PHYSICAL REVIEW D **78,** 083505 (2008)

WMAP haze: Directly observing dark matter?

Michael McNeil Forbes
Nuclear Theory Group, Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

Ariel R. Zhitnitsky
The University of British Columbia, Department of Physics and Astronomy, Vancouver, B.C., V6T 1Z1, Canada
(Received 25 March 2008; published 2 October 2008)

In this paper, we show that dark matter in the form of dense matter/antimatter nuggets could provide a natural and unified explanation for several distinct bands of diffuse radiation from the core of the Galaxy spanning over 13 orders of magnitude in frequency. We fix all of the phenomenological properties of this model by matching to x-ray observations in the keV band, and then calculate the unambiguously predicted thermal emission in the microwave band, at frequencies smaller by 11 orders of magnitude. Remarkably, the intensity and spectrum of the emitted thermal radiation are consistent with—and could entirely explain—the so-called "WMAP haze": a diffuse microwave excess observed from the core of our Galaxy by the Wilkinson Microwave Anisotropy Probe (WMAP). This provides another strong constraint of our proposal, and a remarkable nontrivial validation. If correct, our proposal identifies the nature of the dark matter, explains baryogenesis, and provides a means to directly probe the matter distribution in our Galaxy by analyzing several different types of diffuse emissions.

DOI: 10.1103/PhysRevD.78.083505 PACS numbers: 95.35.+d, 12.38.−t, 95.85.Bh, 98.70.−f

I. INTRODUCTION

In this paper, we discuss a testable and well-constrained model for dark matter [1–4]. In particular, we explain how microwave emissions are an inevitable consequence of our proposal, and test the model against a recent analysis of the Wilkinson Microwave Anisotropy Probe (WMAP) observations that suggest an anomalous emission from the core of our galaxy (dubbed the "WMAP haze") [5–10]. Despite having no free parameters—the model is completely fixed by observations at scales some 10 orders higher—our proposal is consistent with these observations, and could even explain the anomaly if it survives further scrutiny. This provides a highly nontrivial test of our proposal, which remains consistent with all known constraints.

To calculate the emissions, however, requires a careful analysis of several fields of physics, and it is easy to lose track of the overall structure of the calculation. We organize the paper as follows: In Sec. II, we present a short review of the dark-matter proposal, emphasizing the assumptions that underlie the model, the observational constraints that the model must satisfy, and outlining the mechanism by which the dark matter will radiate, thus rendering it observable. In Sec. III, we summarize the calculation of the thermal microwave emission from the dark matter, and show in Sec. IV how this is consistent with the observations. Finally, in Sec. V, we review all of the observational constraints of our proposal and discuss the testable predictions that it makes. To keep the logic clear, some technical details have been omitted from the core of the paper. We include these in the appendix, completing our calculation.

II. PROPOSAL

A. Dark matter as dense quark nuggets

Two of the outstanding cosmological mysteries—the natures of dark matter and baryogenesis—might be explained by the idea that dark matter consists of compact composite objects (CCOs) [1–4] similar to Witten's strangelets [11]. The basic idea is that these CCOs—nuggets of dense matter and antimatter—form at the same quantum chromodynamics (QCD) phase transition as conventional baryons (neutrons and protons), providing a natural explanation for the similar scales $\Omega_{\rm DM} \approx 5\Omega_b$. Baryogenesis proceeds through a charge separation mechanism: both matter and antimatter nuggets form, but the natural *CP* violation of the so-called θ term in QCD[1]—which was of order unity $\theta \sim 1$ during the QCD phase

[1]If θ is nonzero, one must confront the so-called strong *CP* problem whereby some mechanism must be found to make the effective θ parameter extremely small today in accordance with measurements. This problem remains one of the most outstanding puzzles of the standard model, and one of the most natural resolutions is to introduce an axion field. (See the original papers [12–14], and recent reviews [15].) Axion domain walls associated with this field (or ultimately, whatever mechanism resolves the strong *CP* problem) play an important role in forming these nuggets, and may play in important role in their ultimate stability. See [1,2] for details.

transition—drives the formation of more antimatter nuggets than matter nuggets, resulting in the leftover baryonic matter that forms visible matter today (see [2] for details). Note, it is crucial for our mechanism that *CP* violation can drive charge separation. This idea may already have found experimental support through the Relativistic Heavy Ion Collider (RHIC) at Brookhaven [16].

This mechanism requires no fundamental baryon asymmetry to explain the observed matter/antimatter asymmetry. From this, and the observed relation $\Omega_{\rm DM} \approx 5\Omega_b$ (see [17] for a review) we have

$$B_{\rm universe} = 0 = B_{\rm nugget} + B_{\rm visible} - \bar{B}_{\rm antinugget}, \tag{1a}$$

$$B_{\rm dark\ matter} = B_{\rm nugget} + \bar{B}_{\rm antinugget} \approx 5B_{\rm visible}, \tag{1b}$$

where $B_{\rm universe}$ is the total number of baryons *minus* the number of antibaryons in the Universe, $B_{\rm dark\ matter}$ is the total number of baryons *plus* the total number of antibaryons hidden in the nuggets and antinuggets that make up the dark matter, $B_{\rm nugget}$ is the total number of baryons contained in all of the dark-matter nuggets, $\bar{B}_{\rm antinugget}$ is the total number of antibaryons contained in all of the dark antimatter nuggets, and $B_{\rm visible}$ is the total number of residual "visible" baryons (regular matter). Solving Eq. (1) gives the approximate ratios $\bar{B}_{\rm antinugget}:B_{\rm nugget}:B_{\rm visible} \simeq 3:2:1$.

Unlike conventional dark-matter candidates, dark-matter/antimatter nuggets will be strongly interacting, but macroscopically large objects. They do not contradict any of the many known observational constraints on dark matter or antimatter [3,18] for three reasons:

(1) They carry a huge (anti)baryon charge $|B| \approx 10^{20}$–10^{33}, so they have an extremely tiny number density. This explains why they have not been directly observed on earth. The local number density of dark-matter particles with these masses is small enough that interactions with detectors are exceedingly rare and fall within all known detector and seismic constraints [3]. (See also Refs. [19,20] and references therein.[2])

(2) The nuggets have nuclear densities, so their interaction cross section is small $\sigma/M \approx 10^{-13}$–$10^{-9}$ cm^2/g. This is well below typical astrophysical and cosmological limits, which are on the order of $\sigma/M < 1$ cm^2/g. Dark-matter–dark-matter interactions between these nuggets are thus negligible.

(3) They have a large binding energy such that baryons in the nuggets are not available to participate in big bang nucleosynthesis at $T \approx 1$ MeV. In particular, we suspect that the baryons in these nuggets form a superfluid with a gap of the order $\Delta \approx 100$ MeV, and critical temperature $T_c \sim \Delta/\sqrt{2} \approx 60$ MeV, as this scale provides a natural explanation for the observed photon to baryon ratio $n_{\rm B}/n_\gamma \sim 10^{-10}$ [2], which requires a formation temperature of $T_{\rm form} = 41$ MeV [21].[3]

Thus, on large scales, the nuggets are sufficiently dilute that they behave as standard collisionless cold dark matter (CCDM). When the number densities of both dark and visible matter become sufficiently high, however, dark-antimatter–visible-matter collisions may release significant radiation and energy. In particular, antimatter nuggets provide a site at which interstellar baryonic matter—mostly protons and electrons—can annihilate, producing emissions with calculable spectra and energies that should be observable from the core of our Galaxy. These emissions are not only consistent with current observations, but seem to naturally explain several mysterious diffuse emissions observed from the core of our Galaxy, with frequencies ranging over some 12 orders of magnitude.

Although somewhat unconventional, this idea naturally explains several coincidences, is consistent with all known cosmological constraints, and makes definite, testable predictions. Furthermore, this idea is almost entirely rooted in conventional and well-established physics. In particular, there are no "free parameters" that can be—or need to be—"tuned" to explain observations: In principle, everything is calculable from well-established properties of QCD and QED. In practice, fully calculating the properties of these nuggets requires solving the fermion many-body problem at strong coupling, so we must resort to "fitting" a handful of phenomenological parameters from observations.

Nevertheless, these unknown parameters may be determined to within an order of magnitude by observations of processes at the keV scale (described below and in [4]). The model then makes unambiguous predictions about other processes ranging over more than 10 orders of magnitude in scale. The point of this paper is to show that, remarkably, these unambiguous predictions are completely consistent with current observations, providing compelling evidence for our proposal, and explaining another astrophysical puzzle: the origin of the so-called WMAP haze.

[2] It is estimated in [8] that nuggets of mass from ~10 kg to 1 ton (corresponding to $B \sim 10^{28-30}$) must account for less than an order of magnitude of the local dark matter. While our preferable range of $B \sim 10^{25-27}$ is somewhat smaller [18] and does not contradict [19], we still believe that $B \gtrsim 10^{28}$ is not completely excluded by Apollo data, as the corresponding constraint is based on specific model dependent assumptions about the nugget-mass distribution [19], whereas nugget formation due to charge separation as suggested in [2] may lead to a very different distribution.

[3] At temperatures below the gap, incident baryons with energies below the gap would Andreev reflect rather than become incorporated into the nugget.

[5–10][4] We have considered five independent observations of diffuse radiation from the core of our Galaxy:

(1) SPI/INTEGRAL observes 511 keV photons from positronium decay that is difficult to explain with conventional astrophysical positron sources [23–25]. Dark-antimatter nuggets would provide an unlimited source of positrons as suggested in [26,27].

(2) COMPTEL detects a puzzling excess of 1–20 MeV γ-ray radiation. We shall not discuss this here, but it has been shown in [28] that the direct e^+e^- annihilation spectrum could nicely explain this deficit.

(3) Chandra observes a diffuse keV x-ray emission that greatly exceeds the energy from identified sources [29]. Visible-matter/dark-antimatter annihilation would provide this energy.

(4) EGRET/CRGO detects MeV to GeV gamma-rays, constraining antimatter annihilation rates. We shall not discuss these constraints here, but it was shown in [4] that these constraints are consistent with the rates inferred from the other emissions.

(5) Wmap has detected an excess of GHz microwave radiation—dubbed the WMAP haze—from the inner 20° core of our Galaxy [5–9]. Annihilation energy not immediately released by the above mechanisms will thermalize, and subsequently be released as thermal bremsstrahlung emission at the eV scale. Although the eV scale emission will be obscured by other astrophysical sources, the tail of the emission spectrum is very hard, and carries enough energy in the microwave to explain the WMAP haze.

To proceed, we start with the three basic postulates assumed in [4]:

A.1 The antimatter nuggets provide a virtually unlimited source of positrons (e^+) such that impinging electrons (e^-) will readily annihilate at their surface through the formation of positronium [26,27].

About a quarter of the positronium annihilations release back-to-back 511 keV photons. On average, one of these photons will be absorbed by the nugget while the other will be released.

A.2 The nuggets provide a significant source of antibaryonic matter such that impinging protons will annihilate. We assume that the proton annihilation rate is directly related to that of electrons through a suppression factor $f < 1$ as discussed in [4].

Proton annihilation events will release about $2m_p \approx 2$ GeV of energy per event and will occur close to the surface of the nugget creating a hot spot that will radiate x-ray photons with keV energies containing some fraction g of the total annihilation energy. The remaining fraction $1 - g$ will be released at the eV scale, after the energy has thermalized within the nuggets. The tail of this thermal emission will be released in the microwave spectrum and may explain the observed WMAP haze. To further test this theory and connect all of these emissions, we make an additional assumption:

A.3 We assume that the emitted 511 keV photons dominate the observed 511 keV flux, that the emitted keV x-rays dominate the observed diffuse x-ray flux, and that the thermally emitted microwaves dominate the observed WMAP haze.

The basis for this assumption is that none of these fluxes has a convincing explanation. The nuggets may thus provide the missing explanation in each case. This assumption allows us to use the observations at the keV scale—the 511 keV emission measured by INTEGRAL, and the diffuse x-ray emission measured by Chandra—to fix all of the phenomenological parameters [4]. The model then makes unambiguous predictions about the properties of the WMAP haze, allowing it to be tested.

As we shall see, the agreement is remarkable: even though our estimates are only accurate up to the order of magnitude, the picture that dark matter consists mostly of antimatter nuggets can completely explain all three of these puzzling emissions without contradicting a single observation.

B. Observable dark matter: emissions

As discussed in [4], our proposal is that both electrons and protons annihilate on antimatter nuggets, releasing observable radiation from "hot spots" near the annihilation sites. These emissions are best thought of as "jets," and occur sufficiently rapidly that they are produced on a per event basis, thus producing a spectrum that is independent of the local environment [4]. This includes the 511 keV spectrum from positronium annihilation [26,27], the spectrum up to 20 MeV from direct e^+e^- annihilation [28], the diffuse ~10 keV radiation from p^+p^- annihilation [4], and the occasional GeV photon produced directly from proton annihilation [4].

The rates of annihilation and the energies released have been correlated [4] with the observed diffuse 511 keV positronium emission [23,30–34] and diffuse ~10 keV emissions [29] observed from the core of the galaxy, providing a test of the model. It was shown that both of these emissions could be nicely accounted for if the rate of x-ray energy released from p^+p^- annihilation was related

[4] We should remark here that our explanation of the WMAP haze with dark matter is not a new idea. It was suggested previously that self-annihilating weakly interacting massive particles (WIMPs) might explain the WMAP haze [7,8,10]. These WIMPs must be very heavy, $m \sim 100$ GeV, and therefore, annihilation must produce significant amounts of high-energy radiation [22] if it is to also explain microwave emissions with a typical frequency of $\omega \sim 10^{-4}$. For example, if one takes central values for the WIMP parameters, then the microwave intensity from WIMP annihilation would be well below the observed intensity [22]. In any case, this proposal has very different predictions than ours.

to that of e^+e^- annihilation through a suppression factor $f \cdot g \sim 6 \times 10^{-3}$, where the factor f accounts for proton reflection from the sharp nuclear matter interface, and the factor $\frac{1}{10} < g < \frac{1}{2}$ accounts for the fraction of the 2 GeV p^+p^- annihilation energy released at the hot spots.

The topic of this paper is the remaining fraction $1 - g$ of the 2 GeV annihilation energy that will be transmitted deep within the nuggets, ultimately being thermally radiated at a much lower energy scale. We shall show that most of this energy will be released at the eV scale, making it difficult to observe against the bright stellar background. The spectrum of this emission, however, will be shown to be extremely hard, resulting in a significant release of detectable microwave energy ($\sim 10^{-4}$ eV).

Our main point is that this microwave emission could fully account for the recently observed WMAP haze [5–7,35]: a puzzling diffuse emission from the core of our galaxy.

In Sec. III A, we estimate the thermal emissivity of nuggets, and the spectrum of the emitted radiation. In Sec. III B, we show how the nuggets reach thermodynamic equilibrium at an eV scale by balancing the annihilation rate with the emission. Armed with these estimates, in Sec. IV, we compare the predictions of our model with the observations of the WMAP haze, using our previous results [4] to provide the normalizations, and arrive at the remarkable conclusion that our proposal naturally explains the energy budget IV A and spectrum IV B of the observations, even though the predictions are at an energy scale some 10 orders of magnitude smaller than the 511 keV scale at which the normalization was fixed! Finally, in Sec. V B, we reiterate the testable predictions our model makes, thus providing a method with which to confirm or rule out the proposal over the next few years.

III. THERMAL PROPERTIES OF THE NUGGETS

A. Emissivity

Here, we discuss the properties of thermal emission from the "electrosphere" of the nuggets at low temperatures $T \sim$ eV. As we shall show in Sec. III B, this temperature can be established by comparing the rate of annihilation energy deposited in the nugget with the rate of emission. In what follows, we shall present a simple estimate to capture the order of magnitude of the process. In principle, the exact numerical factors can be computed, but such a calculation is extremely tedious, and would be of no use since there are other uncertainties in this problem of a similar magnitude.

The emissivity depends on the density $n(z)$ of the positron cloud, which varies as a function of the distance z from the quark matter core. At the eV scale temperatures, the most important region of emission will be the region of the electrosphere where the kinetic energy of the particles $p^2/2m \simeq T$ is on the same order as the temperature. Closer to the core of the nugget, a well-defined Fermi surface develops with $p_F \gg T$, and the low-energy excitations that can scatter and radiate are confined to an effectively two-dimensional region of momentum space about the Fermi surface. As a result, there is a kinematic suppression of the emissivity from these regions and the emission will not change the order of magnitude estimate we present here. Sufficiently deep into the nugget, the plasma frequency will also be large enough that the emitted eV scale photons will be highly virtual and thus rapidly reabsorbed. A detailed discussion of the suppression of emission from the highly dense regions is presented in Appendix A 4.

Here, we shall estimate the emissivity of a Boltzmann gas of positrons. The Boltzmann approximation is valid where $n \ll p^{-3} \sim (mT)^{3/2}$, and we can neglect both the fermion degeneracy that will suppress the emissivity closer to the core and many-body effects. Thus, we start from the following expression[5] for the cross section for two positrons emitting a photon with $\omega \ll p^2/(2m)$ [36]

$$\mathrm{d}\sigma_\omega = \frac{4}{15}\alpha\left(\frac{\alpha}{m}\right)^2 \cdot \left(17 + 12\ln\frac{p^2}{m\omega}\right)\frac{\mathrm{d}\omega}{\omega}. \tag{2}$$

The emissivity $Q = \mathrm{d}E/\mathrm{d}t/\mathrm{d}V$—defined as the total energy emitted per unit volume, per unit time—and the spectral properties can be calculated from

$$\frac{dQ}{d\omega}(\omega, z) = n_1(z,T)n_2(z,T)\omega\left\langle v_{12}\frac{d\sigma_\omega}{d\omega}\right\rangle = \frac{4\alpha}{15}\left(\frac{\alpha}{m}\right)^2 n^2(z,T)\left\langle v_{12}\left(17 + 12\ln\frac{p_{12}^2}{m\omega}\right)\right\rangle, \tag{3}$$

where $n(z,T)$ is the local density at distance z from the nugget's surface, and $v_{12} = |\vec{v}_1 - \vec{v}_2|$ is the relative velocity. The velocity and momentum p_{12} need to be thermally averaged. To estimate this, we use the Boltzmann ensemble at temperature T with a kinematic cutoff $p^2/(2m) > \omega$, as only particles with sufficient energy can emit photons with energy ω. In principle, one can do better, but the current approach suffices to give the correct order of magnitude (see Appendix A 2 for details):

$$\left\langle v_{12}\left(17 + 12\ln\frac{mv_{12}^2}{\omega}\right)\right\rangle \approx 2\sqrt{\frac{2T}{m\pi}}\left(1 + \frac{\omega}{T}\right)e^{-\omega/T}h\left(\frac{\omega}{T}\right), \tag{4}$$

where (this approximation is accurate to about 25%)

[5] Expression (2) should be contrasted with the well-known dipole type of expression for *different* types of particles emitting soft photons, such as electrons and ions. With identical particles having the same charge to mass ratio e/m, the dipole contribution is zero, and the cross section is dominated by the quadrupole interaction. This quadrupole character explains the appearance of the velocity $\langle v \rangle$ in the numerator of (3) as opposed to the factor $\langle v \rangle^{-1}$ that enters the corresponding expression for electron-ion collisions.

$$h(x) = \begin{cases} 17 - 12\ln(x/2) & x < 1, \\ 17 + 12\ln(2) & x \geq 1. \end{cases} \tag{5}$$

To proceed with our estimates, we need the positron density in the nugget's electrosphere at temperature T. As shown in Appendix A 1, the corresponding expression in the nonrelativistic mean-field approximation is given by

$$n(z, T) \simeq \frac{T}{2\pi\alpha} \cdot \frac{1}{(z + \bar{z})^2}, \tag{6}$$

where $\bar{z}$ is a constant of integration to be determined by some appropriate boundary condition. It is known that the mean-field approximation is not valid for extremely large z, where exponential rather than power-law (5) decay is expected. We could accommodate the corresponding feature by introducing a cutoff at sufficiently large $z = z_{\max}$. The result, however, is not sensitive to this cutoff, so we shall simply take $z_{\max} = \infty$ below to obtain our order of magnitude estimate.

Note: the electrosphere extends well beyond the core of the nugget. To see this, note that the Boltzmann regime (6) is based on an approximation that neglects the curvature of the nuggets surface (see Appendix A 1). This regime terminates only once the curvature becomes significant, i.e. once the electrosphere extends at least to the same order as the macroscopic radius of the nugget. Thus, the electrosphere occupies a significant fraction of the nugget's volume: it is not just a thin outer shell. Corrections from the finite size of the nugget are discussed in Appendix A 3 but do not affect the magnitude of our calculations.

The parameter $\bar{z}$ is not a free parameter, but is fixed by matching the full density profile to the boundary of the nuclear matter core of the nugget, where the lepton chemical potential $\mu_0 \approx 10$ MeV is established by beta-equilibrium in the nuclear matter. A proper computation of $\bar{z}$ thus requires tracking the density through many orders of magnitude from the ultrarelativistic down to the nonrelativistic regime, which is beyond the scope of this work. The order of magnitude, however, is easily estimated by taking $z = 0$ as the onset of the Boltzmann regime

$$n_{z=0} = \frac{T}{2\pi\alpha} \cdot \frac{1}{\bar{z}^2} \simeq (mT)^{3/2}, \tag{7a}$$

$$\bar{z}^{-1} \simeq \sqrt{2\pi\alpha} \cdot m \cdot \sqrt[4]{\frac{T}{m}}. \tag{7b}$$

Numerically, $\bar{z} \sim 0.5 \cdot 10^{-8}$ cm, while the density $n \sim 0.3 \cdot 10^{23}$ cm^{-3} for $T \simeq 1$ eV. (See Appendix A 1 for details of this calculation.)

Our next task is to estimate the surface emissivity (radiant exitance) $F = \int dz Q(z)$—defined as the energy E emitted per unit time dt, per unit area dA (flux)—from the nugget's surface by integrating the emissivity (3) over the Boltzmann regime $z \in [0, z_{\max} \to \infty]$, and introducing an extra factor $1/2$ to account for the fact that only the photons emitted away from the core can actually leave the system.

Our final estimate for spectral surface emissivity can be expressed as follows:

$$\begin{aligned} \frac{dF}{d\omega}(\omega) &= \frac{dE}{dt dA d\omega} \\ &\simeq \frac{1}{2}\int_0^\infty dz \frac{dQ}{d\omega}(\omega, z) \\ &\sim \frac{4}{45}\frac{T^3\alpha^{5/2}}{\pi}\sqrt[4]{\frac{T}{m}}\left(1 + \frac{\omega}{T}\right)e^{-\omega/T}h\left(\frac{\omega}{T}\right). \end{aligned} \tag{8}$$

Integrating over ω contributes a factor of $T \int dx(1 + x) \times \exp(-x)h(x) \approx 60T$, giving the total surface emissivity

$$F_{\text{tot}} = \frac{dE}{dt dA} = \int_0^\infty d\omega \frac{dF}{d\omega}(\omega) \sim \frac{16}{3}\frac{T^4\alpha^{5/2}}{\pi}\sqrt[4]{\frac{T}{m}}. \tag{9}$$

Although a discussion of black-body radiation is inappropriate for these nuggets (for one thing, they are too small to establish thermal equilibrium with low-energy photons), it is still instructive to compare the form of this surface emissivity with that of black-body radiation $F_{BB} = \sigma T^4$

$$\frac{F_{\text{tot}}}{F_{BB}} \simeq \frac{320}{\pi^3}\alpha^{5/2}\sqrt[4]{\frac{T}{m}}. \tag{10}$$

At $T = 1$ eV, the emissivity $F_{\text{tot}} \sim 10^{-6}F_{BB}$ is much smaller than that for black-body radiation. The spectral properties of these two emissions are also very different at low frequencies $\omega \leq T$ as follows from (8).

These two differences are essential to explain the WMAP haze. First, the suppressed total radiant exitance is required to establish the eV temperature scale (this will be discussed in Sec. III B). Second, the extremely long low-frequency tail due to the logarithmic dependence of $h(x)$ is required to ensure that sufficient power is radiated in the microwave. Thus, as we shall show in IV, it is highly nontrivial that the scale of the emitted microwave emission should be consistent with the observed WMAP haze emission: If the nuggets had been simple black-body emitters, the emission would be many orders of magnitude below the observed scale.

Finally, we emphasize here that there are no free parameters in this calculation: all of the scales are set by well-established nuclear and electromagnetic physics. The only unknown parameters that enter are the overall normalizations, which can be fixed by considering the related diffuse x-ray emission. This is the point of the next section.

B. Thermodynamic equilibrium

Armed with an estimate of the total emissivity (9), we may discuss the thermodynamic properties of the nuggets. In order to maintain the overall energy balance, the nuggets must emit energy at the same rate that it is deposited through proton annihilation

$$F_{\rm tot} = (1-g)F_{\rm ann} = (1-g)\frac{dE_{\rm ann}}{dt dA}, \tag{11}$$

where $1-g$ is the fraction of the annihilation energy that is thermalized. Note that both the rate of emission and the rate of annihilation are expressed as per unit area A, so that the equilibrium condition is independent of the nugget size. The rate of annihilation $F_{\rm ann}$ is

$$F_{\rm ann} = 2\ \text{GeV} \cdot f \cdot v \cdot n_{\rm VM}(\vec{\mathbf{r}}), \tag{12}$$

where $2\ \text{GeV} = 2m_p$ is the energy liberated by proton annihilation, v is the speed of the nugget through the visible matter, $n_{\rm VM}(\vec{\mathbf{r}})$ is the local visible matter density, and

$$f = \frac{\sigma_{\rm ann}}{A} \sim 10^{-1}$$

is the factor by which the effective cross section $\sigma_{\rm ann}$ for proton annihilation is reduced from the geometric cross section A due to the possibility of reflection from the sharp quark-matter surface (in contrast, the positron distribution in the electrosphere is very smooth), as discussed in [4].

The typical galactic scale for the speed is $v \sim 100\ \text{km/s} \sim 10^{-3}$ c, while the density at a distance $r \sim$ kpc from the center is

$$n_{\rm VM} \sim \xi \cdot n_{\rm VM}^{\rm disk} = \xi \cdot \frac{3}{\text{cm}^3} \approx \frac{150}{\text{cm}^3},$$

where we have adopt a scaling behavior close to that of an isothermal sphere [37] for the typical visible matter density in the bulge at a distance $r \sim$ kpc from the core, where the observed WMAP haze originates:

$$\xi \approx \left(\frac{8.5\ \text{kpc}}{r}\right)^{1.8} \sim 50.$$

Combining these, we obtain

$$F_{\rm ann} \sim \frac{10^9\ \text{GeV}}{\text{cm}^2 \cdot \text{s}} \cdot \left(\frac{f}{10^{-1}}\right) \cdot \left(\frac{v}{10^{-3}\ \text{c}}\right) \cdot \left(\frac{n_{\rm VM}}{300/\text{cm}^3}\right), \tag{13}$$

which must be compared with the total surface emissivity (9)

$$F_{\rm tot} \sim 10^9 \frac{\text{GeV}}{\text{cm}^2 \cdot \text{s}} \left(\frac{T}{\text{eV}}\right)^{4+1/4}.$$

Taking the typical values $v \sim 10^{-3}$ c and $n_{\rm VM} \sim 300/\text{cm}^3$ gives the relationship between the temperature and typical parameters describing the nuggets f, g,

$$\left(\frac{T}{\text{eV}}\right)^{4+1/4} \approx (1-g)\left(\frac{f}{10^{-1}}\right). \tag{14}$$

As discussed in [4], reasonable values for $f \sim 1/15$ and $g \sim 1/10$ all lead to a $T \sim$ eV equilibrium temperature.

The heat capacity of the nuggets is estimated in Appendix A 5. If the gas of positrons occupies a substantial volume of the nugget, then the heat capacity is "large" in the sense that it will require many annihilations to raise the temperature of the nuggets to the eV scale at which equilibrium is established. Thus, the antimatter nuggets will act as effective thermal integrators, slowly reaching a relatively constant average temperature $T \sim$ eV.

IV. EXPLAINING THE WMAP HAZE

In our proposal, interstellar matter annihilates on antimatter nuggets. The nuggets then radiate this energy over a wide range of frequencies. The model thus makes definite predictions relating these emissions: they should have similar morphologies, and the relative intensities should be related by an overall energy budget determined by the local annihilation rate.

Four types of emission are from hot spots at the annihilation sites, and should be observable from the core of our galaxy:

- B.1 Electron annihilations through positronium produce a well-defined 511 keV emission [26,27] that is consistent with, and could possibly explain the puzzling diffuse 511 keV emission observed by SPI/INTEGRAL.
- B.2 Direct electron annihilation can also produce emission in the 1–20 MeV band [28] that is consistent with, and could explain part of the diffuse gamma-ray emissions observed by COMPTEL.
- B.3 Proton annihilation produces keV x-ray emission from a hot spot at the annihilation site [4] that is consistent with, and could possibly explain the puzzling diffuse x-ray emissions observed by Chandra.
- B.4 Proton annihilation occasionally produces GeV photons [4] that are consistent with, and could partially account for the gamma-ray emissions observed by EGRET.

All of these emissions are "direct" in the sense that the timescale for the emission is much shorter than the time between successive annihilations. Thus, the intensity of these emissions depends only[6] on the rate of annihilation events, which is proportional to $n_{\rm VM}(r)n_{\rm DM}(r)$—the product of the local visible and dark-matter distributions at the annihilation site. The emitted spectrum is also independent of the local density. We emphasize that the model makes two nontrivial predictions: 1) that the morphology of these emissions is very strongly correlated, and 2) that the spectral properties of these emissions are *independent of position*.

A comparison between observations of the direct emissions B.1 through B.4 along the same line of sight is possible because the local emission depends only on the

[6]There additional small dependencies that we neglect here: for example, on the local speed of the nuggets. This introduces only small uncertainties, however, and certainly do not affect the overall magnitude.

local rate of annihilation $\phi(\vec{\mathbf{r}}) \propto n_{\rm VM}(\vec{\mathbf{r}})$. The observed flux thus depends on the uncertain matter distribution through the same line-of-sight integral

$$\Phi_{511,\text{X-ray,etc.}} \propto \int d\Omega dl n_{\rm VM}(l) n_{\rm DM}(l), \tag{15}$$

which cancels when comparing emissions from the same position in the sky.

There is an additional emission from the nuggets:

B.5 Energy not directly released through one of the mechanisms B.1–B.4 heats the nuggets, ultimately being thermally radiated.

As we have shown in Sec. III B and Appendix A 5, the heat capacity and energy budget ensure that the nuggets have a well-defined temperature scale of $T \sim 1$ eV in the core of the Galaxy. The resulting thermal bremsstrahlung emission thus "averages" the annihilations over time, and the resulting emissivity and spectrum will depend on the temperature $T(n_{\rm VM})$, which is a function of the local visible matter density.

The observed flux of this thermal emission exhibits a slightly different dependence because the local emission depends on the temperature $\phi(\vec{\mathbf{r}}) = \phi\{T[n_{\rm VM}(\vec{\mathbf{r}})]\}$. As we shall show in Appendix A 6, however, the difference is small, and can be ignored for the order of magnitude comparisons we present here.

Finally, in principle, we may compare the *total* thermal emission (9) with the direct emissions because thermal equilibrium relates the rate of total emission to the rate of annihilation, both of which are proportional to $n_{\rm VM}(\vec{\mathbf{r}})$. In practice, however, the thermal eV scale emission cannot be seen against the bright stellar background.

A. Energy budget

In [4], the direct emissions B.1, B.3, and B.4 were compared, showing that our proposal is consistent with the current observations, and using the observations to constrain the properties of the nuggets. In particular, two parameters were introduced describe complicated properties of the nuggets: The parameter $f < 1$ was introduced to describe the suppression of the proton annihilation rate with respect to the electron annihilation rate, and the parameter $g < 1/2$ was introduced to describe the fraction of the proton annihilation energy that is directly released as x-rays. We emphasize that these parameters are not free, but they depend on detailed models of the nuggets and are beyond the reach of present day calculational techniques.

By hypothesizing that emissions from the nuggets completely explain the puzzling 511 keV (B.1) and diffuse x-ray (B.3) emissions, one obtains $fg \sim 6 \cdot 10^{-3}$, which is consistent with the theoretical estimates, and provides a nontrivial test of the theory.

Neglecting the small corrections to the line-of-sight averaging discussed above, we may perform a similar analysis of the WMAP haze to see if it is also consistent with the our proposal. The thermal energy input into the antimatter nuggets is the complementary fraction[7] $1 - g$ of the total proton annihilation energy not directly released as x-rays. Thus, if we use the observed x-ray flux $\Phi_{\rm Chandra}$ to provide the energy normalization, then the total thermal emission will be approximately

$$\Phi_T \approx \frac{1-g}{g} \Phi_{\rm Chandra}. \tag{16}$$

The total thermal emission Φ_T may then be used to estimate the observed microwave emission in a specified frequency band by computing the ratio γ of spectral emissivity (8) in the specified band to the total emissivity (9)

$$\gamma = \frac{1}{F_{\rm tot}} \int_\omega^{\omega+\Delta\omega} d\omega \frac{dF}{d\omega}(\omega) \approx \frac{25 - 12\ln(\omega/T_{\rm eff})}{60 T_{\rm eff}} \Delta\omega,$$

where $T_{\rm eff}$ is an "average" temperature that accounts for variations along the line of sight. This is the fraction of the total emitted thermal radiation emitted in the microwave band ω of width $\Delta\omega \ll \omega \ll T_{\rm eff}$. For the typical scale of the emissions we are considering, $T_{\rm eff} \sim$ eV and $\omega \sim h \cdot 30$ GHz $\sim 10^{-4}$ eV, so we have

$$\gamma \approx 2\frac{\Delta\omega}{T_{\rm eff}}.$$

The total observed microwave flux is then related to the total thermal flux (15) by

$$\Delta\omega \frac{d\Phi_{\rm WMAP}}{d\omega} \approx \gamma \Phi_T \approx \gamma \frac{1-g}{g} \Phi_{\rm Chandra},$$

giving us the relationship

$$\frac{d\Phi_{\rm WMAP}}{d\omega} \approx \frac{2}{T_{\rm eff}} \frac{1-g}{g} \Phi_{\rm Chandra}. \tag{17}$$

Observationally, Chandra observes a total flux [29]

$$\Phi_{\rm Chandra} \approx 2 \times 10^{-6} \frac{\rm erg}{\rm cm^2 \cdot s \cdot sr}, \tag{18a}$$

while the WMAP haze flux is [5–8]

$$\frac{d\Phi_{\rm WMAP}}{d\omega} = (3\text{–}6)\frac{\rm kJy}{\rm sr} \approx \frac{(3\text{–}6) \times 10^{-20}\ \rm erg}{\rm cm^2 \cdot s \cdot sr \cdot Hz}. \tag{18b}$$

[7] Technically, we should include the energy from electron annihilations, and subtract the fraction α/α_s of GeV photons B.4 [4]. These are only small corrections to the overall energy budget.

Combining these, and converting $1\ \text{Hz} \approx 4 \times 10^{-15}$ eV, we predict that the observed WMAP haze intensity will be saturated by thermal antinugget emission if the parameters which enter in our estimate satisfy the following constraint:

$$\frac{\text{eV}}{T_{\text{eff}}} \cdot \frac{1-g}{g} \approx (2\text{–}4). \quad (19)$$

Although this relationship is only approximate, it is quite amazing that it is satisfied (without any adjustment) if the previous estimates for T_{eff} and g are used. Thus, the nontrivial relationship (19)—which depends strongly on the observed intensity of the GHz WMAP haze—is satisfied by the phenomenological parameters determined by considering only the keV scale emissions.

B. Spectrum

The observed spectrum of the WMAP haze is extremely hard [5,35]. This feature is easily accommodated in our model by the logarithmic dependence of the thermal bremsstrahlung emission (8). Indeed, the WMAP haze was initially interpreted as thermal bremsstrahlung (free-free emission) from a hot ($T \sim$ eV) gas [22] ($10^4\ \text{K} < T < 10^6$ K), but this interpretation was rejected because a $H\alpha$ recombination line, which should accompany the haze, is not seen. (The possibility of much hotter plasmas $T \gg 10^4$ K has also been ruled out [5–7,35].)

It is quite remarkable that the $T \sim 10^4\ \text{K} \sim \text{eV}$ scale arises naturally in our proposal in two completely independent ways: (14) and (19). Our proposal, thus, naturally fits the observed spectrum of the WMAP haze, without any $H\alpha$ recombination line, since the emission is from a purely positronic gas. It is also remarkable that the spectrum exactly corresponds with bremsstrahlung radiation, as was originally suggested in [5] to fit the data.

C. Morphology

Our proposal also makes a definite prediction about the morphologies of the various emissions. In particular, the morphology of the direct emissions B. 1-B. 4 should be almost identical. As discussed in Appendix A 6, even though the WMAP haze is a thermal emission, the dependence on the line-of-sight matter distribution is quite weak, and our model thus also predicts that the morphology of the WMAP haze be closely correlated with the morphology of the direct emissions.

Ultimately, if our proposal is correct, the morphology of all of the emissions are direct probes of the matter distributions in our Galaxy, and may thus become a useful measuring tool.

V. SUMMARY

We have now demonstrated that our proposal naturally and nontrivially explains diffuse galactic radiations B.1-B. 5 over 13 orders of magnitude from microwave (10^{-4} eV) to GeV scales. The only "parameter" in our model is the overall size of the nuggets, and the dependence of the observations on this parameter can virtually be eliminated by comparing observations along the similar lines of sight to the core of the Galaxy. Comparisons along the same line of sight also virtually eliminate uncertainties related to the distribution of matter in our Galaxy. Removing this uncertainty, our proposal depends on only a couple of parameters: $f < 1$ and $g < 1/2$. These represent presently intractable calculations, but have tight upper bounds and can vary by no more than an order of magnitude or so.[8]

Together with our previous work [4], we now have several constraints on these parameters by postulating that matter annihilation on dark-antimatter nuggets explains significant proportions of puzzling diffuse emissions. These constraints provide a highly nontrivial test of our proposal.

- S.1 The diffuse 511 keV emission B.1 observed by SPI/INTEGRAL has been identified as primarily due to positronium annihilation. The puzzle is how positrons come to be diffusely distributed through the core of our Galaxy. We propose that the positron electrosphere of dark-antimatter nuggets provide the source of positrons. This source is distributed diffusely, and produces a definite spectrum consistent with the observed spectrum [26,27] that is independent of any model-specific parameters. One prediction is that the spectrum is independent of observed direction. Another is that the intensity is determined by the product $n_{\text{VM}}(\vec{\mathbf{r}})n_{\text{DM}}(\vec{\mathbf{r}})$ of the distribution visible and dark matter. We use this emission as a baseline to which we compare the other emissions in order to remove the uncertainties of the nugget size and the matter distribution along the line of sight to the galactic core.
- S.2 Associated with this emission is the MeV spectrum from direct e^+e^- annihilation B.2. This spectrum is model independent, and consistent with observations and background models, possibly explaining the 1–20 MeV energy deficits seen in the COMPTEL data [28].
- S.3 The diffuse keV x-ray emissions B.3 measured by Chandra are puzzling because of the implied energy

[8]The parameters f and g can, in principle, be calculated from the first principles. However, such a computation is extremely difficult as it requires solving the many-body problem in a strongly coupled regime. Indeed, the phase diagram of quark matter in the relevant regime is still largely unknown. Even with these reservations, we still are quite confident that these parameters cannot deviate much from their "natural" values. In this respect our proposal is predictive: there is little freedom to change these parameters. This is in contrast with most other dark-matter proposals where parameters can be arbitrary changed by many orders of magnitude to exploring an enormously large, and largely unconstrained parameter space.

budget. The spectrum looks like a thermal 8keV plasma [29], but such a plasma would not even be gravitationally bound and would require a huge unidentified source of energy to fuel. We propose that this emission is due to bremsstrahlung emission from positrons excited from protons annihilating on the dark-antimatter nuggets [4]. The spectrum for this process is also largely independent of model-specific parameters, is consistent with the observations, and is also independent of position. Comparing this emission, the 511 keV emission gives one constraint on the parameters fg [4],

$$fg \approx 6 \times 10^{-3}, \tag{20}$$

that is satisfied by their natural scales $f \approx 1/15$ and $g \approx 1/10$. The morphologies should also be related, depending on the product $n_{\rm VM}(\vec{\mathbf{r}})n_{\rm DM}(\vec{\mathbf{r}})$.

S.4 The direct emission of GeV photons from proton annihilation B.4 is consistent with gamma-ray observations by EGRET, explaining up to one tenth or so of the observed spectrum [4].

S.5 The annihilation energy not released through one of the previous sources of emission ultimately thermalizes. As shown in Sec. III B, the nuggets reach equilibrium with a typical $T \sim$ eV scale through the constraint (14),

$$\left(\frac{T}{\rm eV}\right)^{4+1/4} \approx (1-g)\left(\frac{f}{10^{-1}}\right), \tag{21}$$

which is again satisfied by the natural parameter scales. We emphasize that this constraint is virtually model independent, depending on only the emissivity calculated in Sec. III A and the properties of the matter distribution in the core of the Galaxy. The emissivity is dominated by the properties of the nugget electrosphere in the low-density regime, where calculations are under order-of-magnitude control. Although not particularly sensitive to the emissivity, the scale set by (14) would be altered by an order of magnitude if the emissivity were black body.

A. Predictions

The constraints described in Sec. V did not involve any measurements of the WMAP haze. Thus, the temperature $T \simeq$ eV (14) allows us to unambiguously predict the energy emitted in the tail of the thermal distribution in the microwave band. Amazingly, comparing this with the observed WMAP haze gives (19)

$$\frac{\rm eV}{T_{\rm eff}} \cdot \frac{1-g}{g} \approx (2-4), \tag{22}$$

which is satisfied with the natural values of the parameters, even though the frequencies are at a scale many orders of magnitude smaller than the scale at which the parameters were constrained.

Unlike (14), this estimate is extremely sensitive to the flat spectral properties of the thermal emission (8) at small frequencies $\omega \ll T$: a very specific feature of thermal bremsstrahlung emission. If the spectrum were not flat, there is no way that this constraint would be satisfied.

Another difference between the two estimates (14) and (19) is that the former is sensitive overall normalization of (9), whereas the latter is sensitive to the detailed shape of the spectrum (8). We also emphasize that the constraints (14) and (19) not only deal with emissions from scales separated by 8 orders of magnitude, but that condition (19) depends on observed intensity of the WMAP haze, which has absolutely nothing to do with the estimate (14). That these two estimates agree with each other and with the estimates in [4] (where the intensity of the diffuse keV x-rays were compared with the 511 keV emission) is truly remarkable.

In addition to satisfying the constraints described in the previous section, our proposal also makes the definite prediction that the morphologies of the 511 keV flux, the 1–20 MeV γ-ray emission, and the x-ray flux should all be identical, following the distribution $n_{\rm VM}(\vec{\mathbf{r}})n_{\rm DM}(\vec{\mathbf{r}})$ of visible and dark matter. The morphology of the WMAP should be similar but may differ slightly because of the different line-of-sight integral (A20). For example, Chandra has detected a diffuse x-ray emission with flux 6.5×10^{-11} erg/cm^2/s/deg^2 from a region of the disk 28° off the center [38]. This is one order of magnitude smaller than the observations from the core of the Galaxy, and so our model predicts that the microwave emission from this region is about one order of magnitude smaller than the WMAP haze from the galactic core, which seems consistent with the observations [5].

Finally, this proposal makes definite testable predictions for the properties of the emitted spectra. In particular, all bands but the WMAP haze are produced on an event-by-event basis, and are thus independent of the rate at which the annihilation processes occur. The observed spectra should thus be largely *independent of the direction of observation.* Only the intensity should vary as a function of the collision rate, and this should be correlated with the visible/dark-matter distribution as discussed above. The WMAP haze will have a slight spectral dependence, but only through the temperature dependence $T(n_{\rm VM}) \propto n_{\rm VM}^{4/17}$, which is quite weak.

B. Conclusion

Our dark-matter proposal not only explains many astrophysical and cosmological puzzles, but makes definite predictions about the correlations of the dark and visible matter distributions $n_{\rm VM}n_{\rm DM}$ with five different bands of radiation ranging over 13 orders of magnitude in frequency. In addition, it makes the definite prediction that these spectra of the emissions should be virtually independent of the local environment. Such correlations and spectral properties would be very difficult to account for with other dark-matter candidates. Future observations may thus easily confirm or rule out this theory. If confirmed, it would provide a key for many cosmological and astrophysical secrets, and finally unlock the nature of dark matter.

ACKNOWLEDGMENTS

The authors would like to thank to members of the UBC astronomy group, especially M. Halpern, H. B. Richer, and D. Scott for providing a forum for vetting the ideas presented in this paper, and for useful discussions. This research was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-97ER41014, and the Natural Sciences and Engineering Research Council of Canada.

APPENDIX A: DETAILED CALCULATIONS

1. Structure of electrosphere

Here, we briefly discuss the properties of the antimatter nugget electrosphere to set the scales of the problem and determine the structure required in Sec. III A.

The radius of the nuggets depends on the mass, but must be larger than $R > 10^{-7}$ cm at the lower limit $|B| > 10^{20}$ set by terrestrial nondetection. We expect that the most likely size is on the order of $|B| \sim 10^{25-27}$ [18]. The quark-matter core of the nuggets ends sharply on a fm scale as set by nuclear physics. Near the surface, as the density falls, the quark matter will definitely be charged due to the relatively large mass of the strange quark $m_s \sim 100$ MeV [39–42], however, depending on the phase of quark matter realized in the core, the matter may be charged throughout. Charge neutrality will be maintained through beta equilibrium, which will establish a positron chemical potential $\mu_{e^+} = \mu_0 \simeq 10$ MeV. (The precise value depends on specific details of the quark-matter phase and may range from a few MeV to hundreds of MeV, but is about an order of magnitude less than the quark chemical potential $\mu_q \simeq 500$ MeV [39,43].) This will induce a thin but macroscopic electrosphere of positrons surrounding the quark-matter core in the transition region as $\mu_{e^+} \to 0$ in the vacuum.

The structure of this electrosphere has been considered for quark matter [39,40], and the existence of this "transition region" is a very generic feature of these systems. It is the direct consequence of Maxwell's equations and chemical equilibrium. The region is called the electrosphere, emphasizing the fact that quarks and other strongly interacting particles are not present. In the case of antimatter nuggets the electrosphere comprises positrons.

The variation of this chemical potential $\mu_{e^+}(z)$, and the density $n(z)$ as a function of distance z from the surface of the nugget may be computed using a mean-field treatment of the Maxwell equations [39,40,44]. For example, in the relativistic regime, one has [42]

$$\mu_{e^+}(z) = \sqrt{\frac{3\pi}{2\alpha}}\frac{1}{(z+z_0)}, \quad n(z) \approx \frac{\mu_{e^+}^3}{3\pi^2} = \frac{1}{3\pi^2}\left(\frac{3\pi}{2\alpha}\right)^{3/2}\frac{1}{(z+z_0)^3}, \quad z_0 = \sqrt{\frac{3\pi}{2\alpha}}\frac{1}{\mu_0}, \tag{A1}$$

where $\mu_0 \equiv \mu_{e^+}(z=0) \sim 10$ MeV is the chemical potential realized in the nugget's bulk. The corresponding results can be obtained outside of the relativistic regime, but they do not have a simple closed form. These calculations treat the electrosphere as a one-dimensional wall rather than including the full radial structure, essentially keeping only the first term in the z/R expansion. This approximation does not affect the order of magnitudes of our calculation and will also be employed here.

The majority of the thermal emission considered in this work comes from the nonrelativistic regime, which we may also analyze analytically using the Boltzmann approximation. The mean-field approximation amounts to solving the Poisson equation

$$\nabla^2\phi(\vec{\mathbf{r}}) = -4\pi e n(\vec{\mathbf{r}}), \tag{A2}$$

where $\phi(\vec{\mathbf{r}})$ is the electrostatic potential and $n(\vec{\mathbf{r}})$ is the density of positrons. Using the spherical symmetry of the nuggets and making the one-dimensional approximation, we can write this as[9]

$$\frac{d^2\phi(z)}{dz^2} = -4\pi e n(z), \tag{A3}$$

where z is the distance from the quark nugget surface. We now introduce the positron chemical potential $\mu_{e^+}(z) = -e\phi(z)$, which is the potential energy of a charge at position z relative to $z = \infty$, where we take $\mu_{e^+}(\infty) = 0$ as a boundary condition. This gives

$$\frac{d^2\mu_{e^+}(z)}{dz^2} = 4\pi\alpha n[\mu_{e^+}(z)] \tag{A4}$$

with the additional boundary conditions $\mu_{e^+}(z=0) = \mu_0 \sim 10$ MeV as established by beta equilibrium in the quark matter. Here, $n[\mu_{e^+}]$ is the density of a free Fermi gas of positrons as a function of the chemical potential for the positrons. The full relativistic form is

$$n[\mu] = 2\int\frac{d^3p}{(2\pi)^3}\left(\frac{1}{1+e^{((\epsilon_p-\mu)/T)}} - \frac{1}{1+e^{((\epsilon_p+\mu)/T)}}\right), \tag{A5}$$

[9]Here, we drop the radial term $2\phi'(z)/r$ on the left-hand side of (A2), assuming that the radius of the nuggets $R \gg z$ is much larger than the thickness of the electrosphere.

where $\epsilon_p = \sqrt{p^2 + m^2}$, which has the property $n[\mu = 0] = 0$ required by our identifying the chemical potential μ with respect to $z = \infty$. This is quite complicated, but is well approximated in the nonrelativistic Boltzmann regime where $n \ll (mT)^{3/2}$ by

$$n[\tilde{\mu}] \approx 2 \int \frac{d^3 p}{(2\pi)^3} e^{[\tilde{\mu} - p2/(2m)]/T} = \sqrt{2}\left(\frac{mT}{\pi}\right)^{3/2} e^{\tilde{\mu}/T}, \tag{A6}$$

where we have performed a nonrelativistic expansion $\sqrt{p^2 + m^2} \approx m + p^2/(2m)$, dropped the antiparticle contribution, and neglected the quantum degeneracy. The effective chemical potential $\tilde{\mu} = \mu_{e^+} - m$ is related to the vacuum chemical potential μ by subtracting the mass. We note that the right boundary condition must now be changed to $n(z = \infty) = 0$ because $\tilde{\mu}$ does not tend to zero under these approximations. The left boundary condition must be determined by matching the density at some point to the full relativistic solution that integrates to the quark matter core and matches (A1). The differential Eq. (A4) may now be expressed in terms of either $\tilde{\mu}(z)$ or $n(z)$, leading to the peculiar solution

$$n(z) = \frac{T}{2\pi\alpha} \frac{1}{(z + \bar{z})^2}, \tag{A7}$$

where $\bar{z}$ is an integration constant determined by matching to a full solution. Here, we make a simple approximation, defining $z = 0$ as the onset of the Boltzmann regime

$$n(z = 0) = \frac{T}{2\pi\alpha\bar{z}^2} = (mT)^{3/2}. \tag{A8}$$

While not an exact matching procedure, this will give the correct parametric dependence and will be valid for the order-of-magnitude estimates required. Thus, we have the following characterization of the density in the Boltzmann regime as used in Sec. III A:

$$\bar{z}^{-1} \simeq \sqrt{2\pi\alpha} \cdot m \cdot \sqrt[4]{\frac{T}{m}}. \tag{A9}$$

The region where $z < 0$ here corresponds to the region of higher density closer to the nugget's surface where the Boltzmann approximation breaks down due to degeneracy effects. One can argue, however, that in this degenerate regime, the emissivity is strongly suppressed for two reasons: 1) only a small portion of the particles close to the Fermi surface can participate in scattering processes, so the phase space for emission is dimensionally reduced, and 2) as the density increases, the plasma frequency increases, and low-energy photons cannot escape.

Our estimates include only emission from the Boltzmann regime. In principle, the emission from denser regions could contribute at the same order: this approximation thus underestimates the emission by a factor, but not by an order of magnitude.

We make one final set of remarks in response to the criticism [45] (version 3) of our proposal. As we have shown, the density profile behaves very differently in different physical regimes. In the ultrarelativistic regime (A1), one has the dependence $n \propto V^3$ where $V(z)$ is the electrostatic potential in the mean-field approximation [39,40,44], whereas in the nonrelativistic Boltzmann regime (A6), one has $n \propto e^{V/T}$, which is the well-known expression for the density in nonrelativistic and nondegenerate systems [46]. In the intermediate regimes, the dependence is quite complicated due to the competing scales that appear in $n[\mu]$ (A5). Thus, one cannot simply apply formulae like $n \propto V^3$ derived in one regime to describe physics in another as was done in [45].

In general, one must also take into account quantum many-body effects—such as charge screening (completely ignored in [45]), the plasma frequency, etc. In the Boltzmann regime discussed here, the density is sufficiently low that many-body corrections may be neglected and vacuum results, such as the cross section (2), employed. At higher densities, however, when the degeneracy becomes important—of order roughly $n \gtrsim (mT)^{3/2}$ in our case—many-body effects can drastically alter the behavior of the system and cannot be neglected as they were in [45], even when considering only the qualitative physics.

2. Boltzmann averages

To evaluate Eq. (3) we need to perform the thermal average

$$\left\langle |v_1 - v_2| \left(17 + 12 \ln \frac{(p_1 - p_2)^2}{m\omega}\right)\right\rangle. \tag{A10}$$

As we are in the Boltzmann regime, we may simplify the calculation by computing this in the Boltzmann ensemble. Formally, we must integrate over both momenta p_1 and p_2, but as we are only interested in the order of magnitude, we simply perform the average over only a single momentum p_2, setting $p_1 = 0$.[10]

$$\left\langle v_{12}\left(17 + 12 \ln \frac{m v_{12}^2}{\omega}\right)\right\rangle = 2\sqrt{\frac{2T}{m\pi}}\left(1 + \frac{\omega}{T}\right) e^{-\omega/T} h\left(\frac{\omega}{T}\right) \tag{A11}$$

where

[10]Including the full angular integrals increases the result by a factor of about $\sqrt{2}$ or so.

$$\left\langle v_{12}\left(17 + 12 \ln \frac{m v_{12}^2}{\omega}\right)\right\rangle = 4\sqrt{\frac{T}{m\pi}}\left(1 + \frac{\omega}{T}\right) e^{-\omega/T} \times \left(17 + 12 \tilde{g}\left(\frac{\omega}{T}\right)\right).$$

The calculation of $\tilde{g}(x)$, however, is somewhat tricky.

$$h(x) = 17 + 12g(x),$$
$$g(x) = \ln(2) + \frac{1 + E_1(x)e^x}{1 + x},$$
$$E_1(x) = \int_1^\infty \frac{e^{-xz}}{z} \mathrm{d}z.$$

The following approximation for $h(x)$ is accurate to within 25% for all x

$$h(x) = \begin{cases} 17 - 12\ln(x/2) & x < 1, \\ 17 + 12\ln(2) & x \geq 1. \end{cases} \tag{A12}$$

3. Finite-size effects

So far, our calculations have assumed that we are working in infinite matter. Here, we estimate the size of the corrections due to the fact that the nuggets have a finite extent on the order of $L \geq 10^{-5}$ cm. We shall demonstrate that properly accounting for these corrections does not significantly affect our estimates of the microwave band emission (though it may drastically suppress emission at much longer wave lengths).

In principle, finite-size effects may change the cross section (2), and therefore, our estimation of the emissivity (3). The cross section (2) was derived using a continuum of plane-wave states, whereas to account for the finite-size effects, one should use the basis of states bound to the quark core. To estimate the size of the corrections, one can imagine confining the positrons to a box of finite extent L.

The electromagnetic field may still be quantized as in free space with states of arbitrarily large size because the photons are not bound to the core, and are not in thermodynamic equilibrium with the positrons. Their mean-free-path is much larger than L, so the low-energy photons produced by the mechanism described above will simply leave the system before they have a chance to interact with other positrons.

Therefore, it is only the positron states that must be considered on a finite-size basis, which will modify the corresponding Green's function used in the calculation of the cross section (2). These modifications occur for momenta of the scale $\delta p \sim \hbar/L$. If $L \geq 10^{-5}$ cm, then this corresponds to shifts in the energies of $\delta E \sim (\delta p)^2/2m \sim 10^{-6}$ eV $\ll 10^{-4}$ eV, which is much smaller than the transitions responsible for the emission at microwave frequencies. Thus, we conclude that finite-size effects do not drastically change the positron Green's function in the region of interests. In other words, the expression for the cross section (2)—derived using the standard (infinite volume) Green's functions—remains valid for the estimation of the emission and spectrum down to the microwave region 10^{-4} eV. We also note that finite-size effects do not change our estimates for the density (A5) and (A6) because the finite-size effects $\delta E \ll T$ are much smaller than the typical energetic scale $T \sim$ eV of the problem. Thus, our expression (2) remains valid for photon energies $\omega \geq 10^{-4}$ eV. To calculate the emission of radiation with much longer wavelengths, however, requires one to account for these finite-size corrections, and we expect the emission of extremely low-energy photons $\omega \ll \delta E \lesssim 10^{-6}$ eV to be suppressed.

One may ask how microwave radiation may be emitted from the nuggets when the wavelength λ is much larger than the size of the nugget $\lambda \gg L$.[11] In general this is not a problem—consider the well-known astrophysical emission of the $\lambda = 21$ cm line from hydrogen with a size $a \simeq 10^{-8}$ cm—but there is a potential suppression: the coherence time τ of the positrons, which must be compared with the formation time $\sim\omega^{-1}$ of the photons. If the coherence time is too short, then multiple scatterings will disrupt the formation of the photons. This suppression is a case of the so-called Landau-Pomeranchuk-Migdal (LPM) effect.

To estimate the coherence time τ, consider the cross section σ_{ee} of the positron-positron interaction. This scales as $\sigma_{ee} \sim \alpha^2/q^2$, where $q \sim b^{-1}$ is the typical momentum transfer, and may be expressed in terms of the impact parameter $b \sim n^{-1/3}$, which is estimated in terms of average interparticle spacing, where n is the local positron density.

The mean-free-path l is thus $l^{-1} \sim \sigma_{ee} n \sim \alpha^2 n^{1/3}$. Therefore, the typical time between collisions (which is the same as coherence time) is $\tau \sim l/v$, where $v \sim \sqrt{T/m}$ is the typical positron velocity.

Collecting all factors together we arrive at the estimate

$$\omega\tau \sim \frac{\omega}{\alpha^2 n^{1/3}}\sqrt{\frac{m}{T}} \sim \frac{\omega}{\alpha^2 T}\left(1 + \frac{z}{\bar{z}}\right)^{2/3} \geq 1. \tag{A13}$$

It is clear that this condition is satisfied for $\omega \geq 10^{-4}$ eV and $T \leq 1$ eV, even for $z = 0$. Thus, we were justified in omitting LPM effect in our estimates in the low-density regime (A7). However, from the same estimate it is clear that this suppression becomes important for either smaller frequencies $\omega \ll 10^{-4}$ eV or at higher densities. We shall now show that there us a much more significant high-density suppression than the LPM effect, which effectively turns off emission from the bulk of the nuggets where the positron density is significantly higher than (A7).

4. Emission from very dense regions

Here, we estimate the emissivity from very dense regions of the nugget when the Boltzmann regime breaks down and degeneracy plays a crucial role. As we shall argue below, the corresponding emission from very dense regions can be neglected in comparison with the estimates (8) and (9) used in the text.

We start from (3), which remains valid for any densities $n(z, T)$. To deal with denser regions, however, one must the

[11] We thank a referee for raising this question.

full expression for (A5) which includes the effects of quantum degeneracy. In these dense regions, only the states close to the Fermi surface are excited and can participate in emission: the states deep within the Fermi surface are "Pauli blocked" and cannot participate in low-energy interactions. It is the density of these "quasiparticles"—not the full density—which enters the emissivity calculations. The other key property for these estimates is the is the plasma frequency ω_p, which characterizes the propagation of photons in the degenerate systems. For ultrarelativistic systems, $\omega_p^2 = \frac{4\alpha\mu^2}{3\pi}$, while for nonrelativistic systems, $\omega_p^2 = \frac{4\pi\alpha n}{m}$. The plasma frequency can be thought as an effective mass for the photon: only photons with energy larger than this mass can propagate outside of the system. Photons with $\omega < \omega_p$ are "off shell" or "virtual": these can only propagate for a short period of time/distance $\sim\omega_p^{-1}$ before they decay.

To derive the analogues of Eqs. (8) and (9) for the denser regions, we must start with (3), but insert the proper form for the expression for $n_1(z, T)n_2(z, T)$, including these effects

$$n_1(z, T)n_2(z, T) \to 4 \int \frac{d^3 p_1}{(2\pi)^3} \frac{d^3 p_2}{(2\pi)^3} \times \frac{\theta(\tilde{\epsilon}_{p_1} + \tilde{\epsilon}_{p_2} - \omega)\theta(\omega - \omega_p)}{\left(1 + e^{\frac{\epsilon_{p_2} - \mu(z)}{T}}\right)\left(1 + e^{\frac{\epsilon_{p_2} - \mu(z)}{T}}\right)}, \tag{A14}$$

where $\tilde{\epsilon}_{p_1}$ and $\tilde{\epsilon}_{p_2}$ are the colliding positron quasiparticle energies above the Fermi surface. The factor of $\theta(\tilde{\epsilon}_{p_1} + \tilde{\epsilon}_{p_2} - \omega)$ accounts for energy conservation: the initial energy must be larger than the energy of emitted photon; and the factor $\theta(\omega - \omega_p)$ accounts for the effects of the plasma frequency. Only photons with $\omega > \omega_p$ can propagate in the dense media: photons with smaller energies will be absorbed on distances $\sim\omega_p^{-1}$. As we shall see, this leads to an exponential suppression of emission $\sim \exp(-\omega_p/T)$ when $\omega_p \gg T$. As such, we have omitted the aforementioned LPM suppression and additional Pauli blocking effects, as these are comparatively insignificant.

The integral in (A14) can be estimated from

$$\int d\tilde{\epsilon}_{p_1} d\tilde{\epsilon}_{p_2} \frac{\theta(\tilde{\epsilon}_{p_1} + \tilde{\epsilon}_{p_2} - \omega)\theta(\omega - \omega_p)}{(1 + e^{\tilde{\epsilon}_{p_1}/T})(1 + e^{\tilde{\epsilon}_{p_2}/T})} \sim T^2 e^{-\omega_p/T}$$

to give

$$n_1(z, T)n_2(z, T) \sim \frac{p_F^2 \mu^2(z) T^2}{\pi^4} \exp\left(-\frac{\omega_p(z)}{T}\right). \tag{A15}$$

The main point is that ω_p grows with the density as $\omega_p \sim \sqrt{n}$ in the nonrelativistic regimes, and as $\omega_p \sim \sqrt[3]{n}$ in the relativistic regimes. This leads to an exponential suppression of emission from the dense regions of the nugget. The suppression is lifted only when $\omega_p \sim T$, which occurs (as can be verified numerically) only when the densities are sufficiently low that the Boltzmann approximation is valid. Our estimates (8) and (9), which are based on this approximation, are thus justified.

One can estimate that the plasma frequency ω_p is a few eV for densities (A17) typical of the Boltzmann regime. Given our previous discussion, one might ask: How can low-energy photons $\omega < \omega_p$ still be emitted? The reason is that, although these photons would be reabsorbed in infinite matter, this reabsorption happens on a length scale of ω_p^{-1}. At the typical densities in the Boltzmann regime, $\omega_p^{-1} \sim 0.3 \cdot 10^{-5}$ cm is sufficiently large compared with the nugget size that many of these photons will have left the nugget before being reabsorbed. One can interpret this effect as a decay of a quasistationary state in quantum mechanics through the tunnelling process, where the barrier has a size comparable with inverse energy of the quasistationary state. The suppression factor only becomes sufficiently large when $\omega_p > L^{-1}$. Then the reabsorption happens well before the photons emerge. This is the origin of the primary suppression in the very dense regions we have just discussed.

5. Heat capacity

Here, we make a rough estimate of the heat capacity of the nuggets. At eV temperatures, the only modes that can be excited are the neutral Nambu-Goldstone superfluid mode, which contributes as T^3, and the gapless positrons at their Fermi surface of $p_F \approx 10$ MeV. It is the gapless positrons that will dominate the low-temperature heat capacity of the nuggets.

To estimate the heat capacity, we simply count the number of low-energy modes. The heat-capacity density for a single mode with dispersion ϵ_p is

$$c_V = \int \frac{d^3 p}{(2\pi)^3} \epsilon_p \frac{d}{dT}\left(\frac{1}{e^{\epsilon_p/T} \pm 1}\right). \tag{A16}$$

The total heat capacity $C_V \approx V c_V$ is obtained by integrating this over the volume V. Here are some approximate contributions to the heat capacity from the nuclear core, assuming the density $\rho \approx 5\ \mathrm{GeV/fm^3}$ is a few times nuclear density so that $V \approx 26B\ \mathrm{GeV}^{-3}$, and taking $T \approx 1$ eV:

Single Boson: For a single boson, $\epsilon_p = p/3$. This gives $c_V \approx 35T^3$ and a total contribution of $C_V \sim 10^{-24}B$.

Two Fermions: For a pair of positrons with Fermi surface p_F we have $\epsilon_p \approx v|p - p_F|$, which gives $c_V \approx p_F^2 T/(3v)$ and a total contribution of $C_V \sim 10^{-12}B$, where we take the Fermi velocity $v \approx c$ for relativistic systems and $p_F \approx 10$ MeV.

In the complete absence of positrons, the heat capacity of a medium nugget $B \approx 10^{24}$ would be about unity, meaning that the addition of 1 GeV of thermal energy would raise the temperature by a GeV, however, even if the core of the nuggets is completely neutral (for example, if the color-flavor–locked (CFL) phase were realized with equal numbers of up, down, and strange quarks), the surface of the quark matter core will still be charged, requiring a large number of positrons in the electrosphere to neutralize the objects.

Once one includes the contribution of the positrons, however, the heat capacity becomes much larger, and the input of 1 GeV of energy will only effect a very small change in temperature. For example, if positrons are present in the core, even the very smallest possible nuggets[12] $B \sim 10^{21}$ would heat by only an eV given 1 GeV of energy. We expect the nuggets have a typical size of $B \sim 10^{25-27}$ [18], so nuggets would require many collisions in order to reach the eV temperature scales at which they would thermally equilibrate by balancing the annihilation energy input with the thermal radiation (14).

Note, however, that while the heat capacity is large in the sense that many collisions are required to heat the nuggets to the equilibrium eV scale, it is small enough to allow the nuggets to heat to this temperature quickly on galactic scales. To see this, consider the total annihilation rate (12).

$$P_{\rm ann} = \frac{dE}{dt} = AF_{\rm ann} \sim \left(\frac{B}{10^{25}}\right)^{2/3} \frac{\rm GeV}{\rm s}, \tag{A17}$$

where A is the surface area of the nuggets

$$A = 4\pi\left(\frac{3B\ {\rm GeV}}{4\pi\rho}\right)^{2/3} = 1.65B^{2/3}\ {\rm fm}^2. \tag{A18}$$

Therefore, the typical time between annihilations is on the order of 1 per second for nuggets with $B \sim 10^{25}$. A nugget with positrons throughout (corresponding to a larger heat capacity) would require

[12] The constraint that the average $B > 10^{21}$ is quite strong and comes from negative terrestrial based search results [47].

$$t_{\rm heat} \sim \frac{1\ {\rm eV} \cdot C_V}{P_{\rm ann}} \sim \left(\frac{B}{10^{25}}\right)^{1/3} \text{hours} \tag{A19}$$

to reach eV temperatures. Thus, the nuggets reach their equilibrium quite rapidly once they enter a region of high visible matter density.

6. Line-of-sight integration

A comparison between the direct emissions B.1-B. 4 is facilitated by the fact that they depend on the same line-of-sight average (15)

$$\Phi_{511,\text{X-ray,etc.}} \propto \int d\Omega dl n_{\rm VM}(l) n_{\rm DM}(l).$$

In principle, comparing these with thermal emissions is more difficult because the line-of-sight integral has an additional dependence on the visible matter distribution. This arises because the emissivity depends on the temperature T of the nuggets, which in turn depends on the annihilation rate through the thermal equilibrium condition (14).

As can be seen from (9), (11), and (12), the temperature of the nuggets is related to the local visible matter density as $T(n_{\rm VM}) \propto n_{\rm VM}^{4/17}$, whereas from (8), we see that the microwave emission scales as $\phi(T) \propto T^{13/4} \propto n_{\rm VM}^{13/17} = n_{\rm VM} n_{\rm VM}^{-4/17}$. Thus, the microwave emission depends on the line-of-sight integral

$$\Phi_{\rm WMAP} \propto \int d\Omega dl [n_{\rm VM}(l) n_{\rm DM}(l)] n_{\rm VM}^{-4/17}(l). \tag{A20}$$

Of course, one must also account for rescattering, absorption and other effects along the line of sight, but our point is that the difference between the line-of-sight averaging of the "direct" emissions and the "thermal" emissions is highly suppressed—depending only on $n_{\rm VM}^{-4/17}$. This allows us to directly compare all emissions in order to estimate the order of magnitude for the energy budget discussed in Sec. IVA. Once the other calculations and observations are better constrained, one might be able to search for this type of scaling to test our proposal more rigorously.

[1] A. R. Zhitnitsky, J. Cosmol. Astropart. Phys. 10 (2003) 010.
[2] D. H. Oaknin and A. Zhitnitsky, Phys. Rev. D **71**, 023519 (2005).
[3] A. Zhitnitsky, Phys. Rev. D **74**, 043515 (2006).
[4] M. M. Forbes and A. R. Zhitnitsky, J. Cosmol. Astropart. Phys. 01 (2008) 023.
[5] D. P. Finkbeiner, Astrophys. J. **614**, 186 (2004).
[6] D. P. Finkbeiner, G. I. Langston, and A. H. Minter, Astrophys. J. **617**, 350 (2004).
[7] D. P. Finkbeiner, arXiv:astro-ph/0409027.
[8] D. Hooper, G. Zaharijas, D. P. Finkbeiner, and G. Dobler, Phys. Rev. D **77**, 043511 (2008).
[9] G. Dobler and D. P. Finkbeiner, arXiv:0712.1038.
[10] D. Hooper, Phys. Rev. D **77**, 123523 (2008).
[11] E. Witten, Phys. Rev. D **30**, 272 (1984).
[12] R. D. Peccei and H. R. Quinn, Phys. Rev. D **16**, 1791 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
[13] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman,

A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B166**, 493 (1980).
[14] M. Dine, W. Fischler, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); A. R. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 260 (1980).
[15] M. Srednicki, arXiv:hep-th/0210172; K. van Bibber and L. J. Rosenberg, Phys. Today **59N8**, 30 (2006); S. J. Asztalos, L. J. Rosenberg, K. van Bibber, P. Sikivie, and K. Zioutas, Annu. Rev. Nucl. Part. Sci.**56**, 293 (2006).
[16] D. Kharzeev and A. Zhitnitsky, Nucl. Phys. **A797**, 67 (2007).
[17] C. Amsler *et al.* (Particle Data Group Collaboration), Phys. Lett. B **667**, 1 (2008).
[18] M. M. Forbes and A. R. Zhitnitsky (unpublished).
[19] E. T. Herrin, D. C. Rosenbaum, and V. L. Teplitz, Phys. Rev. D **73**, 043511 (2006).
[20] E. S. Abers, A. K. Bhatia, D. A. Dicus, W. W. Repko, D. C. Rosenbaum, and V. L. Teplitz, arXiv:0712.4300.
[21] E. W. Kolb and M. S. Turner, *The Early Universe* (Westview Press, Boulder, Colorado, 1994).
[22] A. Bottino, F. Donato, N. Fornengo, and S. Scopel, Phys. Rev. D **77**, 127301 (2008).
[23] J. Knödlseder *et al.*, Astron. Astrophys. **411**, L457 (2003).
[24] J. F. Beacom and H. Yuksel, Phys. Rev. Lett. **97**, 071102 (2006).
[25] H. Yuksel, Nucl. Phys. B, Proc. Suppl. **173**, 83 (2007).
[26] D. H. Oaknin and A. R. Zhitnitsky, Phys. Rev. Lett. **94**, 101301 (2005).
[27] A. Zhitnitsky, Phys. Rev. D **76**, 103518 (2007).
[28] K. Lawson and A. R. Zhitnitsky, J. Cosmol. Astropart. Phys. 01 (2008) 022.
[29] M. P. Muno *et al.*, Astrophys. J. **613**, 326 (2004).
[30] M. Leventhal, C. J. MacCallum, and P. D. Stang, Astrophys. J., Lett. Ed. **225**, L11 (1978).
[31] R. L. Kinzer *et al.*, Astrophys. J. **559**, 282 (2001).
[32] P. Jean, J. Knödlseder, W. Gillard, N. Guessoum, K. Ferriere, A. Marcowith, V. Lonjou, and J. P. Roques, Astron. Astrophys. **445**, 579 (2006).
[33] J. Knödlseder *et al.*, Astron. Astrophys. **441**, 513 (2005)
[34] G. Weidenspointner *et al.*, Astron. Astrophys. **450**, 1013 (2006).
[35] D. Hooper, D. P. Finkbeiner, and G. Dobler, Phys. Rev. D **76**, 083012 (2007).
[36] B. K. Fedyushin, Zh. Eksp. Teor. Fiz. **22**, 140 (1952), uCRL-Trans-10837; A. I. Achiezer and V. B. Berestecky, *Quantum Electrodynamics* (Nauka, Moscow, 1969); E. Haug, Sol. Phys. **178**, 341 (1998).
[37] T. Padmanabhan, *Theoretical Astrophysics* (Cambridge University Press, Cambridge, 2001), Vol. 3.
[38] K. Ebisawa *et al.*, Astrophys. J. **635**, 214 (2005)
[39] C. Alcock, E. Farhi, and A. Olinto, Astrophys. J. **310**, 261 (1986).
[40] C. Kettner, F. Weber, M. K. Weigel, and N. K. Glendenning, Phys. Rev. D **51**, 1440 (1995).
[41] J. Madsen, Phys. Rev. Lett. **87**, 172003 (2001).
[42] V. V. Usov, Phys. Rev. D **70**, 067301 (2004).
[43] A. W. Steiner, S. Reddy, and M. Prakash, Phys. Rev. D **66**, 094007 (2002).
[44] V. V. Usov, T. Harko, and K. S. Cheng, Astrophys. J. **620**, 915 (2005).
[45] D. T. Cumberbatch, J. Silk, and G. D. Starkman, Phys. Rev. D **77**, 063522 (2008).
[46] L. D. Landau and E. M. Lifshitz, *Statistical Physics: Part I*, Course of Theoretical Physics (Butterworth-Heinemann, Oxford, 2000, c1980), 3rd ed, Vol. 5.
[47] I. A. Belolaptikov *et al.*, arXiv:astro-ph/9802223.