Quantum slow-roll and quantum fast-roll inflationary initial conditions: CMB quadrupole suppression and further effects on the low CMB multipoles

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(Received 2 September 2008; published 6 October 2008)

Quantum fast-roll initial conditions for the inflaton which are different from the classical fast-roll conditions and from the quantum slow-roll conditions can lead to inflation that lasts long enough. These quantum fast-roll initial conditions for the inflaton allow for kinetic energies of the same order of the potential energies and nonperturbative inflaton modes with nonzero wave numbers. Their evolution starts with a transitory epoch where the redshift due to the expansion succeeds to assemble the quantum excited modes of the inflaton in a homogeneous (zero mode) condensate, and the large value of the Hubble parameter succeeds to overdamp the fast roll of the redshifted inflaton modes. After this transitory stage the effective classical slow-roll epoch is reached. Most of the e-folds are produced during the slow-roll epoch, and we recover the classical slow-roll results for the scalar and tensor metric perturbations plus corrections. These corrections are important if scales which are horizon size today exited the horizon by the end of the transitory stage and, as a consequence, the lower cosmic microwave background (CMB) multipoles get suppressed or enhanced. Both for scalar and tensor metric perturbations, fast roll leads to a *suppression* of the amplitude of the perturbations (and of the low CMB multipoles), while the quantum precondensate epoch gives an *enhancement* of the amplitude of the perturbations (and of the low CMB multipoles). These two types of corrections can compete and combine in a scale dependent manner. They turn out to be smaller in new inflation than in chaotic inflation. These corrections arise as natural consequences of the quantum nonperturbative inflaton dynamics, and can allow a further improvement of the fitting of inflation plus the ΛCMB model to the observed CMB spectra. In addition, the corrections to the tensor metric perturbations will provide an independent test of this model. Thus, the effects of quantum inflaton fast-roll initial conditions provide a consistent and contrastable model for the origin of the suppression of the quadrupole and for other departures of the low CMB multipoles from the slow-roll inflation-ΛCMB model, which are to be contrasted with the TE and EE multipoles and with the forthcoming and future CMB data.

DOI: 10.1103/PhysRevD.78.083508 PACS numbers: 98.80.Cq, 98.70.Vc

I. INTRODUCTION

Inflation (an epoch of accelerated expansion of the Universe) solves the horizon and flatness problems of the standard big bang model. It naturally generates scalar density fluctuations that seed large scale structure and the temperature anisotropies in the cosmic microwave background (CMB), and tensor perturbations (primordial gravitational waves) [1,2]. Inflation is based on a scalar field (the inflaton) whose homogeneous expectation value drives the dynamics of the scale factor, plus small quantum fluctuations.

A great diversity of inflationary models predict fairly generic features: a Gaussian, nearly scale invariant spectrum of (mostly) adiabatic scalar and tensor primordial fluctuations, and they provide an excellent fit to the highly precise wealth of data of the Wilkinson Microwave Anisotropy Probe (WMAP) [3] making the inflationary paradigm fairly robust. Precise CMB data reveal peaks and valleys in the temperature fluctuations resulting from acoustic oscillations in the electron-photon fluid at recombination. These and future CMB and large scale structure (LSS) observations require more precise theoretical predictions from inflation, and a deeper understanding of how inflation begins and ends.

Amongst the wide variety of inflationary scenarios, the single field slow-roll model provide an appealing, simple, and fairly generic description of inflation. Its simplest implementation is based on a single scalar field (the inflaton). The inflaton potential is fairly flat, and it dominates the Universe energy during inflation. This flatness leads to a slowly varying Hubble parameter (slow roll), ensuring a sufficient number of inflation e-folds to explain the homogeneity, isotropy, and flatness of the Universe, and also

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explains the Gaussianity of the fluctuations as well as the (almost) scale invariance of their power spectrum.

The dynamics of inflation is usually described by the classical evolution of the inflaton in a dynamical spacetime whose scale factor obeys the classical Friedmann equation. This is justified by the enormous stretching of physical lengths during inflation that classicalizes the dynamics. On the other hand, both the perturbations of the metric and of the inflaton are computed considering their quantum nature. The observable consequences of slow-roll inflation are well known [2], in particular, those for the CMB anisotropies. Present observational data agree with its predictions for a class of slow-roll inflaton potentials except for the CMB quadrupole suppression [3,4].

However, the large energy densities during inflation allow for the creation of inflaton particles, and call for a quantum treatment for the inflaton field. References [5–7] considered a quantum treatment to study the dynamics of the inflaton. Quantum excited initial conditions lead to inflation that lasts long enough, provided the quantum inflaton background satisfies the quantum generalized slow-roll conditions [5,7]. In addition, after a transitory precondensate epoch, the excited modes of the inflaton assemble in a homogeneous zero mode condensate [5–7]. Once formed, this condensate obeys the inflaton classical evolution equations, and we therefore recover the classical inflation slow-roll description.

More recently, classical fast-roll initial conditions have been shown to lead to long enough inflation and to the CMB quadrupole suppression [8–10]. In this article we consider more general inflaton initial conditions which lead to long enough inflation, and that we call quantum fast-roll initial conditions. They are different from the quantum slow-roll conditions and include the classical fast-roll conditions as a particular case.

All these general initial conditions lead, after a transitory precondensate stage, to a slow-roll inflationary epoch that admits a classical description, thus recovering the classical slow-roll inflation predictions for the scalar and tensor metric perturbations. Moreover, if the larger cosmologically relevant scales exited the horizon close to the end of the transitory stage, the effect of the modified initial conditions in the bulk modes that dominate the energy density will be imprinted as corrections to the amplitudes of the lower CMB multipoles. These corrections have been computed for classical fast-roll initial conditions [8], and they have been shown to improve the fit of inflation plus the ΛCMB model to the CMB data [9]. For quantum slow-roll initial conditions the corrections were first estimated in Ref. [5], and they will be computed in more detail here.

In the present article we also compute the corrections due to the quantum fast-roll initial conditions introduced here. We show that quantum slow-roll conditions and quantum fast-roll conditions may improve the fit of the CMB data with the ΛCMB model plus inflation. These corrections to the spectrum of scalar and tensor metric perturbations arise as natural consequences of the quantum precondensate dynamics of the inflaton field. The two possible types of transitory epochs present before slow roll, namely, effective classical fast-roll and quantum (fast-roll or slow-roll) precondensates, lead to opposite kinds of corrections for the low CMB multipoles. This quantum precondensate dynamics predict corrections to the low TE and EE multipoles (temperature T and E-polarization mode correlations) and to the low CMB tensor multipoles. Therefore, this is a consistent and contrastable theory for the origin of the suppression of the quadrupole and for other departures of the low CMB multipole data from the classical slow-roll inflation plus the ΛCMB model.

This paper is organized as follows: in Sec. II we present the quantum inflaton field and its evolution equations in the limit of a large number of components, as well as the theory to compute corrections to the spectrum of perturbations due to departures from classical slow-roll evolution, and the relative change in the CMB quadrupole due to these corrections. In Sec. III we compute first the corrections to the primordial spectrum of perturbations due to quantum slow-roll initial conditions. Next, we introduce quantum fast-roll initial conditions and compute the inflationary evolution. Then, we compute the corrections to the primordial spectrum of perturbations due to the quantum fast-roll initial conditions, and the relative change in the low CMB multipoles. Finally, in Sec. IV we further discuss the results and present our conclusions.

We use units $\hbar = 1$, $c = 1$, and in the figures we also take $m = 1$ with m the inflaton mass. For the scale factor of the Friedmann-Robertson-Walker (FRW) metric, we take $a(0) = 1$.

II. QUANTUM INFLATON EVOLUTION

The action for the *quantum inflaton dynamics* can be written as

$$S = S_g + S_m + \delta S_g + \delta S_m \tag{1}$$

where each term describes the dynamics of one of the components: S_g describes the dynamics of the metric background, δS_g that of the cosmologically relevant metric perturbations, S_m accounts for the dynamics of the inflaton field, and δS_m the dynamics of the cosmologically relevant inflaton perturbations. Here, we consider all these terms to be quantum mechanical except for the metric background S_g which is purely classical. This generalizes the classical framework where only the perturbations are quantized.

On one hand, the gravitational terms are

$$S_g + \delta S_g = -\frac{1}{16\pi G}\int \sqrt{-g}\, d^4x R \tag{2}$$

where G is Newton's gravitational constant, and R is the Ricci scalar for the complete metric $g_{\mu\nu}$. By expanding

$g_{\mu\nu}$ in terms of the spatially flat FRW metric and its perturbation, $g_{\mu\nu} = g_{\mu\nu}^{(\mathrm{FRW})} + \delta g_{\mu\nu}$, the corresponding S_g terms (those which do not contain $\delta g_{\mu\nu}$) and the δS_g terms can be recognized (see Refs. [5,11] for more details). We are interested in the metric perturbations at the cosmologically relevant scales.

On the other hand, the inflaton terms in the action are

$$S_m + \delta S_m = \int \sqrt{-g} d^4x \left[\frac{1}{2}\partial_\alpha \vec{\chi} \partial^\alpha \vec{\chi} - V(\vec{\chi})\right] \quad (3)$$

where $\vec{\chi}$ is the inflaton quantum field and $V(\vec{\chi})$ the inflaton potential. We consider a multicomponent inflaton quantum field $\vec{\chi} = (\chi_1, \ldots, \chi_N)$ since it allows us to apply a nonperturbative method to compute the dynamics, namely, the large N limit. It must be stressed that we consider $O(N)$ invariant initial states with an $O(N)$ invariant inflaton Lagrangian, since the Universe as a whole should be expected to be in an $O(N)$ invariant state. This leads to straight trajectories in field space, and therefore, this is effectively a single field inflaton model [5].

It is also convenient to split the contribution from the inflaton terms in the action S_m and the contributions from the cosmologically relevant inflaton and metric perturbations in the term δS_m.

Let us call Λ the k scale in momentum space that separates the modes contributing to the background ($k < \Lambda$) from those that contribute to the perturbations ($k > \Lambda$). Modes with $k \gg m$ cannot be significantly excited since the energy density during inflation must be of the order $\sim 20\ M^4$ in order to have $\gtrsim 60$ e-folds of inflation [7], where $M \sim 10^{16}$ GeV is the energy scale of inflation,

$$m = \frac{M^2}{M_{\mathrm{Pl}}} \quad (4)$$

is the inflaton mass, and M_{Pl} is the Planck mass given by $M_{\mathrm{Pl}} = 1/\sqrt{8\pi G} = 2.4 \times 10^{18}$ GeV.

On the other hand, the modes that are cosmologically relevant, i.e., those corresponding to the scales of large scale structures and the CMB, are, today, in the range from 0.4 Mpc to 4×10^3 Mpc. These scales correspond to physical wave numbers at the beginning of inflation in the range from [9]

$$10 e^{N_Q - 62} m \sqrt{\frac{H}{10^{-4} M_{\mathrm{Pl}}}} \quad \text{to} \quad 10^5 e^{N_Q - 62} m \sqrt{\frac{H}{10^{-4} M_{\mathrm{Pl}}}}, \quad (5)$$

where N_Q is the number of e-folds since the CMB quadrupole modes exit the horizon till the end of inflation, and H is the Hubble parameter during inflation.

Notice that the CMB quadrupole modes are, today, horizon size (4×10^3 Mpc) while at horizon exit their physical wave number corresponds to the left-hand side of Eq. (5).

Thus, Λ is in an intermediate k range of modes whose contributions are irrelevant both for the background and for the perturbations at the cosmologically relevant scales, $\Lambda \sim 10 e^{N_Q - 62} m \sqrt{\frac{H}{10^{-4} M_{\mathrm{Pl}}}}$, and the observable results are independent of the particular value of Λ. (For a more detailed discussion see Ref. [5]).

The quantum evolution equation for the inflaton background is

$$\ddot{\vec{\chi}} + 3H\dot{\vec{\chi}} - \frac{\nabla^2 \vec{\chi}}{a^2} + V'(\vec{\chi}) = 0 \quad (6)$$

with a the scale factor of the FRW metric, $H \equiv \dot{a}/a$ the Hubble parameter, and V' the derivative of the inflaton potential with respect to $\vec{\chi}$, and the dot stands for the derivative with respect to the cosmic time. As the background is homogeneous, this implies that the expectation values of the field and of its momenta are homogeneous. It is important to note that this homogeneity of the expectation values does not prevent excited modes with nonzero wave numbers. These homogeneous states with wave-number excited modes can be thought of as homogeneous seas of particles with nonzero momenta.

The evolution equations for the quantum inflaton are coupled to the Friedmann classical evolution equations for the space-time metric

$$G_{\mu\nu} = \langle T_{\mu\nu} \rangle \Rightarrow H^2 = \frac{\rho}{3M_{\mathrm{Pl}}^2} \quad (7)$$

with $G_{\mu\nu}$ the Einstein tensor for the FRW metric, $\langle T_{\mu\nu} \rangle$ the expectation value of the energy-momentum tensor for the inflaton quantum field, and $\rho = \langle T^{00} \rangle$ the energy density.

The approach of considering the expectation value of the energy-momentum tensor of the matter quantum fields as the source for the metric evolution is usually called semiclassical gravity. This approach is justified provided the quantum gravity corrections can be neglected, as is our case here: they are of order $\sim (m/M_{\mathrm{Pl}})^2 \sim 10^{-9}$.

The evolution equations for the quantum perturbations of the metric and of the inflaton are well known when the inflaton is a classical field (see, for example, Ref. [11]). However, they are not known in the case of quantum inflaton fields. It is only known how to compute the spectrum of perturbations in the regimes where the quantum inflaton can be described by an effective classical field.

A. Quantum evolution equations. The precondensate and the condensate epochs

It is not an easy task to compute the time evolution defined by Eq. (6) for a generic quantum state, and it is only known how to compute this evolution under certain approximations. Here, as we need to compute the evolution of excited nonperturbative states with homogeneous expectation values, we will use the large N limit method [5–7,12]. This method gives the evolution equations in the limit of a large number of components N and provides consistent evolution equations which can be numerically implemented both for the inflaton expectation value and for the nonperturbative inflaton modes. The large N limit captures the relevant physics of inflation.

We consider the inflaton potential

$$V(\vec{\chi}) = \frac{1}{2}\alpha m^2 \vec{\chi}^2 + \frac{\lambda}{8N}(\vec{\chi}^2)^2 + \frac{Nm^4}{2\lambda}\frac{1-\alpha}{2},$$

with $\alpha = \pm 1$. (8)

The last constant term here sets the minimum of $V(\vec{\chi})$ at $V(\vec{\chi}) = 0$. The quartic self-coupling λ is of the order [13]

$$\lambda \sim \left(\frac{M}{M_{\rm Pl}}\right)^4 \sim 10^{-12}. \tag{9}$$

The quantum field Fock state (or density matrix) describing the inflaton field is assumed invariant under translations and rotations, that is, homogeneous and isotropic [6,7]. The quantum field states consist of a distribution of k modes over the Fock vacuum. We choose a mode distribution of width Δk centered around a value $k = k_0 < \Lambda$. The field operator $\vec{\chi}(\vec{x}, t)$ can be Fourier expanded in terms of creation and annihilation operators and mode functions [5–7] as

$$\vec{\chi}(\vec{x}, t) = \hat{e}_1\varphi(t) + \int\frac{d^3k}{(2\pi)^3}[\vec{a}_k f_k(t)e^{i\vec{k}\cdot\vec{x}} + \vec{a}_k^\dagger f_k^*(t)e^{-i\vec{k}\cdot\vec{x}}], \tag{10}$$

where $\hat{e}_1 = (1, 0, \ldots, 0)$ and $\vec{a}_k$ and $\vec{a}_k^\dagger$ obey canonical commutation rules.

In the large N limit the evolution equations for the quantum inflaton background are given by [5,7]

$$\ddot{\varphi} + 3H\dot{\varphi} + \mathcal{M}^2\varphi = 0, \tag{11}$$

$$\ddot{f}_k + 3H\dot{f}_k + \left(\frac{k^2}{a^2} + \mathcal{M}^2\right)f_k = 0 \tag{12}$$

with

$$\mathcal{M}^2 = \alpha m^2 + \frac{\lambda}{2}\varphi^2 + \frac{\lambda}{2}\int_R\frac{d^3k}{2(2\pi)^3}|f_k|^2, \tag{13}$$

where $\varphi \equiv \langle\vec{\chi}_1\rangle$ is the expectation value of the quantum inflaton field, $\langle\vec{\chi}_a\rangle = 0$ for $2 \le a \le N$, and f_k its quantum modes [5]. The inflaton evolution equations are coupled to the evolution equations for the scale factor,

$$\begin{aligned} H^2 &= \frac{\rho}{3M_{\rm Pl}^2}, \\ \frac{\rho}{N} &= \frac{1}{2}\dot{\varphi}^2 + \frac{\mathcal{M}^4 - m^4}{2\lambda} + \frac{m^4}{2\lambda}\frac{1-\alpha}{2} \\ &\quad + \frac{1}{4}\int_R\frac{d^3k}{(2\pi)^3}\left(|\dot{f}_k|^2 + \frac{k^2}{a^2}|f_k|^2\right), \end{aligned} \tag{14}$$

where $\rho = \langle T^{00}\rangle$ is the energy density. The pressure $(p\delta_i{}^j = \langle T_i{}^j\rangle)$ is given by

$$\frac{p+\rho}{N} = \dot{\varphi}^2 + \frac{1}{2}\int_R\frac{d^3k}{(2\pi)^3}\left(|\dot{f}_k|^2 + \frac{k^2}{3a^2}|f_k|^2\right). \tag{15}$$

The index R denotes the renormalized expressions of these integrals [7]. This means that we must subtract the appropriate asymptotic ultraviolet behavior in order to make the integrals convergent in Eqs. (13)–(15).

The characteristic mass scale of the inflaton modes $f_k(t)$ is clearly the inflaton mass m [see Eqs. (12) and (13)]. In order to have an energy density $\rho = \mathcal{O}(M^4)$ and not $\mathcal{O}(m^4) \ll \mathcal{O}(M^4)$, the inflaton modes with $k < \Lambda$ must have a nonperturbative amplitude $|f_k(t)| = \mathcal{O}(\lambda^{-1/2})$. Thus,

$$\rho = \mathcal{O}\left(\frac{m^4}{\lambda}\right) = \mathcal{O}(M^4).$$

where we used Eqs. (4), (9), and (14).

Two types or classes of quantum inflation scenarios have been considered [5–7]:

(i) Quantum chaotic inflation, in which the inflaton field has a large amplitude (in the expectation value and/or in the quantum modes). The inflaton potential energy is due to the potential energy of this large amplitude state.

(ii) Quantum new inflation, in which the inflaton field has a small amplitude, and the inflaton potential energy is due to the large values of the potential for small amplitudes of the field.

In this framework the quantum generalized slow-roll condition is given by [5,7]

$$\dot{\varphi}^2 + \int_R\frac{d^3k}{2(2\pi)^3}|\dot{f}_k|^2 \ll m^2\left(\varphi^2 + \int_R\frac{d^3k}{2(2\pi)^3}|f_k|^2\right). \tag{16}$$

This is a sufficient condition to guarantee inflation ($\ddot{a} > 0$) and to guarantee that it lasts long (for both scenarios). The classical slow-roll condition is a particular case,

$$\dot{\varphi}^2 \ll m^2\varphi^2. \tag{17}$$

The states that verify the quantum generalized slow-roll conditions, Eq. (16), lead to two inflationary epochs separated by a condensate formation [5,7]:

(a) *The precondensate epoch*: During this epoch the term

$$\int_R\frac{d^3k}{(2\pi)^3}\frac{k^2}{a^2}|f_k|^2 \tag{18}$$

gives an important contribution to the energy [Eq. (14)], while it decreases due to the redshift of the excitations ($k/a \to 0$). This epoch ends when all the contributions of k^2/a^2 to the background dynamics are negligible. Then, all the excited modes are effectively assembled in a zero mode condensate.

(b) *The condensate slow-roll epoch*: Once the excited modes behave effectively as a zero mode condensate, the inflaton background can be described as the classical effective field [5,6],

$$\tilde{\varphi}_{\rm eff}(t) = \sqrt{N}\sqrt{\varphi^2(t) + \int \frac{d^3k}{2(2\pi)^3}|f_k(t)|^2}. \quad (19)$$

This effective classical inflaton verifies the classical evolution equations

$$\ddot{\tilde{\varphi}}_{\rm eff} + 3H\dot{\tilde{\varphi}}_{\rm eff} + \alpha\tilde{m}^2\tilde{\varphi}_{\rm eff} + \tilde{\lambda}\tilde{\varphi}^3_{\rm eff} = 0, \quad (20)$$

$$H^2 = \frac{\tilde{\rho}}{3M^2_{\rm Pl}}, \qquad \tilde{\rho} = \frac{1}{2}\dot{\tilde{\varphi}}^2_{\rm eff} + \frac{1}{2}\alpha\tilde{m}^2\tilde{\varphi}^2_{\rm eff} + \frac{\tilde{\lambda}}{4}\tilde{\varphi}^4_{\rm eff} + \frac{m^4}{8\tilde{\lambda}}(1-\alpha), \quad (21)$$

with

$$\tilde{\lambda} = \frac{\lambda}{2N}, \qquad \tilde{m}^2 = m^2, \qquad \alpha = \pm 1. \quad (22)$$

The pressure is given by

$$\tilde{p} + \tilde{\rho} = \dot{\tilde{\varphi}}^2_{\rm eff}. \quad (23)$$

It is important to note that the initial conditions for the classical effective inflaton $\tilde{\varphi}_{\rm eff}$ are fixed by the quantum state (i.e. the quantum precondensate epoch). During the condensate slow-roll epoch the effective inflaton $\tilde{\varphi}_{\rm eff}$ verifies the classical slow-roll condition, Eq. (17), i.e., $\dot{\tilde{\varphi}}^2_{\rm eff} \ll m^2\tilde{\varphi}^2_{\rm eff}$.

In addition, we introduce and study in this article novel quantum initial conditions for the inflaton which are not of quantum slow-roll type, and which we call quantum fast-roll initial conditions. These quantum fast-roll initial conditions also lead to inflation that lasts long enough, as we show in the next section. The effective description, Eqs. (19)–(23), also proves to be valid for quantum fast-roll initial conditions after the evolution has redshifted the modes and assembled them in a zero mode condensate.

These quantum initial conditions concern the nonperturbative inflaton modes that contribute significantly to the background, that is, the $k < \Lambda$ inflaton modes. We choose the usual Bunch-Davies (BD) initial conditions for the cosmologically relevant perturbations $k > \Lambda$. Physical effects of non-Bunch-Davies initial conditions for the cosmologically relevant modes were presented in Ref. [8].

B. Corrections to the spectrum of perturbations. The transfer function $D(k)$

The spectrum of scalar and tensor metric perturbations for quantum inflation (new and chaotic) has been computed in Ref. [5]. There, we found that, provided the cosmologically relevant scales exited the horizon during the condensate epoch, the classical inflation results are recovered at leading order. In addition, this study had identified several sources of quantum corrections and computed their order of magnitude:

(i) Coupling of the cosmologically relevant modes with the quantum inflaton dynamics yields corrections of order $\sim(H/M_{\rm Pl})^2 \sim 10^{-9}$ or smaller; this estimation has been confirmed by the more detailed study in Ref. [14].

(ii) Contributions from higher orders in $1/N$.

(iii) Contributions from the precondensate epoch. These can be important for large scales that exited the horizon before or during the formation of the condensate.

Here we compute these last quantum corrections in more detail and for more general initial conditions. That is, we compute the effects of the quantum precondensate epoch in the primordial spectrum of perturbations and in the observed CMB multipoles.

In addition to the usual method to compute the primordial spectrum of perturbations, we also use here the method developed in Refs. [8] to compute the changes in the primordial spectrum due to generic initial conditions, in particular, the fast-roll conditions. Both of these methods have been obtained for a classical inflaton field. Therefore, they can only be applied, in principle, to regimes where the quantum inflaton can be described by an effective classical field, as in the condensate epoch. In order to obtain estimates of the effects of the precondensate epoch, we use these formalisms when the inflaton is close to the formation of the condensate and thus can be described by the effective inflaton, Eq. (19).

The modes of the curvature perturbations $S_{\mathcal{R}}$ fulfill the equation [8]

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_{\mathcal{R}}(\eta)\right]S_{\mathcal{R}}(k;\eta) = 0, \qquad W_{\mathcal{R}} = \frac{1}{z}\frac{d^2z}{d\eta^2} \quad (24)$$

where $\eta \equiv \int_0^t dt'/a(t')$ is the conformal time and $z(\eta)$ is given in terms of the derivative of the classical inflaton which for our study is the effective inflaton field $\tilde{\varphi}_{\rm eff}(t)$, Eq. (19), i.e.,

$$z(\eta) = a\frac{\dot{\tilde{\varphi}}_{\rm eff}}{H}. \quad (25)$$

$W_{\mathcal{R}}(\eta)$ is the potential felt by the curvature perturbations during the whole evolution: fast roll as well as slow roll.

For the curvature perturbations the quantity that leads to the differences with respect to slow roll is the potential

$$\mathcal{V}_{\mathcal{R}} = W_{\mathcal{R}} - W^{\rm sr}_{\mathcal{R}}, \quad (26)$$

that is, the difference of the potential $W_{\mathcal{R}}$ and its value for slow roll, $W^{\rm sr}_{\mathcal{R}}$, where to first order in slow roll

$$W^{\rm sr}_{\mathcal{R}} = -\frac{2 + 9\epsilon_v - 3\eta_v}{\eta^2}, \quad (27)$$

with ϵ_v, η_v the first order slow-roll parameters evaluated during slow roll:

$$\epsilon_v = \frac{\dot{\tilde{\varphi}}^2_{\rm eff}}{2M^2_{\rm Pl}H^2} \quad \text{and} \quad \eta_v = M^2_{\rm Pl}\frac{V''(\tilde{\varphi}_{\rm eff})}{V(\tilde{\varphi}_{\rm eff})}.$$

The potential $\mathcal{V}_{\mathcal{R}}$, Eq. (26), is determined by the departures of the background dynamics from slow roll. Notice that when the potential $\mathcal{V}_{\mathcal{R}}$ is negative (attractive) the perturbations are suppressed, while when it is positive (repulsive) the perturbations are enhanced [8]. In the purely classical fast-roll case, $\mathcal{V}_{\mathcal{R}}$ is attractive and the perturbations are suppressed [8].

In the slow-roll regime with a classical inflaton, the perturbation modes, Eq. (24), feel the repulsive potential $W^{\rm sr}_{\mathcal{R}}$, Eq. (27), which yields the scale invariant primordial power [11]

$$P^{\rm BD}_{\mathcal{R}}(k) = |\Delta^{\mathcal{R}}_{k_0\, ad}|^2\left(\frac{k}{k_0}\right)^{n_s-1}, \tag{28}$$

when BD initial conditions are imposed on the perturbative modes.

In the classical fast-roll case the primordial power spectrum gets modified by the transfer function $D_{\mathcal{R}}(k)$ as [8,9]

$$P_{\mathcal{R}}(k) = P^{\rm BD}_{\mathcal{R}}(k)[1 + D_{\mathcal{R}}(k)]. \tag{29}$$

The effect of the precondensate regime on the primordial power both for quantum slow-roll and quantum fast-roll initial conditions can also be encoded in an appropriate transfer function $D_{\mathcal{R}}(k)$.

We will consider the effects on the primordial power at the end of the transitory epoch before the effective classical slow-roll epoch. The effects on the low CMB multipoles have two different origins that come from the two kinds of transitory epochs before slow roll:

(i) *Precondensate* effects. For perturbative k modes that exit the horizon *before* the condensate, Eq. (19), is formed and before the classical equations (20) and (21) hold, the transfer function $D_{\mathcal{R}}(k)$ is nonzero and the power spectrum is given by Eq. (29). This entails an enhancement in the low CMB multipoles as we show in Sec. III.

(ii) *Fast-roll* effects. As for the purely classical fast-roll inflation initial conditions [8,9], when the nonperturbative k modes obey quantum fast-roll initial conditions, a nonzero transfer function $D_{\mathcal{R}}(k)$ appears in Eq. (29) and the primordial power is suppressed as we show in Sec. III.

The transfer function in the primordial power due to the deviation from the slow-roll dynamics is given by

$$D(k) = \frac{1}{k}\int_{-\infty}^{0} d\eta\, \mathcal{V}_{\mathcal{R}}(\eta)\left[\sin(2k\eta)\left(1 - \frac{1}{k^2\eta^2}\right) + \frac{2}{k\eta}\cos(2k\eta)\right]. \tag{30}$$

This function allows us to obtain the changes (suppression or enhancement) in the low multipoles resulting from the early fast-roll dynamics [8],

$$\frac{\Delta C_l}{C_l} = \frac{\int_0^\infty D(\kappa x) f_l(x) dx}{\int_0^\infty f_l(x) dx} \tag{31}$$

where $x = k/\kappa$, $\kappa \equiv a_0 H_0/3.3$, with a_0 and H_0 the scale factor and the Hubble parameter today, and

$$f_l(x) = [j_l(x)]^2/x \tag{32}$$

for a nearly scale invariant spectrum of perturbations [8] with $j_l(x)$ the spherical Bessel functions [15]. In particular, for the quadrupole this yields

$$\frac{\Delta C_2}{C_2} = \frac{1}{\kappa}\int_{-\infty}^{0} d\eta\, \mathcal{V}_{\mathcal{R}}(\eta)\Psi(\kappa\eta), \tag{33}$$

with

$$\Psi(x) \equiv 12\int_0^\infty \frac{dy}{y^4}[j_2(y)]^2\left[\left(y^2 - \frac{1}{x^2}\right)\sin(2yx) + \frac{2y}{x}\cos(2yx)\right].$$

Analogously, tensor perturbation modes S_T verify the evolution equation

$$\left[\frac{d^2}{d\eta^2} + k^2 - W_T(\eta)\right]S_T(k;\eta) = 0, \qquad W_T = \frac{a''(\eta)}{a(\eta)}, \tag{34}$$

with $a(\eta)$ the scale factor as a function of the conformal time and $a''(\eta)$ its second derivative with respect to η.

$W_T(\eta)$ is the potential felt by the tensor perturbations during the whole evolution: fast-roll as well as slow-roll inflation.

It is convenient to define the potential

$$\mathcal{V}_T = W_T - W^{\rm sr}_T, \qquad W^{\rm sr}_T \simeq \frac{2 + 3\eta_v}{\eta^2}, \tag{35}$$

where $W^{\rm sr}_T$ is the value of W_T in the slow-roll evolution. The potential $\mathcal{V}_T$ is determined by the fast-roll dynamics. When the potential $\mathcal{V}_T$ is negative (attractive) the tensor perturbations are suppressed, while when it is positive (repulsive) the tensor perturbations are enhanced [8]. In the purely classical fast-roll case, $\mathcal{V}_T$ is attractive and the tensor perturbations are suppressed [8].

III. QUANTUM INITIAL CONDITIONS AND OBSERVABLE CONSEQUENCES

Several initial conditions have been shown to lead to inflationary periods that last long enough (55 e-folds or more) to explain the observed flatness and homogeneity of the Universe. Namely, classical initial conditions that verify the classical slow-roll conditions $\dot{\varphi}^2 \ll m^2\varphi^2$, quantum initial conditions that verify the quantum generalized slow-roll conditions, Eq. (16), and more recently, classical fast-roll initial conditions $\dot{\varphi}^2 \sim m^2\varphi^2$ have been shown to lead to long enough inflation [5,8]. We show in this section that more general quantum initial conditions lead to inflation. These initial conditions are different from the quantum slow-roll conditions and we call them quantum fast-roll initial conditions. The classical fast-roll initial conditions are a particular case.

The observable consequences of slow-roll inflation are well known [2], in particular, those for the CMB fluctuations. Present observational data agree with the slow-roll predictions for appropriate inflaton potentials within the present accuracy, except for the quadrupole of the CMB. All initial conditions mentioned here lead, after a transitory epoch, to a period of slow-roll inflation. If the slow-roll inflation period is large enough, all the details of the previous initial conditions will be erased. However, if the larger scales exited the horizon close to the end of the transitory epoch, the initial conditions imprint features in the low multipoles of the CMB spectrum with respect to the slow-roll results. Such corrections have been computed for the case of classical fast-roll initial conditions in Refs. [8,9]. We compute here the corrections for the quantum slow-roll conditions, and for the quantum fast-roll initial conditions introduced in this paper. This computation shows that the two possible types of transitory epochs present before slow roll, namely, fast roll and precondensate, lead to different kinds of corrections for the low CMB multipoles.

A. Quantum slow-roll initial conditions

We first consider quantum initial conditions that verify the quantum slow-roll condition, Eq. (16). The background dynamics of these quantum initial conditions has been studied in Ref. [7]. These initial conditions have initial equations of state between $p(0)/\rho(0) \simeq -1$ [for classical slow roll, Eq. (17)] and $p(0)/\rho(0) \simeq -1/3$ for the quantum extreme case. The latter equation of state is found when the integral of Eq. (18) dominates the energy ρ [Eq. (14)]. (See Fig. 1).

During the transitory precondensate inflationary epoch, the modes are rapidly redshifted, and at the end of this epoch they are assembled in a zero mode condensate. After that, the condensate epoch starts and the quantum field condensate can be described by the effective classical field Eq. (19) that verifies the classical slow-roll conditions. This allows us to recover the known slow-roll results for the cosmological relevant modes provided they exited the horizon during the condensate epoch [5]. In addition, quantum corrections have been predicted in Ref. [5] for large scales that exited the horizon close to the transition between the quantum precondensate and the classical condensate inflationary epochs.

Here, we numerically compute these quantum corrections to the scalar and tensor metric perturbations. First, we compute the potentials $\mathcal{V}_{\mathcal{R}}$ and $\mathcal{V}_T$ felt by the scalar and tensor fluctuations, respectively.

For quantum chaotic inflation we consider

$$\frac{m}{M_{\rm Pl}} = 1.02 \times 10^{-5} \sim \left(\frac{M}{M_{\rm Pl}}\right)^2 \quad \text{and}$$
$$\tilde{\lambda} = 1.18 \times 10^{-12} \sim \left(\frac{M}{M_{\rm Pl}}\right)^4$$

(best fit to WMAP3 and Sloan Digital Sky Survey data found in Ref. [16]).

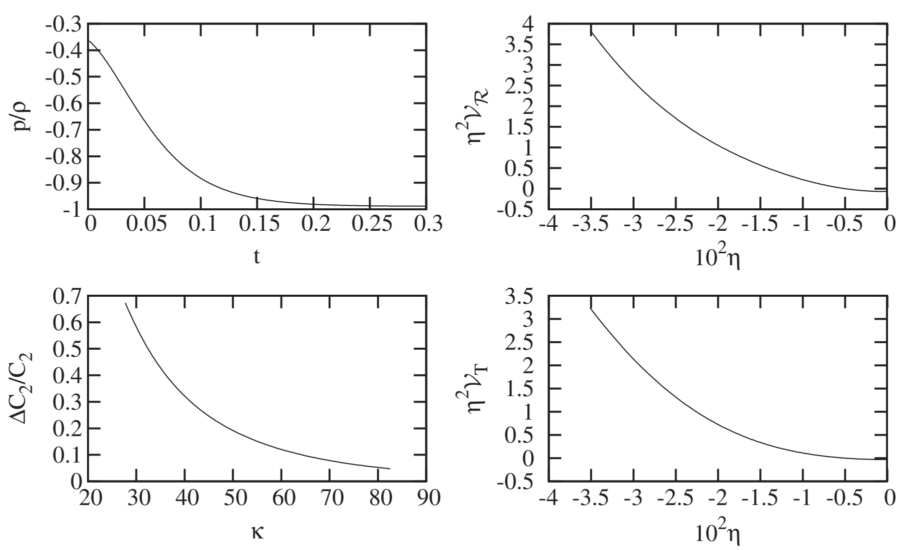

FIG. 1. Quantum chaotic inflation with *quantum slow-roll initial conditions*. The nonperturbative inflaton modes have wave numbers around $k \sim 8m$ with a characteristic width $\Delta k \sim 0.8m$. Top left panel: equation of state p/ρ as a function of comoving time for early times. Top right panel: the potential $\mathcal{V}_{\mathcal{R}}$ felt by the scalar curvature perturbations as a function of the conformal time η. Bottom right panel: the potential $\mathcal{V}_T$ felt by the tensor perturbations as a function of the conformal time η. Bottom left panel: the CMB quadrupole amplitude, $\Delta C_2/C_2$, as a function of the scale κ. $\Delta C_2/C_2$ *increases* due to the effect of the quantum precondensate inflaton epoch.

We consider quantum initial states that verify the quantum slow-roll condition [Eq. (16)] with various wave-number distributions and with amplitudes chosen to lead to an energy density at the beginning of slow roll, $\rho \simeq 6.5m^4/\tilde{\lambda}$. (These values guarantee that the slow-roll part of inflation lasts 55 e-folds). In particular, in Fig. 1 we show the results obtained for a distribution of nonperturbative inflaton modes around $k = k_0 \sim 8m$ with width $\Delta k \sim 0.8m$ and the slow-roll initial conditions

$$\dot{f}_k(0) \simeq -\frac{k^2/a^2(0) + \mathcal{M}^2(0)}{3H(0)} f_k(0),$$

which follow by neglecting $\ddot{f}_k(0)$ in Eq. (12) [7].

The results for quantum chaotic inflation show that *both* the scalar and the tensor metric perturbations became *enhanced* by the quantum slow-roll initial conditions at the large scales that correspond to the end of the precondensate epoch. Figure 1 illustrates this effect. In this case the potentials $\mathcal{V}_\mathcal{R}$ and $\mathcal{V}_T$ felt by the fluctuations are both *repulsive*. This also implies that if the scale corresponding to the CMB quadrupole exited the horizon at the end of the precondensate epoch, its amplitude will be larger than if it exited deep inside the condensate epoch. This enhancement is larger for initial distributions whose energy is dominated by modes with higher momenta. Since the CMB observations show a *suppression* of the quadrupole instead of an enhancement, this simply implies that the CMB quadrupole mode *cannot* exit the horizon during a quantum slow-roll precondensate epoch. We will see below that this statement gets weaker if the scale of the quadrupole exited the horizon during a quantum fast-roll epoch.

For quantum new inflation, on the other hand, initial conditions have small amplitudes for the inflaton modes and for the expectation value of the inflaton. This implies that even the more excited inflaton modes have small amplitudes, and if they have $k \sim m$ the contribution of the integral Eq. (18) to the energy density will be small, and the precondensate corrections to the spectrum will also be small. In addition, if the initial conditions verify the quantum slow-roll condition, then the kinetic contribution

$$\frac{1}{2}\dot{\varphi}^2 + \frac{1}{4}\int_R \frac{d^3k}{(2\pi)^3}|\dot{f}_k|^2$$

is also small. This implies that for new inflation, if the more excited inflaton modes have $k \sim m$, the corrections to the scalar and tensor perturbations from the slow-roll precondensate are negligible.

B. Quantum fast-roll initial conditions

We call quantum fast-roll initial conditions all those quantum initial conditions that are different from the quantum slow-roll condition, Eq. (16). The classical fast-roll initial conditions $\dot{\varphi}^2 \sim m^2\varphi^2$ are particular cases of these quantum fast-roll initial conditions. We have found that quantum fast-roll initial conditions can lead to inflation that lasts long enough, even when their initial kinetic energy is of the order of their initial potential energy. Generalizing the dimensionless variable y defined in Ref. [8], we define here the variable [see Eq. (15)]

$$y^2 \equiv \frac{3}{2}\frac{p+\rho}{\rho}, \qquad 0 \leq y^2 \leq 3. \tag{36}$$

Since inflation requires

$$\frac{\ddot{a}}{a} = H^2(1-y^2) > 0, \tag{37}$$

the range of the variable y^2 for inflationary evolution is $0 < y^2 < 1$.

The classical slow-roll regime corresponds to $y^2 \ll 1$. On the other hand, $y^2 \simeq 3$ can only be reached if the kinetic energy dominates the energy density; in this case the equation of state is $p \simeq \rho$. We consider here quantum fast-roll initial conditions that have kinetic energy of the order of their potential energy and therefore $y^2 \simeq 1$. In this case the equation of state is $p \simeq -\rho/3$.

For quantum chaotic inflation we consider $m/M_{\rm Pl} = 1.02 \times 10^{-5} \sim (\frac{M}{M_{\rm Pl}})^2$ and $\tilde{\lambda} = 1.18 \times 10^{-12} \sim (\frac{M}{M_{\rm Pl}})^4$ (best fit to WMAP3 and Sloan Digital Sky Survey data found in Ref. [16]). We consider initial states obeying the fast-roll condition,

$$\dot{f}_k(0) \sim \sqrt{\frac{k^2}{a^2(0)} + \mathcal{M}^2(0)} f_k(0) \tag{38}$$

with various distributions and with amplitudes chosen to lead to an initial energy density at the beginning of slow roll, $\rho \simeq 6.5m^4/\tilde{\lambda}$. (These values guarantee that inflation lasts at least 55 e-folds). The conditions (38) do not fulfill the quantum slow-roll conditions, Eq. (16).

We find that quantum fast-roll initial conditions lead to an early fast-roll epoch, in which the contribution from the kinetic energy dominates the total energy density. During this fast-roll epoch the large value of the Hubble constant overdamps the derivatives of the inflaton modes and of the inflaton expectation value, rapidly reducing the kinetic energy. This means that generically the fast-roll epoch is followed by the slow-roll epoch. In fact, it is in this slow-roll epoch that most of the required e-folds take place.

Two situations arise depending on whether the initial distribution of nonperturbative inflaton modes is centered around small or large k_0 ($k_0 \ll m$ or $k_0 \sim m$):

(i) When the initially excited nonperturbative inflaton modes are excited only for low wave numbers, $k \sim k_0 \ll m$, they effectively behave as a zero mode condensate, and we have an effective classical description in terms of the effective inflaton field defined in Eq. (19) with the evolution given by Eqs. (20) and (21). For these initial conditions we recover the classical fast-roll epoch described in

Refs. [8,9], followed by the classical slow-roll epoch; in particular, we have from Eqs. (15), (23), and (36)

$$y^2 = \frac{3}{2}\frac{\dot{\tilde{\varphi}}^2_{\rm eff}}{\tilde{\rho}}.$$

Consistently, the corrections for the scalar and tensor metric perturbations are the same as those found in Refs. [8,9]. Both scalar and tensor metric perturbations that exited the horizon by the end of this effective classical condensate fast-roll epoch have their amplitudes *suppressed* with respect to their values for pure slow roll, implying a suppression of the low CMB multipoles, in particular, of the CMB quadrupole. See Fig. 2.

(ii) When the initially excited inflaton modes have $k \sim k_0 \sim m$ or larger, the evolution starts with a quantum precondensate epoch, and the fast-roll and quantum precondensate effects compete. Perturbation modes that exit the horizon during the fast-roll epoch are suppressed, while modes exiting the horizon during the precondensate epoch are enhanced. Their respective intensity depends on the values of y^2 and of the characteristic k scale of the excited inflaton modes. This competition results in that, at some scales, the perturbations are enhanced while at other scales they are suppressed, as can be seen, for example, in Fig. 3. The relative relevance between the kinetic $|\dot{f}_k|^2$ and momentum contributions $\frac{k^2}{a^2}|f_k|^2$ to the energy [Eq. (14)] seems to determine which is the more relevant effect at this k scale, and whether the perturbations exiting the horizon at this scale k are enhanced or suppressed relative to their classical slow-roll amplitude (see right panels of Fig. 3). The enhancement is more notorious for the tensor perturbations. In addition, the fact that the perturbations at *some scales* are *enhanced* while at other scales they are *suppressed* implies that some CMB multipoles can be suppressed while others are enhanced (see bottom right panel of Fig. 3).

If the quantum initial precondensate effects dominate when the quadrupole mode exits the horizon, the quadrupole mode would get enhanced, contrary to the CMB observations which show a suppression for the quadrupole. Therefore, this imposes upper bounds on the characteristic k_0 scale of the nonperturbative inflaton modes. However, increasing the value of y^2 reduces the enhancement of these inflaton k modes, relaxing such upper bounds on k_0.

On the other hand, for quantum new inflation, the quantum initial conditions require initially small amplitudes for the inflaton modes; this makes the contribution of the integral Eq. (18) small, and therefore the precondensate corrections to the spectra will also be small.

We can summarize the time evolution for the quantum fast-roll inflaton initial conditions as follows:

(i) The early precondensate fast-roll epoch where the inflationary dynamics is described by the nonperturbative inflaton k modes $f_k(t)$.

(ii) The first of the two transitions happens. Either the fast roll is damped ending the fast-roll epoch, or the

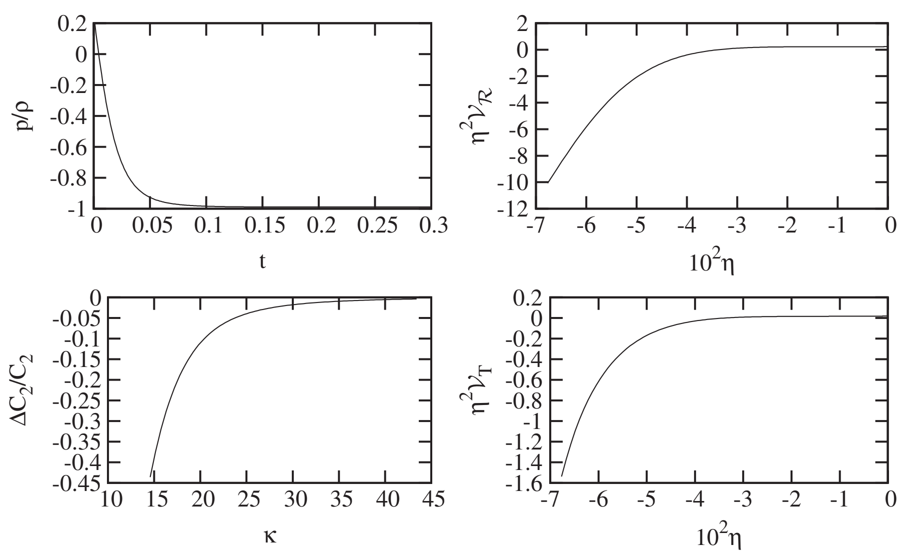

FIG. 2. Quantum chaotic inflation with *quantum fast-roll initial conditions* given by Eq. (38), with low k inflaton excited modes having $k \sim k_0 \ll m$. In this case the quantum fast-roll stage is an effectively classical condensate epoch. Top left panel: equation of state p/ρ as a function of comoving time for early times. Top right panel: the potential $\mathcal{V}_{\mathcal{R}}$ felt by scalar curvature perturbations as a function of the conformal time η. Bottom right panel: the potential $\mathcal{V}_T$ felt by tensor perturbations as a function of the conformal time η. Bottom left panel: change of the CMB quadrupole amplitude—$\Delta C_2/C_2$ as a function of the scale κ. The quadrupole gets suppressed by the fast-roll condensate.

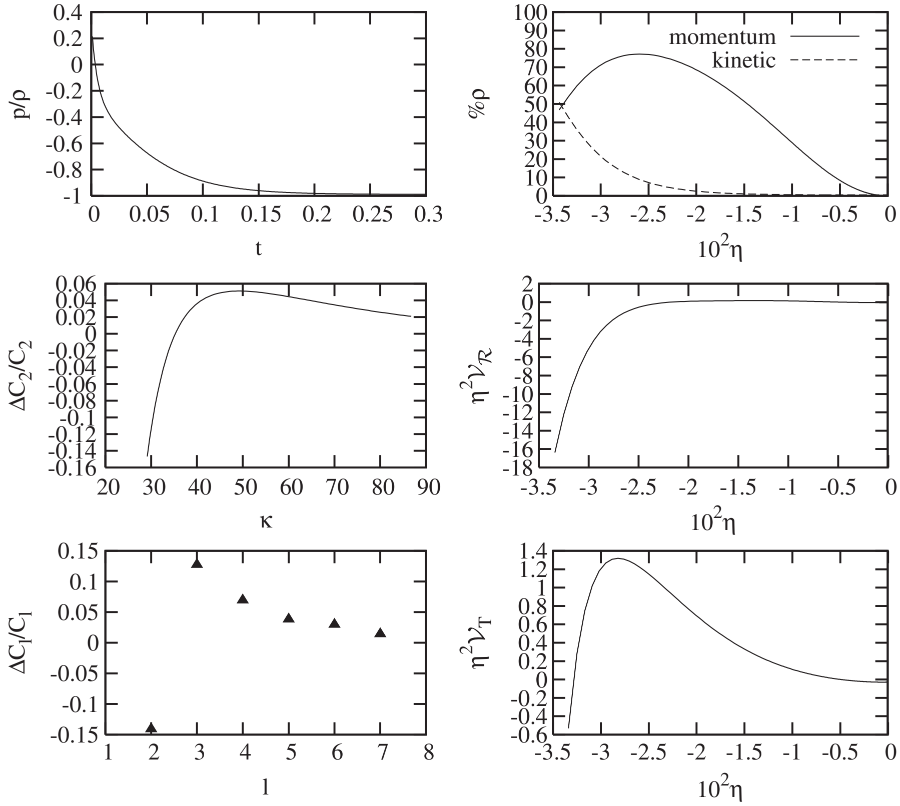

FIG. 3. Quantum chaotic inflation with *quantum fast-roll initial conditions* given by Eq. (38) for excited inflaton modes having $k \sim k_0 \simeq 8m$ and a characteristic width $\Delta k \sim 0.8m$. Top left panel: equation of state p/ρ as a function of comoving time for early times. Top right panel: fraction of the total energy ρ expressed as a percentage (%) that comes from the momentum contribution [the integral Eq. (18)] and from the kinetic term contribution (the derivative of the mode amplitudes $\dot{f}_k$). Middle right panel: the effective potential felt by the scalar perturbations $\mathcal{V}_\mathcal{R}$ as a function of the conformal time η. Bottom right panel: the effective potential felt by the tensor perturbations $\mathcal{V}_T$ as a function of the conformal time η. Middle left panel: change of the quadrupole amplitude of the CMB, $\Delta C_2/C_2$, as a function of the scale κ. $\Delta C_2/C_2$ gets suppressed or enhanced depending on the value of the scale κ. Bottom left panel: change of the low multipoles of the CMB, $\Delta C_l/C_l$, as a function of the multipole order l for $\kappa = 29.12m$.

condensate is formed ending the precondensate epoch. The initial conditions determine what happens first. If the fast roll is stopped first, this second epoch is a precondensate slow-roll epoch, while if the condensate forms first, this second epoch is a fast-roll condensate epoch where the inflationary dynamics is described by the effective inflaton $\tilde{\varphi}_{\rm eff}(t)$ of Eq. (19).

(iii) The second of the two transitions happens. The dynamics reaches a slow-roll condensate regime from its previous precondensate slow-roll epoch or fast-roll condensate epoch. This slow-roll condensate evolves according to the effective description given by Eqs. (20) and (21); i.e., this third epoch is effectively a classical slow-roll epoch.

IV. CONCLUSIONS

We have introduced a new type of inflationary initial condition, namely, quantum fast-roll initial conditions which lead to long enough inflation. They are different from the quantum generalized slow-roll conditions and yield to the classical fast-roll conditions as a particular case. Quantum fast-roll initial conditions are far beyond slow roll, as their kinetic energy can be of the same order as their potential energy.

Two relevant issues play an important role in the quantum inflaton dynamics: the redshift due to the expansion and the large value of the Hubble parameter. The redshift due to the expansion succeeds to suppress the contributions from the quantum inflaton k-terms, and assembles all the nonperturbative inflaton modes in an inflaton condensate after a transitory precondensate epoch. On the other hand, the large value of the Hubble parameter succeeds to overdamp the oscillations of the redshifted inflaton quantum modes, thus stopping the fast roll. Therefore, an effective classical slow-roll inflation epoch is reached after a transitory quantum inflation epoch as a consequence of the combined effect of the redshift due to the expansion and the large values of the Hubble parameter.

Such a transitory epoch can have observable effects. If scales which are, today, horizon size exited the horizon close to the end of the transitory epoch, the spectra of scalar and tensor metric perturbations are similar to the spectra for classical slow roll, but with corrections for the larger scales. These corrections for the larger cosmologically relevant scales of the scalar and tensor metric perturbations are reflected in the amplitudes of the lower multipoles of the CMB spectra. We have found that for both scalar and tensor metric perturbations, the classical or effective classical condensate fast-roll epoch *suppresses* the amplitude of the perturbations (and therefore the amplitude of the respective CMB multipoles), while the quantum precondensate epoch *enhances* the amplitude of the perturbations (and of the respective CMB multipoles).

The corrections to the CMB multipoles from the quantum precondensate turn out to be smaller in new inflation than in chaotic inflation.

In addition, when both fast-roll and quantum precondensate effects compete, they can lead to suppression in some scales and enhancement in other scales. Therefore, these corrections for the lower multipoles can allow one to further improve the fit of the predictions of the inflation plus the ΛCMB model to the observed TT, TE, and EE CMB spectra.

It is important to note that these corrections to the spectrum of scalar metric perturbations arise as natural consequences of the quantum dynamics of the inflaton. Since the quantum dynamics also predicts corrections to the spectrum of tensor perturbations, when the tensor perturbation spectrum is observed, an independent test of the quantum fast-roll model will be possible. Therefore, effects due to quantum fast-roll initial conditions in the nonperturbative inflaton modes ($k > \Lambda$) are a consistent and contrastable explanation for the suppression of the CMB quadrupole and for the other departures of the low CMB multipoles from the slow-roll inflation plus ΛCMB model.

The consequences of our study of quantum fast-roll initial conditions on the low TE and EE multipoles need to be explored. Forthcoming data for low CMB multipoles motivate this work and are needed to reach a clear understanding on these issues.

ACKNOWLEDGMENTS

F. J. C. thanks the LPTHE (Paris) and the Observatoire de Paris for their hospitality, and acknowledges support through Research Project No. FIS2006-05895 (MECD, Spain), No. ESP2007-30785-E (MECD, Spain), and No. CCG07-UCM/ESP-2925 (UCM/CAM, Spain).

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