{abcd}\phi_{IJKL}X_a{}^I X_b{}^J X_c{}^K X_d{}^L, \tag{3.1}$$

where D_μ is the $SO(4)$ covariant derivative defined by [2,3]

$$D_\mu X_a{}^I \equiv \partial_\mu X_a{}^I + A_{\mu a}{}^b X_b{}^I, \quad D_\mu \psi_a \equiv \partial_\mu \psi_a + A_{\mu a}{}^b \psi_b. \tag{3.2}$$

As in the original system in [2,3,5], c is a nonzero real constant. All the m-dependent terms are new in (3.1); e.g., its last term contains ϕ^{IJKL}. Because of the presence of ϕ^{IJKL}, our action is *no* longer invariant under the original $SO(8)_{\text{global}}$, but is invariant under $SO(7)_{\text{global}}$, as will be explicitly given in (3.8).

Our action I is also invariant under reduced $N = 1$ supersymmetry

$$\delta_Q X_a{}^I = +i(\bar{\epsilon}\mathcal{P}\Gamma^I\psi_a) \equiv +i(\mathcal{P})_A{}^B(\Gamma^I)_B{}^{\dot{C}}(\bar{\epsilon}_A\psi_{\dot{C}a}), \tag{3.3a}$$

$$\delta_Q \psi_a = -(\gamma^\mu\Gamma_I\mathcal{P}\epsilon)D_\mu X_a{}^I + \tfrac{2}{3}c\epsilon^{abcd}(\Gamma_{IJK}\mathcal{P}\epsilon)X_b{}^I X_c{}^J X_d{}^K + m(\Gamma_I\mathcal{P}\epsilon)X_a{}^I, \tag{3.3b}$$

$$\delta_Q A_\mu{}^{ab} = -4ic\epsilon^{abcd}(\bar{\epsilon}\mathcal{P}\gamma_\mu\Gamma_I\psi_c)X_d{}^I. \tag{3.3c}$$

The reduction of supersymmetry $N = 8 \rightarrow N = 1$ is due to the projector $\mathcal{P}$ always multiplying the parameter ϵ.

As is easily seen from our lagrangian and transformation rule, the total on shell degrees of freedom $X_a{}^I(4 \times 8)$ plus $\psi_{\alpha\dot{A}a}(4 \times 8)$ will be the same as the original BL theory with $SO(4)_{\text{local}} \times SO(8)_{\text{global}}$ symmetry [3].

The reduction of $N = 8 \rightarrow N = 1$ by the parameter $\epsilon \equiv \mathcal{P}\epsilon$ is in a sense similar to the reduced supersymmetry $N = (1/8, 1)$ for "self-dual" supersymmetric Yang-Mills theory in Euclidean 8D [20]. In fact, the reduced supersymmetry 1/8 out of 1 is precisely caused by the restriction of $\mathcal{Q}\epsilon_+ \equiv 0$ equivalent to $\epsilon_+ \equiv \mathcal{P}\epsilon_+$ in 8D Euclidean space [20].

The confirmation of $\delta_Q I = 0$ is rather straightforward, as in [3], because the only new contributions are from the m-dependent terms. For example, the identity (2.12e) is crucial for the $m\chi\varphi^3$ terms in $\delta_Q I = 0$.

Our explicit field equations are[4]

$$\frac{\delta\mathcal{L}}{\delta X_{aI}} = +D_\mu^2 X^{aI} + 2ic\epsilon^{abcd}(\bar{\psi}_b\Gamma^{IJ}\psi_c)X_{dJ} - 48c^2 X^b{}_J X^c{}_K X^{a[I}X_b{}^J X_c{}^{K]} - m^2 X^{aI} - \frac{8}{3}cm\epsilon^{abcd}\phi^I{}_{JKL}X_b{}^J X_c{}^K X_d{}^L \doteq 0, \tag{3.4a}$$

$$\frac{\delta\mathcal{L}}{\delta\psi^a} = +i(\gamma^\mu D_\mu\psi_a) + 2ic\epsilon_a{}^{bcd}(\Gamma_{IJ}\psi_b)X_c{}^I X_d{}^J + im\psi_a \doteq 0, \tag{3.4b}$$

$$\frac{\delta\mathcal{L}}{\delta A_\mu{}^{ab}} = +\frac{1}{32}c^{-1}\epsilon^{\mu\nu\rho}\epsilon_{abcd}F_{\nu\rho}{}^{cd} + X_{[a|I}D^\mu X_{|b]}{}^I + \frac{i}{2}(\bar{\psi}_a\gamma^\mu\psi_b) \doteq 0. \tag{3.4c}$$

The on shell closure of two supersymmetry transformations

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_P(\xi) + \delta_G(\alpha) \equiv \delta_\xi + \delta_\alpha, \tag{3.5}$$

can be also confirmed, where δ_P and δ_G are the translation $\delta_P(\xi) \equiv \xi^\mu\partial_\mu$ and $SO(4)_{\text{local}}$ transformation with the parameters

$$\xi^\mu \equiv +2i(\bar{\epsilon}_1\mathcal{P}\gamma^\mu\epsilon_2), \qquad a^{ab} \equiv -\xi^\mu A_\mu{}^{ab}. \tag{3.6}$$

Compared with the massless case [3], there is no X^2 term involved in α^{ab}, because of $\mathcal{P}\Gamma^{IJ}\mathcal{P} \equiv 0$ in (2.12d). Because of the on shell feature of the system, the closure is valid only up to terms vanishing by the use of field equations (3.4)

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]X_a{}^I = \xi^\mu\partial_\mu X_a{}^I - \alpha_a{}^b X_b{}^I, \tag{3.7a}$$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]\psi_a = \xi^\mu\partial_\mu\psi_a - \alpha_a{}^b\psi_b + \frac{i}{2}\xi^\mu\left(\gamma_\mu\frac{\delta\mathcal{L}}{\delta\psi^a}\right), \tag{3.7b}$$

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]A_\mu{}^{ab} = \xi^\nu\partial_\nu A_\mu{}^{ab} + D_\mu\alpha^{ab} - 4c\epsilon_{\mu\nu\rho}\epsilon^{abcd}\xi^\nu\left(\frac{\delta\mathcal{L}}{\delta A_\rho{}^{cd}}\right). \tag{3.7c}$$

[4] We use the symbol $\doteq$ for a field equation, or for its solution (s).

The closure of supersymmetry with $SO(7)_{\rm global}$ transformation is

$$\delta_\beta X_a{}^I = -\beta^{IJ}_{(+)} X_{aJ}, \tag{3.8a}$$

$$\delta_\beta \psi_{\dot A a} = -\tfrac{1}{4}\beta^{IJ}_{(+)}(\Gamma_{IJ}\psi_a)_{\dot A} \equiv -\tfrac{1}{4}\beta^{IJ}_{(+)}(\Gamma_{IJ})_{\dot A}{}^{\dot B}\psi_{\dot B a}, \tag{3.8b}$$

$$\delta_\beta A_\mu{}^{ab} = 0. \tag{3.8c}$$

Note that the parameter $\beta^{IJ}_{(+)}$ is restricted by $\beta^{IJ}_{(+)} \equiv P^{IJ}{}_{KL}\beta^{KL}$. Here, the symbol $(+)$ denotes the **21** of $SO(7)$ out of the original **28** components in β^{IJ} projected out by $P_{IJ}{}^{KL}$. The spinor charge and the original $SO(8)$ generators T^{IJ} satisfy

$$[Q_{\alpha A}, T^{IJ}] = +\tfrac{1}{2}(\Gamma^{IJ})_A{}^B Q_{\alpha B}. \tag{3.9}$$

However, δ_Q and δ_β commute, due to the restriction of supersymmetry $\epsilon = \mathcal{P}\epsilon$. In fact, from (3.9) we get[5]

$$[\delta_Q, \delta_\beta] = [(\bar\epsilon\mathcal{P}Q), -\tfrac{1}{4}\beta^{IJ}_{(+)}T_{IJ}] = -\tfrac{1}{4}\beta^{IJ}_{(+)}(\bar\epsilon\mathcal{P}\Gamma^{(+)}_{IJ}Q) \equiv 0, \tag{3.10}$$

due to the identity [18,19]

$$\mathcal{P}\Gamma^{(+)}_{IJ} \equiv 0, \qquad \Gamma^{(+)}_{IJ} \equiv P_{IJ}{}^{KL}\Gamma_{KL}. \tag{3.11}$$

Under $SO(8) \rightarrow SO(7)$, the original **28** of $SO(8)$ decomposes into β^{IJ} into **21** + **7** of $SO(7)$, where the **7** for $\beta^{IJ}_{(-)}$ completely disappears from our system. The explicit confirmation of $[\delta_Q, \delta_\beta] = 0$ on all the fields is rather straightforward. All of these are consistent with the fact that the parameter $\mathcal{P}\epsilon$ reduces the original $N = 8$ supersymmetry to $N = 1$, while $\beta^{IJ}_{(+)}$ reduces the original $SO(8)_{\rm global}$ to $SO(7)_{\rm global}$.

Because of the mass parameter m in our system, the original global conformal symmetry [2,3,5] is manifestly broken, as is easily seen from the terms in our lagrangian with m.

IV. POTENTIAL, VACUUM CONFIGURATIONS AND SELF-DUAL VECTOR

Our bosonic potential U in our lagrangian is positive definite:

$$\begin{aligned} U &= +\frac{1}{2}m^2(X_a{}^I)^2 + \frac{4}{3}c^2(\epsilon^{abcd}X_b{}^IX_c{}^JX_d{}^K)^2 \\ &\quad + \frac{2}{3}cm\epsilon^{abcd}\phi_{IJKL}X_a{}^IX_b{}^JX_c{}^KX_d{}^L \\ &= +\frac{1}{2}\left(mX_a{}^I + \frac{2}{3}c\epsilon^{abcd}\phi_{IJKL}X_b{}^JX_c{}^KX_d{}^L\right)^2 \\ &= +\frac{1}{2}\left(\frac{\partial W}{\partial X_a{}^I}\right)^2 \geq 0, \end{aligned} \tag{4.1}$$

[5]The spinor charge Q here should not be confused with the $\mathcal{Q}$'s in (2.11).

where W is a "superpotential"

$$W \equiv +\tfrac{1}{2}m(X_a{}^I)^2 + \tfrac{1}{6}c\epsilon^{abcd}\phi_{IJKL}X_a{}^IX_b{}^JX_c{}^KX_d{}^L. \tag{4.2}$$

In order to reach the perfect-square form (4.1), we need a nontrivial identity

$$\phi_{JKLM}Y_{aI}{}^{JK}Y^{aILM} \equiv 0, \tag{4.3}$$

for $Y^{aJKL} \equiv \epsilon^{abcd}X_b{}^JX_c{}^KX_d{}^L$. This can be confirmed as follows: First define $Z^{IJ} \equiv X_a{}^IX^{aJ}$, and note that

$$Y_{aIJK}Y^{aILM} = +2Z_I{}^IZ_{[J}{}^LZ_{K]}{}^M + 4Z^I{}_{[J}Z_{K]}{}^{[L}Z_I{}^{M]}. \tag{4.4}$$

Because of the symmetry $Z^{IJ} = +Z^{JI}$, if we multiply (4.4) by $\phi^{JK}{}_{LM}$, we will get the vanishing result, confirming (4.3).

For the vacuum-structure analysis, it is advantageous to use the superpotential W, because $\delta_Q\psi_a$ is simplified as

$$\delta_Q\psi a = -(\gamma^\mu\Gamma_I\mathcal{P}\epsilon)D_\mu X_a{}^I + (\Gamma^I\mathcal{P}\epsilon)\left(\frac{\partial W}{\partial X^{aI}}\right). \tag{4.5}$$

Although this simplification looks similar to that in [17], it is the result of peculiar features of octonion structure constants such as (2.12e).

The $N = 1$ supersymmetric vacuum structure can be analyzed by $\partial W/\partial X_a{}^I \doteq 0$,

$$mX_a{}^I \doteq -\tfrac{2}{3}c\epsilon_a{}^{bcd}\phi^I{}_{JKL}X_b{}^JX_c{}^KX_d{}^L. \tag{4.6}$$

As the simplest nontrivial ansatz, we put

$$X_a{}^I \doteq \begin{cases} X_a{}^{b'} \neq 0 & (\text{for } I = b' = 1, 2, 3, 8), \\ X_a{}^{b''} \doteq 0 & (\text{for } I = b'' = 4, 5, 6, 7). \end{cases} \tag{4.7}$$

The expression $X_a{}^{b'} \neq 0$ means that the matrix $(X_a{}^{b'})$ is *not a zero matrix*. Since ϕ^{IJKL} is totally antisymmetric and $\phi^{1238} = +1$, $\phi^{a'b'c'd'}$ can be regarded as an ϵ tensor $\epsilon^{a'b'c'd'}$, even though the index 8 instead of 4 is the last index for $a', b', \ldots = 1, 2, 3, 8$. Equation (4.6) is now related to $\det X \equiv \det(X_a{}^{b'})$,

$$\begin{aligned} mX_a{}^{a'} &\doteq -\frac{2}{3}c\epsilon_a{}^{bcd}\epsilon^{a'}{}_{b'c'd'}X_b{}^{b'}X_c{}^{c'}X_d{}^{d'} \\ &= -\frac{1}{6}c\frac{\delta}{\delta X_a{}^{a'}}[(4!)\det(X_b{}^{b'})] \\ &= -4c[\det(X_b{}^{b'})]X_{a'}{}^a, \end{aligned} \tag{4.8}$$

where $(X_{a'}{}^a) \equiv X^{-1}$. In terms of matrices, this is equivalent to

$$mX^2 \doteq -4c(\det X)I \qquad (I \equiv \mathrm{diag}(1, 1, 1, 1)). \tag{4.9}$$

We can get a class of solutions to (4.9) as follows. First, take the determinant of both sides to get

$$\det X \doteq \pm\frac{m^2}{16c^2} = \pm\eta^4 \qquad \left(\eta \equiv +\frac{1}{2}\sqrt{\left|\frac{mc}{c}\right|}\right). \tag{4.10}$$

Substituting this back into (4.9) yields

$$X^2 \doteq \mp \frac{m}{4c} I = \mp \mathrm{sgn}(mc)\eta^2 I. \tag{4.11}$$

Next, we look for diagonal solutions[6]

$$X = \mathrm{diag}(\mu_1, \mu_2, \mu_3, \mu_4) \qquad (\mu_i \epsilon \mathbb{R}). \tag{4.12}$$

Then its substitution into (4.11) fixes $\mu_i = \pm\eta$, so that

$$X \doteq +\eta \mathrm{diag}(\pm 1, \pm 1, \pm 1, \pm 1), \tag{4.13}$$

where all the double signs are arbitrary.

As the simplest example, we consider

$$X \doteq +\eta I. \tag{4.14}$$

Then the original $SO(4)_{\mathrm{local}} \times SO(4)_{\mathrm{global}}$ symmetry is broken down to the diagonal $SO(4)_{\mathrm{global}}$, because (4.14) is invariant under the $SO(4)_{\mathrm{global}}$ transformation

$$\delta_\zeta X_a{}^{b'} = -\zeta_a{}^c X_c{}^{b'} - \zeta^{b'c'} X_{ac'}. \tag{4.15}$$

Here (ζ^{ab}) is a 4×4 antisymmetric constant parameter matrix for $SO(4)_{\mathrm{global}}$. If instead we choose

$$X \doteq +\eta \mathrm{diag}(+1, +1, -1, -1), \tag{4.16}$$

the remaining symmetry is the product of the diagonal $SO(2)$'s: $SO(2)_{\mathrm{global}} \times SO(2)_{\mathrm{global}}$. Alternatively, the choice of

$$X \doteq +\eta \mathrm{diag}(+1, +1, +1, -1). \tag{4.17}$$

corresponds to $SO(3)_{\mathrm{global}}$.

For all of these solutions (4.14), (4.16), and (4.17), $N = 1$ supersymmetry is intact, because $\delta_Q \psi_a = 0$ is preserved for $\partial W / \partial X_a{}^I \doteq 0$ with the constant $X_a{}^I$ in (4.5).

Once the vacuum expectation value (v.e.v.) $\langle X_a{}^I \rangle$ has been developed, the vector field $A_\mu{}^{ab}$ is no longer "auxiliary" but propagating. This is associated with the self-duality condition on a vector field in 3D [21]. For example, the v.e.v. (4.14) provides the mass term for $A_\mu{}^{ab}$ *via* the X-kinetic term as in the Higgs mechanism, so that

$$\frac{\delta \mathcal{L}}{\delta A_{\mu ab}} = + \frac{1}{32c} \epsilon^{\mu\nu\rho} \epsilon^{ab}{}_{cd} F_{\nu\rho}{}^{cd} - \frac{m}{4c} A_\mu{}^{ab} + \mathcal{O}(\varphi^2) \doteq 0, \tag{4.18a}$$

$$\Rightarrow F_{\mu\nu}{}^{ab} + m \epsilon_{\mu\nu}{}^\rho \epsilon^{ab}{}_{cd} A_\rho{}^{cd} + \mathcal{O}(\varphi^2) \doteq 0, \tag{4.18b}$$

where $\mathcal{O}(\varphi^2)$ stands for any interaction terms. Equation (4.18b) is the so-called self-duality condition on $A_\mu{}^{ab}$ [21], with the only difference that ϵ^{abcd} is in the second term. As in [21], applying ∂^μ to (4.18b) implies $A_\mu{}^{ab}$ propagating with the mass $2m$,

$$\partial_\mu^2 A_\nu{}^{ab} - 4m^2 A_\nu{}^{ab} + \mathcal{O}(\varphi^2) \doteq 0. \tag{4.19}$$

Interestingly, our mass deformation results in the self-dual and massive propagating vector field for the nontrivial $\langle X_a{}^I \rangle$. This resembles the massless case [13], where the nontrivial v.e.v. $\langle X^{A(I)} \rangle$ results in the propagation of $A_\mu{}^{ab}$.

V. CONCLUDING REMARKS

In this brief report, we have presented a mass-deformed BL theory, in terms of octonion structure constants ϕ^{IJKL}. We have seen that while the original $SO(8)_{\mathrm{global}}$ is reduced to $SO(7)_{\mathrm{global}}$, the original $N = 8$ supersymmetry is reduced to $N = 1$. We have also confirmed the positive definiteness of the bosonic potential U, as well as the superpotential W whose derivative is involved in $\delta_Q \psi_a$, as usual in supersymmetric theories. Compared with the model [17] in which the original $N = 8$ is maintained, and $SO(8)_{\mathrm{global}}$ is broken into $SO(4)_{\mathrm{global}} \times SO(4)_{\mathrm{global}}$, our lagrangian explicitly breaks both $N = 8 \rightarrow N = 1$ and $SO(8)_{\mathrm{global}} \rightarrow SO(7)_{\mathrm{global}}$.

We have given the condition (4.6) on the constant $\langle X_a{}^I \rangle$ for nontrivial $N = 1$ supersymmetric vacuum configurations. A class of solutions is given by (4.13), if we restrict $X_a{}^I$ to be a square matrix $(X_a{}^{b'})$. The first vacuum configuration is (4.14) with the reduced symmetry $SO(4)_{\mathrm{global}}$, as the diagonal of the original $SO(4)_{\mathrm{local}} \times SO(8)_{\mathrm{global}}$. Other solutions in (4.16) and (4.17), respectively, preserve the symmetries $SO(2)_{\mathrm{global}} \times SO(2)_{\mathrm{global}}$ and $SO(3)_{\mathrm{global}}$ as the subgroups of the original $SO(4)_{\mathrm{local}} \times SO(8)_{\mathrm{global}}$. The $N = 1$ supersymmetry is also preserved by these configurations.

Analyzing the $A_\mu{}^{ab}$-field equation for the nontrivial vacuum configuration (4.14), we have found that $A_\mu{}^{ab}$ starts propagating with the mass $2m$. This mechanism is well known as the self-dual vector fields in 3D [21]. It is interesting that such a self-duality is implied by BL theory [2,3] after our mass deformation.

We emphasize that these are the consequences of the special role played by the octonion structure constants ϕ^{IJKL}, via nontrivial relationships, such as (2.12). These indicate many other possible unexpected aspects of BL theory [2,3] yet to be discovered.

If no matter multiplets are present in 3D, there is no upper limit for the number of extended supersymmetric CS theories, which are called $\mathcal{N}_0$ supersymmetric CS theories [22]. In contrast, once matter multiplets are included, $N = 8$ supersymmetry [2,3] is now supposed to be the maximal global supersymmetry. However, this present common wisdom itself might be evaded, in a way similar to BL theory [2,3] that circumvented the widely-held belief [23] about the limit $N \leq 3$ for matter-coupled supersymmetric CS theory.

The new links between BL theory [2,3] and octonions shown in this paper suggest further unknown features of

[6]Because of the index range $b' = 1, 2, 3, 8$, the last column of (4.12) corresponds to $b' = 8$.

BL theory with M theory [1] or M2-branes that has yet to be discovered. We believe that our result here will be helpful in applying the BL formulation [2,3] to more complicated system or studying multiple M2-branes.

After this work had been completed, we encountered a new paper [24], where all the possible Bogomol'nyi-Prasad-Sommerfield states of BL theory are systematically analyzed in the massless case [2,3] *before* mass deformations.

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