PHYSICAL REVIEW D **78,** 081503(R) (2008)

Overspinning a nearly extreme black hole and the weak cosmic censorship conjecture

Maurício Richartz*

Instituto de Física Gleb Wataghin, UNICAMP, C.P. 6165, 13083-970 Campinas, SP, Brazil

Alberto Saa+

Departamento de Matemática Aplicada, UNICAMP, C.P. 6065, 13083-859 Campinas, SP, Brazil

(Received 24 April 2008; published 15 October 2008)

We revisit here the recent proposal for overspinning a nearly extreme black hole by means of a quantum tunneling process. We show that electrically neutral massless fermions evade possible backreaction effects related to superradiance, confirming the view that it would be indeed possible to form a naked singularity due to quantum effects.

DOI: 10.1103/PhysRevD.78.081503 PACS numbers: 04.20.Dw, 04.62.+v, 04.70.Dy

I. INTRODUCTION

The weak cosmic censorship conjecture (WCCC) states basically that any spacetime singularity originated after a gravitational collapse must be hidden inside an event horizon [1]. The conjecture, which is believed to be true at the *classical level*, is one of the most important open problems in general relativity. We remind that, without a full description of spacetime singularities, the WCCC must be true in order to assure the predictability of the laws of physics [2]. Gravitational collapse and gedanken experiments trying to destroy the event horizons of black holes are common theoretical tests of WCCC. According to well-known no-hair theorems [3], all stationary black hole solutions of Einstein-Maxwell equations are uniquely determined by three parameters: the mass M, the electrical charge Q, and the angular momentum J, which satisfy

$$M^2 \geq Q^2 + (J/M)^2, \tag{1}$$

with the equality corresponding to the case of an extreme black hole (we adopt here natural units in which $G = \hbar = c = 1$). Solutions for which $M^2 < Q^2 + (J/M)^2$ have no event horizon; their central singularities are exposed and they are named naked singularities [4]. Many *classical* results [5] have established that it is impossible for a physical process to increase Q (to overcharge) or J (to overspin) a black hole in order to violate (1). Such results have strongly supported the belief that the WCCC is true at classical level.

Quantum effects, on the other hand, have already altered our understanding of the classical laws of black holes in the past. Hawking radiation [6], for instance, implies the decreasing of the black hole area, a process known to be classically forbidden [4]. This was precisely the main motivation behind the recent work of Matsas and Silva [7], where a quantum tunneling process leading to the overspinning of a black hole is proposed. They consider a nearly extreme Reissner-Nordstrom black hole ($Q/M \approx 1$ and $J = 0$) and show that the probability of absorbing low energy massless scalar particles with high angular momentum is nonvanishing. By conservation of energy and angular momentum, they conclude that (1) could be violated after the absorption. In particular, they show that for a black hole with $M = 100$ Planck mass units and $Q = M - e$, $e = 1/\sqrt{\alpha} \approx 1/\sqrt{137}$ being the elementary charge in Planck charge units, a particle with very low energy and total angular momentum $L = \sqrt{\ell(\ell + 1)}$, with $\ell = 413$, would be enough to overspin the black hole and produce a naked singularity. Larger nearly extreme Reissner-Nordstrom black holes would require larger total angular momentums in order to produce naked singularities. In fact, $\ell \sim M^{3/2}$ for large M.

As Matsas and Silva stress in their paper, the transfer of a high amount of angular momentum to an initially static black hole raises some doubts about the role of possible backreaction effects in such a kind of process. This is the central point of Hod's contribution [8] to Matsas and Silva's process: when a wave (or particle) with large angular momentum approaches a black hole, higher order backreaction interactions could trigger the rotation of the black hole before the tunneling. Hod shows then that superradiance effects would imply that only those modes with frequency ω and azimuthal number m such that

$$\omega > m\Omega, \tag{2}$$

where Ω is the angular velocity of the black hole, could be really absorbed by the black hole, leading eventually to the conclusion that it is impossible for such a process to increase J without increasing M simultaneously, preserving (1) and saving the WCCC. We remind that superradiance was proposed by Misner as a version for waves of the Penrose process to extract energy from a rotating black hole. Superradiance is known to affect in similar ways (bosonic) fields of spins $s = 0$ (scalar), $s = 1$ (photons), and $s = 2$ (gravitons) [9]. Besides, there is an equivalent of the superradiance effect for charged situations [10], imply-

*richartz@ifi.unicamp.br
+asaa@ime.unicamp.br

ing that it is also impossible to overcharge a black hole by the absorption of low energy scalar charged particles.

In this paper, we go a step further in this problem by showing that it is possible to reduce arbitrarily the back-reaction effects raised by Hod [8] by considering the quantum tunneling of neutral fermions into nearly extreme rotating black holes, implying that quantum effects could indeed lead to the appearance of naked singularities. Physically, the validity of the process presented here rests on the well-known fact that there is no superradiance effect for electrically neutral massless fermions [11]. We notice also that some recent observations suggest that rapidly spinning black holes could be rather common in the universe [12].

II. FERMIONS AROUND BLACK HOLES

The Dirac equation is known to be separable in a Kerr-Newman spacetime [13]. By using Boyer-Lindquist coordinates, a neutral massless Dirac fermion can be separated in terms of the modes

$$u_{s\omega\ell m}(t, r, \theta, \phi) = e^{-i\omega t}R_{s\omega\ell m}(r)S_{s\omega\ell m}(\theta)e^{im\phi}, \quad (3)$$

where the radial $R_{s\omega\ell m}(r) = R$ and angular $S_{s\omega\ell m}(\theta) = S$ functions satisfy the (spin weight $s = \pm\frac{1}{2}$) Teukolsky equations [13]

$$\Delta^{-s}\frac{d}{dr}\left(\Delta^{s+1}\frac{dR}{dr}\right) + \left(\frac{K^2 - 2is(r-M)K}{\Delta} + 4is\omega r + 2am\omega - a^2\omega^2 - \lambda\right)R = 0, \quad (4)$$

$$\frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{dS}{d\theta}\right) + \left[(a\omega\cos\theta - s)^2 - \left(\frac{s\cos\theta + m}{\sin\theta}\right)^2 - s(s-1) + \lambda\right]S = 0, \quad (5)$$

with $\Delta = r^2 - 2Mr + a^2 + Q^2$, $a = J/M$, and $K = (r^2 + a^2)\omega - am$, where ω, $\ell \geq \frac{1}{2}$, and $-\ell \leq m \leq \ell$ are, respectively, the frequency of the mode and the spheroidal and azimuthal spin-weighted harmonic indexes. In the limit $a\omega \ll 1$, the angular dependence of (3) reduces to the spin-weighted spherical harmonic ${}_sY_\ell^m(\theta, \phi) = S_{s\omega\ell m}(\theta)e^{im\phi}$, with corresponding eigenvalues $\lambda = (\ell - s)(\ell + s + 1)$.

The low energy sector $M\omega \ll 1$ of the modes (3) can be considered analogously to the scalar case [9,14]. The field configuration associated to the tunneling of fermions into the black hole corresponds to the physical boundary conditions of purely ingoing modes at the event horizon $r_+ = M + \sqrt{M^2 - a^2 - Q^2}$,

$$R_{r\to r_+} \approx \Delta^{-s}e^{-i(\omega - m\Omega)r^*}, \quad (6)$$

and a mixture of both ingoing and outgoing modes at infinity

$$R_{r\to\infty} \approx Y_{\rm in}^{(s)}e^{-i\omega r^*}r^{-1} + Y_{\rm out}^{(s)}e^{i\omega r^*}r^{-(2s+1)}, \quad (7)$$

where r^* is the usual tortoise coordinate, defined as $dr^* = (r^2 + a^2)dr/\Delta$. In contrast to the scalar ($s = 0$) case, the calculation of the transmission $\mathcal{T}_{s\omega\ell m}$ and reflection $\mathcal{R}_{s\omega\ell m}$ coefficients, which obey $|\mathcal{T}_{s\omega\ell m}|^2 + |\mathcal{R}_{s\omega\ell m}|^2 = 1$, is rather tricky since it involves s-dependent normalization factors which are different for ingoing and outgoing modes. In particular, we do not have simply $|\mathcal{T}_{s\omega\ell m}|^2 = 1 - |Y_{\rm out}^{(s)}/Y_{\rm in}^{(s)}|^2 = 1/|Y_{\rm in}^{(s)}|^2$ as we do for the $s = 0$ case. However, a property of the solutions of (4) and (5) discovered by Teukolsky and Press [15] leads to

$$|\mathcal{T}_{s\omega\ell m}|^2 = 1 - \left|\frac{Y_{\rm out}^{(s)}Y_{\rm out}^{(-s)}}{Y_{\rm in}^{(s)}Y_{\rm in}^{(-s)}}\right|, \quad (8)$$

facilitating our task considerably.

Incidentally, the transmission coefficient (8) has been already calculated by Page [16] in the limit of low frequency modes ($M\omega \ll 1$) for the case $Q = 0$ in the context of particle emission by black holes. We have

$$|\mathcal{T}_{s\omega\ell m}|^2 = \left[\frac{(\ell - s)!(\ell + s)!}{(2\ell)!(2\ell + 1)!!}\right]^2 \times \prod_{n=1}^{\ell+1/2}\left[1 + \left(\frac{\omega - m\Omega}{n\kappa - \frac{1}{2}\kappa}\right)^2\right]\left(\frac{A\kappa}{2\pi}\omega\right)^{2\ell+1}, \quad (9)$$

for $s = \pm\frac{1}{2}$, where A stands for the black hole area and κ to its surface gravity. The quantity (9) corresponds to the probability that a mode (3) with a (low) frequency ω, spheroidal and azimuthal numbers ℓ and m, respectively, be absorbed by the black hole. As one can see, it is positive for arbitrary small values of ω, in contrast to the bosonic cases where superradiance effects are present [17].

III. OVERSPINNING THE BLACK HOLE

Now, we can finally show how to overspin a black hole by means of the quantum tunneling of fermions, evading Hod's back-reaction issues [8]. Let us suppose, first, we have the "nearest extreme" possible Kerr black hole: mass M (in Planck units), angular momentum $J = M^2 - 1$, and electrical charge $Q = 0$. This black hole can absorb a mode with arbitrarily small frequency ω and $\ell = m = 3/2$. The nonvanishing probability for this process is

$$|\mathcal{T}_{(1/2)\omega(3/2)(3/2)}|^2 = \frac{(M\omega)^4}{36}\left(1 + 8\frac{a^2}{M^2}\right), \quad (10)$$

valid in the limit $M\omega \ll 1$. From the conservation laws, after the tunneling the black hole will have mass $M_f = M + \omega$, angular momentum $J_f = J + m = M^2 + 1/2$, and electrical charge $Q_f = 0$. It would be enough to choose

$$\omega < M\left(\sqrt{1+\frac{1}{2M^2}}-1\right) \sim \frac{1}{4M} \tag{11}$$

in order to violate (1) and induce the formation of a naked singularity. The last term in (11) corresponds to the dominant term for large M. In contrast with Matsas and Silva's original process [7], the total amount of angular momentum that must be transferred to the black hole in order to form a naked singularity does not depend on M and, moreover, is small. A rotating black hole with large mass M and large angular momentum $J = M^2 - 1$ is not expected to be significatively disturbed by an infalling wave with very small frequency ω and $\ell = m = 3/2$. By choosing large values of M, and consequently large values of J, we can reduce arbitrarily any issue related to rotation higher order interactions raised by Hod in [8]. By dealing with large black holes we can also avoid any complication due to possible interactions between the infalling wave and the emitted Hawking radiation [6].

Nevertheless, we can improve even more our result by considering a slightly charged nearly extreme rotating black hole. Let us add, for instance, a charge of $Q = 1$ Planck unit (corresponding to about $12e$) to the black hole with large mass M and angular momentum $J = M^2 - 1$. Such a black hole, which will not be discharged by Schwinger pair production processes [18], can be converted into a naked singularity by absorbing a single low energy fermion with minimal angular momentum. Since electrically neutral fermions do not couple directly to the black hole electric field, the transmission coefficient for low frequencies modes in a Kerr-Newman black hole is given essentially by the same Page's formula (9), leading to the following nonvanishing probability for the tunneling of a neutral massless fermion with low frequency ω and $\ell = m = 1/2$

$$|\mathcal{T}_{(1/2)\omega(1/2)(1/2)}|^2 = (M^2 - Q^2)\omega^2, \tag{12}$$

valid in the limit $M\omega \ll 1$. Because of the conservation laws, after the absorption the black hole will have mass $M_f = M + \omega$, angular momentum $J_f = M^2 - 1/2$, and charge $Q_f = 1$. In order to violate (1) and produce a naked singularity, ω must be chosen as

$$\omega < M\left(\sqrt{\frac{1+\sqrt{1+4(M^2-\frac{1}{2})^2}}{2M^2}}-1\right) \sim \frac{1}{16M^3}, \tag{13}$$

where the last term also corresponds to the dominant term for large M. This process could lead to the formation of a naked singularity by the tunneling of a single neutral fermion with *minimal* angular momentum into a slightly charged nearly extreme rotating black hole with arbitrary mass M. Again, by considering large black hole masses one can minimize all the backreaction effects raised by Hod.

There is, nevertheless, a remaining possible source of backreaction effects in our scenario: the sudden disappearance of the event horizon due to the particle tunneling. A complete description of these effects would certainly require a full quantum gravity theory. In spite of that, some string theoretical results do indeed suggest that the disappearance of the event horizon could be a smooth process without, consequently, any sudden backreaction effect that could alter significantly our conclusions. In the D-brane picture used by Maldacena to describe near extremal black holes [19] the (essentially nonperturbative) vanishing of the event horizon is mapped into a well-defined perturbative process of a low energy effective string model. In particular, the absorption and emission rates of photons and fermions by nearly extreme black holes do not show any evidence of sudden phenomena [20].

Although the probabilities of the absorption for both cases considered here are extremely small, we have shown that it would be indeed possible *in principle* to overspin a nearly extreme black hole through the quantum tunneling of low energy fermions. Moreover, we have shown that the process proposed here, in contrast to Matsas and Silva's original one [7], involves the transfer of a minimal amount of angular momentum from the quantum field to the black hole. Since we deal with neutral fermion fields, which are known to be free from superradiance effects, and manipulate only minimal amounts of energy and angular momentum, we can evade all the backreaction issues pointed out by Hod [8], rendering the overspinning of a black hole by quantum effects a quite robust conclusion.

Spacetime singularities belong naturally to the realm of quantum gravity. We believe that only a complete quantum gravity theory will be able to describe naked singularities properly, dissecting them conclusively or even restoring the WCCC in a more fundamental level.

ACKNOWLEDGMENTS

The authors are grateful to G. Matsas, R. A. Mosna, and C. A. S. Maia for enlightening discussions. This work was supported by FAPESP and CNPq.

[1] R. Penrose, Riv. Nuovo Cimento Soc. Ital. Fis. **1**, 252 (1969); in *General Relativity, an Einstein Centenary Survey*, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England, 1969); C. J. S. Clarke, Classical Quantum Gravity **11**, 1375 (1994); R. M. Wald, arXiv:gr-qc/9710068.

[2] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[3] M. Heusler, *Black Hole Uniqueness Theorems* (Cambridge University Press, Cambridge, England, 1996).
[4] R. M. Wald, *General Relativity* (University of Chicago, Chicago, 1984).
[5] C. V. Vishveshwara, Phys. Rev. D **1**, 2870 (1970); R. Price, Phys. Rev. D **5**, 2419 (1972); **5**, 2439 (1972); B. S. Kay and R. M. Wald, Classical Quantum Gravity **4**, 893 (1987); B. F. Whiting, J. Math. Phys. (N.Y.) **30**, 1301 (1989); D. G. Boulware, Phys. Rev. D **8**, 2363 (1973); R. M. Wald, Ann. Phys. (N.Y.) **82**, 548 (1974); T. Needham, Phys. Rev. D **22**, 791 (1980); S. Hod, Phys. Rev. D **60**, 104031 (1999); **66**, 024016 (2002).
[6] S. W. Hawking, Nature (London) **248**, 30 (1974); Commun. Math. Phys. **43**, 199 (1975).
[7] G. E. A. Matsas and A. R. R. da Silva, Phys. Rev. Lett. **99**, 181301 (2007).
[8] S. Hod, Phys. Rev. Lett. **100**, 121101 (2008).
[9] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Oxford University, New York, 1998).
[10] J. D. Bekenstein, Phys. Rev. D **7**, 949 (1973).
[11] W. Unruh, Phys. Rev. Lett. **31**, 1265 (1973); R. Guven, Phys. Rev. D **16**, 1706 (1977); B. R. Iyer and Arvind Kumar, Phys. Rev. D **18**, 4799 (1978); S. M. Wagh and N. Dadhich, Phys. Rev. D **32**, 1863 (1985).
[12] J.-M. Wang, Y.-M. Chen, L. C. Ho, and R. J. McLure, Astrophys. J. **642**, L111 (2006); R. S. Nemmen, R. G. Bower, A. Babul, and T. Storchi-Bergmann, Mon. Not. R. Astron. Soc. **377**, 1652 (2007).
[13] S. A. Teukolsky, Phys. Rev. Lett. **29**, 1114 (1972); Astrophys. J. **185**, 635 (1973); D. Page, Phys. Rev. D **14**, 1509 (1976).
[14] A. A. Starobinsky and S. M. Churilov, Sov. Phys. JETP **38**, 1 (1974).
[15] S. A. Teukolsky and W. H. Press, Astrophys. J. **193**, 443 (1974).
[16] D. Page, Phys. Rev. D **13**, 198 (1976).
[17] Compare the following formulas of [16]: (13), valid for bosons ($2s$ even, Hod's [8] formula (9) corresponds to $s = 0$), and (14) [our (9)], valid for fermions.
[18] W. T. Zaumen, Nature (London) **247**, 530 (1974); G. W. Gibbons, Commun. Math. Phys. **44**, 245 (1975).
[19] J. Maldacena, Phys. Rev. D **55**, 7645 (1997).
[20] J. Maldacena and A. Strominger, Phys. Rev. D **56**, 4975 (1997); S. S. Gubser, Phys. Rev. D **56**, 7854 (1997).