

Cosmological equivalence principle and the weak-field limit

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The strong equivalence principle is extended in application to averaged dynamical fields in cosmology to include the role of the average density in the determination of inertial frames. The resulting cosmological equivalence principle is applied to the problem of synchronization of clocks in the observed universe. Once density perturbations grow to give density contrasts of order 1 on scales of tens of megaparsecs, the integrated deceleration of the local background regions of voids relative to galaxies must be accounted for in the relative synchronization of clocks of ideal observers who measure an isotropic cosmic microwave background. The relative deceleration of the background can be expected to represent a scale in which weak-field Newtonian dynamics should be modified to account for dynamical gradients in the Ricci scalar curvature of space. This acceleration scale is estimated using the best-fit nonlinear bubble model of the universe with backreaction. At redshifts $z \lesssim 0.25$ the scale is found to coincide with the empirical acceleration scale of modified Newtonian dynamics. At larger redshifts the scale varies in a manner which is likely to be important for understanding dynamics of galaxy clusters, and structure formation. Although the relative deceleration, typically of order 10^{-10} ms^{-2} , is small, when integrated over the lifetime of the universe it amounts to an accumulated relative difference of 38% in the rate of average clocks in galaxies as compared to volume-average clocks in the emptiness of voids. A number of foundational aspects of the cosmological equivalence principle are also discussed, including its relation to Mach's principle, the Weyl curvature hypothesis, and the initial conditions of the universe.

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I. INTRODUCTION

The strong equivalence principle (SEP) stands at the conceptual core of general relativity, as a physical principle which limits the choice of our physical theory of gravitation among all possible metric theories of gravitation one can construct. In this paper I will argue that the ramifications of this principle have not been fully explored, and that its physical interpretation requires further clarification to deal with the dynamical properties of spacetime inherent in Einstein's theory when the nonequilibrium situation is considered. In particular, the problem of how to synchronize clocks in the absence of a spacetime background with specific symmetries does not have a general solution in general relativity. In this paper I will show that at least for universes which began with a great deal of symmetry, as ours did, the reasoning of the equivalence principle can be extended: the average regional density provides a clock in expanding regions. This has particular consequences for cosmological models and the definition of gravitational energy. It underlies the author's proposal that dark energy is a misidentification of cosmological gravitational energy gradients in an inhomogeneous void-dominated universe [1–3]. Broader foundational consequences may also follow.

To set the scene, it pays to recall that, historically, the equivalence principle [4], and indeed general relativity [5], was formulated at a time before the dynamical properties of spacetime were understood. The conceptual route that Einstein took began with the *weak equivalence principle* or the *principle of uniqueness of free fall*, known since the experiments of Galileo, that all bodies (subject to no forces other than gravity) will follow the same paths given the same initial positions and velocities. Realizing that this observational statement embodies a feature of universality of the gravitational interaction, Einstein created a theory in which gravity is a property of spacetime itself.

Einstein's identification of what the true gravitation field should be began 101 years ago with first identifying what it is not, based on thought experiments with elevators, concerning what motions of particles cannot be distinguished operationally. His 1907 principle of equivalence [4] may be translated as follows: *All motions in an external static homogeneous gravitational field are identical to those in no gravitational field if referred to a uniformly accelerated coordinate system.* A uniformly accelerated reference frame may be found operationally in empty Minkowski spacetime by firing rockets; if matter is the source of the true gravitational field, then such choices of frame cannot represent gravity.

More generally, since special relativity with nongravitational forces appears to always be valid in small regions, one should always be able to get rid of gravity near a point.

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This is embodied in the SEP: *At any event, always and everywhere, it is possible to choose a local inertial frame such that in a sufficiently small spacetime neighborhood all nongravitational laws of nature take on their familiar forms appropriate to the absence of gravity, namely, the laws of special relativity.* This means that gravity is made to be universal, as it is contained in spacetime structure. The true gravitational field strength is encoded in the Riemann curvature tensor, and is determined regionally by the tidal effects of geodesic deviation.

One of the most profound and difficult consequences of the SEP is that gravitational energy and momentum cannot be described by a local density, and so are not local quantities. General relativity overcomes the nonlocality problem of Newtonian gravity: there is no action at a distance. General relativity is an entirely local theory in the sense of *propagation* of the gravitational interaction, which is causal. However, the background on which the interaction propagates may contain its own energy and momentum, when integrated over sufficiently large regions, and this has to be understood in the calibration of local rods and clocks at widely separated events.

Whereas the calibration of rods and clocks is mathematically determined by invariants of the *local* metric, and the spacetime connection which relates local invariants at widely separated events, in practice we cannot analytically solve Einstein's equations for the most general distribution of matter to unambiguously determine the metric and its connection. A slicing of spacetime into hypersurfaces, and a threading of these hypersurfaces by timelike worldlines of observers or by null geodesics, is inevitable for any operational description of spacetime in terms of rods and clocks. Such splittings of space and time, together with additional symmetry assumptions, are also necessary for analytically solving Einstein's equations in particular cases, or more generally, for numerical modeling.

The definition of quasilocal gravitational energy and momentum [6] then turns out to depend on the choice of slicing, associated surfaces of integration, and the identification of observers that thread the slices. These procedures are inherently noncovariant and nonunique, and many questions of naturalness of any particular definition inevitably arise. (See Ref. [7] for a recent discussion.) We have the dilemma that a spacetime split inevitably breaks any given particle motion into a motion of the background and a motion with respect to the background; and this may involve a degree of arbitrariness. The viewpoint that will be adopted here is that since quasilocal gravitational energy gradients have their origin in the equivalence principle, the primary criterion for making canonical identifications of physically relevant classes of observer frames is that the equivalence principle itself must be properly formulated, and respected, when making macroscopic cosmological averages.

If the SEP is applied to macroscopic objects, then, strictly speaking, we can only apply it to systems in which

the gravitational interaction is ignored. Yet we implicitly apply the SEP to scales at least as large as galaxies which are treated as particles of dust in an expanding fluid—with an expansion rate given by a Friedmann-Lemaître-Robertson-Walker (FLRW) model—subject to additional Newtonian gravitational interactions. The Newtonian approximation, which is made in present-day numerical simulations of structure formation, must, by the rules of general relativity, presuppose that a sufficiently large static Minkowski frame can be found.

I shall argue in this paper that to correctly derive a Newtonian limit, without prior assumptions about the cosmological background geometry, we must first correctly apply the SEP. If galaxies are to be treated as particles of dust, we must address the following question: given that the background is not static, what is the largest scale on which the SEP can be applied? An attempt to answer this question, which is unavoidable if we are to consistently apply the principles of general relativity to cosmology, means that we have to deal with the relationship of inertial frames to averages of matter fields and motions. In other words, we must address Mach's principle, which may be stated as [8,9] follows: "*Local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions.*" The leading questions in this statement are what is to be understood by "local," and what is the "suitable weighted average"?

The problem with any process of averaging in general relativity is that by coarse graining we can lose information about the calibration of local rods and clocks within the coarse-grained cells relative to average quantities. Rods and clocks are related to invariants of the local metric, can vary greatly within an averaging cell, and will not, in general, coincide with some average calibration of rods and clocks once averaging cells become sufficiently large in the nonlinear regime of general relativity. Galaxies, for example, contain supermassive black holes, in whose local neighborhood the determination of proper lengths and times for typical observers differs extremely from typical observers in the outskirts of a galaxy.

It is commonly believed that as long as we are "in the weak-field limit" we do not have to worry about complications such as the extreme ones posed by black holes. However, the weak-field limit is always taken about a *background*, and once inhomogeneities develop in the universe there are no exact symmetries to describe the background. One set of uniformly calibrated rods and clocks is no longer sufficient to describe the background itself. Given an initially homogeneous isotropic universe with small scale-invariant density perturbations, once one is within the nonlinear regime of structure formation, homogeneity and isotropy are only defined in a statistically average sense.

Within a cell of statistical homogeneity—which can be taken to be [1] of the same order as the baryon acoustic

oscillation (BAO) scale, $100h^{-1}$ Mpc, h being the dimensionless parameter related to the Hubble constant by $H_0 = 100h$ km sec $^{-1}$ Mpc $^{-1}$ —there are density contrasts of order unity over scales of tens of megaparsecs. Since the universe is inhomogeneous over these scales *not every observer is the same average observer*. Different classes of canonical observers are required to interpret cosmological parameters [1]. In particular, we and the objects we see are typically in galaxies in locally nonexpanding regions which formed from density perturbations which were greater than the critical density. In such regions local spatial curvature can differ markedly from the volume-average location in voids, where space is freely expanding. Small differences of spatial curvature and rates of clocks can accumulate to give large differences over the lifetime of the universe [1]. Dynamical gradients in the curvature of space are a physical reality which must be properly understood.

In this paper I will estimate the scale of the small relative deceleration of the background in regions of different density, by proposing an extension of the SEP to cosmology. I will demand an equivalence between the particular example of geodesic flow of a congruence of particles “at rest” in a dynamically expanding universe, whose average volume expansion is governed by an average over all masses and motions of the particles, and the equivalent “volume-expanding” motion of point particles in a Minkowski space. In other words, the local Ricci scalar curvature due to the volume average of pressureless dust can always be “renormalized away” on sufficiently small scales, but in a way which may lead to relative recalibrations of local rods and clocks between different spacetime regions.

Equivalently, for volume expansion we cannot *locally* distinguish between particles at rest in a dynamic spacetime and particles moving in a static spacetime: the two situations are equivalent in a sense which deserves the designation *cosmological principle of equivalence*. This might be viewed as a further Machian style refinement of the notion of inertia. Although we measure geodesic deviation, in terms of the scalar curvature part of the Riemann tensor and volume expansion, we are unable to distinguish whether the geodesics are deviating because of local accelerations of particles in a static space, or whether the particles are at rest in an expanding space which is decelerating due to the gravitational attraction of the average density of matter.

Historically, it might be said that although Einstein was conceptually guided by Mach’s principle, he never quite succeeded in fully implementing it in general relativity, because when he first formulated the theory he did not fully appreciate the dynamical nature of spacetime. His first attempt to study cosmology indicated that for any usual source of matter the theory was not stable, but intrinsically dynamical [10]. Famously, he invoked the cosmological

constant—his “*größte Eselei*”—to avoid the issue. This paper attempts to lay the conceptual groundwork for an alternative first principles route to the physical interpretation of cosmological general relativity, taking account of observational evidence that is immeasurably better now than it was in 1917.

The plan of the paper is as follows. Preliminary definitions, motivations, and a statement of the cosmological equivalence principle (CEP) are presented in Sec. II. In Sec. III the thought experiments introduced in Sec. II are generalized to the case of regions of different density, and an estimate of clock rate variance is given based on presently observed density contrasts. The definition of the cosmic rest frame, and its relation to other frames used in cosmological averaging, is clarified in Sec. IV. In Sec. V a numerical estimate of the time-varying relative deceleration of voids relative to walls, where galaxies are located, is computed over the lifetime of the universe, and its cosmological implications discussed. The role of initial conditions and a possible conceptual relationship to Penrose’s Weyl curvature hypothesis are discussed in Sec. VI. The paper concludes with a summary discussion, including some further speculations, in Sec. VII.

II. AVERAGING AND THE EQUIVALENCE PRINCIPLE

The SEP addresses physics on relatively small scales in which matter can be treated by field theories in a local inertial frame (LIF). In a LIF fundamental nongravitational interactions are described by Lagrangian field theories, and in four spacetime dimensions the mass m and spin s are given by the Casimir invariants of representations of the local Lorentz group, $P^\mu P_\mu = -m^2 c^2$ and $W^\mu W_\mu = s(s+1)m^2 c^2 \hbar^2$, where P^μ is a particle 4-momentum and W^μ the Pauli-Lubanski pseudovector. To make a transition to macroscopic scales we have to take averages of these quantities to obtain a hydrodynamic limit: a fluid description with an effective energy-momentum tensor, not derivable from a field Lagrangian.

Following the end of the radiation dominated epoch, on cosmological scales of averaging, we largely deal with situations in which the massive matter fields form condensates for which the average spin is zero, and can be treated as pressureless dust described by one scalar parameter, the density ρ . Neutrinos, on account of their ultralight masses, near relativistic speeds, and their interesting property that mass eigenstates do not coincide with flavor eigenstates, are an exception to this rule. However, neutrinos are so light that from the point of view of the cosmological background they can be effectively treated as a massless species.

Massless particles which are relevant for cosmology will be considered. Gravitational waves will not be considered here, as their cosmological contribution at late epochs is negligible. Electromagnetic waves are considered, as they

are the means by which almost all our information about the universe is transmitted. For the purpose of cosmological averages it is sufficient that light propagation can be treated in the geometric optics limit, and that the cosmic radiation background has a perfect fluid equation of state $P = \frac{1}{3}\rho c^2$.

The question of the largest scale on which the SEP can apply has not, to the best of my knowledge, been addressed in a fundamental way. Taken literally, as soon as we are dealing with scales on which particle masses must be treated as an integrated density, then the SEP can only apply if we ignore the gravitational interaction. Of course, in practice, for many practical purposes we can treat the gravitational interaction by Newtonian gravity on the scale of the solar system, and the scale of galaxies. However, the true nature of general relativity is to replace the Newtonian gravitational force by a dynamical spacetime, not to replace spacetime by Newtonian gravity in the limit of weak fields.

It must be recalled that, whereas Newtonian gravity is a nonlocal interaction on a static space, general relativity is a local dynamical theory of spacetime. The curvature of the spacetime background is not local, and the Newtonian limit picks up the aspect of the nonlocal curvature of the background generated by slowly moving point particles, but in full general relativity, changes in the curvature of the background can only be built up over time by locally propagating processes. This means that even in the weak-field limit there exist circumstances in which Newtonian gravity on a static background is not an appropriate limit. In cosmology the background is not static, and thus clearly the Newtonian approximation may break down. While much has been achieved numerically by assuming N -body Newtonian interactions on a preexisting FLRW background, the universe is clearly inhomogeneous on scales of at least tens of megaparsecs at the present epoch. To consider the dynamical gravitational processes which lead to macroscopic variations of the spacetime background over these scales, we need to go back to first principles.

A. The cosmological equivalence principle

My proposed answer to the question of the largest scale on which the SEP can apply is to deal with the average effects of density by extending the SEP to potentially larger scales while removing the time-translation and boost symmetries of the background as follows:

At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighborhood, the cosmological inertial frame, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation,

$$ds_{\text{CIF}}^2 = a^2(\eta)[-d\eta^2 + dr^2 + r^2d\Omega^2], \quad (1)$$

where $d\Omega^2$ is the metric on a 2-sphere.

The standard SEP is obtained in the limit that $a(\eta)$ is constant, which physically corresponds to virialized systems that can be effectively thought of as asymptotically flat. Alternatively, during very short time intervals over which the time variation of the scale factor can be neglected, the standard SEP is also retrieved. The idea here is that even when spacetime is dynamical in cosmology, one can always find a spacetime neighborhood whose average volume expansion can be characterized by a metric of the form (1) for time intervals over which $a(\eta)$ varies. The metric (1) is of course that of the spatially flat FLRW geometry in conformal coordinates.

The rationale for the statement of the CEP is twofold. First, if we are to demand a smooth Newtonian gravitational limit in all circumstances, we have to deal with the fact that Newtonian gravity deals with just one scalar source, the density, whereas general relativity is tensorial. This means that we must be dealing with an average spacetime with symmetries in taking a Newtonian gravity limit. Since gravitation with matter is dynamical, it is the symmetries involving time that should be removed from the SEP to account for the average density of matter. The metric (1) does this while preserving the isotropy and homogeneity of space regionally within a cosmological inertial frame (CIF).

Second, at the core of the equivalence principle is the notion that we can always choose a LIF, for example, by specifying Riemann normal coordinates. However, in the case of the volume-expanding and contracting motions of the metric (1), as illustrated by Fig. 1, it is impossible to *locally* distinguish between the case of comoving particles at rest in an expanding metric (1) and the case of particles in motion in the static Minkowski space of the relevant LIF, a point which has been emphasized by a number of authors recently from various points of view [11]. On local scales, both yield the Hubble law redshift

$$z \simeq \frac{H_0 \ell_r}{c}, \quad H_0 = \left. \frac{\dot{a}}{a} \right|_{t_0}$$

where ℓ_r is the radial proper distance from an observer at the origin to a source, and an overdot denotes a derivative with respect to t where $cdt = ad\eta$. This is true whether the exact relation, $z + 1 = a_0/a$, is used or the radial Doppler formula, $z + 1 = [(c + v)/(c - v)]^{1/2}$, of special relativity is used, before making a local approximation [12]. Mathematically, the equivalence of the two situations might be viewed as a consequence of $\partial/\partial\eta$ being a conformal Killing vector of (1).

The aim of the CEP is to go beyond the limit of the static special relativistic LIF, to consider arbitrarily long time intervals over which the motion of the particles is decelerated, $\ddot{a} < 0$, by the average density of matter. As Einstein himself stated [10], “In a consistent theory of relativity there can be no inertia relatively to ‘space,’ but only an inertia of masses relatively to one another.” Since the

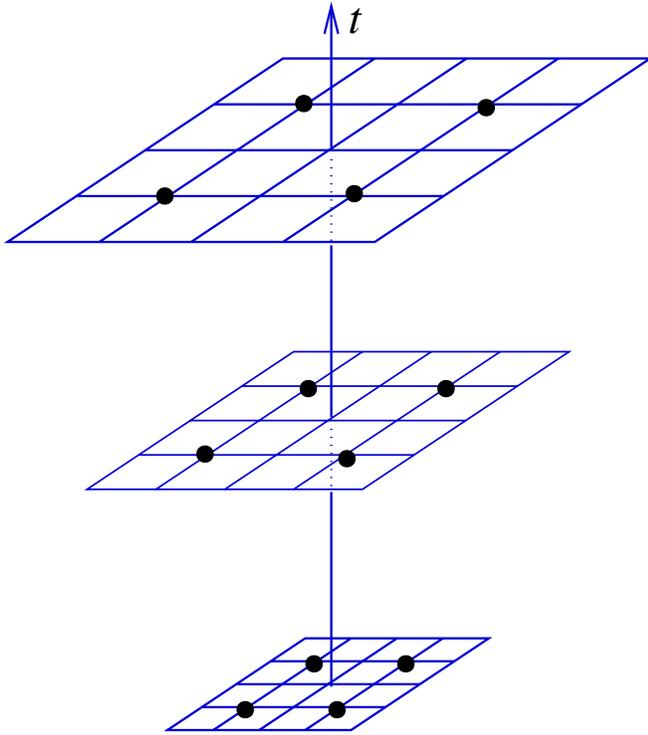


FIG. 1 (color online). A set of particles undergoes an isotropic spatial 3-volume expansion in a spatially flat local region. For the same initial conditions, provided that we consider time intervals over which deceleration of the expansion is negligible, it is impossible to locally distinguish the case of particles at rest in a dynamically expanding cosmological space from particles moving isotropically in a static Minkowski space. One spatial dimension is suppressed.

deceleration of the volume expansion is due to the back-reaction of the average density of the matter particles in defining their average background, the CEP thus represents a refinement in the definition of inertial frames. To demonstrate this at a conceptual level, we will first show that for localized regions a suitable equivalent of decelerated Minkowski particles can always be found for the motion of a congruence of comoving particles in (1), even for arbitrarily long time intervals.

B. Thought experiment: The semi-tethered lattice

Given a spatial point, taken to coincide with the origin of the coordinates (1), we might be tempted to think that we can mimic the situation of an arbitrary decelerating expansion of (1) in Minkowski space by attaching rockets to a set of test particles initially chosen to be equidistant from the given center and with identical radial velocities. The rockets could then be fired in unison at the same rate to give an inward radial acceleration while maintaining equidistance of the particles from the center and equality of instantaneous radial velocities as seen from the frame of the central observer with global Minkowski time, t .

The rocket analogy is flawed, however. To see the flaw, let us consider a second Minkowski space thought experiment which is directly equivalent to the cosmological situation of comoving particles moving in the geometry (1), even in the case of deceleration of the volume flow. Let us construct what I will call the *semi-tethered lattice* by the following means. Take a lattice of Minkowski observers, initially moving isotropically away from each nearest neighbor at uniform initial velocities [13]. The lattice observers are chosen to be equidistant along mutual oriented \hat{x} , \hat{y} , and \hat{z} axes. Now suppose that the observers are each attached to six others by strings of negligible mass and identical tension along the mutually oriented spatial axes, as in Fig. 2. The string in each observer’s negative \hat{x} , \hat{y} , and \hat{z} directions is held fixed and extends to the observer’s nearest neighbor in those directions. The string extending towards each nearest neighbor in the positive \hat{x} , \hat{y} , and \hat{z} directions unreels freely from a spool at the observer’s site on which an arbitrarily long supply of string is wound. The strings initially unreel at the same uniform rate, representing a “recession velocity.” Each observer carries synchronized clocks [14], and at a prearranged local proper time, all observers apply a braking mechanism, the braking mechanisms having been preprogrammed to deliver the same impulse as a function of local time.

The semi-tethered lattice is directly analogous to the case of the decelerating volume expansion of (1) due to some average homogeneous matter density, because it maintains the homogeneity and isotropy of space over a region as large as the lattice. In the case of the firing of rockets, the act of firing a rocket means that each observer with a rocket feels a net force in a particular direction, while also breaking the symmetry of the homogeneity of

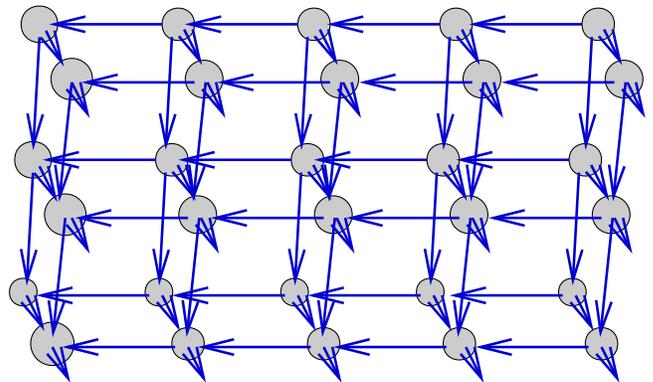


FIG. 2 (color online). The semi-tethered lattice. Point particle observers in a homogeneous spatial lattice, initially expanding with uniform velocity, are attached to nearest neighbors by strings. Each string is fixed at one end. The arrowheads denote the free end of each string wound on a spool at a neighboring particle, to which the observers apply brakes in a synchronized fashion according to a predetermined plan. The time evolution of the lattice follows a course similar to that of the spatial grid in Fig. 1, but with deceleration.

space. In the case of the semi-tethered lattice, work is done in applying the brakes, and energy can be extracted from this. However, since brakes are applied in unison (according to local proper time at each lattice site), there is *no net force on any observer in the lattice*. Although the rate at which the brakes are applied can be an arbitrary prearranged function of local proper time, provided the braking function is applied uniformly at every lattice site, the clocks will remain synchronous in the comoving sense, as all observers have undergone the same relative deceleration.

The semi-tethered lattice is also a useful analogy because the kinetic energy of the particles is converted directly to heat in the brakes, which might then be converted to other forms. It is a direct analogue for the conversion of kinetic energy of particles in the expansion of the universe to other forms of energy via gravitational collapse. Apart from the energy of massless and near massless species released during recombination and earlier phase transitions, all the useful forms of energy in the present observable universe have undergone such a process of transformation: from the kinetic energy of expansion to gravitational energy, and then to other forms.

The semi-tethered lattice and the CEP might be said to extend the elevator thought experiments and the 1907 Einstein equivalence principle [4] in a natural fashion to the case of a homogeneous isotropic *nonstatic* gravitational field [15]. The important point in the present situation is that, in both the cosmological case and the semi-tethered lattice analogy, the observers feel *no net force* from the relative deceleration, which justifies the description of (1) as a cosmological *inertial frame*.

C. Cosmological inertial frames and averaging

Although (1) is simply the standard spatially flat FLRW geometry, the important physical difference from the usual treatment of cosmology is that here it is not to be viewed as a global metric of spacetime, but as an average cosmological frame over some region. The FLRW geometries with spatial sections of positive and negative spatial Gaussian curvature, k ,

$$ds_{\text{FLRW}}^2 = \tilde{a}^2(\tilde{\eta}) \left[-d\tilde{\eta}^2 + \frac{d\tilde{r}^2 + \tilde{r}^2 d\Omega^2}{(1 + \frac{1}{4}k\tilde{r}^2)^2} \right], \quad (2)$$

can of course also be brought into the form (1) over small spatial domains as compared to the radius of curvature. This follows directly from the fact that the FLRW geometries are conformally flat and coordinates can be found to bring (2) into the form [16]

$$ds_{\text{FLRW}}^2 = a^2(\eta, r) [-d\eta^2 + dr^2 + r^2 d\Omega^2]. \quad (3)$$

The new physical interpretation is that no single FLRW geometry (2) is to be taken as a global average geometry for the whole universe for all times. However, with an extension of the SEP to account for the cosmological

average effects of the density of matter, regional cosmological inertial frames with average geometry (1) can always be found. It should also be emphasized that since these are average frames, with differing regional scale factors and local coordinates, no metric of the form (2) is substituted into the Einstein equations and solved. Rather, an appropriate average of the full inhomogeneous Einstein equations, such as a Buchert average [17], should be applied to solve for the background average of the inhomogeneous geometry. One can solve the Buchert equations for a realistic approximation to the observed universe [2], but care must be taken in interpreting the solution, as we must account for where the observers are within the inhomogeneous structure when it comes to the relative calibration of their rods and clocks [1].

In the case of a semi-tethered lattice which is confined to a finite region, the relative deceleration of the region would give a local proper time to comoving observers in the lattice different from that of a global Minkowski observer, even if this observer had a synchronous clock before the volume deceleration began. By the CEP the average homogeneous density in different regions likewise sets a standard of local time for CIFs, and this may vary significantly when regional density contrasts grow.

Since any CIF (1) is an average, the relationship of the average to the inhomogeneous geometry needs to be stated. For example, in terms of the proper time, $cdt = ad\eta$, of an observer “comoving” with the CIF, the uniform expansion of the CIF should be viewed as an average only:

$$\frac{\dot{a}}{a} \equiv \frac{1}{3} \langle \theta \rangle_{\text{CIF}} = H_{\text{CIF}}, \quad (4)$$

where θ is the volume expansion, angle brackets denote the appropriate average, and an overdot denotes a derivative with respect to t . In relation to the averaging scheme, the specification of a CIF should incorporate a notion of “center-of-mass motion” in the sense that the variance in the CIF volume expansion is an average of shear and vorticity fluctuations for which the net backreaction *within* a CIF vanishes. In the notation of Buchert and Carfora [18],

$$\delta^2 H_{\text{CIF}} = \frac{1}{9} (\langle \theta^2 \rangle_{\text{CIF}} - \langle \theta \rangle_{\text{CIF}}^2) = \frac{1}{3} \langle \sigma^2 \rangle_{\text{CIF}}, \quad (5)$$

in Buchert’s scheme with vanishing vorticity, where $\sigma^2 = \frac{1}{2} \sigma_{\mu\nu} \sigma^{\mu\nu}$ is the scalar shear. The condition (5) ensures that the average kinematic backreaction within a CIF vanishes in Buchert’s scheme. For CIFs containing galaxies, vorticity will be important, and so Buchert’s scheme needs to be generalized to include vorticity before a precise statement can be made. As the contributions of vorticity and shear are of opposite sign in the Raychaudhuri equation, their average contributions may even be largely self-canceling, for virialized systems at least. However, a precise formalism including vorticity remains to be developed. It is possible that ideas could be incorporated from Zalaletdinov’s averaging scheme [19], though the physical content of that

scheme first needs to be more clearly elucidated in a cosmological setting [20].

For the present paper it is sufficient that the backreaction can be neglected in characterizing the average properties within a CIF. Thus a CIF is characterized by a single scalar, the volume expansion (4). In an expanding universe with collapsing substructure, this volume expansion is best viewed as a macroscopic property of a given CIF, but often not of its more finely grained subregions. In particular, if a CIF contains a galaxy or a galaxy cluster it will contain virialized regions and may also contain collapsing regions. For cosmological averages, when one is interested in comparing deceleration rates in expanding regions of different density, the notion of a finite infinity cutoff scale [1,21] is suggested as a minimum region for a CIF in relevant averages.

III. THOUGHT EXPERIMENT: RELATIVE HOMOGENEOUS ISOTROPIC DECELERATION

The beauty of the equivalence principle is that it allows one to quantitatively deduce the order of magnitude of simple effects, such as the leading order of gravitational redshift [4], by simple thought experiments alone. The basic physical effect that is of interest here—the gravitational energy cost of a spatial curvature gradient—can likewise be understood by a simple thought experiment.

From the evidence of the cosmic microwave background (CMB) radiation, we know that, apart from tiny fluctuations of order $\delta\rho/\rho \sim 10^{-5}$ in photons and the baryons they couple to, and density fluctuations perhaps an order of magnitude larger in nonbaryonic dark matter particles, the observable universe was close to spatially flat, homogeneous, and isotropic at the epoch of last scattering. Assuming the Copernican principle, it was sufficient to describe the universe by a single frame (1) at that epoch. However, since a CIF is a local regional frame, we should be careful never to construct a CIF with spatial extent larger than the particle horizon at any epoch. Rather, it is better physically to think of a class of local CIFs at disjoint spatial locations at the surface of last scattering, in which there exist geodesic congruences of observers like those depicted in Fig. 1, which all have an identical uniform expansion away from each other, on account of the initial uniformity of the Hubble flow at that epoch.

Let us first analogously consider two sets of disjoint semi-tethered lattices, with identical initial local expansion velocities, in a static background Minkowski space. [See Fig. 3(a).] The observers in the first congruence apply brakes in unison to decelerate homogeneously and isotropically, with inward 4-acceleration of magnitude, $\alpha_1(\tau_1)$, as measured by the force applied to the brakes in the frame of any of the observers, where τ_1 is the proper time measured by any of them. From the viewpoint of a global Minkowski observer, members of the congruence will agree on their measurements of time, τ_1 , though this

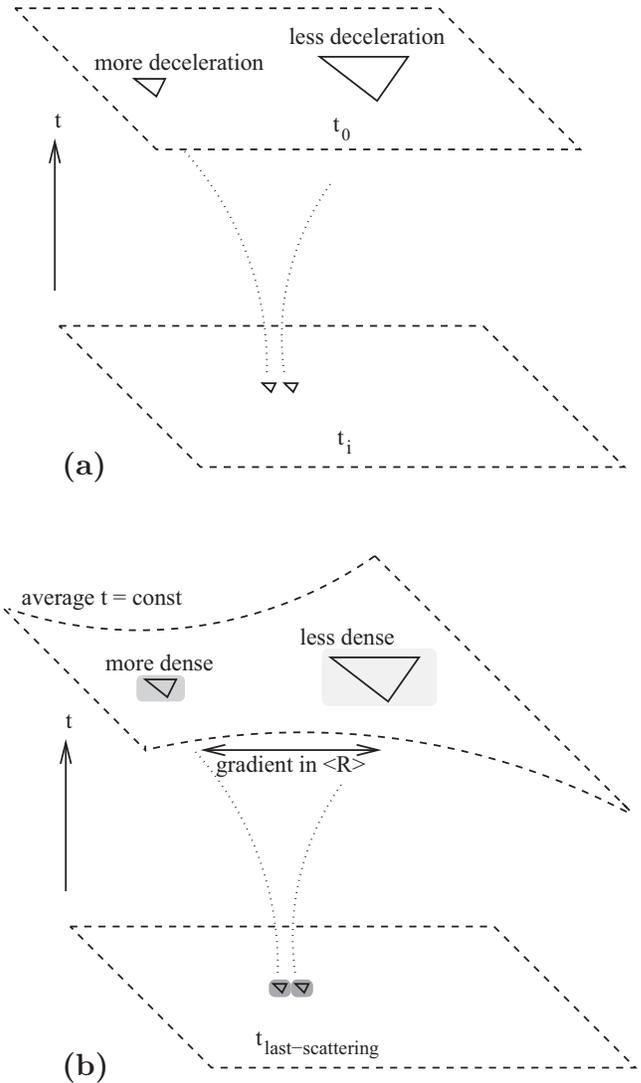


FIG. 3. Two equivalent situations: (a) in Minkowski space observers in separate semi-tethered lattices, initially expanding at the same rate, apply brakes homogeneously and isotropically within their respective regions but at different rates; (b) in the universe which is close to homogeneous and isotropic at last scattering, observers in separated regions initially recede from each other at the same rate, but experience different locally homogeneous isotropic decelerations as local density contrasts grow. In both cases there is a relative deceleration of the observer congruences, and those in the region which has decelerated more will age less.

time of course differs from the global Minkowski observer's time, t .

Now take a second semi-tethered lattice, with the same initial expansion speed, where brakes are applied with a force corresponding to a 4-acceleration of magnitude $\alpha_2(\tau_2)$. At any global Minkowski time t , we will assume that when transformed from their proper frames to that of the global Minkowski observer, at each time step $\alpha_1(t) > \alpha_2(t) > 0$. It is then the case that the members of the first

congruence decelerate more than the members of the second congruence, and at any time t the proper times satisfy $\tau_1 < \tau_2$. The members of the first congruence age less quickly than members of the second congruence.

By the CEP, the case of volume expansion of two regions of different average density at late times is entirely analogous. The fact that we are able to apply the equivalence of the two circumstances rests on the fact that the expansion of the universe was extremely uniform at the time of last scattering, by the evidence of the CMB. At this epoch all regions effectively have the same density—apart from negligible fluctuations—and the same uniform Hubble flow. At late epochs, suppose that in the frame of any average cosmological observer there are regions of different density which have decelerated by different amounts by a given time, t , according to that observer. Then, by the CEP, the local proper time of the isotropic observers in the denser region, which has decelerated more, will be less than that of the equivalent observers in the less dense region which has decelerated less. [See Fig. 3(b).] Consequently the *proper time of the observers in the more dense CIF will be less than that of those in the less dense CIF*, by equivalence of the two situations.

The fact that a global Minkowski observer does not exist in the second case does not invalidate the argument. The global Minkowski time is just a coordinate label, and in the cosmological case the only restriction on the $t = \text{const}$ slices is that *the expansion of both average congruences must remain homogeneous and isotropic in local regions of different average density* in the global $t = \text{const}$ slice. Of course, we need to be careful to patch together different CIFs continuously to specify the slice, as we will further discuss in Sec. IV. In this way the equivalence to the Minkowski space case is maintained. Thus in the cosmological case, provided that we refer to *local* homogeneous isotropic expansion in different regions on any average $t = \text{const}$ slice (where t is some coordinate label), then if such regions *are still expanding* and have a significant density contrast, we can expect a significant clock rate variance.

This equivalence directly establishes the idea of a *gravitational energy cost for a spatial curvature gradient*, since the existence of expanding regions of different density within an average $t = \text{const}$ slice implies a gradient in the average Ricci scalar curvature, $\langle \mathcal{R} \rangle$, on one hand, while the fact that the local proper time varies on account of the relative deceleration implies a gradient in gravitational energy on the other.

A. Order of magnitude estimate of clock rate variance

An order of magnitude estimate of present epoch clock rate variances due to gravitational energy gradients induced by relative volume deceleration of the background can now be made by directly using observationally measured density contrasts. Although a CIF is a frame (1) within which backreaction can be neglected, to determine

its scale factor over long periods of time one must consider the evolution of the universe within which the CIF is embedded. Such evolution will, in general, include the effects of backreaction. However, if the backreaction is small, an order of magnitude estimate of the clock rate variance can be made assuming that regions with observed strong density contrasts evolve independently by solutions of the local Friedmann equation for regions of different density. There will be a relative deceleration of the local background of such regions, which via equivalence to the Minkowski space semi-tethered lattices, will accumulate clock rate differences.

Galaxies formed from perturbations which were greater than critical density and if space is negatively curved on average, they must always be bounded by a region which is spatially flat. These on-average spatially flat locally expanding bounding regions are called *finite infinity* regions [1,21], and a union of such regions is called a *wall*. Since they are spatially flat, neglecting backreaction, the evolution of the wall CIFs can be approximated by spatially flat Einstein-de Sitter regions with local scale factor $a_w = a_i (\frac{3}{2} H_i \tau_w)^{2/3}$, where H_i is the common initial Hubble parameter at last scattering, and a_i is a constant. On the other hand, CIFs within voids can be approximated by portions of spatially open FLRW solutions, given parametrically by

$$a_v = \frac{a_i \tilde{\Omega}_i}{2(1 - \tilde{\Omega}_i)} (\cosh \tilde{\eta} - 1), \quad (6)$$

$$H_i \tau_v = \frac{\tilde{\Omega}_i}{2(1 - \tilde{\Omega}_i)^{3/2}} (\sinh \tilde{\eta} - \tilde{\eta}), \quad (7)$$

where $cd\tau_v = a_v d\tilde{\eta}$, $\tilde{\Omega}_i = 1 - \epsilon_i$ is an initial density parameter at last scattering, $\epsilon_i \ll 1$, and a_i is a constant. Using (6) and (7) in the Friedmann equation, one obtains a standard parametric relation for the density parameter,

$$\tilde{\Omega}(\tilde{\eta}) = \frac{2(\cosh \tilde{\eta} - 1)}{\sinh^2 \tilde{\eta}}, \quad (8)$$

which is to be viewed here as a regional parameter in a CIF inside a void.

We now follow the analysis of Ref. [22], where the author's proposal was first advanced. There an attempt was made to estimate cosmological parameters by making the approximation that the evolution of the observable universe was entirely due to the voids. In fact, backreaction between the walls and voids must be included to obtain more reliable estimates of cosmological parameters [1,23]. However, if we confine attention to small regional CIFs, then the argument of Ref. [22] can give an estimate of clock rate variance from *observed* density contrasts. In particular, since the critical density also defines the Einstein-de Sitter standard of time of the wall CIFs, it also follows that

$$\tilde{\Omega} = \tilde{\Omega}_i \left(\frac{a_w}{a_v} \right)^3 = \frac{18H_i^2(1 - \tilde{\Omega}_i)^3 \tau_w^2}{\tilde{\Omega}_i^2 (\cosh \tilde{\eta} - 1)^3}. \quad (9)$$

Combining (8) and (9) we find

$$H_i \tau_w = \frac{\tilde{\Omega}_i (\cosh \tilde{\eta} - 1)^2}{3(1 - \tilde{\Omega}_i)^{3/2} \sinh \tilde{\eta}}. \quad (10)$$

Differentiating both (7) and (10) we find a relative clock rate, which we will call the *lapse function*, given by

$$\gamma(\tilde{\eta}) \equiv \frac{d\tau_v}{d\tau_w} = \frac{3(\cosh \tilde{\eta} + 1)}{2(\cosh \tilde{\eta} + 2)}. \quad (11)$$

A relative clock rate variance due to the relative volume deceleration between CIFs in walls and voids can now be estimated since $\tilde{\Omega} = 1 + \tilde{\delta}$, where $\tilde{\delta}$ is the density contrast, and inverting (8) we have

$$\cosh \tilde{\eta} = \frac{2 - \tilde{\Omega}}{\tilde{\Omega}} = \frac{1 - \tilde{\delta}}{1 + \tilde{\delta}}. \quad (12)$$

Hoyle and Vogeley [24] estimate that 40%–50% of the present-epoch universe is in voids of diameter about $30h^{-1}$ Mpc, with the statistics summarized in Table I. The density contrasts quoted are an average, and are of greater magnitude in the centers of the voids which are extremely empty. Taking these values as indicative, if we assume $\tilde{\delta} = -0.9$ then (11) and (12) give $\gamma = 1.42$, while if we assume $\tilde{\delta} = -0.95$ then $\gamma = 1.46$. For larger density contrasts, in the center of the void the lapse would approach the limiting value $\gamma \rightarrow 1.5$, which represents the relative local expansion rates of an empty Milne universe [25] to an Einstein-de Sitter one.

These clock rate variances of 42%–46% are large, and counterintuitive given we usually encounter large clock rate differences only for large local boosts or for density contrasts from extremely compact sources in static backgrounds, such as black holes. However, the effect described is neither of these familiar situations, as the *background is not static*. Rather, it is the clock rate variance due to the cumulative effect of a very small relative deceleration of the background. The above variances are simply those demanded by the CEP taking present epoch density contrasts observed in the actual universe.

TABLE I. Dominant void statistics in the Point Source Catalogue Survey (PSCz), the Updated Zwicky Catalogue (UZC), and the 2 degree Field Survey (2dF) North Galactic Pole (NGP) and South Galactic Pole (SGP), from Refs. [24].

Survey	Void diameter	Density contrast
PSCz	$(29.8 \pm 3.5)h^{-1}$ Mpc	$\tilde{\delta} = -0.92 \pm 0.03$
UZC	$(29.2 \pm 2.7)h^{-1}$ Mpc	$\tilde{\delta} = -0.96 \pm 0.01$
2dF NGP	$(29.8 \pm 5.3)h^{-1}$ Mpc	$\tilde{\delta} = -0.94 \pm 0.02$
2dF SGP	$(31.2 \pm 5.3)h^{-1}$ Mpc	$\tilde{\delta} = -0.94 \pm 0.02$

Taking backreaction into account to determine a full present epoch average [1,2], the present epoch lapse difference between walls and a volume-average location in a void based on luminosity distance data fits is $\bar{\gamma} = 1.38^{+0.06}_{-0.05}$ [23]. This is an average value; the relative lapse would be larger in the void centers. Thus the estimates made without backreaction are reasonably accurate, showing that the effect is not a direct consequence of backreaction in the evolution equations but rather of relative volume deceleration alone. From (11) and (12) without backreaction the density contrast estimate for a local CIF at the volume-average position is $\tilde{\delta} = -0.83$ for a lapse of $\gamma = 1.38$. If such a clock variance were produced by a uniform acceleration in Minkowski space, a simple calculation shows it would require a tiny relative acceleration of order $5.5 \times 10^{-10} \text{ ms}^{-2}$ over the lifetime of the universe. Of course, such a relative acceleration is not uniform: a better estimate is presented in Sec. V.

The argument above relies on it being possible to choose a locally uniform Hubble flow gauge, as will be discussed in the next section. Such a gauge can be maintained outside finite infinity regions, but not within them where collapsing regions are located, and where vorticity and tidal torques become important. Thus there are no obvious inferences analogous to those above that can be made with respect to bound structures from observations of the magnitude of their positive density contrasts.

IV. AVERAGE FRAMES

A. The cosmic rest frame

In taking cosmological averages with inhomogeneous structures, the question arises as to which average frames have the most utility. One must generally make a choice of gauge in specifying such frames. One way of viewing the SEP is that we can always set the first derivatives of the metric to zero near a point. In particular, the volume expansion θ , which involves first derivatives of the metric, can always be set to zero. The CEP extends this by the statement that in the dynamical situation a gauge can be chosen in which the volume expansion in a CIF is spatially uniform, but varying with time.

As was pointed out in Sec. III in order to compare CIFs in regions of different densities, we need to specify suitable spacelike slices [26] by patching together different CIFs in a continuous manner. Operationally, the way to do this is first to choose an orientation of the 4-velocity fields, $\partial/\partial\tau_I$, of comoving observers in a CIF *such that the CMB radiation is isotropic* in each frame. In terms of the local proper time τ_I of such observers, the metric (1) is rewritten

$$ds_{\text{CIF}}^2 = -c^2 d\tau_I^2 + a_I^2(\tau_I)[dr_I^2 + r_I^2 d\Omega^2], \quad (13)$$

where the index I runs over CIFs. Second the spacelike slice is specified by the demand that the *locally measured value* of the volume expansion remains uniform as one

moves from the patch of one CIF to the next. In other words, the “local Hubble flow” remains uniform in this gauge even though the proper lengths and proper time scales will change as one moves between CIFs of different density. As discussed in Ref. [1], although the proper volume of voids increases faster than that of wall regions, this is compensated for *locally* by the faster relative rate of the void clocks. Relative to any one set of clocks, such as our own, it will always appear that voids expand faster than walls. So the average Hubble flow over both walls and voids—by one set of clocks—will generally differ from the underlying uniform flow. Its value is a choice of gauge depending on the choice of fiducial observer.

The uniform Hubble flow slices defined in this manner constitute the *cosmic rest frame*: surfaces within which the CMB is isotropic, even though the mean value of the CMB temperature, and angular scale of CMB anisotropies, will vary from point to point as spatial volumes vary in relation to proper radius with changes in spatial curvature. The proper times of CIF observers within the slice will also vary. We can choose the clocks of a canonical set of observers in expanding regions with the same average local density to label the slices, provided we realize that this time labeling is only a proper time in particular locations on the slice and is just a coordinate label elsewhere.

The uniform Hubble flow gauge is one of the standard gauges of perturbation theory in FLRW models [27], and has been further refined with the addition of a minimal shift distortion condition by Bičak, Katz, and Lynden-Bell [9]. These authors recognize the resulting “Mach 1 gauge” as one of three possible gauges which best incorporates Mach’s principle, and within which there is a minimal amount of residual gauge freedom. Bičak, Katz, and Lynden-Bell work within the framework of perturbations of a global FLRW geometry. The viewpoint of the present paper is that there is no single global FLRW geometry. However, on account of the CEP the spatially flat and negatively curved FLRW geometries can be considered as local regional geometries on spacelike slices as the density varies relative to the critical density. Thus a modification of the formalism of Bičak, Katz, and Lynden-Bell may be an appropriate starting point to deal with an averaging formalism for the full nonlinear inhomogeneous problem, as an alternative to Buchert’s scheme.

B. Buchert averaging

Buchert’s averaging scheme [17] is based on the starting point that, in the case of an energy-momentum tensor for irrotational dust particles in the presence of inhomogeneities, one can choose Gaussian normal coordinates

$$ds^2 = -c^2 dt^2 + {}^3g_{ij} dx^i dx^j, \quad (14)$$

comoving with the dust. The scalar density appearing in the energy-momentum tensor,

$$T^{\mu\nu} = \rho c^2 \bar{n}^\mu \bar{n}^\nu$$

where $\bar{n}^\mu = \frac{dX^\mu}{dt}$, then represents the rest mass density of the dust, and one averages over spatial slices of constant t orthogonal to the flow, over regions which conserve the rest mass of a portion of the fluid in a domain, \mathcal{D} , with continuity equation

$$\partial_t \langle \rho \rangle + \frac{\dot{\bar{a}}}{\bar{a}} \langle \rho \rangle = 0, \quad (15)$$

where $\bar{a}(t) \equiv [\mathcal{V}(t)/\mathcal{V}(t_0)]^{1/3}$ with $\mathcal{V}(t) \equiv \int_{\mathcal{D}} d^3x \sqrt{\det^3 g}$. Here angle brackets denote the spatial volume average of a quantity, so that $\langle \mathcal{R} \rangle \equiv (\int_{\mathcal{D}} d^3x \sqrt{\det^3 g} \mathcal{R}(t, \mathbf{x})) / \mathcal{V}(t)$ is the average spatial curvature, for example. The Buchert equations consist of (15) and

$$\frac{3\dot{\bar{a}}^2}{\bar{a}^2} = 8\pi G \langle \rho \rangle - \frac{1}{2} c^2 \langle \mathcal{R} \rangle - \frac{1}{2} \mathcal{Q}, \quad (16)$$

$$\frac{\ddot{\bar{a}}}{\bar{a}} = -4\pi G \langle \rho \rangle + \mathcal{Q}, \quad (17)$$

$$\partial_t (\bar{a}^6 \mathcal{Q}) + \bar{a}^4 c^2 \partial_t (\bar{a}^2 \langle \mathcal{R} \rangle) = 0, \quad (18)$$

where $\mathcal{Q} = \frac{2}{3} (\langle \theta^2 \rangle - \langle \theta \rangle^2) - 2 \langle \sigma^2 \rangle$ is the kinematic backreaction. Equation (18) is an integrability condition which ensures closure of the other equations.

Since the backreaction term \mathcal{Q} includes variance in volume expansion, and this is to be evaluated on a constant t slice, it is clear that as compared to the cosmic rest frame of Sec. IVA, different physical premises underlie the interpretation implicitly assumed by Buchert’s scheme. My approach is therefore different from other approaches to cosmological building that have been adopted in the context of Buchert averaging [28–31]. The differences may be understood from the fact that, in talking about the “rest mass density of the dust,” one is actually dealing with a concept which depends on the manner in which dust “particles” are coarse grained. Since all forms of energy have a rest mass equivalent, the kinetic energy of particles within a dust particle is included as rest mass. Similarly, since Ricci curvature affects spatial volumes relative to their diameter, the concept of a rest density depends on the scale of coarse graining relative to the curvature scale.

In general, the notion of “comoving with the dust” implicit in Buchert’s scheme can be very distinct from “comoving with the background,” although the notions coincide for FLRW models. This is well illustrated by the exact spherically symmetric Lemaître-Tolman-Bondi (LTB) models [32] for pressureless dust, with a prescribed inhomogeneous density $\rho(t, r)$. These can be written in Gaussian normal form (14), making it straightforward to compute a Buchert average [33–35]. At fixed comoving proper time t , as the radial coordinate r varies the LTB dust

shells have different densities, different spatial curvature, nonzero shear, and, in general, observers at $r > 0$ would not expect to see an isotropic CMB. Since the solution is completely dynamical, there is no average homogeneous isotropic background with respect to which one could be comoving, unless one puts in such a background by hand by making the model asymptotic to an FLRW model at large r .

With respect to fixed FLRW backgrounds an alternative, simple way to treat spherical inhomogeneities is by the spherical top hat model, using concentric spherical shells [36,37]. In the case of a void in a background Einstein-de Sitter universe, for example, a spherical underdense shell will acquire a peculiar velocity with respect to the background which tends to 50% of the background Hubble rate at late times [38]. One can account for the kinetic energy of the shell, but in view of the large peculiar velocity of the shell, there is a limit to the extent to which it can be considered comoving with respect to the background with a synchronous clock.

Once one averages on the scale of statistical homogeneity, as in (15), one wants to have a sense of “comoving with the background”; i.e., different observers in different averaging cells should have a notion of determining the same average density at the same cosmological epoch, and one should be able to talk about motion with respect to canonically defined observers.

In general, when the background is only *statistically* homogeneous and isotropic, there is an ambiguity in distinguishing between motion of the background and motion with respect to the background. The spirit of the CEP is that in the case of volume expansion, this is because there is a fundamental indistinguishability: the Hubble parameter is a gauge choice. The definition of the uniform Hubble expansion gauge of Sec. IVA makes an unambiguous separation of the “kinetic energy of the volume expansion of space” for regions which are *locally spatially flat* from other forms of energy. In the standard interpretation of Buchert averaging, depending on the choice of averaging volume and the manner in which one coarse grains over dust cells, the kinetic energy of the volume expansion of space can be intermingled with other forms of energy.

The view that I adopt is that one can either use Buchert averaging in defining CIFs on small scales by the requirement that the kinematic backreaction can be neglected, as in (5), or alternatively, on dust cells of at least the scale of statistical homogeneity, of about $100h^{-1}$ Mpc, at which scale the volume expansion is statistically uniform. In both cases the time parameter and $\langle \mathcal{R} \rangle$ can be regarded as relevant parameters which describe the *collective* degrees of freedom of the cell. However, they should not be regarded as representative for observers in finer partitions within the cell. For CIFs containing galaxies with black holes, this imperative is obvious. In the case of statistically homogeneous cells in cosmology, the case has been less

obvious, but I believe it is equally imperative on account of the density contrasts that are observed between voids and walls below the scale of homogeneity, together with the arguments of Sec. III.

I will take the view that the Buchert time parameter is the relevant one for an observer at a volume-average position in a statistically homogeneous cell. As the present universe is dominated by voids, this will be in a void but not at the void centers. Kinematic backreaction between voids and walls from the volume-average perspective must be included in determining the average evolution. Einstein’s equations are causal and depend on all events within the past light cone. Thus some sort of spatial averaging, such as Buchert averaging, is required to determine cosmic evolution. Buchert’s equations should thus be viewed as evolution equations.

Observations are made on null cones, however. Thus a Buchert average is not the one we perform operationally in determining cosmic averages. Instead, a radial null geodesic average of a solution to the Buchert equations, combined with an operational identification of relevant classes of observers within the inhomogeneous structure, is required to make comparisons with observations. This is the approach adopted in Refs. [1,2,23].

V. RELATIVE DECELERATION OF THE BACKGROUND AND THE WEAK-FIELD LIMIT

In Refs. [1,2] a model universe was constructed based on a regional division of cells of average homogeneity into voids, and the bubble walls that surround them, assuming that backreaction within the walls and voids can be neglected, but not in the combined average [39,40]. Space within the walls is assumed to be spatially flat on average, and space within the voids is negatively curved. Technically a “wall” is understood to be a union of connected finite infinity regions [1], namely, CIFs in regions of average critical density. Observationally, the walls would include all morphological types of extended structures containing galaxy clusters: namely, “sheets,” “filaments,” and “knots” [41].

Qualitatively, the regional division into voids and walls may well be a consequence of the evolution of the scales with statistical excesses of power in the primordial angular power spectrum. The $100h^{-1}$ Mpc scale of statistical homogeneity would correspond to the first Doppler peak (BAO scale): on account of the finite sound speed in the primordial plasma, no structures in excess of this scale are expected statistically, with the exception of those random structures that arise by percolation. The scale of the $30h^{-1}$ Mpc dominant voids would correspond to the evolution of the second Doppler peak, namely, the first rarefaction peak, which is well within the nonlinear regime of structure formation. The third Doppler peak, which is the first compression peak within the nonlinear regime, would give the scale of the largest bound structures that have

broken from the Hubble flow, namely, clusters of galaxies. Finite infinity represents a demarcation scale of on-average spatially flat regions between clusters of galaxies and voids. The fourth Doppler peak, the next rarefaction peak, may possibly give an independent scale corresponding to minivoids. In the two-scale approximation [42] of Refs. [1,2], we assume that the average curvature of minivoids and dominant voids is the same. A further refinement might separate these scales. Ultimately these qualitative speculations about the correspondence of the Doppler peaks to the observed scales of present epoch structures should be verified from a numerical model of structure formation.

An exact solution to the Buchert evolution equations of the two scales was found [2], which by a Bayesian analysis fits the Riess07 gold supernovae data set [43] at a level which is statistically indistinguishable from the standard spatially flat Λ CDM model [23]. The same best-fit parameters also fit the angular scale of the sound horizon seen in CMB data, and the effective comoving baryon acoustic oscillation scale seen in angular diameter tests of galaxy clustering statistics [1,23].

Those sceptical of the proposal sometimes question how the relatively large present lapse of $\bar{\gamma} = 1.38$ between wall observers and the volume average can have possibly arisen. As pointed out already in Sec. III A a relatively small relative deceleration of the background for the lifetime of the universe is sufficient to achieve this. Since the accumulated average lapse function $\bar{\gamma}$ is not uniform in time, the equivalent relative deceleration of the background is not uniform. In this section we will estimate the relative deceleration which would produce the tracker solution mean lapse function [2].

In principle, one is trying to compare the relative deceleration of regions of different density as a function of the expansion age of the universe of some fiducial observer such as ourselves. Ideally, if sources did exist freely falling in voids unbound to condensed structures, then by the assumptions of Ref. [1] they would have different redshifts to sources in bound structures that coexist “at the same epoch” on account of the accumulated gravitational energy differences. We would have

$$1 + z_w = \frac{(-\mathbf{k} \cdot \mathbf{U}_w)}{(-\mathbf{k} \cdot \mathbf{U}_{\text{obs}})} \quad (19)$$

for the redshift of a wall source with 4-velocity \mathbf{U}_w as seen by an observer with 4-velocity \mathbf{U}_{obs} , where \mathbf{k} is the 4-velocity of a radial null geodesic. Similarly

$$1 + z_v = \frac{(-\mathbf{k} \cdot \mathbf{U}_v)}{(-\mathbf{k} \cdot \mathbf{U}_{\text{obs}})} \quad (20)$$

where \mathbf{U}_v is a volume-average void source, and $z_v \neq z_w$. In general [1],

$$1 + z_w = (1 + z_v)/\bar{\gamma} \quad (21)$$

where similarly to (11) $dt = \bar{\gamma}d\tau_w$ represents the relative clock rates of volume-average observers to wall observers of emission. Unfortunately, we have a mass biased view of the universe and only observe sources in bound systems in regions which are locally spatially flat on average; otherwise the relative blueshift $-1 + \bar{\gamma}^{-1}$ of a volume-average void clock relative to a wall one would have an obvious observational signature nearby.

To determine the relative deceleration, we will invoke the CEP by considering the Minkowski space equivalent semi-tethered lattice analogy of Fig. 3. Ideally, we should have to calculate the difference in the rate of extraction of energy in applying brakes at different rates, to represent regions of different density in the actual universe. However, this should be equivalent to asking what relative volume deceleration would be required to produce $\bar{\gamma}(\tau_w)$ at any epoch, if $\bar{\gamma}(\tau_w)$ is treated as a Lorentz gamma factor in special relativity, beginning from two regions with the same initial expansion velocity (which is the situation at last scattering). In relation to (21), this is equivalent to treating the blueshift $-1 + \bar{\gamma}^{-1}$ of voids relative to walls as if it were a standard transverse Doppler shift in special relativity. Since we are dealing with an isotropic volume deceleration, there is no directional significance associated with a “direction of motion” in the special relativistic analogy.

We assume equivalence to the special relativistic 4-acceleration $\alpha = \frac{dU}{d\tau}$, where $U^\mu = \gamma(c, v^i)$, which has a magnitude [44]

$$\frac{\alpha}{c} = \frac{1}{\sqrt{\gamma^2 - 1}} \frac{d\gamma}{d\tau} = \frac{d}{dt} \sqrt{\gamma^2 - 1}, \quad (22)$$

with $dt = \gamma d\tau$. Of course, in the present case we are not really dealing with a proper acceleration in a single direction, as the appropriate analogy is that of two semi-tethered lattices in which all directions contribute. Assuming equivalence of the situations the relative acceleration of the background here has a magnitude

$$\frac{\alpha}{c} = \frac{d}{dt} \sqrt{\bar{\gamma}^2 - 1} = \frac{\bar{\gamma}(\bar{\gamma} \dot{\bar{H}} - \dot{H})}{\sqrt{\bar{\gamma}^2 - 1}} \quad (23)$$

where, following the notation of Refs. [1,2], t is the time parameter of the volume-average observer in a void, and $dt = \bar{\gamma}d\tau_w$, where τ_w is the time for an observer at finite infinity in a wall, which will be close to the time in an average galaxy. Furthermore, \bar{H} is the bare or locally measured Hubble parameter, while on account of Eq. (42) of Ref. [1], $H = \bar{\gamma} \bar{H} - \dot{\bar{\gamma}}$ is the dressed Hubble parameter obtained by averaging over both walls and voids on the past light cone.

Using the tracker solution of Ref. [2],

$$\bar{\gamma} = \frac{3}{2}t\bar{H}(t) \quad (24)$$

$$= 1 + \frac{1}{2}f_v(t) \quad (25)$$

$$= \frac{9f_{v0}\bar{H}_0 t + 2(1-f_{v0})(2+f_{v0})}{2[3f_{v0}\bar{H}_0 t + (1-f_{v0})(2+f_{v0})]}, \quad (26)$$

where \bar{H}_0 is the present epoch value of the bare Hubble parameter $\bar{H}(t)$, $f_v(t)$ is the void fraction, and f_{v0} its present epoch value. The void fraction here is that of all voids, including minivoids [45], and not just the dominant voids of Table I. The dressed Hubble parameter satisfies

$$H = \frac{2}{3t} + \frac{f_v(t)[4f_v(t) + 1]}{6t} = \bar{H}(t) + \frac{f_v(t)[4f_v(t) - 1]}{6t}, \quad (27)$$

while the time parameter τ_w of wall observers is related to that of volume-average ones by

$$\tau_w = \frac{2}{3}t + \frac{4\Omega_{M0}}{27f_{v0}\bar{H}_0} \ln\left(1 + \frac{9f_{v0}\bar{H}_0 t}{4\Omega_{M0}}\right), \quad (28)$$

where $\Omega_{M0} = \frac{1}{2}(1-f_{v0})(2+f_{v0})$ is the present epoch dressed matter density. From (23)–(26) we find

$$\frac{\alpha}{c} = \frac{3(1-f_{v0})(2+f_{v0})f_v(t)\bar{H}(t)}{2\sqrt{3f_{v0}\bar{H}_0 t[15f_{v0}\bar{H}_0 t + 4(1-f_{v0})(2+f_{v0})]}}. \quad (29)$$

A. Estimate of relative deceleration scale

Using the estimates of Ref. [23], at the present epoch the void fraction is $f_{v0} = 0.76^{+0.12}_{-0.09}$, while the dressed Hubble constant is $H_0 = 61.7^{+1.2}_{-1.1}$ km sec⁻¹ Mpc⁻¹, where the uncertainties are 1σ uncertainties from a fit to the Riess07 gold data set [43]. The bare Hubble constant is then $\bar{H}_0 = 48.2^{+2.0}_{-2.4}$ km sec⁻¹ Mpc⁻¹. From these values the present epoch magnitude of the relative deceleration (29) is $\alpha_0 = 6.7^{+2.4}_{-3.4} \times 10^{-11}$ ms⁻². Furthermore, since α is a time-varying quantity, its best-fit value is plotted as a function of redshift for recent epochs in Fig. 4.

It is interesting to note that over the range of redshifts $z \leq 0.25$ in Fig. 4, the curve for the best-fit parameters with present epoch void fraction $f_{v0} = 0.76$ precisely traverses a range of values for α that has been used for the empirical acceleration scale of the modified Newtonian dynamics (MOND) scenario [46]. In particular, using a range of recently quoted values [47], the MOND acceleration scale is $\alpha_{\text{mond}} = 1.2^{+0.3}_{-0.2} \times 10^{-10} h_{75}^2 \text{ ms}^{-2}$, where $h_{75} = H_0 / (75 \text{ km sec}^{-1} \text{ Mpc}^{-1})$. Using our best fit $H_0 = 61.7^{+1.2}_{-1.1}$ km sec⁻¹ Mpc⁻¹, this range corresponds to $\alpha_{\text{mond}} = 8.1^{+2.5}_{-1.6} \times 10^{-11} \text{ ms}^{-2}$.

The fact that α is larger at higher redshifts reflects the property that it scales in proportion to the bare Hubble parameter $\bar{H}(t)$, as can be seen from (29). Both the bare Hubble parameter and dressed Hubble parameter are of course larger at earlier epochs at higher redshift, since the universe is always decelerating in the present model. (As demonstrated in Refs. [1,2] cosmic acceleration is a purely apparent effect related to clock rate variance.) From (29)

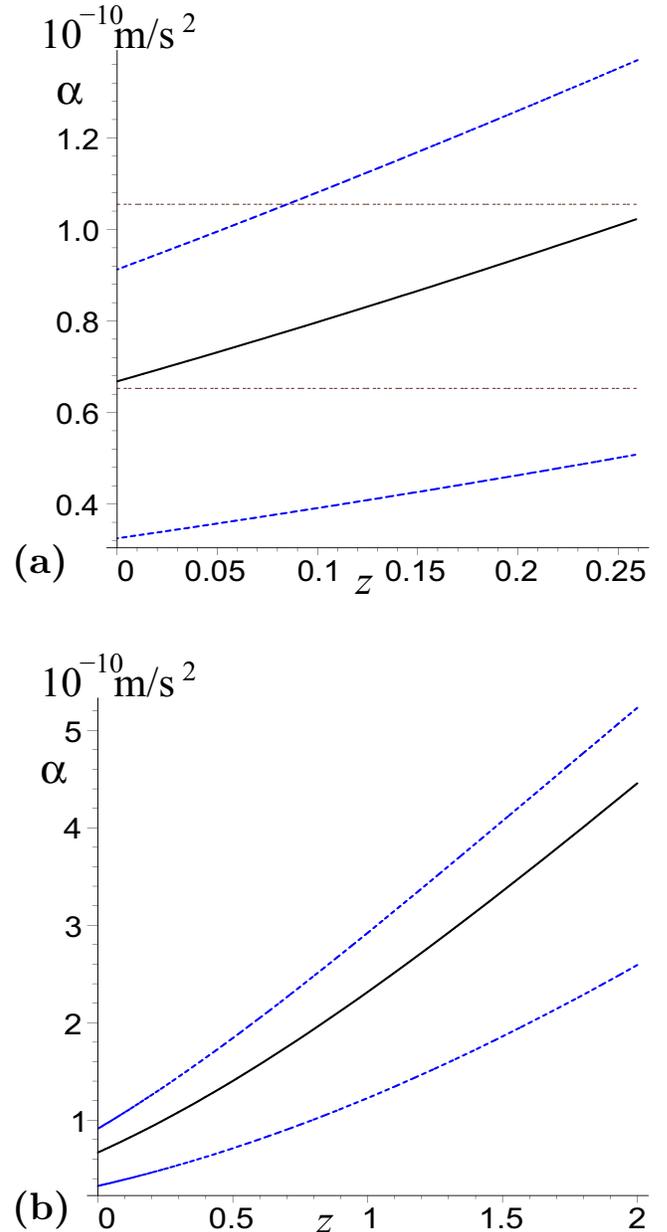


FIG. 4 (color online). The magnitude of the relative deceleration scale, α , as a function of redshift: (a) for redshifts $z < 0.25$; (b) for redshifts $z < 2$. The central curve shows the value for the best-fit parameters $f_{v0} = 0.76$, $H_0 = 61.7$ km sec⁻¹ Mpc⁻¹ ($\bar{H}_0 = 48.2$ km sec⁻¹ Mpc⁻¹) from Ref. [23]. The dashed curves show the corresponding results with 1σ uncertainties, which are largely due to the large uncertainty on f_{v0} , which is not tightly constrained by supernovae data. The upper dashed curve corresponds to $f_{v0} = 0.67$ and the lower dashed curve to $f_{v0} = 0.88$. In panel (a) the horizontal dotted lines indicate the bounds of the empirical acceleration scale of MOND when normalized for $H_0 = 61.7^{+1.2}_{-1.1}$ km sec⁻¹ Mpc⁻¹.

one sees that α also scales in proportion to $f_v(t)$ which is smaller at earlier times, contributing a term in competition with both the term $\bar{H}(t)$ in the numerator of (29) and with terms in the denominator. As a proportion of the Hubble

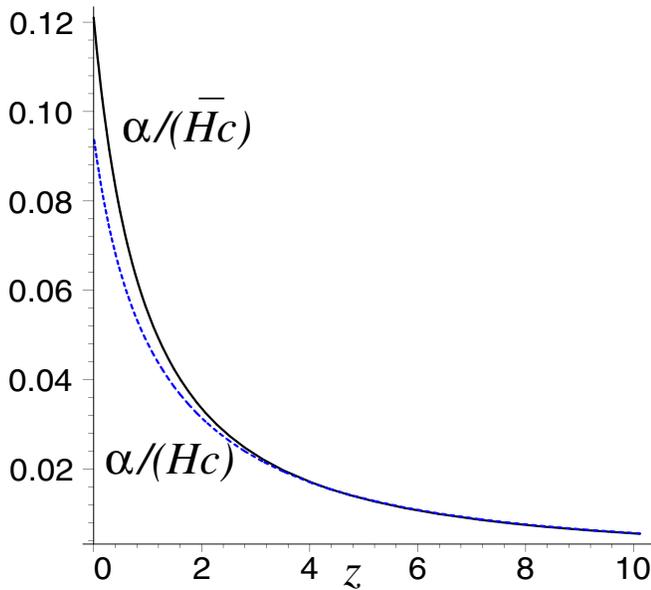


FIG. 5 (color online). The magnitude of the dimensionless ratios $\alpha/(c\bar{H})$ (solid curve) and $\alpha/(cH)$ (dashed curve), where $\alpha(z)$, $\bar{H}(z)$, and $H(z)$ are varied with redshift, for best-fit values of H_0 , f_{v0} .

flow at any epoch, the relative deceleration is *suppressed* at higher redshifts, as is shown in Fig. 5, where the dimensionless ratios $\alpha/(\bar{H}c)$ and $\alpha/(Hc)$ with respect to the bare and dressed Hubble parameters are plotted. At last scattering, $z \approx 1100$, $\alpha/(\bar{H}c) \sim 6 \times 10^{-6}$. As $t \rightarrow 0$, $\alpha/(\bar{H}c) \propto t^{1/2}$.

The object of estimating the relative deceleration scale was to determine whether its magnitude is physically acceptable. This is the case. Although the physical magnitude of the relative deceleration is larger at earlier epochs, over most of the history of the universe α is of the order of 10^{-10} ms^{-2} , which is tiny. From Fig. 4, at $z = 2$ we have $\alpha \approx 4.5 \times 10^{-10} \text{ ms}^{-2}$, which corresponds to an expansion age of 4 Gyr, 27% of the current age of the universe in wall time. By comparison, the Pioneer anomaly in the solar system [48] occurs at an acceleration scale of $(8.74 \pm 1.33) \times 10^{-10} \text{ ms}^{-2}$ [49], a value which α attains only at a redshift of 3.85 equivalent to an expansion age of 2.1 Gyr in wall time. Furthermore, the relative deceleration scale should largely affect dynamics in the transition zones between walls and voids. At earlier epochs the void fraction is less: at $z = 2$ it is 44%, and at $z = 3.85$ it is 28%. At $z = 10$, when $\alpha \approx 2 \times 10^{-9} \text{ ms}^{-2}$, it is 10%.

In the absence of an exact timelike symmetry of the background, there is no obvious solution to the problem of how to keep two clocks synchronized in general relativity. The CEP proposes a solution to this conundrum: the evolving average density provides the relevant regional clock. Even though we are talking about weak fields in cosmology, and small relative decelerations between expanding regions of different densities, the fact that the relative

decelerations are integrated over the lifetime of the universe means that the cumulative clock variance can be large for the density contrasts that are observed.

B. Modified Newtonian dynamics?

The fact that the present epoch value of α turns out to coincide with the MOND scale [46,47] is intriguing. To start thinking about possible connections, it pays to recall that α is an estimate of the relative deceleration of wall regions, at finite infinity where space is still expanding, with respect to the cosmological volume average at any epoch. It is certainly an acceleration scale at which dynamical gradients in the Ricci curvature of space are likely to affect dynamics of particles between walls and void regions.

Since we are no longer dealing with asymptotically flat geometries with exact timelike Killing vectors, it is quite possible that the solution to the Kepler problem for bound geodesics in galaxies should be modified. Therefore the possibility that the MOND phenomenon is related is a reasonable hypothesis. However, the estimate we have made of the relative deceleration scale, α , refers specifically to the finite infinity scale relative to the volume average. Given that the outskirts of galaxies are expected to lie within finite infinity, it means that no detailed quantitative comparisons can be made until the transition zone around finite infinity is more directly modeled.

I will not attempt to look at the problem of the rotation curves of galaxies in the present paper, but will make a few observations that would follow if the effects of MOND are simply a modification of Newtonian dynamics in a static background which arise from dynamical density gradients affecting the Ricci curvature of space.

The first important point is that, since we are dealing with an effect which is most pronounced between walls and voids, then the distance of a galaxy to a finite infinity boundary should play a role, at least naively. One might expect more pronounced effects for galaxies in filaments [50] as compared to rich clusters of galaxies in thick walls, where the distance to finite infinity is greater. In fact, rich clusters of galaxies often tend to harbour elliptical galaxies for which the MOND results differ little from Newtonian expectations. But apart from this, it appears that for the nearby redshifts over which it is tested, the MOND scale [46] is uniform for many different galaxy environments. This runs counter to naïve intuition about the distance to finite infinity, and suggests that some key physical insight remains to be found.

The second point is that, although the magnitude of the relative deceleration is larger at higher redshifts, the frequency of voids is also less. There may be a trade-off between these two competing factors as to an optimal redshift at which dynamical effects would be most evident. For the best-fit parameters [23], the void fraction reaches 50% at a redshift $z = 1.52$ when $\alpha \approx 3.4 \times 10^{-10} \text{ ms}^{-2}$. It

is possible that effects resulting from the relative deceleration of the background may be important for structure formation. Toy model calculations based on exact inhomogeneous solutions of Einstein's equations show that the nonlinear treatment of inhomogeneities in general relativity can considerably enhance the rate of structure formation [51].

Given that the expected distance between finite infinity and the outskirts of a galaxy does not suggest a direct link between α_0 and α_{mond} , the close match between α_0 and α_{mond} may be purely coincidental, even if both effects are related to Ricci curvature gradients which are typically of the same order of magnitude. As no direct link to MOND has yet been established, it is not possible to say whether the redshift dependence of α should be linked to a redshift dependence in the MOND scale. MOND is purely empirical and the fitting of rotation curves requires both an acceleration scale and an empirical function, $\mu(x)$, which interpolates between the Newtonian and "modified dynamics" regimes. Any first principles treatment would need to describe the transition zone between finite infinity and its interior, which might conceivably be related empirically to the interpolating function. We are at an early stage of gathering the pieces of a puzzle, where all observations have to be treated with rigorous scrutiny, and where no firm conclusions should be drawn until a compelling theoretical case is assembled.

Finally, it should be noted that the best-fit parameters of Ref. [23] indicate that nonbaryonic dark matter is likely to be the dominant component of matter in the universe by mass, even if the relative fraction is generally somewhat lower than in the Λ CDM model. If there is any link to MOND, then it is the phenomenology of MOND that one might hope to explain, rather than an alternative to nonbaryonic dark matter.

VI. WEYL CURVATURE AND INITIAL CONDITIONS

The cosmological equivalence principle has been applied here principally on the scale of macroscopic cosmological averages. A question remains as to what extent we should take it as a universal principle? Given that the universe is well approximated by frames of the sort (1) at the earliest epochs which are observationally tested, such as the epochs relevant to big bang nucleosynthesis, it is certainly tempting to try to apply the CEP on arbitrarily small scales within the past light cone at the earliest epochs. For any perfect fluid the pressure is determined by the density and no modification of the CEP is required. However, if one is dealing with spinning fluids or gravitational waves which cannot be treated by a single collective scalar degree of freedom, then modifications to the stated principle would be necessary. Is the process of "getting rid of local Ricci curvature" to recalibrate rods and clocks so that the effects of gravity disappear in a small region, as the

equivalence principle demands, related to the mathematical problem of Ricci flow? This is certainly a possibility, which has been discussed in the context of cosmological averaging by Buchert and Carfora [52].

As far as the rest of the curvature degrees of freedom are concerned, the CEP in fact incorporates a strong statement about average Weyl curvature, since the CIF (1) has vanishing Weyl curvature. Of course, Weyl curvature does not vanish everywhere identically—it is nonzero in the vicinity of collapsed structures such as stars and black holes, or in gravitational wave ripples. However, the statement of the CEP means that average effects of Weyl curvature need not be considered in the calibration of rods and clocks of generic cosmological frames. This "average Weyl curvature condition" may, at first sight, seem stronger than Penrose's "Weyl curvature hypothesis" [53] that the universe *began* in an initial state with total Weyl curvature exactly zero. However, at the epoch of last scattering the universe was very close to a standard FLRW model, and all FLRW models have vanishing Weyl curvature. Thus the "average Weyl curvature condition" incorporated within the CEP could be viewed as a relatively weak phenomenological statement about the evolution of a universe whose initial state at last scattering is consistent with the predictions of inflation. Alternatively, it is also consistent with any other cosmological scenario which solves the flatness and horizon problems to give a close to spatially flat FLRW universe with near scale-free perturbations by the epoch of big bang nucleosynthesis, whether such a scenario satisfies Penrose's Weyl curvature hypothesis or not.

It is my view, however, that in searching for potential scenarios for initial conditions, Penrose's Weyl curvature hypothesis needs to be taken seriously, and could be related to a generalized cosmological equivalence principle, if it can be formulated to apply at earlier epochs in the very early universe. To understand my rationale let us recall that the Weyl curvature tensor includes any nonlocal curvature in a manifold, whereas the Ricci tensor encodes purely local curvature since it is directly related to the energy-momentum tensor via the Einstein equations.

Since general relativity is a local causal theory, given that the observable universe was in a global state very close to a FLRW geometry with zero Weyl curvature at last scattering, then the only Weyl curvature we are allowed today is that which has accumulated by local causal processes within the past light cone at any event: in particular, by gravitational collapse and production of gravitational waves. The Weyl tensor encodes tidal curvature information on local scales which grows as matter clumps. On the large scales where the universe is still expanding, Weyl curvature cannot be important in defining the average geometry, since the universe has only had a finite time over which to evolve from its state at last scattering. This is also the reason why there is a statistical scale of average homogeneity.

If a version of the CEP can be taken to apply at even the earliest epochs, then it amounts to the statement that even throughout its earliest history the background universe contains no “nonlocal curvature” that cannot have evolved causally within the past light cone at any event. For as far back as a 4-dimensional spacetime continuum has any meaning, it would then make sense to be able to choose a CIF in which the average effects of density are volume-contracting. Large classes of models, such as Bianchi models with anisotropic flows, would be cosmologically irrelevant. If such a CEP should survive through the epoch of inflation or any other very early universe scenario, then it would coincide with Penrose’s Weyl curvature hypothesis. Ultimately these conceptual issues might inform quantum gravity and quantum cosmology.

VII. DISCUSSION

In this paper I have extended the strong equivalence principle to account for the average effect of the density of matter in the definition and relative calibration of clocks in inertial frames on cosmological scales. Since the resulting cosmological equivalence principle relates the single scalar degree of freedom of Newtonian gravity to the framework of general relativity, it may provide a means to better understand the calibration of cosmological weak fields once density perturbations have grown large to form a universe that is very inhomogeneous on scales of tens of megaparsecs. It should thereby give a setting for better understanding the Newtonian limit in the dynamical situation of cosmology. The numerical estimate of the relative deceleration between observers in the walls around galaxy clusters and volume-average observers in voids, typically of order 10^{-10} ms^{-2} , is acceptably small for weak-field scales and yet leads cumulatively to the present epoch clock rate variance of 38% found in Refs. [1,23]. Intriguingly, at redshifts $z \lesssim 0.25$ the relative deceleration required by the CEP coincides precisely with the empirical acceleration scale of MOND [46,47].

At a conceptual level I have attempted to present a framework for the consistent definition of average inertial frames in relation to average dynamically varying matter densities in cosmological general relativity. The hope is that the cosmological equivalence principle is thereby a key step to the incorporation of Mach’s principle into general relativity, in the way that Einstein intended but never quite realized. Mach’s principle is most commonly invoked in distinguishing inertial frames from rotating frames [9,54–56]. What is studied here is a different aspect of Mach’s principle: the role of the average volume deceleration of the local geometry in defining the standard of time of inertial frames. In the absence of a timelike Killing vector, the evolution of the average density provides a relevant clock. To fully incorporate Mach’s principle in general relativity, it is of course necessary to deal with the other dynamical gravitational degrees of freedom which

can affect the distinction of inertial frames from rotating ones, such as gravitational waves [56].

In this paper I have expounded the view that to deal with the volume-contracting average dynamical effects of matter density, a reduction to a frame (1) is the relevant step in the normalization of gravitational energy before the final step to a static Minkowski space. It is quite possible that when average degrees of freedom in addition to the scalar Ricci curvature are considered, in order to deal with gravitational waves and spinning matter fluids, there are other steps in the relevant relative calibrations of rods and clocks. After all, energy, momenta, and angular momenta are only defined with respect to a frame. In the dynamical regime of general relativity the question arises as to which collective average frames have physical utility in the absence of exact symmetries described by Killing vectors. My view is that a truly deep understanding of quasilocal gravitational energy and momentum is still to be found, but the path to such enlightenment requires a better conceptual understanding of the equivalence principle in application to collective dynamical degrees of freedom of matter fields.

Historically speaking, in the early stages of the development of general relativity Einstein did not fully appreciate the dynamical importance of the energy and momentum of spacetime itself. Spacetime is inevitably dynamical for matter obeying the strong energy condition. Einstein’s first journey through the conceptual landscape of cosmological general relativity had him worrying about boundary conditions at spatial infinity [10], as he overlooked the possibility that the universe had a beginning.

Since general relativity is causal the geometry at any event can only depend on events within its past light cone, and is independent of what lies beyond the particle horizon. Thus boundary conditions at spatial infinity beyond the particle horizon are physically irrelevant if the universe had a beginning, a possibility that Einstein did not consider when he first formulated his static universe [10]. For a universe like ours which had a beginning, the initial conditions are of vital importance in determining the relevant weighted average of the apparent motions, as I have discussed in Sec. VI.

The conceptual journey discussed in this paper arose in an effort to model the universe more realistically [1,2,23], to account for the structure we actually observe, by realizing that the quasilocal gravitational energy of a dynamical spacetime geometry—which has real effects on the calibration of clocks—should be an essential feature of a universe with large dynamical density gradients. If successful, this will eliminate the need for a cosmological constant or other fluidlike vacuum energy as the source of “dark energy,” but it still leaves the other cosmological problem, why $\Lambda = 0$, unsolved. If we take the strong equivalence principle literally then Λ must be zero since otherwise we could not have a vacuum Minkowski spacetime for our local inertial frames. My own personal view is

that quantum field theoretic calculations based in a flat spacetime which suggest that $\Lambda \sim M_{\text{Planck}}^4$ miss the mark, because the spacetime vacuum cannot be understood without accounting for the intrinsically dynamic nature of spacetime. It is not a problem for flat space quantum field theory. While the cosmological constant problem is no doubt a problem for quantum gravity, I believe that quantum gravity research might benefit from a more physical understanding of dynamical gravitational energy and the equivalence principle.

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