Dynamic domain walls in a Maxwell-dilaton background

Debaprasad Maity*

The Institute of Mathematical Sciences, CIT Campus, Tharamani, Chennai, 600 113, India

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Motivated by the well-known Chamblin-Reall solutions of n-dimensional background spacetime in a dilaton gravity and the dynamics of a domain wall in the same backgrounds, we have tried to generalize those solutions by including an electromagnetic field in the bulk. The electromagnetic field is assumed to be coupled with the scalar field in an exponential way. Under the specific relations among the various parameters in our model, we have found five different types of solutions. For every case, the solution has singularity. In these backgrounds, we have also studied the dynamics of the domain wall. The energy densities, which play the role of these interesting dynamics, are known to be induced from the bulk fields through the Israel junction condition. In this more general background field configuration, we have found that static bulk spacetime exists consistently with the dynamic domain wall. In several cases, depending on the values of the parameters, in the early stage of evolution, the domain wall is found to have an inflationary phase for finite period of its proper time followed by usual decelerated expansion.

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I. INTRODUCTION

Our universe as a four dimensional subspace in an extra dimensional spacetime, has long been the subject of interest from the theoretical as well as phenomenological point of view. However, with the present day experimental resolution, we have not seen yet the extra dimensions. This leads to a long-time general belief that if extra dimensions exist, then that should be compactified to a very tiny scale (down to a Planck scale). Parallel to this notion of compactified dimension, the references [1–4] came up with a novel idea of our universe as a domain wall of spatial dimension $n - 2$ in n dimensional spacetime with uncompactified extra dimension (more recently [5]). This means our four dimensional universe is a hypersurface moving in an extra spacelike dimension. The primary assumption of all these models is the localization of standard model fields on this hypersurface. The localized fields can be thought of as either the zero modes of all the bulk fields peaked at the position of the domain wall [3,6], or by some mechanism, the fields being polarized parallel to the domain wall world volume. For the later case we know that string theory gives a possible explanation of localization by identifying the domain wall as D-brane on which an open string ends [7]. As a result of this new idea of a "domain wall universe," many works have explored the notion in the context of theoretical generalization as well as in various cosmological and particle phenomenological model building [6,8–10].

Motivated by these ideas of domain wall for the past few years, the embedding of a four dimensional Friedmann-Robertson-Walker (FRW) universe was considered in generic bulk spacetime background with cosmological constant and various other fields [11,12]. It is generically true from the Israel junction condition [13] that various fields in the bulk under consideration induce energy density with different equation of states on the domain wall. Furthermore, these equation of states appear to be functions of various bulk parameters. So, by tuning these various parameters in the bulk action, one can in principle construct a viable cosmology. In certain settings, people have also found bouncing cosmology which has received much interest recently. The unique feature of this bouncing universe is the nonsingular transition between a contracting phase of the scale factor of the wall and a following expanding stage [14,15]. However in the recent studies, people have found some kind of inherent instability in this bouncing cosmological model showing the very presence of the singularity [16]. This also leads to a new direction to the study of circumventing the singularity in the extra dimensional scenario [17,18].

In this report, we are not going to construct any cosmological model. We will first try to generalize the constructions given in [19]. The explicit model for the cosmology, we leave for our future publications. Before going into the motivation of our work, we ought to mention that the authors of [19] have discussed the dynamics of the domain wall which is coupled only with a dilaton in the bulk spacetime. It was shown by suitably choosing various parameters of the model, that a domain wall coupled to a dilaton field can be dynamic even within the static bulk spacetime background. As we stated earlier, all these important aspects came from the so called Israel junction conditions across the domain wall. The condition tells us the specific relation between the extrinsic curvature of the domain wall and the localized energy-momentum tensor of the wall. The boundary condition can be written as

$$\{K_{MN} - Kh_{MN}\} = \mu h_{MN}, \tag{1}$$

where h_{MN} is the induced metric on the domain wall, and

*debu@imsc.res.in

K_{MN} is the extrinsic curvature with its trace $K = h^{MN}K_{MN}$. Finally the main implication of their study was to produce successful inflation on a domain wall through bulk energy induction. At this point we are not going to elaborate on this. In our subsequent analysis, we will be showing their results analytically as well as graphically at every stage in the appropriate limits.

In the context of standard four dimensional cosmology, the domain wall had been studied extensively. These types of domain walls were supposed to have been produced as a stable topological defect due to phase transition in the early universe [20,21]. However, finally it appeared that in the context of structure formation the topologically stable domain walls are not compatible with the recent cosmological observations as opposed to the inflationary stage in the early universe [22].

So, in this report we will be discussing dynamic domain wall solutions in a more general background field configuration along the line of [19]. We consider a general n dimensional action with a usual Einstein-Hilbert term and a dilaton field ϕ nonminimally coupled with a $U(1)$ gauge field A_μ. The motivation of choosing this kind of Maxwell-dilaton system is to relate with a more fundamental theory, specifically string theory. It is generically true that the low energy limit of any string theory turns out to be the supergravity. So then, by doing suitable truncation and dimensional reduction of this supergravity action one can get a dilaton-Maxwell system having exponential coupling between them. In addition to this low energy action, we also assume the dilaton to be coupled with a domain wall in the same way as was in [19]. Now, for a suitable solution ansatz for the bulk scalar field, we first analytically find five different types of metric solutions under specific relation among the various constant parameters in the theory. At this point, it is important to note that we have taken into account the full backreaction of the domain wall for the bulk spacetime metric. However, apart from studying the properties of these various solutions, we also discuss the dynamics of a domain wall in those bulk backgrounds in the spirit of Chamblin-Reall's paper. A topic of particular interest in these kinds of scenarios is how inflation occurs on the wall. As we have mentioned earlier that generically, the motion of a domain wall in a higher dimensional background can be written as a Hubble expansion equation with various kinds of positive as well as negative energy density equation of states. As was shown in [19] and also here we will again see in a more general setting that for a wide range of parameter space of the model under consideration, the domain wall indeed inflates in the early stage of the evolution followed by standard decelerated expansion. The bulk spacetime can also be set to a static background for this dynamic domain wall. The inflation can either be of exponential or power law type depending upon the kind of bulk solution we are considering. In the context of a large extra dimensional brane world scenario [5,23], there exists a long list of papers [10,12,24] which have been devoted to study these cosmological aspects. One important feature in our model as opposed to the general large extra dimensional brane world model is that it can naturally accommodate the inflation as well as decelerated expansion phase of the universe. The energy density which drives this inflation on the domain wall, strictly come from the bulk.

The paper is organized as follows: In section II, we will start with an action corresponding to a domain wall moving in Maxwell-dilaton background. After this we explicitly write down the equations of motion and its boundary conditions at the position of the domain wall. In section III, taking the static metric ansatz, we shortly restate the parametrization of the domain wall and the expression for the extrinsic curvature. From the various components of the extrinsic curvature, the consistency condition is derived in order to have the dynamic domain wall coupled to the scalar field. In section IV, we explicitly solve the metric and study its structure in great detail in the various limits of radial coordinate. We obtain five different types of bulk background solutions. In section V, we study the dynamics of the domain wall in those various types of metric backgrounds. The induced metric on the domain wall is similar to the FRW cosmological metric. So, naturally, the equation describing the dynamics will be a Hubble equation which has been derived from the Israel junction condition. Then following the line of [19], we again plot the various forms of the potential encountered by the domain wall and qualitatively study the dynamics under these potentials. In some cases we show that the bulk metric becomes time dependent. Furthermore, in many cases, for finite range of the scale factor the domain wall inflates for a finite period of the proper time followed by the usual decelerated expansion in the static background. Finally, in section VI, we state some concluding remarks some future possible extensions.

II. ACTION AND EINSTEIN EQUATIONS

We start with an action of Einstein-Maxwell-Dilaton system in the bulk with arbitrary dimension n and a codimension one domain wall coupled with the bulk dilaton field,

$$S = \int d^n x\sqrt{-g}\bigg(\frac{1}{2}R - \frac{1}{2}\partial_A\phi\partial^A\phi - V(\phi) - \frac{\lambda}{2}e^{-2\gamma\phi}F_{AB}F^{AB}\bigg) + S_{DW}, \quad (2)$$

where $S_{DW} = -\int d^{n-1}x\sqrt{-h}(\{K\} + \bar{V}(\phi))$, in the above equations R is the curvature scalar. h_{AB} is the induced metric on the domain wall. As is clear in the limit $\lambda = 0$, we get back the action studied in [19].

Now, corresponding Einstein equations are

$$R_{AB} = \partial_A\phi\partial^A\phi + \frac{2}{n-2}V(\phi)g_{AB} + \lambda e^{-2\gamma\phi}\left(2F_{AC}F^C_B - \frac{1}{n-2}F_{CD}F^{CD}g_{AB}\right) \quad (3)$$

$$D_C\partial^C\phi - \frac{\partial(\phi)}{\partial\phi} + \lambda\gamma e^{-2\gamma\phi}F_{AB}F^{AB} = 0 \quad (4)$$

$$D_A(e^{-2\gamma\phi}F^{AB}) = 0, \quad (5)$$

where D_A is a covariant derivative with respect to the bulk metric. The boundary conditions at the position of the domain wall are

$$\{K_{MN}\} = -\frac{1}{n-2}\bar{V}(\phi)h_{MN} \quad (6)$$

$$\{n^M\partial_M\phi\} = \frac{\partial\bar{V}(\phi)}{\partial\phi}, \quad (7)$$

where n^M is the unit normal to the domain wall. The first boundary condition comes from the Israel junction condition across the wall.

III. THE DOMAIN WALL AND ITS EXTRINSIC CURVATURE

In this section, we will shortly review a few steps in calculating the extrinsic curvature of the domain wall and the boundary conditions for various fields across the domain wall following the paper [19]. Once again we will consider reflection symmetry (Z_2) across the wall. So, under this symmetry, the above boundary condition Eq. (6) for the extrinsic curvature turns out to be

$$K_{MN} = -\frac{1}{2(n-2)}\bar{V}(\phi)h_{MN}. \quad (8)$$

Our aim is to find the solution for the dynamic domain wall in a static bulk background. So, keeping this in mind, we consider the static spherically symmetric bulk metric ansatz as

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + R(r)^2d\Omega^2_\kappa, \quad (9)$$

where we have taken $d\Omega^2_\kappa$ as the line element on a $(n-2)$ dimensional space of constant curvature with the metric $\tilde{g}_{ij}$. The Ricci curvature of this subspace is $\tilde{R}_{ij} = k(n-3)\tilde{g}_{ij}$ with $k \in \{-1, 0, 1\}$.

We want to get spherically symmetric bulk solutions corresponding to a homogeneous and isotropic induced metric on the domain wall. Now, let us parametrize the position of the domain wall by giving $r = r(t)$. Equivalently, we can introduce a new time parameter τ and specify the functions

$$r = r(\tau), \quad ; \quad t = t(\tau), \quad ; \quad R = R(\tau). \quad (10)$$

We choose the domain wall proper time τ such that the following relation is satisfied

$$N(r)\left(\frac{dt}{d\tau}\right)^2 - \frac{1}{N(r)}\left(\frac{dr}{d\tau}\right)^2 = 1. \quad (11)$$

This condition ensures that the induced metric on the wall takes the standard Robertson-Walker form,

$$ds^2_{\mathrm{wall}} = -d\tau^2 + R(\tau)^2d\Omega^2_\kappa. \quad (12)$$

So, the size of our domain wall universe is determined by the radial distance, R, which in turn determines the position of it in the bulk spacetime.

However, the unit normal pointing into $r < r(t)$ and the unit tangent to the moving wall is read as

$$n_M = \frac{\sqrt{N}}{\sqrt{N^2 - \dot{r}^2}}(\dot{r}, -1, 0, \ldots, 0), \quad (13)$$

$$u^M = \frac{\sqrt{N}}{\sqrt{N^2 - \dot{r}^2}}(1, \dot{r}, 0, \ldots, 0) \quad (14)$$

respectively. Where $\dot{r} = \frac{\partial r}{\partial t}$. Defining these tangent and normal to the domain wall, we can readily express the induced metric on the domain wall and its extrinsic curvature as

$$h_{MN} = g_{MN} - n_Mn_N \quad (15)$$

$$K_{MN} = h^P_Mh^Q_N\nabla_Pn_Q. \quad (16)$$

Now, the expressions for the components of the extrinsic curvature by using the bulk metric come out to be

$$K_{ij} = -\frac{R'}{R}\frac{N^{3/2}}{\sqrt{N^2 - \dot{r}^2}}h_{ij} \quad (17a)$$

$$K_{00} = \frac{1}{\dot{r}}\frac{d}{dt}\left(\frac{N^{3/2}}{\sqrt{N^2 - \dot{r}^2}}\right). \quad (17b)$$

By substituting the above expressions Eqs. (17) in the Israel junction condition Eq. (6) we get from K_{ij} and K_{00} components

$$\frac{R'}{R} = \frac{V(\phi)}{2(n-2)}\frac{\sqrt{N^2 - \dot{r}^2}}{N^{3/2}} \quad (18)$$

$$\frac{1}{\dot{r}}\frac{d}{dt}\left(\frac{N^{3/2}}{\sqrt{N^2 - \dot{r}^2}}\right) = \frac{V(\phi)}{2(n-2)}, \quad (19)$$

which gives us the equations of motion for the domain wall. 'Prime' is derivative with respect the bulk radial coordinate r

Now, using the expression for K_{ij} into K_{00}, and then integrating one gets

$$R' = C\bar{V}(\phi) \quad (20)$$

and again by using the above equation Eq. (20) into the boundary condition for the scalar field gives us

$$\frac{\partial\phi}{\partial R} = -\frac{n-2}{R}\frac{1}{\bar{V}}\frac{\partial\bar{V}}{\partial\phi}. \tag{21}$$

This equation has to hold at every point in the bulk visited by the domain wall. So, if the wall visits a range of R, then the above equation can be solved to yield ϕ as a function of R without specifying the bulk potential. This gives us a consistency condition for the dynamic domain wall coupled with the bulk scalar field to exist. In the subsequent section, we will be using this to find the solution for the metric.

IV. THE SOLUTIONS FOR BULK METRIC

In this section we will calculate various solutions of the metric assuming static bulk metric configuration. From the above action Eq. (2) and using the metric ansatz Eq. (9), one can read out the equations of motion as

$$\frac{R''}{R} = -\frac{1}{n-2}\phi'^2 \tag{22}$$

$$\frac{1}{2R^{n-2}}\{N(R^{n-2})'\}' - \frac{k(n-3)(n-2)}{2R^2} = -V - \frac{2Q^2\lambda}{R^{2n-4}}e^{2\gamma\phi} \tag{23}$$

$$\frac{n-2}{4R^{n-2}}(N'R^{n-2})' = -V + \frac{(n-3)Q^2\lambda}{R^{2n-4}}e^{2\gamma\phi} \tag{24}$$

$$\frac{1}{R^{n-2}}(\phi' N R^{n-2})' = \frac{\partial V(\phi)}{\partial\phi} + \frac{2\lambda\gamma Q^2}{R^{n-2}}e^{2\gamma\phi}. \tag{25}$$

Now, we will employ the Eqs. (20) and (21), to seek the solution of the Einstein equations of motion. So, taking the Liouville-type brane potential

$$\bar{V}(\phi) = \bar{V}_0 e^{\alpha\phi}, \tag{26}$$

one can easily get the solution for the scalar field without specifying the bulk potential, as well as for the radius $R(r)$ of the unit sphere Ω_k as

$$\phi = \phi_0 - \frac{\alpha(n-2)}{\alpha^2(n-2)+1}\log(r), \tag{27a}$$

$$R(r) = C\bar{V}_0 e^{\alpha\phi_0} r^{1/(\alpha^2(n-2)+1)}, \tag{27b}$$

where ϕ_0 and C are the integration constants. Furthermore, in order to have the solution for the bulk metric, we need to specify the dilaton potential $V(\phi)$. So, again we take the same Liouville-type bulk potential,

$$V(\phi) = V_0 e^{\beta\phi}, \tag{28}$$

where V_0 is constant. However, in our subsequent analysis, we will use the above two expressions Eqs. (27) for R and ϕ as solutions ansatz with respect to the bulk equations of motion. Making use of the bulk potential for the scalar field, we find five types of solutions to the equation of motions Eq. (22). At this point it is worth mentioning that, for $\lambda = 0$ case corresponding to no electromagnetic field in the bulk [19], one had three types of solutions for the bulk metric incompatible with the dynamic domain wall. However in what follows, we will be extensively discussing the nature of these various types of solutions and subsequently the dynamics of the domain wall under the same bulk spacetime backgrounds.

Type-I solution: When, $\alpha = \beta = \gamma = 0$. We note that the bulk and brane potential play the role of cosmological constant and brane tension, respectively. So, effectively, the action is a Einstein-Maxwell system with a bulk cosmological constant and a domain wall with tension.

By choosing this particular set of value of the parameters, the solution turns out to be

$$N(r) = k - 2Mr^{-(n-3)} - \frac{2V_0}{(n-2)(n-1)}r^2 + \frac{2\lambda Q^2}{(n-3)(n-2)}r^{-2(n-3)} \tag{29}$$

$$R(r) = r \quad ; \quad \phi = \phi_0, \tag{30}$$

where M and ϕ_0 are integration constants. The solution for the scalar field becomes constant. In general, it is difficult to extract the horizon structure for this kind of metric solution. So, we have plotted this solution in Fig. 1 for several possibilities of parameter values. Now, it is easy to read off the horizon structure from these various figures. For all practical purposes we have plotted solid lines depicting $\lambda = 0$ case which corresponds to Einstein-dilaton system in the bulk [19]. Whereas, dashed and dotted lines for different values of the parameters correspond to the full solution with dilaton-Maxwell fields present in the bulk. As is seen from Fig. 1 that for every case, there exists singularity at $r = 0$, which is timelike.

For all cases, we have four possibilities for different region of the parameter space (V_0, M). When $V_0 > 0$, $M > 0$, one has two different cases, one for $k = 0, -1$ (left panel of the figure) and the another one for $k = 1$ (right panel). We have noted that for each case the bulk spacetime has horizon. As in the first case we have cosmological horizon, but for the second case there could be a Risner-Nordstrom (RN) black hole inside the cosmological horizon [25]. In all these cases asymptotically, the metric becomes de Sitter space where, V_0 is playing the role of cosmological constant.

When $V_0 > 0$, $M < 0$, the only possibility is a cosmological horizon with an asymptotically de Sitter space which is again defined by the value of V_0. So, $r = 0$ is naked singularity.

If $V_0 < 0$, $M > 0$, for every case $k = 0, \pm 1$, the metric has either naked singularity at $r = 0$ or the same is hidden

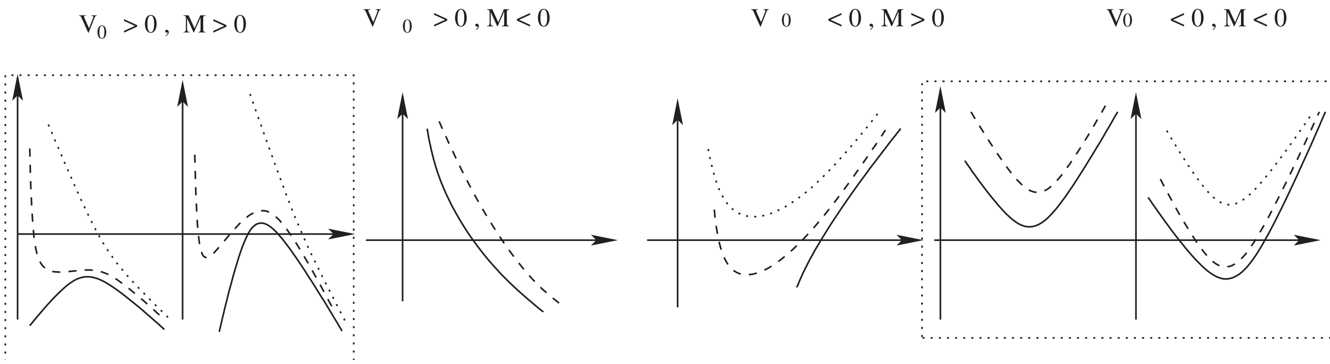

FIG. 1. $N(r)$ for type-I solutions. The solid line indicates $\lambda = 0$ solution [19]. Dashed and dotted lines represent the modified solution with $\lambda \neq 0$ for different sets of parameter values.

by an event horizon depending upon the values of various parameters. The black hole is charged in an asymptotically anti–de Sitter spacetime [25]. If one takes r_+ to be the outer horizon radius, then it should satisfy

$$(n-3)kr_+^{2n-6} + \frac{2|V_0|}{n-2}r_+^{2n-4} - \frac{2\lambda Q^2}{(n-3)(n-2)} \geq 0, \tag{31}$$

otherwise there is no horizon. When above inequality saturates, the black hole becomes extremal. The ADM mass $M_{\rm ADM}$ [26] of this black hole is related to the integration constant M by

$$M_{\rm ADM} = \frac{2n\omega_{n-1}}{2}M, \tag{32}$$

where ω_{n-1} is the volume of the unit n-sphere.

When $V_0 < 0$, $M < 0$, for $k = 0$, 1, the metric has a timelike naked singularity at $r = 0$ for any other value of the parameters present in the metric as is clear from the expression for the metric. However, $k = 0$ leads to a possibility of having a RN black hole in an asymptotically anti–de Sitter spacetime for certain range of parameter space (V_0, M) and λQ^2, otherwise it has also naked singularity at $r = 0$.

Type-II solution: For $\alpha = \frac{\beta}{2} = \gamma$; $k = 0$, the bulk metric has only a flat spatial section. The solution comes out to be

$$N(r) = -(1+c^2)^2 r^{2/(1+c^2)}\bigg[\frac{2\Lambda}{n-1-c^2} + 2Mr^{-((n-1-c^2)/(1+c^2))} - \frac{2\lambda\Omega}{c^2+n-3}r^{-((2(n-2))/(1+c^2))}\bigg] \tag{33}$$

$$R(r) = r^{1/(1+c^2)};$$
$$\phi(r) = \sqrt{n-2}\bigg(\phi_0 * - \frac{c}{1+c^2}\log(r)\bigg), \tag{34}$$

where $\phi_0^* = \phi_0/\sqrt{n-2}$, the integration constants. The various other notations are given below,

$$c = \frac{1}{2}\beta\sqrt{(n-2)}; \quad \Lambda = \frac{V_0 e^{2c\phi_0^*}}{n-2}; \quad \Omega = \frac{Q^2 e^{2c\phi_0^*}}{n-2}. \tag{35}$$

For this type of solution also, we have figured out for Fig. 2 the various possibilities for different values of the parameters present in the expression for $N(r)$. This kind of solution had been derived previously in [27] in four dimensional space. In another work [28], an explicit solution for the metric for $M = 0$ has been derived for arbitrary number of spacetime dimensions. Once again we note that for this particular choice of parameters, the Einstein equations of motion are invariant under constant scale transformation $g_{MN} \rightarrow \omega^2 g_{MN}$, $\phi \rightarrow \phi - \frac{2}{\beta}\log\omega$.

All the detailed structure can easily be read off from the corresponding Fig. 2. The asymptotic structures remain the same as was in $\lambda = 0$ case (extensively studied in [19]. As is seen from the figures, various structures of the spacetime are depending upon the value of c. If $c^2 < n-1$, asymptotically for some range of the parameter values, we can have either FRW universe ($V_0 > 0$) with flat spatial section

$$ds^2 \sim -dT^2 + T^{2/c^2}dx^2 \tag{36}$$

by defining r as time variable T or a black $(n-2)$ brane solution for $V_0 < 0$ such as

$$ds^2 \sim d\rho^2 + \rho^{2/c^2}dx^2 \tag{37}$$

by defining r as space variable ρ Furthermore, if $c^2 > n-1$, metric has a curious property that the mass determines the asymptotic structure. For $M > 0$, we have a Kasner type anisotropic cosmological metric

$$ds^2 \sim -dT^2 + T^{(2(c^2-n+3)/(c^2+n-1))}dt^2 + T^{(4/(c^2+n-1))}dx^2, \tag{38}$$

where the coordinate r is changed to time coordinate T and t becomes radial coordinate.

We have noted that the singularity structures at $r = 0$ for all these solutions have drastically changed due to the presence of an electromagnetic field and for every case,

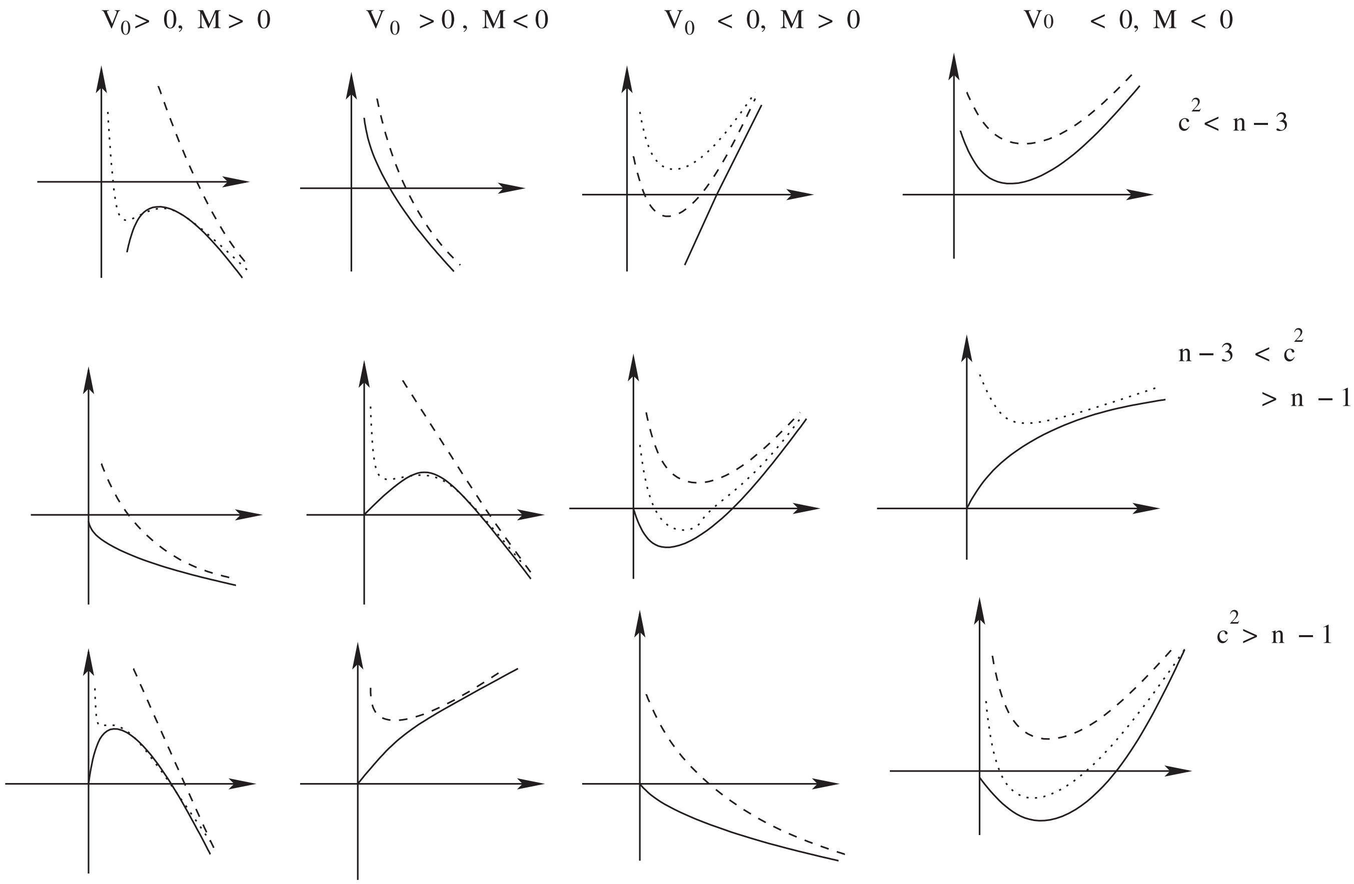

FIG. 2. $N(r)$ for type-II solutions. $k = 0$.

it is timelike. When $V_0 < 0$, $M > 0$, one has a possibility of having two horizon black holes for $c^2 < n - 1$. The same structure appears again if we take $V_0 < 0$, $M < 0$ and $c^2 > n - 1$. As an alternative behavior, we note that the bulk spacetime has naked singularity for the above cases.

Type-III solution: For $\alpha = \frac{2}{\beta(n-2)} = \gamma$; $k \neq 0$, the metric has no solution with flat spatial section. The solution looks like

$$N(r) = -(1+c^2)^2 r^{(2/(1+c^2))}\bigg[\frac{2\Lambda}{(n-3)c^2+1} + 2Mr^{-(((n-3)c^2+1)/(1+c^2))} - \frac{2\lambda\Omega r^{-((2(n-3)c^2+2)/(1+c^2))}}{\xi^{2n-4}c^2\{c^2(n-3)+1\}}\bigg] \tag{39}$$

$$R(r) = \xi r^{c^2/(1+c^2)}; \qquad \phi(r) = \sqrt{n-2}\bigg(\phi_0^* - \frac{c}{1+c^2}\log r\bigg), \tag{40}$$

where we use the notation

$$\xi = \sqrt{\frac{k(n-3)}{2\Lambda(1-c^2)}}. \tag{41}$$

Again all the solutions are singular at $r = 0$. The above Fig. 3 says the detailed asymptotic structure of the spacetime. For this metric, we can analytically solve for the location of the horizon $r = r_h$ where, $N(r_h) = 0$. So, the expression for r_h is

$$r_h^{(1+c^2(n-3))/(1+c^2)} = -\frac{M(1+c^2(n-3))}{2\Lambda} \pm \sqrt{\frac{M^2(1+c^2(n-3))^2}{4\Lambda^2} + \frac{4\lambda\Omega}{\xi^{2n-4}c^2\Lambda}}. \tag{42}$$

However, as is clear from the above expression and figures that for $V_0 > 0$, there exists only one horizon. On the other hand, if we consider $V_0 < 0$ then depending upon the sign of parameter M and also value of the various other parameters, we have either a two horizon black hole with open spatial section or a spacetime with naked timelike singularity at $r = 0$. As for $V_0 < 0$ and $M > 0$, the condition of having the two horizons, among the various parameters would be

$$\frac{M^2(1+c^2(n-3))^2}{4\Lambda^2} - \frac{4\lambda\Omega}{\xi^{2n-4}c^2|\Lambda|} \geq 0. \tag{43}$$

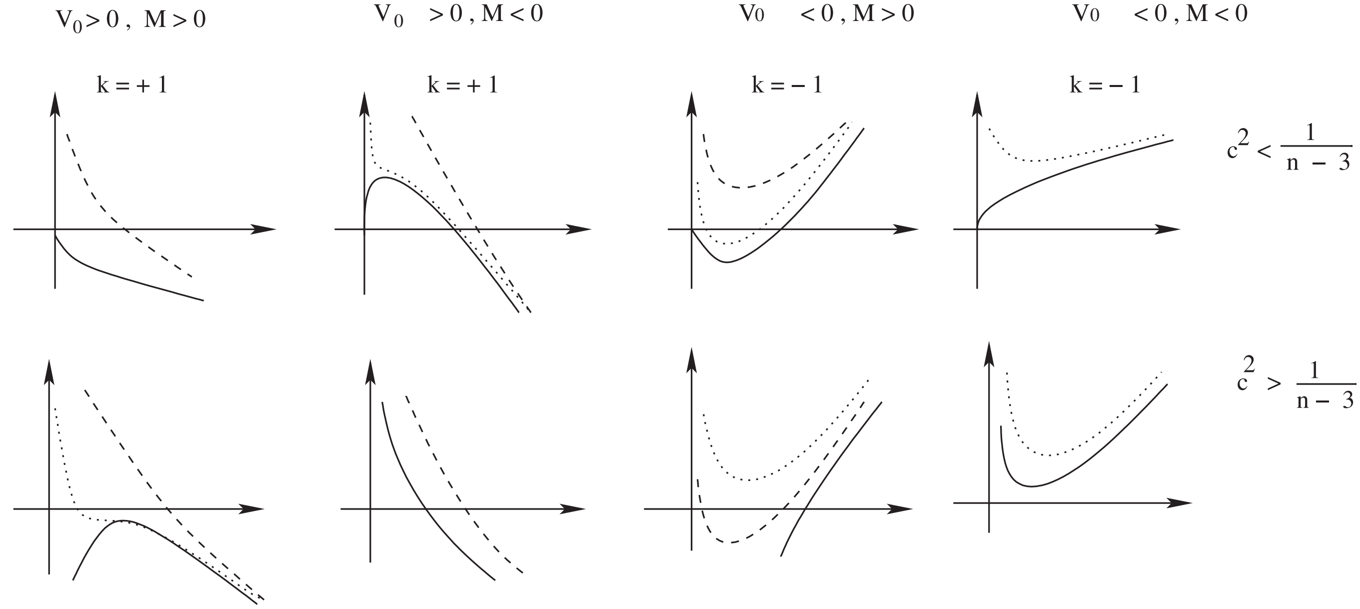

FIG. 3. $N(r)$ for type-III solutions. The value of k in the second row is the same as in the first row when $c^2 < 1$ and opposite of this when $c^2 > 1$.

When the above inequality saturates then the black hole becomes extremal.

For every case, the asymptotic limit of the solutions depends upon the sign of the parameter V_0 for the bulk scalar field. This is true even for $\lambda = 0$ which has extensively been studied in [19].

Type-IV solution: $\alpha = \frac{\beta}{2} = -\frac{(n-3)}{\gamma(n-2)}$; $k = 1$ for $\lambda > 0$ is considered throughout without mentioning further. So, in this case, the metric with closed spatial section consistent with the dynamic domain wall is allowed. However, the metric solution is

$$N(r) = -(1+c^2)^2 r^{2/(1+c^2)}\left[\frac{2\Lambda}{n-1-c^2} + 2Mr^{-((n-1-c^2)/(1+c^2))} - \frac{2(n-3)\lambda\Omega}{(c^2+n-3)c^2\chi^{2n-4}} r^{((2(c^2-1))/(1+c^2))}\right] \quad (44)$$

$$R(r) = \chi r^{1/(1+c^2)};$$
$$\phi(r) = \sqrt{n-2}\left(\phi_0^* - \frac{c}{1+c^2}\log r\right), \quad (45)$$

where M and ϕ_0^* are the integration constants and

$$\chi^{2n-6} = \frac{2((n-3)+c^2)\lambda\Omega}{kc^2(n-3)}, \quad (46)$$

where c, Ω and λ are defined above. As stated earlier, it is clear from the expression for the χ, that the only possibility could be $k = 1$ for $\lambda > 0$. Now, at this point we want to mention that if we take λ to be negative, the energy-momentum tensor turns out to be that of the Kalb-Ramon field [18] with a different overall numerical coefficient. We will elaborate on this as a separate note at the end.

Here, also all the solutions are singular $r = 0$. Depending upon the value c^2, we have four possibilities.

If $c^2 > 1$, the asymptotic structure of this metric surprisingly depends on the electric charge Q^2 irrespective of the value of mass parameter M and scalar field potential V_0. By rescaling t and the spatial sections of the metric and changing the variable $r \rightarrow \rho$, the form of the asymptotic metric comes out to be

$$ds^2 \sim -\rho^{2c^2}dt^2 + d\rho^2 + \rho^2 d\Omega_k^2 \quad (47)$$

for $k = 1$. So, the spatial section of this metric is of cylindrical topology. In a large region of parameter space, we have static bulk metric in the large R limit. For a few cases, metric has a naked singularity but otherwise it is hidden behind the black hole horizon.

On the other hand, when $c^2 < 1$, the singularity behavior at $r = 0$ is characterized by the sign of V_0. As is clear from Fig. 4 that for $V_0 > 0$, the singularity is spacelike and vice versa. On the other hand the asymptotic structure of the metric is characterized by the sign of M. If, $M > 0$ then there are two possibilities. For one r is time coordinate every where. In the asymptotic limit, by suitably rescaling the various coordinates, and changing the variable $r \rightarrow T(r)$ we can write the metric as

$$ds^2 \sim -dT^2 + T^{2/c^2}dt^2 + T^{2/c^2}d\Omega_k^2, \quad (48)$$

where $k = 1$. So, the metric describes accelerating universe with spatial sections of cylindrical topology. In the asymptotic limit, the spatial section of the bulk spacetime inflates much faster than that of axial direction. In an

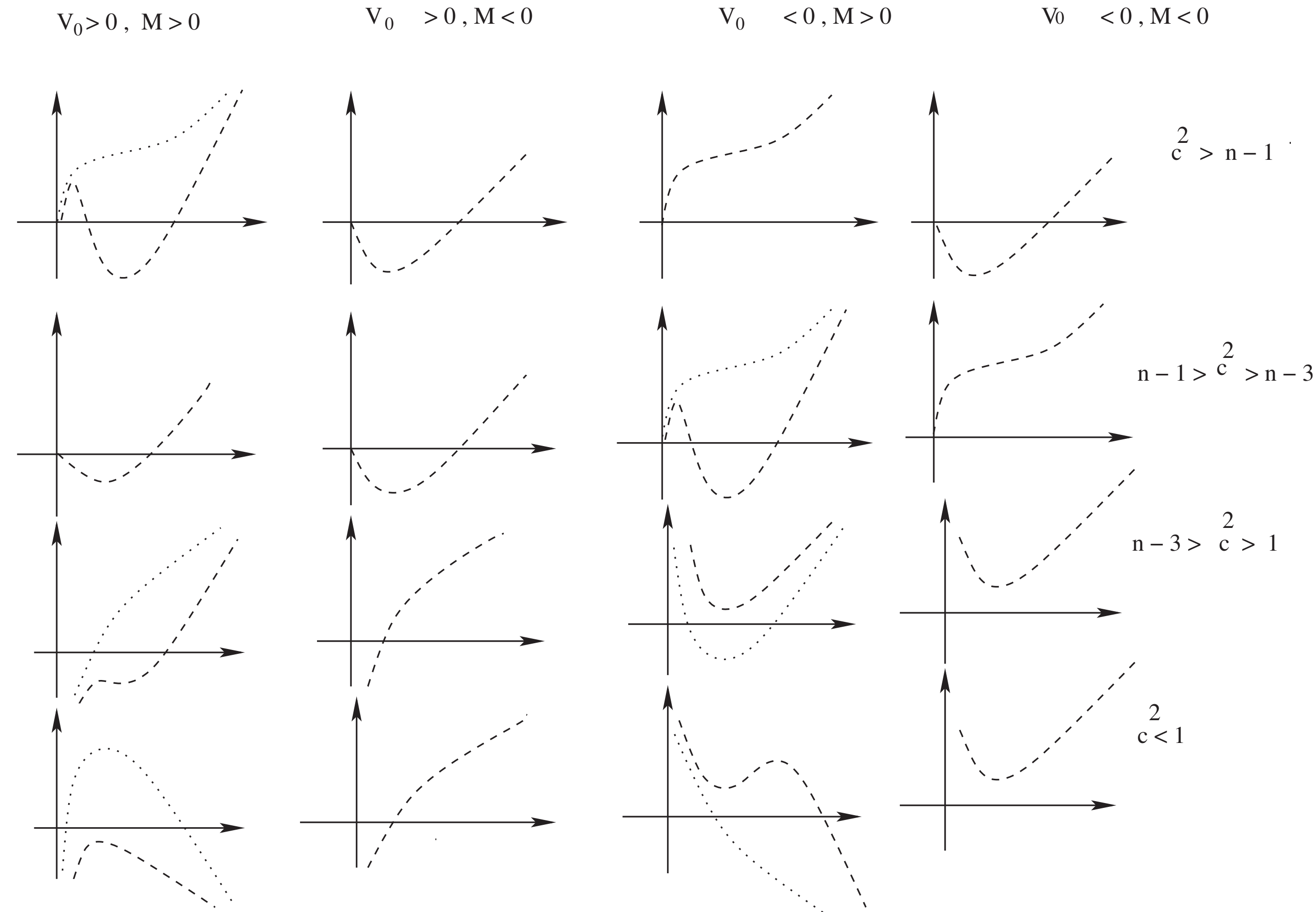

FIG. 4. $N(r)$ for type-IV solutions. The value of $k = 1$ for $\lambda > 0$.

another possibility, we have a Schwarzschild–de Sitter like solution in the bulk but asymptotic structure remains the same as Eq. (48). Whereas if $M < 0$ then r remains spatial coordinate and the asymptotic solution is the same as the above with the signs of the first two terms interchanged.

Type-V solution: When $\alpha = \frac{2}{\beta(n-2)} = -\frac{n-3}{\gamma(n-2)}$, the metric has again three types of spatial geometry as was in the first case viz. $k = 0, \pm 1$.

For $k = 0$, the expression for the $N(r)$ turns out to be

$$N(r) = r^2\left[Mr^{-(((n-2)c^2)/(1+c^2))} + \frac{2\Lambda(1+c^2)^2(n-4)}{(1+c^2(n-3))^2} r^{-(2c^2/(1+c^2))}\right] \tag{49}$$

$$R(r) = \eta r^{(c^2/1+c^2)}; \quad \phi(r) = \sqrt{n-2}\left(\phi_0^* - \frac{c}{1+c^2}\log r\right), \tag{50}$$

where M and ϕ_0^* are the integration constants and the expression for η is

$$\eta^{2n-4} = -\frac{2\lambda\Omega(1+c^2(n-3))}{2\Lambda(1-c^2)}, \tag{51}$$

Ω and Λ are already defined earlier. So, it is clear from the above expression for η to be positive, either $\Lambda < 0$, $(1 - c^2) > 0$ or vise versa.

On the other hand, when $k \neq 0$, the solution looks the same but the coefficients are different as

$$N(r) = r^2\left[Mr^{-(((n-2)c^2)/(1+c^2))} - 2c^2(d-3) \times\left(\Lambda + \frac{(n-3)\lambda\Omega}{\zeta^{2n-4}}\right) r^{-(2c^2/(1+c^2))}\right] \tag{52}$$

$$R(r) = \zeta r^{c^2/(1+c^2)}; \quad \phi(r) = \sqrt{n-2}\left(\phi_0^* - \frac{c}{1+c^2}\log r\right), \tag{53}$$

where the expression for ζ would be the solution of the following algebraic equation

$$\mathcal{A}\,\zeta^{2n-4} - \zeta^{2n-6} + \mathcal{B} = 0, \tag{54}$$

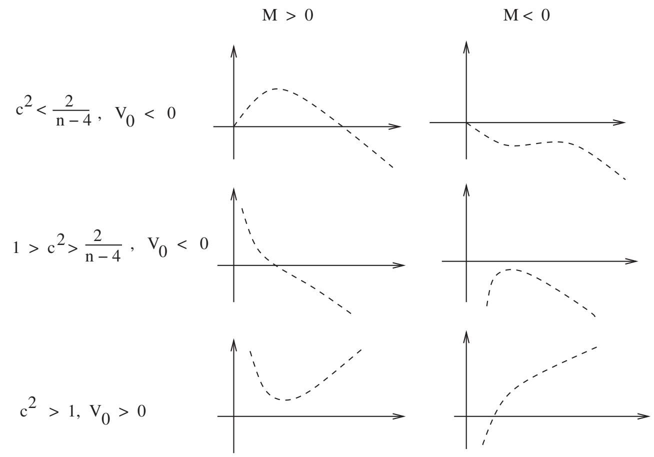

FIG. 5. $N(r)$ for type-V solutions.

where

$$\mathcal{A} = \frac{2\Lambda(1-c^2)}{k(n-3)}; \qquad \mathcal{B} = \frac{2\lambda\Omega(1+c^2(n-3))}{k(n-3)}. \tag{55}$$

Now, depending upon the sign of k, Λ and $(1-c^2)$, $\mathcal{A}$, $\mathcal{B}$ could be positive or negative. So, we have different possibilities which correspond to different dilaton profiles. For example, if $\mathcal{A} > 0$, $\mathcal{B} > 0$ and

$$\mathcal{A}\left(\frac{n-3}{\mathcal{A}(n-2)}\right)^{(n-2)} - \left(\frac{n-3}{\mathcal{A}(n-2)}\right)^{(n-3)} + \mathcal{B} < 0 \tag{56}$$

above inequality holds then the Eq. (54) has two different roots corresponding to the same value of the parameters we have started with. Surprisingly, if the metric has event or cosmological type of horizon, then the same numerical value of the parameters give rise to different value of horizon radius and scalar field profile. For every other possibility, the Eq. (54) has only one positive root.

Now, for both types of metric, one can solve analytically the structure of metric function $N(r)$. As is clear from both Eqs. (49) and (52), that if the metric has either black hole or cosmological type of horizon, the expression for the horizon radius r_h would be

$$r_h = \left(-\frac{M}{\mathcal{Z}}\right)^{(1+c^2)/((n-4)c^2)}, \tag{57}$$

where either M and $\mathcal{Z}$ should be negative. The expression for $\mathcal{Z}$ is

$$\mathcal{Z} = \begin{cases} \frac{2\Lambda(1+c^2)^2(n-4)}{(1+c^2(n-3))^2} & \text{for } k=0 \\ -2c^2(d-3)(\Lambda + \frac{(n-3)\lambda\Omega}{\zeta^{2n-4}}) & \text{for } k\neq 0 \end{cases}. \tag{58}$$

So, for $k=0$, as is also clear from the above Fig. 5 as well Eq. (58), the metric has cosmological type of horizon for $M>0$ and $\mathcal{Z}<0$ implying $V_0<0$ when $1>c^2>\frac{2}{n-4}$ and black hole type of event horizon for $M<0$ and $\mathcal{Z}>0$ implying $V_0>0$ when $c^2>1$.

Now, for $k \neq 0$, the structure of the spacetime remains the same as for $k=0$, but in this case for every constraint relation among the parameters, one has four possibilities corresponding to the value of $\mathcal{Z}$ in place of V_0 and M.

It is clear from the metric Eqs. (49) and (52), that for any value of c^2, the asymptotic structure of the bulk spacetime is determined by the sign of $\mathcal{Z}$. For $\mathcal{Z}>0$, by changing $r \longrightarrow \rho$ and suitably rescaling the time and space coordinate we have a $n-2$ brane solution as

$$ds^2 \sim d\rho^2 + \rho^{2c^2}(-dt^2 + d\Omega_k^2), \tag{59}$$

where the geometry of the spatial section would be any one of $k=0, \pm 1$. On the other hand for $\mathcal{Z}<0$, the metric will be of FRW cosmological type with any one of the allowed spatial sections. The scale factor is T^{2c^2}.

So far we have discussed about five possible types of solutions for the bulk metric in an Einstein-Maxwell-Dilaton background. The detailed thermodynamic studies of some of these solutions can be obtained [27,28]. In the subsequent section will use these metrics to study the dynamics of the domain wall.

V. THE DOMAIN WALL AND ITS DYNAMICS

Without going into further details, we will just state the expression for equation of motion of the domain wall as

$$\frac{1}{2}\left(\frac{dR}{d\tau}\right)^2 + F(R) = 0, \tag{60}$$

where $F(R)$ is the expression for the potential encounter by the domain wall moving in the bulk. The potential is expressed as

$$F(R) = \frac{1}{2}NR'^2 - \frac{1}{8(n-2)^2}\bar{V}^2R^2, \tag{61}$$

where 'prime' is derivative with respect to bulk radial

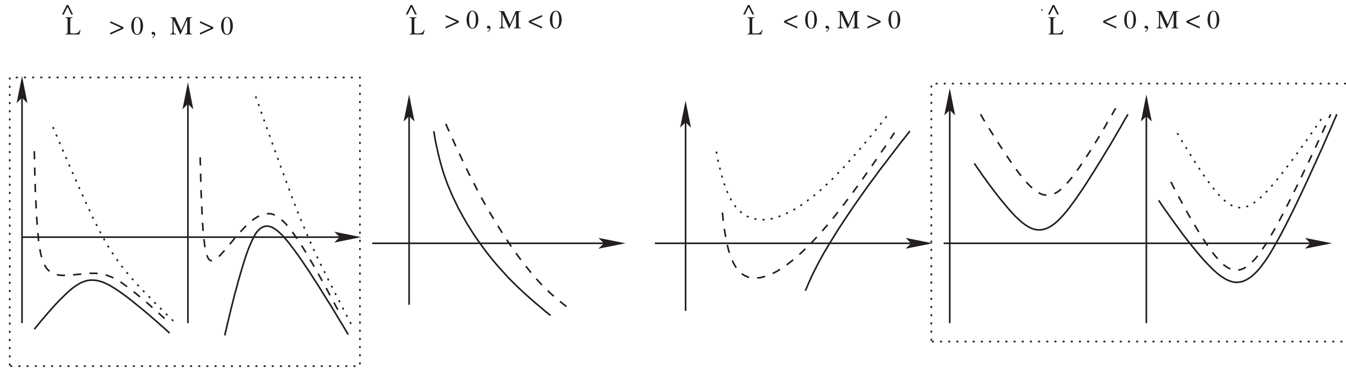

FIG. 6. $F(R)$ for the type-I solutions.

coordinate r. It is clear from the above equation of motion that the solution exists only when $F(R) < 0$.

Now, in what follows, we consider different types of bulk solutions in the expression for the potential of the domain wall and study its structure. We cannot solve for the equation of motion analytically for its complicated expressions. So, we will try to analyze the potential on the basis of graphical representation. Furthermore, for every case, we will try to display the analytic solutions of the evolution equation Eq. (60) in the asymptotic and near singularity region of the bulk spacetime.

Type-I potential: The potential encountered by the domain wall moving in the bulk spacetime for the type-I solution, is

$$F(R) = k - 2MR^{-(n-3)} - \left(\frac{2V_0}{(n-2)(n-1)} + \frac{\bar{V}_0^2}{8(n-2)^2}\right)R^2 + \frac{2\lambda Q^2}{(n-3)(n-2)}R^{-(2(n-3))}, \qquad (62)$$

where the effective cosmological constant on the brane is

$$\hat{L} = \frac{1}{n-2}\left[\frac{V_0}{n-1} + \frac{\bar{V}_0^2}{8(n-2)}\right]. \qquad (63)$$

By tuning the bulk and brane potential parameters, we can set the cosmological constant to be zero. However, the qualitative features of this potential can be extracted from the Fig. 6. The very fact is that for every case, corresponding to a fixed potential structure, there exists two distinct bulk background spacetimes depending upon the value of cosmological constant V_0 and the domain wall tension $\bar{V}_0$.

The plots that correspond to these cases are as follows, $\hat{L} > 0$, $M < 0$. In connection with the choice of parameters, there exists many possibilities for the structure of the potential. As we can say that the first figure corresponds to $k \neq 1$ and $k = 1$ with specific relation of the parameters. So, the potential can take very distinct form depending on the region of the parameter space. The bulk can either be an asymptotically de Sitter spacetime with a single horizon or a topological Reissner-Nordstrom bulk hole spacetime. Now in the asymptotic limit, the domain wall goes through an exponential expansion as is clear from the potential. So, collapsing from the infinity, the domain wall can either be stopped by the repulsive singularity at finite value of R and then reexpands to infinity, or after climbing up the maximum of the potential, falls into the local minimum of the potential for finite value of R and oscillates. If $k = 1$, the background bulk can either be RN–de Sitter or simply de Sitter depending upon the region of parameter space when $V_0 > 0$. On the other hand for $V_0 < 0$ the bulk can be either RN-anti–de Sitter or a spacetime which has naked singularity, with asymptotically anti–de Sitter spacetime. In this case, one has a possibility of bouncing back the collapsing domain wall at finite value of R which is greater than outer horizon of the RN black hole in the bulk. In the lower R limit, there exists a region for finite value of the scale factor, in which the domain wall oscillates and in this region the domain wall passes through inflation for finite period followed by standard deceleration.

Asymptotically, for any parameter value, the metric becomes anti–de Sitter space. In the both the cases, the expansion in the large R limit is of exponential inflationary type $R(\tau) \sim e^{\sqrt{2\hat{\Lambda}}\tau}$.

When $\hat{L} > 0$, $M < 0$, the asymptotic expansion of the domain wall world volume is of the same form as in the previous case. But in this case, for all value of k, the structure of the potential remains more or less the same. So, the motion of the domain wall has the same behavior as it starts collapsing from infinity and then reexpands to infinity by bouncing back from the certain value of R.

For the other two cases in the parameter space of $(\hat{L}, M)$, the structure of the potential is more or less the same. There exists a dip in the potential $F(R)$, in which the domain wall oscillates in the region of finite value of the scale factor. During this course of motion, the domain wall appears to be residing inside the black hole region. It has again inflation for the finite period of time followed by deceler-

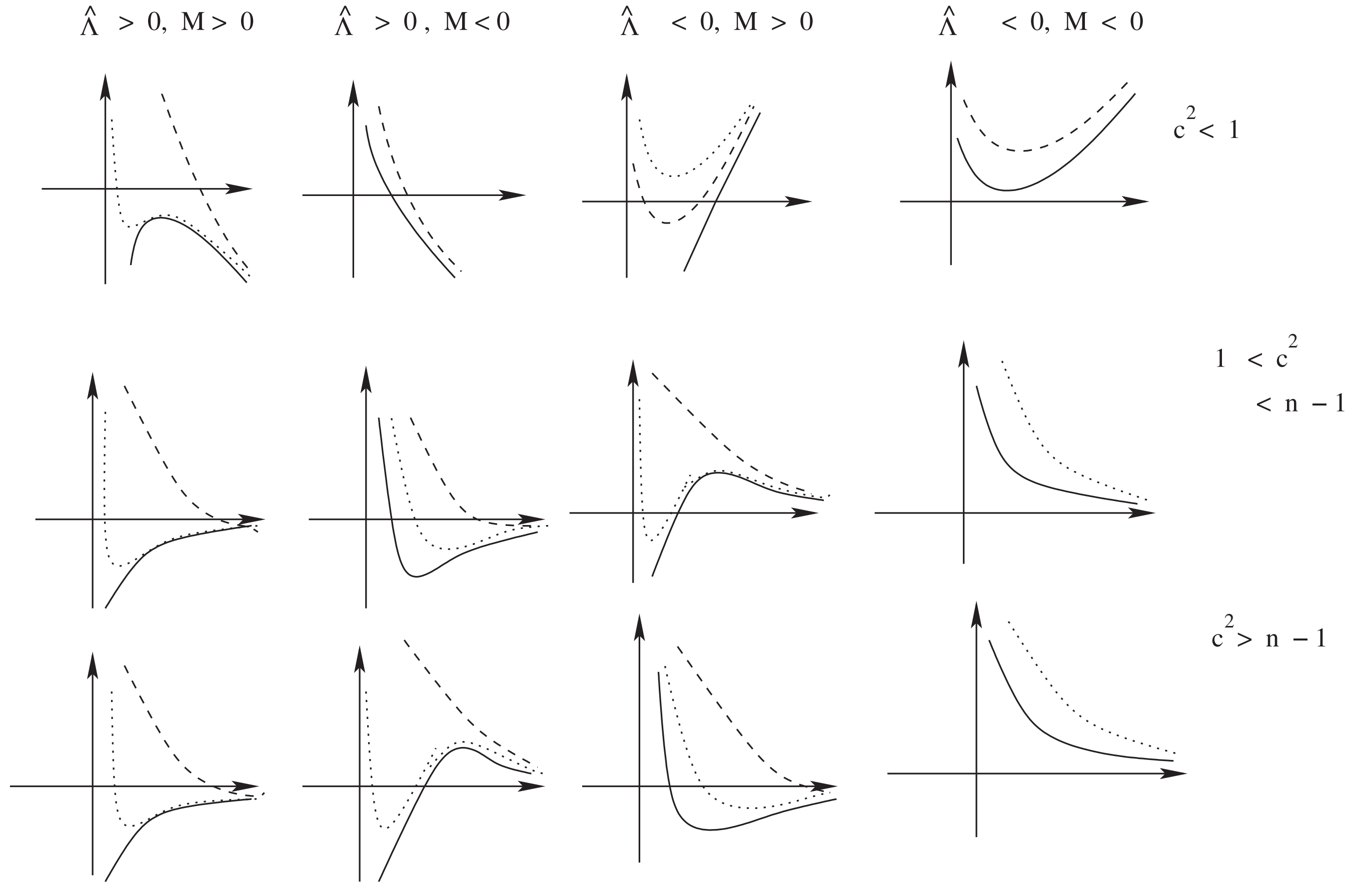

FIG. 7. $F(R)$ for type-II solutions $k = 0$.

ated expansion. Otherwise there exists no solution for the dynamics.

Type-II potential: For the type-II solution one gets this expression for the potential

$$F(R) = -R^{2(1-c^2)}\Big[MR^{-(n-1-c^2)} + \hat{\Lambda} - \frac{\lambda\Omega}{c^2+n-3}R^{-(2n-2)}\Big], \tag{64}$$

where

$$\hat{\Lambda} = \frac{e^{2c\phi_0^*}}{n-2}\Big[\frac{V_0}{n-1-c^2} + \frac{\bar{V}_0^2}{8(n-2)}\Big]. \tag{65}$$

In this case, the structure of the potential is seen from Fig. 7. In general, asymptotically the potential function tends to zero value for $c^2 > 1$ and its form depends upon the value of c^2 and of course the sign of M and $\hat{\Lambda}$. Three classes of behavior are apparent from the figures.

If the potential function $F(R)$ is positive everywhere. This subjects to no solution to the domain wall motion. For $\hat{\Lambda} < 0$, when $M < 0$, the potential is positive for all values of R irrespective of the value of c^2 but when $M > 0$, the same depends upon the constrained region of the full parameter space with $c^2 < n - 1$.

As is clear from the figures, for every case $F(R)$ is singular but positive in $R \to 0$ limit. So, the potential function is always positive for small value of R. Furthermore if the potential is negative for the higher value of R then that amounts to a bounce of the domain wall at finite value of R. Asymptotically, the dynamics of the domain wall for this type of potential structure, is guided by the following respective expressions

$$R(\tau) = \Big(c^2\sqrt{2\hat{\Lambda}}\Big)\tau^{1/c^2} \quad \text{when } c^2 < 1 \quad \text{with } \hat{\Lambda} > 0 \tag{66}$$

$$R(\tau) = \Big(c^2\sqrt{2\hat{\Lambda}}\Big)\tau^{1/c^2} \quad \text{when } 1 < c^2 < n-1 \quad \text{with } \hat{\Lambda} > 0, \tag{67}$$

$$R(\tau) = \Big((c^2+n-2)\sqrt{2M}\Big)\tau^{1/(c^2+n-2)} \quad \text{when } c^2 > n-1 \quad \text{with } M > 0. \tag{68}$$

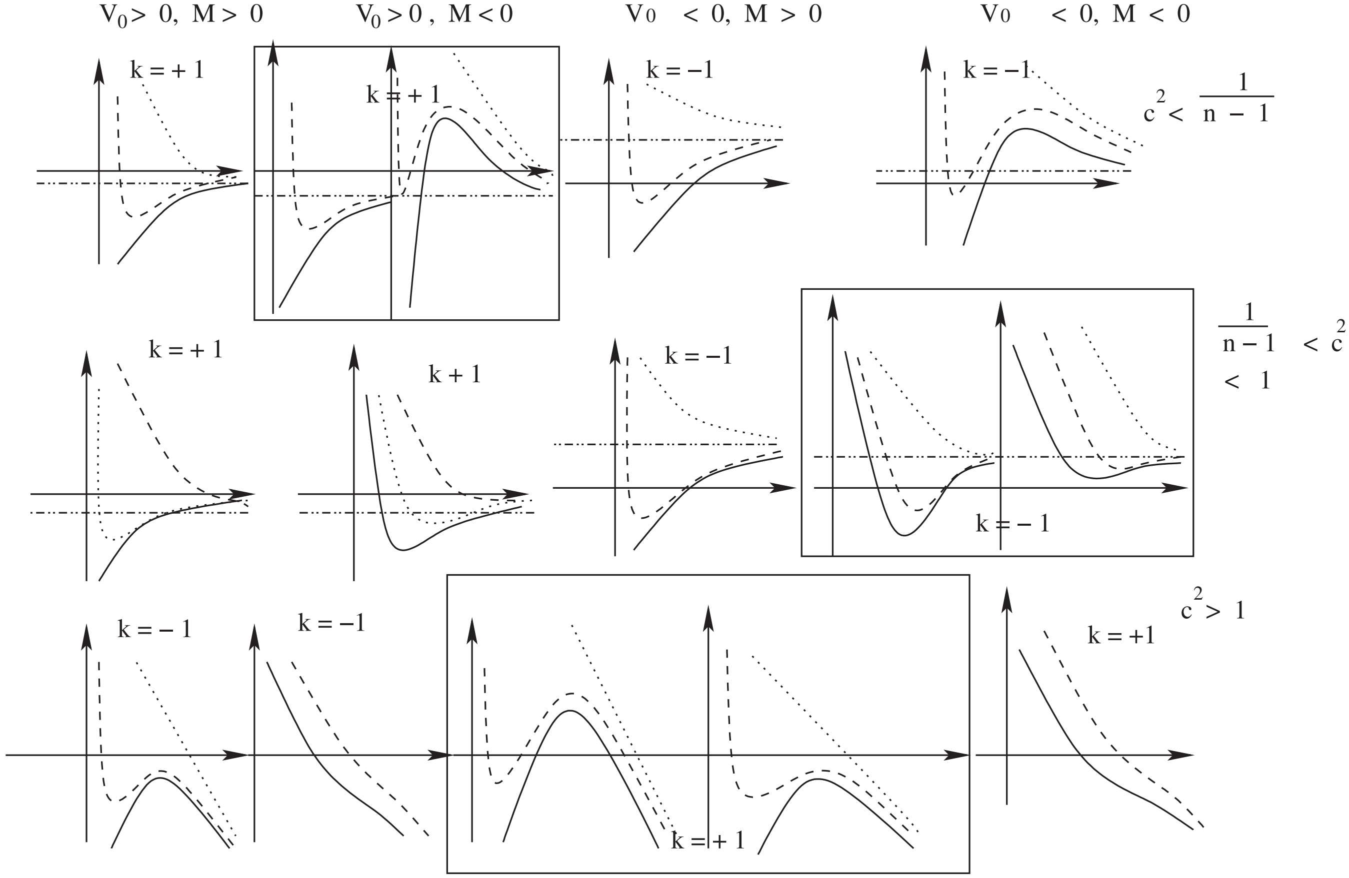

FIG. 8. $F(R)$ for type-III solutions.

The first equation corresponds to inflation. The other two correspond to decelerated expansion of the universe. In the last two cases, namely, $1 < c^2 < n - 1$, $\hat{\Lambda} > 0$ and $c^2 > n - 1$, $M > 0$ solutions, we have finite period of inflation of the domain wall world volume in low R limit. In both of these, the domain wall collapses from infinity, gets repelled by the timelike naked singularity in the bulk background, and then expands. Inflation occurs when the expansion starts.

If $F(R)$ is negative in the intermediate region of the scale factor R, there exists two zeros of the potential function. This amounts to oscillating as well as a bouncing universe. The domain wall does not expand to infinity. For a very particular region of the parameter space, this oscillating phase of the universe appears.

Type-III potential: For the type-III solution one gets this expression for the potential

$$
\begin{aligned}
F(R) = &-\frac{k(n-3)c^4}{2(1-c^2)(1+c^2(n-3))} \\
&- M\xi^2 c^4\left(\frac{R}{\xi}\right)^{-((1+c^2(n-3))/c^2)} \\
&- \frac{\bar{V}_0^2 e^{2(\phi_0^*/c)}\xi^2}{8(n-2)^2}\left(\frac{R}{\xi}\right)^{-2((1/c^2)-1)} \qquad (69)\\
&+ \frac{\lambda\Omega c^2}{\xi^{2n-6}(c^2(n-3)+1)}\left(\frac{R}{\xi}\right)^{-2((1+c^2(n-3))/c^2)}. \qquad (70)
\end{aligned}
$$

In this case also, we can classify the potential into four different types of behavior as is seen from Fig. 8.

Class (i) $F(R)$ is positive everywhere. A solution does not exist.

Class (ii) $F(R)$ is positive for small value of R and negative for large value of R. For this class of potential, we have two different behaviors in the asymptotic limit depending upon the value of c^2. When $c^2 > 1$, asymptotically, the domain wall is driven by its energy density parametrized by $\bar{V}_0$ and inflates towards infinity under power law of proper time τ

$$R(\tau) = \left(\frac{\bar{V}_0 e^{\phi_0^*/b}}{2(n-2)c^2}\right)^{c^2}\xi\tau^{c^2}. \qquad (71)$$

On the other hand, when $c^2 < 1$, the asymptotic dynamics depend upon the sign of either k or V_0. So, when $k = 1$, the potential tends to a constant negative value. At late time, the domain wall energy density is dominated by the curvature of the spatial section and the scale factor of the domain wall grows as

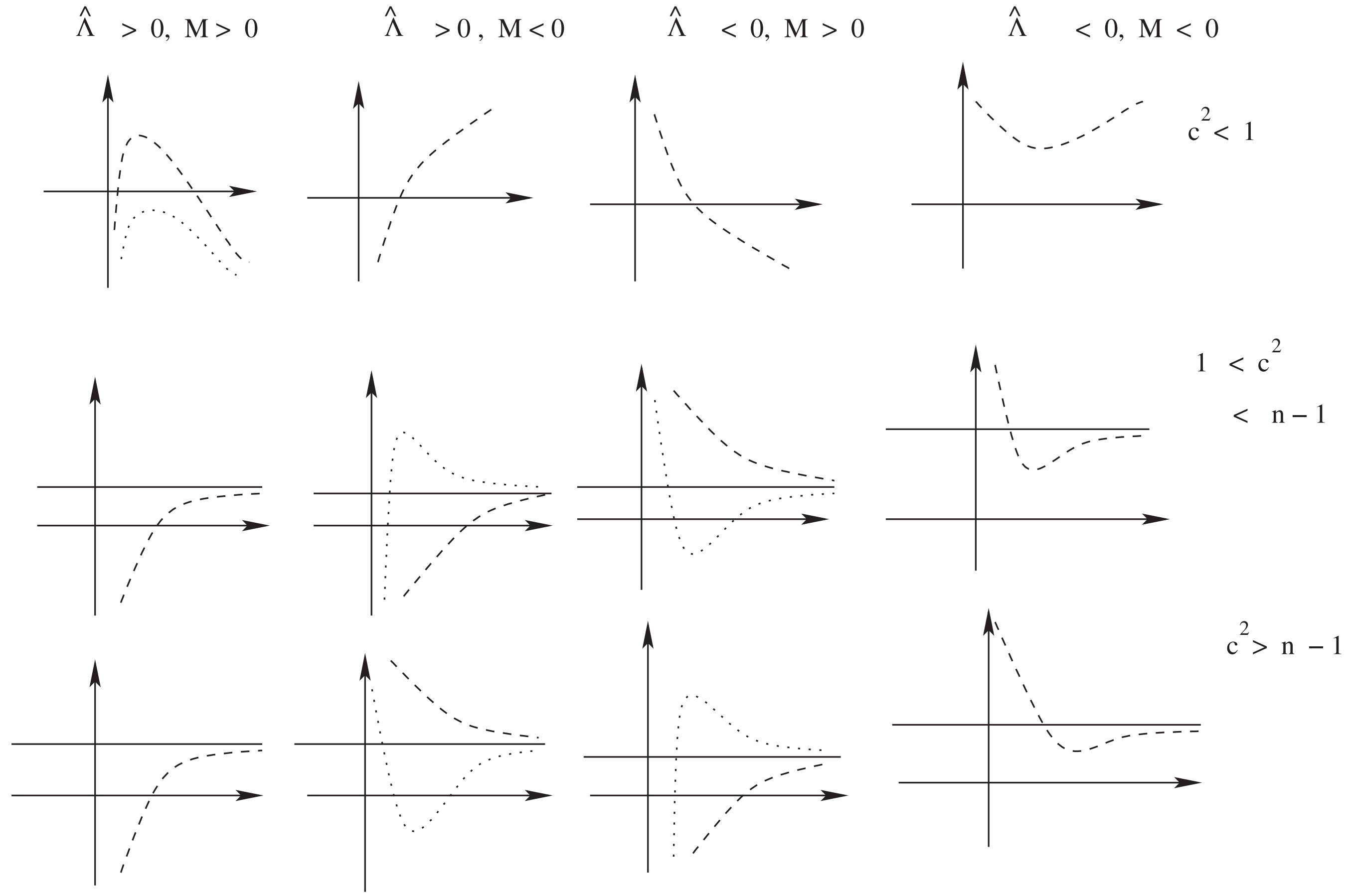

FIG. 9. $F(R)$ for type-IV solutions.

$$R(\tau) = \sqrt{\frac{2(n-3)c^4}{(1-c^2)(1+c^2(n-3))}}\tau. \qquad (72)$$

Furthermore, there exists a finite amount of inflation in the low R limit. The domain wall is repelled by a time singularity, inflates for a brief period of time and then decelerates. In an alternative behavior for the somewhat different parameter range, the inflationary period does not exist.

Class (iii) $F(R)$ is negative for a finite range of R. This situation occurs when $c^2 < 1$ and $k = -1$ which is governed by the sign of V_0. The domain wall world volume describes an open 'oscillating' as well as a 'bouncing' universe as was explained in [19].

Class (iv) $F(R)$ is negative for finite range of R and followed by positive value again for finite range of R. We have noted this kind of behavior of the potential for only some specific range of values of the various parameters. For $c^2 > 1$ and $V_0 < 0$, $M > 0$, depending upon the initial position, the domain wall world volume can either be described by a closed 'bouncing' universe or a collapsing wall being stopped at some finite value of R and again bounced back to infinity. For the later case, again asymptotically, the domain wall inflates according to Eq. (71).

On the other hand, when $c^2 < 1$ and $V_0 > 0$, $M < 0$, qualitatively, the dynamics of the domain wall remain the same as above, but in the asymptotic limit the scale factor expands linearly with proper time τ following Eq. (72).

Type-IV potential: For the type-IV solution one gets this expression for the potential

$$F(R) = -\chi^2\left(\frac{R}{\chi}\right)^{2(1-c^2)}\left[M\left(\frac{R}{\chi}\right)^{-(n-1-c^2)} + \hat{\Lambda}\right] + \frac{k(n-3)^2}{(c^2+n-3)^2}, \qquad (73)$$

where expression for $\hat{\Lambda}$ is mentioned above.

For $\lambda > 0$, as we have already mentioned that the domain wall has closed spatial section. In this case also, we have many different types of potential structures corresponding to the values of various parameters. Depending upon the value of c^2, the dynamics is determined by M or $\hat{\Lambda}$. From Fig. 9, we note six different types of structures of the potential as follows:

Class (i) $F(R)$ is positive everywhere. For $\hat{\Lambda} < 0$, $M < 0$, $F(R)$ is positive irrespective of the value of c^2. As we have mentioned several times that we do not have any dynamical solution of the domain wall.

Class (ii) $F(R)$ is negative everywhere. For a constrained set of parameters $c^2 < 1$ and $\hat{\Lambda} > 0$, $M > 0$ and as an alternative behavior, we get this kind of potential. The bulk spacetime may be either black hole or it has naked singularity at $r = 0$. So, the domain wall starts collapsing from infinity and falls into bulk spacetime singularity. Its asymptotic dynamics are guided by the total brane cosmological constant $\hat{\Lambda}$. So, the expression for the scale factor would be

$$R(\tau) = (2c^4\hat{\Lambda})^{2/c^2}\chi\tau^{1/c^2}, \tag{74}$$

which is inflating.

Class (iii) $F(R)$ positive for small value of R but negative for the large value of R. In this case, the domain wall starts collapsing from infinity, gets repelled by the timelike singularity at finite value of R and reexpands again to infinity. The background bulk may have a naked singularity or a topological black hole with single or double horizon. Asymptotic dynamics of the domain wall is the same as Eq. (74).

Class (iv) $F(R)$ is negative for small value of R but positive for large value of R. In a large region of the parameter space of $(\hat{\Lambda}, M)$, the domain wall encounters this specific potential. So, as the Hubble equation tells, for almost all cases, the dynamics of the domain wall is confined inside the black hole region and is attracted by the singularity at $r = 0$.

Class (v) $F(R)$ is positive for finite value of R. For $c^2 < 1$ and $\hat{\Lambda} > 0$, $M > 0$, we have this kind of potential structure. The form of the $\hat{\Lambda}$ suggests that for the bulk we have either two horizon or a single horizon topological black hole. Dynamics are of two kinds, either it is similar to class (iv) for small value of the scale factor R or in the large R, it is similar to class (ii). But for the later case, asymptotically the domain wall inflates following the power low in terms of proper time τ as $\sim\tau^{1/c^2}$.

Class (vi) $F(R)$ is negative for finite range of R. This kind of potential structure occurs for $\hat{\Lambda} < 0$, $M > 0$ with $1 < c^2 < n - 1$ and $\hat{\Lambda} > 0$, $M < 0$ with $c^2 > n - 1$. For most of the cases, the bulk background has singularity hidden by the event horizon and outside the horizon the spacetime is static. So, again in this case, the domain wall has a finite period of inflation at low value of R and then decelerates and then stopped at some point. The domain wall describes a bouncing universe.

Type-V potential: For type-V solutions one gets this expression for the potential

$$F(R) = \frac{1}{2}\Theta^2\left(\frac{c^2}{1+c^2}\right)^2\left[M\left(\frac{R}{\Theta}\right)^{-(n-4)} + \mathcal{Z}\right] - \frac{\bar{V}_0^2 e^{2\alpha\phi_0^*}}{8(n-2)^2\Theta^2}, \tag{75}$$

where $\mathcal{Z}$ is defined in Eq. (58) and

$$\Theta = \begin{cases} \eta & \text{for } k = 0 \\ \zeta & \text{for } k \neq 0 \end{cases}. \tag{76}$$

Now, as is clear from the above expression for the potential, we can solve the dynamics of the domain wall analytically. The equation we need to solve is

$$\frac{dR}{d\tau} = \sqrt{\mathcal{D}R^{-(n-4)} + \mathcal{F}}, \tag{77}$$

where

$$\mathcal{D} = -\left(\frac{c^2}{1+c^2}\right)^2 M\left(\frac{1}{\Theta}\right)^{-(n-6)};$$
$$\mathcal{F} = \frac{\bar{V}_0^2 e^{2\alpha\phi_0^*}}{4(n-2)^2\Theta^2} - \Theta^2\left(\frac{c^2}{1+c^2}\right)^2\mathcal{Z}. \tag{78}$$

In general for arbitrary dimension n, the solution of the above Eq. (77) will be the hypergeometric function of R

$$R^{n-2/2}{}_2F_1\left[\frac{n-2}{2(n-4)}, \frac{1}{2}, \frac{3n-10}{2(n-4)}, -\frac{\mathcal{F}}{\mathcal{D}}R^{n-4}\right] = \frac{\sqrt{\mathcal{D}}(n-2)}{2}\tau. \tag{79}$$

So, it is very difficult to get the inverse of the above solution in terms of proper time τ. However, interestingly, the R dependent part of the potential seems to appear from an effective dustlike matter field on the brane. But we note that the energy density is a function of bulk electromagnetic charge Q^2. However, in the early stage of the evaluation, for say $n = 5$, the domain wall world volume expands like pressure less matter dominated universe

$$R(\tau) = \mathcal{D}^{1/3}\left(\frac{3\tau}{2}\right)^{2/3}. \tag{80}$$

After passing through this matter dominated phase of evolution, at late time the domain wall world volume expands linearly with proper time.

VI. CONCLUSION

To summarize, in this report we have tried to generalize the construction of [19] by introducing a $U(1)$ gauge field in the bulk. The bulk dilaton field is also assumed to couple exponentially with the electromagnetic field with an arbitrary coupling parameter γ. Under this somewhat general background field configuration, we have first tried to find out the possible background solutions taking into account the domain wall backreaction. We have analytically found five different types of solutions in accord with the specific relations among the various parameters. The analytical study of these various metrics is very difficult. So, we have adopted the same line as in [19] by plotting all the metric functions and studied its structure in various limits along the radial coordinate. For consistency check, we have the same background metric of [19] under the λ tends to zero limit for the first three solutions. For the other two cases, we do not have such limits. Finally, after getting details of the background spacetime we have tried to study the dynamics of the domain wall in those bulk spacetime configurations. In this case also, there exists specific relation among the various coupling parameters so that one can have a static bulk spacetime background inconsistent with the dynamic domain wall.

In many cases again, we also found inflation to exist for a finite period of proper time with respect to domain wall world volume followed by a standard decelerated expansion phase. These kind of features might lead to constructing the viable cosmological model in the domain wall scenario. One important aspect which we have already mentioned earlier is that in the domain wall expansion equation (which is basically the Hubble equation), we have encountered the negative energy density. An important aspect regarding this negative energy density is to lead to a bouncing cosmology which avoids the bulk singularity for finite minimum value of the scale factor $R(\tau)$. This has already been discussed in many situations, but only for the first solution. We have several solutions with different asymptotic as well as near singularity structure for the same kind of background field configurations. So, it would be interesting to study the other solutions on this particular context of bouncing cosmology in detail.

Other significant attempts would be to interpret the various bulk energy density playing the role of different types of unseen energy density with respect to the domain wall point of view. For example, this interesting behavior may help us to construct dark matter and dark energy [29] model building [30] in solving discrepancies with standard general relativity predictions for the galaxy rotation curves [31], late time acceleration of the universe [32], and gravitational lensing [33]. Another possible interesting extension of this work would be to analyze stability under perturbation in the domain wall world volume. An interesting point to analyze would be whether all these types of solutions are compatible in addition to external matter sources such as radiation and baryonic matter, restricted to the domain world volume.

ACKNOWLEDGMENTS

I would like to acknowledge Professor Soumitra SenGupta for carefully reading the manuscript and also for valuable suggestions. I also thank Professor Sudipta Mukherji for his valuable comments during the course of this work.

Note added in proof.—At this stage we would like to illustrate a point regarding a recent study [18] in which the author has found the solution for the bouncing universe by using the Kalb-Ramnod (KR) (second rank antisymmetric tensor B_{MN}) field in the static bulk. In doing so, the author has chosen a particular solution ansatz for the KR field. We can reproduce those results just by changing the sign of the parameter λ in our action with an appropriate numerical coefficient. We take the Kalb-Ramond field instead of the Maxwell field in the bulklike $F_{MN}F^{MN} \rightarrow H_{MNP}H^{MNP}$ where $H_{MNP} = \partial_{[M}B_{NP]}$ being strength of the B_{MN}. After getting equations of motion, we make a particular solution ansatz (only for $n = 5$ dimension) such as $H^{MNP} = e^{2\gamma\phi}\epsilon^{MNPQR}\partial_Q C_R$ where C_M is some dual vector field resembling the electromagnetic potential. This particular ansatz leads us to the solutions that we have discussed throughout this report, with (λ) to be replaced by ($-\lambda$) up to a numerical factor depending upon the rank of the KR field. The structure of the bulk spacetime solutions get reversed at $r = 0$ for the first three types of solutions and corresponding potentials. For the fourth type, the form of the solutions remains the same but in this case, geometry of the spatial section for the bulk spacetime will be open ($k = -1$) as opposed to the electromagnetic case. The structure of all the solutions for the finite range of R will get changed drastically.

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