

Scale dependence of cosmological backreaction

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Because of the noncommutation of spatial averaging and temporal evolution, inhomogeneities and anisotropies (cosmic structures) influence the evolution of the averaged Universe via the cosmological backreaction mechanism. We study the backreaction effect as a function of averaging scale in a perturbative approach up to higher orders. We calculate the hierarchy of the critical scales, at which 10% effects show up from averaging at different orders. The dominant contribution comes from the averaged spatial curvature, observable up to scales of ~ 200 Mpc. The cosmic variance of the local Hubble rate is 10% (5%) for spherical regions of radius 40 (60) Mpc. We compare our result to the one from Newtonian cosmology and Hubble Space Telescope Key Project data.

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Various cosmological observations, interpreted in the framework of spatially flat, homogeneous, and isotropic cosmologies, have now confirmed the accelerated expansion of the Universe. The most direct evidence comes from the study of supernova (SN) of type Ia [1]. Many attempts have been proposed to understand this mystery, e.g., dark energy in the form of a cosmological constant, quintessence field or modification of gravity. However, these suggestions always rely on the homogeneity and isotropy of the cosmic medium, which are rather rough approximations.

The Universe hosts enormous structures. In our neighborhood, there seem to exist two voids, both 35 to 70 Mpc across, associated with the so-called velocity anomaly [2], a large filament known as the Sloan great wall about 400 Mpc long [3] and the Shapely supercluster with a core diameter of 40 Mpc at a distance of ~ 200 Mpc from us [4]. Furthermore, based on the Hubble Space Telescope (HST) Key Project data [5], evidence for a significant anisotropy in the local Hubble expansion at distances of ~ 100 Mpc was found [6], and an anisotropy of SN Ia Hubble diagrams extending to larger distances has been reported recently [7]. Therefore, spatial homogeneity and isotropy seem to be valid only on scales larger than ~ 100 Mpc [8], and effects of local inhomogeneities are worthy of investigation. More specifically, observables from within a few 100 Mpc must be revisited critically. The most fundamental of those are cosmic distances and the Hubble constant H_0 .

In this paper, we study the averaging of the inhomogeneous and anisotropic Universe over a local domain in space-time. We stick to the idea of cosmological inflation, assuming that the Universe approaches homogeneity and isotropy at scales as large as the Hubble distance.

Many cosmological observables are averaged quantities. For instance, the matter power spectrum is a Fourier transform and thus a volume average weighted by a factor $e^{i\mathbf{k}\cdot\mathbf{x}}$. Another very important example is the idealized measurement of H_0 [9]. One picks N standard candles in a local volume V (e.g., SN Ia in the Milky Way's neighborhood out to ~ 100 Mpc), measures their luminosity distances d_i and recession velocities $v_i = cz_i$ (z_i being the redshift of each candle) and performs the average $H_0 \equiv \frac{1}{N} \sum_{i=1}^N \frac{v_i}{d_i}$. In the limit of a very big sample, it turns into a volume average $H_0 = \frac{1}{V} \int_V \frac{v}{d} dV$.

Cosmological observations are made on the past light cone, so one should average over a light-cone volume. However, for objects at $z \ll 1$, spatial averaging on a constant-time-hypersurface is a good approximation, as the Universe does not change significantly on the temporal scale involved.

Because of the nonlinearity of the Einstein equations, spatial averaging and temporal evolution do not commute. Hence, inhomogeneities and anisotropies affect the evolution of the averaged Universe via the so-called “backreaction mechanism” [10–16]. Below, we utilize Buchert's averaging method [12] to estimate the order of magnitude of backreaction effects and study the signatures of averaging from the local measurement of H_0 .

Buchert's setup is well adapted to the situation of a real observer, if we are allowed to neglect the difference between baryons and cold dark matter (CDM). On scales ≥ 10 Mpc, baryon pressure is insignificant, and a real observer comoves with matter, uses her own clock and regards space to be time-orthogonal. These conditions define a comoving synchronous coordinate system. There are no primordial vector perturbations from cosmological inflation, so we assume the Universe to be irrotational. As we are concerned about the present Universe, radiation is thus neglected. Moreover, the cosmological constant is also supposed to vanish, as we ask whether averaging could mimic a component of dark energy. Following Buchert, we

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use physically comoving boundaries to thoroughly fix the averaging procedure.

In the synchronous coordinates, the metric of the inhomogeneous and anisotropic Universe is $ds^2 = -dt^2 + g_{ij}(t, \mathbf{x})dx^i dx^j$. The spatial average of an observable $O(t, \mathbf{x})$ at time t is defined as

$$\langle O \rangle_D \equiv \frac{1}{V_D(t)} \int_D O(t, \mathbf{x}) \sqrt{\det g_{ij}} dx. \quad (1)$$

$V_D(t) \equiv \int_D \sqrt{\det g_{ij}} dx$ is the volume of a comoving domain D , introducing an effective scale factor

$$\frac{a_D}{a_{D_0}} \equiv \left(\frac{V_D}{V_{D_0}} \right)^{1/3}. \quad (2)$$

The subscript 0 denotes the present time. The effective Hubble rate is thus defined as $H_D \equiv \dot{a}_D/a_D = \langle \theta \rangle_D/3$ (θ being the volume expansion rate) [12].

Effective Friedmann equations for a dust Universe follow from averaging Einstein's equations [12],

$$\left(\frac{\dot{a}_D}{a_D} \right)^2 = \frac{8\pi G}{3} \rho_{\text{eff}}, \quad -\frac{\ddot{a}_D}{a_D} = \frac{4\pi G}{3} (\rho_{\text{eff}} + 3p_{\text{eff}}). \quad (3)$$

Here ρ_{eff} and p_{eff} are the energy density and pressure of an effective fluid,

$$\rho_{\text{eff}} \equiv \langle \rho \rangle_D - \frac{1}{16\pi G} (\langle Q \rangle_D + \langle \mathcal{R} \rangle_D), \quad (4)$$

$$p_{\text{eff}} \equiv -\frac{1}{16\pi G} \left(\langle Q \rangle_D - \frac{1}{3} \langle \mathcal{R} \rangle_D \right), \quad (5)$$

where ρ is the energy density of dust. $\langle Q \rangle_D \equiv \frac{2}{3} (\langle \theta^2 \rangle_D - \langle \theta \rangle_D^2) - 2\langle \sigma^2 \rangle_D$ denotes the kinematical backreaction (σ^2 being the shear scalar) and $\langle \mathcal{R} \rangle_D$ the averaged spatial curvature. They are related by an integrability condition [12],

$$(a_D^6 \langle Q \rangle_D)' + a_D^4 (a_D^2 \langle \mathcal{R} \rangle_D)' = 0. \quad (6)$$

We further define an effective equation of state,

$$w_{\text{eff}} \equiv \frac{p_{\text{eff}}}{\rho_{\text{eff}}} = \frac{\langle \mathcal{R} \rangle_D - 3\langle Q \rangle_D}{2\langle \theta \rangle_D^2}.$$

So we find that cosmological backreaction gives rise to a nontrivial equation of state, even for a dust Universe [16].

Alternatively, we may map this effective fluid on a model with dust and dark energy. Let n be the number density of dust particles, and m be their mass. For any comoving domain, $\langle n \rangle_D = \langle n \rangle_{D_0} (a_{D_0}/a_D)^3$. In the dust Universe, $\rho(t, \mathbf{x}) \equiv mn(t, \mathbf{x})$, and we identify $\rho_m \equiv \langle \rho \rangle_D = m\langle n \rangle_D$. From Eq. (4), dark energy is consequently $\rho_{\text{de}} = -(\langle Q \rangle_D + \langle \mathcal{R} \rangle_D)/(16\pi G)$, with the relevant equation of state reading

$$w_{\text{de}} \equiv \frac{p_{\text{de}}}{\rho_{\text{de}}} = \frac{p_{\text{eff}}}{\rho_{\text{de}}} = -\frac{1}{3} + \frac{4\langle Q \rangle_D}{3(\langle Q \rangle_D + \langle \mathcal{R} \rangle_D)}.$$

It is -1 , iff $\langle Q \rangle_D = -\frac{1}{3}\langle \mathcal{R} \rangle_D$ [14], corresponding to a cosmological constant $\Lambda = \langle Q \rangle_D$.

Equations (3) and (6) are not closed, as the four unknown variables $\langle Q \rangle_D$, $\langle \mathcal{R} \rangle_D$, $\langle \rho \rangle_D$ and a_D are constrained by only three equations. Below, we close these dynamical equations for the averaged Universe by means of cosmological perturbation theory.

We wish to estimate the scale dependence of $\langle Q \rangle_D$, $\langle \mathcal{R} \rangle_D$, $\langle \rho \rangle_D$, H_D , and w_{eff} . We start from a spatially flat dust model. In the comoving synchronous gauge, the linear perturbed metric is $ds^2 = -dt^2 + a^2(t)[(1 - 2\Psi)\delta_{ij} + (\partial_i \partial_j - \frac{1}{3}\delta_{ij}\Delta)\chi]dx^i dx^j$. Here, its scale factor $a(t)$ ($a_0 \equiv 1$) is different from a_D , and their relation was provided in Ref. [16]. Ψ and χ are the scalar metric perturbations, and Δ is the three-dimensional Laplace operator. The solutions for Ψ and χ are given in terms of the time-independent peculiar gravitational potential $\varphi(\mathbf{x})$: $\Psi = \frac{1}{2}\Delta\varphi t_0^{4/3} t^{2/3} + \frac{5}{3}\varphi$ and $\chi = -3\varphi t_0^{4/3} t^{2/3}$ (only growing modes are taken into account) [16]. Moreover, φ is related to the hypersurface-invariant variable ζ [17] by $\zeta = \frac{1}{2}\Delta\varphi t_0^{4/3} t^{2/3} - \frac{5}{3}\varphi$.

Following Ref. [16], we use the metric perturbations attained from linear perturbation theory together with the nonperturbative integrability condition to obtain the averaged physical observables up to second order. We focus on the dominant contributions from the growing modes and neglect the decaying ones, since we are interested in the late-time effects of cosmic averaging. Thus, we find

$$\langle Q \rangle_D = \frac{a_{D_0}}{a_D} B(\varphi) t_0^2, \quad (7)$$

$$\langle \mathcal{R} \rangle_D = \frac{20}{3} \frac{a_{D_0}^2}{a_D^2} \langle \Delta\varphi \rangle - 5 \frac{a_{D_0}}{a_D} B(\varphi) t_0^2, \quad (8)$$

$$\langle \rho \rangle_D = \frac{1}{6\pi G t_0^3} \frac{a_{D_0}^3}{a_D^3}, \quad (9)$$

$$H_D = \frac{2}{3t_0} \frac{a_{D_0}^{3/2}}{a_D^{3/2}} \left[1 - \frac{5}{4} \frac{a_D}{a_{D_0}} t_0^2 \langle \Delta\varphi \rangle + \frac{3}{4} \frac{a_D^2}{a_{D_0}^2} t_0^4 \left(B(\varphi) - \frac{25}{24} \langle \Delta\varphi \rangle^2 \right) \right], \quad (10)$$

$$w_{\text{eff}} = \frac{5}{6} \frac{a_D}{a_{D_0}} t_0^2 \langle \Delta\varphi \rangle - \frac{a_D^2}{a_{D_0}^2} t_0^4 \left(B(\varphi) - \frac{25}{12} \langle \Delta\varphi \rangle^2 \right), \quad (11)$$

with $B(\varphi) \equiv \langle \partial^i (\partial_i \varphi \Delta\varphi) - \partial^i (\partial_j \varphi \partial^j \partial_i \varphi) \rangle - \frac{2}{3} \langle \Delta\varphi \rangle^2$ and $\langle O \rangle \equiv \int_D O d\mathbf{x} / \int_D d\mathbf{x}$. We see from Eqs. (7)–(11) that these quantities are polynomials of surface terms. Thus, all information is encoded on the boundaries of the comov-

ing domain D . The temporal dependence of these averaged quantities can be found in Ref. [16], and their leading terms are gauge invariant [16].

Our perturbative results suggest that we could write $\langle Q \rangle_D$ and $\langle \mathcal{R} \rangle_D$ in a Laurent series of a_D . (Recently, a power-law ansatz for the integrability condition was investigated in Ref. [18].) We know from Eqs. (7) and (8) that $\langle Q \rangle_D$ and $\langle \mathcal{R} \rangle_D$ start from different powers: a_D^{-1} and a_D^{-2} , so $\langle Q \rangle_D = \sum_{n=-1} Q_n (a_D/a_{D_0})^n$ and $\langle \mathcal{R} \rangle_D = \sum_{n=-2} \mathcal{R}_n (a_D/a_{D_0})^n$. The integrability condition then connects the coefficients: $(n+6)Q_n + (n+2)\mathcal{R}_n = 0$. Thus, $Q_0 = -\frac{1}{3}\mathcal{R}_0$ at third order in perturbation theory. Therefore, cosmological backreaction can mimic a cosmological constant, but induces extra terms as well. The third order results will be presented elsewhere [19].

The effect of cosmological backreaction in the early Universe is tiny and is undistinguishable from that of a homogeneous curvature, as $w_{\text{de}} \rightarrow -1/3$ when $a_D \rightarrow 0$. This result seems inconsistent with our intuition of a vanishing cosmological backreaction at early times, suggesting that w_{de} should also vanish. However, as we have seen above, cosmological averaging gives rise to extra degrees of freedom in the dynamics of the averaged Universe.

The effect of averaging over a typical domain is provided by the ensemble average. From Eq. (11), we find $\bar{w}_{\text{eff}} = \frac{11}{4} \frac{(a_D/a_{D_0})^2 t_0^4 \langle \Delta \varphi \rangle^2}{a_{D_0}} > 0$. This means that cosmological backreaction is expected to lead to a positive definite equation of state. However, we should pay attention that \bar{w}_{eff} is of second order, but the root of its variance $[\text{Var}(w_{\text{eff}})]^{1/2} = \frac{5}{6} \frac{a_D}{a_{D_0}} t_0^2 \langle \Delta \varphi \rangle^{1/2}$ is of first order and therefore larger than \bar{w}_{eff} . Thus, the possibilities of $w_{\text{eff}} < 0$ and the effective acceleration of the averaged Universe cannot be easily excluded. Looking only at mean values of the ensemble obviously causes an underestimation of the possible backreaction effects, as we often observe just one particular domain in the Universe.

We now turn to estimate the order of magnitude of cosmological backreaction as a function of the averaging scale $r \sim V_{D_0}^{1/3}$. We show that cosmological averaging produces important modifications to local physical observables and determine the averaging scale, at which corrections show up at a 10% level.

Effective acceleration of the averaged Universe occurs if $\rho_{\text{eff}} + 3p_{\text{eff}} < 0$, i.e., $\langle Q \rangle_D > 4\pi G \langle \rho \rangle_D$. From Eqs. (7) and (9) we have

$$\left| \frac{\langle Q \rangle_D}{4\pi G \langle \rho \rangle_D} \right| = \frac{3}{2} \frac{a_D^2}{a_{D_0}^2} B(\varphi) t_0^4 = \frac{8}{27} \frac{R_H^4}{(1+z)^2} B(\varphi), \quad (12)$$

with $R_H = 2.998 \times 10^3 h^{-1}$ Mpc being the present Hubble distance. In Eq. (12), we can safely use the results for the background Universe: $a_D/a_{D_0} = 1/(1+z)$ and $t_0 = 2R_H/3$, because $B(\varphi)$ is of second order. Since the ratio

in Eq. (12) is dimensionless, a dimensional analysis immediately implies $|\langle Q \rangle_D / 4\pi G \langle \rho \rangle_D| \propto (R_H/r)^4$, where for an almost scale-invariant power spectrum the unique relevant scale is the averaging scale r . The order of magnitude of Eq. (12) can be estimated as

$$\left| \frac{\langle Q \rangle_D}{4\pi G \langle \rho \rangle_D} \right| \sim \frac{8}{75} \frac{1}{(1+z)^2} \left(\frac{R_H}{r} \right)^4 \mathcal{P}_\zeta. \quad (13)$$

$\mathcal{P}_\zeta = 2.457 \times 10^{-9}$ is the dimensionless power spectrum [20]. We pick the second term in $B(\varphi)$, $\langle \partial^i (\partial_j \varphi \partial^j \partial_i \varphi) \rangle$, to demonstrate how to obtain this estimate. In the Fourier space, $\partial^i \varphi \rightarrow ik^i \varphi \sim \varphi/r$. The latter step comes from the observation that only structure of the size of the averaged volume cannot be averaged out. At much smaller scales, structures contribute a negligible amount to $B(\varphi)$, because it is not positive definite and is expected to fluctuate on small scales. Thus,

$$\langle \partial^i (\partial_j \varphi \partial^j \partial_i \varphi) \rangle \rightarrow \frac{1}{r^4} \langle \varphi^2 \rangle \sim \frac{1}{r^4} \mathcal{P}_\varphi = \frac{9}{25} \frac{1}{r^4} \mathcal{P}_\zeta,$$

i.e., each derivative in $B(\varphi)$ contributes a factor $1/r$. Also, $\langle \varphi^2 \rangle$ in the Fourier space is estimated as the power spectrum \mathcal{P}_φ . Since φ is constant in time, and $\zeta \approx -5\varphi/3$ on superhorizon scales, we can identify today's \mathcal{P}_φ with $9\mathcal{P}_\zeta/25$. Similar estimation works for the other two terms in $B(\varphi)$.

The kinematical backreaction induces 10% and larger modifications if $|\langle Q \rangle_D / 4\pi G \langle \rho \rangle_D| \gtrsim 0.1$. This happens if

$$r_Q \lesssim \frac{21h^{-1}}{\sqrt{1+z}} \text{ Mpc}. \quad (14)$$

For observations at $z \ll 1$, $r_Q \lesssim 30$ Mpc ($h = 0.7$).

The averaged spatial curvature $\langle \mathcal{R} \rangle_D$ is the most important correction to energy density. The criterion for the scale, at which its effect emerges, is estimated analogously by

$$\left| \frac{\rho_{\text{eff}}}{\langle \rho \rangle_D} - 1 \right| \approx \left| \frac{\langle \mathcal{R} \rangle_D}{16\pi G \langle \rho \rangle_D} \right| \sim \frac{2}{3} \frac{1}{1+z} \left(\frac{R_H}{r} \right)^2 \sqrt{\mathcal{P}_\zeta}. \quad (15)$$

We find effects larger than 10% within

$$r_{\mathcal{R}} \lesssim \frac{54h^{-1}}{\sqrt{1+z}} \text{ Mpc}. \quad (16)$$

At small redshifts, $r_{\mathcal{R}} \lesssim 77$ Mpc. Furthermore, effects above 1% are expected up to a scale of ~ 240 Mpc. Note that the curvature of the Universe has been measured at the few per cent accuracy in the cosmic microwave background (CMB) [20]. It was shown in Ref. [21] that even small curvature might affect the analysis of high- z SNe significantly.

Finally, we turn to the Hubble rate. To go beyond the order of magnitude estimates above, we calculate the ensemble mean and its variance (cosmic variance) of the

relative fluctuation of the Hubble rate $\delta_H \equiv (H_D - H_0)/H_0$. Before doing so, let us stress that the analogous order of magnitude estimate for δ_H agrees with the result for $[\text{Var}(\delta_H)]^{1/2}$ given below up to a factor of ~ 2 . For a spherical domain of radius r , we find from Eq. (10),

$$\bar{\delta}_H = -\frac{41}{32} \frac{a_D^2}{a_{D_0}^2} t_0^4 \overline{\langle \Delta \varphi \rangle^2}, \quad \text{Var}(\delta_H) = \frac{25}{16} \frac{a_D^2}{a_{D_0}^2} t_0^4 \overline{\langle \Delta \varphi \rangle^2}, \quad (17)$$

where

$$\begin{aligned} \overline{\langle \Delta \varphi \rangle^2} &= \int \frac{d\mathbf{x}_1 d\mathbf{x}_2}{V^2} \frac{d\mathbf{k}_1 d\mathbf{k}_2}{(2\pi)^6} k_1^2 k_2^2 \overline{\varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2}} e^{i(\mathbf{k}_1 \cdot \mathbf{x}_1 + \mathbf{k}_2 \cdot \mathbf{x}_2)} \\ &= \int \frac{d\mathbf{x}_1 d\mathbf{x}_2}{V^2} \frac{d\mathbf{k}}{32\pi^4} k \mathcal{P}_\varphi(k) e^{i\mathbf{k} \cdot (\mathbf{x}_1 + \mathbf{x}_2)}, \end{aligned}$$

with $V = 4\pi r^3/3$ (a top-hat window). Above, we introduce the dimensionless power spectrum as $\overline{\varphi_{\mathbf{k}_1} \varphi_{\mathbf{k}_2}} \equiv 2\pi^2 \delta(\mathbf{k}_1 + \mathbf{k}_2) \mathcal{P}_\varphi(k_1)/k_1^3$ ($k \equiv |\mathbf{k}|$). So

$$\begin{aligned} \text{Var}(\delta_H) &= \frac{25}{144\pi^2} \frac{1}{(1+z)^2} \left(\frac{R_H}{r}\right)^4 \\ &\quad \times \int_0^\infty dx \mathcal{P}_\varphi(x/r) J_{3/2}^2(x). \quad (18) \end{aligned}$$

$J_{3/2}(x)$ is the Bessel function of first kind ($x \equiv kr$). For a scale-invariant power spectrum, we must introduce an ultraviolet cutoff $\mathcal{P}_\varphi(k) = \mathcal{P}_\varphi e^{-k/k_c}$. No cutoff is required for a red-tilted spectrum $\mathcal{P}_\varphi(k) = \mathcal{P}_\varphi (k/k_0)^{n_s-1}$ ($n_s < 1$ being the spectrum index), consistent with WMAP5 [20]. Here, let us stress that although $[\text{Var}(\delta_H)]^{1/2}$ is only a first order quantity, the next contribution is already of third order, if we consult the perturbed metric to second order. Since we constrain our attention to the leading order effects, these higher order terms are negligible [19].

Now we can link the effect of cosmological backreaction in Buchert's setup (evaluated in a perturbative approach up to second order) to actual cosmological observations. The trick is to consider the scale dependence but not the time dependence. The value of the relative fluctuation of the Hubble rate in Eq. (17) is dominated by its variance, and thus the sign of the observed value of δ_H cannot be predicted. A comparison of the mean and the root of the variance of δ_H tells us that perturbation theory breaks down below ~ 20 Mpc.

The scale dependence of the cosmic variance of δ_H has previously been studied in the context of Newtonian cosmology [22,23], largely based on CDM simulations. In this setting, the variance of δ_H is due to peculiar motions (besides sampling variance and observational errors). In a relativistic and comoving approach, peculiar velocities vanish identically, and the cosmic variance of the Hubble rate turns into a curvature effect, because Eqs. (8) and (17) give $\text{Var}(\delta_H) \propto \overline{\langle \mathcal{R} \rangle_D^2}$.

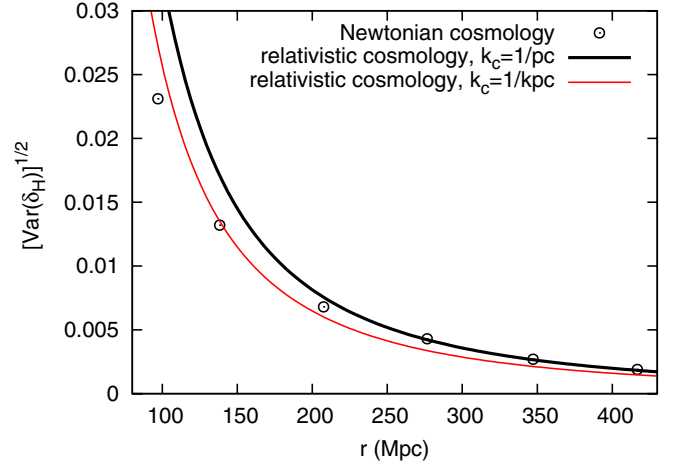


FIG. 1 (color online). Scale dependence of the cosmic variance of the Hubble rate. Data are from the Newtonian CDM model in Ref. [23], with $h = 0.5$, $\Omega_m = 1$ and a COBE-normalized power spectrum. Thick and thin lines correspond to the relativistic result Eq. (18) for a scale-invariant power spectrum with cutoffs at $k_c = 1/\text{kpc}$ (simulation) and $1/\text{pc}$ (physical), respectively.

In Fig. 1, we compare the relativistic (correct up to second order) result Eq. (18) to Newtonian “standard CDM” case in Ref. [23]. We find that up to ~ 400 Mpc, our results for scale-invariant power spectra ($k_c = 1/\text{kpc}$ corresponding to a typical cutoff in CDM simulations and $1/\text{pc}$ to the physical cutoff in the primordial CDM spectrum) agree with Newtonian simulations. This agreement is not unexpected, as metric perturbations and peculiar velocities are small at ~ 100 Mpc scales.

The consistency between the relativistic and Newtonian approaches encourages the comparison of our perturbative results with experimental data. We compare Eq. (18) with observations from the HST Key Project [5]. We use 54 individual measurements of H_0 in the CMB rest frame (corrected for local flow) from SN Ia and the Tully-Fisher relation (Tables (6) and (7) in Ref. [5]). We have checked explicitly that the SN and Tully-Fisher measurements of H_0 are consistent with each other, while we cannot confirm that for the fundamental plane method and thus dropped them from a former analysis.

We restrict our analysis to objects between 31.3 to 467.0 Mpc, as Eq. (18) can be trusted only above 30 Mpc. Be r_i , H_i and σ_i the distance, Hubble rate and 1σ error for the i 'th datum, with distances increasing. We calculate the mean distance for the nearest k objects by $\bar{r}_k = \sum_{i=1}^k g_i r_i / \sum_{i=1}^k g_i$, with weights $g_i = H_0^2 / \sigma_i^2$. An analogue holds for the averaged Hubble rate \bar{H}_k , i.e., H_D for different subsets. The empirical variance of each subset is $\bar{\sigma}_k^2 = [\sum_{i=1}^k g_i (H_i - \bar{H}_k)^2] / [H_0^2 (k-1) \sum_{i=1}^k g_i]$. Notice that Eq. (18) is insensitive to global calibration issues.

The comparison of the result Eq. (18) with the HST Key Project data is shown in Fig. 2. We now normalize to the WMAP5 best-fit power-law spectrum, with pivot

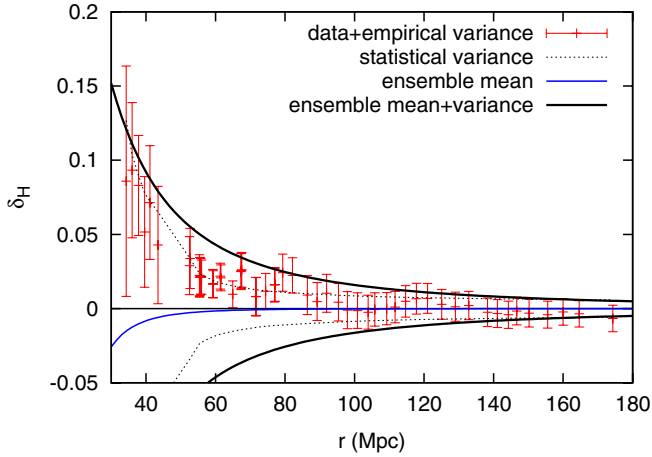


FIG. 2 (color online). Relative fluctuation of the Hubble rate from cosmological backreaction and its cosmic variance band (thick lines) compared to the empirical mean and variance of δ_H obtained from the HST Key Project data [5] as a function of averaging radius. The thin line shows the ensemble mean of δ_H . The band enclosed by the thick lines indicates the effect of the inhomogeneities ($\propto 1/r^2$), and the dashed lines are the effect from sampling with given measurement errors in an otherwise perfectly homogeneous Universe.

$k_0 = 0.002/\text{Mpc}$ and spectral index $n_s = 0.960$ and use $H_0 = 72 \text{ km/s/Mpc}$ [20]. We see that the theoretical band matches the experimental data well, without any fit parameter in the panel. Moreover, we see from Fig. 2 that the value of δ_H is positive within ~ 100 Mpc. This is consistent with the result in a recent paper [24] that we are located in a 200–300 Mpc underdense void, which is expanding faster than the global Hubble rate.

Before we can claim that we have observed the expected $1/r^2$ behavior in Eq. (18) and thus the evidence for cosmological backreaction, we must make sure that statistical noise cannot account for it. In the case of a perfectly homogeneous coverage of the averaged domain with standard candles, we would expect a $1/r^{3/2}$ behavior. In Fig. 2, we show the statistical noise for the actual data set ($1/(\sum_{i=1}^k g_i)^{1/2}$), which is smaller than our result Eq. (18). It turns out that the sampling noise for this small data set is still too large to claim that the inhomogeneity of the Universe can be detected in the relative fluctuation of the Hubble rate observed by the HST Key Project. However, it is fully consistent with our theoretical expectations. Actually the fluctuation δ_H appears to be smaller

than expected, and one might wonder why that is so. From the theoretical expectation plotted in Fig. 2, we find that at $\sim 40(60)$ Mpc, the value of H_D differs from its global value 72 km/s/Mpc (WMAP5) by about 10% (5%), whereas the expected variance for a perfectly homogeneous and isotropic Universe is 8% (2%).

A similar analysis of the Hubble diagram was pioneered in Refs. [25–28], in which the velocity field of the local Universe and its influence on the correlated fluctuations in luminosity distance and the Hubble rate was analyzed. Two essential differences to this work are that our analysis includes effects to higher orders and we study the scale dependence of the averaged observables. Although the relative fluctuation of the Hubble rate was not explicitly analyzed in Refs. [25–27], it seems to us that our results are consistent with those findings.

To summarize, we argue that cosmological averaging (backreaction) gives rise to observable effects up to scales of ~ 200 Mpc. However, it is not sufficient to explain the observed accelerated expansion at this point.

We find a hierarchy of backreaction effects. The averaged spatial curvature $\langle \mathcal{R} \rangle_D$ leads to 10% (1%) effects up to $\sim 80(240)$ Mpc in a dust model with $h = 0.7$. Below ~ 40 Mpc, the cosmic variance of the Hubble rate is larger than 10%, which coincides with the estimate from the effect of peculiar motions in Newtonian setup. Within ~ 30 Mpc, the kinematical backreaction $\langle Q \rangle_D$, due to second order perturbations caused by local inhomogeneities and anisotropies, enters the game. Cosmological backreaction may put some of the steps on the cosmological distance ladder in question, as they are deeply in the domain of large backreaction, i.e., large fluctuations among small averaged volumes.

Our findings call for revisiting local observations, like galaxy redshift surveys, in terms of possible backreaction signatures. The large scale physics of primordial CMB anisotropies is not affected. However, this statement cannot be made for secondary effects, e.g., the late integrated Sachs-Wolfe effect.

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