Charm and bottom baryon masses in the combined $1/N_c$ and $1/m_Q$ expansion versus the quark model

Claude Semay* and Fabien Buisseret+
Groupe de Physique Nucléaire Théorique, Université de Mons-Hainaut, Académie universitaire Wallonie-Bruxelles, Place du Parc 20, B-7000 Mons, Belgium

Florica Stancu‡
University of Liège, Institute of Physics B5, Sart Tilman, B-4000 Liège 1, Belgium
(Received 26 August 2008; published 15 October 2008)

A good agreement between a flux tube-based quark model of light baryons (strange and nonstrange) and the $1/N_c$ expansion mass formula has been found in previous studies. In the present work a larger connection is established between the quark model and the $1/N_c$ and $1/m_Q$ expansion method by extending the previous procedure to baryons made of one heavy and two light quarks. The compatibility between both approaches is shown to hold in this sector too.

DOI: 10.1103/PhysRevD.78.076003 PACS numbers: 11.15.Pg, 12.39.Ki, 12.39.Pn, 14.20.−c

I. INTRODUCTION

The recent discoveries of the Ξ_b, Σ_b, and Σ_b^* baryons at the Tevatron have incited to a new analysis of heavy baryons both within the combined $1/N_c$ and $1/m_Q$ expansion [1] and the quark model; see, for example, Refs. [2–4]. The combined $1/N_c$ and $1/m_Q$ expansion is a model-independent method. It is thus important to search for a link between this method and the quark model. In previous studies [5,6] we have investigated the possibility to establish a connection between the two approaches, and we have found that a remarkable compatibility exists between them when dealing with nonstrange [5] or strange baryons [6].

Presently we extend the ideas of our previous studies [5,6] to the case of heavy baryons made of one heavy quark (c or b) and two light ones (u, d, or s). This is the first step of a larger project and we view it as an exploratory work where we search for the compatibility between the spin-independent part of a quark model Hamiltonian and the corresponding terms in the combined $1/N_c$ and $1/m_Q$ expansion mass formula for the ground state. The spin-dependent part as well as the excited states will be analyzed subsequently.

As previously, the comparison of the quark model results with those of the $1/N_c$ expansion, presently combined with a $1/m_Q$ expansion, will be based on the introduction of a quantum number N, which is the same as in the harmonic oscillator potential and which is treated as a band number in baryon phenomenology. The introduction of N in the eigenvalues of the Hamiltonian was quite simple for identical quarks; the procedure becomes more involved for baryons containing heavy quarks, as we shall see.

The paper is organized as follows. After a summary of the charm and bottom baryon flavor states given in Sec. II, the mass formula used by combining the $1/N_c$ and $1/m_Q$ expansions for such baryons is presented in Sec. III. Section IV gives a corresponding mass formula obtained from a Hamiltonian quark model where the confinement is of Y-junction type and where one gluon exchange and quark self-energy contributions are added perturbatively. In that section the excitation quantum number N is introduced and its meaning is discussed. A comparison between results obtained, on one hand, in the combined $1/m_Q$ and $1/N_c$ expansion and, on the other hand, in the quark model is then made in Sec. V. Conclusions are finally drawn in Sec. VI.

In the following, the symbol q will denote a light quark (u, d, s) and the symbol Q will denote a heavy quark (c, b). Moreover, the symbol n will be used for u and d quarks since both particles are assumed to have the same mass, as in our previous works.

II. FLAVOR STATES

A. Charm baryons

Here we introduce the classification of ground state charmed baryons based on SU(4) as due to Glashow, Iliopoulos, and Maiani [7]. Although the SU(4) symmetry is badly violated by the mass difference between heavy and light quarks, this classification scheme is very convenient because it helps in dividing baryons into submultiplets of fixed charm (or beauty). These submultiplets appear naturally from the decomposition of SU(4) irreps into SU(3) irreps describing baryons where the mass difference is essentially due to SU(3) breaking (see e.g. Ref. [8]). A possible extension of this study to, for example, doubly heavy baryons will follow this classification scheme as well.

In the following the total spin of a baryon is denoted by $\vec{J}$, the spin of the light subsystem by $\vec{J}_{qq}$, and that of the heavy quark by $\vec{J}_Q$. In SU(4) the baryon multiplets arise

*claude.semay@umh.ac.be
+fabien.buisseret@umh.ac.be
‡fstancu@ulg.ac.be

from the direct product decomposition $\mathbf{4}\otimes\mathbf{4}\otimes\mathbf{4}=\mathbf{20}\oplus\mathbf{20}\oplus\mathbf{20}\oplus\mathbf{4}$. All baryons in the symmetric multiplet $\mathbf{20}$ have $J^P=\frac{3}{2}^+$. The lightest SU(3) submultiplet is the well-known Gell-Mann-Ne'eman decuplet. The single charm baryons form a sextet where the Fermi statistics requires $J_{qq}=1$. The six baryons Σ_c^{*++}, Σ_c^{*+}, Σ_c^{*0}, $\Xi_c^{*\prime+}$, $\Xi_c^{*\prime0}$, and Ω_c^{*0} have the flavor structure given in Table I. The remaining members of the symmetric multiplet are the three double charm baryons Ξ_{cc}^{*+}, Ξ_{cc}^{*++}, Ω_{cc}^{*+} and the triple charm baryon Ω_{ccc}^{++}.

The experimental masses of single charm baryons with $J^P=\frac{3}{2}^+$ are [9]

$$\begin{aligned}\Sigma_c^* &= 2518.0\pm0.8\ \text{MeV},\\ \Xi_c^* &= 2646.4\pm0.9\ \text{MeV},\\ \Omega_c^* &= 2768.3\pm3.0\ \text{MeV},\end{aligned}\tag{1}$$

which represent mass averages when the hadron appears with different charges. Note that here and below none of the quantum numbers assigned to the charm baryons have been measured experimentally, but are based on quark model expectations.

The mixed symmetric representation $\mathbf{20}$ has $J^P=\frac{1}{2}^+$. The lowest submultiplet is the SU(3) Gell-Mann-Ne'eman octet. The single charm baryons Σ_c^{++}, Σ_c^+, Σ_c^0, $\Xi_c^{\prime+}$, $\Xi_c^{\prime0}$, and Ω_c^0 form a sextet with flavor states indicated in Table I and $J_{qq}=1$. Λ_c^+, Ξ_c^+, and Ξ_c^0 form an antitriplet with flavor states shown in Table II having $J_{qq}=0$.

The experimental masses of single charm baryons with $J^P=\frac{1}{2}^+$ are [9]

$$\begin{aligned}\Lambda_c &= 2286.46\pm0.14\ \text{MeV},\\ \Sigma_c &= 2453.56\pm0.16\ \text{MeV},\\ \Xi_c &= 2469.5\pm0.3\ \text{MeV},\\ \Xi_c' &= 2576.9\pm2.1\ \text{MeV},\\ \Omega_c &= 2697.5\pm2.6\ \text{MeV},\end{aligned}\tag{2}$$

where, again, mass averages are made when the hadron appears with different charges. In the observed Ξ_c and Ξ_c' baryons it is expected that the light quarks are mostly in a state with $J_{qq}=0$ and $J_{qq}=1$, respectively.

The mixed symmetric multiplet also contains three double charm baryons, Ξ_{cc}^+, Ξ_{cc}^{++}, and Ω_{cc}^+, from which only Ξ_{cc}^+ has been observed by the SELEX Collaboration with a mass of 3518.9 ± 0.9 MeV [9], but needs confirmation.

TABLE II. Flavor states of the single charm antitriplet baryons with $J^P=\frac{1}{2}^+$ in the mixed symmetric representation. They all have $J_{qq}=0$.

Baryon	Flavor state
Λ_c^+	$\sqrt{\frac{1}{2}}(ud-du)c$
Ξ_c^+	$\sqrt{\frac{1}{2}}(us-su)c$
Ξ_c^0	$\sqrt{\frac{1}{2}}(ds-sd)c$

B. Bottom baryons

Despite the large symmetry breaking, for the sake of classification one can also assume an SU(4) classification of bottom baryons. Similarly, for single bottom baryons there is a sextet shown in Table III and an antitriplet shown in Table IV. The mass of Λ_b has been previously measured [9],

$$\Lambda_b = 5620.2\pm1.6\ \text{MeV}.\tag{3}$$

Recent measurements have been made for Ξ_b [10,11], Σ_b, and Σ_b^* [12]. The measured masses are

$$\begin{aligned}\Xi_b^- &= 5774\pm11\pm15\ \text{MeV}\,[10],\\ &\quad 5792.9\pm2.5\pm1.7\ \text{MeV}\,[11],\\ \Sigma_b^\pm &= 5811.5\pm1.7\ \text{MeV}\,[12],\\ \Sigma_b^{*\pm} &= 5832.7\pm1.8\ \text{MeV}\,[12].\end{aligned}\tag{4}$$

The first observation of the doubly strange baryon Ω_b^- has just been reported by D0 Collaboration [13] with a mass of

TABLE I. Flavor states of the single charm sextet baryons: $J^P=\frac{1}{2}^+$ (no star) for baryons in the mixed representation and $J^P=\frac{3}{2}^+$ (with star) for baryons in the symmetric representation. They all have $J_{qq}=1$. Members of the same doublet become degenerate at $m_Q\to\infty$.

Baryon doublet	Flavor state
Σ_c^{++}, Σ_c^{*++}	uuc
Σ_c^{+}, Σ_c^{*+}	$\frac{1}{\sqrt{2}}(ud+du)c$
Σ_c^{0}, Σ_c^{*0}	ddc
$\Xi_c^{\prime+}$, $\Xi_c^{*\prime+}$	$\frac{1}{\sqrt{2}}(us+su)c$
$\Xi_c^{\prime0}$, $\Xi_c^{*\prime0}$	$\frac{1}{\sqrt{2}}(ds+sd)c$
Ω_c^{0}, Ω_c^{*0}	ssc

TABLE III. Flavor states of the single bottom sextet baryons: $J^P=\frac{1}{2}^+$ (no star) for baryons in the mixed representation and $J^P=\frac{3}{2}^+$ (with star) for baryons in the symmetric representation. They all have $J_{qq}=1$. Members of the same doublet become degenerate at $m_Q\to\infty$.

Baryon doublet	Flavor state
Σ_b^{+}, Σ_b^{*+}	uub
Σ_b^{0}, Σ_b^{*0}	$\frac{1}{\sqrt{2}}(ud+du)b$
Σ_b^{-}, Σ_b^{*-}	ddb
$\Xi_b^{\prime0}$, $\Xi_b^{*\prime0}$	$\frac{1}{\sqrt{2}}(us+su)b$
$\Xi_b^{\prime-}$, $\Xi_b^{*\prime-}$	$\frac{1}{\sqrt{2}}(ds+sd)b$
Ω_b^{-}, Ω_b^{*-}	ssb

TABLE IV. Flavor states of the single bottom antitriplet baryons with $J^P = \frac{1}{2}^+$ in the mixed symmetric representation. They all have $J_{qq} = 0$.

Baryon	Flavor state
Λ_b^+	$\sqrt{\frac{1}{2}}(ud - du)b$
Ξ_b^+	$\sqrt{\frac{1}{2}}(us - su)b$
Ξ_b^0	$\sqrt{\frac{1}{2}}(ds - sd)b$

$6165 \pm 10 \pm 13$ MeV. The remaining undiscovered single bottom baryons are $\Xi_b'^-$, $\Xi_b^{*\prime -}$, and Ω_b^{*-}.

III. GROUND STATE HEAVY BARYONS IN THE $1/m_Q$ AND $1/N_c$ EXPANSION

The approximate spin-flavor symmetry for large N_c baryons containing light $q = u$, d, or s quarks and heavy $Q = c$ or b quarks is $SU(6) \times SU(2)_c \times SU(2)_b$; i.e. there is a separate spin symmetry for each heavy flavor. Over a decade ago the $1/N_c$ expansion has been generalized to include an expansion in $1/m_Q$ and light-quark flavor symmetry breaking [14].

Let us first consider that SU(3)-flavor symmetry is exact. In this case the mass operator is a flavor singlet. In the combined $1/m_Q$ and $1/N_c$ expansion to order $1/m_Q^2$ the ground state mass operator $M^{(1)}$ takes the following form:

$$M^{(1)} = m_Q N_Q \mathbb{1} + \Lambda_{qq} + \lambda_Q + \lambda_{qqQ}, \tag{5}$$

where N_Q is the number of heavy quarks. The leading order term is m_Q at all orders in the $1/N_c$ expansion. Next we have

$$\Lambda_{qq} = c_0 N_c \mathbb{1} + \frac{c_2}{N_c} J_{qq}^2, \tag{6}$$

where $\vec{J}_{qq}$ is the total spin of the light-quark pair. This operator contains the dynamical contribution of the light quarks and is independent of m_Q. Then, λ_Q gives the $1/m_Q$ corrections due to N_Q heavy quarks,

$$\lambda_Q = N_Q \frac{1}{2m_Q}\left(c_0' \mathbb{1} + \frac{c_2'}{N_c^2} J_{qq}^2\right). \tag{7}$$

In the following we shall deal with $N_Q = 1$ only. Lastly, λ_{qqQ} contains the heavy-quark spin-symmetry violating (chromomagnetic) operator which is of order $1/m_Q$ as well,

$$\lambda_{qqQ} = 2\frac{c_2''}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q, \tag{8}$$

$\vec{J}_Q$ being the spin of the heavy quark. This is the term responsible for the splitting between states which form degenerate doublets in the heavy-quark limit; see Tables I and III.

The unknown coefficients c_0, c_2, c_0', c_2', and c_2'' are functions of $1/N_c$ and of a QCD scale Λ. Each coefficient has an expansion in $1/N_c$ where the leading term is of order unity and does not depend on $1/m_Q$. Without loss of generality one can set $c_0 \equiv \Lambda$. The other coefficients contain a dimensional power of Λ and a dimensionless function of $1/N_c$ beginning at order unity and have to be fitted to the available experimental data. In agreement with Ref. [14], we can take

$$c_0 = \Lambda, \qquad c_2 \sim \Lambda, \qquad c_0' \sim c_2' \sim c_2'' \sim \Lambda^2. \tag{9}$$

At the dominant order, the value of Λ can be extracted from the mass combinations

$$\Lambda_Q = m_Q + N_c\Lambda, \tag{10a}$$

$$\frac{1}{3}(\Sigma_Q + 2\Sigma_Q^*) - \Lambda_Q = 2\frac{\Lambda}{N_c}, \tag{10b}$$

$$\Sigma_Q^* - \Sigma_Q = \frac{3}{2}\left(\frac{2\Lambda^2}{N_c m_Q}\right), \tag{10c}$$

resulting from the mass definition (5). Equations (10a) and (10b) imply that the dimensionless expansion coefficients are taken approximately equal to 1 and are thus only functions of the QCD scale Λ [see Eqs. (9)], in agreement with Ref. [14]. They also express the fact that λ_Q is negligible with respect to the other terms in (5). Here and below the particle label represents its mass.

A slightly more complicated mass combination, involving light baryons as well as heavy ones, directly leads to m_Q, that is [1],

$$\frac{1}{3}(\Lambda_Q + 2\Xi_Q) - \frac{1}{4}\left[\frac{5}{8}(2N + 3\Sigma + \Lambda + 2\Xi) - \frac{1}{10}(4\Delta + 3\Sigma^* + 2\Xi^* + \Omega)\right] = m_Q. \tag{11}$$

This mass combination gives

$$m_c = 1315.1 \pm 0.2 \text{ MeV}, \qquad m_b = 4641.9 \pm 2.1 \text{ MeV}, \tag{12a}$$

while the value

$$\Lambda \approx 324 \text{ MeV} \tag{12b}$$

ensures that the mass combinations (10) are optimally compatible with the experimental values for $Q = c$ and

TABLE V. Mass combinations resulting from the heavy quark and large N_c limit and their experimental values [1].

Mass combination	Experiment (MeV) $Q = c$	Experiment (MeV) $Q = b$
Λ_Q	2286.46 ± 0.14	5620.2 ± 1.6
$\frac{1}{3}(\Sigma_Q + 2\Sigma_Q^*) - \Lambda_Q$	210.0 ± 0.5	205.4 ± 2.1
$\Sigma_Q^* - \Sigma_Q$	64.4 ± 0.8	21.2 ± 2.5
$\Xi_Q - \Lambda_Q$	183.0 ± 0.3	172.7 ± 3.4

$Q = b$ indicated in Table V. Note also that the heavy-quark flavor symmetry predicts that the observed $(\Lambda_b - \Lambda_c) = 3333.7 \pm 1.6$ MeV splitting [1] can give a measure of the quark mass difference $m_b - m_c$ up to corrections of the order $\Lambda^2(1/2m_c - 1/2m_b) \approx 23$ MeV [14]. The values given by Eqs. (12) satisfy this constraint.

The operator analysis including SU(3)-flavor breaking leads to an expansion in the SU(3) violating parameter ϵ which contains the singlet $M^{(1)}$, an octet $M^{(8)}$, and a 27-plet $M^{(27)}$. The last term brings contributions proportional to ϵ^2 and we neglect it. For $M^{(8)}$ we retain its dominant contribution T^8 to order N_c^0. Then the mass formula becomes

$$M = M^{(1)} + \epsilon T^8. \tag{13}$$

The flavor breaking parameter ϵ is governed by the mass difference $m_s - m$ (where m is the average of the m_u and m_d masses) and therefore is $\epsilon \sim 0.2$–0.3. It is measured in units of the chiral symmetry breaking scale parameter $\Lambda_\chi \sim 1$ GeV. A measure of the SU(3)-flavor breaking factor can be given by [14]

$$\Xi_Q - \Lambda_Q = \frac{\sqrt{3}}{2}(\epsilon\Lambda_\chi). \tag{14}$$

The value $(\epsilon\Lambda_\chi) = 206$ MeV leads to $\Xi_Q - \Lambda_Q = 178$ MeV, which is the average value of the corresponding experimental data listed in Table V.

IV. QUARK MODEL FOR HEAVY BARYONS

A. Hamiltonian

The potential model used to describe heavy baryons is the same as that which has been proposed in Ref. [6] for light baryons. Let us recall its main features.

In quark models, a baryon is a bound state of three valence quarks which can be described at the dominant order by the spinless Salpeter Hamiltonian

$$H = \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} + V_Y, \tag{15}$$

where m_i is the current (bare) mass of the quark i and V_Y is the confining interaction potential. Both the flux tube model [15] and lattice QCD [16,17] suggest that the flux tubes form a Y junction: A flux tube starts from each quark and the three tubes meet at the Torricelli point of the triangle formed by the three quarks. This point, located in $\vec{x}_T$, minimizes the sum of the flux tube lengths, leading to the following confining potential:

$$V_Y = a \sum_{i=1}^{3} |\vec{x}_i - \vec{x}_T|. \tag{16}$$

The position of the quark i is denoted by $\vec{x}_i$, and a is the energy density of the flux tubes. Such a Hamiltonian can also be obtained in the framework of the field correlator method [18].

As $\vec{x}_T$ is a complicated three-body function, it is interesting to approximate the confining potential by a more tractable form. In the following, we will use

$$H_R = \sum_{i=1}^{3} \sqrt{\vec{p}_i^{\,2} + m_i^2} + V_R, \tag{17}$$

$$V_R = ka \sum_{i=1}^{3} |\vec{x}_i - \vec{R}|, \tag{18}$$

where $\vec{R}$ is the position of the center of mass and k is a corrective factor [19]. The eigenvalues corresponding to potentials V_Y and V_R differ from each other only by about 5% in most cases. The accuracy of the formula (18) is thus rather satisfactory, and has already led to relevant results in Ref. [6]. For light (symmetrical) qqq baryons, a good value for the corrective factor is $k_0 = 0.952$. For very asymmetrical qqQ baryons, a good choice is $k_1 = 0.930$ [19]. This last value corresponds actually to the case $m_q/m_Q \to 0$.

Besides the confining potential (16), other contributions are necessary to reproduce the baryon masses. We shall add them as perturbations to the dominant Hamiltonian (17). The most widespread correction is a Coulomb interaction term of the form

$$\Delta H_{\text{oge}} = -\frac{2}{3}\sum_{i<j}\frac{\alpha_{S,ij}}{|\vec{x}_i - \vec{x}_j|}, \tag{19}$$

arising from one gluon exchange processes, where $\alpha_{S,ij}$ is the strong coupling constant between the quarks i and j. Actually, one should deal with a running form $\alpha_S(r)$, but it would considerably increase the difficulty of the computations. Typically, we need two values: $\alpha_0 = \alpha_{S,qq}$ for a qq pair and $\alpha_1 = \alpha_{S,qQ}$ for a qQ pair, in the spirit of what has been done in a previous study describing mesons in the relativistic flux tube model [20]. There it was found that $\alpha_1/\alpha_0 \approx 0.7$ describes rather well the experimental data of $q\bar{q}$ and $Q\bar{q}$ mesons.

Another perturbative contribution to the mass is the quark self-energy. This is due to the color-magnetic moment of a quark propagating through the vacuum background field. It adds a negative contribution to the hadron masses [21]. The quark self-energy contribution for a baryon is given by

$$\Delta H_{\text{qse}} = -\frac{fa}{2\pi}\sum_i \frac{\eta(m_i/\delta)}{\mu_i}. \tag{20}$$

The factors f and δ have been computed in quenched and unquenched lattice QCD studies [22,23]. Although it is not known with great accuracy, it seems well established that $3 \le f \le 4$ and $(1.0 \le \delta \le 1.3)$ GeV [22,23]. The function $\eta(\epsilon)$ is analytically known; we refer the reader to Ref. [21] for the explicit formula. For typical values of the light-quark masses, we have $0 \le m_q/\delta \lesssim 0.3$, while for heavy quarks, we have $1.0 \lesssim m_Q/\delta \lesssim 6.0$. The function $\eta(\epsilon)$ is such that

$$\eta(\epsilon) \approx 1 + \left(4 + 3\ln\frac{\epsilon}{2}\right)\epsilon^2 \quad \text{for } \epsilon \ll 1, \qquad \approx \frac{2}{\epsilon^2} \quad \text{for } \epsilon \to \infty. \tag{21}$$

For the relevant values of $\epsilon = m_i/\delta$ a better accuracy is obtained with the following simple forms:

$$\eta(\epsilon) \approx 1 - \beta\epsilon^2 \quad \text{with} \quad \beta = 2.85 \quad \text{for } 0 \le \epsilon \lesssim 0.3, \qquad \approx \frac{\gamma}{\epsilon^2} \quad \text{with} \quad \gamma = 0.79 \quad \text{for } 1.0 \lesssim \epsilon \lesssim 6.0. \tag{22}$$

Let us note that the corrections depending on the parameter γ appear at order $1/m_Q^3$ in the mass formula, so they are not considered in this work. Finally, μ_i is the dynamical mass of the quark i, defined as [21]

$$\mu_i = \left\langle \sqrt{\vec{p}_i^{\,2} + m_i^2} \right\rangle. \tag{23}$$

This dynamical mass is state dependent: It represents the kinetic energy of the quark i averaged with the wave function of the unperturbed spinless Salpeter Hamiltonian (17).

B. General formulas

We are mainly interested in analytical expressions, so that a comparison with the large N_c mass formula will be straightforward. To this aim, the auxiliary field technique will be used in order to transform the Hamiltonian (17) into an analytically solvable one [24,25]. With $\lambda = ka$, we obtain

$$H(\mu_i, \nu_j) = \sum_{j=1}^{3}\left[\frac{\vec{p}_j^{\,2} + m_j^2}{2\mu_j} + \frac{\mu_j}{2}\right] + \sum_{j=1}^{3}\left[\frac{\lambda^2(\vec{x}_j - \vec{R})^2}{2\nu_j} + \frac{\nu_j}{2}\right]. \tag{24}$$

The auxiliary fields, denoted as μ_i and ν_j, are operators, and $H(\mu_i, \nu_j)$ is equivalent to H up to their elimination thanks to the constraints

$$\delta_{\mu_i} H(\mu_i, \nu_j) = 0 \Rightarrow \mu_{i,0} = \sqrt{\vec{p}_i^{\,2} + m_i^2}, \qquad \delta_{\nu_j} H(\mu_i, \nu_j) = 0 \Rightarrow \nu_{i,0} = \lambda|\vec{x}_i - \vec{R}|. \tag{25}$$

$\langle \mu_{i,0} \rangle$ is the dynamical quark mass introduced in Eq. (23), and $\langle \nu_{i,0} \rangle$ is the energy of the flux tube linking the quark i to the center of mass.

Although the auxiliary fields are operators, the calculations are considerably simplified if one considers them as real numbers. They are finally eliminated by a minimization of the masses [24], and the extremal values of μ_i and ν_j are logically close to $\langle \mu_{i,0} \rangle$ and $\langle \nu_{j,0} \rangle$, respectively. This technique can give approximate results very close to the exact ones (see Ref. [26] for a comparative study of baryons with the auxiliary fields introduced only in the kinetic part of the Hamiltonian).

In Ref. [27], it has been shown that the eigenvalues of a Hamiltonian of the form (24) can be analytically found by making an appropriate change of variables, the quark coordinates $\vec{x}_i = \{\vec{x}_1, \vec{x}_2, \vec{x}_3\}$ being replaced by new coordinates $\vec{x}'_k = \{\vec{R}, \vec{\xi}, \vec{\eta}\}$. The center of mass is defined as

$$\vec{R} = \frac{\mu_1\vec{x}_1 + \mu_2\vec{x}_2 + \mu_3\vec{x}_3}{\mu_t}, \tag{26}$$

with $\mu_t = \mu_1 + \mu_2 + \mu_3$. $\{\vec{\xi}, \vec{\eta}\}$ are two relative coordinates: $\vec{\xi} \propto \vec{x}_1 - \vec{x}_2$ and $\vec{\eta} \propto \frac{\mu_1\vec{x}_1 + \mu_2\vec{x}_2}{\mu_1 + \mu_2} - \vec{x}_3$. As we only consider baryons built from two different quarks, the general formulas obtained in Ref. [27] can be simplified. In the case of two quarks with mass m and another with mass m_3, the mass spectrum of the Hamiltonian (24) is given by ($\mu_1 = \mu_2 = \mu$, $\nu_1 = \nu_2 = \nu$)

$$M(\mu, \mu_3, \nu, \nu_3) = \omega_\xi(N_\xi + 3/2) + \omega_\eta(N_\eta + 3/2) + \mu + \nu + \frac{\mu_3 + \nu_3}{2} + \frac{m^2}{\mu} + \frac{m_3^2}{2\mu_3}, \tag{27}$$

where

$$\omega_\xi = \frac{\lambda}{\sqrt{\mu\nu}}, \qquad \omega_\eta = \frac{\lambda}{\sqrt{2\mu + \mu_3}}\sqrt{\frac{\mu_3}{\mu\nu} + \frac{2\mu}{\mu_3\nu_3}}. \tag{28}$$

The integers $N_{\xi/\eta}$ are given by $2n_{\xi/\eta} + \ell_{\xi/\eta}$, where $n_{\xi/\eta}$ and $\ell_{\xi/\eta}$ are the radial and orbital quantum numbers relative to the variable $\vec{\xi}/\vec{\eta}$, respectively. One can also easily check that [27]

$$\langle \vec{\xi}^{\,2} \rangle = \frac{N_\xi + 3/2}{\phi\omega_\xi}, \qquad \langle \vec{\eta}^{\,2} \rangle = \frac{N_\eta + 3/2}{\phi\omega_\eta}, \tag{29}$$

with

$$\phi = \sqrt{\frac{\mu^2\mu_3}{2\mu + \mu_3}}. \tag{30}$$

These last identities provide relevant information about the structure of the baryons, since

$$\langle \vec{X}^2 \rangle = \langle (\vec{x}_1 - \vec{x}_2)^2 \rangle = \sqrt{\frac{4\mu_3}{2\mu + \mu_3}}\langle \vec{\xi}^{\,2} \rangle, \tag{31}$$

$$\langle \vec{Y}^2 \rangle = \left\langle \left(\frac{\vec{x}_1 + \vec{x}_2}{2} - \vec{x}_3\right)^2 \right\rangle = \sqrt{\frac{2\mu + \mu_3}{4\mu_3}}\langle \vec{\eta}^{\,2} \rangle. \tag{32}$$

Moreover, by symmetry, we can assume the following equality

$$\langle (\vec{x}_1 - \vec{x}_3)^2 \rangle = \langle (\vec{x}_2 - \vec{x}_3)^2 \rangle \approx \frac{\langle \vec{X}^2 \rangle}{4} + \langle \vec{Y}^2 \rangle, \tag{33}$$

which will be useful in the computation of the one gluon exchange contribution.

The case of qqq baryons, studied in our previous papers [5,6], is obtained by taking $m = m_n = 0$ and $m_3 = m_s$, and by setting $\lambda = k_0 a$. If the three quarks are identical, then $m_3 = m$, $\mu_3 = \mu$, $\nu_3 = \nu$. For qqQ baryons, we explicitly write $m_3 = m_Q$, $\mu_3 = \mu_Q$, $\nu_3 = \nu_Q$, and we set $\lambda = k_1 a$, $m = 0$ or m_s for n or s-quarks, respectively. Let us note that different values of k_0 have been previously used: $k_0 = (1/2 + \sqrt{3}/4)$ in Ref. [5] and $k_0 = 1$ in Ref. [6]. In this work, we choose phenomenological values computed in Ref. [19] in order to obtain the best possible simulation of the Y junction for both qqq and qqQ baryons with the potential (18).

C. Mass formula for heavy baryons

In this section, we focus our attention on ssQ baryons. The mass formula for nnQ baryons is obtained simply by setting $m_s = 0$, and the case of nsQ baryons will be discussed in the next section. The four auxiliary fields appearing in the mass formula (27) have to be eliminated by solving simultaneously the four constraints:

$$\partial_\mu M(\mu, \mu_Q, \nu, \nu_Q) = 0, \qquad \partial_{\mu_Q} M(\mu, \mu_Q, \nu, \nu_Q) = 0,$$
$$\partial_\nu M(\mu, \mu_Q, \nu, \nu_Q) = 0, \qquad \partial_{\nu_Q} M(\mu, \mu_Q, \nu, \nu_Q) = 0. \tag{34}$$

This cannot be done exactly in an analytical way, but solutions can be obtained by assuming that $1/m_Q$ and m_s are small quantities. After some algebra, a solution was found by working at order $1/m_Q$ and m_s^2 (all contributions proportional to m_s are vanishing). By denoting

$$N = N_\xi + N_\eta, \qquad \mu_1 = \sqrt{\frac{k_1 a(N+3)}{2}},$$
$$G(N, N_\eta) = \sqrt{2N_\eta + 3}\left(\sqrt{2(N+3)} - \sqrt{2N_\eta + 3}\right), \tag{35}$$

we have obtained

$$\mu = \mu_1 + \frac{3m_s^2}{4\mu_1} - \frac{k_1 a}{4m_Q} G(N, N_\eta),$$
$$\nu = \mu_1 - \frac{m_s^2}{4\mu_1} - \frac{k_1 a}{4m_Q}(2N_\eta + 3),$$
$$\mu_Q = m_Q + \frac{k_1 a}{2m_Q} G(N, N_\eta), \tag{36}$$
$$\nu_Q = \frac{k_1 a}{m_Q}\sqrt{\frac{(2N_\eta + 3)(N+3)}{2}}.$$

Logically, $\mu_Q \approx m_Q$ since this auxiliary field is dominated by the effective mass of the heavy quark. The length of the flux tube joining the heavy quark to the center of mass is smaller than the other ones, so $\lim_{m_Q\to\infty} \nu_Q = 0$ as expected.

The mass formula (27), in which the auxiliary fields are replaced by the expressions (36), reads at orders $1/m_Q$ and m_s^2 as

$$M = m_Q + 4\mu_1 + \frac{m_s^2}{\mu_1} + \frac{k_1 a}{2m_Q} G(N, N_\eta). \tag{37}$$

It is interesting to look at the magnitude of the various terms in this formula. Let us choose typical values for the parameters: $k_1 = 1$, $a = 0.2\ \mathrm{GeV}^2$, $m_s = 0.3$ GeV, $m_c = 1.5$ GeV, $m_b = 5.0$ GeV. For the ground state ($N = 0$), $\mu_1 = 0.548$ GeV. The contribution of the kinetic energy and of the confinement in M, given by $4\mu_1 = 2.191$ GeV, is of the order of m_Q. The contribution of the strange quark is given by $\frac{m_s^2}{\mu_1} = 0.164$ GeV, while the term $\frac{k_1 a}{2m_Q} G(0,0)$ is 0.083 GeV and 0.025 GeV, respectively, for the charm and bottom masses. These values justify *a posteriori* the use of the power expansion in m_s and in $1/m_Q$. Formulas (36) and (37) giving the optimal values of the auxiliary fields and the corresponding minimal mass are approximate solutions of Eq. (27). In Table VI, these values are compared with the exact solutions obtained numerically. In all cases, the error on the mass is quite small, even if the error on some auxiliary fields is larger. The auxiliary fields μ and μ_Q are used to compute perturbatively the self-energy. Fortunately, the error on these fields is small. As expected,

TABLE VI. Relative error (%) on auxiliary fields (36) and mass (37) for typical values of the physical parameters ($k_1 = 1$, $a = 0.2\ \mathrm{GeV}^2$). Quark masses are given in GeV.

m_s/m_Q	0/1.5			0.3/1.5			0/5.0			0.3/5.0		
(N, N_η)	(0,0)	(4,0)	(4,4)	(0,0)	(4,0)	(4,4)	(0,0)	(4,0)	(4,4)	(0,0)	(4,0)	(4,4)
μ	0.007	6.6	5.2	2.8	5.6	5.4	0.2	0.7	1.4	2.9	0.02	2.0
ν	8.9	7.1	29.5	10.2	7.2	30.1	1.1	1.0	3.6	2.3	1.1	3.5
μ_Q	0.2	4.8	4.6	0.8	4.5	5.3	0.04	0.2	0.4	0.1	0.1	0.5
ν_Q	44.5	74.0	67.4	33.7	68.8	61.9	12.8	21.8	18.7	1.8	16.4	13.3
M	0.2	1.4	1.7	0.5	1.7	1.7	0.05	0.1	0.3	0.6	0.3	0.2

the accuracy is improved for large values of m_Q, while m_s has only a little influence.

The contribution of the one gluon exchange term can be computed with the help of relations (31) and (32). One obtains

$$\Delta M_{\rm oge} \approx -\frac{2}{3}\left[\frac{\alpha_0}{\sqrt{\langle\vec{X}^2\rangle}} + \frac{2\alpha_1}{\sqrt{\langle\vec{X}^2\rangle/4 + \langle\vec{Y}^2\rangle}}\right]$$
$$= -\frac{2}{3}\alpha_0\sqrt{\frac{k_1 a}{2N_\xi+3}}\left[1+\frac{m_s^2}{4\mu_1^2}+\frac{\sqrt{k_1 a}}{8m_Q}\times\sqrt{2N_\eta+3}\left(\sqrt{\frac{2(2N_\eta+3)}{N+3}}-1\right)\right]$$
$$-\frac{4}{3}\alpha_1\sqrt{\frac{2k_1 a}{N+3}}\left[1+\frac{m_s^2}{4\mu_1^2}-\frac{\sqrt{k_1 a}}{2m_Q}\frac{2N_\eta+3}{\sqrt{2(N+3)}}\right]. \tag{38}$$

For values of the parameters defined above together with $\alpha_0 = \alpha_1 = 0.4$, the contribution of the dominant term in $\Delta M_{\rm oge}$ is -0.264 GeV for the ground state. The m_s^2/μ_1^2 term brings -0.020 GeV while the $1/m_c$ term brings 0.034 GeV. Again, the use of the power expansion in m_s and in $1/m_Q$ seems relevant.

The relations (36) defining μ and μ_Q allow us to write down the contribution of the quark self-energy (20). Using the approximation (22) one obtains

$$\Delta M_{\rm qse} = -\frac{fa}{\pi\mu_1}\left[1-\left(\frac{3}{4\mu_1^2}+\frac{\beta}{\delta^2}\right)m_s^2 + \frac{k_1 a}{4\mu_1 m_Q}G(N,N_\eta)\right]. \tag{39}$$

We recall that the correction proportional to βm_s^2 comes from a convenient parametrization of the $\eta(\epsilon)$ function, while the term proportional to m_s^2/μ_1^2 is due to the expansion of the auxiliary field μ. For the values of the parameters defined above and the typical values $f = 3.5$ and $\delta = 1$ GeV, the contribution of the dominant term in $\Delta M_{\rm qse}$ is -0.302 GeV for the ground state. The m_s^2/μ_1^2 term brings 0.092 GeV while the $1/m_c$ term brings -0.031 GeV. The use of the power series expansion in m_s and in $1/m_Q$ seems more questionable here, mostly for the contribution of the strange quark. This is due to the particular nature of the self-energy interaction which can be defined only as a perturbation [21].

If we now look at the dominant terms in $M - m_Q$, $\Delta M_{\rm oge}$, and $\Delta M_{\rm qse}$, we find, respectively, 2.191 GeV, -0.264 GeV, and -0.302 GeV for the ground state with parameters defined above. These numbers show that it is *a posteriori* justified to treat the Coulomb interaction and the self-energy interaction as perturbations.

D. Mass formulas for general *qqq* and *qqQ* baryons

In this section we gather mass formulas obtained for both light and heavy baryons. The qqq mass formula is given in Ref. [6] and is recalled here for completeness,

$$\mu_0 = \sqrt{\frac{k_0 a(N+3)}{3}}, \qquad M_{qqq} = M_0 + n_s \Delta M_{0s}$$
$$(n_s = 0, 1, 2, 3), \qquad M_0 = 6\mu_0 - \frac{2k_0 a\alpha_0}{\sqrt{3}\mu_0} - \frac{3fa}{2\pi\mu_0},$$
$$\Delta M_{0s} = \left[\frac{1}{2} - \frac{k_0 a\alpha_0}{6\sqrt{3}\mu_0^2} + \frac{fa}{2\pi}\left(\frac{3}{4\mu_0^2}+\frac{\beta}{\delta^2}\right)\right]\frac{m_s^2}{\mu_0}. \tag{40}$$

All parameters were already presented above, except the number n_s of s-quarks in the baryons. The mass formula M_{qqq} depends only on $N = N_\xi + N_\eta$ since the contribution of terms proportional to $N_\xi - N_\eta$, vanishing for $n_s = 0$ and 3, was found to be very weak in general [6].

In the previous section, only the case of a heavy baryon containing two identical light quarks was treated ($n_s = 0$ or $n_s = 2$). It has been shown that every s-quark brings the same contribution ΔM_{0s} to the mass of a light baryon [see Eq. (40)]. So, we can reasonably assume that the same situation occurs for qqQ baryons. To take into account the contribution of n_s-quarks to the mass of these baryons, it is enough to replace the term m_s^2 by $n_s m_s^2/2$ in Eqs. (37)–(39). Let us note that it is not necessarily true for the auxiliary fields μ and ν [6]. In the following formulas, we keep explicitly the dependence on both N_ξ and N_η:

$$\mu_1 = \sqrt{\frac{k_1 a(N+3)}{2}}, \qquad M_{qqQ} = m_Q + M_1 + n_s\Delta M_{1s} + \Delta M_Q \qquad (n_s = 0, 1, 2),$$
$$M_1 = 4\mu_1 - \frac{2}{3}\left(\alpha_0\sqrt{\frac{k_1 a}{2N_\xi+3}} + 2\alpha_1\sqrt{\frac{2k_1 a}{N+3}}\right) - \frac{fa}{\pi\mu_1},$$
$$\Delta M_{1s} = \frac{m_s^2}{\mu_1}\left[\frac{1}{2} - \frac{1}{12\mu_1}\left(\alpha_0\sqrt{\frac{k_1 a}{2N_\xi+3}} + 2\alpha_1\sqrt{\frac{2k_1 a}{N+3}}\right) + \frac{fa}{2\pi}\left(\frac{3}{4\mu_1^2}+\frac{\beta}{\delta^2}\right)\right],$$
$$\Delta M_Q = \frac{k_1 a}{2m_Q}\left[\left(1-\frac{fa}{2\pi\mu_1^2}\right)G(N,N_\eta) - \frac{\alpha_0}{6}\sqrt{\frac{2N_\eta+3}{2N_\xi+3}}\left(\sqrt{\frac{2(2N_\eta+3)}{N+3}}-1\right) + \frac{4\alpha_1}{3}\frac{2N_\eta+3}{N+3}\right]. \tag{41}$$

E. What is the good quantum number?

At the lowest order, the mass formula (37), with the rescaling $a \leftrightarrow \sigma$ (see next section), leads to

$$(M - m_Q)^2 = \frac{4\pi\sigma}{3}\frac{k_1}{k_0}(N + 3). \tag{42}$$

The model thus predicts Regge trajectories for heavy baryons, with a slope of $4\pi\sigma k_1/(3k_0) \approx 1.3\pi\sigma$ instead of $2\pi\sigma$ for light baryons. At this dominant order, the mass formula depends only on N. However, when corrections are added, the mass formula is no longer symmetric under interchange of N_η and N_ξ. Is it still possible to find a single quantum number? There are three possibilities:

(i) As in Ref. [6], we could assume that $N_\xi \approx N_\eta$. But, the presence of a heavy quark makes the system rather asymmetric in the $\vec{\xi}$ and $\vec{\eta}$ variables. So this solution seems unnatural.

(ii) Another possibility is to impose $N_\eta = 0$ and $N_\xi = N$. With no excitation in the $\vec{\eta}$ variable, the two light quarks are moving around a static heavy quark in the configuration $q - Q - q$, as proposed in Ref. [28].

(iii) The opposite possibility can also be assumed: $N_\eta = N$ and $N_\xi = 0$. With no excitation in the $\vec{\xi}$ variable, the two light quarks behave as a diquark orbiting around the heavy quark by forming a $Q - (qq)$ system, as considered in Ref. [29].

At order $1/m_Q$, the dominant term (37) depends on the function $G(N, N_\eta)$. The baryon mass is lowered when $G(N, N_\eta)$ is minimal, that is to say, for $N_\eta = N$. In this case

$$F(N) = G(N, N) = \sqrt{3 + 2N}[\sqrt{2(3 + N)} - \sqrt{3 + 2N}], \tag{43}$$

with $F(0) = 3(\sqrt{2} - 1) \leq F(N) < 3/2$, this upper bound being the limit of $F(N)$ for N going to infinity. The analysis of the dominant part of the Coulomb term (38) shows that the baryon mass is also lowered when $N_\eta = N$. So it is natural to assume that the favored configuration, minimizing the baryon energy, is $N_\eta = N$ and $N_\xi = 0$, as in Ref. [29]. In this case a light-diquark–heavy-quark structure for the baryon is favored.

It is also possible to reach the same conclusion by looking at the mean values of the variables $\vec{X}$ and $\vec{Y}$. At the dominant order, we have

$$\langle \vec{X}^2 \rangle = \frac{3 + 2N_\xi}{a}, \qquad \langle \vec{Y}^2 \rangle = \frac{3 + 2N_\eta}{4a}. \tag{44}$$

Because of the particular shape of the potential (a Cornell type), the more the system is small, the more its mass will be small. Indeed, the energy of the flux tubes increases with the size of the baryon, while the attractive Coulomb-like forces are larger for small quark separations. Equations (44) show that an excitation of type N_η will keep the baryon smaller than the corresponding excitation in N_ξ. Thus, the most favored possibility, at least for the small excitation numbers, is also $N_\eta = N$ and $N_\xi = 0$.

As for light baryons, heavy baryons can be labeled by a single harmonic oscillator excitation number, and the emergence of this quantum number can be understood within a relativistic quark model framework. However, we only discuss the ground state in the following, that is, $N_\xi = 0$ and $N_\eta = N = 0$. Excited states will be studied in subsequent papers.

F. Determination of the parameters

The parameters needed for qqq baryons have been obtained in our previous papers [5,6] but, since we use a new value for k_0, we prefer to determine a set of new values for the parameters which are gathered in Table VII. The new values are very close to the previous ones and do not alter the good results obtained in Refs. [5,6]. The auxiliary field method systematically overestimates the absolute scale of the mass spectrum [24]. In order to obtain a good accuracy for the baryon masses, it is necessary to perform the rescaling $a = \pi\sigma/(6k_0)$ throughout the mass formulas, where σ is the physical string tension for a meson [5]. As u and d current quark masses are expected to be very small, we also take a vanishing current mass for the quark n. The parameters σ and f are fitted on the nnn baryon Regge trajectory. As it is not possible to determine independently α_0 and f, we choose for α_0 a value in agreement with other potential models. More details can be found in Ref. [5]. It is worth noting that the value 3.6 for f is in the range [3–4] and that the string tension value of 0.165 GeV^2 is in good agreement with the value predicted by the flux tube model [30]. The s-quark mass is fitted to the strange baryon masses in the band $N = 0$ [6]. The value found for m_s is larger than the PDG value of 104^{+26}_{-34} MeV [9]. However, a strange quark mass in the range 0.2–0.3 GeV is quite common in potential models [31–33].

The parameters linked to heavy quarks are m_c, m_b, k_1, and α_1. We fix $\alpha_1 = 0.7\alpha_0$ from the quark model study of Ref. [20]. The value $k_1 = 0.930$ has been computed in Ref. [19]. Because of the rescaling $a = \pi\sigma/(6k_0)$, only the ratio $k_1/k_0 \approx 0.98$ is relevant. Let us note that fixing this ratio to 1 does not noticeably change the other parameters. The heavy-quark masses can be fitted to the experimental data as follows. The quark model mass formula (41) is spin independent; it should thus be suitable to reproduce the masses of heavy baryons for which $J^2_{qq} = 0$. Typically,

TABLE VII. Parameters for qqq baryons.

Fixed parameters		Fitted parameters
$m_n = 0$	$\alpha_0 = 0.4$	$m_s = 0.240$ GeV
$k_0 = 0.952$	$\delta = 1.0$ GeV	$\sigma = 0.165$ GeV^2
$a = \pi\sigma/(6k_0)$	$\beta = 2.85$	$f = 3.60$

one expects that

$$M_{nnc}|_{N=0} = \Lambda_c = 2286.46 \pm 0.14 \text{ MeV}, \quad (45)$$

$$M_{nnb}|_{N=0} = \Lambda_b = 5620.2 \pm 1.6 \text{ MeV}. \quad (46)$$

These values are reproduced by formula (41) with $m_c = 1.252$ GeV and $m_b = 4.612$ GeV. These masses, obtained by a comparison of the quark model to the experimental data, are clearly compatible with those obtained from the mass combination (11)—both determinations actually differ by less than 5%. This is a first evidence of the compatibility between the quark model and large N_c expansion in the heavy baryon sector. The supplementary parameters for qqQ baryons are gathered in Table VIII. One can notice that we predict $M_{nsc}|_{N=0} = 2433$ MeV and $M_{nsb}|_{N=0} = 5767$ MeV with these parameters. These values are very close to the experimentally observed masses of Ξ_c and Ξ_b, respectively.

V. COMPARISON OF THE TWO APPROACHES

First we recall that the heavy-quark masses can be obtained in two different ways. On the one hand, the large N_c inspired mass combination (11) leads to $m_c = 1315$ MeV and $m_b = 4642$ MeV. On the other hand, the quark model mass formula (41) is compatible with the experimental data, provided $m_c = 1252$ MeV and $m_b = 4612$ MeV. Both approaches lead to quark masses that differ by less than 5%, as pointed out in Sec. IV F. Thus the two approaches that are considered in this paper agree at least at the dominant order, where only m_Q is present.

The other parameter involved in the large N_c mass formula is Λ. A comparison of the spin-independent part of the mass formulas (5) and (41) leads to the following identification for $N_c = 3$:

$$c_0 = \frac{1}{3} M_1|_{N=0} = \frac{4}{3}\mu_1 - \frac{2}{9}\sqrt{\frac{k_1 a}{3}}(\alpha_0 + 2\sqrt{2}\alpha_1) - \frac{fa}{3\pi\mu_1}, \quad (47)$$

where $\mu_1 = \sqrt{3k_1 a/2}$. According to Eqs. (9) and (12b) one has $c_0 = \Lambda \simeq 0.324$ GeV. The quark model parameters of Tables VII and VIII give 0.333 GeV for the expression after the second equality sign in Eq. (47), which means a very good agreement for the QCD scale Λ. In this quantity, 0.475 GeV comes from the dynamics of the confinement $(4\mu_1/3)$, while the Coulomb interaction (term containing α_0 and α_1) contributes with -0.044 GeV and the self-energy (term proportional to f) with -0.097 GeV. The mass shift yielded by these two residual interactions is quite significant and their presence improves the value of Λ.

Next the terms of order $1/m_Q$ lead to the identity

$$c_0' = 2m_Q \Delta M_Q|_{N=0} = k_1 a\left[3(\sqrt{2}-1)\left(1 - \frac{fa}{2\pi\mu_1^2}\right) - \frac{\alpha_0}{6}(\sqrt{2}-1) + \frac{4\alpha_1}{3}\right]. \quad (48)$$

Note that to test this relation the value of m_Q is not needed, like for the identity (47). The large N_c parameter $\Lambda = 0.324$ GeV gives, for the left-hand side of (48), $c_0' \sim \Lambda^2 = 0.096$ GeV^2, and the quark model gives, for the right-hand side, 0.091 GeV^2, which is again a good agreement. In this quantity, the contribution of the dynamics of the confinement $[k_1 a F(0)]$ is 0.105 GeV^2, while the contributions of the Coulomb interaction and of the self-energy are 0.029 GeV^2 and -0.043 GeV^2, respectively. The relative magnitude of these two terms compared to the first one is larger here but they nearly cancel each other.

The SU(3)-flavor breaking term is proportional to the factor $\epsilon\Lambda_\chi \sim m_s - m$ in the $1/N_c$ mass formula (13). This is also the case in our quark mass formula since $m = 0$. Using Eqs. (13), (14), and (41) one obtains

$$\epsilon\Lambda_\chi = \frac{2}{\sqrt{3}}\Delta M_{1s}|_{N=0} = \frac{2m_s^2}{\sqrt{3}\mu_1}\left[\frac{1}{2} - \frac{1}{12\mu_1}\sqrt{\frac{k_1 a}{3}}(\alpha_0 + 2\sqrt{2}\alpha_1) + \frac{fa}{2\pi}\left(\frac{3}{4\mu_1^2} + \frac{\beta}{\delta^2}\right)\right]. \quad (49)$$

From phenomenology, Eq. (14) implies that $\epsilon\Lambda_\chi = 0.206$ GeV and the quark model estimate is 0.170 GeV, which compares satisfactorily with the value used in the combined $1/N_c$ and $1/m_Q$ expansion [14]. In the quark model, the contribution of the dynamics of the confinement (term proportional to 1/2) is 0.093 GeV, while the contributions of the Coulomb interaction and of the self-energy are -0.009 GeV and 0.085 GeV, respectively. Thus the effect of the self-energy is as large as that of the confinement.

Let us recall that, except for m_c and m_b, all the model parameters are determined from theoretical arguments combined with phenomenology, or are fitted on light baryon masses. The comparison of our results with the $1/N_c$ expansion coefficients c_0, c_0', and $\epsilon\Lambda_\chi$ is independent of the m_Q values. So we can say that this analysis is parameter-free.

So far, our formalism is spin independent. An evaluation of the coefficients c_2, c_2', and c_2'' through a computation of the spin-dependent effects within a three-body quark model is then *de facto* out of the scope of the present approach. If included, the spin-dependent interactions be-

TABLE VIII. Supplementary parameters for qqQ baryons.

Fixed parameters	Fitted parameters
$k_1 = 0.930$	$m_c = 1.252$ GeV
$\alpha_1 = 0.7\alpha_0$	$m_b = 4.612$ GeV

tween quarks i and j would appear as relativistic corrections to the Coulomb potential, proportional to $1/\mu_i\mu_j$ (μ_i is the dynamical mass of the quark i) [5]. At the dominant order, one expects that $c_2 \propto \mu_1^{-2}$ and $c_2'' \propto \mu_1^{-1}$. The ratio c_2''/c_2 should thus be of order $\mu_1 = 356$ MeV, which is roughly in agreement with Eq. (9) stating that $c_2''/c_2 \sim \Lambda$. This gives an indication that the quark model and the $1/N_c$ expansion method would remain compatible if the spin-dependent effects were included, as we already pointed out in the light baryon sector [5,6].

Charm and bottom baryons have been studied with a Hamiltonian similar to ours in Ref. [33]. All parameters (a, α_S, m_n, etc.) have values very close to ours, but some differences exist: A genuine junction Y is used for the confinement instead of the approximation (17) and (18), the auxiliary fields are introduced only at the level of the kinetic part, the Coulomb potential is not treated perturbatively, and the color-magnetic interaction is taken into account. The consequence of this procedure is that no analytical mass formula can be derived explicitly. But, the numerical results obtained in that paper are in good agreement with experiment, which reinforces our approach. Moreover, it was also found that a unit of angular momentum between the heavy quark and the two light quarks is energetically favored with respect to a unit of angular momentum between the two light quarks. This corresponds to our choice $N_\xi = 0$.

VI. CONCLUSIONS

Our previous studies establishing a connection between the quark model and the $1/N_c$ expansion for light baryons have been successfully extended to baryons containing a heavy quark. Accordingly, the $1/N_c$ expansion was supplemented by a $1/m_Q$ expansion due to the heavy quark. As in the light baryon sector, there is a clear correspondence between various terms appearing in our mass formula (41) and those of the mass formula in the combined $1/N_c$ and $1/m_Q$ expansion described in Sec. III. First, both methods lead to compatible values for the heavy-quark masses. Second, the typical QCD scale involved in the $1/N_c$ expansion is well reproduced by the quark model without any free parameter: All necessary parameters have been previously fitted on light baryons. Finally, the dominant term in SU(3)-flavor breaking expansion is satisfactorily reproduced. The spin-dependent terms, seen as relativistic effects, deserve a special study, to be considered in the future.

This study, completing the two previous ones [5,6], brings reliable QCD-based support in favor of the constituent quark model assumptions due to the compatibility of its mass formula and the mass formula derived from the model-independent $1/N_c$ expansion. Moreover, a better insight into the coefficients c_i encoding the QCD dynamics in the mass operator is obtained: the dependence on the quark content and on the excitation number.

We presently focused on ground state heavy baryons. For excited states, the quark model suggests that the band number N classifying the heavy baryon resonances should be associated to the quantum of excitation of the heavy-quark–light-diquark pair in a harmonic oscillator picture. We leave a detailed study of excited heavy baryons for future studies.

ACKNOWLEDGMENTS

Financial support from F. R. S.-FNRS (Belgium) is acknowledged by C. S. and F. B..

[1] E. E. Jenkins, Phys. Rev. D **77**, 034012 (2008).
[2] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008).
[3] M. Karliner and H. J. Lipkin, Phys. Lett. B **660**, 539 (2008).
[4] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, arXiv:0804.1575, and references therein.
[5] C. Semay, F. Buisseret, N. Matagne, and Fl. Stancu, Phys. Rev. D **75**, 096001 (2007).
[6] C. Semay, F. Buisseret, and Fl. Stancu, Phys. Rev. D **76**, 116005 (2007).
[7] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D **2**, 1285 (1970).
[8] Fl. Stancu, *Group Theory in Subnuclear Physics*, Oxford Stud. Nucl. Phys. Vol. 19 (Oxford University Press, Oxford, 1996), pp 1–421.
[9] W.-M. Yao *et al.*, J. Phys. G **33**, 1 (2006), and partial update for 2008.
[10] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **99**, 052001 (2007).
[11] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **99**, 052002 (2007).
[12] T. Aaltonen *et al.* (CDF Collaboration), Phys. Rev. Lett. **99**, 202001 (2007).
[13] V. M. Abazov *et al.* (D0 Collaboration), arXiv:0808.4142.
[14] E. E. Jenkins, Phys. Rev. D **54**, 4515 (1996).
[15] J. Carlson, J. Kogut, and V. R. Pandharipande, Phys. Rev. D **27**, 233 (1983).
[16] Y. Koma, E. M. Ilgenfritz, T. Suzuki, and H. Toki, Phys. Rev. D **64**, 014015 (2001).
[17] F. Okiharu *et al.*, Phys. Rev. D **72**, 014505 (2005).
[18] H. G. Dosch, Phys. Lett. B **190**, 177 (1987); H. G. Dosch and Yu. A. Simonov, Phys. Lett. B **205**, 339 (1988); Yu. A. Simonov, Nucl. Phys. **B307**, 512 (1988).
[19] B. Silvestre-Brac, C. Semay, I. M. Narodetskii, and A. I. Veselov, Eur. Phys. J. C **32**, 385 (2004).

[20] C. Semay and B. Silvestre-Brac, Phys. Rev. D **52**, 6553 (1995).
[21] Yu. A. Simonov, Phys. Lett. B **515**, 137 (2001); F. Buisseret and C. Semay, Phys. Rev. D **71**, 034019 (2005).
[22] A. Di Giacomo and H. Panagopoulos, Phys. Lett. B **285**, 133 (1992).
[23] A. Di Giacomo and Yu. A. Simonov, Phys. Lett. B **595**, 368 (2004).
[24] C. Semay, B. Silvestre-Brac, and I. M. Narodetskii, Phys. Rev. D **69**, 014003 (2004).
[25] B. Silvestre-Brac, C. Semay, and F. Buisseret, J. Phys. A **41**, 275301 (2008); J. Phys. A **41**, 425301 (2008).
[26] I. M. Narodetskii, C. Semay, and A. I. Veselov, Eur. Phys. J. C **55**, 403 (2008).
[27] F. Buisseret and C. Semay, Phys. Rev. D **73**, 114011 (2006).
[28] J.-P. Lee, C. Liu, and H. S. Song, Phys. Rev. D **59**, 034002 (1998).
[29] J.-P. Lee, C. Liu, and H. S. Song, Phys. Rev. D **62**, 096001 (2000).
[30] M. G. Olsson and S. Veseli, Phys. Rev. D **51**, 3578 (1995).
[31] W. Lucha, F. Schöberl, and D. Gromes, Phys. Rep. **200**, 127 (1991).
[32] A. M. Badalian and B. L. G. Bakker, arXiv:hep-ph/0702229.
[33] I. M. Narodetskii, M. A. Trusov, and A. I. Veselov, arXiv:0801.1980.