

# Variants of the standard model with electroweak-singlet quarks

Robert Shrock

C. N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, New York 11794, USA

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The successful description of current data provided by the standard model includes fundamental fermions that are color-singlets and electroweak-nonsinglets, but no fermions that are electroweak-singlets and color-nonsinglets. In an effort to understand the absence of such fermions, we construct and study *gedanken* models that do contain electroweak-singlet chiral quark fields. These models exhibit several distinctive properties, including the absence of any neutral lepton and the fact that both the  $(uud)$  and  $(ddu)$  nucleons are electrically charged. We also explore how such models could arise as low-energy limits of grand unified theories and, in this more restrictive context, we show that they exhibit further exotic properties.

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## I. INTRODUCTION

The fundamental fermions in nature, as probed up to energies reached in experiments so far, exhibit an intriguing asymmetry. The asymmetry with respect to fermion chirality is well-known. This is evident in the fact that the fermion content of the standard model (SM) is chiral with respect to its gauge group,  $G_{\text{SM}} = \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ . The asymmetry that we focus on here is the fact that there are fermions, namely, the leptons, that are color-singlets but nonsinglets under the electroweak (EW) subgroup of  $G_{\text{SM}}$ ,  $G_{\text{EW}} = \text{SU}(2)_L \times \text{U}(1)_Y$ , but there is no evidence for fermions that are singlets under  $G_{\text{EW}}$  while being nonsinglets under  $\text{SU}(3)_c$ . Can one understand this property of nature at a deeper level? To address this question, we construct and study *gedanken* models that are variants of the standard model and that include electroweak-singlet quarks. Our aim here is not to try to find another model that fits current data but instead to work out properties of these *gedanken* models and determine in what general ways these properties differ from those observed in the real world. Our methods of analysis are simply those of quantum field theory and group theory; we do not include any results from anthropic arguments.

One class of variants involves the addition of a vector-like set of electroweak-singlet, color-nonsinglet fermions to the standard model. A second class of variants is obtained by altering the hypercharges and thus also the electric charges of the quarks in the standard model so that either the  $d_R$ -type or  $u_R$ -type quarks of each generation have  $Y = Q = 0$ . This can be done in a manner consistent with constraints from anomaly cancellation so long as one also makes corresponding changes in the charges of the leptons [1]. Within this class of models we discuss three particular cases. In two of these, the electric charges of left- and right-handed Weyl components of fermions satisfy  $q_{f_L} = q_{f_R}$  and (i)  $q_{d_R} = 0$  or (ii)  $q_{u_R} = 0$ . In the third, all fermions have  $Y = 0$ . We find several ways in which the properties of such variants differ from those of the standard

model, including the absence of any neutral leptons and the fact that both the  $(uud)$  and  $(ddu)$  nucleons are electrically charged. We then consider possible “ultraviolet completions” of these models [2]. There are various motivations, including gauge coupling unification, quark-lepton unification, and charge quantization, to believe that the SM is a low-energy effective field theory resulting from a grand unified theory (GUT) based on a (semi)simple gauge group,  $G_{\text{GUT}}$ , with  $G_{\text{SM}} \subset G_{\text{GUT}}$ . Modern grand unified theories usually entail a supersymmetric extension of the SM [3], although examples of gauge coupling unification in nonsupersymmetric contexts have also been found [4]. In the grand unified theories that we consider, we show that such models would exhibit further exotic properties; for example, in an  $\text{SU}(5)$  theory, we find breaking of  $U(1)_{\text{em}}$  by QCD quark condensates.

We recall certain basic properties and fix some notation. The fermion content of the standard model consists of  $N_g = 3$  generations of the quarks

$$Q_{n,L}^a = \begin{pmatrix} u_n^a \\ d_n^a \end{pmatrix}_L,$$

$u_{n,R}^a$ , and  $d_{n,R}^a$ , transforming, respectively, as  $(3, 2)_{1/3}$ ,  $(3, 1)_{4/3}$ , and  $(3, 1)_{-2/3}$ , and the leptons

$$L_{n,L} = \begin{pmatrix} \nu_{e_n} \\ e_n \end{pmatrix}_L$$

and  $e_{n,R}$  transforming as  $(1, 2)_{-1}$  and  $(1, 1)_{-2}$ . Here  $a$  is the color index, the numbers in parentheses are the dimensions of the representations of  $\text{SU}(3)_c$  and  $\text{SU}(2)_L$ , the subscripts are the weak hypercharge  $Y$ ,  $n$  is the generational index, and we use a compact notation in which  $u_1 \equiv u$ ,  $u_2 \equiv c$ ,  $u_3 \equiv t$ , etc. To accommodate massive neutrinos, we also include a number  $n_s$  of electroweak-singlet neutrinos,  $\nu_{\ell_n,R}$  transforming as  $(1, 1)_0$  and will usually take  $n_s = N_g = 3$ . We have  $Q = T_3 + (Y/2)$  and will consider theories with values of  $Y$  and hence  $Q$  different from those in the SM itself. For supersymmetric extensions of the SM, we stress

that our aim is to study models with EW-singlet, color-nonsinglet matter fermions contained in chiral superfields; of course, such models automatically include electroweak-singlet color-adjoint fermions in vector superfields, namely, the gluinos.

## II. MODEL WITH ADDITIONAL VECTORLIKE FERMIONS

One way to construct a variant of the standard model with electroweak-singlet color-nonsinglet matter fermions is simply to add a vectorlike set of  $SU(2)_L$ -singlet fermions  $\{f_L, f_R\}$ , i.e. a set in which  $f_L$  and  $f_R$  transform according to the same representation of  $SU(3)_c$  and have the same  $Y/2 = Q$ , including some with  $Y = Q = 0$ . If one starts with the minimal supersymmetric standard model (MSSM), possibly augmented with  $G_{SM}$ -singlet chiral superfields, then one would add the set of (left-handed) chiral superfields  $\{\hat{F}, \hat{F}^c\}$ . It is easy to see why one would not have observed such particles at energies probed so far, since the bare fermion mass term  $m_F \bar{F}_L F_R + \text{H.c.}$  or corresponding superfield term  $\hat{F} \hat{F}^c$  is invariant under  $G_{SM}$ , and hence  $m_F$  would be expected to be of order the scale characterizing the ultraviolet completion of the theory, such as the GUT scale. This is a special case of the general result that fields that can form bare mass terms consistent with gauge symmetry group describing the theory at a given scale do form such terms at this scale, and are integrated out in the effective field theory below this scale [5]. Rather than adding such fields to the SM or MSSM, one can, instead, change the hypercharge assignments of the SM or MSSM fields themselves, as we discuss next.

## III. SM WITH ALTERED FERMION HYPERCHARGES

### A. Models with $q_{f_L} = q_{f_R}$

A minimal way to obtain electroweak-singlet quarks in a SM-like model or extension thereof is to change the hypercharge assignments for the SM fermions. To keep  $U(1)_{em}$  vectorial in the simplest manner, we maintain the relations for the electric charges

$$q_{f_L} = q_{f_R}, \quad (3.1)$$

where  $f$  runs over the quarks and leptons. Since the  $T_3 = 1/2$  component of the  $SU(2)_L$ -doublet lepton field will have a nonzero charge for the models of interest here, we avoid the SM notation

$$L_{n,L} = \begin{pmatrix} \nu_{e_n} \\ e_n \end{pmatrix}_L$$

and instead write

$$L_L = \begin{pmatrix} \ell_1 \\ \ell_2 \end{pmatrix}_L, \quad (3.2)$$

where here and below we shall often suppress the genera-

tional index  $n$ . The altered hypercharge assignments are subject to the constraint of cancellation of anomalies in gauged currents. The  $SU(3)_c^3$  and  $SU(3)_c^2 U(1)_Y$  triangle anomalies vanish because of the vectorial property of  $SU(3)_c$  and  $U(1)_{em}$ . The condition that the  $SU(2)_L^2 U(1)_Y$  triangle anomaly vanishes is

$$N_c Y_{Q_L} + Y_{L_L} = 0, \quad (3.3)$$

where we display the general  $N_c$  dependence. This is equivalent to the condition [1]

$$q_u = q_d + 1 = \frac{1}{2} \left( 1 - \frac{(2q_{\ell_2} + 1)}{N_c} \right). \quad (3.4)$$

This provides two ways to get electroweak-singlet quarks. We discuss these for the relevant case  $N_c = 3$ .

The first way entails the  $Y$  assignments and corresponding  $SU(2)_L$ -doublets (with electric charges in parentheses and suppressing generation indices)

$$\begin{aligned} Y_{Q_L} &= 1; & Q_L^a &= \begin{pmatrix} u^a(1) \\ d^a(0) \end{pmatrix}_L \\ Y_{L_L} &= -3; & L_L &= \begin{pmatrix} \ell_1(-1) \\ \ell_2(-2) \end{pmatrix}_L \end{aligned} \quad (3.5)$$

and  $SU(2)_L$  singlets having  $Y_{f_R} = 2q_{f_R}$ ,

$$u_R^a(1), \quad d_R^a(0), \quad \ell_{1,R}(-1), \quad \ell_{2,R}(-2), \quad (3.6)$$

so that  $d_R^a$  is an EW singlet. We denote this case as DRS, standing for “ $d_R$  singlet.”

The second case has  $SU(2)_L$ -doublets

$$\begin{aligned} Y_{Q_L} &= -1; & Q_L^a &= \begin{pmatrix} u^a(0) \\ d^a(-1) \end{pmatrix}_L \\ Y_{L_L} &= 3; & L_L &= \begin{pmatrix} \ell_1(2) \\ \ell_2(1) \end{pmatrix}_L \end{aligned} \quad (3.7)$$

and  $SU(2)_L$  singlets

$$u_R^a(0), \quad d_R^a(-1), \quad \ell_{1,R}(2), \quad \ell_{2,R}(1), \quad (3.8)$$

so that  $u_R^a$  is an electroweak-singlet (denoted case URS). Both the DRS and URS cases also satisfy the conditions of vanishing  $U(1)_Y^3$  and  $G^2 U(1)_Y$  triangle anomalies, where  $G = \text{graviton}$ . The DRS and URS cases correspond to cases  $C4_q$  and  $C5_q$  with  $N_c = 3$  in the classification of Ref. [1].

The DRS and URS models exhibit several properties that differ from those of the standard model. First, they do not have any neutral leptons. Second, not just the proton,  $p = (uud)_{J=1/2}$ , but also its isospin partner nucleon,  $n = (ddu)_{J=1/2}$  (the neutron in the SM), would be charged and would have charges  $q_p$  and  $q_n = q_p - 1$  of the same sign:

$$q_p = 2, \quad q_n = 1 \quad \text{DRS case}, \quad (3.9)$$

$$q_p = -1, \quad q_n = -2 \quad \text{URS case}. \quad (3.10)$$

For arbitrary  $q_{\ell_2}$  it follows from Eq. (3.4) that

$$q_p = -q_{\ell_2}, \quad q_n = -q_{\ell_1}. \quad (3.11)$$

(This is true more generally for the analogues of  $p$  and  $n$  for higher  $N_c$  [1,6].)

One can construct a supersymmetric extension of either the DRS or URS SM-like model. The usual Higgs mechanism in its SM or MSSM form can be implemented for these DRS and URS models, considered in isolation. One could also choose one of the various scenarios for supersymmetry breaking, so that, in the observable sector this occurs at the electroweak level, as in the MSSM. Alternatively, as *gedanken* theories, one might use dynamical electroweak symmetry breaking (EWSB) via technicolor (TC) [7] and extended technicolor (ETC) [8]. If the residual nuclear force had the same strength as in the real world, then the binding of nucleons to form nuclei would be somewhat reduced because of the increased Coulomb repulsion between nucleons resulting from the fact that both types of nucleons have nonzero electric charges of the same sign (and indeed one of these is double the usual proton charge in magnitude). This would tend to destabilize some nuclei that are stable in the real world. Although both members of the nucleon isodoublet are charged, there are spin-1/2 baryons that are neutral in this model. For the DRS and URS cases these include, for example, the spin-1/2 baryons

$$(dds), \quad (dss), \quad (ddb), \quad \text{etc.} \quad (\text{DRS case}) \quad (3.12)$$

$$(uuc), \quad (ucc), \quad (uut), \quad \text{etc.} \quad (\text{URS case}). \quad (3.13)$$

If one keeps the masses of the quarks equal or similar to their values in the real world, then these are heavier than the nucleons and would beta decay. We shall show below how the matter fermion content of the DRS and URS models can arise from a grand unified theory, where their structure is more tightly constrained.

### B. Model with all fermions having $Y = 0$

A different modification of the SM with electroweak-singlet quarks that is allowed by anomaly constraints is for each fermion generation to have the  $SU(2)_L$  doublets (again suppressing the  $n$  index)

$$\begin{aligned} Y_{Q_L} = 0; \quad Q_L^a &= \begin{pmatrix} u^a(1/2) \\ d^a(-1/2) \end{pmatrix}_L, \\ Y_{L_L} = 0; \quad L_L &= \begin{pmatrix} \ell_1(1/2) \\ \ell_2(-1/2) \end{pmatrix}_L \end{aligned} \quad (3.14)$$

and  $SU(2)_L$  singlets  $\{f_R\}$  with

$$Y_{f_R} = q_{f_R} = 0 \quad \forall f_R. \quad (3.15)$$

To keep  $SU(3)_c$  vectorial, the set  $\{f_R\}$  includes two color

triplets for each generation, which we denote  $\eta_R^a$  and  $\eta_R'^a$ . (We avoid denoting these as  $u_R^a$  and  $d_R$  since they have different electric charges than  $u_L^a$  or  $d_L^a$ .) The remainder of the set  $\{f_R\}$  is comprised of (two or some other number of)  $G_{SM}$ -singlets. We denote this model with the matter fermion content in Eqs. (3.14) and (3.15) as the YZ ( $Y$  zero) case; it corresponds to the case  $C2_{q,\text{sym}}$  (equivalently  $C2_{\ell,\text{sym}}$ ) in the classification of Ref. [1]. Although  $SU(3)_c$  (with other interactions turned off) and  $U(1)_{\text{em}}$  (with other interactions turned off) are vectorial symmetries in the YZ model, this occurs in a “twisted” manner, in which there is not a 1-1 correspondence between a left-handed Weyl field and a right-handed Weyl field with the same color and charge [9]. From Eq. (3.11), it follows that both members of the nucleon isodoublet are charged:

$$q_p = -q_n = \frac{1}{2} \quad (\text{YZ case}). \quad (3.16)$$

If one considers the YZ model in isolation without trying to construct an ultraviolet completion, then one can include a  $I = 1/2$ ,  $Y = 1$  SM Higgs field  $\phi$  or, in an MSSM context,  $I = 1/2$   $Y = \pm 1$  Higgs chiral superfields. With either of these one can break  $SU(2)_L \times U(1)_Y$  to  $U(1)_{\text{em}}$  via Higgs vacuum expectation values (VEV's). However, one cannot construct  $G_{SM}$ -invariant Yukawa couplings and use these to generate masses for the matter fermions. For example, the Yukawa term  $\bar{Q}_{a,n,L} f_{n',R}^a \phi + \text{H.c.}$  is forbidden by  $U(1)_Y$  gauge invariance, since it transforms as a  $Y = 1$  operator. Assigning any value of  $Y$  other than  $\pm 1$  to the Higgs field(s) would not allow EWSB, since the Higgs would not have any neutral components.

In this YZ model, QCD confines and spontaneously breaks chiral symmetry. The most attractive channel for condensate formation,  $3 \times \bar{3} \rightarrow 1$ , yields the condensates (suppressing  $n$  indices)  $\langle \bar{u}_{a,L} f_R^a \rangle$  and  $\langle \bar{d}_{a,L} f_R^a \rangle$ , where  $f_R$  refers to  $\eta_R$  or  $\eta_R'$ . Without loss of generality, we can write these as  $\langle \bar{u}_{a,L} \eta_R^a \rangle$  and  $\langle \bar{d}_{a,L} \eta_R^a \rangle$ . Since  $q_{u_L} = 1/2$ ,  $q_{d_L} = -1/2$ , and  $q_{\eta_R} = q_{\eta_R'} = 0$ , these condensates break not just  $SU(2)_L$ , but also  $U(1)_{\text{em}}$ . This model is thus strikingly different from the real world. We shall show below how the matter fermion content of the YZ model [but not  $I = 1/2$ ,  $Y = \pm 1$  Higgs field(s)] arises naturally as a low-energy effective field theory if one requires electroweak-singlet fermions in an  $SU(5)$  GUT.

## IV. GRAND UNIFICATION IN $SU(5)$

We now analyze electroweak-singlet quarks in the context of grand unified theories. Much modern work on grand unified theories has focused on meeting constraints from proton decay and deriving models from a presumed underlying string theory. Our purpose here is somewhat different; we are not trying to account in detail for the experimentally observed values of gauge couplings or limits on proton decay. Instead, we wish to explore the properties of *gedanken* grand unified theories containing

electroweak-singlet quarks, accepting that these would entail changes in the measured values of  $\sin^2\theta_W$ , etc. We first consider the case where the GUT group has the minimal rank, namely, 4, the same as  $G_{\text{SM}}$ . For this case, the canonical choice is  $G_{\text{GUT}} = \text{SU}(5)$  [10]. One assigns the left-handed matter fermions of each generation to a  $\bar{5}$  and 10 representation. Under  $\text{SU}(3)_c \times \text{SU}(2)_L$  these decompose as  $\bar{5} = (\bar{3}, 1) \oplus (1, 2)$  and  $10 = (\bar{3}, 1) \oplus (3, 2) \oplus (1, 1)$ . In order to make the  $(\bar{3}, 1)$  in the  $\bar{5}$  of  $\text{SU}(5)$  an EW-singlet (anti)quark, we assign it zero hypercharge. In terms of the equivalent  $5_R$ , we write

$$\psi_R = \begin{pmatrix} \eta^a \\ L^c \end{pmatrix}_R \quad (4.1)$$

where  $a$  is again the color index and  $Y_{\eta_R} = 0$ . As before, we use the symbol  $\eta_R$  rather than  $u_R$  or  $d_R$  for this quark because it will not have the charge of either  $u_L$  or  $d_L$ . If the GUT group is  $\text{SU}(5)$ , then  $Y$  (and hence  $Q$ ) are (linear combinations of) generators of the Lie algebra of  $\text{SU}(5)$  and hence satisfy  $\text{Tr}(Y) = \text{Tr}(Q) = 0$ . Therefore,  $Y_{L^c_R} = 0$ , and

$$L_L = \begin{pmatrix} \ell_1(1/2) \\ \ell_2(-1/2) \end{pmatrix}_L$$

(charges listed in parentheses). Consequently, as operators,  $Y = \text{diag}(0, 0, 0, 0, 0)$  and thus

$$Q = \text{diag}(0, 0, 0, 1/2, -1/2). \quad (4.2)$$

These operators are to be contrasted with the forms in conventional  $\text{SU}(5)$  [10],

$$Y_{\text{conv.}} = \text{diag}(-2/3, -2/3, -2/3, 1, 1) \quad (4.3)$$

and thus

$$Q_{\text{conv.}} = \text{diag}(-1/3, -1/3, -1/3, 1, 0). \quad (4.4)$$

It follows that the  $Y$  assignments in the 10 of  $\text{SU}(5)$  in the present case are

$$10 = (\bar{3}, 1)_0 \oplus (3, 2)_0 \oplus (1, 1)_0, \quad (4.5)$$

with respective component fields

$$\eta_{a,L}^c(0), \quad \begin{pmatrix} u^a(1/2) \\ d^a(-1/2) \end{pmatrix}_L, \quad \chi_L^c(0). \quad (4.6)$$

Thus, for each generation (suppressing the generation index) the fermions from the  $\bar{5}_L$  and  $10_L$  comprise the representations

$$\begin{aligned} Q_L^a &= \begin{pmatrix} u^a(1/2) \\ d^a(-1/2) \end{pmatrix}_L : (3, 2)_0, \\ L_L &= \begin{pmatrix} \ell_1(1/2) \\ \ell_2(-1/2) \end{pmatrix}_L : (1, 2)_0 \end{aligned} \quad (4.7)$$

and

$$\eta_R^a(0), \quad \eta_R^a(0) : (3, 1)_0, \quad \chi_R(0) : (1, 1)_0. \quad (4.8)$$

This therefore yields a YZ-type model. There could also be  $\text{SU}(5)$ -singlet matter fermions.

One envisions that the  $\text{SU}(5)$  gauge symmetry is spontaneously broken to  $G_{\text{SM}}$  at the GUT scale,  $M_{\text{GUT}}$ . The resultant theory below  $M_{\text{GUT}}$  has several properties that are quite different from those of the real world. First, since all of the particles have  $Y = 0$ , the effective gauge group is just  $\text{SU}(3)_c \times \text{SU}(2)_L$ , without a  $\text{U}(1)_Y$  factor. Thus, in this model  $Q = T_3$  and  $\text{U}(1)_{\text{em}}$  is a subgroup of  $\text{SU}(2)_L$ . Second, as noted above, although the  $\text{SU}(3)_c$  and  $\text{U}(1)_{\text{em}}$  gauge interactions are individually vectorial, this is realized in a manner different from that of the SM. Third, one cannot construct a SM- or MSSM-type Higgs sector in this theory because the operator  $Q$  has no color-singlet,  $\text{SU}(2)_L$ -doublet, electrically neutral component. For example, the Higgs  $\text{SU}(2)_L$  doublet contained in an  $\text{SU}(5)$  5 of Higgs is

$$\phi = \begin{pmatrix} \phi_1(1/2) \\ \phi_2(-1/2) \end{pmatrix}.$$

A VEV for either component of this Higgs doublet breaks  $\text{SU}(2)_L$  completely, including its  $\text{U}(1)_{\text{em}}$  subgroup. For the same reason, usual SM-type Yukawa couplings and their supersymmetric extensions are not possible in this theory. The ( $\text{SU}(3)_c$ -invariant) mass terms that one might consider for the quarks,

$$\sum_{n,n'} \bar{q}_{a,n,L} M_{nn'}^{(q)} f_{n',R}^a + \text{H.c.} \quad (4.9)$$

with  $q = u, d$  and  $f = \eta, \eta'$ , break  $\text{SU}(2)_L$  and hence also  $\text{U}(1)_{\text{em}}$ . The same is true of the lepton Dirac mass terms

$$\sum_{n,n'} \bar{\ell}_{j,n,L} M_{nn'}^{(\ell_j)} \chi_{n',R}^a + \text{H.c.}, \quad (4.10)$$

where  $j = 1, 2$ . The  $\text{SU}(2)_L$ -doublet leptons can have the  $G_{\text{SM}}$ -invariant Majorana bare mass terms

$$\sum_{n,n'} \epsilon_{ij} L_{n,L}^{iT} C M_{nn'}^{(L)} L_{n',L}^j + \text{H.c.} \quad (4.11)$$

where  $i, j$  denote  $\text{SU}(2)_L$  indices. The structure of this operator implies that  $M_{nn'}^{(L)} = -M_{n'n}^{(L)}$ , so for the relevant case of odd  $N_g$  there is at least one zero eigenvalue, i.e., a massless charged lepton at this level. Via diagrams involving the exchange of GUT-scale gauge bosons, the proton and  $\ell_2^c$  will mix, as will the  $(ddu)_{J=1/2}$  nucleon and  $\ell_1^c$ , which will give extremely small masses to these leptons. The effect of the very low-mass charged unconfined leptons is reduced by the fact that  $\text{U}(1)_{\text{em}}$  is broken, as we discuss next. The  $\chi_{n,R}$ 's could have bare Majorana mass terms  $\sum_{n,n'} \chi_{n,R}^T C M_{nn'}^{(\chi)} \chi_{n',R} + \text{H.c.}$

Since the  $\text{SU}(3)_c$  gauge interaction is asymptotically free, as the energy scale  $\mu$  decreases below  $M_{\text{GUT}}$ , the  $\text{SU}(3)_c$  coupling grows. As  $\mu$  decreases through  $\Lambda_{\text{QCD}}$  and  $\alpha_s = g_s^2/(4\pi)$  reaches values of order unity, the QCD

sector exhibits confinement. Since mass terms for the quarks would violate  $U(1)_{\text{em}}$ , we take them to be massless. Then in the limit where one neglects electroweak interactions, the theory has a global flavor symmetry

$$G_{\text{fl}} = \text{SU}(2N_g)_L \times \text{SU}(2N_g)_R \quad (4.12)$$

The QCD interaction spontaneously breaks  $G_{\text{fl}}$  by the formation of the bilinear quark condensates (in the  $3 \times \bar{3} \rightarrow 1$  channel)

$$\sum_{n,n'} \langle \bar{q}_{a,n,L} f_{n',R}^a \rangle + \text{H.c.}, \quad (4.13)$$

where  $q = u, d$  and  $f = \eta, \eta'$ . As with the mass terms, these condensates are invariant under  $\text{SU}(3)_c$  but break  $\text{SU}(2)_L$  and hence  $U(1)_{\text{em}}$ . This is the same as the YZ model, now seen in a GUT context.

## V. GRAND UNIFICATION IN $\text{SO}(10)$

One can construct grand unified theories with electroweak-singlet quarks that avoid the breaking of  $U(1)_{\text{em}}$  by using  $G_{\text{GUT}} = \text{SO}(10)$  [11] and taking advantage of the additional freedom of having  $Y$ , and hence  $Q$ , be generators of  $\text{SO}(10)$  but not of  $\text{SU}(5)$ .  $\text{SO}(10)$  models in which  $Y$  and  $Q$  are not generators of  $\text{SU}(5)$  were constructed in Ref. [12] with conventional quark and lepton charges and the  $u_R^a$  assigned to the  $5_R$ , so that  $Y = \text{diag}(4/3, 4/3, 4/3, 1, 1)$ . We will use this freedom in a different way here. We denote  $\tilde{Y}$  as the generator of  $\text{SU}(5)$  that commutes with  $\text{SU}(3)_c$  and  $\text{SU}(2)_L$ . Now  $\text{SO}(10)$  contains, as a maximal subgroup,  $\text{SU}(5) \times U(1)_X$ , and the spinor 16 of  $\text{SO}(10)$  transforms as  $16 = 1_5 \oplus 5_{-3} \oplus 10_1$ , where subscripts denote  $X$  values. We set

$$\tilde{Y} = aY + bX. \quad (5.1)$$

Since the electroweak-singlet quark is assigned to the first  $N_c = 3$  components of  $\psi_R$ , it follows that for the 5 of  $\text{SU}(5)$ ,

$$Y = \text{diag}(0, 0, 0, y, y), \quad y = Y_{L_5} = -Y_{L_3}. \quad (5.2)$$

Hence  $\text{Tr}(\tilde{Y}) = 2ay + 15b$ . We can take  $a = 1$ , and solve to get  $b = -2y/15$ , so that

$$\tilde{Y} = Y - \frac{2y}{15}X. \quad (5.3)$$

For the DRS case, with  $d_R$  in  $\psi_R$  and  $Y_{L_3} = -3$ , we thus have  $\tilde{Y} = Y - (2/5)X$ . For the 5 of  $\text{SU}(5)$  this is

$$\text{DRS: } \tilde{Y} = \frac{1}{5} \text{diag}(-6, -6, -6, 9, 9). \quad (5.4)$$

For a representation  $R$  with a given value of  $X$ , one then calculates  $Y$  and  $Q$  by using Eq. (5.3). For the 5 of  $\text{SU}(5)$ ,  $Y = \text{diag}(0, 0, 0, 3, 3)$  and  $Q = \text{diag}(0, 0, 0, 2, 1)$ . The components of the 10 of  $\text{SU}(5)$  have the  $Y$  values indicated,

$$\text{DRS: } 10_L: (\bar{3}, 1)_{-2} \oplus (3, 2)_1 \oplus (1, 1)_4, \quad (5.5)$$

with component fields given by  $u_{a,L}^c(-1)$ ,

$$\begin{pmatrix} u^a(1) \\ d^a(0) \end{pmatrix}_L$$

and  $(\ell_2^c)_L(2)$ . The remaining component of the 16 of  $\text{SO}(10)$  is an  $\text{SU}(5)$  singlet,  $(\ell_1^c)_L(1)$ . These fields thus comprise the set in Eqs. (3.5) and (3.6).

For the URS case, with  $u_R$  in  $\psi_R$  and  $Y_{L_3} = 3$ , we have  $\tilde{Y} = Y + (2/5)X$ . For the 5 of  $\text{SU}(5)$  this is

$$\text{URS: } \tilde{Y} = \frac{1}{5} \text{diag}(6, 6, 6, -9, -9). \quad (5.6)$$

Thus for the 5 of  $\text{SU}(5)$ ,  $Y = \text{diag}(0, 0, 0, -3, -3)$  and  $Q = \text{diag}(0, 0, 0, -1, -2)$ . The components of the 10 of  $\text{SU}(5)$  have the  $Y$  values indicated,

$$\text{URS: } 10_L: (\bar{3}, 1)_2 \oplus (3, 2)_{-1} \oplus (1, 1)_{-4} \quad (5.7)$$

with component fields given by  $d_{a,L}^c(1)$ ,

$$\begin{pmatrix} u^a(0) \\ d^a(-1) \end{pmatrix}_L$$

and  $(\ell_1^c)_L(-2)$ . The remaining component of the 16 of  $\text{SO}(10)$  is an  $\text{SU}(5)$  singlet,  $(\ell_2^c)_L(-1)$ . These fields make up the set in Eqs. (3.7) and (3.8).

Using the relation  $\sin^2 \theta_W = \text{Tr}_R(T_3^2)/\text{Tr}_R(Q^2)$ , we find

$$\sin^2 \theta_W = \frac{1}{2(1 + 3Y_{Q_L}^2)} \quad \text{at } M_{\text{GUT}}. \quad (5.8)$$

Thus, at  $M_{\text{GUT}}$ ,  $\sin^2 \theta_W \leq 1/2$ , and this value is reached for the  $Y$  choices leading to the YZ low-energy field theory. The  $Y$  choices leading to the DRS or URS models yield  $\sin^2 \theta_W = 1/8$  at  $M_{\text{GUT}}$ .

In these  $\text{SO}(10)$ -based DRS and URS models the 16-dimensional spinor representation has no color-singlet,  $\text{SU}(2)_L$ -doublet, electrically neutral entries, in contrast to both conventional [11] and ‘‘flipped’’ [12]  $\text{SO}(10)$  models, and the same follows for the  $\bar{5}$  and 10 representations of  $\text{SU}(5)$  arising from this spinor. A Higgs (super)field that could give rise to electroweak symmetry breaking is thus problematic. One could consider adding a (super)field transforming as a vector representation of  $\text{SO}(10)$ . Under  $\text{SU}(5) \times U(1)_X$  the 10 of  $\text{SO}(10)$  decomposes as  $10 = 5_{-2} \oplus \bar{5}_2$ . In the DRS and URS cases the 5 of Higgs resulting from this would have charges  $(-1, -1, -1, 1, 0)$  and  $(1, 1, 1, 0, -1)$ , respectively. Although these Higgs fields thus contain color-singlet,  $\text{SU}(2)_L$ -doublet, neutral entries, they can form bare,  $\text{SO}(10)$ -invariant mass terms with masses naturally of order the GUT scale and hence would be integrated out of the low-energy theory operative below this scale. Moreover, if such mass terms were not present, then color-triplet Higgs components in the Higgs 10 of  $\text{SO}(10)$  could contribute to overly rapid nucleon decay. For the purposes of our further analysis, we assume

that GUT-scale mass terms are present for Higgs (super) fields transforming as the 10 of SO(10).

If one were to depart from this simple GUT and adjoin to the theory a technicolor sector, then one could use this to break electroweak symmetry at the 250 GeV scale (where in this *gedanken* model we would accept resultant technicolor modifications of  $Z$  and  $W$  propagators) [13]. However, if we consider the theory by itself, without such an addition, then we may ask if this resultant theory would break electroweak symmetry. The answer is yes, and this breaking is dynamical. There would not be Higgs-based Yukawa couplings or TC/ETC contributions to give fermions masses. In the absence of these and in the limit where one turns off electroweak interactions, the QCD sector would have the global chiral symmetry  $G_{\text{fl}}$  in Eq. (4.12). (Note that there would not be any strong  $CP$  problem in QCD because the massless quarks would allow one to rotate away the  $\bar{\theta}$  angle.) Because the  $SU(3)_c$  gauge interaction is asymptotically free,  $\alpha_s$  increases as the energy scale decreases below  $M_{\text{GUT}}$ , reaching  $O(1)$  at  $\Lambda_{\text{QCD}}$ , at which scale this interaction produces the bilinear quark condensates in the  $3 \times \bar{3} \rightarrow 1$  channel. We now turn the electroweak interactions back on. By vacuum alignment arguments, the condensates preserve  $U(1)_{\text{em}}$  and the diagonal, vectorial subgroup  $SU(2N_g)_V$  of  $G_{\text{fl}}$ , thereby producing  $(2N_g)^2 - 1$  Nambu-Goldstone bosons (NGB's). Without loss of generality, we can define the ordering of the generational bases for left- and right-handed quarks so that these condensates take the form (summed on  $a$ , not on  $n$ ),

$$\langle \bar{u}_{a,n,L} u_{n,R}^a \rangle + \text{H.c.} \quad \langle \bar{d}_{a,n,L} d_{n,R}^a \rangle + \text{H.c.}, \quad 1 \leq n \leq N_g. \quad (5.9)$$

These condensates transform with weak  $I = 1/2$ ,  $|Y| = 1$  and break  $SU(2)_L \times U(1)_Y$  to  $U(1)_{\text{em}}$ , so three of the NGB's are absorbed to produce longitudinal components and masses for the  $W^\pm$  and  $Z$ , leaving the remaining  $4(N_g^2 - 1)$  (P)NGB's in the spectrum. Denoting  $f_\pi$  as the generalized pion decay constant and  $g$  and  $g'$  as the  $SU(2)_L$  and  $U(1)_Y$  gauge couplings, one has  $m_W^2 = N_g g^2 f_\pi^2 / 4$  and  $m_Z^2 = N_g (g^2 + g'^2) f_\pi^2 / 4$ , satisfying  $m_W^2 = m_Z^2 \cos^2 \theta_W$ . The realization that QCD quark condensates break electroweak symmetry and that three of the resultant NGB's would be absorbed to give the  $W^\pm$  and  $Z$  masses was, indeed, one of the main motivations for the original development of technicolor [7].

The confinement and spontaneous chiral symmetry breaking in the QCD sector generate dynamical, constituent masses of order  $\Lambda_{\text{QCD}}$  for the quarks. Provided that the quarks have zero hard masses, the resultant constituent masses would be equal for  $u$ -type and  $d$ -type quarks, up to electromagnetic corrections. We assume that if one begins with a supersymmetric grand unified theory, the supersymmetry is broken at a higher scale. The spectrum of the theory depends sensitively on the number of matter fermion generations. In the hypothetical case  $N_g = 1$ ,

since the NGB's (pions) are absorbed by the  $W^\pm$  and  $Z$ , the low-lying hadron spectrum would be comprised of the isovector  $\rho$  and isoscalar  $\omega$ , the nucleons, the (non-NGB) isoscalar pseudoscalar analogue of  $\eta'$ , and so forth. For  $N_g = 2$  or  $N_g = 3$ , the spectrum would be qualitatively different because of the (i) residual (P)NGB's and (ii) electrically neutral, spin-1/2, ground state baryons. Indeed, the absence of hard, current-quark masses would mean that, up to electromagnetic effects, the various ground-state baryons of a given spin would be essentially degenerate. Thus, although the  $(uud)$  and  $(ddu)$  baryons would be charged, there would be the spin-1/2 ground-state baryons listed in Eqs. (3.12) and (3.13) for the respective DRS and URS cases.

Concerning the charged baryons, one expects a Coulombic energy contribution roughly proportional to  $q^2/R$  to a hadron of charge  $q$  and size  $R$ . Hence, in the absence of hard quark masses, for  $N_g = 1$ , the  $p$  and  $n$  would be the lightest baryons, while for  $N_g = 2, 3$  the lightest baryons would be the neutral ones in the respective Eqs. (3.12) and (3.13). Among ground-state baryons, electromagnetic mass differences would be of order  $\approx \alpha_{\text{em}} \Lambda_{\text{QCD}}$ , i.e., a few MeV if one continues to take  $\Lambda_{\text{QCD}}$  to have its real-world value of  $\sim 200$  MeV. Since the size of the proton  $p$  and its isospin partner nucleon  $n$  would be essentially the same, and since  $q_p = 2$  and  $q_n = 1$  for the DRS case, one infers that

$$m_p > m_n \quad \text{DRS case.} \quad (5.10)$$

For the URS case, since  $q_p = -1$  while  $q_n = -2$ , one has

$$m_n > m_p \quad \text{URS case.} \quad (5.11)$$

Given that the leptons have very small masses (see below), the following beta decays would occur:

$$p \rightarrow n \ell_1 \ell_2^c \quad (\text{DRS}) \quad (5.12)$$

and

$$n \rightarrow p \ell_2 \ell_1^c \quad (\text{URS}). \quad (5.13)$$

In each of these cases, one could make the formal observation that the lighter nucleon could form a neutral Coulombic bound state that are stable with respect to strong and weak decays, namely

$$[n(1)\ell_1(-1)] \quad (\text{DRS}) \quad (5.14)$$

and

$$[p(-1)\ell_2(1)] \quad (\text{URS}), \quad (5.15)$$

where we have indicated charges in parentheses and implicitly refer to the lightest mass eigenstates in the relevant interaction eigenstates  $\ell_j$ . Also, formally, there could be stable three-body leptonic Coulomb bound states,

$$\begin{aligned} & [\ell_2(-2)\ell_1^c(1)\ell_1^c(1)] \quad (\text{DRS}), \\ & [\ell_1(2)\ell_2^c(-1)\ell_2^c(-1)] \quad (\text{URS}). \end{aligned} \quad (5.16)$$

However, in the absence of Lagrangian mass terms for the matter fermions, both the DRS and URS models resulting from this SO(10) GUT have charged, unconfined  $\ell_1$  and  $\ell_2$  leptons with zero Lagrangian masses. There is mixing between the  $p$  and  $\ell_2^c$  and, separately, mixing between the  $n$  and  $\ell_1^c$ . These mixings generate nonzero, although extremely small, masses for these leptons. To illustrate this, let us take the DRS case for definiteness (similar statements hold for the URS case) and  $N_g = 1$  for simplicity. Here the GUT gauge boson sector includes bosons ( $Y^a(-1), X^a(-2)$ ) transforming as  $(3, 2)_{-3}$  and their adjoints. A tree-level amplitude contributing to proton decay is  $u + u \rightarrow d^c + \ell_2^c$ , mediated by the  $s$ -channel exchange of a  $X_a^\dagger$ . The corresponding  $uud\ell_2$  operator also gives rise to the mixing  $u + u + d \leftrightarrow \ell_2^c$ . The matrix element of this operator between the states  $|p\rangle$  and  $|\ell_2^c\rangle$  is  $\delta \equiv \text{Amp}(p \leftrightarrow \ell_2^c) \simeq (g_{\text{GUT}}^2/M_{\text{GUT}}^2)\Lambda_{\text{QCD}}^3$ , where  $g_{\text{GUT}}$  is the GUT gauge coupling. Diagonalizing the corresponding  $2 \times 2$  mixing matrix, one finds a negligibly small shift in the proton mass  $m_p$  and a nonzero mass

$$m_{\ell_2} = \frac{|\delta|^2}{m_p}. \quad (5.17)$$

Using  $\Lambda_{\text{QCD}} = 200$  MeV and the illustrative values  $\alpha_{\text{GUT}} \simeq 1/24$  and  $M_{\text{GUT}} \simeq 10^{16}$  GeV, we have  $m_{\ell_2} \simeq 10^{-72}$  GeV. A similar mass is generated for  $\ell_1$ . Since these masses are so small, they are obviously quite sensitive to additional ingredients in an ultraviolet completion of the theory and should only be considered as illustrative. Formally, an isolated Coulombic state such those in Eqs. (5.14) or (5.15) would have a size set by the Bohr radius  $a \sim (\alpha_{\text{em}}m_{\ell_j})^{-1}$ ,  $j = 1, 2$ . However, this is only formal, since with the tiny  $\ell_j$  masses, this size would be many orders of magnitude larger than the present size of the (real-world) universe. Unconfined charged leptons with such small masses would cause an infrared instability in the theory, so that the above-mentioned Coulombic bound states would be replaced by a plasma, similar to the situation in the hypothetical case of the SM with conventional fermion charges but without a Higgs field [14].

In view of these various results, we may conclude that even though our SO(10) GUT constructions yielding DRS and URS models as low-energy effective field theories avoid the breaking of  $U(1)_{\text{em}}$  that afflicted the YZ model, they still exhibit exotic properties and striking differences as compared with the real world. Some further remarks may be in order. In the absence of a usual SM Higgs or the pair of MSSM Higgs chiral superfields, the perturbatively calculated partial wave amplitudes for the scattering of longitudinally polarized vector bosons in the DRS and URS models would exceed unitarity upper bounds at a center-of-mass energy somewhat above the EWSB scale. The unitarization of these amplitudes would depend on the ultraviolet completion of the theory. If the only source of

EWSB is QCD, then this unitarization would involve exchanges of the scalar and vector hadrons of QCD at the scale of  $O(1)$  GeV. If one were to adjoin a TC/ETC sector to obtain EWSB at the usual physical scale of 250 GeV, then this unitarization would involve exchanges of technihadrons. One could also consider larger grand unified groups. In particular, we have studied electroweak-singlet quarks in the context of a GUT based on the gauge group  $E_6$ , which contains SO(10) as a subgroup, and have obtained similar conclusions.

Our finding that the DRS and URS models are more tightly constrained when considered as low-energy effective field theories resulting from an SO(10) grand unified theory than when considered in isolation is understandable, since the larger structure of a GUT provides a more predictive theoretical framework. This is analogous to the fact that the ratios of gauge couplings for the three factor groups in the standard model are essentially arbitrary when this theory is considered in isolation, but are predicted when it (or its supersymmetric extension) is embedded in a grand unified theory. In principle, one might further extend the present analysis of electroweak-singlet quarks to the case of GUT's containing an extension of the SM with  $N_c$  different from 3. However, although one can satisfy anomaly constraints in an  $N_c$ -extended standard model, the natural embedding of such a theory in an  $\text{SO}(2(N_c + 2))$  grand unified theory would require

$$2^{N_c+1} = 4(N_c + 1), \quad (5.18)$$

which only has a solution for  $N_c = 3$  [1,15]. Accordingly, we do not pursue such an  $N_c$ -generalization here.

## VI. CONCLUSIONS

In this paper we have sought to gain a deeper understanding of one property of the standard model, namely, the absence of electroweak-singlet quarks. For this purpose we have constructed and studied *gedanken* models that are similar to the standard model but do contain electroweak-singlet quarks. We have found that these models exhibit properties fundamentally different from those of the standard model, including the absence of neutral leptons and the fact that both the ( $uud$ ) and ( $ddu$ ) nucleons are charged. Furthermore, working in the context of a grand unified theory, we have shown that (i) an SU(5) theory with electroweak-singlet quarks leads to a low-energy field theory which, among other things, violates  $U(1)_{\text{em}}$ ; and (ii) SO(10) grand unified theories in which  $Y$  and  $Q$  are generators of SO(10) but not SU(5) can avoid breaking of  $U(1)_{\text{em}}$ , but still generically lead to low-energy effective field theories quite different from the standard model.

## ACKNOWLEDGMENTS

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- $$L_L = \begin{pmatrix} \ell_1(1) \\ \ell_2(0) \end{pmatrix}_L,$$
- $10_L$  containing
- $$Q_L^a = \begin{pmatrix} u^a(1/3) \\ d^a(-2/3) \end{pmatrix}_L,$$
- $d_{a,L}^c(2/3)$ , and  $\ell_{1,L}^c(-1)$ , with  $Y = \text{diag}(2/3, 2/3, 2/3, -1, -1)$  and a possible SU(5)-singlet  $\ell_{2,L}^c(0)$ .
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