Pion distribution amplitude extracted from the experimental data with the local duality sum rule

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The photon-to-pion transition form factor is investigated using the form of the renormalon-based twist-four pion distribution amplitude (DA) in the framework of the light-cone local-duality QCD sum rule, which, with suitable parameters, is insensitive to the higher-order Gegenbauer coefficients. With a careful determination for the insertion parameters so that the contribution from the higher-order Gegenbauer expansions is suppressed, the best-fit central values of the first two nontrivial Gegenbauer coefficients of the pion distribution amplitude are extracted out from the CLEO data to be $a_2(1\ \mathrm{GeV}^2) = 0.145 \pm 0.055$ and $a_4(1\ \mathrm{GeV}^2) = -(0.125 \pm 0.085)$, respectively. The rescaled photon-to-pion transition form factor with our best-fit parameters is consistent very well with both the CELLO data and the prediction of the interpolation formula in all the experimental accessible region of the momentum transfer. The shape of the pion distribution amplitude based on the two-parameter model favors the camel-like type, where the near-end-point values are suppressed more than the asymptotic DA, and satisfies the midpoint constraint from light-cone sum rules approximately.

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I. INTRODUCTION

The production of one neutral pion by two virtual photon fusion plays a crucial role in the study of QCD exclusive processes. For photons with large virtualities, the transition form factor $F^{\gamma^*\pi}$, which relates to the vacuum-to-pion transformation through two electromagnetic currents located at different positions with a lightlike separation, can be expanded with the operator product expansion (OPE) technique near the light cone. It has been turned out that this transition form factor can be factorized, and expressed as a convolution of two factors [1], in which the first one is a process-dependent hard-scattering kernel arising from the short distance dynamics of QCD, and the second one names as the pion distribution amplitude (DA), a universal nonperturbative quantity, coming from the large distance dynamics. Experimentally, the most favorable situation is when one of the photons is nearly real. The corresponding transition form factor, $F^{\gamma\pi}$, has been measured first by the CELLO Collaboration [2], and then extended up to the momentum transfer of 9 GeV^2 by the CLEO Collaboration [3] with high precision. Theoretically, $F^{\gamma\pi}$ has been investigated up to now in different frameworks, such as the light-cone quark models [4], the QCD sum rule methods [5], the instanton-vacuum-based chiral quark models [6–8], etc. Among these, the light-cone QCD sum rules (LCSR) have been developed to be a useful tool to extract the information of the pion DA from the experimental data [9–14]. The main outcome of these theoretical analyses shows that the most famous asymptotic form of the pion DA [1] and the Chernyak-Zhitnitsky (CZ) one [15] are excluded from their first two extracted nontrivial Gegenbauer coefficients of the pion DA. To our knowledge, all the LCSR's results are based on the assumption that the truncated conformal expansion of the pion DA up to the forth Gegenbauer coefficient would be valid. However, the uncertainty using this assumption is ambiguous, and extremely difficult to estimate.

In the present work, we reanalyze the $\gamma\gamma^* \to \pi^0$ transition form factor, with sum rules of a quite different type, namely the light-cone sum rules based on the analytic continuation by duality (ACD) [16–21]. Because of strictly local duality, the ACD sum rules need less phenomenological information, do not suffer from stability problems, and become accurate at large momentum transfer, and of course, they can give a more reliable estimate to uncertainties. Moreover, a suitable choice of the radius of local duality in the ACD sum rules will lead to a great suppression of the magnitude of the factors in front of higher-order Gegenbauer coefficients, so that it gives a possibility to obtain an accurate estimate for the first two nontrivial Gegenbauer coefficients.

The paper is organized as follows: In the next section we present the theoretical expression of the form factor, $F_{\mathrm{QCD}}^{\gamma^*\pi}(Q^2, q^2)$, based on a light-cone OPE and the consistent expressions of the twist-two and twist-four parts of the pion DAs, which are connected with each other by the renormalon model approach. In Sec. III we make a comparison between the Borel LCSR and the light-cone ACD sum rules, and clarify the reasons why we use the ACD sum rules in our work. An error analysis for the ACD sum rules is also discussed in this section. Section IV is devoted to the detailed error estimate for our ACD sum rules, and the optimized parameters used in these sum rules are carefully determined. In Sec. V we present our fit results for the first two nontrivial Gegenbauer coefficients, compare them with other models. Finally, in Sec. VI a summary of our

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conclusions is given, and some open questions are discussed as well.

II. THE TRANSITION FORM FACTOR $F^{\gamma^*\pi}$

The transition form factor for $\gamma^*\gamma^* \to \pi^0$, $F^{\gamma^*\pi}(q^2, Q^2)$, is defined through the correlation function [1]:

$$\Pi_{\mu,\nu}(q_1, q_2) \equiv \int d^4x e^{-iq_1\cdot x}\langle \pi^0(p) \mid T\{j_\mu(x)j_\nu(0)\} \mid 0\rangle = i\epsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta F^{\gamma^*\pi}(Q^2, q^2), \tag{1}$$

where q_1 and q_2 are the momenta of two incident photons, $p = q_1 + q_2$ is the momentum of pion, $Q^2 = -q_1^2 > 0$, $q^2 = -q_2^2 > 0$, and $j_\mu = (2/3)\bar{u}\gamma_\mu u - (1/3)\bar{d}\gamma_\mu d$ is the electromagnetic current of the light quarks.

If the virtualities of the photons are far from zero, the correlation function can be expanded using the OPE technique near the light cone, and the resultant expression turns to be factorizable [1], namely the transition form factor can be expressed as a convolution of a perturbative hard-scattering kernel T, and a universal nonperturbative DA ($\varphi_\pi^{(i)}$ of twist i) [9,10]

$$F^{\gamma^*\pi}(Q^2, q^2) = f_\pi \int_0^1 du\bigg\{\bigg[T_0 + \frac{\alpha_s(\mu_R^2)}{4\pi}T_1\bigg]\varphi_\pi^{(2)}(u, \mu_F^2) - \frac{1}{2}N_T^{-1}T_0^2\varphi_\pi^{(4)}(u, \mu_F^2)\bigg\}, \tag{2}$$

where $f_\pi = 132$ MeV is the pion decay constant $N_T = (e_u^2 - e_d^2)/\sqrt{2} = \sqrt{2}/6$, the QCD normalization factor, u the quark longitudinal momentum fraction, and μ_F and μ_R are the factorization scale and the renormalization scale, respectively. The leading term, T_0, in the hard-scattering kernel is [1]

$$T_0 = \frac{N_T}{\bar{Q}^2}\bigg[\frac{1}{1 + \omega(\bar{u} - u)} + (u \to \bar{u})\bigg], \tag{3}$$

where $\bar{u} = 1 - u$, $\omega = (Q^2 - q^2)/(Q^2 + q^2)$, and $\bar{Q}^2 = -(q_1 - q_2)^2/4 = (Q^2 + q^2)/2$ is the average virtuality of the valence quarks. The next-to-leading order (NLO) radiative correction, T_1, to the leading twist part has been calculated in the Feynman gauge [22] and is represented by [23]

$$\begin{aligned} T_1 = 4T_0\bigg\{&\frac{1}{6}[(1 + \xi\omega)\ln(1 - \xi\omega) + 4(1 - \omega)\ln(1 - \omega) + (1 + \xi\omega)\ln^2(1 - \xi\omega) \\ &- (1 - \omega)\ln^2(1 - \omega) - 9(1 + \xi\omega)] \\ &+ \frac{1}{6}\ln\frac{\bar{Q}^2}{\mu_F^2}[2(1 + \xi\omega)\ln(1 - \xi\omega) - 2(1 - \omega)\ln(1 - \omega) + 3(1 + \xi\omega)] \\ &+ \frac{1}{6\omega^2(1 - \xi^2)}[2(1 + \xi\omega)(1 + \xi\omega - 2\omega^2)\ln(1 - \xi\omega) - 2(1 + \omega)(1 + \omega - 2\omega^2)\ln(1 - \omega) \\ &- (1 + \xi\omega)(1 - \omega^2)\ln^2(1 - \xi\omega) + (1 + \omega)(1 - \omega^2)\ln^2(1 - \omega)] \\ &- \frac{1 - \omega^2}{3\omega^2(1 - \xi^2)}\ln\frac{\bar{Q}^2}{\mu_F^2}[(1 + \xi\omega)\ln(1 - \xi\omega) - (1 + \omega)\ln(1 - \omega)] + (\omega \to -\omega)\bigg\}, \end{aligned} \tag{4}$$

with $\xi = 2u - 1$.

The dependence of the twist-two DA $\varphi_\pi^{(2)}(u; \mu_F^2)$ [$\varphi_\pi(u; \mu_F^2)$ hereafter] on the factorization scale μ_F is governed by the Efremov-Radyushkin-Brodsky-Lepage (ERBL) evolution equation [1]:

$$\frac{d\varphi_\pi(u; \mu_F^2)}{d\ln\mu_F^2} = \int_0^1 dv V(u, v; \alpha_s(\mu_F^2))\varphi_\pi(v; \mu_F^2), \tag{5}$$

with the ERBL kernel

$$V(u, v; \alpha_s(\mu_F^2)) = \frac{\alpha_s(\mu_F^2)}{4\pi}V_0(u, v) + \frac{\alpha_s^2(\mu_F^2)}{16\pi^2}V_1(u, v) + \cdots, \tag{6}$$

where V_0 and V_1 come from one- and two-loop contributions, respectively.

With the knowledge of the ERBL evolution kernel in one-loop approximation [1], it is useful to expand the twist-two DA, $\varphi_\pi(u, \mu_F^2)$, in the eigenfunctions of ERBL equation, namely to make the conformal expansion in terms of Gegenbauer polynomials $C_n^{3/2}(\xi)$,

$$\varphi_\pi(u; \mu_F^2) = \Omega(u){\sum_{n\geq 0}}' a_n(\mu_F^2)C_n^{3/2}(\xi), \qquad \Omega(u) \equiv 6u(1 - u), \qquad a_0 = 1, \tag{7}$$

where the summation ${\sum}'_{n\geq 0}$ is taken only over even indices $n \geq 0$ accounting for the symmetry under isospin transformation and charge conjugation. The scale dependence of the DA is determined by

$$a_n^{\text{1-loop}}(\mu_F^2) = a_n(\mu_0^2)\bigg[\frac{\alpha_s(\mu_F^2)}{\alpha_s(\mu_0^2)}\bigg]^{\gamma(n)}, \tag{8}$$

where $\gamma(n) \equiv \gamma_0(n)/2b_0$ with $\gamma_i(n)$ and b_i being the anomalous dimensions and the β-function coefficients, respectively.

When one evolves over a large interval in μ_F or when α_s at the starting scale μ_0 is large, the NLO evolution will become more important. However, to the NLO accuracy, the $C_n^{3/2}(\xi)$ are no longer eigenfunctions of the evolution, so that their coefficients do not evolve independently. Namely, $a_n(\mu_F)$ at $\mu_F > \mu_0$ depends on all coefficients $a_2(\mu_0), \ldots, a_n(\mu_0)$. The solution to the evolution equation at the two-loop level is [24]

$$\varphi_\pi^{\text{2-loop}}(u, \mu_F^2) = \Omega(u)\sum_n{}' a_n(\mu_0^2)E_n(\mu_F^2, \mu_0^2)\bigg[\psi_n(u) + \frac{\alpha_s(\mu_F^2)}{4\pi}\sum_{j>n}{}' d_{n,j}(\mu_F^2, \mu_0^2)\psi_j(u)\bigg]. \tag{9}$$

It implies that

$$a_n^{\text{2-loop}}(\mu_F^2) = E_n(\mu_F^2, \mu_0^2)a_n(\mu_0^2) + \frac{\alpha_s(\mu_F^2)}{4\pi}\sum_{0\le j<n}{}' E_j(\mu_F^2, \mu_0^2) \times d_{j,n}(\mu_F^2, \mu_0^2)a_j(\mu_0^2), \tag{10}$$

where

$$E_n(\mu_F^2, \mu_0^2) = \left[\frac{\alpha_s(\mu_F^2)}{\alpha_s(\mu_0^2)}\right]^{\gamma(n)} \times \left[\frac{b_0 + b_1\alpha_s(\mu_F^2)/(4\pi)}{b_0 + b_1\alpha_s(\mu_0^2)/(4\pi)}\right]^{(\gamma_1(n)b_0-\gamma_0(n)b_1)/2b_0b_1}, \tag{11}$$

for the "diagonal" part, and

$$d_{j,n}(\mu_F^2, \mu_0^2) = \frac{M_{j,n}}{2b_0[\gamma(n) - \gamma(j) - 1]} \times \left\{1 - \left[\frac{\alpha_s(\mu_F^2)}{\alpha_s(\mu_0^2)}\right]^{\gamma(n)-\gamma(j)-1}\right\}, \tag{12}$$

for the mixing coefficients, and the numerical values of the anomalous dimensions and the first few elements of the matrix M_{jn} are [25]

$$\begin{aligned}
&\gamma_0(0) = 0, \quad \gamma_1(0) = 0, \quad \gamma_0(2) = \frac{100}{9}, \quad \gamma_1(2) = \frac{34\,450}{243} - \frac{830}{81}N_f, \quad \gamma_0(4) = \frac{728}{45},\\
&\gamma_1(4) = \frac{662\,846}{3375} - \frac{31\,132}{2025}N_f, \quad \gamma_0(6) = \frac{2054}{105}, \quad \gamma_1(6) = \frac{958\,337\,651}{4\,116\,000} - \frac{3\,745\,727}{198\,450}N_f;\\
&M_{02} = -11.2 + 1.73N_f, \quad M_{04} = -1.41 + 0.565N_f, \quad M_{24} = -22.02 + 1.65N_f,\\
&M_{06} = 0.0259 + 0.0259N_f, \quad M_{26} = -7.765 + 0.823N_f, \quad M_{46} = -22.77 + 1.39N_f,
\end{aligned} \tag{13}$$

respectively, with N_f being a number of active quark flavors.

The higher-twist component of the transition form factor depends on the higher quark-gluon Fock state and the quark transverse momentum. In the LCSR, the asymptotic form of the twist-four DA is usually used for simplicity [9–12]. Dorokhov [26] has calculated the transition form factor up to twist-four in the instanton-vacuum-based effective quark-meson model, and extracted the twist-four DA of pion. Recently, the renormalon approach [27], which relates the leading twist DA to the twist-four DA, has been used to explore the twist-four DA beyond the asymptotic approximation. This kind of renormalon-motivated twist-four DA of pion has been adopted by Agaev first to analyze the experimental data [14]. The analysis is then improved by Bakulev *et al.* [13], and a consistent expression in the renormalon approach is given in the form of

$$\varphi_\pi^{(4)}(u, \mu_F^2) = \delta^2(\mu_F^2)\int_0^1 dv K(u, v)\varphi_\pi(v), \tag{14}$$

where the kernel K is

$$K(u, v) = -\frac{2}{3}\left\{\theta(v > u)\left[\frac{u\bar{u}}{v^2} + \frac{1}{v}\ln\left(1 - \frac{u}{v}\right)\right] + (u \to \bar{u}, v \to \bar{v})\right\}, \tag{15}$$

and the coupling δ^2 is defined by [28]

$$\begin{aligned}&\langle\pi(p) \mid g_s\bar{d}\tilde{G}_{\alpha\mu}\gamma^\alpha u \mid 0\rangle = i\delta^2 f_\pi p_\mu,\\ &\tilde{G}_{\alpha\mu} = \tfrac{1}{2}\epsilon_{\alpha\mu\rho\sigma}G^{\rho\sigma}, \qquad G_{\rho\sigma} = G_{\rho\sigma}^a\lambda^a/2,\end{aligned} \tag{16}$$

with the one-loop scale dependence being given by [28]

$$\delta^2(\mu^2) = \left[\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right]^{\gamma_{T_4}/b_0}\delta^2(\mu_0^2), \qquad \gamma_{T_4} = \frac{32}{9}. \tag{17}$$

For the sake of consistency between the twist-two and four DAs of pion in the renormalon model, we prefer to use Eq. (14) in this paper. For latter convenience, we complete the integration over v in Eq. (14), and obtain an explicit expression of the twist-four DA in terms of the first three

Gegenbauer polynomial expansion coefficients:

$$\begin{aligned}\varphi_\pi^{(4)}(u,\mu_F^2) = \delta^2(\mu_F^2)\Big\{u\bar{u}\Big[-5a_2(\mu_F^2)(-2+u\bar{u}) - \frac{35}{10}a_4(\mu_F^2)(-8-35u\bar{u}+94u^2\bar{u}^2)\\ + a_6(\mu_F^2)\Big(54+\frac{8882}{10}u\bar{u} - \frac{51\,832}{10}u^2\bar{u}^2 + \frac{75\,647}{10}u^3\bar{u}^3\Big)\Big] + 4u^2[-1+a_2(\mu_F^2)(-1+10\bar{u})\\ + a_4(\mu_F^2)(-1+28\bar{u}-63\bar{u}^2+126\bar{u}^3) + 3a_6(\mu_F^2)\bar{u}(18+99\bar{u}-484\bar{u}^2-858\bar{u}^3+572\bar{u}^4)]\ln u + (u\to\bar{u})\Big\}.\end{aligned} \tag{18}$$

For the case where one photon is nearly real, i.e. $q^2 \to 0$, the interaction induced by this photon at large distance will play an important role [5], and such nonperturbative effects cannot be sufficiently suppressed by using only the truncated light-cone OPE in Eq. (2). One way to avoid this disease is to work with the LCSR method [9], which allows the QCD theoretical calculation at sufficient large q^2 at first, and then returns to the real photon case by means of the dispersion relation. We will follow this stratagem, but with a different type of sum rules, the ACD sum rules.

III. THE ACD SUM RULES

A. The reason for using the local-duality sum rules

The ACD sum rules belong to a class of strictly local QCD sum rules. The key point is that the transition form factor $F^{\gamma^*\pi}(Q^2, t\equiv q_2^2)$ is an analytic function in the complex t plane with a cut on the positive real axis running from the lowest resonance threshold to infinity. Then, the dispersion relation may be written as

$$\begin{aligned}F^{\gamma^*\pi}(Q^2,t_0) &= \frac{1}{2\pi i}\oint_\Gamma dt\frac{F^{\gamma^*\pi}(Q^2,t)}{t-t_0}\\ &= \frac{1}{\pi}\int_0^R dt\frac{\mathrm{Im}F^{\gamma^*\pi}(Q^2,t)}{t-t_0} + \frac{1}{2\pi i}\\ &\quad\times\oint_{|t|=R} dt\frac{F^{\gamma^*\pi}(Q^2,t)}{t-t_0},\end{aligned} \tag{19}$$

where Γ denotes the integral contour composed of a circle C_R and a cut on the positive real axis as shown in Fig. 1. Because of the asymptotic behavior of $F^{\gamma^*\pi}$, we choose the dispersion relation without subtraction.

It is obvious that, when the radius of the circle becomes infinite, and the integral along the circle vanishes, we get the usual sum rule

$$F^{\gamma^*\pi}(Q^2,t_0) = \frac{1}{\pi}\int_0^\infty dt\frac{\mathrm{Im}F^{\gamma^*\pi}(Q^2,t)}{t-t_0}. \tag{20}$$

Then, the spectral density can be calculated from the imaginary part of the transition form factor according to the global or semilocal quark-hadron duality above some threshold s_0,

$$\int_{s_0}^\infty dt\frac{\rho^h(Q^2,t)}{t-t_0} = \frac{1}{\pi}\int_{s_0}^\infty dt\frac{\mathrm{Im}F^{\gamma^*\pi}(Q^2,t)}{t-t_0}. \tag{21}$$

The global duality assumption introduces a source of systematic uncertainties, which can be controlled qualitatively after the Borel transformation if there is a stability window of the Borel mass and the duality threshold. When the radius R of the circle C_R is finite but still sufficiently large, the pointwise quark-hadron duality, namely the local duality, is valid, and the uncertainty of duality assumption can be estimated in a controlled and quantitative way.

Usually, the pion DA can be parametrized by only two variables, $a_2(\mu_0^2)$ and $a_4(\mu_0^2)$. This two-parameter model enables one to fit the experimental constraints for the coefficients, $\langle\xi^N\rangle_\pi \equiv \int_0^1 \varphi_\pi(x)(2x-1)^N dx$, up to $N=10$ with the nonlocal-condensate (NLC) QCD sum rules [29]. Although the two-loop evolution of the two-parameter model generates the higher Gegenbauer harmonics, the contribution of the corresponding higher coefficients still remain numerically negligible, so that the analysis of the transition form factor can be safely carried out within the framework of this two-parameter model [11,13]. However, as an initial input, the validity of the two-parameter model analysis from the CLEO data strongly depends on the value of the asymmetric parameter $\omega = (Q^2-q^2)/(Q^2+q^2)$, namely the kinematic region

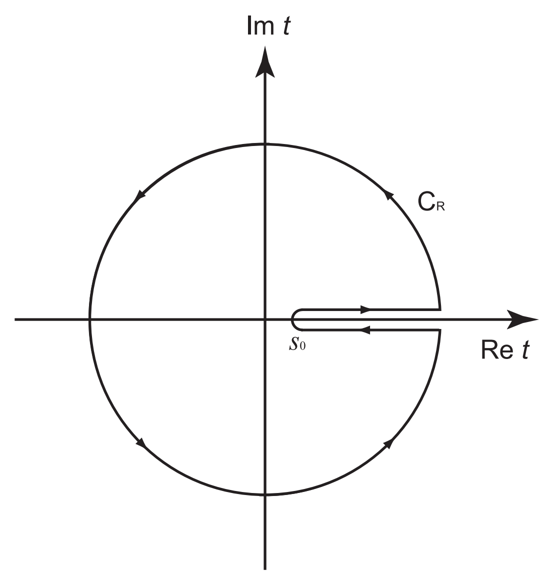


FIG. 1. The contour of the integration in the complex t plane.

of ω. For instance, the transition form factor in the spacelike region can be expressed as

$$F^{\gamma^*\pi}(Q^2, q^2) = \frac{f_\pi}{\sqrt{2}\cdot\bar{Q}^2}\bigg[W_0(\omega, \mu_R) + \sum_{n>0}{}^{\prime} W_n(\omega, \mu_R) a_n(\bar{Q}^2)\bigg], \tag{22}$$

where, for simplicity, we set the factorization scale $\mu_F^2 = \bar{Q}^2$. In the small $|\omega|$ region, $W_n \sim \omega^n$, so that the conformal expansion converges fast. However, in the real photon limit, $|\omega| \to 1$, all the Gegenbauer coefficients are seized of nearly equal weights, $W_n(1, \mu_R) \approx 1$ [23]. The two-parameter model is thus not suitable in this kinematic region if the Gegenbauer coefficients decrease not so fast with n, just as in the instanton-vacuum-based chiral quark model [6–8] and the light-cone quark model [4]. For the usual LCSR, the value of ω is an average over the global duality region, and eventually $|\omega| \to 1$ when q^2 grows. However, in the ACD sum rules, the exact local duality allows us to choose an appropriate R, so that the values of $W_{n>4}$ corresponding to the local-duality point are suppressed, and the estimate of the first two nontrivial Gegenbauer coefficients from the sum rules is reliable.

B. Description of the ACD sum rules

The integral in Eq. (19) along the real t axis from $4m_\pi^2$ to $t_{\rm th}$ is the low energy physical absorptive part, and the corresponding spectral density contains resonances in the interval $[4m_\pi^2, t_{\rm th}]$, which cannot be produced from the light-cone OPE in general. As necessary phenomenological information, these resonances' contribution is assumed to be dominated by the lower vector mesons, and described as the finite-width Breit-Wigner form [9],

$$F_R(Q^2, t_0) \equiv \frac{1}{\sqrt{2}\pi}\sum_{V=\rho,\omega}\int_{4m_\pi^2}^{t_{\rm th}} dt \times \frac{m_V\Gamma_V f_V F^{V\pi}(Q^2)}{[(m_V^2 - t)^2 + m_V^2\Gamma_V^2](t - t_0 - i\epsilon)}, \tag{23}$$

where $F^{V\pi}(Q^2)$ and f_V are defined through the matrix elements of electromagnetic current

$$\langle\pi^0(p)|j_\mu|V(q_2)\rangle = F^{V\pi}(Q^2)m_V^{-1}\epsilon_{\mu\nu\alpha\beta}e^\nu q_1^\alpha q_2^\beta, \tag{24}$$

and

$$\langle V|j_\nu|0\rangle = \frac{f_V}{\sqrt{2}}m_V e_\nu^*, \tag{25}$$

respectively, with e_ν being the polarization vectors of the corresponding vector mesons. The isospin symmetry leads to

$$\tfrac{1}{3}\langle\pi^0(p)|j_\mu|\omega(q_2)\rangle \approx \langle\pi^0(p)|j_\mu|\rho^0(q_2)\rangle, \tag{26}$$

which implies

$$F^{\omega\pi}(Q^2) \approx 3F^{\rho\pi}(Q^2). \tag{27}$$

Assuming the spectral density is smooth in t within the interval $[t_{\rm th}, R]$ and can be represented by a polynomial of degree $m \le N_1$, an auxiliary analytic function $D_{N_1}^{(1)}(t)$ with $D_{N_1}^{(1)}(t_0) = 1$ may be introduced as a weight factor to suppress the Cauchy kernel by the least-square fit routine [16–19],

$$F^{\gamma^*\pi}(Q^2, t_0) = \frac{1}{\sqrt{2}\pi}\sum_{V=\rho,\omega}\int_{4m_\pi^2}^{t_{\rm th}} dt \times \frac{m_V\Gamma_V f_V F^{V\pi}(Q^2)D_{N_1}^{(1)}(t)}{[(m_V^2 - t)^2 + m_V^2\Gamma_V^2](t - t_0 - i\epsilon)} + \frac{1}{2\pi i}\oint_{|t|=R} dt\frac{D_{N_1}^{(1)}(t)F^{\gamma^*\pi}(Q^2, t)}{t - t_0} + \Delta^{\rm fit}, \tag{28}$$

where a specific form for the weight factor is chosen to be

$$D_{N_1}^{(1)}(t) = 1 - \frac{t - t_0}{R}\sum_{n=0}^{N_1} c_n\left(\frac{t}{R}\right)^n, \tag{29}$$

the coefficients c_n are determined by the conditions

$$\int_{t_{\rm th}}^R dt\frac{D_{N_1}^{(1)}(t)}{t - t_0}t^m = 0, \qquad \text{for } m = 0, 1, \ldots, N_1, \tag{30}$$

and the fit error is

$$\Delta^{\rm fit} \equiv \frac{1}{\pi}\int_{t_{\rm th}}^R dt\left[\frac{1}{t - t_0} - \frac{1}{R}\sum_{n=0}^{N_1} c_n\left(\frac{t}{R}\right)^n\right]{\rm Im}F^{\gamma^*\pi}(Q^2, t). \tag{31}$$

which goes to zero for $N_1 \to \infty$. On the other hand, on the circle C_R, only the truncated terms in the asymptotic expansion of $F^{\gamma^*\pi}$ are known in practice. It gives rise to an additional source of error named asymptotic error, $\Delta^{\rm asy}$. Choosing a larger value of N_1 will increase the contribution from the truncated terms, and thus enhances the asymptotic error [19], while the fit error will be larger for N_1 being smaller. The order of magnitude of N_1 is usually chosen to be less than or equal to the order of the truncated expansions, see Sec. IV for more details.

There may be an additional error in the ACD trick generated by an inappropriate procedure of the analytic continuation of the theoretical expansion from the spacelike region of t, the negative real axis of the t plane, to the whole circle C_R. As shown in Ref. [18], if one neglects the t dependence of the expansion coefficients on the circle C_R, and just replaces them with their expressions in the spacelike region, for example

$$F^{\gamma^*\pi}(Q^2, t) \simeq F^{\gamma^*\pi}(Q^2, t = q^2 < 0), \tag{32}$$

in our case, it will then introduce the so-called analytic continuation error, Δ^{AC}, and may be out of control [19]. To avoid the influence from such analytic continuation error, we try to take the t dependence of $F^{\gamma^*\pi}$ on the circle C_R into account, and analytically extrapolate the expression of $F^{\gamma^*\pi}$ from the negative real axis of t to the circle C_R by considering t as a complex variable. This simple process for the analytical continuation exhibits also one disease, namely the expression for the perturbative part must be divergent at some fixed ω in the timelike region where $|\omega| > 1$, as seen from Eqs. (3) and (4). Moreover, this singularity may be far from the end points, and thus, it cannot be suppressed by the DA. To cure this disease, another weight function of the same type as before is introduced [20,21]:

$$D^{(2)}_{N_2}(t) = 1 - \frac{t - t_0}{R}\sum_{n=0}^{N_2} d_n\left(\frac{t}{R}\right)^n, \tag{33}$$

with the coefficients d_n determined by

$$\frac{d^m}{dt^m}D^{(2)}_{N_2}(t)(t = R) = 0, \qquad m = 0, 1, 2, \ldots, N_2. \tag{34}$$

which leads to a replacement of the weight factor in Eq. (28):

$$D^{(1)}_{N_1}(t) \to D_{N_1,N_2}(t) \equiv D^{(1)}_{N_1}(t)D^{(2)}_{N_2}(t). \tag{35}$$

Finally, in the limit $t_0 \to 0$, we obtain the ACD expression for the transition form factor,

$$F^{\gamma\pi}(Q^2) \equiv F^{\gamma\pi}(Q^2, q^2 = 0) = F^{\rho\pi}(Q^2)I_R + I(Q^2), \tag{36}$$

where

$$I_R \equiv \frac{1}{\sqrt{2}\pi}\sum_{V=\rho,\omega}\int_{4m_\pi^2}^{t_{\rm th}} {\rm d}t\frac{c_V m_V\Gamma_V f_V D_{N_1,N_2}(t)}{[(m_V^2 - t)^2 + m_V^2\Gamma_V^2](t - t_0)},$$

$$c_\rho = 1, \qquad c_\omega = 3$$

$$I(Q^2) \equiv \frac{1}{2\pi}\int_0^{2\pi} {\rm d}\phi D_{N_1,N_2}(Re^{i\phi})F^{\gamma^*\pi}(Q^2, Re^{i\phi}). \tag{37}$$

The two equations for $F^{\gamma\pi}(Q^2)$ and $F^{\rho\pi}(Q^2)$, Eq. (36) with (N_1, N_2) and (N_1', N_2') respectively, can be solved to obtain

$$F^{\gamma\pi}(Q^2) = \frac{I_R' I(Q^2) - I_R I'(Q^2)}{I_R' - I_R}, \tag{38}$$

$$F^{\rho\pi}(Q^2) = \frac{I(Q^2) - I'(Q^2)}{I_R' - I_R}, \tag{39}$$

where the unprimed quantities and the primed ones correspond to the different choices of a pair of the indices appearing in $D_{N_1}(t)$ and $D_{N_2}(t)$, namely (N_1, N_2) and (N_1', N_2'), respectively. Equations (38) and (39) are our local ACD sum rules for determining the form factors $F^{\gamma\pi}(Q^2)$ and $F^{\rho\pi}(Q^2)$.

IV. DETERMINATION OF THE PARAMETERS

To determine the values of $t_{\rm th}$, the vector-meson masses, widths

$$m_\rho = 770\ \text{MeV}, \qquad \Gamma_\rho = 151\ \text{MeV};$$
$$m_\omega = 782\ \text{MeV}, \qquad \Gamma_\omega = 8\ \text{MeV}, \tag{40}$$

and the coupling $f_\pi = 132$ MeV, $f_\rho = 216$ MeV are used, while the coupling of ω meson has not been fixed unambiguously. From the approximate relations $3\langle\omega|j_\nu|0\rangle \approx \langle\rho^0|j_\nu|0\rangle$, we have $f_\omega \approx \frac{1}{3}f_\rho$, which is consistent with the extraction of the resonance parameters from the experimental data [30]. It is found numerically that the spectral density of ρ and ω mesons in our expression becomes negligible when s goes beyond 1 GeV^2, which is consistent with the conclusion that the duality radius in the rho-meson channel was 1.5 GeV^2 [31]. Thus, we take $t_{\rm th} = 1.25$ GeV^2 for the duality radius in our estimation. The coupling $\delta^2(\mu_0^2)$ was originally estimated in [28,32], reestimated in [11], and found to be $\delta^2(1\ \text{GeV}^2) = 0.19 \pm 0.02$ GeV^2. We use 0.19 GeV^2 for $\delta^2(1\ \text{GeV}^2)$ in the numerical analysis except for explicit statement. The ERBL evolution will be executed at the two-loop level with the two-loop strong coupling constant as in [11]. Up to now, there are still three parameters in our ACD sum rule, i.e. R, N_1, and N_2, left to be determined.

The determination of the R value is not a trivial task. Choosing a larger value of R could reduce the uncertainty resulting from the higher-twist DAs, and the uncertainty from the next-to-next-to-leading order (NNLO) perturbative correction since the renormalization scale μ_R is proportional to $(Q^2 + R)$ in some renormalization schemes. However, if R exceeds some value, the corresponding absolute value of $\omega = (Q^2 - R)/(Q^2 + R)$ with some fixed Q^2 will be larger than 0.8 in the spacelike region, and the neglected higher Gegenbauer coefficients will take an important role when those Gegenbauer coefficients $a_{n>4}$ are not very small as usually expected. Therefore, within the experimentally acquirable range of Q^2, the larger R, and consequently the larger $|\omega|$, may lead to the invalidation of the two-parameter model of the pion DA. Furthermore, we hope to avoid/suppress the influence from the resonances in the J/ψ region. Therefore, an upper bound for R is set to be 9 GeV^2, and the other values, namely $R = 3$ and 5.76 GeV^2, are also checked to obtain the optimized R for the sum rule.

To avoid an additional running due to the renormalization, the natural renormalization scale, μ_R^2, may be chosen to be lower than the usual average virtuality of the valence quarks, $(Q^2 + R)/2$, since the large transferred momentum is, in fact, partitioned among many propagators in the hard-scattering amplitude. In these attempts, the Brodsky-Lepage-Mackenzie scale [33,34] is expected to decrease the influence from the NNLO radiative correction, but its rather low value $Q^2/9$ [33] questions the applicability of the perturbative QCD prediction at experimental accessible momentum transfers. In the so-called physically based scheme, the α_V scheme [35], the renormalization scale is determined to be $Q^2/1.7$ for the amplitude of $\gamma^*\gamma \to \pi$ calculated with the asymptotic distribution and $\overline{\mathrm{MS}}$ renormalization scheme, which becomes as, say $(Q^2+q^2)/4$ roughly, corresponding to our case, $\gamma^*\gamma^* \to \pi$. It is expected that the higher-order QCD corrections are minimized due to the fact that this scale reflects the mean (NLO) virtuality of the exchanged gluons [33]. In the spirit of the α_V scheme, the natural renormalization scale μ_R^2 is set to be $(Q^2+R)/4$ in our ACD sum rule approach.

Now we deal with the estimation of N_1 and N_2. In the ACD sum rules, N_1, N_2 are determined by minimizing the amount of magnitude of the fit and asymptotic errors [17,21], and play the role similar to the stability window in the Borel sum rules. The error of the transition form factor is

$$|F^{\gamma\pi}_{\mathrm{exact}}(Q^2) - F^{\gamma\pi}_{\mathrm{theory}}(Q^2)| \le \Delta^{\mathrm{asy}} + \Delta^{\mathrm{fit}}, \qquad (41)$$

where the analytic continuation error is negligible after making an appropriate analytic continuation for t. The fit error is bounded by

$$\Delta^{\mathrm{fit}} \le \Delta^{\mathrm{fit\text{-}max}} \equiv \frac{\mathcal{M}}{\pi}\int_{t_{\mathrm{th}}}^{R} dt \left|\frac{D_{N_1,N_2}(t)}{t}\right|, \qquad (42)$$

where $\mathcal{M}$ denotes the maximal mean value of the spectral density $\mathrm{Im}F_R(t,Q^2)$ within the interval $[t_{\mathrm{th}}, R]$, and is set to be $1.0\ \mathrm{GeV}^{-1}$ since the maximal value of the spectral density $\mathrm{Im}F_R(t,Q^2)$ in our analysis is, at most, $9.4\ \mathrm{GeV}^{-1}$. The resultant upper bounds of the fit error corresponding to different values of N_1, N_2, and R are shown in Fig. 2 and Table I. It is obvious from this estimate that the larger the values of N_1 and N_2, the smaller the fit errors. We will choose $N_1 = 2$, 3 and $N_2 = 5$, 6 to make the fit errors to be minimal and negligible.

TABLE I. The $\Delta^{\mathrm{fit\text{-}max}}$ with some parameters.

$\Delta^{\mathrm{fit\text{-}max}}$ [GeV^{-1}]	$R = 5.76\ \mathrm{GeV}^2$	$R = 9\ \mathrm{GeV}^2$
$N_1 = 3$, $N_2 = 5$	1.43×10^{-4}	1.16×10^{-3}
$N_1 = 3$, $N_2 = 6$	8.4×10^{-5}	7.75×10^{-4}

The asymptotic error in ACD sum rule is defined as

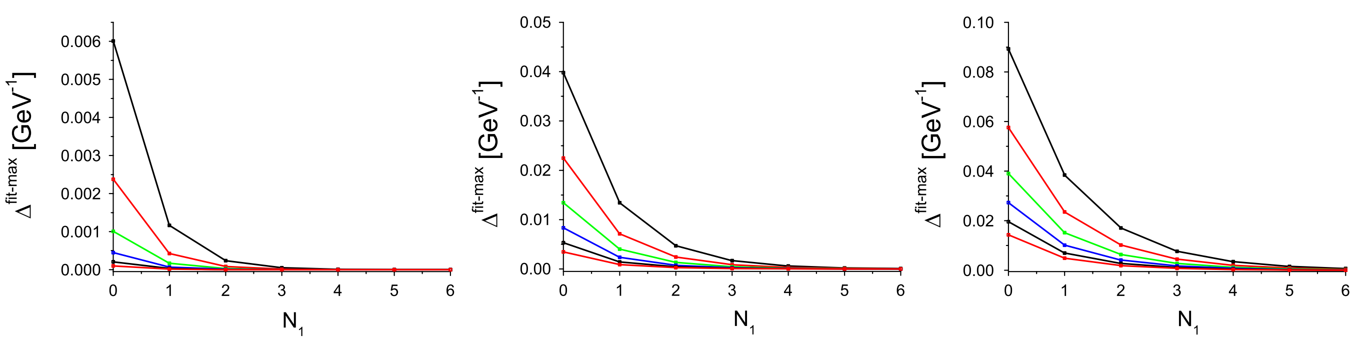
$$\Delta^{\mathrm{asy}} = \frac{1}{2\pi}\left|\int_0^{2\pi} D_{N_1,N_2}(Re^{i\phi})[F^{\gamma^*\pi}(Q^2, Re^{i\phi}) - F^{\gamma^*\pi}_{\mathrm{QCD}}(Q^2, Re^{i\phi})]d\phi\right|, \qquad (43)$$

where the QCD prediction, $F^{\gamma^*\pi}_{\mathrm{QCD}}(Q^2,t)$, is considered as an asymptotic expression of the physical farm factor, $F^{\gamma^*\pi}(Q^2,t)$. One of the upper bounds for the asymptotic error is traditionally estimated to be [17,21]

$$\Delta^{\mathrm{asy\text{-}max}} = \frac{\epsilon}{2\pi}\int_0^{2\pi} d\phi |D_{N_1,N_2}(Re^{i\phi})|, \qquad (44)$$

where ϵ stands for the upper bound of $|F^{\gamma^*\pi}(Q^2,t) - F^{\gamma^*\pi}_{\mathrm{QCD}}(Q^2,t)|$ in the spacelike region. However, it is much overestimated so that $\Delta^{\mathrm{asy\text{-}max}} \gg \Delta^{\mathrm{asy}}$ because of the strong enhancement due to the phase decoherence in $\Delta^{\mathrm{asy\text{-}max}}$. We prefer to trace back to the original definition (43).

The difference between $F^{\gamma^*\pi}(Q^2,t)$ and $F^{\gamma^*\pi}_{\mathrm{QCD}}(Q^2,t)$ comes from two sources, one of which is the NNLO radiative correction, $F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)$, for the hard kernel of the leading twist part, and the other one the neglected higher twists. The latter will not be considered in this paper for two reasons: First, the contributions higher than twist-four to the photon-to-pion transition form factor have not been calculated up to now to our knowledge; second, the (factorizable) twist-six contribution to the electromagnetic pion form factor is only about 2% of the twist-four one, and thus negligible [36]. We thus assume that such higher-twist corrections are similar or even more negligible since the experimental accessible momentum transfers in our case



(a) $R = 3\mathrm{GeV}^2$ (b) $R = 5.76\mathrm{GeV}^2$ (c) $R = 9\mathrm{GeV}^2$

FIG. 2 (color online). The dependence of $\Delta^{\mathrm{fit\text{-}max}}$ on N_1 and N_2 for $R = 3$, 5.76, and 9 GeV^2. From above to below, the plots (discrete data connected by straight lines) correspond to $N_2 = 1$, 2, 3, 4, 5, and 6, respectively.

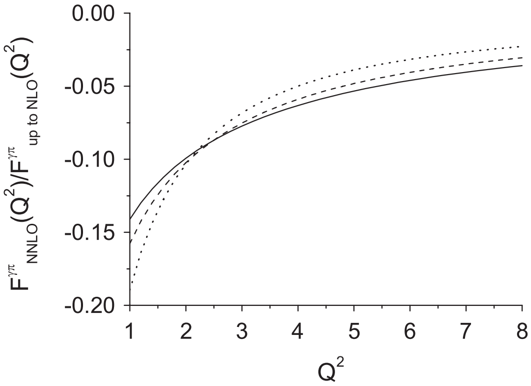


FIG. 3. The ratio $F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)/F^{\gamma\pi}_{\mathrm{up\,to\,NLO}}(Q^2)$ for the renormalization scale, $\mu_R^2 = (Q^2 + R)/4$, with R being 3 GeV^2 (dotted line), 5.76 GeV^2 (dashed line), and 9 GeV^2 (solid line).

are higher. Therefore, we have approximately

$$\Delta^{\mathrm{asy}} \approx \frac{1}{2\pi}\left|\int_0^{2\pi} D_{N_1,N_2}(Re^{i\phi})F^{\gamma^*\pi}_{\mathrm{NNLO}}(Q^2, Re^{i\phi})d\phi\right| = |F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)|_{\mu_R^2=(Q^2+R)/4,\,\mu_F^2=2Q^2}, \tag{45}$$

where the fact that the average of any analytical function on a circle is just its value at the center is used. It is worth noting that, after the integration over the circle c_R, the chosen value of μ_R in our ACD sum rules will remain to be invariant, and occur in the expression of $F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)$. The remarkable characteristic of Δ^{asy} is that it is, in fact, independent of N_1 and N_2, which are simply determined by minimizing Δ^{fit}.

Now as a quantitative estimate, let us use the result for the radiative correction up to the NNLO order in the $\overline{\mathrm{MS}}$ scheme [see Eq. (4.3) in [34]] including only a large part of $F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)$ proportional to the leading β function, and calculate the ratio of $F^{\gamma\pi}_{\mathrm{NNLO}}(Q^2)$ to $F^{\gamma\pi}_{\mathrm{up\,to\,NLO}}(Q^2)$ which are shown in Fig. 3. In most of the Q^2 region of the data, the influence from NNLO correction is below 10%. Another observation is that $R = 9$ GeV^2 is a better choice in the whole Q^2 region. The magnitudes of the asymptotic error for R to be the Schmedding-Yakovlev (SY) scale $\mu_{\mathrm{SY}}^2 \equiv 5.76$ GeV^2 are listed in Table II. From Tables I and II, we can see that the upper bound of the total error of the ACD sum rules is about 10^{-4} GeV^{-1} for smaller N_1 and N_2, and R being 5.76 or 9 GeV^2.

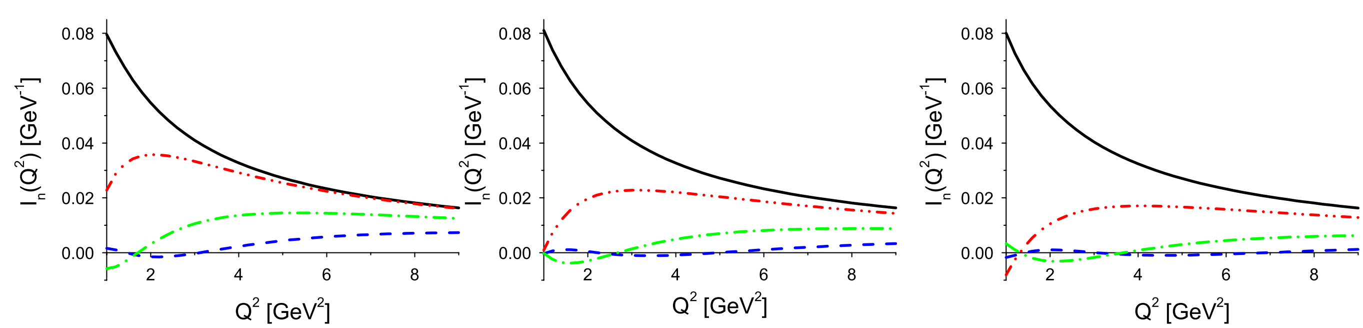
As the end of this section, we note that the magnitude of the R value has an important role to control the convergence of the conformal expansion for the transition form factor in the numerical simulation. Rewriting the integral expression on the circle in terms of the first three Gegenbauer coefficients

$$\begin{aligned} I(Q^2)|_{|t|=R} = {} & I_0(Q^2)|_{|t|=R} + I_2(Q^2, \mu_F^2)\cdot a_2(\mu_F^2)|_{|t|=R} \\ & + I_4(Q^2, \mu_F^2)\cdot a_4(\mu_F^2)|_{|t|=R} \\ & + I_6(Q^2, \mu_F^2)\cdot a_6(\mu_F^2)|_{|t|=R}. \end{aligned} \tag{46}$$

We can show in Fig. 4 the dependence of the different weights, I_n, on Q^2 in the summation $I(Q^2)$ for different choices of the R value. The absolute value of I_6 is nearly negligible compared with others when R is not less than $\mu_{\mathrm{SY}}^2 \equiv 5.76$ GeV^2, so that the first two nontrivial Gegenbauer coefficients of pion DA extracted from the experiment can be creditable.

TABLE II. The NNLO radiative corrections for $R = 5.76$ GeV^2.

Q^2 [GeV^2]	1.00	2.00	3.00	5.76	8.00
Δ^{asy} [GeV^{-1}]	1.2×10^{-2}	3.8×10^{-3}	1.9×10^{-3}	5.6×10^{-4}	2.9×10^{-4}



(a) R = 3GeV^2 (b) R = 5.76GeV^2 (c) R = 9GeV^2

FIG. 4 (color online). The dependence of the weights I_n ($n \le 6$) on Q^2 for $\mu_R^2 = \mu_F^2 = (Q^2 + R)/2$, $N_1 = 3$, $N_2 = 5$ and (a) $R = 3$ GeV^2, (b) $R = 5.76$ GeV^2, (c) $R = 9$ GeV^2. From above to below, the plots correspond to the weights I_0 (solid), I_2 (dot-dot-dashed), I_4 (dash-dotted), and I_6 (dashed), respectively.

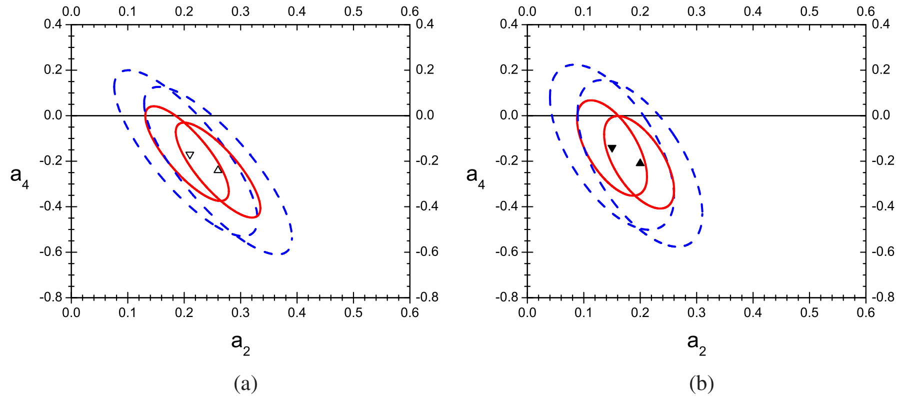


FIG. 5 (color online). The fit at $\mu_0^2 = 1\ \mathrm{GeV}^2$ with (a) $R = 5.76\ \mathrm{GeV}^2$, (b) $R = 9\ \mathrm{GeV}^2$. The solid and dashed error ellipses correspond to the 68% and 95% confidential regions, respectively.

V. EXTRACTING THE PION DA FROM THE CLEO DATA

The first two nontrivial Gegenbauer coefficients are extracted from the CLEO experimental data with the fit of the form

$$Q^2 \cdot F^{\gamma\pi}(Q^2) = A_0(Q^2) + A_2(Q^2, \mu_0^2) \cdot a_2(\mu_0^2) + A_4(Q^2, \mu_0^2) \cdot a_4(\mu_0^2), \tag{47}$$

where the expressions of the factors A_0, A_2, and A_4 can be read off from Eq. (2), the initial scale μ_0^2 is fixed at $1\ \mathrm{GeV}^2$ for the comparison with other LCSR results, and the contribution of a_6 is neglected according to the previous discussion. We consider only the experimental uncertainty in the fit procedure in order to obtain a strict constrain in the parameter space of (a_2, a_4) [11]. The natural renormalization scale is taken to be $\mu_R^2 = (Q^2 + R)/4$, while the factorization scale is chosen to be higher than μ_R^2, say $\mu_F^2 = (Q^2 + R)/2$. The parameters in the auxiliary weight functions are set to be $N_1 = 3$, $N_2 = 5$ and $N_1' = 3$, $N_2' = 6$ to make the fit error in Eq. (38) negligible. The two different R values, $5.76\ \mathrm{GeV}^2$ and $9\ \mathrm{GeV}^2$, are used in the numerical analysis for comparison. The neglected higher-twist contributions are small enough in most of the experimental energy region, and the largest potential theoretical uncertainty comes from the renormalon-based twist-four contribution. In fact, a_4 predicted in the renormalon model may be considered to be as its upper limit due to neglecting the contribution coming from the twist-four anomalous dimensions as mentioned in Ref. [27]. In the following, the lower limit of the twist-four coupling, $\delta^2(1\ \mathrm{GeV}^2) = 0.17\ \mathrm{GeV}^2$ for $R = 5.76\ \mathrm{GeV}^2$ and $R = 9\ \mathrm{GeV}^2$ respectively, will be used to check the influence of the uncertainty of the twist-four coupling.

The fit results of our numerical simulation are shown in Fig. 5, where the experimental data are taken from Ref. [3], and the experimental systematic and statistic uncertainties, σ_{sys} and σ_{sta}, have been combined in quadrature, and the weight $1/(\sigma_{\mathrm{sys}}^2 + \sigma_{\mathrm{sta}}^2)$ is put for every datum point in the error analysis. The central values of the fits and the corresponding $\chi^2_{\mathrm{DOF}} \equiv \chi^2/13$ are listed in Table III, in which the different symbols are used to designate the different parameter insertions leading to the different locations in the a_2-a_4 plane. Note that our χ^2 values are comparable with those in LCSR [11,13]. As shown in Fig. 5, the variation with R and δ^2 is moderate, and the 1σ confidential regions largely overlap, which exhibits a stability of the ACD local sum rule in the variation of the parameters. The average ratio of $A_4(Q^2, \mu_0^2)/A_2(Q^2, \mu_0^2)$, which determines the orientation of the large axis of the fiducial ellipse, is insensitive to the variation of δ^2, and will be smaller when

TABLE III. The extracted central values of the Gegenbauer coefficients a_2 and a_4 evaluated at the normalization scale $\mu_0^2 = 1\ \mathrm{GeV}^2$ for different parameter insertions.

R-value	Twist-four coupling (GeV^2)	Symbols	a_2	a_4	$\langle u^{-1}\rangle_\pi^R$	χ^2_{DOF}
$R = 5.76\ \mathrm{GeV}^2$	$\delta^2(\mu_0^2) = 0.17$	▽	0.21	−0.17	0.04	0.467
	$\delta^2(\mu_0^2) = 0.19$	△	0.26	−0.24	0.02	0.466
$R = 9\ \mathrm{GeV}^2$	$\delta^2(\mu_0^2) = 0.14$	◀	0.09	−0.04	0.05	0.472
	$\delta^2(\mu_0^2) = 0.17$	▼	0.15	−0.14	0.01	0.466
	$\delta^2(\mu_0^2) = 0.19$	▲	0.2	−0.21	−0.01	0.466
	$\delta^2(\mu_0^2) = 0.21$	▶	0.25	−0.28	−0.03	0.468

TABLE IV. Estimation of the Gegenbauer coefficients a_2 and a_4, and the reduced inverse moment $\langle u^{-1}\rangle_\pi^R$ normalized at $\mu_0^2 = 1\ \mathrm{GeV}^2$ in different two-parameter models of pion DA.

DAs	Symbols	$a_2(\mu_0^2)$	$a_4(\mu_0^2)$	$\langle u^{-1}\rangle_\pi^R$
Asymptotic	■	0	0	0
CZ [15]	■	0.56	0	0.56
PR [7]	□	0.09	−0.02	0.07
ADT [8]	○	0.05	−0.04	0.01
SY [10]	★	0.27	−0.22	0.05
BMS03 [11]	×	0.31	−0.35	−0.04
Agaev [14]	☆	0.27	−0.3	−0.03
BMS06 [13]	+	0.44	−0.40	0.04
BZ [37]	●	0.12	−0.02	0.1
NLC01 [29]	◆	0.2	−0.14	0.06
NLC06 [38]	◇	0.29	−0.21	0.08
QCDSF/UKQCD06 [39]		0.28 ± 0.16		

R becomes larger. The values of this ratios at $\mu_0^2 = 1\ \mathrm{GeV}^2$ in Figs. 5(a) and 5(b) are about 0.30 and 0.19, respectively (corresponding to 0.36 and 0.23 at $\mu_0^2 = 5.76\ \mathrm{GeV}^2$, respectively), which are smaller than the latest LCSR result [13], and indicate that the factors in front of the neglected Gegenbauer coefficients, say a_6, should be very small as shown in Fig. 4. For comparison, the values of a_2, a_4 and $\langle u^{-1}\rangle_\pi^R$ for the different DAs of the pion are listed in Table IV, where most of the rows are taken from the Table I of Ref. [13] except for the results from Agaev's revised renormalon-based model [14], the NLC sum rules with the improved Gaussian model of the nonperturbative QCD vacuum [38], and the latest lattice QCD estimate of the QCDSF/UKQCD collaboration [39], and we have evolved the values of the last two rows down to the scale of $\mu_F^2 = 1\ \mathrm{GeV}^2$ using the exact two-loop coupling and the two-loop ERBL kernel.

The reduced inverse moment is defined by

$$\langle u^{-1}\rangle_\pi^R(\mu_F^2) \equiv \frac{\langle u^{-1}\rangle_\pi(\mu_F^2)}{3} - 1 = \int_0^1 du\varphi_\pi(u,\mu_F^2)u^{-1} = a_2 + a_4 + \cdots, \tag{48}$$

which is relevant in leading-order perturbative calculations for the photon-to-pion transition form factor and the pion electromagnetic form factor. We can see that our fit is consistent, in the signs of a_2 and a_4 as well as in magnitude of the reduced inverse moment, with all results based on the two-parameter model listed in Table IV.

The curves of the transition form factor with different parameters in Table III are displayed in Figs. 6(a) and 6(b). For comparison, the plots of the interpolation formula for both the perturbative and the nonperturbative regions [40,41],

$$F^{\gamma\pi}(Q^2) = \frac{\sqrt{2}}{4\pi^2 f_\pi(1+Q^2/s_0)}\left(1 - \frac{5}{3}\frac{\alpha_s(Q^2)}{\pi}\right), \quad s_0 = 0.67\ \mathrm{GeV}^2, \tag{49}$$

are drawn as well, where the exact two-loop coupling is used for consistency. The interpolation formula without the radiative correction is consistent with the chiral anomaly constraint, and thus is more suitable in the low energy region, while the one with the radiative correction should be more reliable in the asymptotic region. We note that our strategy is to derive the $t = 0$ prediction from the incomplete $t = R$ QCD expression working at relatively large Q^2, and thus we do not want to exactly arrive at the normalization point of the axial anomaly, but prefer to take it as a standard for choosing the best-fit parameters. As usual, the higher-order radiative contributions and the

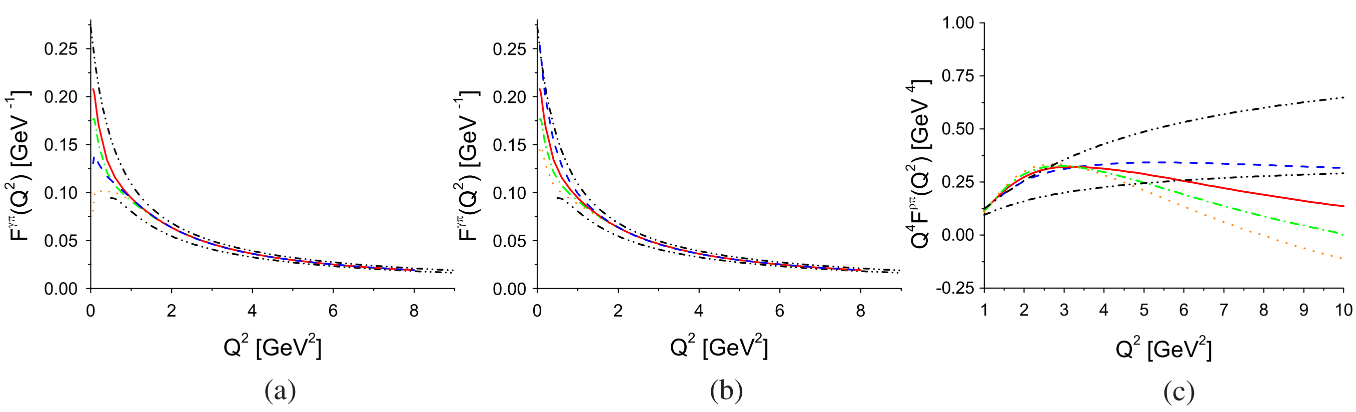


FIG. 6 (color online). (a) The transition form factor $F^{\gamma\pi}(Q^2)$ from the ACD sum rules with different parameter insertions △ (dotted), ▽ (dashed), ▲ (dash-dotted), and ▼ (solid). The interpolation formula without the radiative correction is plotted by the upper dot-dot-dashed line, while that formula with the radiative correction by the lower dot-dot-dashed line. (b) $F^{\gamma\pi}(Q^2)$ from the ACD sum rules with other parameter insertions ▶ (dotted), ▲ (dash-dotted), ▼ (solid), and ◀ (dashed). The interpolation plots are the same as (a). (c) The rescaled $\rho - \pi$ transition form factor from the ACD sum rules with the same parameters and designations as (b), and from the LCSR [42] (lower dot-dot-dashed line), [9] (upper dot-dot-dashed line).

power corrections are suppressed at the larger R value, and the theoretical results for $F^{\gamma\pi}(Q^2)$ from the ACD sum rules should be more reliable if the higher resonance contributions are carefully dropped by choosing an appropriate weight function $D^{(1)}_{N_1}(t)$. Therefore, we fix R to be $9\ \mathrm{GeV}^2$. To get a reliable value region of the twist-four coupling, we vary it within a large interval of $[0.14\ \mathrm{GeV}^2, 0.21\ \mathrm{GeV}^2]$. From Table III, we see that the a_4 becomes relatively large with the increasing of δ^2. Then, from Fig. 6(b), one can see that the fit plots with larger R and $\delta^2(1\ \mathrm{GeV}^2) \leq 0.19\ \mathrm{GeV}^2$ are more consistent with the interpolation without radiative correction, and the plot with ◀ even has nearly the same value as the chiral anomaly.

Another constraint for our fit parameters is arising from the transition form factor $F^{\rho\pi}(Q^2)$ derived from Eq. (39) in our ACD sum rules. We can see from Fig. 6(c) that this form factor with the corresponding a_2 and a_4 extracted from the experimental data of $F^{\gamma\pi}(Q^2)$ is sensitive to the variation of δ^2, and the larger δ^2 value will lead to a bad behavior in the asymptotic region. On the other hand, $F^{\rho\pi}(Q^2)$ has been calculated in the LCSR [9,42], and turns out to be related with the decay rate of $J/\Psi \to \gamma \to \pi^0\omega$,

$$\mathrm{Br}(J/\Psi \to \gamma \to \pi^0\omega/J/\Psi \to e^+e^-) = \frac{9}{32}(3M_{J/\Psi}F^{\rho\pi}(M^2_{J/\Psi})/m_\rho)^2, \tag{50}$$

which is predicted from our ACD sum rules with the parameter insertion ◀ to be about 4.9×10^{-4}, and agrees very well with the experimental data $(4.5 \pm 0.5) \times 10^{-4}$ [43]. In consideration with all of the above, we obtain our best fit and the uncertainty, mainly caused from the variation of the twist-four coupling, as a conservative estimate for $\delta^2(1\ \mathrm{GeV}^2)$ varying from $0.14\ \mathrm{GeV}^2$ to $0.19\ \mathrm{GeV}^2$,

$$\begin{aligned} a_2(1\ \mathrm{GeV}^2) &= 0.145 \pm 0.055, \\ a_4(1\ \mathrm{GeV}^2) &= -(0.125 \pm 0.085). \end{aligned} \tag{51}$$

The best-fit error ellipses of $R = 9\ \mathrm{GeV}^2$ with some typical model predictions are shown in the Fig. 7. The latest LCSR estimates with *the same renormalon-based twist-four DA* [13] are far from the 3σ confidential region even with $\delta^2(1\ \mathrm{GeV}^2) = 0.21\ \mathrm{GeV}^2$, while those for the instanton-vacuum-based chiral quark model [7], the earlier NLC condensate QCD sum rules [29], and the Ball-Zwicky (BZ) DA [37] exhibit a nice compatibility between themselves and ours in our 2σ confidential region when $\delta^2(1\ \mathrm{GeV}^2) \leq 0.19\ \mathrm{GeV}^2$.

As mentioned before, our local ACD sum rule and the usual light-cone sum rule use the dispersion relations (28) with (31) and (20) with (21), respectively. We note that in the former the resonances' contributions in the interval $[t_{\mathrm{th}}, R]$, Δ^{fit}, are eliminated by introducing an appropriate

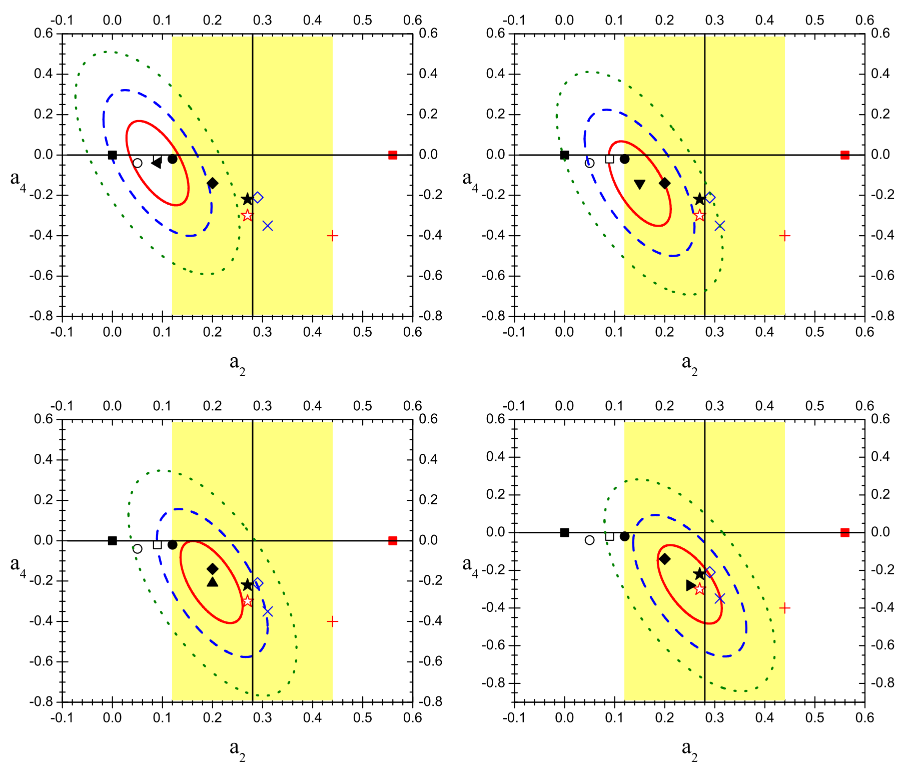


FIG. 7 (color online). The error ellipses of our best fit at $\mu_0^2 = 1\ \mathrm{GeV}^2$ with different parameters ◀ (top left), ▼ (top right), ▲ (bottom left), ▶ (bottom right), compared with other models listed in Table IV, where solid error ellipses correspond to 1σ, dashed to 2σ, and dotted to 3σ.

weight function $D^{(1)}_{N_1}(t)$ into the integral, and the higher resonances' contributions, such that J/ψ etc. are avoided by chosen R to be $\leq 9\ \mathrm{GeV}^2$. However, these complicated high resonance contributions are only approximated by the assumption of the global or semilocal quark-hadron duality above some threshold s_0 with Borel suppression in the usual light-cone sum rules. Moreover, as we already discussed, the usual LCSR is working in the kinematical region of ω being an average over the global duality region, eventually $|\omega| \to 1$, where all the Gegenbauer coefficients are seized of nearly equal weights, and the two-parameter model may not be suitable. By contrast, the ACD sum rules allow us to choose an appropriate R to work in the small $|\omega|$ region where the conformal expansion (22) converges fast, so that the estimate of the first two nontrivial Gegenbauer coefficients from the sum rules is reliable [as indicated by the low values of $A_4(Q^2, \mu_0^2)/A_2(Q^2, \mu_0^2)$ shown in Fig. 5]. Thus, we believe that the absolute values of a_2 and a_4 may be overestimated in the latest LCSR evaluation though the signs of a_2, a_4, and $\langle u^{-1}\rangle^R_\pi$ are the same as ours and many other estimates [13].

In Fig. 8(a), we present the shapes of our two-parameter DAs corresponding to four different insertions of the twist-four couplings δ^2, each of which is camel-like, and has one minimum at the midpoint and two maxima on its two sides. They are similar to the CZ DA, but not a simple copy of the CZ DA because of their stronger suppression near the end points. This characteristic indicates that the soft gluon exchange effect in the perturbative QCD hard kernel is suppressed so that the Sudakov suppression is not as necessary as for the case of the asymptotic DA [44,45], and that there may be a nonperturbative suppression for DA at the end points, as suggested by Dorokhov [46]. We note here that our two-parameter DAs of the best fit may also be camel-like despite the better numerical compatibility with some quasiasymptotic ones.

It should be emphasized that another constraint on the pion DA is coming from the decay couplings, such as $g_{\pi NN}$ and $g_{\rho\omega\pi}$, which are related to the value of $\varphi_\pi(1/2)$ in the LCSR. It was further suggested by Braun and Filyanov that this midpoint constraint should be $\varphi_\pi(u = 1/2, \mu_0^2 = 1\ \mathrm{GeV}^2) = 1.2 \pm 0.3$ [47], which conflicts heavily with the camel-like DA from the latest LCSR, and matches approximately to our best fits for the parameter insertion $0.14\ \mathrm{GeV}^2 \leq \delta^2(1\ \mathrm{GeV}^2) \leq 0.19\ \mathrm{GeV}^2$.

VI. CONCLUSIONS

In this study, we use the light-cone ACD sum rule and the renormalon-based twist-four pion DA to extract the first two nontrivial Gegenbauer coefficients from the CLEO data of the photon-to-pion transition form factor. The ACD sum rule provides a flexible tool to study the different kinematic regions of ω, some of which are very suitable for the truncated conformal expansion. With a careful determination for the parameters so that the contribution from the higher-order Gegenbauer expansions is suppressed, the best-fit central values of the first two nontrivial Gegenbauer coefficients, $a_2(1\ \mathrm{GeV}^2)$ and $a_4(1\ \mathrm{GeV}^2)$, are found to be 0.145 ± 0.055 and $-(0.125 \pm 0.085)$, respectively. They are consistent not only in signs of a_2 and a_4 but also in the magnitude of the reduced inverse moment $\langle u^{-1}\rangle^R_\pi$ with all results based on the two-parameter model listed in Table IV. The rescaled photon-to-pion transition form factor with our best-fit parameters is consistent very well with both the CELLO data and the prediction of the interpolation formula in all the experimental accessible region of the momentum transfer. Moreover, the decay rate of $J/\Psi \to \gamma \to \pi^0\omega$, which is related to $F^{\rho\pi}(Q^2)$, is predicted from our ACD sum rules with the parameter insertion ◀ (see Table III) to be about 4.9×10^{-4}, and agrees very well with the experimental data $(4.5 \pm 0.5) \times 10^{-4}$ [43].

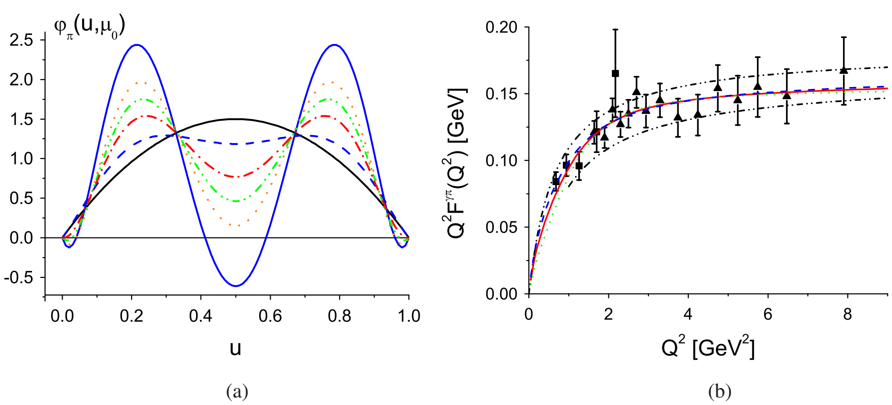


FIG. 8 (color online). (a) Some two-parameter DAs at $\mu_0^2 = 1\ \mathrm{GeV}^2$. The plots with midpoint values from above to below correspond to the asymptotic DA, our best fit with parameters ◀, ▼, ▲, ▶, LCSR [13], respectively. (b) The rescaled photon-to-pion transition form factor. Our best fit with ◀ (dashed line), ▼ (solid line), and ▲ (dotted line). The interpolation formula without the radiative correction (upper dot-dot-dashed line), and with that correction (lower dot-dot-dashed line). The triangles denote the CLEO data and the squares denote the CELLO data.

Our extracted pion DA based on the two-parameter model belongs to the camel-like type, where the near-end-point values are suppressed more than the asymptotic DA, and it is helpful for the collinear factorization assumption adopted in this up-to-NLO order study. Our best-fit error ellipses show that the latest LCSR estimates with *the same renormalon-based twist-four DA* [13] are far from our 3σ confidential region even with $\delta^2(1\ \mathrm{GeV}^2) = 0.21\ \mathrm{GeV}^2$, while those for the instanton-vacuum-based chiral quark model [7], the earlier NLC condensate QCD sum rules [29], and the BZ DA [37] lie in the 2σ confidential region when $\delta^2(1\ \mathrm{GeV}^2) \leq 0.19\ \mathrm{GeV}^2$.

More information about the pion DA can be deduced from other experiment sources, such as the pion electromagnetic form factor [44], the diffractive dissociation of a pion into jets [12], and the Drell-Yan process $\pi^- N \to \mu^+\mu^- X$ [48]. All the analyses favor the camel-like type, but cannot single out the pion DA without ambiguity. Our investigation also supports these results but with more confidence for the first two coefficients of the Gegenbauer polynomial expansion compared with the usual LCSR. Importantly, our pion DA based on the two-parameter model, which is different from the latest LCSR's result [13], satisfies approximately the midpoint constraint for the pion DA from the vertex LCSR [47]. However, the constraint from the decay couplings in LCSR is still an open question up to now. For example, with $\varphi_\pi(1/2, 1\ \mathrm{GeV}^2) = 1.2$, the coupling $g_{D^*D\pi}$ extracted from the LCSR is about 12 [49], and decreases to 10.5 by taking the radiative correction into account [50], which are smaller than the experimental value $g_{D^*D\pi} = 17.9 \pm 0.3 \pm 1.9$. This decay coupling increases only by 9% even for the asymptotic DA [51], which has, to our knowledge, the largest midpoint value among the two-parameter models. Although the key to this problem is uncertain, a relative larger midpoint value for the pion DA is yet necessary [51], and the two-parameter model of the pion DA may need to be generalized to, for example, the powerlike falloff model [52].

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