Collective neutrino oscillations in matter and *CP* violation

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We explore *CP* violation effects on the neutrino propagation in dense environments, such as in core-collapse supernovae, where the neutrino self-interaction induces nonlinear evolution equations. We demonstrate that the electron (anti)neutrino fluxes are not sensitive to the *CP* violating phase if the muon and tau neutrinos interact similarly with matter. On the other hand, we numerically show that new features arise, because of the nonlinearity and the flux dependence of the evolution equations, when the muon and tau neutrinos have different fluxes at the neutrinosphere (due to loop corrections or physics beyond the standard model). In particular, the electron (anti)neutrino probabilities and fluxes depend upon the *CP* violating phase. We also discuss the *CP* effects induced by radiative corrections to the neutrino refractive index.

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I. INTRODUCTION

One of the major open issues in neutrino physics is the possible existence of *CP* violation. As with the recent crucial experimental discoveries on neutrino oscillations, the answer to this question has fundamental implications in high-energy physics, astrophysics, and cosmology, e.g. to understand the matter versus antimatter asymmetry in the Universe. The observation that the weak interaction violates the *CP* symmetry in the quark sector was first established in 1964 [1]. Future strategies to search for *CP* violation in the lepton sector depend upon the actual value of one yet unknown neutrino oscillation parameter, i.e. θ_{13} [2], and require long-term accelerator projects producing very intense neutrino beams [3]. It is therefore essential to explore alternative avenues to get clues on this fundamental question, such as indirect effects in dense environments like core-collapse supernovae.

Core-collapse supernovae emit about 10^{53} erg as neutrinos of all flavors during their rapid gravitational collapse. Such neutrinos might play a role in the two major supernova unsolved problems, namely, understanding how the explosion finally occurs and where the nucleosynthesis of the heavy elements, produced during the *r* process, takes place. While neutrinos from a massive star were first observed during the SN1987A explosion, future observations of (extra)galactic or relic supernova neutrinos will help unravel supernova physics and/or unknown neutrino properties. For example, in [4–7] the imprint of the shock wave on the neutrino time signal is investigated, while avenues for extracting information on the third neutrino mixing angle are discussed in [8,9]. These searches require advances in the modeling of supernova dynamics, our understanding of neutrino propagation in dense environments, and our knowledge of neutrinos.

Impressive developments are ongoing in our understanding of neutrino propagation in dense matter. While solar experiments [10–13] have beautifully confirmed the oscillation enhancement induced by the coupling with matter—the Mikheev-Smirnov-Wolfenstein, or MSW, effect [14,15]—recent theoretical investigations have shown that the inclusion of the neutrino self-interactions in dense environments introducing a nondiagonal neutrino refractive index [16] gives rise to a wealth of new phenomena, as first pointed out in [17]. Various regimes have been identified: the synchronized one [18,19], the bipolar oscillations [19,20], and the spectral split phenomenon [21,22]. Since numerical calculations have become more involved, analytical treatments for the three flavor case are being proposed (see e.g. [23]). The importance of the loop corrections to the neutrino refractive index, the $V_{\mu\tau}$ potential [24], is underlined in [25]. Moreover constraints on neutrino mixing from shock reheating and the *r*-process nucleosynthesis, including the neutrino-neutrino interaction, are investigated in [26,27]. The impact of the neutrino-neutrino interaction on the electron fraction relevant for the *r* process is investigated in [28,29].

In a previous work [30] we have investigated the *CP* effects on the neutrino fluxes and on the electron fraction in a supernova (relevant for the *r* process), as well as the possible impact on the supernova neutrino signal in an observatory on Earth. In particular we have shown analytically that no effects can be found on the electron (anti) neutrino fluxes, when muon and tau neutrinos have the same fluxes at the neutrinosphere, while significant effects are obtained numerically on the fluxes when they differ. The calculations in [30] are obtained considering interaction with matter at tree level only. In this work we explore for the first time *CP* violation effects on the neutrino propagation in dense environments, including the standard MSW effect, the neutrino self-interactions, and the $V_{\mu\tau}$ refractive index. We first show analytically that if the muon and tau neutrinos interact similarly, the electron (anti) neutrinos are not sensitive to the *CP* violating phase, even in the presence of neutrino self-interactions. This result is general and valid for any matter density profile and/or initial neutrino luminosity. We present numerical results, obtained within the three flavor formalism, of the neutrino oscillation probabilities and fluxes within the star. In particular we show that both the probabilities and the fluxes become sensitive to the *CP* violating phase if muon and tau neutrinos interact differently with matter (e.g. because of loop corrections or physics beyond the standard model.) The paper is structured as follows. Section II presents the theoretical framework for describing the neutrino propagation, including the coupling with matter as well as the neutrino-neutrino interaction. Section III gives the analytical and numerical results. Conclusions are drawn in Sec. IV.

II. THEORETICAL FRAMEWORK

In a dense environment the nonlinear coupled neutrino evolution equations with neutrino self-interactions are given by (we follow the formalism of Ref. [31])

$$i\frac{d}{dt}\psi_{\nu_{\underline{\alpha}}} = [H_0 + H_m + H_{\nu\nu}]\psi_{\nu_{\underline{\alpha}}}, \tag{1}$$

where $\psi_{\nu_{\underline{\alpha}}}$ denote a neutrino created at the neutrinosphere initially in a flavor state $\alpha = e, \mu, \tau$, $H_0 = UH_{\mathrm{vac}}U^\dagger$ is the Hamiltonian describing the vacuum oscillations $H_{\mathrm{vac}} = \mathrm{diag}(E_1, E_2, E_3)$, $E_{i=1,2,3}$ being the energies of the neutrino mass eigenstates, and U the unitary Maki-Nakagawa-Sakata-Pontecorvo matrix

$$\begin{aligned} U &= T_{23}T_{13}T_{12} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \\ &\quad\times\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \end{aligned} \tag{2}$$

$c_{ij} = \cos\theta_{ij}$ ($s_{ij} = \sin\theta_{ij}$) with θ_{12}, θ_{23}, and θ_{13} the three neutrino mixing angles. The presence of a Dirac δ phase in Eq. (2) renders U complex and introduces a difference between matter and antimatter. The U matrix relates the mass and the flavor basis

$$\psi_{\nu_{\underline{\alpha}}} = \sum_i U_{\alpha i}\psi_i. \tag{3}$$

The neutrino interaction with matter is taken into account through an effective Hamiltonian which corresponds, at tree level, to the diagonal matrix $H_m = \mathrm{diag}(V_c, 0, 0)$, where the $V_c(x) = \sqrt{2}G_F N_e(x)$ potential, due to the charged-current interaction, depends on the electron density $N_e(x)$ (note that the neutral current interaction introduces an overall phase only).

The neutrino self-interaction term is

$$H_{\nu\nu} = \sqrt{2}G_F \sum_\alpha \sum_{\nu_\alpha, \bar\nu_\alpha} \int \rho_{\nu_{\underline{\alpha}}}(\mathbf{q}')(1 - \hat{\mathbf{q}}\cdot\hat{\mathbf{q}}')dn_\alpha dq', \tag{4}$$

where G_F is the Fermi coupling constant, $\rho = \rho_{\nu_{\underline{\alpha}}}$ $(-\rho^*_{\nu_{\underline{\alpha}}})$ is the density matrix for neutrinos (antineutrinos)

$$\rho_{\nu_{\underline{\alpha}}} = \begin{pmatrix} |\psi_{\nu_e}|^2 & \psi_{\nu_e}\psi^*_{\nu_\mu} & \psi_{\nu_e}\psi^*_{\nu_\tau} \\ \psi^*_{\nu_e}\psi_{\nu_\mu} & |\psi_{\nu_\mu}|^2 & \psi_{\nu_\mu}\psi^*_{\nu_\tau} \\ \psi^*_{\nu_e}\psi_{\nu_\tau} & \psi^*_{\nu_\mu}\psi_{\nu_\tau} & |\psi_{\nu_\tau}|^2 \end{pmatrix}, \tag{5}$$

$\mathbf{q}$ ($\mathbf{q}'$) denotes the momentum of the neutrino of interest (background neutrino), and dn_α is the differential number density. In the single-angle approximation, which assumes that the neutrinos are all emitted with the same angle, i.e. $\rho(\mathbf{q}) = \rho(q)$, Eq. (5) reduces to

$$\begin{aligned} H_{\nu\nu} = \frac{\sqrt{2}G_F}{2\pi R_\nu^2} D(r/R_\nu) \sum_\alpha \int [\rho_{\nu_{\underline{\alpha}}}(q')L_{\nu_{\underline{\alpha}}}(q') \\ - \rho^*_{\bar\nu_{\underline{\alpha}}}(q')L_{\bar\nu_{\underline{\alpha}}}(q')]dq' \end{aligned} \tag{6}$$

with the geometrical factor

$$D(r/R_\nu) = \frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{R_\nu}{r}\right)^2}\right]^2, \tag{7}$$

where the radius of the neutrino sphere is $R_\nu = 10$ km, and

$$L_{\nu_{\underline{\alpha}}}(r, E_\nu) = \frac{L^0_{\nu_{\underline{\alpha}}}}{T^3_{\nu_{\underline{\alpha}}}\langle E_{\nu_{\underline{\alpha}}}\rangle F_2(\eta)} \frac{E^2_{\nu_{\underline{\alpha}}}}{1 + \exp(E_{\nu_{\underline{\alpha}}}/T_{\nu_{\underline{\alpha}}} - \eta)}, \tag{8}$$

where $F_2(\eta)$ is the Fermi integral and $L^0_{\nu_{\underline{\alpha}}}$ and $T_{\nu_{\underline{\alpha}}}$ are the luminosity and temperature at the neutrinosphere.

III. *CP* EFFECTS IN THE PRESENCE OF THE ν-ν INTERACTION AND $V_{\mu\tau}$ REFRACTIVE INDEX

A. Analytical results

One way to study under which conditions the electron (anti)neutrino survival probabilities depend upon the *CP* violating phase δ is to demonstrate that the δ dependence of the total Hamiltonian $H_T = H_0 + H_m + H_{\nu\nu}$ factorizes as follows [30]:

$$\tilde{H}_T(\delta) = S\tilde{H}_T(\delta = 0)S^\dagger \tag{9}$$

in the T_{23} basis which is

$$\begin{aligned} \tilde\psi_\mu &= \cos\theta_{23}\psi_\mu - \sin\theta_{23}\psi_\tau, \\ \tilde\psi_\tau &= \sin\theta_{23}\psi_\mu + \cos\theta_{23}\psi_\tau. \end{aligned} \tag{10}$$

Here the whole dependence is in the unitary diagonal matrix $S = \mathrm{diag}(1, 1, e^{-i\delta})$. In fact, it is straightforward to show that for any such Hamiltonian the corresponding evolution operator also factorizes as

$$\tilde{U}(\delta) = S^\dagger \tilde{U}(\delta = 0) S \leftrightarrow \tilde{H}(\delta) = S^\dagger \tilde{H}(\delta = 0) S. \quad (11)$$

As a consequence one can demonstrate that the electron (anti)neutrino survival probabilities satisfy $P(\nu_e \to \nu_e, \delta \neq 0) = P(\nu_e \to \nu_e, \delta = 0)$ and that the appearance probabilities satisfy

$$\begin{aligned} &P(\nu_\mu \to \nu_e, \delta \neq 0) + P(\nu_\tau \to \nu_e, \delta \neq 0) \\ &\quad = P(\nu_\mu \to \nu_e, \delta = 0) + P(\nu_\tau \to \nu_e, \delta = 0). \end{aligned} \quad (12)$$

Since the ν_e ($\bar{\nu}_e$) fluxes are given by

$$\begin{aligned} \phi_{\nu_e}(\delta) &= L_{\nu_{\underline{e}}} P(\nu_e \to \nu_e) + L_{\nu_{\underline{\mu}}} P(\nu_\mu \to \nu_e) \\ &\quad + L_{\nu_{\underline{\tau}}} P(\nu_\tau \to \nu_e), \end{aligned} \quad (13)$$

where $L_{\nu_{\underline{x}}}$ are the neutrino fluxes at the neutrinosphere, from Eqs. (12) and (13) one can see that ϕ_{ν_e} ($\phi_{\bar{\nu}_e}$) are not sensitive to the CP violating phase if muon and tau neutrinos interact with matter in the same way (i.e. $L_{\nu_{\underline{\mu}}} = L_{\nu_{\underline{\tau}}}$). This is demonstrated in [30] in the case of the standard MSW case only, i.e. $H = H_0 + H_m$. Note that a similar conclusion is drawn in [32], using a different procedure. Besides, from Eqs. (12) and (13), one can see that if the condition $L_{\nu_{\underline{\mu}}} \neq L_{\nu_{\underline{\tau}}}$ is relaxed, then ϕ_{ν_e} and $\phi_{\bar{\nu}_e}$ become dependent on delta, as first pointed out in [30].

We now show that Eq. (9) is indeed satisfied for the total Hamiltonian of Eq. (1) including the nonlinear $H_{\nu\nu}$ term of Eq. (4). Let us start with the Liouville–von Neumann equation for the density matrix ($\hbar = 1$):

$$i\frac{d\rho_{\nu_{\underline{\alpha}}}(\delta)}{dt} = [U H_{vac} U^\dagger + H_m + H_{\nu\nu}(\delta), \rho_{\nu_{\underline{\alpha}}}(\delta)]. \quad (14)$$

To prove our result (that the CP violating phase can be factorized out of the total Hamiltonian which includes $H_{\nu\nu}$), one has to rotate in the T_{23} basis, since the S matrix contained in T_{13} (which can be rewritten as $T_{13} = S^\dagger T_{13}^0 S$) does not commute with T_{23}. We then obtain

$$\begin{aligned} i\frac{dS\tilde{\rho}_{\nu_{\underline{\alpha}}}(\delta)S^\dagger}{dt} &= [T_{13}^0 T_{12} H_{\rm vac} T_{12}^\dagger T_{13}^{0\dagger} + H_m \\ &\quad + S\tilde{H}_{\nu\nu}(\delta)S^\dagger, S\tilde{\rho}_{\nu_{\underline{\alpha}}}(\delta)S^\dagger], \end{aligned} \quad (15)$$

where

$$\tilde{\rho}_{\nu_{\underline{\alpha}}} = \begin{pmatrix} P(\nu_\alpha \to \nu_e) & \psi_{\nu_e}\tilde{\psi}^*_{\nu_\mu} & \psi_{\nu_e}\tilde{\psi}^*_{\nu_\tau} \\ \psi^*_{\nu_e}\tilde{\psi}_{\nu_\mu} & P(\nu_\alpha \to \tilde{\nu}_\mu) & \tilde{\psi}_{\nu_\mu}\tilde{\psi}^*_{\nu_\tau} \\ \psi^*_{\nu_e}\tilde{\psi}_{\nu_\tau} & \tilde{\psi}^*_{\nu_\mu}\tilde{\psi}_{\nu_\tau} & P(\nu_\alpha \to \tilde{\nu}_\tau) \end{pmatrix}. \quad (16)$$

Let us now consider the evolution equation of the linear combination $\sum_{\nu_\alpha} L_{\nu_{\underline{\alpha}}} S\tilde{\rho}_{\nu_{\underline{\alpha}}}(\mathbf{q}, \delta)S^\dagger$ at a given momentum $\mathbf{q}$. At the initial time, this quantity reads, in the T_{23} basis of Eq. (10), as

$$\begin{aligned} &\sum_{\nu_\alpha} L_{\nu_{\underline{\alpha}}} S\tilde{\rho}_{\nu_{\underline{\alpha}}}(\mathbf{q}, \delta, t=0)S^\dagger \\ &= \begin{pmatrix} L_{\nu_{\underline{e}}} & 0 & 0 \\ 0 & c_{23}^2 L_{\nu_{\underline{\mu}}} + s_{23}^2 L_{\nu_{\underline{\tau}}} & c_{23}s_{23}e^{-i\delta}(L_{\nu_{\underline{\mu}}} - L_{\nu_{\underline{\tau}}}) \\ 0 & c_{23}s_{23}e^{i\delta}(L_{\nu_{\underline{\mu}}} - L_{\nu_{\underline{\tau}}}) & s_{23}^2 L_{\nu_{\underline{\mu}}} + c_{23}^2 L_{\nu_{\underline{\tau}}} \end{pmatrix}. \end{aligned} \quad (17)$$

One immediately sees that this quantity does not depend on δ if and only if $L_{\nu_{\underline{\mu}}} = L_{\nu_{\underline{\tau}}}$. Moreover, the total Hamiltonian of Eq. (15) is independent of δ at initial time since $T_{13}^0 T_{12} H_{\rm vac} T_{12}^\dagger T_{13}^{0\dagger} + H_m$ does not depend on δ (at any time) and

$$\begin{aligned} S\tilde{H}_{\nu\nu}(t=0, \delta)S^\dagger &= \sqrt{2}G_F \sum_\alpha \int (1 - \hat{\mathbf{q}} \cdot \hat{\mathbf{q}}') \\ &\quad \times [S\tilde{\rho}_{\nu_{\underline{\alpha}}}(t=0, q')S^\dagger L_{\nu_{\underline{\alpha}}}(q') \\ &\quad - S\tilde{\rho}^*_{\bar{\nu}_{\underline{\alpha}}}(t=0, q')S^\dagger L_{\bar{\nu}_{\underline{\alpha}}}(q')]dq' \end{aligned} \quad (18)$$

is equal to $\tilde{H}_{\nu\nu}(t=0, \delta=0)$ initially when $L_{\nu_{\underline{\mu}}} = L_{\nu_{\underline{\tau}}}$ (and $L_{\bar{\nu}_{\underline{\mu}}} = L_{\bar{\nu}_{\underline{\tau}}}$). In that case, one can see by recurrence from the Liouville–von Neumann equation [Eq. (14)] that the evolution of the term $\sum_{\nu_\alpha} L_{\nu_{\underline{\alpha}}} S\tilde{\rho}_{\nu_{\underline{\alpha}}}(\mathbf{q}, \delta)S^\dagger$ is exactly the same as the term $\sum_{\nu_\alpha} L_{\nu_{\underline{\alpha}}} \tilde{\rho}_{\nu_{\underline{\alpha}}}(\mathbf{q}, \delta = 0)$, since they have the same initial conditions (for any $\mathbf{q}$) and the same evolution equations. Indeed, with the exact same relation applying at the same time for the antineutrino case (where the sign of δ has to be changed), one simultaneously obtains that at any time:

$$\tilde{H}_{\nu\nu}(\delta) = S\tilde{H}_{\nu\nu}(\delta = 0)S^\dagger, \quad (19)$$

hence,

$$\tilde{H}_T(\delta) = S\tilde{H}_T(\delta = 0)S^\dagger. \quad (20)$$

Note that the derivation holds both for the multiangle case Eq. (4) and the single-angle case Eq. (6).

This implies that the (anti)electron neutrino survival probability is independent of δ and Eq. (12) is valid; therefore $\phi_{\nu_e}(\delta) = \phi_{\nu_e}(\delta = 0)$ and $\phi_{\bar{\nu}_e}(\delta) = \phi_{\bar{\nu}_e}(\delta = 0)$, even considering the presence of neutrino-neutrino interaction, if muon and tau neutrino fluxes at the neutrinosphere are equal. When the fluxes $L_{\nu_{\underline{\mu}}}$ and $L_{\nu_{\underline{\tau}}}$ are different, the derivation does not hold anymore, since $\sum_{\nu_\alpha} L_{\nu_{\underline{\alpha}}} S\tilde{\rho}_{\nu_{\underline{\alpha}}}(\mathbf{q}, \delta)S^\dagger$ initially depend on δ.

In the case where radiative corrections to the neutrino scattering are considered, there is an extra term in $H_m = \mathrm{diag}(V_c, 0, V_{\mu\tau})$. For the corresponding Hamiltonian the δ dependence cannot be factorized anymore and in general nothing prevents the electron (anti)neutrino survival probabilities to be sensitive to the CP violating phase.[1] These cases will be studied in the following.

B. Numerical results

The main goal of this section is to investigate numerically the effects that can arise when the factorization Eq. (9) is not satisfied. In particular this occurs in three cases: (i) if the MSW $H_0 + H_m$ Hamiltonian does not satisfy Eq. (11) because the $V_{\mu\tau}$ refractive index is included; (ii) if the initial conditions [Eq. (17)] of the $H_{\nu\nu}$ Hamiltonian are not independent of δ, which happens when $L_{\nu_{\underline{\mu}}} \neq L_{\nu_{\underline{\tau}}}$, which implies that the neutrino self-interaction term does not follow Eq. (19); (iii) when (i) and (ii) occur. Therefore the results that we present here correspond to the following possibilities:

(a) $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} \neq 0$ with the condition $L_{\nu_{\underline{\mu}}} = L_{\nu_{\underline{\tau}}}$;

(b) $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} = 0$ with the condition $L_{\nu_{\underline{\mu}}} \neq L_{\nu_{\underline{\tau}}}$;

(c) $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} \neq 0$ with the condition $L_{\nu_{\underline{\mu}}} \neq L_{\nu_{\underline{\tau}}}$.

It is important to note that a new feature arises in the (a), (b), and (c) cases: the $P(\nu_e \rightarrow \nu_e)$ becomes dependent on δ. (It is a well known fact that the electron survival probability does not depend on CP violation in a vacuum and in the presence of the interaction of matter at tree level.) In the following we investigate the CP effects by varying $\delta \in [0°, 180°]$ while we show results for $\delta = 180°$ when at such value the effects are maximal.

The numerical results we present are obtained by solving the three flavor evolution equation [Eq. (1)] with a supernova density profile having a $1/r^3$ behavior that fits the numerical simulations shown in [29]. Note that with such a density profile the region of the first 100 km, where the neutrino self-interaction dominates, is well separated from the one of the MSW (high and low) resonances, produced by the interaction with ordinary matter.

The oscillation parameters are fixed at the present best fit values [34], namely, $\Delta m^2_{12} = 8 \times 10^{-5}$ eV^2, $\sin^2 2\theta_{12} = 0.83$ and $\Delta m^2_{23} = 3 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{23} = 1$ for the solar and atmospheric differences of the mass squares and mixings, respectively. For the third still unknown neutrino mixing angle θ_{13}, we take either the present upper limit $\sin^2 2\theta_{13} = 0.19$ at 90% C.L. (L) or a very small value of $\sin^2 2\theta_{13} = 3 \times 10^{-4}$ (S) that might be attained at the future (third generation) long-baseline experiments [3]. To include the neutrino-neutrino interaction we use the single-angle approximation of Eqs. (6) and (7) with $R_\nu = 10$ km (considering that all neutrinos are emitted radially). The neutrino fluxes at the neutrinosphere $L_{\nu_{\underline{\alpha}}}$ [Eq. (8)] are taken as Fermi-Dirac distributions with typical average energies of $\langle E_{\nu_e} \rangle = 10$ MeV, $\langle E_{\bar{\nu}_e} \rangle = 15$ MeV, and $\langle E_{\nu_x} \rangle = 24$ MeV with $\nu_x = \nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau$, unless stated otherwise (the chemical potentials are assumed to be zero for simplicity). We take the neutrino luminosity $L^0_{\nu_{\underline{\alpha}}} = 10^{51}$ erg $\cdot$ s^{-1}.

Our numerical results in three flavors present the collective oscillations induced by the neutrino-neutrino interaction, already discussed in the literature (see e.g. [17–22,25,35]). Figure 1 presents the (anti)neutrino oscillation probabilities within the star. One recognizes the synchronized regime in the first 50 km outside the neutrinosphere R_ν (assumed here to be equal to the neutron-star surface). In this regime the strong neutrino-neutrino interaction makes neutrinos of all energies oscillate with the same frequency so that flavor conversion is frozen, as discussed e.g. in [18,36]. When the neutrino self-interaction term becomes smaller, the ordinary matter term starts to dominate, producing large bipolar oscillations (between 50 and 80 km) that produce strong flavor conversion for both neutrinos and antineutrinos, in particular, for the case of inverted hierarchy, independently of the θ_{13} value [20]. Finally neutrinos show complete (no) flavor conversion for energies larger (smaller) than a characteristic energy $E_c = 7.4$ MeV, due to lepton number conservation [21]. This is known as the spectral split phenomenon (apparent around 150 km in Fig. 1, left). (Note that the effect of the partially nonadiabatic MSW low resonance can be seen at around 270 km in Fig. 1, right.)

The neutrino-neutrino interaction might have an important impact on the neutrino spectra as well. If in the case of normal hierarchy the flavor evolution of both electron neutrinos and antineutrinos is essentially the same as in the case where matter only is included, for the case of inverted hierarchy, important modifications are found compared to the MSW case [37]. While electron neutrinos swap their spectra with muon and tau neutrinos (Fig. 2, left), the electron antineutrinos show a complete spectral swapping (Fig. 2, right). Such behaviors are found for both large and small values of the third neutrino mixing angle, in contrast with the standard MSW effect. (Note that antineutrinos of energies less than 2 MeV have already undergone the MSW low resonance at 200 km, as can be seen from Fig. 2, right.)

Let us now discuss the CP violation effects in the presence of the neutrino-neutrino interaction[2] and of the loop corrections to the neutrino refractive index, with the condition that the muon and tau fluxes at the neutrinosphere are equal ($L_{\nu_\mu} = L_{\nu_\tau}$). Figure 3 shows the ratios of the electron neutrino oscillation probabilities for different

[1]Note that the possible inclusion of nonstandard neutrino interactions in the flavor neutrino mixing, as e.g. in [33], implies that Eq. (9) does not hold anymore.

[2]Note that a comment is made in [38,39] on the δ effects on the neutrino fluxes in the presence of the neutrino self-interaction in a core-collapse supernova.

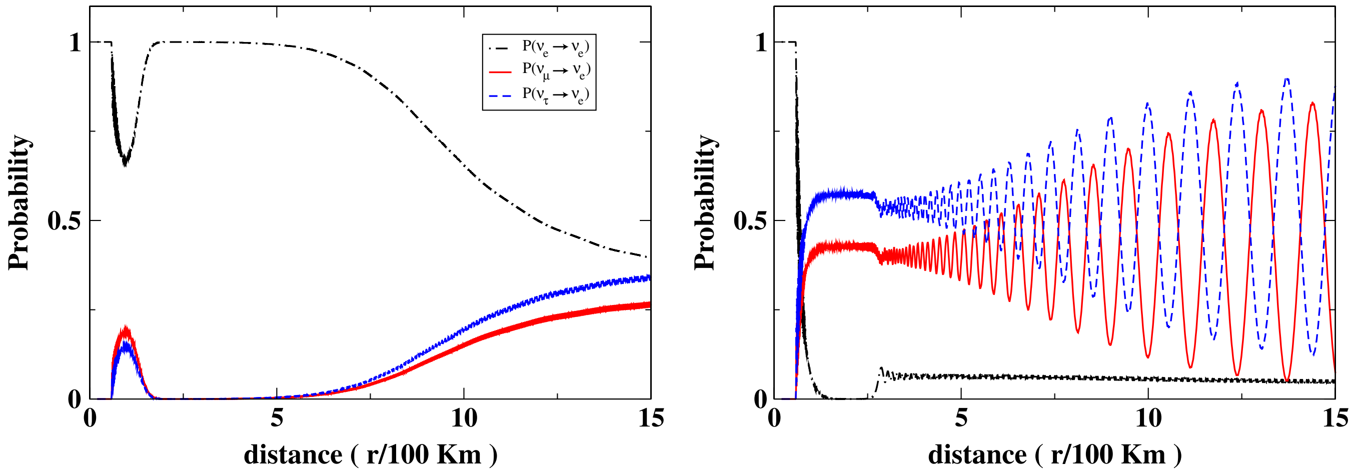

FIG. 1 (color online). Neutrino (left) and antineutrino (right) oscillation probabilities in three flavors, as a function of the distance from the neutron-star surface (10 km), including the neutrino-neutrino interaction and $V_{\mu\tau}$ refractive index. The different curves correspond to electron (anti)neutrinos (dash-dotted line), muon (solid line), and tau (dashed line) (anti)neutrinos. The results are obtained solving Eqs. (1)–(7) numerically for a neutrino energy of 5 MeV as an example. The case of inverted hierarchy and small neutrino mixing angle θ_{13} is shown where the neutrino self-interaction effects are particularly impressive: the regimes of synchronized and bipolar oscillations can be recognized in the first 100 km. In the case of the electron neutrinos (left figure), the spectral split is also apparent.

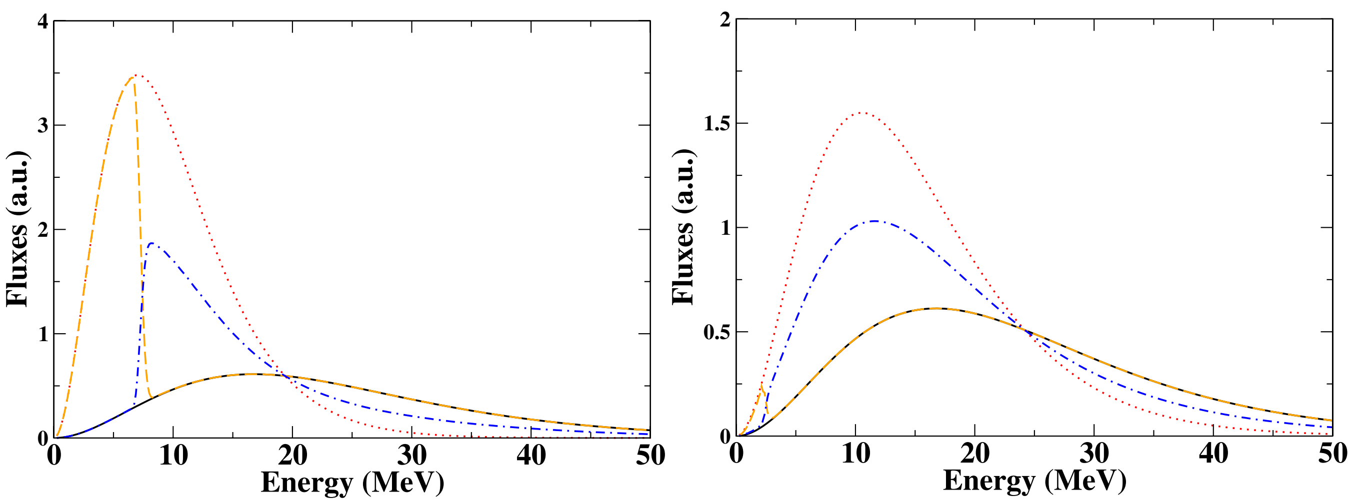

FIG. 2 (color online). Neutrino (left) and antineutrino (right) spectra, at 200 km from the neutron-star surface. The different curves correspond to the original Fermi-Dirac distributions for ν_e (dotted line) and ν_μ (solid line), and the ν_e (dashed line) and ν_μ (dash-dotted line) fluxes after the evolution in the star with the neutrino self-interaction. The results are obtained for an inverted hierarchy and a small third neutrino mixing angle. While neutrinos show a spectral split, antineutrinos undergo full flavor conversion.

δ values, as a function of the distance within the star. A 5 MeV neutrino is taken, as an example. One can see that the δ effects are at the level of 1%. Note that the presence of $H_{\nu\nu}$ with $V_{\mu\tau}$ amplifies these effects that are at the level of less than 0.1% and smaller, when $V_{\mu\tau}$ only is included.[3] One can also see that in the synchronized regime the *CP* effects are "frozen," while they develop with the bipolar oscillations. Similar modifications are also found in the case of electron antineutrinos, with effects up to 10% for low energies (less than 10 MeV). Note that the latter might be partially modified in a multiangle calculation, since it has been shown that the decoherence effects introduced by multiangles modify the electron antineutrino energy spectra, in particular, at low energies [37]. Multiangle decoherence is also discussed in [31,36,41,42]. To predict how these effects modify the numerical results presented in this paper would require a full multiangle calculation.

[3]Note that it was first pointed out in [40] that the inclusion of the $V_{\mu\tau}$ refractive index renders the electron neutrino survival probability slightly δ dependent.

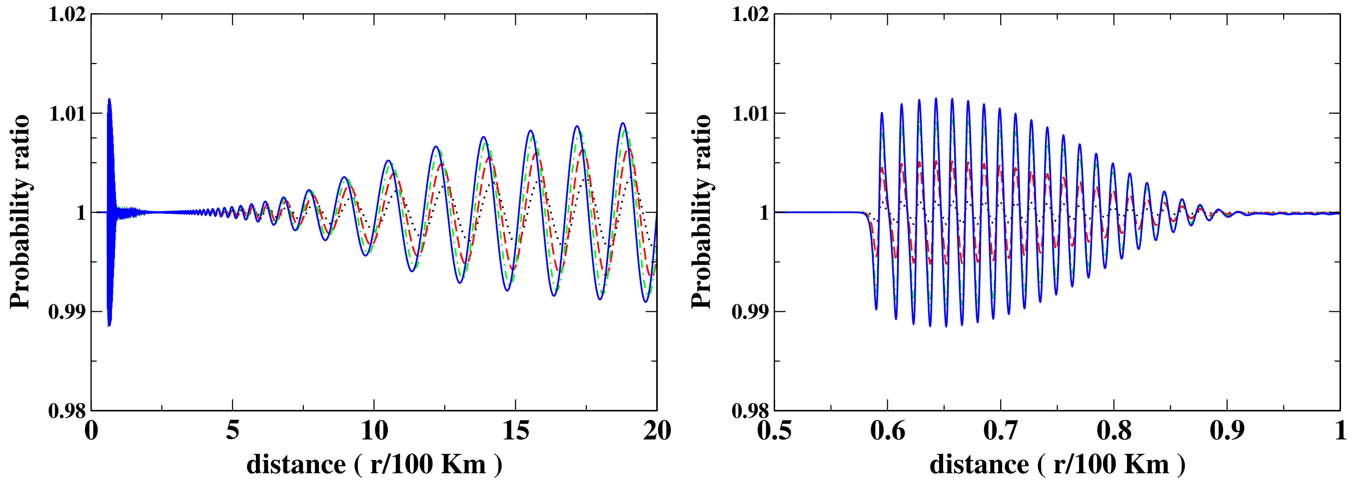

FIG. 3 (color online). Ratios of the electron neutrino oscillation probabilities for a CP violating phase $\delta = 45°$ (dotted line), 90° (dashed line), 135° (dash-dotted line), 180° (solid line) over $\delta = 0°$, as a function of the distance from the neutron-star surface. The left figure shows the ratios up to 2000 km, while the right figure presents the region between 50 to 100 km where collective effects induced by the neutrino self-interaction are maximal. The results correspond to the case of inverted hierarchy and small third neutrino mixing angle, for a neutrino energy of 5 MeV.

The modifications induced by δ on the electron neutrino fluxes are shown in Fig. 4 for the (a), (b), and (c) cases, in comparison with a calculation within the MSW effect at tree level only as investigated in a previous work [30]. Figure 5 shows how the CP effects evolve as a function of the distance from the neutron-star surface for the (a) and (c) cases. To differentiate the muon and tau neutrino fluxes at the neutrinosphere here we take as an example $T_{\nu_\mu} = 1.05\, T_{\nu_\tau}$ (note that in [30] differences of 10% are considered). In general, we have found that the inclusion of the neutrino self-interaction in the propagation reduces possible effects from δ compared to the case without neutrino-neutrino interaction, as can be seen in Fig. 4. In all studied cases both for ν_e and $\bar{\nu}_e$ we find effects up to a few percent at low neutrino energies, and at the level of 0.1% at high energies (60–120 MeV). Our numerical results show deviations at low energies that can sometimes be larger than in the absence of neutrino self-interaction (Fig. 4); those at high energies turn out to be much smaller. This effect of the neutrino-neutrino interaction might be due to the presence of the synchronized regime that freezes possible flavor conversion at initial times and therefore also reduces the modifications coming from a nonzero CP violating phase at later times.

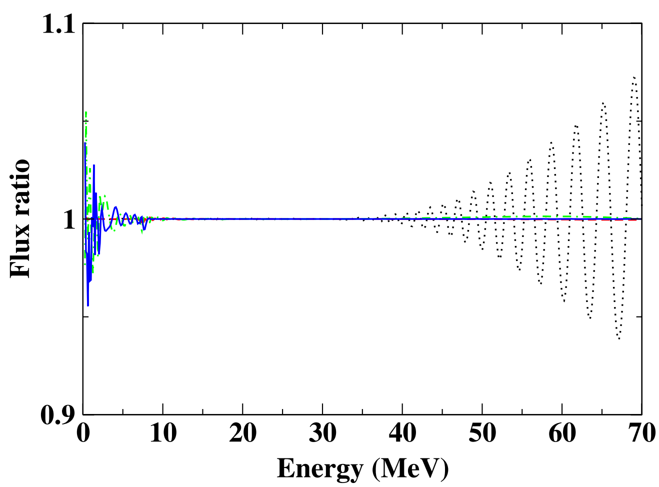

FIG. 4 (color online). Ratios of the ν_e fluxes for a CP violating phase $\delta = 180°$ over $\delta = 0°$ as a function of neutrino energy, at 1000 km within the star. The curves correspond to the following cases: $H_{\nu\nu} = 0$ and $V_{\mu\tau} = 0$ (dotted line), $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} = 0$ (dash-dotted line), and $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} \neq 0$ (solid line). These are obtained with $L_{\nu_{\underline{\mu}}} \neq L_{\nu_{\underline{\tau}}}$, e.g. $T_{\nu_\mu} = 1.05\, T_{\nu_\tau}$. The case $H_{\nu\nu} \neq 0$ and $V_{\mu\tau} \neq 0$ (dash-dotted) with $L_{\nu_{\underline{\mu}}} = L_{\nu_{\underline{\tau}}}$ is also shown. The results correspond to an inverted hierarchy and a small θ_{13}.

IV. CONCLUSIONS

We have investigated possible effects of the CP violating phase on the neutrino propagation in dense matter when interaction with matter without/with loop corrections and the neutrino self-interaction are included. Our analytical results demonstrate that, at tree level, even when the neutrino-neutrino interaction is included there are no CP violating effects on the electron (anti)neutrino fluxes in the star unless ν_μ and ν_τ fluxes differ at the neutrinosphere. If such condition is not satisfied, a totally new feature arises, namely, that the electron (anti)neutrino oscillation probabilities (and fluxes) become sensitive to the CP violating phase δ. The latter is also true when the loop corrections to the refractive index are included. We find numerically that, in most cases studied, the modifications introduced by the CP violating phase are larger (smaller) at low (high) en-

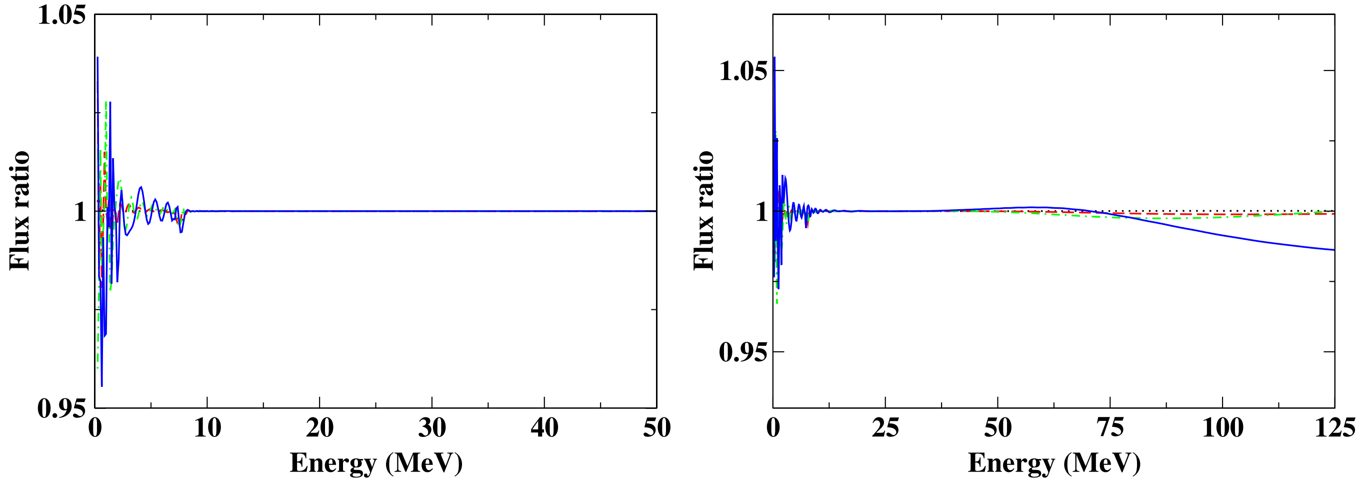

FIG. 5 (color online). Ratios of the ν_e fluxes for a *CP* violating phase $\delta = 180°$ over $\delta = 0°$ as a function of neutrino energy. They correspond to inverted hierarchy and small θ_{13} and different distances from the neutron-star surface, i.e. 200 km (dotted line), 500 km (dashed line), 750 (dash-dotted line), 1000 (solid line). The results include the ν-ν interaction and the $V_{\mu\tau}$ refractive index. They are obtained using equal ν_μ and ν_τ fluxes at the neutrinosphere (left) or taking $T_{\nu_\mu} = 1.05\, T_{\nu_\tau}$ (right). For the $\bar{\nu}_e$ fluxes deviations up to 10% are found at energies lower than 10 MeV.

ergies than in the case where the neutrino-neutrino interaction is not included, and at the level of a few percent. We also find numerically that, even assuming that the muon and tau neutrinos have the same fluxes at the neutrinosphere, the *CP* effects induced by $V_{\mu\tau}$ only are amplified by the neutrino self-interactions up to several percent.

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