# Structure-dependent radiative corrections to $\phi \to K^+K^-/K_L K_S$ decays

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Current predictions for the ratio of $\phi \to K^+K^-/K_L K_S$ decay rates exceed the corresponding experimental value in about 5 standard deviations. By far, the dominant sources of isospin breaking to this ratio are the phase-space (52%) and the electromagnetic radiative (4.3%, computed within scalar QED) corrections. Here we estimate the effects of the electromagnetic structure of kaons and other model-dependent contributions into the radiative corrections.

## I. INTRODUCTION

Precise knowledge of the ratio for $P^+P^-/P^0\bar{P}^0$ production ($P = K$, $D$ or a $B$ pseudoscalar meson) is very important for measurements of branching fractions and determination of fundamental parameters at the $\phi$, $\psi(3770)$, and $\Upsilon(4S)$ resonances [1]. It has long been known that standard theoretical calculations overestimate the isospin breaking corrections to the ratio of $\phi \to K^+K^-/K_L K_S$ decay rates [2]. Thus, the isospin breaking corrections induced by the mass difference of kaons and the electromagnetic radiative corrections change the ratio

$$R \equiv \frac{\Gamma(\phi \to K^+K^-)}{\Gamma(\phi \to K^0\bar{K}^0)}, \qquad (1)$$

from unity to about 1.59 [2]. This value lies $4.7\sigma$'s above the corresponding experimental value [3]:

$$R_{\text{exp}} = 1.45 \pm 0.03, \qquad (2)$$

quoted by the PDG from an *overall fit* to measured branching fractions of $\phi$ decays. This large discrepancy is still missing a convincing explanation.

Large isospin breaking corrections to the $\phi \to K^+K^-/K^0\bar{K}^0$ ratio are naturally expected since both decay modes occur near threshold in a $p$ wave. However, it is rather difficult to identify additional contributions (beyond conventional phase-space and radiative corrections) which may render theory and experiment into agreement. For instance, calculations of the isospin breaking to the ratio of $\phi K^+K^-$ and $\phi K^0\bar{K}^0$ coupling constants, carried out in the context of effective hadronic interactions, may increase further the theoretical prediction up to $R^{\text{theory}} = 1.62$ [2,4]. Related to this problem, the effects of strong scattering phases on the isospin breaking effects to $P^+P^-/P^0\bar{P}^0$ ($P = K$, $D$, or $B$ meson states) production in $e^+e^-$ annihilation near threshold where considered in Ref. [5]. Finally, we should mention that a nonconventional mechanism to solve this discrepancy which proposes corrections

to the Fermi golden rule formula of decay rates was discussed in Ref. [6].

In this paper we revisit the calculation of radiative corrections to the $\phi \to K^+K^-$, $K^0\bar{K}^0$ decays. The corrections of order $\alpha$ to the decay into charged kaons were calculated long ago by Cremmer and Gourdin [7] using scalar QED. Since the electromagnetic structure of kaons is ignored in the framework of scalar QED, no corrections to the neutral mode are induced in this case. In the present paper we focus on the electromagnetic form factors of kaons and compute their effects in the observable $R$ defined in Eq. (1). One should note that this calculation was first considered in [2], where a correction of order $2 \times 10^{-3}$ was found for the decay rate into charged kaons. In that paper, the correction to the neutral mode is mentioned to be negligible. We find good agreement with the findings of Ref. [2] and provide an independent test of the model by considering that the dominant contributions come from the region of validity of the vector-meson dominance model. In addition, we also consider the effects of the subleading contributions due to hard-photon emission. Since structure-dependent effects have been found to be important in other calculations of radiative corrections, for instance in $\tau \to \pi\nu_\tau$ and $\pi \to \mu\nu_\mu$ decays [8], it is worth seeing if they can be important in the $\phi$ decays of our interest.

## II. ISOSPIN BREAKING CORRECTIONS TO $R$ IN SCALAR QED

We start by defining the tree-level amplitude for the $\phi(q, \eta) \to K(p)\bar{K}(p')$ decays:

$$\mathcal{M}_0(K\bar{K}) = ig^j(p - p') \cdot \eta, \qquad (3)$$

where $g^+$ ($g^0$) refers to the strong coupling constant for the $K^+K^-$ ($K^0\bar{K}^0$) final state, and $\eta^\mu$ is the polarization four-vector of the $\phi$ vector meson ($q \cdot \eta = 0$). The corresponding ratio of decay rates reads

$$R_0^{\text{theory}} = \left(\frac{g^+}{g^0}\right)^2 \cdot \frac{v_+^3}{v_0^3}, \qquad (4)$$

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where $v_+$ $(v_0)$ denotes the velocity of the charged (neutral) kaons in the rest frame of the $\phi$ meson. In the limit of isospin symmetry, $g^+ = g^0$ and $m_{K^+} = m_{K^0}$, thus the ratio of decay rates becomes $R_0^{\text{theory}} = 1$.

The largest source of isospin breaking corrections to $R_0^{\text{theory}}$ arises from the mass difference of charged and neutral kaons $[(v_+/v_0)^3 = 1.5225]$. The next most important breaking effect comes from radiative corrections. The corrections of $O(\alpha)$ to the $\phi \to K^+K^-$ decay rate, $\delta_{\text{QED}} = 2\delta_{\text{point}}^v + \delta^r$, were calculated in Ref. [7] in the context of scalar QED. They include the sum of virtual corrections ($\delta_{\text{point}}^v$) to the nonradiative amplitude and the real ($\delta^r$) photonic corrections. The virtual corrections are divergent for infrared photons but are finite in the ultraviolet region owing to the Ward identity satisfied by the vertex and self-energy corrections.

The real photon corrections $\delta^r$ contain an infrared divergent piece $\delta_I^r$ and a regular contribution $\delta_R^r$, namely $\delta^r = \delta_I^r + \delta_R^r$. For consistency $\delta_I^r$ is computed by summing over the longitudinal and transverse degrees of freedom of a massive photon [9]. The sum of virtual and soft-photon corrections, $2\delta_{\text{point}}^v + \delta_I^r$, is explicitly free from infrared divergences, as it should be. The explicit expressions of $\delta_{\text{point}}^v$ and $\delta_I^r$ can be found in Ref. [10].

The calculation of the regular term $\delta_R^r$ can be done in a numerical way. It can receive contributions from intermediate states other than $K^\pm$ mesons (for instance, $\phi \to K^+K^{*-} \to K^+K^-\gamma$). However, these model-dependent corrections are expected to be very small either for charged (which we found to be $-7.1 \times 10^{-8}$ for the contribution of $K^*$ intermediate state) or neutral [11] channels because the maximum photon energy is very small compared to the masses of other hadrons. Thus we obtain

$$\delta_R^r = 7.96 \times 10^{-5}. \tag{5}$$

When we include the phase-space corrections, $(v^+/v^-)^3$, and the radiative corrections of scalar QED, $\delta_{\text{QED}} = 2\delta_{\text{point}}^v + \delta_I^r + \delta_R^r = 0.043\,15$, one gets

$$R^{\text{theory}} = R_0^{\text{theory}}(1 + \delta_{\text{QED}}) = 1.588, \tag{6}$$

which is about $5\sigma$'s above the experimental value shown in Eq. (2).

## III. STRUCTURE-DEPENDENT EFFECTS IN RADIATIVE CORRECTIONS TO $R$

Measurements of the electromagnetic interactions of kaons at low [12] and intermediate [13] energies exhibit a structure which can be well described within a vector dominance model. As usual, we define the kaon electromagnetic vertex $\gamma^*(k) \to K(p)\bar{K}(p')$:

$$-ieF_K(k^2)(p - p')_\mu, \tag{7}$$

where $k^2 = (p + p')^2$ is the squared momentum of the virtual photon.

Following Refs. [8,13] we write the form factors in the vector dominance model as

$$F_K(k^2) = \sum_{V=\rho,\omega,\phi} \frac{g_{VK\bar{K}}}{f_V} \cdot \frac{m_V^2}{\hat{m}_V^2 - k^2}, \tag{8}$$

where $em_V^2/f_V$ denotes the photon–vector-meson couplings, $m_V$ is the mass of intermediate vector meson, and $\hat{m}_V^2 = m_V^2 - im_V\Gamma_V\theta(k^2 - k_{\text{threshold}}^2)$ for timelike $k^2$ ($k_{\text{threshold}}^2$ is the square of virtual momentum that allows one to generate imaginary parts in the one-loop corrections to the $V$ meson propagator). At $k^2 = 0$, the form factors are normalized to the electric charges of kaons and the following condition must be satisfied:

$$\sum_{V=\rho,\omega,\phi} \frac{g_{VK^+K^-}}{f_V} = 1 \tag{9}$$

$$\sum_{V=\rho,\omega,\phi} \frac{g_{VK^0\bar{K}^0}}{f_V} = 0. \tag{10}$$

A simplifying assumption (also used in the experimental analysis of Ref. [13]) is implemented by using the SU(3)-invariant Lagrangian for the $VPP'$ interaction which gives the couplings:

$$g_{\rho K^+K^-} = -g_{\rho K^0\bar{K}^0\bar{K}^0} = \frac{1}{2}G_{VPP'}$$

$$g_{\omega K^+K^-} = g_{\omega K^0\bar{K}^0} = \frac{\sqrt{3}}{2}G_{VPP'}\sin\theta_V \tag{11}$$

$$g_{\phi K^+K^-} = g_{\phi K^0\bar{K}^0} = \frac{\sqrt{3}}{2}G_{VPP'}\cos\theta_V,$$

where $\theta_V$ is the $\omega - \phi$ mixing angle (we will use here $\tan\theta_V = 1/\sqrt{2}$ as in Ref. [13]) and $G_{VPP'}$ is the SU(3)-invariant strong coupling constant. Solving Eqs. (9) and (10) with the constraints given in (11) leads to

$$T_\rho^+ = -T_\rho^0 = \frac{1}{2} \qquad T_\omega^+ = T_\omega^0 = \frac{f_\phi}{2[f_\phi + \sqrt{2}f_\omega]}$$

$$T_\phi^+ = T_\phi^0 = \frac{f_\omega}{\sqrt{2}[f_\phi + \sqrt{2}f_\omega]}, \tag{12}$$

where we have introduced the notation $T_V^{+,0} \equiv g_{VK^{+,0}K^{-,0}}/f_V$ for $V = \rho$, $\omega$, $\phi$ mesons. Finally, if we specify the values of the electromagnetic couplings ($T_\omega = 0.1790$, $T_\phi = 0.3210$, using the measured rates of $\omega$, $\phi \to e^+e^-$ decays), the form factors do not contain further free parameters.

Now, let us write the form factors by separating explicitly the point and structure-dependent contributions:

$$F_{K^+}(k^2) = 1 + \sum_{V=\rho,\omega,\phi} T_V^+\left(\frac{k^2 + m_V^2 - \hat{m}_V^2}{\hat{m}_V^2 - k^2}\right), \tag{13}$$

$$F_{K^0}(k^2) = 0 + \sum_{V=\rho,\omega,\phi} T_V^0\left(\frac{k^2 + m_V^2 - \hat{m}_V^2}{\hat{m}_V^2 - k^2}\right). \tag{14}$$

Note that for small values of $k^2$ ($k^2 \leq k_{\text{threshold}}^2$), the nu-

merator of the second term in the right-hand side (rhs) of the above equations is linear in $k^2$.

The Feynman diagrams contributing to the virtual corrections within this meson dominance model are displayed in Fig. 1 for the generic $\phi \to K\bar{K}$ decay. The contributions of self-energies should be added to these virtual corrections. Since the structure-dependent piece of form factors—second term in the rhs of Eqs. (13) and (14)—falls linearly as $k^2$ approaches zero , their contributions to radiative corrections are free from infrared divergences. As in the corresponding case of scalar QED, the result is also free of ultraviolet divergences.

When we insert the form factors in the calculation of the virtual corrections, the expressions for the one-loop amplitudes are the same as in the point case but with an additional factor $|F_K(k^2)|^2$ in the integrand over virtual momenta. The structure-dependent pieces of the radiative corrected amplitudes (due to $T_V^{+,0} \neq 0$) become

$$
\begin{aligned}
\mathcal{M}_{SD}^v(K^+K^-) = \mathcal{M}_0(K^+K^-)\left(\frac{\alpha}{4\pi}\right)&\{[4.37 - i0.44](T_\rho^+)^2 \\
&+ [4.39 + i0.04](T_\omega^+)^2 + [5.84 + i0.02] \\
&\times (T_\phi^+)^2 + [10.25 - i0.34]T_\rho^+ T_\phi^+ \\
&+ [9.05 - i0.40]T_\rho^+ T_\omega^+ + [10.27 - i0.03] \\
&\times T_\omega^+ T_\phi^+ + [-6.30 + i0.53]T_\rho^+ + [-6.31 \\
&+ i0.03]T_\omega^+ + [-7.94 + i0.01]T_\phi^+\} \quad (15)
\end{aligned}
$$

$$
= \mathcal{M}_0(K^+K^-) \times \delta_{VMD}^+, \quad (16)
$$

and

$$
\begin{aligned}
\mathcal{M}_{SD}^v(K^0\bar{K}^0) = \mathcal{M}_0(K^0\bar{K}^0)\left(\frac{\alpha}{4\pi}\right)&\{[4.34 - i0.46](T_\rho^0)^2 \\
&+ [4.36 + i0.02](T_\omega^0)^2 + [5.81 + i0.01] \\
&\times (T_\phi^0)^2 + [10.16 - i0.34]T_\rho^0 T_\phi^0 + [8.96 \\
&- i0.40]T_\rho^0 T_\omega^0 + [10.18 - i0.03]T_\omega^0 T_\phi^0\} \\
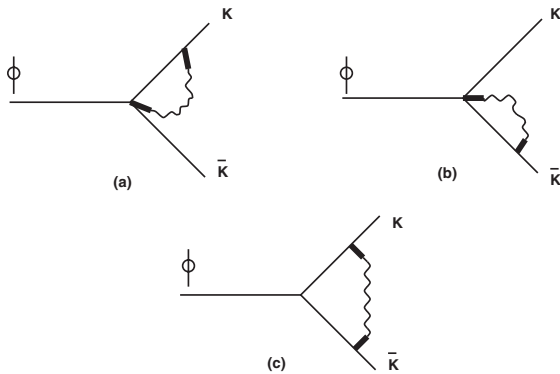& \quad (17)
\end{aligned}
$$



FIG. 1. Feynman graphs for virtual corrections to $\phi \to K\bar{K}$ decays within the vector-meson dominance model.

$$
= \mathcal{M}_0(K^0\bar{K}^0) \times \delta_{VMD}^0, \quad (18)
$$

where $\mathcal{M}_0(K\bar{K})$ denote the amplitudes at the tree level defined in Eq. (3). In the above results, the terms linear in $T_i^{+,0}$ appear from the interference between the point and the structure-dependent terms in the square of Eqs. (13) and (14). Note that the imaginary parts arising from the finite decay width of vector mesons give a small contribution in radiative corrections. Finally, when we insert the numerical values for the couplings constants in the above expressions, we get

$$
\delta_{VMD}^+ = -1.13 \times 10^{-3}, \quad (19)
$$

$$
\delta_{VMD}^0 = -1.37 \times 10^{-5}. \quad (20)
$$

Thus, once we include the effects induced by the electromagnetic structure of kaons, Eq. (5) gets modified to

$$
R^{\text{theory}} = R_0^{\text{theory}}(1 + \delta_{QED} + 2[\delta_{VMD}^+ - \delta_{VMD}^0]) = 1.585. \quad (21)
$$

Therefore, the structure-dependent effects in virtual radiative corrections are tiny but larger than hard-photon contributions [(Eq. (5)].

In view of the above result, one may wonder how appropriate is using the meson dominance model, Eqs. (13) and (14), in the full range of virtual photon momenta $k$. To address this question we introduce a modified photon propagator according to (see, for example, Ref. [8])

$$
\frac{1}{k^2} \to \frac{1}{k^2} \cdot \frac{\mu^2}{\mu^2 - k^2}, \quad (22)
$$

where $\mu$ is a cutoff scale which suppresses the contributions of high $k^2$ values. Clearly, in the limit $\mu^2 \to \infty$ one should recover the previous results, while for finite values of $\mu^2$ the contributions of very high $k^2$ get suppressed.
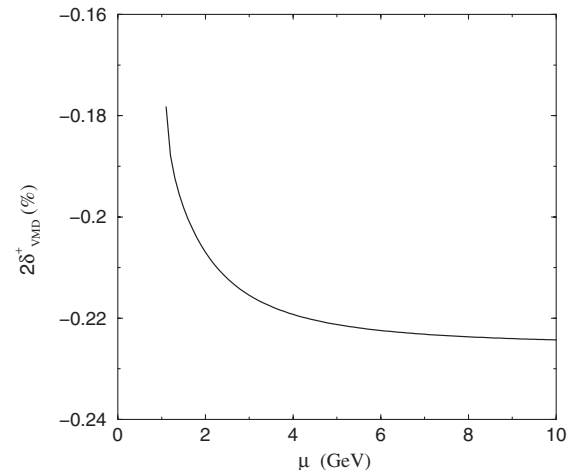


FIG. 2. Structure-dependent virtual corrections $2\delta_{VMD}^+$ to the decay rate of $\phi \to K^+K^-$ decays as a function of the cutoff scale $\mu$.

Using in loop calculations the modified photon propagator of Eq. (22), we can compute again the structure-dependent parts of radiative corrections. Following the recipe given in Eqs. (111)–(113) of Ref. [8], we can easily compute these corrections in the case of $\phi \rightarrow K^+ K^-$ decays for several values of the cutoff $\mu$. Our results, displayed in Fig. 2, show that the values of $\delta_{\mathrm{VMD}}^+(\mu)$ reach the previous value already for low values of $\mu$, i.e. the most important contributions arise in the region of validity of the model (intermediate energies).

## IV. SUMMARY AND CONCLUSIONS

We have computed the radiative corrections to the ratio $R = \Gamma(\phi \rightarrow K^+ K^-)/\Gamma(\phi \rightarrow K_L K_S)$ by taking into account the electromagnetic structure of the $K$ mesons within a vector-meson dominance model. Our results are in agreement with previous calculations of Ref. [2] for the charged kaon channel. Although tiny, the structure-dependent corrections are larger than hard real photon corrections. We have shown that the main contribution of structure-dependent corrections arises from the region of energies where the vector-meson dominance of kaon form factors is expected to hold.

Clearly, structure-dependent corrections do not resolve the discrepancy between theory and the experimental value of $R$ [see Eq. (2) above]. It is worth noticing, however, that a weighted average of direct measurements of $\phi \rightarrow K\bar{K}$ decay rates gives $R_{\mathrm{exp}} = 1.49 \pm 0.05$ [3], which is only 1.9$\sigma$ below the theoretical prediction [Eq. (21)]. This weighted average of direct measurements is dominated by results of the CMD2 Collaboration [13]. It may happen that this result of direct measurements will turn out to be more reliable than the *indirect* value obtained from a constrained fit which requires that the sum of dominant decay modes saturates the total decay width of the $\phi$ meson [14].

If the experimental value shown in Eq. (2) is confirmed by new measurements, still another possibility to solve the discrepancy can be provided by the short-distance effects in radiative corrections. Short-distance corrections can be induced by highly virtual photons coupled to $q\bar{q}$ quark pairs ($q = u, d, s$), in other words, by the quark components of the photon wave function [15]. Since the light $q\bar{q}$ pair required to produce $K^+ K^-/K^0 \bar{K}^0$ in $\phi$ decays is $u\bar{u}/d\bar{d}$, an additional isospin breaking correction to $R$ may be induced. In the case of radiative corrections to semileptonic weak decays, short-distance effects provide a universal correction (they do not break isospin symmetry) to all semileptonic processes [16] since they affect only the underlying quark decay process (isospin breaking arises only from long-distance corrections). Thus, although short-distance corrections could eventually contribute to isospin breaking corrections in $R$, we restrict here only to long- and intermediate-distance radiative corrections.

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