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Probing the effective number of neutrino species with the cosmic microwave background

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We discuss how much we can probe the effective number of neutrino species N_ν with the cosmic microwave background alone. Using the data of the WMAP, ACBAR, CBI, and BOOMERANG experiments, we obtain a constraint on the effective number of neutrino species as $0.96 < N_\nu < 7.94$ at 95% C.L. for a power-law ΛCDM flat universe model. The limit is improved to be $1.39 < N_\nu < 6.38$ at 95% C.L. if we assume that the baryon density, N_ν, and the helium abundance are related by the big bang nucleosynthesis theory. We also provide a forecast for the Planck experiment using a Markov chain Monte Carlo approach. In addition to constraining N_ν, we investigate how the big bang nucleosynthesis relation affects the estimation for these parameters and other cosmological parameters.

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I. INTRODUCTION

Cosmology is now becoming a precision science, and cosmological observations can give us a lot of information for our understanding of the Universe. Moreover, the interplay between cosmology and particle physics in various contexts has also been discussed vigorously. One of such examples is the effective number of neutrino species N_ν. Although collider experiments such as the CERN LEP have measured the number of light active neutrino types to be 2.92 ± 0.06 [1], it is important to cross-check this value because cosmological measurements may lead to different values. This could be due to an extra radiation component which is predicted by some models of particle physics such as sterile neutrinos (see Ref. [2] and references therein), or due to incomplete thermalization of neutrinos in the low-scale reheating universe in which the reheating temperature $T_{\rm reh}$ can be as low as $T_{\rm reh} \sim \mathcal{O}(1)$ MeV and N_ν is predicted to be less than 3 [3–6]. If such a nonstandard ingredient exists, it can affect big bang nucleosynthesis (BBN), the cosmic microwave background (CMB), large scale structure (LSS) and so on; thus precise cosmological observations can probe these scenarios through the effective number of neutrino species.

Constraints on N_ν have been investigated in the literature using the information of the CMB and LSS, sometimes with priors on the Hubble constant, cosmic age, and helium abundance [7–30]. Although the CMB, in general, can constrain various quantities severely, since the effects of N_ν on the CMB are degenerate with some cosmological parameters, the studies so far have combined CMB data with some other observations such as LSS to obtain a sensible constraint on N_ν. However, when one uses the data from LSS, constraints can become different depending on how one treats nonlinear correction/bias on small scales for the matter power spectrum [23]. Furthermore, different LSS data seem to give different constraints on N_ν [16,17,20,21,23]. Regarding the prior on the Hubble constant H_0, as is summarized in Ref. [20], it can yield some constraints on N_ν when combined with CMB data (without LSS data) [8–10,12,13], but they depend on the H_0 prior adopted. One may consider that we can use the usually assumed prior on the Hubble constant based on the result by Freedman *et al.* $H_0 = 72 \pm 8$ [31], but another group reported a somewhat lower value as $H_0 = 62.3 \pm 5.2$ [32]. If the lower value for H_0 is adopted as the prior, a resulting constraint on N_ν would be different. Having these considerations in mind, it is desirable to investigate a constraint on N_ν without these kinds of uncertainties.

In this paper, we study a constraint on N_ν from CMB experiments alone. By making the analysis of CMB data alone, we can avoid such subtleties as the galaxy-bias/nonlinear corrections and the value for the prior on the Hubble constant. However, as is mentioned above, the effects of N_ν are strongly degenerate in the CMB with other cosmological parameters such as energy density of matter, the Hubble constant, and the scalar spectral index, and, in fact, we could not obtain a meaningful bound only with Wilkinson Microwave Anisotropy Probe 3-year result (WMAP3) [20,23]. The recent WMAP5-alone analysis gives a better constraint but it still cannot give an upper bound [27,28]. As we will discuss later, the degeneracy is significant up to about the 2nd/3rd peak of the CMB power spectrum where the observation of WMAP has been precisely measured. To break this degeneracy to some extent, it would be helpful to have the information at higher multipoles where signals unique to relativistic neutrinos are expected to appear [33]. Recently, the data from Arcminute Cosmology Bolometer Array Receiver (ACBAR), which probes the CMB at higher multipoles than those of WMAP, have been updated [34]. By using these data in addition to other small scale observations such as Balloon Observations of Millimetric Extragalactic Radiation and Geophysics

(BOOMERANG) and Cosmic Background Imager (CBI), we can obtain a relatively severe constraint on N_ν which is comparable to that obtained previously with LSS data.

The organization of this paper is as follows. In the next section, we start with a discussion of how N_ν affects the CMB power spectrum, which helps us to understand our results for the constraint on N_ν. In Sec. III, we study the current constraint on N_ν using observations of the CMB alone. We use the data from WMAP5, the recent ACBAR, BOOMERANG, and CBI. Furthermore, we forecast the constraint from the future Planck experiment. In the final section, we summarize our results and discuss their implications for some models of particle physics/the early universe.

II. EFFECTS OF N_ν ON THE CMB

The effective number of neutrino species N_ν represents the energy density stored in relativistic components as

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_x = \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu\right]\rho_\gamma \qquad (1)$$

where ρ_γ, ρ_ν, and ρ_x are energy densities of photons, three species of massless active neutrinos, and some possible extra radiation components, respectively. In this paper, we assume that neutrinos are massless and have no chemical potential. For the case with the standard three neutrino flavors without an extra relativistic component, the effective number of neutrinos is $N_\nu = 3.046$, where some corrections from the incomplete decoupling due to a slight interaction of neutrinos with electrons/positrons and finite temperature QED effects to the electromagnetic plasma are taken into account [35]. Any deviation of N_ν from this value implies that there exists an extra relativistic component and/or some nonstandard thermal history takes place such as the low-reheating temperature scenario.

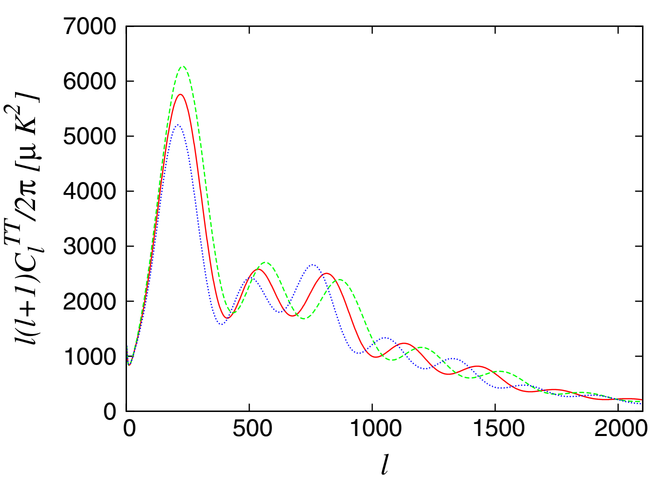

FIG. 1 (color online). CMB power spectra for the cases with $N_\nu = 1$ (blue dotted line), 3 (red solid line), and 5 (green dashed line). Other cosmological parameters are taken as the mean value from the WMAP5-alone analysis for a power-law flat ΛCDM model.

To illustrate the effects of N_ν on the CMB, we plot CMB power spectra for several values of N_ν in Fig. 1. Other cosmological parameters are assumed as the mean values of the WMAP5-alone analysis for a power-law flat ΛCDM model. As seen from the figure, as we increase the value of N_ν, the height of the 1st peak is strongly enhanced and the positions of acoustic peaks are shifted to higher multipoles. Also, the amplitude on small scales (higher multipoles) is suppressed. Below, we discuss where these changes are coming from.

One of the main effects of N_ν comes from the change of the epoch of the radiation-matter equality. By increasing (decreasing) the value of N_ν, the radiation-matter equality occurs later (earlier). Thus the increase (decrease) of N_ν gives almost the same effect of the decrease (increase) of energy density of matter. One of the noticeable features is that the height of the first acoustic peak is enhanced by increasing the value of N_ν. This is due to the early integrated Sachs-Wolfe (ISW) effect in which fluctuations of the corresponding scale, having crossed the sound horizon in the radiation-dominated epoch, are boosted by the decay of the gravitational potential. Thus a larger amount of relativistic species drives the first peak higher. Another effect is the shift of the position of acoustic peaks due to the change of the radiation-matter equality through the change of N_ν. The position of acoustic peaks is well captured by the so-called acoustic scale θ_A which is inversely proportional to the peak position and written as

$$\theta_A = \frac{r_s(z_{\rm rec})}{r_\theta(z_{\rm rec})} \qquad (2)$$

where $r_\theta(z_{\rm rec})$ and $r_s(z_{\rm rec})$ are the comoving angular diameter distance to the last scattering surface and the sound horizon at the recombination epoch $z_{\rm rec}$, respectively. Although $r_\theta(z_{\rm rec})$ almost remains the same for different values of N_ν, $r_s(z_{\rm rec})$ becomes smaller when N_ν is increased. Thus the positions of acoustic peaks are shifted to higher multipoles (smaller scales) by increasing the value of N_ν. Furthermore, since the position of the nth peak can be roughly written as $l_n \sim n\pi/\theta_A$, separations of the peaks also become greater for larger N_ν.

Another important effect is free-streaming of ultrarelativistic neutrinos [33]. The perturbation of ultrarelativistic neutrinos propagates with the speed of light, which is faster than the sound speed of acoustic oscillations of photon-baryon fluid. The coupled photon-baryon component oscillates like a compressional fluid; on the other hand, ultrarelativistic neutrinos free-stream to erase their fluctuations. These two components are coupled via gravity; thus the fluctuations of photons can also be affected by the free-streaming of neutrinos, which results in the damping of the amplitude and the shift of the acoustic oscillations. The effects are significant on small scales where fluctuations of a given scale enter the horizon while the energy density of ultrarelativistic neutrinos takes a significant portion of that of the universe.

Although the effects of the standard cosmological parameters on the heights and positions of acoustic peaks are well known, here we discuss them in some phenomenological way including the effects of N_ν. For this purpose, we calculated the response of the heights and positions of the acoustic peaks to the change of the cosmological parameters up to the 5th peak around the fiducial values. As a fiducial parameter set, we take those of the mean value from the WMAP5-alone analysis for a power-law flat ΛCDM model. The shifts of the positions of the acoustic peaks are

$$\Delta l_1 = 15.58\frac{\Delta\omega_b}{\omega_b} - 26.99\frac{\Delta\omega_m}{\omega_m} + 36.01\frac{\Delta n_s}{n_s} + 0.94\frac{\Delta Y_p}{Y_p} - 44.58\frac{\Delta h}{h} + 15.53\frac{\Delta N_\nu}{N_\nu}, \quad (3)$$

$$\Delta l_2 = 62.57\frac{\Delta\omega_b}{\omega_b} - 74.90\frac{\Delta\omega_m}{\omega_m} + 14.69\frac{\Delta n_s}{n_s} + 2.81\frac{\Delta Y_p}{Y_p} - 108.60\frac{\Delta h}{h} + 47.73\frac{\Delta N_\nu}{N_\nu}, \quad (4)$$

$$\Delta l_3 = 74.23\frac{\Delta\omega_b}{\omega_b} - 143.47\frac{\Delta\omega_m}{\omega_m} + 9.88\frac{\Delta n_s}{n_s} + 4.64\frac{\Delta Y_p}{Y_p} - 152.39\frac{\Delta h}{h} + 81.82\frac{\Delta N_\nu}{N_\nu}, \quad (5)$$

$$\Delta l_4 = 110.84\frac{\Delta\omega_b}{\omega_b} - 181.89\frac{\Delta\omega_m}{\omega_m} + 7.29\frac{\Delta n_s}{n_s} + 6.77\frac{\Delta Y_p}{Y_p} - 220.21\frac{\Delta h}{h} + 112.76\frac{\Delta N_\nu}{N_\nu}, \quad (6)$$

$$\Delta l_5 = 136.88\frac{\Delta\omega_b}{\omega_b} - 237.18\frac{\Delta\omega_m}{\omega_m} + 6.20\frac{\Delta n_s}{n_s} + 7.79\frac{\Delta Y_p}{Y_p} - 276.02\frac{\Delta h}{h} + 145.34\frac{\Delta N_\nu}{N_\nu}, \quad (7)$$

where l_i means the position of the ith peak. ω_b, ω_m, n_s, Y_p, and h are the energy densities of the baryon and matter, the scalar spectral index, the primordial helium abundance, and the normalized Hubble constant. n_s is defined at the wave number $k = 0.05\ \mathrm{Mpc}^{-1}$. The positive derivatives of the peak positions with respect to N_ν demonstrate the decrease in $r_s(z_{\rm rec})$ due to the increase in N_ν.

The responses of the heights of acoustic peaks to the change of various cosmological parameters are

$$\frac{\Delta\mathcal{C}_{l_1}}{\mathcal{C}_{l_1}} = 0.429\frac{\Delta\omega_b}{\omega_b} - 0.632\frac{\Delta\omega_m}{\omega_m} - 0.947\frac{\Delta n_s}{n_s} - 0.0065\frac{\Delta Y_p}{Y_p} + 0.141\frac{\Delta N_\nu}{N_\nu}, \quad (8)$$

$$\frac{\Delta\mathcal{C}_{l_2}}{\mathcal{C}_{l_2}} = -0.211\frac{\Delta\omega_b}{\omega_b} - 0.579\frac{\Delta\omega_m}{\omega_m} - 0.034\frac{\Delta n_s}{n_s} - 0.035\frac{\Delta Y_p}{Y_p} + 0.083\frac{\Delta N_\nu}{N_\nu}, \quad (9)$$

$$\frac{\Delta\mathcal{C}_{l_3}}{\mathcal{C}_{l_3}} = 0.026\frac{\Delta\omega_b}{\omega_b} - 0.136\frac{\Delta\omega_m}{\omega_m} + 0.276\frac{\Delta n_s}{n_s} - 0.071\frac{\Delta Y_p}{Y_p} - 0.080\frac{\Delta N_\nu}{N_\nu}, \quad (10)$$

$$\frac{\Delta\mathcal{C}_{l_4}}{\mathcal{C}_{l_4}} = -0.044\frac{\Delta\omega_b}{\omega_b} - 0.229\frac{\Delta\omega_m}{\omega_m} + 0.587\frac{\Delta n_s}{n_s} - 0.125\frac{\Delta Y_p}{Y_p} - 0.108\frac{\Delta N_\nu}{N_\nu}, \quad (11)$$

$$\frac{\Delta\mathcal{C}_{l_5}}{\mathcal{C}_{l_5}} = 0.149\frac{\Delta\omega_b}{\omega_b} - 0.006\frac{\Delta\omega_m}{\omega_m} + 0.776\frac{\Delta n_s}{n_s} - 0.172\frac{\Delta Y_p}{Y_p} - 0.216\frac{\Delta N_\nu}{N_\nu}, \quad (12)$$

where $\mathcal{C}_l = l(l+1)C_l^{TT}/2\pi$. Since h gives only negligible changes to the heights of the peaks, we omit it. By increasing the value of N_ν, the height of the 1st peak is enhanced due to the early ISW effect, as previously mentioned. For the 3rd and higher peaks, the heights are damped more by increasing N_ν, which is inferred from the negative coefficients. This is due to the effect of free-streaming of neutrinos [33].

In addition, for later convenience, we also show the derivatives of the peak heights relative to the first peak height following Ref. [36]. Here, $H_i \equiv \mathcal{C}_i/\mathcal{C}_1$ for $i = 2$–5. They are useful quantities when we interpret degeneracies since the dependence on the overall amplitude is canceled out.

$$\Delta H_2 = -0.291\frac{\Delta\omega_b}{\omega_b} + 0.023\frac{\Delta\omega_m}{\omega_m} + 0.396\frac{\Delta n_s}{n_s} - 0.013\frac{\Delta Y_p}{Y_p} - 0.026\frac{\Delta N_\nu}{N_\nu}, \quad (13)$$

$$\Delta H_3 = -0.177\frac{\Delta\omega_b}{\omega_b} + 0.206\frac{\Delta\omega_m}{\omega_m} + 0.514\frac{\Delta n_s}{n_s} - 0.028\frac{\Delta Y_p}{Y_p} - 0.098\frac{\Delta N_\nu}{N_\nu}, \quad (14)$$

$$\Delta H_4 = -0.102\frac{\Delta\omega_b}{\omega_b} + 0.082\frac{\Delta\omega_m}{\omega_m} + 0.317\frac{\Delta n_s}{n_s} - 0.025\frac{\Delta Y_p}{Y_p} - 0.054\frac{\Delta N_\nu}{N_\nu}, \quad (15)$$

$$\Delta H_5 = -0.040\frac{\Delta\omega_b}{\omega_b} + 0.084\frac{\Delta\omega_m}{\omega_m} + 0.236\frac{\Delta n_s}{n_s} - 0.023\frac{\Delta Y_p}{Y_p} - 0.052\frac{\Delta N_\nu}{N_\nu}. \quad (16)$$

III. CONSTRAINT ON N_ν FROM OBSERVATIONS OF THE CMB

In this section, we present our result for the constraint on N_ν from the CMB alone. First, we give some details of our analysis. We use the CMB data of WMAP5 [27,28,37–39], BOOMERANG [40–42], CBI [43], and ACBAR [34]. We performed a Markov chain Monte Carlo (MCMC) analysis to obtain constraints on cosmological parameters using the COSMOMC code [44] with some modifications which are described in the following. We explore a 9 dimensional parameter space which consists of ω_b, ω_c, τ, θ_s, n_s, A_s, A_{SZ}, Y_p, and N_ν. Here, ω_c is the energy density of dark matter, τ is the optical depth of reionization, θ_s is the acoustic peak scale [45], A_s is the amplitude of primordial fluctuations, and A_{SZ} is the amplitude of thermal Sunyaev-Zel'dovich (SZ) effect which is normalized to the C_l^{SZ} template from Ref. [46].

As far as the CMB is concerned, these cosmological parameters can be considered to be independent. However, when we take into account the BBN theory, Y_p is determined once ω_b and N_ν are given. In this case, we should relate these parameters to each other and sample an 8 dimensional parameter space. We shall refer to this relation among Y_p, ω_b, and N_ν as the BBN relation. For this purpose, we calculate Y_p as a function of ω_b and N_ν using the Kawano BBN code [47] with some updates in the nuclear reaction network part based on Ref. [48]. Such a relation is considered in the CMB analyses in Refs. [26,49–52]. In passing, we would like to make a comment on the fitting formula for the BBN calculation presented in Ref. [53], which has been used in the authors' previous work [50]. We do not adopt the formula here since, as we will see later, our MCMC chains for the constraints from the present data sets sometimes go to the region beyond the range over which their fitting formula is valid, $0 \leq N_\nu \leq 6$. For the Planck forecast, where the chains are contained in that region, we obtain the same results with the Kawano code and the fitting formula of Ref. [53]. Furthermore, we also consider the case where the helium abundance is fixed to $Y_p = 0.24$ since, in most analyses, the primordial helium abundance is fixed to this value. Finally, it should be noted that this BBN relation is not necessarily realized in some cases. We can think of more exotic scenarios in which the BBN theory cannot relate those parameters. For example, ω_b and Y_p may vary between BBN and CMB epochs [54], or an increase in N_ν may take place [55].

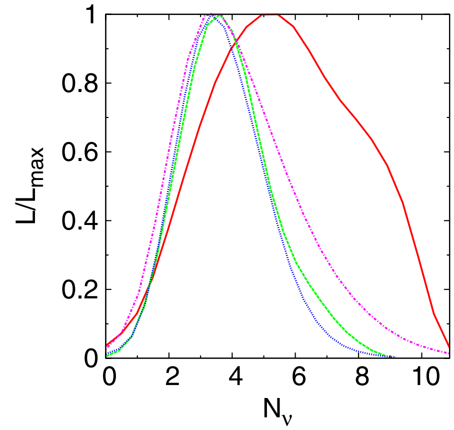

FIG. 2 (color online). The 1D posterior distributions of N_ν. The red solid line uses WMAP5 alone (with $Y_p = 0.24$ fixed) and the other lines use WMAP5 + ACBAR + BOOMERANG + CBI with different assumptions on Y_p. The green dashed line fixes it to be $Y_p = 0.24$, the blue dotted line uses the BBN relation to fix Y_p from ω_b and N_ν, and the magenta dot-dashed line treats Y_p as a free parameter. For the analysis with WMAP5 alone, we assumed the prior on the cosmic age as 10 Gyr $< t_0 <$ 20 Gyr.

Now, we present our results in order. In Fig. 2, the posterior 1D distributions for N_ν are shown for the analysis with WMAP5 alone and WMAP5 + ACBAR + BOOMERANG + CBI (CMB all). The former is shown by a red solid line. For the latter case, the results for different assumptions on Y_p are depicted: Y_p being fixed as $Y_p = 0.24$ (green dashed line), Y_p determined from the BBN relation (blue dotted line), and Y_p being treated as a free parameter (magenta dot-dashed line). Corresponding constraints on N_ν are summarized in Table I. We also show 2D contours of 68% and 95% C.L. in the planes of N_ν vs

TABLE I. The mean values, and 68% and 95% limits of N_ν for several current CMB data sets and assumptions of Y_p.

	Mean	68% ↑ 68% ↓	95% ↑ 95% ↓
WMAP5	5.65	7.88	9.96
($Y_p = 0.24$: fixed)		3.02	1.92
CMB all	4.24	5.47	7.94
(Y_p: free)		2.03	0.96
CMB all	3.71	4.80	6.38
(Y_p: BBN relation)		2.27	1.39
CMB all	3.89	4.89	6.84
($Y_p = 0.24$: fixed)		2.19	1.28

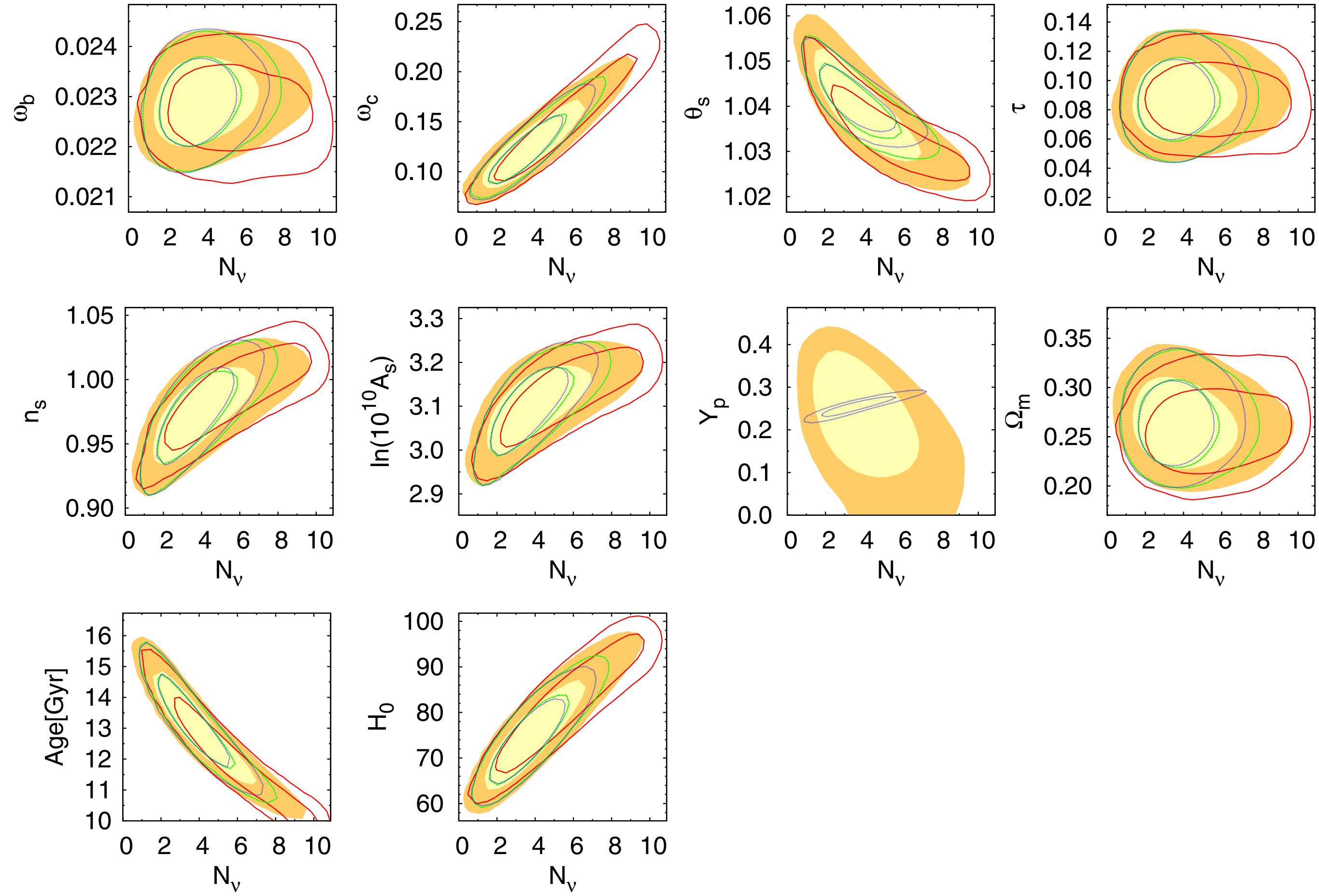

FIG. 3 (color online). The 68% and 95% allowed regions in the plane of N_ν vs other parameters when WMAP5 alone is used with $Y_p = 0.24$ (red solid line), WMAP5 + ACBAR + BOOMERANG + CBI are used with $Y_p = 0.24$ (green dashed line), Y_p is fixed by the BBN relation (blue dotted line), and Y_p is treated as a free parameter (orange and yellow shaded regions).

several other cosmological parameters in Fig. 3. Table II summarizes the derived constraints on these parameters.

As seen from Fig. 2, the likelihood for N_ν from WMAP alone has an irregular shape, far from Gaussian. It has the maximum around $N_\nu \sim 5$, declines slowly as N_ν increases, and drops abruptly at $N_\nu \sim 9$. The abrupt cut can be traced to the prior on the cosmic age t_0 which is implicitly assumed as 10 Gyr $< t_0 <$ 20 Gyr in the analysis. In par-

TABLE II. Mean values and 68% C.L. errors from current observations of the CMB for the cases with WMAP5 alone and all data combined.

Parameters	CMB all (Y_p: free)	CMB all (Y_p: BBN relation)	CMB all ($Y_p = 0.24$)	WMAP5 ($Y_p = 0.24$)
ω_b	$0.0229^{+0.000\,56}_{-0.000\,54}$	$0.02291^{+0.000\,52}_{-0.000\,59}$	$0.02291^{+0.000\,58}_{-0.000\,53}$	$0.02275^{+0.000\,60}_{-0.000\,62}$
ω_c	$0.132^{+0.018}_{-0.037}$	$0.124^{+0.017}_{-0.027}$	$0.127^{+0.017}_{-0.029}$	$0.153^{+0.036}_{-0.047}$
θ_s	$1.0390^{+0.0071}_{-0.0088}$	$1.0413^{+0.0039}_{-0.0051}$	$1.0402^{+0.0044}_{-0.0060}$	$1.0334^{+0.0042}_{-0.0084}$
τ	$0.088^{+0.016}_{-0.019}$	$0.088^{+0.016}_{-0.017}$	$0.088^{+0.015}_{-0.019}$	$0.088^{+0.016}_{-0.019}$
n_s	$0.977^{+0.026}_{-0.022}$	$0.975^{+0.024}_{-0.023}$	$0.975^{+0.024}_{-0.021}$	$0.989^{+0.030}_{-0.019}$
$\ln(10^{10}A_s)$	$3.104^{+0.0067}_{-0.0061}$	$3.097^{+0.066}_{-0.061}$	$3.098^{+0.065}_{-0.059}$	$3.128^{+0.080}_{-0.054}$
Y_p	$0.220^{+0.105}_{-0.085}$	$0.256^{+0.015}_{-0.016}$	...	...
N_ν	$4.24^{+1.23}_{-2.21}$	$3.72^{+1.07}_{-1.45}$	$3.89^{+1.00}_{-1.70}$	$5.65^{+2.63}_{-2.23}$
A_{SZ}	$1.04^{+0.96}_{-0.34}$	$1.07^{+0.93}_{-0.33}$	$1.05^{+0.95}_{-0.34}$	$1.00^{+0.85}_{-0.50}$
Ω_m	$0.265^{+0.026}_{-0.032}$	$0.266^{+0.025}_{-0.030}$	$0.265^{+0.025}_{-0.030}$	$0.260^{+0.028}_{-0.033}$
Age (Gyr)	$12.9^{+1.3}_{-1.2}$	$13.2^{+1.0}_{-1.0}$	$13.1^{+1.0}_{-1.1}$	$12.1^{+7.7}_{-1.9}$
H_0	$76.5^{+6.4}_{-9.7}$	$74.5^{+5.5}_{-6.7}$	$75.2^{+5.6}_{-7.3}$	$82.0^{+9.9}_{-8.9}$

ticular, the lower limit $t_0 > 10$ Gyr makes the cut (see Fig. 3). We can regard this prior to be very conservative on observational grounds since it is far looser than the astrophysical lower bound of the cosmic age, e.g., $t_0 > 11.2$ Gyr (95% C.L.) from the age estimates of globular clusters [56]. Moreover, we should include such a prior for practical reasons. As can be seen by the relatively slow decline of the likelihood for $5 \lesssim N_\nu \lesssim 9$ or the elongated contours in Fig. 3, the degeneracy is so severe that we cannot produce MCMC chains which are well converged within a reasonable time. Although we can formally calculate a constraint using this data as shown in Table I, since the likelihood is so irregular, we would conclude that it is not meaningful to constrain N_ν from WMAP5 alone.

However, it may be instructive to understand how the degeneracies arise in the WMAP-alone analysis. As clearly shown in Fig. 3, N_ν most notably degenerates with ω_c and H_0 (or θ_s). There are also some degeneracies with n_s and A_s but not as severe as ω_c or H_0. These degeneracies are understood as follows. First, to produce the same amount of the early ISW effect, ω_c has to be increased as N_ν increases. It roughly scales as $(\omega_b + \omega_c) \propto N_\nu$ to make the matter-radiation equality the same. At the same time, under the flatness assumption, Ω_m has to be preserved in order to have the same distance to the last scattering surface. Then, since $\Omega_m = (\omega_b + \omega_c)/h^2$, h has to increase for larger N_ν. The slight enhancement in n_s and A_s can be attributed to their effects to cancel the suppression around the diffusion damping scales due to the increase in N_ν. A more quantitative argument based on the derivatives presented in the previous section may be useful. The degeneracy as regards the same matter-radiation equality is given by setting $\Delta\mathcal{C}_{l_1} = 0$ to be $\Delta\omega_m/\omega_m \sim 0.2\Delta N_\nu/N_\nu$. This is equivalent to $\Delta\omega_c/\omega_c \sim 0.3\Delta N_\nu/N_\nu$, which roughly gives the slope in the $N_\nu - \omega_c$ plane in Fig. 3. Using this relation with $\Delta l_1 = 0$ shows the $N_\nu - h$ degeneracy. From Eq. (3), we obtain $\Delta h/h \sim 0.2\Delta N_\nu/N_\nu$, which appears in the $N_\nu - h$ contour in Fig. 3. The $N_\nu - n_s$ degeneracy is given by further requiring $\Delta H_2 = 0$. Plugging $\Delta\omega_m/\omega_m$ and $\Delta h/h$ into Eq. (13) yields $\Delta n_s/n_s \sim 0.05\Delta N_\nu/N_\nu$. This 5% increase in the best fit value of n_s for $\Delta N_\nu = 3$ is consistent with the $N_\nu - n_s$ contour in Fig. 3. Although WMAP has measured the CMB power spectrum very precisely, since it is just up to around the 2nd peak, the effects of N_ν are absorbed in the changes of ω_c, h, n_s, and A_s, and we cannot constrain N_ν.

For a visual illustration of the degeneracy, in Fig. 4, we show CMB power spectra for the case with $N_\nu = 1$ (blue dotted line), 3 (red solid line), and 5 (green dashed line), with other cosmological parameters being chosen such that they give the degenerate spectra up to the 2nd/3rd peak. We can see that these curves coincide up to the 2nd peak but begin to separate around the 3rd or higher peaks. On small scales, the change caused by N_ν cannot be fully canceled just by tuning other parameters. In particular, n_s affects the spectrum in the whole scale; thus, even if the spectra are almost degenerate up to the 2nd peak by tuning the value of n_s, they cannot cancel the damping on small scales.

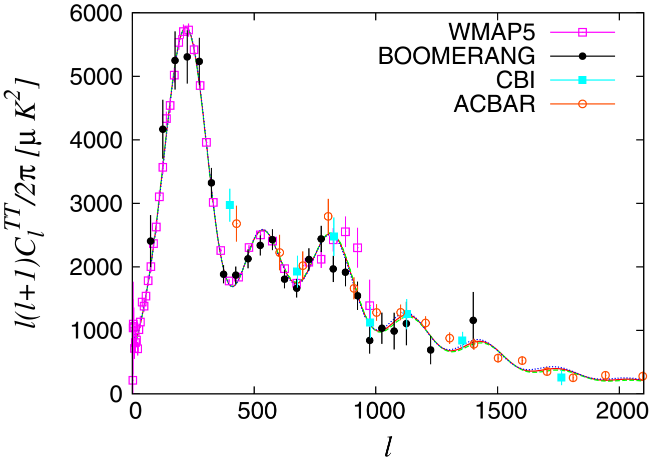

FIG. 4 (color online). An illustration of the degeneracy of N_ν with other cosmological parameters. Here the values of the effective number of neutrinos are assumed as $N_\nu = 1$ (blue dotted line), 3 (red solid line), and 5 (green dashed line), and other cosmological parameters are chosen such that the CMB spectra become the same as that with the fiducial parameters.

When we include the data at higher multipoles such as ACBAR, BOOMERANG, and CBI, the likelihood functions now have a well-behaved peak close to Gaussian and we can obtain meaningful constraints. The bound on N_ν is $0.96 \leq N_\nu \leq 7.94$ at 95% C.L. when Y_p is treated as a free parameter. At higher multipoles, the free-streaming of neutrinos damps the spectrum, which cannot be compensated by the above-mentioned parameters. Hence the degeneracy can be removed to some extent. That is the reason why we can have a severer constraint on N_ν by including the data on small scales. In fact, Y_p also suppresses the amplitude on small scales via diffusion damping [see Eqs. (11) and (12)]; thus the constraint on N_ν slightly changes for different treatments of Y_p, but the differences are very small as seen in Fig. 2 and Table I. Since current CMB observations on small scales are not so precise, it does not make much difference how we treat Y_p. Imposing the BBN relation tightens the constraint to $1.39 \leq N_\nu \leq 6.38$ at 95% C.L., but it is not so different from the Y_p-free case. Also, the limit does not differ much even if we set $Y_p = 0.24$. Similarly, the estimates for the other cosmological parameters are not affected by the assumption on Y_p as shown in Table II.

Up to now, we have assumed no prior on N_ν. However, if we consider an extra radiation component such as sterile neutrinos and so on, the effective number of neutrino species just increases. In this case, N_ν cannot be less than the standard value of 3.046. Thus it may be appropriate to study adopting the prior $N_\nu > 3.046$ to constrain a scenario with such an extra radiation component. We denote it as $\Delta N_\nu^{\rm ext} \equiv N_\nu - 3.046$. With this prior, we obtain an upper bound on an extra radiation component as $N_\nu < 8.19$ (or $\Delta N_\nu^{\rm ext} < 5.14$) at 95% C.L. when Y_p is taken as a free parameter, and improve it to be $N_\nu < 6.35$ (or $\Delta N_\nu^{\rm ext} < 3.30$) at 95% C.L. when the BBN relation is assumed. Notice that these limits are weaker than those with no priors on N_ν, which are $N_\nu < 7.94$ ($\Delta N_\nu^{\rm ext} < 4.89$) and $N_\nu < 6.38$ ($\Delta N_\nu^{\rm ext} < 3.33$), respectively. This somewhat peculiar fact stems from the shape of the likelihood shown in Fig. 2, which is not symmetric with respect to $N_\nu = 3.046$. Since the differences due to the N_ν prior are not negligible, caution is needed when we use these constraints regarding the prior on N_ν.

Even when we limit ourselves to the case with three active neutrino species, a deviation from the standard value of $N_\nu = 3.046$ is possible. In a scenario with low (MeV scale) reheating temperature, N_ν can be less than 3.046. In this case, N_ν only takes the value less than the standard one. Thus it may be interesting to investigate a constraint on N_ν assuming $N_\nu < 3.046$. As regards the treatment of Y_p, we do not consider the case of adopting the BBN relation here because, in a scenario with an MeV reheating temperature, Y_p should be calculated taking into account the nonthermal neutrino distribution functions and oscillation effects [3–6]. These effects drive Y_p to increase as N_ν decreases contrary to the usual case where N_ν just represents a measure of the expansion rate. (This is why we are not showing constraints for the prior $N_\nu < 3.046$ with the BBN relation in Table III.) Since taking into account this effect is beyond the scope of this paper, we show the constraint for the case with Y_p being varied freely, which can be considered as the conservative one for the prior $N_\nu < 3.046$. We obtained constraints $N_\nu > 1.27$ and $N_\nu > 1.17$ at 95% C.L. for the cases with Y_p being fixed as $Y_p = 0.24$ and Y_p being assumed as a free parameter, respectively. For a scenario with an MeV scale reheating temperature, these limits are translated into the lower bound on the reheating temperature as $T_{\rm reh} > 2.0$ MeV [6].

Finally, we investigate a future constraint on cosmological parameters paying particular attention to N_ν and its effects on constraints on other parameters. We use the data expected from the future Planck experiment and make a MCMC analysis following the method in Ref. [57]. As for the specification of Planck, we adopt the following parameters for the instrument. For the frequency channels of $\nu = 100$, 143, and 217 GHz, the width of the beam and the sensitivities per pixel for temperature and polarization are adopted as $(\theta_{\rm FWHW}[{\rm arcmin}], \sigma_T[\mu{\rm K}], \sigma_P[\mu{\rm K}]) = (9.5, 6.8, 10.9)$, (7.1, 6.0, 11.4), and (5.0, 13.1, 26.7), respectively. Other frequency channels are assumed to be used to remove foregrounds. We make use of the data up to $l = 2500$ so that our results will not be affected by the SZ effect and the marginalization over $A_{\rm SZ}$ is not performed. These setups for the Planck forecast are similar to the recent works performed in Refs. [26,30,51], but the explored parameter spaces are different. We make a simple extension by adding N_ν and Y_p to the standard 6 dimensional parameter space, but theirs includes neutrino masses and/or lepton asymmetry. When one would like to check the constraint on an extra radiation component in a simple scenario, one can refer to our results here. However, when some other particular setups are considered such as a scenario with large lepton asymmetry and massive neutrinos, the above-mentioned references should be consulted.

Our results are summarized in Tables III and IV. As seen from Table III, the constraint is most stringent when the BBN relation is adopted, and in this case, we obtained a future constraint as $2.68 \le N_\nu \le 3.44$ at 95% C.L. Another point which should be noted is that fixing $Y_p = 0.24$ can bias the determination of some other cosmological parameters such as ω_b and n_s, which was already pointed out in Refs. [51,52]. However, when we vary the value of N_ν, the effect of fixing $Y_p = 0.24$ is partly can-

TABLE III. The mean values, and 68% and 95% limits of N_ν for current and future CMB data.

	No priors on N_ν			$N_\nu \ge 3.046$		$N_\nu \le 3.046$	
	Mean	68% ↑ 68% ↓	95% ↑ 95% ↓	68% ↑ 68% ↓	95% ↑ 95% ↓	68% ↑ 68% ↓	95% ↑ 95% ↓
CMB all	4.24	5.47	7.94	5.51	8.19	(3.046)	(3.046)
(Y_p: free)		2.03	0.96	(3.046)	(3.046)	2.05	1.17
CMB all	3.71	4.80	6.38	4.70	6.35		⋯
(Y_p: BBN relation)		2.27	1.39	(3.046)	(3.046)		
CMB all	3.89	4.89	6.84	4.87	6.88	(3.046)	(3.046)
($Y_p = 0.24$: fixed)		2.19	1.28	(3.046)	(3.046)	2.12	1.27
Planck	3.11	3.44	3.83	3.45	3.87	(3.046)	(3.046)
(Y_p: free)		2.72	2.41	(3.046)	(3.046)	2.72	2.43
Planck	3.06	3.26	3.44	3.25	3.44		⋯
(Y_p: BBN relation)		2.87	2.68	(3.046)	(3.046)		
Planck	3.19	3.43	3.67	3.37	3.63	(3.046)	(3.046)
($Y_p = 0.24$: fixed)		2.95	2.72	(3.046)	(3.046)	2.87	2.67

TABLE IV. Forecasts on mean values and 68% errors of N_ν and other cosmological parameters.

Parameters	Planck (Y_p: free)	Planck (Y_p: BBN relation)	Planck ($Y_p = 0.24$)
ω_b	$0.02275^{+0.00025}_{-0.00028}$	$0.02275^{+0.00026}_{-0.00027}$	$0.02273^{+0.00027}_{-0.00026}$
ω_c	$0.1108^{+0.0046}_{-0.0056}$	$0.1101^{+0.0028}_{-0.0028}$	$0.1120^{+0.0033}_{-0.0036}$
θ_s	$1.0404^{+0.0014}_{-0.0014}$	$1.04060^{+0.00044}_{-0.00049}$	$1.04000^{+0.00055}_{-0.00062}$
τ	$0.0881^{+0.0050}_{-0.0064}$	$0.0881^{+0.0053}_{-0.0063}$	$0.0880^{+0.0056}_{-0.0059}$
n_s	$0.964^{+0.009}_{-0.010}$	$0.964^{+0.010}_{-0.010}$	$0.963^{+0.010}_{-0.009}$
$\ln(10^{10}A_s)$	$3.066^{+0.016}_{-0.016}$	$3.065^{+0.014}_{-0.015}$	$3.068^{+0.015}_{-0.015}$
Y_p	$0.246^{+0.020}_{-0.018}$	$0.2488^{+0.0027}_{-0.0027}$	...
N_ν	$3.11^{+0.33}_{-0.39}$	$3.06^{+0.20}_{-0.19}$	$3.19^{+0.24}_{-0.24}$
Ω_m	$0.256^{+0.010}_{-0.010}$	$0.256^{+0.009}_{-0.010}$	$0.255^{+0.009}_{-0.010}$
Age (Gyr)	$13.63^{+0.34}_{-0.31}$	$13.67^{+0.20}_{-0.21}$	$13.56^{+0.22}_{-0.25}$
H_0	$72.3^{+2.2}_{-2.4}$	$72.0^{+1.7}_{-1.6}$	$72.7^{+1.8}_{-1.9}$

celed by the change in N_ν. In fact, this in turn results in biases of other cosmological parameters such as ω_c and θ_s which are strongly correlated with N_ν. Therefore, Y_p should be treated carefully in investigating cosmological constraints with future CMB data.

Finally, we would like to comment on how our discussion so far can be affected by theoretical uncertainties in the recombination process [58–66]. Since the change of Y_p can influence the recombination process, its uncertainties might affect the cosmological parameter determination in some way. Thus it may be worth commenting here on the effects. For this purpose, we proceed with the same analysis that has been done in Ref. [52] but varying N_ν here. Two parameters F_H and b_{He}, which represent the uncertainties in the recombination modeling, are included among other free parameters. (See Ref. [52] and references therein for more details). We impose top-hat priors, $0 < F_H < 2$ and $0 < b_{He} < 1.5$, which are very conservative ones, to take into account the uncertainties in the recombination theory. We made the analyses for the two cases where Y_p is from the BBN relation and Y_p is treated as a free parameter. In both cases, we found that the constraints on other cosmological parameters including N_ν are scarcely affected even by very conservative priors on F_H and b_{He}. The mean values are unchanged, and errors increase only very slightly (no more than 10% for any parameters other than F_H and b_{He}). Therefore we can say that the uncertainties parametrized with F_H and b_{He} do not change much our results of the Planck forecast discussed above. However, we would need more understanding of the uncertainties in the recombination theory for the precise determination of cosmological parameters in future CMB surveys.

IV. SUMMARY

We discussed the issue of probing the effective number of neutrino species N_ν from CMB data alone. Although a constraint on N_ν has been investigated by many authors, in most analyses, some combinations of data sets such as CMB + LSS, CMB + H_0, CMB + LSS + H_0 have been used to constrain N_ν. This is partly because N_ν has severe degeneracies in WMAP with some other cosmological parameters such as ω_m and h; thus N_ν can be more constrained by combining some data sets. However, when we combine data from LSS, some subtleties can arise: a constraint from LSS data depends on how we treat nonlinear corrections/bias. Furthermore, different galaxy data lead to slightly different constraints on N_ν. In addition, as for the Hubble prior, the prior usually adopted is $H_0 = 72 \pm 8$, which is from the result of Freedman *et al.* [31]. However, another group has reported a somewhat different value as $H_0 = 62.3 \pm 5.2$ [32]. Since different priors on the Hubble constant can give different results; in this respect, the constraint obtained by assuming some prior on H_0 should be regarded by taking into account the above uncertainty. Taking these issues into consideration, it may be interesting to study a constraint on N_ν, removing such subtleties, which can be done by using CMB data alone.

In this paper, first we discussed the effects of N_ν on the CMB and the issues of degeneracies with some other cosmological parameters. Phenomenological descriptions of its effects on the heights and the positions of acoustic peaks were also given. Then, in Sec. III, a constraint on N_ν was studied by using CMB data alone. We made use of the data from WMAP5, ACBAR, BOOMERANG, and CBI. As discussed there, although the WMAP measurement is very accurate, its precision is limited up to the 2nd/3rd peak. We have explicitly shown that the information up to the 2nd/3rd peak is not enough to constrain N_ν severely. This was demonstrated by making the analysis with WMAP data alone, in which a sensible constraint cannot be obtained. However, if we include the data on small scales, the degeneracies of N_ν with some other cosmological parameters can be removed to some extent; then a stronger constraint can be obtained. In fact, on small scales, the amplitude is suppressed due to the free-streaming effect by increasing N_ν, which is similar to the effects of Y_p through the diffusion damping. Thus we have studied the constraint on N_ν assuming different priors on Y_p: adopting the BBN relation to derive Y_p for given N_ν

and ω_b, assuming Y_p as a free parameter, and fixing $Y_p = 0.24$ as usual. Depending on the prior, the constraint slightly changes. We obtained the 95% limits as $0.96 \leq N_\nu \leq 7.94$ for the case with Y_p being free, $1.28 \leq N_\nu \leq 6.84$ for Y_p being fixed as $Y_p = 0.24$, and $1.39 \leq N_\nu \leq 6.38$ when the BBN relation is adopted. It should be noted that these constraints are comparable to that obtained using CMB + LSS in previous works.

One of the main purposes of constraining the effective number of neutrino species using cosmological data is to check the standard value of N_ν independently from particle physics experiments. Thus we primarily focus on the analysis with no prior on N_ν. However, from the viewpoint of constraining extra radiation which may be motivated by some particle physics models, a constraint obtained by assuming the prior $N_\nu > 3.046$ may be interesting since extra radiation always increases the value of N_ν. In this respect, we also made an analysis adopting this prior and obtained the constraint on the effective number of neutrinos as $N_\nu \leq 6.35$ and $N_\nu \leq 8.14$ for the cases where the BBN relation is adopted and Y_p is treated as a free parameter.

On the other hand, in a scenario with low-reheating temperature, the effective number of neutrino species can be reduced. In this case, another prior may be motivated to be assumed for a simple scenario of low-reheating temperature with three relativistic neutrino species. In this regard, we have also studied the case with the prior $N_\nu < 3.046$ and obtained the constraints as $N_\nu > 1.17$ for the cases with Y_p being assumed as a free parameter. This can be translated into the lower bound on the reheating temperature as $T_{\rm reh} > 2.0$ MeV.

We have also discussed a future constraint on N_ν using the expected data from the Planck experiment. It was shown that the attainable constraint on N_ν from Planck is $2.68 \leq N_\nu \leq 3.44$ at 95% C.L. when the BBN relation is adopted for Y_p, which is most stringent compared to the other cases. Since the Planck experiment can probe the CMB down to smaller scales than WMAP, Planck alone can give a stringent constraint on N_ν.

The interplay between particle physics and cosmology is now becoming more important in the era of precision cosmology. One such example is the number of neutrino species, which was investigated in this paper. In light of upcoming, more precise observations of cosmology, research of this kind will bring us fruitful insight into particle physics and cosmology.

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