

Meson decay in a corrected 3P_0 model

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The 3P_0 model, in which $q\bar{q}$ pair production is the dominant mechanism, is extensively applied to both light and heavy meson decay and stands as one of the most successful strong decay models. The pair production can be obtained from the nonrelativistic limit of a microscopic interaction Hamiltonian involving Dirac quark fields. The evaluation of the decay amplitude can be performed by a diagrammatic technique for drawing quark lines. In this paper we use an alternative approach which consists of a mapping technique, the Fock-Tani formalism, in order to obtain an effective Hamiltonian starting from the same microscopic interaction. An additional effect is manifested in this formalism associated to the extended nature of mesons: bound-state corrections. A corrected 3P_0 is obtained and applied, as an example, to $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$ decays.

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I. INTRODUCTION

A great variety of quark-based models are known that describe with reasonable success single-hadron properties. A natural question that arises is to what extent a model which gives a good description of hadron properties is, at the same time, able to describe the complex hadron-hadron interaction or by the same principles hadron decay. In particular, the theoretical aspects of strong decay have been challenged by QCD exotica (glueballs and hybrids) where a consistent understanding of the mixing schemes for these states is still an open question [1–3]. The nature of the family of “new mesons” X , Y , Z [4] is another unsolved puzzle: are they actually new $q\bar{q}$ mesons, hadronic molecules, or something else? In the direction of clarifying these questions is the successful decay model, the 3P_0 model, which considers only Okubo-Zweig-Iizuka (OZI)-allowed strong-interaction decays. This model was introduced over 30 years ago by Micu [5] and applied to meson decays in the 1970s by LeYaouanc *et al.* [6]. This description is a natural consequence of the constituent quark model scenario of hadronic states.

T. Barnes *et al.* [7–10] have made an extensive survey of meson states in light of the 3P_0 model. Two basic parameters of their formulation are γ (the interaction strength) and β (the wave function’s extension parameter). Although they found the optimum values near $\gamma = 0.5$ and $\beta = 0.4$ GeV, for light $1S$ and $1P$ decays, these values lead to overestimates of the widths of higher- L states. In this

perspective a modified $q\bar{q}$ pair-creation interaction with $\gamma = 0.4$ was preferred.

In the present work, we employ a mapping technique in order to obtain an effective interaction for meson decay. A particular mapping technique long used in atomic physics [11], the Fock-Tani formalism (FTf), has been adapted, in previous publications [12–16], in order to describe hadron-hadron scattering interactions with constituent interchange. Now this technique has been extended in order to include meson decay. We start from the microscopic $q\bar{q}$ pair-creation interaction, as will be shown, in lower order, the 3P_0 results are reproduced. An additional and interesting feature appears in higher orders of the formalism: corrections due to the bound-state nature of the mesons and a natural modification in the $q\bar{q}$ interaction strength.

In the Fock-Tani formalism one starts with the Fock representation of the system using field operators of elementary constituents which satisfy canonical (anti)commutation relations. Composite-particle field operators are linear combinations of the elementary-particle operators and do not generally satisfy canonical (anti)commutation relations. “Ideal” field operators acting on an enlarged Fock space are then introduced in close correspondence with the composite ones. Next, a given unitary transformation, which transforms the single composite states into single ideal states, is introduced. Application of the unitary operator on the microscopic Hamiltonian, or on other Hermitian operators expressed in terms of the elementary constituent field operators, gives equivalent operators which contain the ideal field operators. The effective Hamiltonian in the new representation has a clear physical interpretation in terms of the processes it describes. Since all field operators in the new representation satisfy canoni-

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cal (anti)commutation relations, the standard methods of quantum field theory can then be readily applied.

In this paper we shall extend the FTf to meson decay processes. In the next section we review the basic aspects of the formalism. Section III is dedicated to obtain an effective decay Hamiltonian. In Sec. IV, two light meson decays examples are calculated: $b_1 \rightarrow \omega \pi$ and $a_1 \rightarrow \rho \pi$. The summary and conclusions are followed by appendices which detail the method employed throughout this work.

II. MAPPING OF MESONS

This section reviews the formal aspects of the mapping procedure and how it is implemented to quark-antiquark meson states [12]. The starting point of the Fock-Tani formalism is the definition of single composite bound states. We write a single-meson state in terms of a meson creation operator M_α^\dagger as

$$|\alpha\rangle = M_\alpha^\dagger |0\rangle, \quad (1)$$

where $|0\rangle$ is the vacuum state. The meson creation operator M_α^\dagger is written in terms of constituent quark and antiquark creation operators q^\dagger and \bar{q}^\dagger ,

$$M_\alpha^\dagger = \Phi_\alpha^{\mu\nu} q_\mu^\dagger \bar{q}_\nu^\dagger, \quad (2)$$

$\Phi_\alpha^{\mu\nu}$ is the meson wave function and $q_\mu |0\rangle = \bar{q}_\nu |0\rangle = 0$. The index α identifies the meson quantum numbers of space, spin, and isospin. The indices μ and ν denote the spatial, spin, flavor, and color quantum numbers of the constituent quarks. A sum over repeated indices is implied. It is convenient to work with orthonormalized amplitudes,

$$\langle \alpha | \beta \rangle = \Phi_\alpha^{*\mu\nu} \Phi_\beta^{\mu\nu} = \delta_{\alpha\beta}. \quad (3)$$

The quark and antiquark operators satisfy canonical anti-commutation relations,

$$\begin{aligned} \{q_\mu, q_\nu^\dagger\} &= \{\bar{q}_\mu, \bar{q}_\nu^\dagger\} = \delta_{\mu\nu}, \\ \{q_\mu, q_\nu\} &= \{\bar{q}_\mu, \bar{q}_\nu\} = \{q_\mu, \bar{q}_\nu^\dagger\} = \{q_\mu, \bar{q}_\nu\} = 0. \end{aligned} \quad (4)$$

Using these quark anticommutation relations, and the normalization condition of Eq. (3), it is easily shown that the meson operators satisfy the following noncanonical commutation relations:

$$[M_\alpha, M_\beta^\dagger] = \delta_{\alpha\beta} - \Delta_{\alpha\beta}, \quad [M_\alpha, M_\beta] = 0, \quad (5)$$

where

$$\Delta_{\alpha\beta} = \Phi_\alpha^{*\mu\nu} \Phi_\beta^{\mu\sigma} \bar{q}_\sigma^\dagger \bar{q}_\nu + \Phi_\alpha^{*\mu\nu} \Phi_\beta^{\rho\nu} q_\rho^\dagger q_\mu. \quad (6)$$

In addition,

$$\begin{aligned} [q_\mu, M_\alpha^\dagger] &= \Phi_\alpha^{\mu\nu} \bar{q}_\nu^\dagger, & [\bar{q}_\nu, M_\alpha^\dagger] &= -\Phi_\alpha^{\mu\nu} q_\mu^\dagger, \\ [q_\mu, M_\alpha] &= [\bar{q}_\nu, M_\alpha] = 0. \end{aligned} \quad (7)$$

The presence of the operator $\Delta_{\alpha\beta}$ in Eq. (5) is due to the composite nature of the mesons. This term enormously complicates the mathematical description of processes that involve the hadron and quark degrees of freedom. The usual field theoretic techniques used in many-body problems, such as the Green's functions method, Wick's theorem, etc., apply to creation and annihilation operators that satisfy canonical relations. Similarly, the nonvanishing of the commutators $[q_\mu, M_\alpha^\dagger]$ and $[\bar{q}_\nu, M_\alpha^\dagger]$ is a manifestation of the lack of kinematic independence of the meson operator from the quark and antiquark operators. Therefore, the meson operators M_α and M_α^\dagger are not convenient dynamical variables to be used.

A transformation is defined such that a single-meson state $|\alpha\rangle$ is redescribed by an (ideal) elementary-meson state by

$$|\alpha\rangle \rightarrow U^{-1} |\alpha\rangle = m_\alpha^\dagger |0\rangle, \quad (8)$$

where m_α^\dagger an ideal meson creation operator. The ideal meson operators m_α^\dagger and m_α satisfy, by definition, canonical commutation relations

$$[m_\alpha, m_\beta^\dagger] = \delta_{\alpha\beta}, \quad [m_\alpha, m_\beta] = 0. \quad (9)$$

The state $|0\rangle$ is the vacuum of both q and m degrees of freedom in the new representation. In addition, in the new representation the quark and antiquark operators $q^\dagger, q, \bar{q}^\dagger$, and \bar{q} are kinematically independent of the m_α^\dagger and m_α

$$[q_\mu, m_\alpha] = [q_\mu, m_\alpha^\dagger] = [\bar{q}_\mu, m_\alpha] = [\bar{q}_\mu, m_\alpha^\dagger] = 0. \quad (10)$$

The unitary operator U of the transformation is

$$U(t) = \exp[tF], \quad (11)$$

where F is the generator of the transformation and t a parameter which is set to $-\pi/2$ to implement the mapping. The next step is to obtain the transformed operators in the new representation. The basic operators of the model are expressed in terms of the quark operators. Therefore, in order to obtain the operators in the new representation, one writes

$$q(t) = U^{-1} q U, \quad \bar{q}(t) = U^{-1} \bar{q} U. \quad (12)$$

The generator F of the transformation is

$$F = m_\alpha^\dagger \tilde{M}_\alpha - \tilde{M}_\alpha^\dagger m_\alpha, \quad (13)$$

where

$$\tilde{M}_\alpha = \sum_{i=0}^n \tilde{M}_\alpha^{(i)}, \quad (14)$$

with

$$\begin{aligned} [\tilde{M}_\alpha, \tilde{M}_\beta^\dagger] &= \delta_{\alpha\beta} + \mathcal{O}(\Phi^{n+1}), \\ [\tilde{M}_\alpha, \tilde{M}_\beta] &= [\tilde{M}_\alpha^\dagger, \tilde{M}_\beta^\dagger] = 0. \end{aligned} \quad (15)$$

It is easy to see from (13) that $F^\dagger = -F$ which ensures that U is unitary. The index i in (14) represents the order of the expansion in powers of the wave function Φ . The \tilde{M}_α operator is determined up to a specific order n consistent with (15). The examples studied in [12] required the determination of $\tilde{M}_\alpha^{(i)}$ up to order 3 as shown below

$$\begin{aligned} \tilde{M}_\alpha^{(0)} &= M_\alpha; & \tilde{M}_\alpha^{(1)} &= 0, \\ \tilde{M}_\alpha^{(2)} &= \frac{1}{2}\Delta_{\alpha\beta}M_\beta; & \tilde{M}_\alpha^{(3)} &= \frac{1}{2}M_\beta^\dagger T_{\alpha\beta\gamma}M_\gamma, \end{aligned} \quad (16)$$

with $T_{\alpha\beta\gamma} = -[M_\alpha, \Delta_{\beta\gamma}]$. In the ‘‘zero-order’’ approximation, overlap among mesons is neglected and terms of the same power in the bound-state wave function Φ_α (Φ_α^*) are collected. In order to have a consistent power counting scheme, the implicit Φ_α (Φ_α^*) entering via Eq. (2) are not counted. The consequence of this is that the equations for m_α and \tilde{M}_α are manifestly symmetric,

$$\begin{aligned} \frac{dm_\alpha(t)}{dt} &= [m_\alpha(t), F] = \tilde{M}_\alpha(t), \\ \frac{d\tilde{M}_\alpha(t)}{dt} &= [\tilde{M}_\alpha(t), F] = -m_\alpha(t), \end{aligned} \quad (17)$$

and their solutions involve only trigonometric functions of t ,

$$\begin{aligned} m_\alpha(t) &= \tilde{M}_\alpha \text{sint} + m_\alpha \text{cost}, \\ \tilde{M}_\alpha(t) &= \tilde{M}_\alpha \text{cost} - m_\alpha \text{sint}. \end{aligned} \quad (18)$$

The equations of motion for the quark operators q and \bar{q} can be obtained by making use of Eq. (7) in a similar way,

$$\frac{dq_\mu(t)}{dt} = [q_\mu(t), F]; \quad \frac{d\bar{q}_\mu(t)}{dt} = [\bar{q}_\mu(t), F]. \quad (19)$$

In the zero-order approximation, the effects of the meson structure are neglected resulting in

$$\begin{aligned} q_\mu^{(0)}(t) &= q_\mu, & \bar{q}_\nu^{(0)}(t) &= \bar{q}_\nu, \\ m_\alpha^{(0)}(t) &= m_\alpha \text{cost} + M_\alpha \text{sint}, \\ M_\alpha^{(0)}(t) &= M_\alpha \text{cost} - m_\alpha \text{sint}. \end{aligned} \quad (20)$$

In first order one has

$$\begin{aligned} q_\mu^{(1)}(t) &= -\Phi_\alpha^{\mu\nu_1} \bar{q}_{\nu_1}^\dagger [m_\alpha \text{sint} + M_\alpha(1 - \text{cost})], \\ \bar{q}_\nu^{(1)}(t) &= \Phi_\alpha^{\mu_1\nu} q_{\mu_1}^\dagger [m_\alpha \text{sint} + M_\alpha(1 - \text{cost})], \\ m_\alpha^{(1)}(t) &= 0, & M_\alpha^{(1)}(t) &= 0. \end{aligned} \quad (21)$$

The second and third order solutions to (19) were calculated in Ref. [12] and appear again, for completeness, in Appendix A, together with the higher order operators required in our calculation.

Once a microscopic interaction Hamiltonian H is defined, at the quark level, a new transformed Hamiltonian can be obtained. This effective interaction we shall call the *Fock-Tani Hamiltonian* and is evaluated by the application

of the unitary operator U on the microscopic Hamiltonian $\mathcal{H}_{\text{FT}} = U^{-1}HU$. The transformed Hamiltonian \mathcal{H}_{FT} describes all possible processes involving mesons and quarks. The general structure of \mathcal{H}_{FT} is of the following form:

$$\mathcal{H}_{\text{FT}} = \mathcal{H}_q + \mathcal{H}_m + \mathcal{H}_{mq}, \quad (22)$$

where the first term involves only quark operators, the second one involves only ideal meson operators, and \mathcal{H}_{mq} involves quark and meson operators.

In \mathcal{H}_{FT} there are higher order terms that provide bound-state corrections (also called orthogonality corrections) to the lower order ones. The basic quantity for these corrections is the *bound-state kernel* $\Delta(\rho\tau; \lambda\nu)$ defined as

$$\Delta(\rho\tau; \lambda\nu) = \Phi_\alpha^{\rho\tau} \Phi_\alpha^{*\lambda\nu}. \quad (23)$$

To discuss the physical meaning of the bound-state corrections and how they modify the fundamental quark interaction we shall present an example, in a toy model similar to the model studied in [12], where the basic arguments are outlined. In this example, the starting point is a two-body microscopic quark-antiquark Hamiltonian of the form

$$\begin{aligned} H_{2q} &= T(\mu)q_\mu^\dagger q_\mu + T(\nu)\bar{q}_\nu^\dagger \bar{q}_\nu + V_{q\bar{q}}(\mu\nu; \sigma\rho)q_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho q_\sigma \\ &\quad + \frac{1}{2}V_{qq}(\mu\nu; \sigma\rho)q_\mu^\dagger q_\nu^\dagger q_\rho q_\sigma \\ &\quad + \frac{1}{2}V_{\bar{q}\bar{q}}(\mu\nu; \sigma\rho)\bar{q}_\mu^\dagger \bar{q}_\nu^\dagger \bar{q}_\rho \bar{q}_\sigma. \end{aligned} \quad (24)$$

The transformation $\mathcal{H}_{\text{FT}} = U^{-1}H_{2q}U$ is implemented again by transforming each quark and antiquark operator in Eq. (24), where a similar structure to Eq. (22) is obtained. In free space, the wave function Φ of Eq. (2) satisfies the following equation:

$$H(\mu\nu; \sigma\rho)\Phi_\alpha^{\sigma\rho} = \epsilon_{[\alpha]} \Phi_{[\alpha]}^{\mu\nu}, \quad (25)$$

where $H(\mu\nu; \sigma\rho)$ is the Hamiltonian matrix

$$\begin{aligned} H(\mu\nu; \sigma\rho) &= \delta_{\mu[\sigma]}\delta_{\nu[\rho]}[T([\sigma]) + T([\rho])] \\ &\quad + V_{q\bar{q}}(\mu\nu; \sigma\rho), \end{aligned} \quad (26)$$

$\epsilon_{[\alpha]}$ is the total energy of the meson. There is no sum over repeated indices inside square brackets.

The effective quark Hamiltonian \mathcal{H}_q has an identical structure to the microscopic quark Hamiltonian, Eq. (24), except that the term corresponding to the quark-antiquark interaction is modified as follows:

$$\mathcal{V}_{q\bar{q}} = [V_{q\bar{q}} - H\Delta - \Delta H + \Delta H\Delta], \quad (27)$$

where $V_{q\bar{q}} \equiv V_{q\bar{q}}(\mu\nu; \sigma\rho)$ and the contraction $H\Delta \equiv H(\mu\nu; \tau\xi)\Delta(\tau\xi; \sigma\rho)$. An important property of the bound-state kernel is

$$\Delta(\mu\nu; \sigma\rho)\Phi_\alpha^{\sigma\rho} = \Phi_\alpha^{\mu\nu}, \quad (28)$$

which follows from the wave function’s orthonormalization, Eq. (3). In the case that Φ is a solution of Eq. (25), the

new quark-antiquark interaction term becomes

$$\mathcal{V}_{q\bar{q}}(\mu\nu; \sigma\rho) = V_{q\bar{q}}(\mu\nu; \sigma\rho) - \sum_{\alpha} \epsilon_{\alpha} \Phi_{\alpha}^{*\mu\nu} \Phi_{\alpha}^{\sigma\rho}. \quad (29)$$

The spectrum of the modified quark Hamiltonian, \mathcal{H}_q , is positive semidefinite and hence has no bound states [11]. This result is exactly the same as in Weinberg's quasiparticle method [17], where the bound states are redescribed by ideal particles. The new $\mathcal{V}_{q\bar{q}}$ is a weaker potential, modified in such a way that no new bound states are formed.

In the quark-meson sector of Eq. (22) in $\mathcal{H}_{m\bar{q}}$ appears a term related to spontaneous meson breakup

$$H_{m \rightarrow q\bar{q}} = V(\mu\nu; \alpha) q_{\mu}^{\dagger} \bar{q}_{\nu}^{\dagger} m_{\alpha} \quad (30)$$

with

$$V(\mu\nu; \alpha) = H(\mu\nu; \sigma\rho) \Phi_{\alpha}^{\sigma\rho} - \Delta(\mu\nu; \sigma\rho) H(\sigma\rho; \tau\lambda) \Phi_{\alpha}^{\tau\lambda}. \quad (31)$$

Again, in the case that Φ is a solution of Eq. (25), a straightforward calculation demonstrates that $H_{m \rightarrow q\bar{q}} = 0$. When there is no external interaction, this result is a direct consequence of the bound state's stability against spontaneous breakup. This term can be interesting in studies related to dense hadronic mediums. For these systems the wave function is, in general, not a solution of Eq. (25) and the strength of the potential $V(\mu\nu; \alpha)$ is now only decreased [13].

In the ideal meson sector \mathcal{H}_m many similar approaches to FTf [12] have obtained the meson-meson scattering interaction in the Born approximation: resonating group method (RGM) [18], quark Born diagram formalism (QBDF) [19],

$$H_{mm} = T_{mm} + V_{mm}, \quad (32)$$

where T_{mm} is the kinetic term and V_{mm} is the meson-meson interaction potential with constituent interchange. This potential is given by

$$V_{mm} = V_{mm}^{\text{dir}} + V_{mm}^{\text{exc}} + V_{mm}^{\text{int}}, \quad (33)$$

where V_{mm}^{dir} is the direct potential (no quark interchange), V_{mm}^{exc} is the quark exchange term, and V_{mm}^{int} is the intra-exchange term. As shown in Refs. [12,13], if one extends the FT calculation to higher orders a new meson-meson Hamiltonian is obtained

$$\bar{H}_{mm} = H_{mm} + \delta H_{mm}, \quad (34)$$

where δH_{mm} is the bound-state correction Hamiltonian. If the wave function Φ is chosen to be an eigenstate of the microscopic quark Hamiltonian, then the intraexchange term V_{mm}^{int} is cancelled

$$V_{mm}^{\text{int}} + \delta H_{mm} = 0. \quad (35)$$

In summary, these examples reveal an important and common feature of bound-state corrections: they weaken the quark-antiquark potential. In the next section we shall follow the same procedure for a quark pair-creation interaction, which is fundamental for the description of meson decay. Similar to the toy model, the resulting interaction that describes meson decay will contain a Born order contribution and a bound-state correction.

III. THE 3P_0 DECAY MODEL IN THE FOCK-TANI FORMALISM

In the paper of E. S. Ackleh, T. Barnes and E. S. Swanson [7] a formulation of the 3P_0 model is presented. It regards the decay of an initial state meson in the presence of a $q\bar{q}$ pair created from the vacuum. The pair production is obtained from the nonrelativistic limit of the interaction Hamiltonian H_I involving Dirac quark fields

$$H_I = 2m_q \gamma \int d\vec{x} \bar{\psi}(\vec{x}) \psi(\vec{x}), \quad (36)$$

where γ is the pair production strength. For a $q\bar{q}$ meson A to decay to mesons $B + C$ we must have $(q\bar{q})_A \rightarrow (q\bar{q})_B + (q\bar{q})_C$. To determine the decay rate a matrix element of (36) is evaluated

$$\langle BC|H_I|A\rangle = \delta(\vec{P}_A - \vec{P}_B - \vec{P}_C) h_{fi}. \quad (37)$$

The evaluation of h_{fi} is performed by diagrammatic technique for drawing quark lines. The h_{fi} decay amplitude is combined with relativistic phase space, resulting in the differential decay rate

$$\frac{d\Gamma_{A \rightarrow BC}}{d\Omega} = 2\pi P \frac{E_B E_C}{M_A} |h_{fi}|^2, \quad (38)$$

which after integration in the solid angle Ω , a usual choice for the meson momenta is made: $\vec{P}_A = 0$ ($P = |\vec{P}_B| = |\vec{P}_C|$).

In our approach, the starting point for the Fock-Tani h_{fi} is also the microscopic Hamiltonian H_I in (36). The quark fields can be written as (color and flavor are not represented explicitly)

$$\psi(\vec{x}) = \sum_s \int \frac{d^3k}{(2\pi)^{3/2}} [u_s(\vec{k}) q_s(\vec{k}) + v_s(-\vec{k}) \bar{q}_s^{\dagger}(-\vec{k})] e^{i\vec{k}\cdot\vec{x}}. \quad (39)$$

In the product $\bar{\psi}(x) \psi(x)$ we shall retain only the $q^{\dagger} \bar{q}^{\dagger}$ term, which yields from Eq. (36) a Hamiltonian in a compact form,

$$H_I = V_{\mu\nu} q_{\mu}^{\dagger} \bar{q}_{\nu}^{\dagger}, \quad (40)$$

where sum (integration) is again implied over repeated indexes. In the compact notation, the quark and antiquark momentum, spin, flavor, and color are written as $\mu = (\vec{p}_{\mu}, s_{\mu}, f_{\mu}, c_{\mu})$; $\nu = (\vec{p}_{\nu}, s_{\nu}, f_{\nu}, c_{\nu})$, while the pair-

creation potential $V_{\mu\nu}$ is given by

$$V_{\mu\nu} \equiv 2m_q\gamma\delta(\vec{p}_\mu + \vec{p}_\nu)\bar{u}_{s_\mu f_\mu c_\mu}(\vec{p}_\mu)v_{s_\nu f_\nu c_\nu}(\vec{p}_\nu). \quad (41)$$

It should be noted that since Eq. (36) is meant to be taken in the nonrelativistic limit, Eq. (41) should be as well. In the meson decay calculations of the next section, this limit is considered.

Applying the Fock-Tani transformation to H_I one obtains the effective Hamiltonian

$$\mathcal{H}_{\text{FT}} = U^{-1}H_I U. \quad (42)$$

The physical quantities in the FTf appear in a second quantization notation. The effective decay amplitude will be a product of the ideal meson operators with the following structure in the ideal meson sector: $m^\dagger m^\dagger m$. To obtain this product one has to expand up to third order in powers of the wave function. A Hamiltonian that describes this decay process, which we shall call H_m , can be extracted from the mapping (42) by the following products:

$$H_m = V_{\mu\nu}q_\mu^{\dagger(3)}\bar{q}_\nu^{\dagger(0)} + V_{\mu\nu}q_\mu^{\dagger(1)}\bar{q}_\nu^{\dagger(2)}. \quad (43)$$

After the substitution of Eqs. (20), (21), (A1), and (A3) into (43) results in the effective meson decay Hamiltonian

$$H_m = f^{\mu\nu}(\alpha, \beta, \gamma)V_{\mu\nu}m_\alpha^\dagger m_\beta^\dagger m_\gamma, \quad (44)$$

where

$$f^{\mu\nu}(\alpha, \beta, \gamma) = -\Phi_\alpha^{*\mu\tau}\Phi_\beta^{*\rho\nu}\Phi_\gamma^{\rho\tau}. \quad (45)$$

In the ideal meson space the new initial and final states involve only ideal meson operators $|A\rangle = m_\gamma^\dagger|0\rangle$ and $|BC\rangle = m_\alpha^\dagger m_\beta^\dagger|0\rangle$. The 3P_0 amplitude is obtained in the FTf by an expression equivalent to Eq. (37),

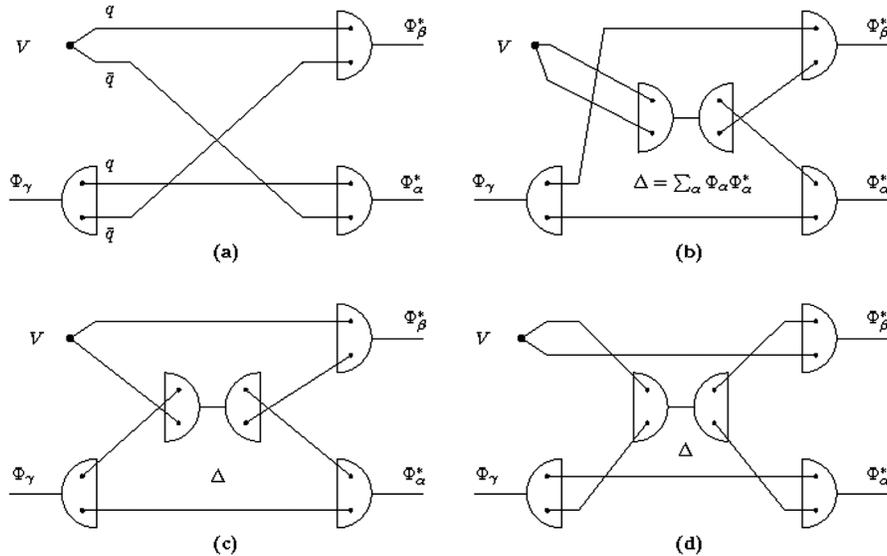


FIG. 1. Diagrams representing the C^3P_0 model. Diagram (a) corresponds to the 3P_0 amplitude. Diagrams (b), (c), and (d) are the bound-state corrections. The complete h_{fi} amplitude includes the diagrams above plus diagrams with $\alpha \leftrightarrow \beta$.

$$\langle BC|H_I|A\rangle = \langle 0|m_\alpha m_\beta H_m m_\gamma^\dagger|0\rangle$$

$$= f^{\mu\nu}(\alpha, \beta, \gamma)V_{\mu\nu} + f^{\mu\nu}(\beta, \alpha, \gamma)V_{\mu\nu}. \quad (46)$$

The term $f^{\mu\nu}(\beta, \alpha, \gamma)$ of (46) is shown in Fig. 1(a), and the term $f^{\mu\nu}(\alpha, \beta, \gamma)$ corresponds to the same diagram with $\alpha \leftrightarrow \beta$.

In the FTf perspective a new aspect is introduced to meson decay: bound-state corrections. The lowest order correction is one that involves only one bound-state kernel $\Delta(\mu\nu; \sigma\rho)$. This implies that the Hamiltonian representing this correction must be of fifth order in the power expansion of the wave function.

We shall call this new Hamiltonian, with the same basic operatorial structure $m_\alpha^\dagger m_\beta^\dagger m_\gamma$, of δH_m . The only combinations $q^{\dagger(i)}\bar{q}^{\dagger(j)}$ that result in a fifth order Hamiltonian are

$$\delta H_m = [q_\mu^{\dagger(3)}\bar{q}_\nu^{\dagger(2)} + q_\mu^{\dagger(1)}\bar{q}_\nu^{\dagger(4)} + q_\mu^{\dagger(5)}\bar{q}_\nu^{\dagger(0)}]V_{\mu\nu}. \quad (47)$$

Details of this calculation are found in Appendix B. The bound-state corrected 3P_0 Hamiltonian, which shall be called the 3P_0 Hamiltonian, is

$$H^{C3P0} = H_m + \delta H_m = -\Phi_\alpha^{*\rho\xi}\Phi_\beta^{*\lambda\tau}\Phi_\gamma^{\omega\sigma}V^{C3P0}m_\alpha^\dagger m_\beta^\dagger m_\gamma \quad (48)$$

where V^{C3P0} is a condensed notation for

$$V^{C3P0} = [\delta_{\mu\lambda}\delta_{\xi\nu}\delta_{\omega\rho}\delta_{\sigma\tau} - \frac{1}{2}\delta_{\sigma\xi}\delta_{\lambda\omega}\Delta(\rho\tau; \mu\nu) + \frac{1}{4}\delta_{\sigma\xi}\delta_{\mu\lambda}\Delta(\rho\tau; \omega\nu) + \frac{1}{4}\delta_{\xi\nu}\delta_{\lambda\omega}\Delta(\rho\tau; \mu\sigma)]V_{\mu\nu}. \quad (49)$$

IV. LIGHT MESON DECAY EXAMPLES

The light meson sector is an interesting test ground where the effects of the bound-state correction can be compared to the usual 3P_0 model. In particular, as examples, two specific decay processes will be studied: $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$. The wave function and details of the matrix elements are found in Appendix C. The general decay amplitude can be written as

$$h_{fi}^{C^3P_0} = \frac{\gamma}{\pi^{1/4}\beta^{1/2}} \mathcal{M}_{fi}^{C^3P_0}. \quad (50)$$

For the first decay process, $b_1 \rightarrow \omega\pi$ results in a decay amplitude given by

$$\mathcal{M}_{fi}^{C^3P_0} = C_{01}Y_{00}(\Omega_x) + C_{21}Y_{20}(\Omega_x), \quad (51)$$

with

$$\begin{aligned} C_{01} &\equiv -\frac{2^4}{3^{5/2}} \left[1 - \frac{2}{9}x^2\right] e_1(x) + \frac{2^5}{7^{5/2} \cdot 3} \left[1 - \frac{8}{21}x^2\right] e_2(x), \\ C_{21} &\equiv -x^2 \left[\frac{2^{11/2}}{3^{9/2}} e_1(x) - \frac{2^{17/2}}{7^{7/2} \cdot 9} e_2(x) \right], \end{aligned} \quad (52)$$

where $x = P/\beta$ and

$$e_1(x) = \exp\left(-\frac{x^2}{12}\right); \quad e_2(x) = \exp\left(-\frac{9x^2}{28}\right). \quad (53)$$

The decay rate has a straightforward evaluation, by substituting (51) and (52) in (50) and then in (38) obtaining

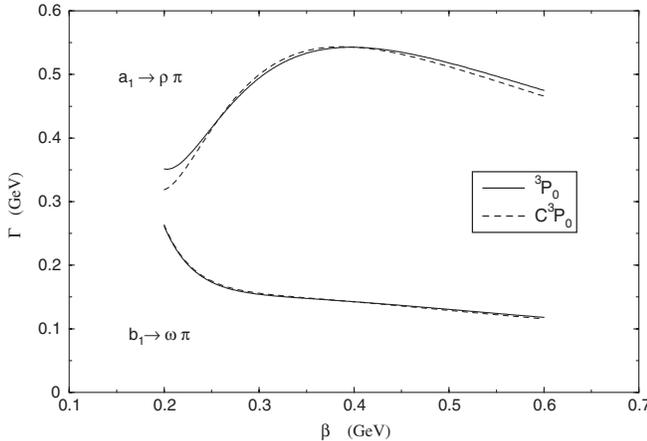


FIG. 2. Decay rates for $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$ decays, for 3P_0 ($\gamma = 0.506$) and C^3P_0 ($\gamma = 0.539$) models.

$$\Gamma_{b_1 \rightarrow \omega\pi} = 2\sqrt{\pi}x \frac{E_\omega E_\pi}{M_{b_1}} \gamma^2 (C_{01}^2 + C_{21}^2). \quad (54)$$

The second decay process, $a_1 \rightarrow \rho\pi$, is similar to the former one and results in the following amplitude:

$$\mathcal{M}_{fi}^{C^3P_0} = C_{01}Y_{00}(\Omega_x) + C_{21}Y_{20}(\Omega_x), \quad (55)$$

with

$$\begin{aligned} C_{01} &\equiv \frac{2^{9/2}}{3^{5/2}} \left[1 - \frac{2}{9}x^2\right] e_1(x) - \frac{2^{11/2}}{7^{5/2} \cdot 3} \left[1 - \frac{8}{21}x^2\right] e_2(x), \\ C_{21} &\equiv -x^2 \left[\frac{2^5}{3^{9/2}} e_1(x) - \frac{2^{17/2}}{3^2 \cdot 7^{7/2}} e_2(x) \right], \end{aligned} \quad (56)$$

and by a similar procedure one obtains

$$\Gamma_{a_1 \rightarrow \rho\pi} = 2\sqrt{\pi}x \frac{E_\rho E_\pi}{M_{a_1}} \gamma^2 (C_{01}^2 + C_{21}^2). \quad (57)$$

In the former equations, $e_2(x) = 0$ recovers the original 3P_0 results.

In addition to the decay widths Γ , b_1 , and a_1 mesons have D/S ratios, which give a sensitive test for decay models. By definition, these quantities are obtained from the ratios of C_{21} and C_{01} coefficients, in Eqs. (52) and (56),

$$\begin{aligned} \frac{D}{S} \Big|_{a_1 \rightarrow \rho\pi} &= \frac{-x^2 \left[\frac{2^{11/2}}{3^2} e_1(x) - \frac{3^{1/2} 2^{5/2} 5}{7^{7/2}} e_2(x) \right]}{\left[1 - \frac{2}{9}x^2\right] e_1(x) - \frac{3^{3/2} 2}{7^{5/2}} \left[1 - \frac{8}{21}x^2\right] e_2(x)}, \\ \frac{D}{S} \Big|_{b_1 \rightarrow \omega\pi} &= \frac{x^2 \left[\frac{2^{3/2}}{3^2} e_1(x) - \frac{2^{9/2} 3^{1/2}}{7^{7/2}} e_2(x) \right]}{\left[1 - \frac{2}{9}x^2\right] e_1(x) - \frac{3^{3/2} 2}{7^{5/2}} \left[1 - \frac{8}{21}x^2\right] e_2(x)}. \end{aligned} \quad (58)$$

The meson masses assumed in the numerical calculation were $M_\pi = 138$ MeV; $M_\rho = 775$ MeV; $M_{a_1} = 1230$ MeV; $M_{b_1} = 1229$ MeV; $M_\omega = 782$ MeV [20].

The choice of simple harmonic oscillator (SHO) wave functions allow exact evaluations of the decay amplitudes even in the corrected model. A first new aspect that appears is the presence of a new dependence in the exponential of the corrected term. This implies a different range for the bound-state correction due to the fact that $e_2(x)/e_1(x) \rightarrow 0$ as $x \rightarrow \infty$.

The correction introduces the bound-state kernel, Eq. (23), to the calculation of the decay processes. A general sum over the meson index α is present and as stated before, this index represents the quantum numbers

TABLE I. Decay rates 3P_0 ($\gamma = 0.506e$, $\beta = 0.397$ GeV and C^3P_0 ($\gamma = 0.539e$, $\beta = 0.396$ GeV).

Decay	Γ (MeV)			D/S		
	Exp [20]	3P_0	C^3P_0	Exp [20]	3P_0	C^3P_0
$b_1 \rightarrow \omega\pi$	142	143	142	0.277(27)	0.288	0.273
$a_1 \rightarrow \rho\pi$	250 to 600	543	543	-0.108(16)	-0.149	-0.113

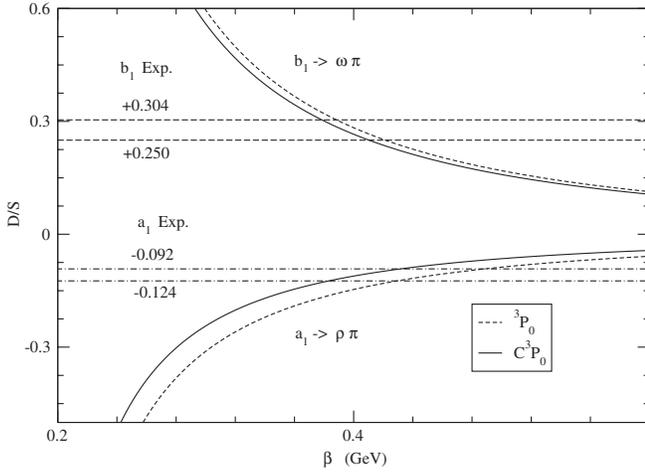


FIG. 3. D/S ratios in $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$ decays, for 3P_0 and C^3P_0 models.

of space, spin, and isospin. The OZI-allowed decays represent flavor conserved continuous (anti)quark lines. A direct consequence of this fact is the possibility to sum over a larger set of mesons in the α index. In our calculation the sum was restricted only to the final state mesons. In the $b_1^+ \rightarrow \omega\pi^+$ decay, there are two bound-state kernel contributions one associated to the ω meson and the other to π^+ . Similarly, the $a_1^+ \rightarrow \rho^+\pi^0$ decay has two bound-state kernel contributions: one associated to the ρ^+ meson and the other to the π^0 .

In this example, the parameters were chosen in order to give a closer fit to the experimental data. In the b_1 decay, width and partial waves are known with accuracy. The 3P_0 model's optimum fit for the b_1 data (Γ and D/S ratio) is achieved with $\gamma = 0.506$ and $\beta = 0.397$ GeV. In the C^3P_0 model a similar fit is obtained with $\gamma = 0.539$ and $\beta = 0.396$ GeV. These parameters are used in the two models to describe the a_1 decay. The results for Γ as a function of β appear in Fig. 2 and specific values are presented in Table I. In Fig. 3, the D/S ratios for the two models are plotted.

V. SUMMARY AND CONCLUSIONS

In this paper we have presented an alternative approach for meson decay which consists of a mapping technique, known as the Fock-Tani formalism, long used in atomic physics. This formalism has been applied to hadron-hadron scattering interactions with constituent interchange. The challenge resided in extending the approach to include meson decay. After demonstrating that in lower order the result obtained was equivalent to the 3P_0 model, an additional feature pointed out was the appearance of bound-state corrections in the effective decay Hamiltonian. These corrections present a natural modification in the $q\bar{q}$ interaction strength. As an example, we studied two decay processes $b_1 \rightarrow \omega\pi$ and $a_1 \rightarrow \rho\pi$. The D/S ratios in

Fig. 3 show that a common range of β values for mesons is obtained. In a new calculation with the inclusion of other decay processes it might require different β values [21]. The corrected model presents an interesting feature that for these two mesons the decay width differs slightly when compared with the 3P_0 , but D/S ratios are improved. The examples studied here are encouraging but a more extensive survey of the light meson sector would be a necessary next step. The inclusion of the full meson octet, in the evaluation of the bound-state correction, may provide a fine-tuning for the model.

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APPENDIX A: SECOND AND THIRD ORDER OPERATORS

The second order operators are

$$\begin{aligned} q_\mu^{(2)}(t) &= \frac{1}{2}\Phi_\alpha^{*\mu_2\nu_1}\Phi_\beta^{\mu\nu_1}M_{\alpha\beta}q_{\mu_2}, \\ \bar{q}_\nu^{(2)}(t) &= \frac{1}{2}\Phi_\alpha^{*\mu_1\nu_2}\Phi_\beta^{\mu_1\nu}M_{\alpha\beta}\bar{q}_{\nu_2}, \end{aligned} \quad (\text{A1})$$

where

$$\begin{aligned} M_{\alpha\beta} &= m_\alpha^\dagger M_\beta \text{sint cost} - m_\alpha^\dagger m_\beta \text{sin}^2 t \\ &\quad - M_\alpha^\dagger M_\beta (2 - 2 \text{cost} - \text{sin}^2 t) \\ &\quad - M_\alpha^\dagger m_\beta (2 \text{sint} - \text{sint cost}). \end{aligned} \quad (\text{A2})$$

The third order operators are

$$\begin{aligned} q_\mu^{(3)}(t) &= \frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\beta^{\mu\sigma}\Phi_\gamma^{\rho\sigma_1}q_{\sigma_1}^\dagger M_{\alpha\beta\gamma} \\ &\quad - \frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\alpha^{\mu\nu_1}\Phi_\beta^{\rho\sigma_1}q_{\nu_1}^\dagger q_{\sigma_1}^\dagger \bar{q}_\sigma \bar{M}_\beta \\ &\quad - \frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\alpha^{\mu\nu_1}\Phi_\beta^{\rho_1\sigma}q_{\nu_1}^\dagger q_{\rho_1}^\dagger q_\rho \bar{M}_\beta, \\ \bar{q}_\nu^{(3)}(t) &= -\frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\beta^{\rho\nu}\Phi_\gamma^{\rho_1\sigma}q_{\rho_1}^\dagger M_{\alpha\beta\gamma} \\ &\quad + \frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\alpha^{\mu_1\nu}\Phi_\beta^{\rho\sigma_1}q_{\mu_1}^\dagger q_{\rho_1}^\dagger \bar{q}_\sigma \bar{M}_\beta \\ &\quad + \frac{1}{2}\Phi_\alpha^{*\rho\sigma}\Phi_\alpha^{\mu_1\nu}\Phi_\beta^{\rho_1\sigma}q_{\mu_1}^\dagger q_{\rho_1}^\dagger q_\rho \bar{M}_\beta, \end{aligned} \quad (\text{A3})$$

where

$$\begin{aligned}
M_{\alpha\beta\gamma} &= m_\alpha^\dagger m_\beta m_\gamma \sin^3 t + M_\alpha^\dagger M_\beta m_\gamma (\sin t - \sin^3 t) \\
&\quad + M_\alpha^\dagger m_\beta M_\gamma (2 \sin t - \sin t \cos t - \sin^3 t) \\
&\quad + (M_\alpha^\dagger m_\beta m_\gamma + m_\alpha^\dagger M_\beta m_\gamma) (-\cos t + \cos^3 t) \\
&\quad + m_\alpha^\dagger m_\beta M_\gamma (-\cos t + \cos^3 t + \sin^2 t) \\
&\quad + M_\alpha^\dagger M_\beta M_\gamma (2 - \cos t - \cos^3 t - \sin^2 t) \\
&\quad + m_\alpha^\dagger M_\beta M_\gamma (\sin t - \sin t \cos t - \sin^3 t), \\
\bar{M}_\beta &= 2M_\beta (1 - \cos t) + m_\beta \sin t. \tag{A4}
\end{aligned}$$

APPENDIX B: THE δH_m HAMILTONIAN

The δH_m Hamiltonian is evaluated from Eq. (47). The $q_\mu^{\dagger(3)} \bar{q}_\nu^{\dagger(2)}$ combination can be obtained from (A1) and (A3)

$$\begin{aligned}
\delta H_1 &= q_\mu^{\dagger(3)} \bar{q}_\nu^{\dagger(2)}, \\
V_{\mu\nu} &= \delta f_1^{\mu\nu}(\alpha, \beta, \gamma) V_{\mu\nu} m_\alpha^\dagger m_\beta^\dagger m_\gamma \tag{B1}
\end{aligned}$$

with

$$\delta f_1^{\mu\nu}(\alpha, \beta, \gamma) = \frac{1}{4} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\mu\tau} \Delta(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma}. \tag{B2}$$

The $q_\mu^{\dagger(1)} \bar{q}_\nu^{\dagger(4)}$ combination has an important feature: a contribution from a higher order operator. A new generator \tilde{M}_α has to be evaluated, with the inclusion of the following fourth order term:

$$\begin{aligned}
\tilde{M}_\alpha^{(4)} &= \frac{3}{8} \Delta_{\alpha\beta} \Delta_{\beta\gamma} M_\gamma - \frac{1}{8} M_\beta^\dagger [\Delta_{\alpha\gamma}, \Delta_{\beta\delta}] M_\delta M_\gamma \\
&\quad - \frac{1}{4} M_\beta^\dagger [M_\alpha, T_{\delta\gamma\beta}^\dagger] M_\gamma M_\delta. \tag{B3}
\end{aligned}$$

The only relevant term in the $\bar{q}_\nu^{\dagger(4)}$ for meson decay is

$$\bar{q}_\nu^{(4)}(t) \approx -\frac{1}{8} \Phi_\alpha^{*\sigma\eta} \Delta(\sigma\nu; \rho\tau) \Phi_\beta^{\rho\eta} M_\alpha^{\dagger(0)}(t) \bar{q}_\tau M_\beta^{(0)}(t). \tag{B4}$$

The resulting contribution is then

$$\begin{aligned}
\delta H_2 &= q_\mu^{\dagger(1)} \bar{q}_\nu^{\dagger(4)}, \\
V_{\mu\nu} &= \delta f_2^{\mu\nu}(\alpha, \beta, \gamma) V_{\mu\nu} m_\alpha^\dagger m_\beta^\dagger m_\gamma, \tag{B5}
\end{aligned}$$

where

$$\delta f_2^{\mu\nu}(\alpha, \beta, \gamma) = -\frac{1}{8} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\mu\tau} \Delta(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma}. \tag{B6}$$

The $q_\mu^{\dagger(5)} \bar{q}_\nu^{\dagger(0)}$ combination implies in a fifth order generator to obtain the complete set of equations of motion (17) and (19)

$$\tilde{M}_\alpha^{\dagger(5)} = -M_\beta^\dagger Z_{\alpha\gamma\beta}^\dagger M_\gamma + \frac{1}{8} M_\omega^\dagger M_\beta^\dagger W_{\alpha\omega\beta\gamma\delta}^\dagger M_\delta M_\gamma, \tag{B7}$$

where

$$\begin{aligned}
Z_{\alpha\gamma\beta}^\dagger &= -\frac{3}{8} T_{\alpha\delta\beta}^\dagger \Delta_{\delta\gamma} - \frac{5}{8} \Delta_{\beta\delta} T_{\alpha\gamma\delta}^\dagger - \frac{1}{4} T_{\delta\gamma\beta}^\dagger \Delta_{\delta\alpha}, \\
W_{\alpha\omega\beta\gamma\delta}^\dagger &= [M_\alpha^\dagger, Q_{\omega\beta\gamma\delta}] - [\Delta_{\omega\gamma}, T_{\alpha\delta\beta}^\dagger], \\
Q_{\alpha\beta\gamma\delta} &= -\frac{1}{2} [\Delta_{\alpha\gamma}, \Delta_{\beta\delta}] - [M_\alpha, T_{\delta\gamma\beta}^\dagger]. \tag{B8}
\end{aligned}$$

The only relevant terms in the $q_\mu^{\dagger(5)}$ for meson decay are

$$\begin{aligned}
q_\mu^{\dagger(5)}(t) &\approx [\frac{1}{2} \Delta(\rho\tau; \mu\omega) \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\lambda\tau} \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{1}{4} \Delta(\rho\tau; \mu\sigma) \Phi_\alpha^{*\rho\omega} \Phi_\beta^{*\lambda\tau} \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{3}{8} \Delta(\rho\tau; \lambda\omega) \Phi_\alpha^{*\mu\tau} \Phi_\beta^{*\rho\sigma} \Phi_\gamma^{\lambda\sigma}] \\
&\quad \times M_\alpha^{\dagger(0)}(t) M_\beta^{\dagger(0)}(t) \bar{q}_\omega M_\gamma^{(0)}(t). \tag{B9}
\end{aligned}$$

The resulting contribution is

$$\delta H_3 = \delta f_3^{\mu\nu}(\alpha, \beta, \gamma) V_{\mu\nu} m_\alpha^\dagger m_\beta^\dagger m_\gamma, \tag{B10}$$

where

$$\begin{aligned}
\delta f_3^{\mu\nu}(\alpha, \beta, \gamma) &= \frac{1}{2} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\lambda\tau} \Delta(\rho\tau; \mu\nu) \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{1}{4} \Phi_\alpha^{*\sigma\tau} \Phi_\beta^{*\rho\nu} \Delta(\rho\tau; \mu\lambda) \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{3}{8} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\mu\tau} \Delta(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma}. \tag{B11}
\end{aligned}$$

The complete δH_m Hamiltonian is

$$\begin{aligned}
\delta H_m &= \delta H_1 + \delta H_2 + \delta H_3 \\
&= \delta f^{\mu\nu}(\alpha, \beta, \gamma) V_{\mu\nu} m_\alpha^\dagger m_\beta^\dagger m_\gamma \tag{B12}
\end{aligned}$$

with

$$\begin{aligned}
\delta f^{\mu\nu}(\alpha, \beta, \gamma) &= \delta f_1^{\mu\nu} + \delta f_2^{\mu\nu} + \delta f_3^{\mu\nu} \\
&= \frac{1}{2} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\lambda\tau} \Delta(\rho\tau; \mu\nu) \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{1}{4} \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\mu\tau} \Delta(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma} \\
&\quad - \frac{1}{4} \Phi_\alpha^{*\sigma\tau} \Phi_\beta^{*\rho\nu} \Delta(\rho\tau; \mu\lambda) \Phi_\gamma^{\lambda\sigma}. \tag{B13}
\end{aligned}$$

APPENDIX C: DECAY, WAVE FUNCTION, AND MATRIX ELEMENTS

We will use the decay $b_1^+(+\hat{z}) \rightarrow \omega(+\hat{z})\pi^+$ to illustrate the nature of our formalism and simply quote the other case in the text.

1. The wave function

The general meson wave function can be written as

$$\Phi_\alpha^{\mu\nu} = \chi_{S_\alpha}^{s_1 s_2} f_{f_\alpha}^{f_1 f_2} C^{c_1 c_2} \Phi_{nl}^{\vec{p}_\alpha - \vec{p}_1 - \vec{p}_2}, \tag{C1}$$

a direct product of the spin $\chi_{S_\alpha}^{s_1 s_2}$ [the indexes s_1 and s_2 are the quark (antiquark) spin projections with ($s = 1 \Rightarrow \uparrow$ and $s = 2 \Rightarrow \downarrow$); the index S_α denotes the meson spin]; flavor $f_{f_\alpha}^{f_1 f_2}$; color $C^{c_1 c_2}$, and space $\Phi_{nl}^{\vec{p}_\alpha - \vec{p}_1 - \vec{p}_2}$ components.

In all our calculations the color component will be given by

$$C^{c_1 c_2} = \frac{1}{\sqrt{3}} \delta^{c_1 c_2}. \quad (\text{C2})$$

The spatial part is defined as harmonic oscillator wave functions

$$\Phi_{nl}^{\vec{p}_\alpha - \vec{p}_1 - \vec{p}_2} = \delta(\vec{P}_\alpha - \vec{p}_1 - \vec{p}_2) \Phi_{nl}(\vec{p}_1, \vec{p}_2), \quad (\text{C3})$$

where $\Phi_{nl}(\vec{p}_i, \vec{p}_j)$ is given by

$$\begin{aligned} \Phi_{nl}(\vec{p}_i, \vec{p}_j) &= \left(\frac{1}{2\beta}\right)^l N_{nl} |\vec{p}_i - \vec{p}_j|^l \exp\left[-\frac{(\vec{p}_i - \vec{p}_j)^2}{8\beta^2}\right] \\ &\times \mathcal{L}_n^{l+(1/2)}\left[\frac{(\vec{p}_i - \vec{p}_j)^2}{4\beta^2}\right] Y_{lm}(\Omega_{\vec{p}_i - \vec{p}_j}) \end{aligned} \quad (\text{C4})$$

with $p_{i(j)}$ the internal momentum, the spherical harmonic Y_{lm} , β a scale parameter, and N_{nl} the normalization constant dependent on the radial and orbital quantum numbers

$$N_{nl} = \left[\frac{2(n!)}{\beta^3 \Gamma(n+l+3/2)}\right]^{1/2}. \quad (\text{C5})$$

The Laguerre polynomials $\mathcal{L}_n^{l+(1/2)}(p)$ are defined as

$$\mathcal{L}_n^{l+(1/2)}(p) = \sum_{k=0}^n \frac{(-)^k \Gamma(n+l+3/2)^{(n-k)!}}{k! \Gamma(k+l+3/2)} p^k. \quad (\text{C6})$$

In this paper two kinds of light nonstrange mesons will be studied:

(1) $L_{q\bar{q}} = 0$

$$\varphi(\vec{p}) \equiv \Phi_{00}(\vec{p}) = \frac{1}{\pi^{3/4} \beta^{3/2}} \exp\left[-\frac{p^2}{8\beta^2}\right] \quad (\text{C7})$$

(2) $L_{q\bar{q}} = 1$

$$\Phi_{1m}(\vec{p}) = \phi(\vec{p}) Y_{1m}(\Omega_{\vec{p}}) \quad (\text{C8})$$

with

$$\phi(\vec{p}) = \left[\frac{2}{3\sqrt{\pi}\beta^5}\right]^{1/2} p \exp\left[-\frac{p^2}{8\beta^2}\right]. \quad (\text{C9})$$

Returning to our example the pion has $J = 0$ and b_1 $J = 1$. We choose the $(+\hat{z})$ direction for this calculation, so the spin wave functions become

$$\begin{aligned} |b_1\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\bar{\downarrow}\rangle - |\downarrow\bar{\uparrow}\rangle), & |\omega\rangle &= |\uparrow\bar{\uparrow}\rangle, \\ |\pi\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\bar{\downarrow}\rangle - |\downarrow\bar{\uparrow}\rangle) \end{aligned} \quad (\text{C10})$$

or in the χ notation

$$\begin{aligned} \chi_\omega^{11} &= 1, & \chi_\omega^{12} &= \chi_\omega^{21} = \chi_\omega^{22} = 0, \\ \chi_{\pi, b_1}^{11} &= \chi_{\pi, b_1}^{22} = 0; & \chi_{\pi, b_1}^{12} &= -\chi_{\pi, b_1}^{21} = \frac{1}{\sqrt{2}}. \end{aligned} \quad (\text{C11})$$

The flavor component $f_{f_\alpha}^{f_\mu f_\nu}$ follows the same logic as the spin part

$$\begin{aligned} |b_1^+\rangle &= |\pi^+\rangle = -|u\bar{d}\rangle, \\ |\omega\rangle &= \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle). \end{aligned} \quad (\text{C12})$$

The b_1^+ and the π^+ mesons have the same flavor contribution

$$f_{b_1^+, \pi^+}^{12} = -1; \quad f_{b_1^+, \pi^+}^{11} = f_{b_1^+, \pi^+}^{21} = f_{b_1^+, \pi^+}^{22} = 0. \quad (\text{C13})$$

For ω , one has

$$f_\omega^{11} = f_\omega^{22} = \frac{1}{\sqrt{2}}; \quad f_\omega^{12} = f_\omega^{21} = 0. \quad (\text{C14})$$

2. The spin matrix elements

In the evaluation of a decay amplitude, the following spin matrix element is necessary:

$$\chi_{s'}^*(\vec{\sigma} \cdot \vec{P}) \chi_s^c \quad (\text{C15})$$

with

$$\begin{aligned} \chi_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}; & \chi_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}; \\ \chi_1^c &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}; & \chi_2^c &= \begin{pmatrix} -1 \\ 0 \end{pmatrix}. \end{aligned} \quad (\text{C16})$$

By direct calculation one can show

$$\begin{aligned} \chi_1^*(\vec{\sigma} \cdot \vec{P}) \chi_1^c &= P_x - iP_y, & \chi_1^*(\vec{\sigma} \cdot \vec{P}) \chi_2^c &= -P_z, \\ \chi_2^*(\vec{\sigma} \cdot \vec{P}) \chi_1^c &= -P_z, & \chi_2^*(\vec{\sigma} \cdot \vec{P}) \chi_2^c &= -(P_x + iP_y). \end{aligned} \quad (\text{C17})$$

3. Matrix elements: $b_1^+ \rightarrow \omega \pi^+$ decay

The transition considered is of the form $m_\gamma \rightarrow m_\alpha + m_\beta$, where the initial state is $|A\rangle = m_\gamma^\dagger |0\rangle$ and the final state is given by $|BC\rangle = m_\alpha^\dagger m_\beta^\dagger |0\rangle$. The matrix element of the uncorrected part results in

$$\langle BC | H_m | A \rangle = -d_1 - d_2; \quad (\text{C18})$$

d_1 and d_2 are defined as

$$d_1 \equiv \Phi_\alpha^{*\rho\nu} \Phi_\beta^{*\mu\lambda} \Phi_\gamma^{\rho\lambda} V_{\mu\nu}, \quad d_2 \equiv \Phi_\alpha^{*\mu\lambda} \Phi_\beta^{*\rho\nu} \Phi_\gamma^{\rho\lambda} V_{\mu\nu}. \quad (\text{C19})$$

Equations (C19) can be decomposed according to the

sector of the wave function to which they correspond: flavor, color, spin space:

$$d_1 = d_1^f d_1^c d_1^{s-e}, \quad d_2 = d_2^f d_2^c d_2^{s-e}. \quad (\text{C20})$$

The matrix elements of the bound-state correction refer to diagrams (b), (c), and (d) of Fig. 1. The bound-state kernel's definition as $\Phi_\alpha^{\rho\tau} \Phi_\alpha^{*\lambda\nu}$ implies in an additional element, due to the contraction in the α index, a *sum over species* requirement [11]. A question that naturally arises is which states to include in this sum. We shall adopt in our calculation a restrictive choice: include in the sum only the particles that are present in the final state. For the b_1^+ decay, $\Delta(\rho\tau; \lambda\nu)$ will have two contributions: ω and π^+ . Similarly, the a_1^+ decay shall be corrected by the final state mesons ρ^+ and π^0 .

Because of the parity assignment of the spatial part, the integration of diagram (1b) is zero. Spatial symmetry also implies that the matrix elements of diagrams (1c) and (1d) are equal. This simplifies our calculation, reducing the problem to the evaluation of diagram (1c) only. The bound-state correction (bsc) matrix element reduces to evaluate the following expression:

$$\langle BC | \delta H_m | A \rangle = -d_1^{\text{bsc}} - d_2^{\text{bsc}}, \quad (\text{C21})$$

where

$$d_1^{\text{bsc}} = \frac{1}{4}(d_{1\omega} + d_{1\pi}), \quad d_2^{\text{bsc}} = \frac{1}{4}(d_{2\omega} + d_{2\pi}) \quad (\text{C22})$$

with

$$\begin{aligned} d_{1j} &= \Phi_\alpha^{*\rho\sigma} \Phi_\beta^{*\mu\tau} \Delta_j(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma} V_{\mu\nu} \equiv d_{1j}^f d_{1j}^c d_{1j}^{s-e}, \\ d_{2j} &= \Phi_\alpha^{*\mu\tau} \Phi_\beta^{*\rho\sigma} \Delta_j(\rho\tau; \lambda\nu) \Phi_\gamma^{\lambda\sigma} V_{\mu\nu} \equiv d_{2j}^f d_{2j}^c d_{2j}^{s-e}. \end{aligned} \quad (\text{C23})$$

In (C23) j refers to mesons ω and π^+ .

4. $b_1^+ \rightarrow \omega\pi^+$ decay (uncorrected)

(i) Flavor:

$$d_1^f = d_2^f = f_\pi^{f\rho f\nu} f_\omega^{f\mu f\lambda} f_{b_1}^{f\rho f\lambda} \delta_{f_\mu f_\nu} = \frac{1}{\sqrt{2}}. \quad (\text{C24})$$

(ii) Color:

$$d_1^c = d_2^c = \frac{1}{3\sqrt{3}} \delta^{c_\rho c_\nu} \delta^{c_\mu c_\lambda} \delta^{c_\rho c_\lambda} \delta^{c_\mu c_\nu} = \frac{1}{3}. \quad (\text{C25})$$

(iii) Spin space: The spin matrix element is

$$d_1^s = d_2^s = \chi_\pi^{s_\rho s_\nu} \chi_\omega^{s_\mu s_\lambda} \chi_{b_1}^{s_\rho s_\lambda} V_{s_\mu s_\nu}^{s-e} = \frac{1}{2} V_{11}^{s-e}(\vec{p}_\mu, \vec{p}_\nu), \quad (\text{C26})$$

where

$$\begin{aligned} V_{11}^{s-e}(\vec{p}_\mu, \vec{p}_\nu) &= -\gamma \delta(\vec{p}_\mu + \vec{p}_\nu) \\ &\quad \times \chi_1^*[\vec{\sigma} \cdot (\vec{p}_\mu - \vec{p}_\nu)] \chi_1^c. \end{aligned} \quad (\text{C27})$$

Using (C17) to evaluate (C27) and after integrating momentum conservation deltas one arrives at

$$\begin{aligned} d_1^{s-e} &= -\gamma \int d^3 K(K_x - iK_y) \varphi(\vec{P} - 2\vec{K}) \\ &\quad \times \phi(2\vec{P} - 2\vec{K}) Y_{11}(\Omega_{2\vec{P}-2\vec{K}}) \varphi(\vec{P} - 2\vec{K}). \end{aligned} \quad (\text{C28})$$

Introducing the spatial wave function and integrating

$$\begin{aligned} d_1^{s-e} &= \left(\frac{2^{7/2}}{3^{5/2}}\right) \left(\frac{\gamma}{\pi^{1/4} \beta^{1/2}}\right) \left\{ \left[1 - \frac{2}{9}x^2\right] Y_{00}(\Omega_x) \right. \\ &\quad \left. + \frac{2}{3^2\sqrt{5}} x^2 Y_{20}(\Omega_x) \right\} e_1(x), \end{aligned} \quad (\text{C29})$$

d_2^{s-e} is obtained from d_1^{s-e} by $\vec{P} \rightarrow -\vec{P}$. The decay amplitude results in

$$\begin{aligned} h_{fi} &= -\left(\frac{2^4}{3^{5/2}}\right) \left(\frac{\gamma}{\pi^{1/4} \beta^{1/2}}\right) \left\{ \left[1 - \frac{2}{9}x^2\right] Y_{00}(\Omega_x) \right. \\ &\quad \left. + \frac{2}{3^2\sqrt{5}} x^2 Y_{20}(\Omega_x) \right\} e_1(x). \end{aligned} \quad (\text{C30})$$

5. $b_1^+ \rightarrow \omega\pi^+$ decay (bound-state corrected)

The quantities between [...] in the following expressions are related to the bound-state kernel.

(i) Flavor:

$$\begin{aligned} d_{1\omega}^f &= f_\pi^{f\rho f\sigma} f_\omega^{f\mu f\tau} [f_\omega^{f\rho f\tau} f_\omega^{f\lambda f\mu}] f_{b_1}^{f\lambda f\sigma} = \frac{1}{2\sqrt{2}}, \\ d_{1\pi}^f &= f_\pi^{f\rho f\sigma} f_\omega^{f\mu f\tau} [f_\pi^{f\rho f\tau} f_\pi^{f\lambda f\mu}] f_{b_1}^{f\lambda f\sigma} = \frac{1}{\sqrt{2}}, \\ d_{2\omega}^f &= f_\pi^{f\mu f\tau} f_\omega^{f\rho f\sigma} [f_\omega^{f\rho f\tau} f_\omega^{f\lambda f\mu}] f_{b_1}^{f\lambda f\sigma} = \frac{1}{2\sqrt{2}}, \\ d_{2\pi}^f &= f_\pi^{f\mu f\tau} f_\omega^{f\rho f\sigma} [f_\pi^{f\rho f\tau} f_\pi^{f\lambda f\mu}] f_{b_1}^{f\lambda f\sigma} = 0. \end{aligned} \quad (\text{C31})$$

(ii) Color:

$$\begin{aligned} d_{1\omega}^c &= d_{1\pi}^c = d_{2\omega}^c = d_{2\pi}^c \\ &= \frac{1}{9\sqrt{3}} \delta^{c_\rho c_\sigma} \delta^{c_\mu c_\tau} [\delta^{c_\rho c_\tau} \delta^{c_\lambda c_\nu}] \delta^{c_\lambda c_\sigma} \delta^{c_\mu c_\nu} = \frac{1}{3\sqrt{3}}. \end{aligned} \quad (\text{C32})$$

(iii) Spin space: The spin matrix element is

$$\begin{aligned}
d_{1\omega}^s &= \chi_{\pi}^{s_{\rho} s_{\sigma}} \chi_{\omega}^{s_{\mu} s_{\tau}} [\chi_{\omega}^{s_{\rho} s_{\tau}} \chi_{\omega}^{s_{\lambda} s_{\nu}}] \chi_{b_1}^{s_{\lambda} s_{\sigma}} V_{s_{\mu} s_{\nu}}^{s-e} \\
&= \frac{1}{2} V_{11}^{s-e}(\vec{p}_{\mu}, \vec{p}_{\nu}), \\
d_{1\pi}^s &= \chi_{\pi}^{s_{\rho} s_{\sigma}} \chi_{\omega}^{s_{\mu} s_{\tau}} [\chi_{\pi}^{s_{\rho} s_{\tau}} \chi_{\pi}^{s_{\lambda} s_{\nu}}] \chi_{b_1}^{s_{\lambda} s_{\sigma}} V_{s_{\mu} s_{\nu}}^{s-e} \\
&= \frac{1}{4} V_{11}^{s-e}(\vec{p}_{\mu}, \vec{p}_{\nu}), \\
d_{2\omega}^s &= \chi_{\pi}^{s_{\mu} s_{\tau}} \chi_{\omega}^{s_{\rho} s_{\sigma}} [\chi_{\omega}^{s_{\rho} s_{\tau}} \chi_{\omega}^{s_{\lambda} s_{\nu}}] \chi_{b_1}^{s_{\lambda} s_{\sigma}} V_{s_{\mu} s_{\nu}}^{s-e} = 0, \\
d_{2\pi}^s &= \chi_{\pi}^{s_{\mu} s_{\tau}} \chi_{\omega}^{s_{\rho} s_{\sigma}} [\chi_{\pi}^{s_{\rho} s_{\tau}} \chi_{\pi}^{s_{\lambda} s_{\nu}}] \chi_{b_1}^{s_{\lambda} s_{\sigma}} V_{s_{\mu} s_{\nu}}^{s-e} \\
&= \frac{1}{4} V_{11}^{s-e}(\vec{p}_{\mu}, \vec{p}_{\nu}).
\end{aligned} \tag{C33}$$

Because of symmetries in the spatial part the following relations are true:

$$d_{1\pi}^{s-e} = d_{2\pi}^{s-e} = \frac{1}{2} d_{1\omega}^{s-e}; \quad d_{2\omega}^{s-e} = 0, \tag{C34}$$

where $d_{1\pi}^{s-e}$ is given by

$$\begin{aligned}
d_{1\pi}^{s-e} &= \frac{\gamma}{2} \int d^3 K d^3 q (K_x - iK_y) \varphi(2\vec{q} - \vec{P}) \\
&\quad \times \varphi(2\vec{K} - \vec{P}) [\varphi(\vec{q} + \vec{K} - 2\vec{P}) \\
&\quad \times \varphi(\vec{q} + \vec{K})] \phi(2\vec{q}) Y_{11}(\Omega_{2\vec{q}}).
\end{aligned} \tag{C35}$$

After integration one finds

$$\begin{aligned}
d_{1\pi}^{s-e} &= -\left(\frac{2^{11/2}}{7^{5/2}}\right) \left(\frac{\gamma}{\pi^{1/4} \beta^{1/2}}\right) \left\{ \left[1 - \frac{8}{21} x^2 \right] Y_{00}(\Omega_x) \right. \\
&\quad \left. + \frac{2^{7/2}}{21} \sqrt{\frac{1}{10}} x^2 Y_{20}(\Omega_x) \right\} e_2(x).
\end{aligned} \tag{C36}$$

The decay amplitude for the bound-state correction is

$$\begin{aligned}
h_{fi}^{\text{bsc}} &= \left(\frac{2^4}{7^{5/2} 3}\right) \left(\frac{\gamma}{\pi^{1/4} \beta^{1/2}}\right) \left\{ \left[1 - \frac{8}{21} x^2 \right] Y_{00}(\Omega_x) \right. \\
&\quad \left. + \frac{2^{7/2}}{21} x^2 Y_{20}(\Omega_x) \right\} e_2(x).
\end{aligned} \tag{C37}$$

The total amplitude will be

$$h_{fi}^{C^3P_0} = h_{fi} + 2h_{fi}^{\text{bsc}} = \frac{\gamma}{\pi^{1/4} \beta^{1/2}} \mathcal{M}_{fi}^{C^3P_0}, \tag{C38}$$

which are expressions (50) and (51).

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