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Lepton flavor violating $l \to l'\gamma$ and $Z \to l\bar{l}'$ decays induced by scalar leptoquarks

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Motivated by the recent muon $g-2$ data, we study the lepton flavor violating (LFV) $l \to l'\gamma$ and $Z \to l\bar{l}'$ ($l, l' = e, \mu, \tau$ decays with $l \neq l'$) in a scalar leptoquark model. Leptoquarks can produce sizable LFV $l \to l'\gamma$ decay rates that can be easily reached by present or near future experiments. Leptoquark masses and couplings are constrained by the muon $g-2$ data and the current $l \to l'\gamma$ bounds. We predict $\mathrm{Br}(Z \to \tau^{\mp} e^{\pm})$ reaching the present limit (10^{-5}) and $\mathrm{Br}(Z \to \mu^{\mp}\tau^{\pm})$ reaching 2×10^{-8}, which will be accessible by future linear colliders, whereas, the current bounds on LFV impose very strong constraints on the $\mathrm{Br}(Z \to \mu^{\mp} e^{\pm})$ and the ratio is too low to be observed in the near future.

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I. INTRODUCTION

The excess value of the anomalous magnetic moment of muon was reported by the E821 Collaboration at BNL [1]

$$a_\mu^{\exp} = 116\,592\,080(63) \times 10^{-11}. \tag{1}$$

The standard model prediction for a_μ^{SM} with QED, hadronic, and electroweak contributions is [2,3]

$$a_\mu^{\mathrm{SM}} = 116\,591\,785(61) \times 10^{-11}. \tag{2}$$

With the experimental value of $(g-2)/2$, the comparison gives

$$\Delta a_\mu \equiv a_\mu^{\exp} - a_\mu^{\mathrm{SM}} = (295 \pm 87.7) \times 10^{-11} \quad (3.4\sigma). \tag{3}$$

The 3.4 standard deviation difference between the two may be a hint of new physics contribution.

It has been shown that contributions from leptoquark (LQ) exchanges are capable to resolve the above deviation [4–6]. Leptoquarks are vector or scalar particles carrying both lepton and baryon numbers. LQs can be quite naturally introduced in the low-energy theory as a relic of a more fundamental theory at some high-energy scale, such as grand unified theories [7,8]. In some models, it is possible to have leptoquarks at TeV scale [9]. The low-energy LQ phenomenology has received considerable attention. Possible LQ manifestations in various processes have been extensively investigated [9–25]. Various constraints on LQ masses and couplings have been deduced from existing experimental data and prospects for the forthcoming experiments have been estimated. Direct searches of LQs as s-channel resonances in deep inelastic ep scattering and pair production in hadron colliders placed lower limits on their mass $M_{\mathrm{LQ}} \geq 73$–298 GeV [18] depending on the LQ types and couplings. The interest on leptoquarks has been renewed during the last few years since ongoing collider experiments have good prospects for searching these particles [26]. For a recent review of leptoquarks, one is referred to [27].

Lepton flavor violation (LFV) are powerful tools to search for new physics. The present experimental limits give [18]

$$\mathrm{Br}\,(\mu \to e\gamma) < 1.2 \times 10^{-11}, \tag{4}$$

$$\mathrm{Br}\,(\tau \to e\gamma) < 1.1 \times 10^{-7}, \tag{5}$$

$$\mathrm{Br}\,(\tau \to \mu\gamma) < 6.8 \times 10^{-8}. \tag{6}$$

Since effects of leptoquark interactions can manifest in a_μ, it is very likely that they can also give interesting contributions to these $l \to l'\gamma$ processes [5,6]. There are considerable efforts on experiments that aim at pushing the sensitivity of $\mathrm{Br}(\mu \to e\gamma)$ down by two orders of magnitude [28]. B factories and the upgraded super-B factory can probe the $\tau \to e\gamma$, $\mu\gamma$ decays at better sensitivities.

The $Z \to \ell\bar{\ell}'$ decays are among the LFV interactions and the theoretical predictions of their branching ratios in the framework of the SM are extremely small [29–31]. These results are far from the experimental limits obtained at LEP1 [18]:

$$\mathrm{Br}\,(Z \to e^{\pm}\mu^{\mp}) < 1.7 \times 10^{-6}, \tag{7}$$

$$\mathrm{Br}\,(Z \to e^{\pm}\tau^{\mp}) < 9.8 \times 10^{-6}, \tag{8}$$

$$\mathrm{Br}\,(Z \to \mu^{\pm}\tau^{\mp}) < 1.2 \times 10^{-5}. \tag{9}$$

Better sensitivities are expected from the Giga-Z modes at future colliders, such as the International Linear Collider (ILC), to have [32–34]

$$\mathrm{Br}\,(Z \to e^{\pm}\mu^{\mp}) < 2 \times 10^{-9}, \tag{10}$$

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$$\mathrm{Br}(Z \to e^{\pm}\tau^{\mp}) < \kappa \times 6.5 \times 10^{-8}, \tag{11}$$

$$\mathrm{Br}(Z \to \mu^{\pm}\tau^{\mp}) < \kappa \times 2.2 \times 10^{-8}, \tag{12}$$

with $\kappa \simeq 0.2$–1.0. It will be interesting to study the leptoquark contributions to the $Z \to l\bar{l}'$ processes.

The aim of the present paper is to study the leptoquark effects in various LFV processes including $l \to l'\gamma$ and $Z \to l\bar{l}'$ decays, while considering leptoquark contribution to a_μ as a solution to the muon anomalous moment discrepancy. The layout of the present paper is as follows. In Sec. II we introduce the formalism. We then use it in Sec. III to study the leptoquark contributions to a_μ and LFV processes including $l \to l'\gamma$ and $Z \to l\bar{l}'$ decays. Section IV contains our conclusions. Some formulas and low-energy constraints are given in appendices.

II. FORMALISM

A. Scalar leptoquark interactions

In this section we list the relevant parts of the scalar leptoquark Lagrangian. We consider isosinglet scalar leptoquarks. The effective Lagrangian describing the leptoquark interactions in the mass basis is given by [10,24]

$$\begin{aligned}\mathcal{L}_{\mathrm{LQ}} = {} & \bar{u}^c_a(h'_{ai}\Gamma_{k,S_R}P_L + h_{ai}\Gamma_{k,S_L}P_R)e_i S^*_k \\ & + \bar{e}_j(h'^*_{aj}\Gamma^\dagger_{S_R,k}P_R + h^*_{aj}\Gamma^\dagger_{S_L,k}P_L)u^c_a S_k \\ & - eQ_{(u^c)}A_\mu \bar{u}^c_a\gamma^\mu u^c_a - ieQ_S A_\mu S^*_k \overleftrightarrow{\partial}^\mu S_k \\ & + ieQ_S \tan\theta_W Z_\mu S^*_k \overleftrightarrow{\partial}^\mu S_k \\ & - \frac{e}{s_W c_W} Z_\mu \bar{u}^c_a \gamma^\mu((T_{3(u^c)} - Q_{(u^c)}s^2_W)P_R \\ & - Q_{(u^c)}s^2_W P_L)u^c_a,\end{aligned} \tag{13}$$

where $k = 1, 2$ are the indices of leptoquark, $T_3 = -1/2$, $Q_{u^c} = -2/3$ are quark's isospin and electric charge, $Q_S = -1/3$ is the electric charge of scalar leptoquarks S_k, a, and i, j are quarks and leptons flavor indices and we use $c_W = \cos\theta_W$ and $s_W = \sin\theta_W$. The $\Gamma_{k,S_{L,R}}$ are elements of leptoquark mixing matrix that brings $S_{L,K}$ to the mass basis S_k:

$$S_L = \Gamma^\dagger_{S_L,k}S_k, \qquad S^*_R = \Gamma_{k,S_R}S^*_k, \tag{14}$$

where the $S_{L(R)}$ is the field that associates with the $\bar{e}_j P_{L(R)} u^c_a$ terms in $\mathcal{L}_{\mathrm{LQ}}$ [24]. Note that in the no-mixing case ($\Gamma = 1$), $S_{1,2}$ reduce to $S_{L,R}$, which are called chiral leptoquarks, as they only couple to quarks and leptons in certain chirality structures. Finally, the couplings h and h' are 3 by 3 matrices, which give rise to various LFV processes and must be subject to experimental constraints.

In this work we do not aim at a comprehensive study of the effects of all possible leptoquark interactions. Instead, we try to demonstrate that a simple scalar leptoquark model can provide rich and interesting LFV phenomenons.

B. Muon anomalous magnetic moment $(g-2)_\mu$

The LQ interaction is capable of generating a muon anomalous magnetic moment and resolving the discrepancy between theoretical and experimental results. The one-loop diagrams are shown in Figs. 1(a) and 1(b) with $l = l' = \mu$. The extra contribution to a_μ arises from the LQ model due to quark and scalar leptoquark one-loop contribution and is given by

$$\begin{aligned}a^{\mathrm{LQ}}_\mu = {} & -\frac{N_c m^2_\mu}{8\pi^2}\sum_{q=1}^{3}\sum_{k=1}^{2}\frac{1}{M^2_{S_k}}[(|h_{q\mu}\Gamma_{k,S_L}|^2 + |h'_{q\mu}\Gamma_{k,S_R}|^2) \\ & \times (Q_{(u^c)}F_2(x) - Q_S F_1(x)) \\ & - \frac{m_{(u^c_a)}}{m_\mu}\mathrm{Re}(h'_{q\mu}h^*_{q\mu}\Gamma^+_{S_R,k}\Gamma_{k,S_L})(Q_{(u^c)}F_3(x_{ka}) \\ & - Q_S F_4(x_{ka}))],\end{aligned} \tag{15}$$

In the above expression, $N_c = 3$, $Q_S = -1/3$, $Q_{u^c} = -2/3$. Our expression agrees with that in [6,15]. The kinematic loop functions F_i $(i = 1, \ldots, 4)$ depend on the variable $x = m^2_{(u^c_a)}/m^2_{S_k}$ and are given in Appendix A.

Using leptoquark contribution to saturate the deviation given in Eq. (3), the leptoquark masses $M_{S_{1,2}}$, mixing angle θ_{LQ}, and couplings $h^{(\prime)}_{q\mu}$ will be constrained.

C. $\ell \to \ell'\gamma$

In this subsection we give the amplitude of $\ell \to \ell'\gamma$ from leptoquark exchange. According to the gauge invariance, the amplitude can be written as

$$i\mathcal{M}^\gamma = ie\bar{u}(p_2)(F^\gamma_{2RL}P_L + F^\gamma_{2LR}P_R)(i\sigma_{\mu\nu}q^\nu)u(p_1)\varepsilon^{\mu*}_\gamma, \tag{16}$$

where ε_γ is the polarization vector and $q = p_1 - p_2$ is the momentum transfer. For the amplitude of leptoquark exchange at one-loop level as depicted in Figure. 1, we have

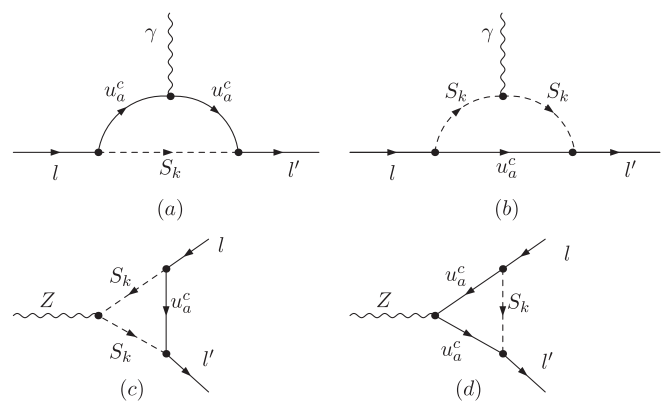

FIG. 1. Feynman diagrams contributing to $\ell \to \ell'\gamma$ and $Z \to \ell\bar{\ell}'$, S_k are the scalar leptoquark $k = 1, 2$, u^c_a are quark up with $a = 1, 2, 3$.

$$F^{\gamma}_{2LR} = \frac{N_c}{16\pi^2}\sum_{q=1}^{3}\sum_{k=1}^{2}\frac{1}{M^2_{S_k}}[(m_l h'_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_R} + m_{l'}h_{q\ell}h^{*}_{q\ell'}\Gamma^{\dagger}_{S_L,k}\Gamma_{k,S_L})(Q_{(u^c)}F_2(x) - Q_S F_1(x)) - m_{(u^c_a)}(h_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_L})(Q_{(u^c)}F_3(x) - Q_S F_4(x))], \tag{17}$$

$$F^{\gamma}_{2RL} = F^{\gamma}_{2LR}(h \leftrightarrow h', R \leftrightarrow L), \tag{18}$$

with $x = m^2_{(u^c_a)}/m^2_{S_k}$. The branching ratio of $\ell \to \ell'\gamma$ is

$$\mathrm{Br}\,(\ell \to \ell'\gamma) = \frac{\alpha_{em}}{4\Gamma(\ell)}\frac{(m^2_\ell - m^2_{\ell'})^3}{m^3_\ell}(|F^{\gamma}_{2LR}|^2 + |F^{\gamma}_{2RL}|^2). \tag{19}$$

In our numerical calculations we analyze the Brs of the decays under consideration by using the total decay widths of the decaying leptons $\Gamma(\ell)$.

D. $Z \to \ell\bar{\ell}'$

The Feynman diagrams of LFV Z decay process are shown in Fig. 1. The total contribution of all diagrams (c) and (d) can be written as

$$i\mathcal{M}^Z_\mu = iem^2_Z\bar{u}(p_2)\bigg[(F^Z_{1L}P_R + F^Z_{1R}P_L)\Big(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2_Z}\Big)\gamma^\nu + \frac{1}{m^2_Z}(F^Z_{2RL}P_L + F^Z_{2LR}P_R)(i\sigma_{\mu\nu}q^\nu)\bigg]u(p_1)\varepsilon^Z_\mu(q), \tag{20}$$

where q_μ is the Z four-momentum. The decay rates involve both $F^Z_{1L(R)}$ and $F^Z_{2LR(RL)}$:

$$\mathrm{Br}(Z \to \ell\bar{\ell}') = \frac{\alpha_{em}}{6}\frac{m_Z}{\Gamma_Z}\bigg[(|F^Z_{1L}|^2 + |F^Z_{1R}|^2) + \frac{1}{2m^2_Z}(|F_{2LR}(Z)|^2 + |F_{2RL}(Z)|^2)\bigg], \tag{21}$$

where the form factors $F^Z_{1L(R)}$ and $F^Z_{2LR(RL)}$ are given by

$$F^Z_{1L} = \frac{N_c}{16\pi^2}\frac{1}{M^2_{S_k}}\bigg[h'_{q\ell}h'^{*}_{q\ell'}\Gamma^{+}_{S_R,k}\Gamma_{k,S_R}(g_S G_1(x) + g_R G_2(x)) - \frac{m_{u_a}}{m^2_\ell - m^2_{\ell'}}(g_L - g_R)(h_{q\ell}h'^{*}_{q\ell'}\Gamma^{+}_{S_R,k}\Gamma_{k,S_L}m_\ell - h'_{q\ell}h^{*}_{q\ell'}\Gamma^{+}_{S_L,k}\Gamma_{k,S_R}m_{\ell'})G_3(x)\bigg], \tag{22}$$

$$F^Z_{1R} = F^Z_{1L}(h \leftrightarrow h', L \leftrightarrow R), \tag{23}$$

and

$$F^Z_{2LR} = \frac{N_c}{16\pi^2}\frac{1}{M^2_{S_k}}[h_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_L}m_{u_a}(g_R + g_L)G_3(x) + (g_R h'_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_R}m_\ell + g_L h_{q\ell}h^{*}_{q\ell'}\Gamma^{\dagger}_{S_L,k}\Gamma_{k,S_L}m_{\ell'})G_4(x) - g_S((h'_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_R}m_\ell + h_{q\ell}h^{*}_{q\ell'}\Gamma^{\dagger}_{S_L,k}\Gamma_{k,S_L}m_{\ell'})G_5(x) + m_{u_a}h_{q\ell}h'^{*}_{q\ell'}\Gamma^{\dagger}_{S_R,k}\Gamma_{k,S_L}G_6(x))], \tag{24}$$

$$F^Z_{2RL} = F^Z_{2LR}(h \leftrightarrow h', L \leftrightarrow R), \tag{25}$$

where we have $x = m^2_{u_a}/m^2_{S_k}$ and the couplings $g_{R,L}$ and g_S are given by

$$g_R = -\frac{2}{\sin\theta_W\cos\theta_W}(T_{3(u^c)} - Q_{(u^c)}\sin^2\theta_W), \tag{26}$$

$$g_L = Q_{(u^c)}\tan\theta_W, \qquad g_S = Q_S\tan\theta_W. \tag{27}$$

In the above expressions of $F^Z_{1L(R)}$, we keep only the leading term in $m^2_Z/m^2_{S_k}$. The explicit expressions of one-loop functions G_n $(n = 1, ..6)$ can be found in Appendix A.

III. NUMERICAL RESULTS AND DISCUSSION

We are now ready to give some numerical results. Quark masses are evaluated at the scale of the $\mu = 300$ GeV [35], which is the typical leptoquark mass used in this work,

$$m_t = 161.4\ \mathrm{GeV}, \qquad m_c = 0.55\ \mathrm{GeV}, \qquad m_u = 11.4\times 10^{-3}\ \mathrm{GeV}, \tag{28}$$

and for the following quantities we use [18]

$$\alpha_{em} = 1/137.0359, \qquad M_W = 80.45\ \mathrm{GeV}, \qquad M_Z = 91.1875\ \mathrm{GeV}. \tag{29}$$

For simplicity, we assume that the couplings h and h' are real and equal to each other, i.e.

$$h = h' = h^*. \tag{30}$$

We use leptoquark mass splitting $\Delta = 500$ GeV in our analysis, where Δ is defined as $\sqrt{M^2_{S_2} - M^2_{S_1}}$. Consequently, the remaining parameters in the leptoquark model are the mass of the light scalar leptoquark M_{S_1}, the mixing angle θ_{LQ}, and the couplings $h_{q\ell}$.

A. Muon anomalous magnetic moment a_μ

In this section we discuss a few phenomenological aspects of the leptoquark contributions to a_μ. In the left panel of Fig. 2, we present a scatter plot in the $(M_{S_1} - |h_{q\mu}|^2)$ plane for top quark contribution (red) and charm quark

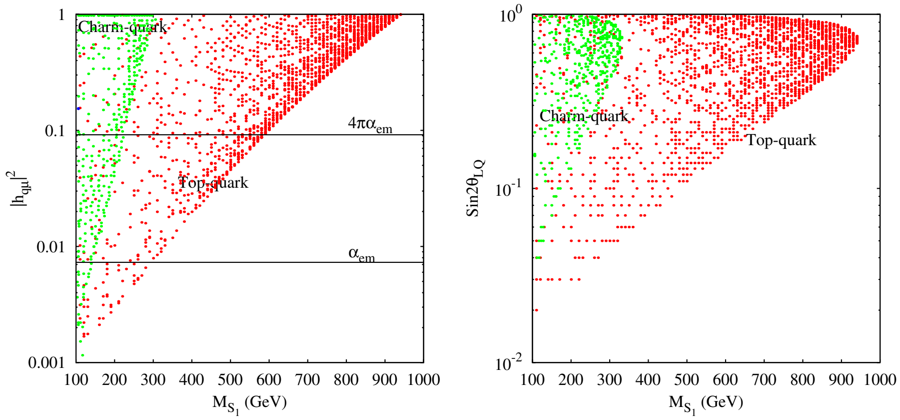

FIG. 2 (color online). Scatter plot in the plane $(M_{S_1} - |h_{q\mu}|^2)$ in the left panel, $(M_{S_1} - \sin 2\theta_{\rm LQ})$ in the right panel. These are allowed regions in the parameter space that give $a_\mu^{\rm LQ} = \Delta a_\mu = (295 \pm 87.7) \times 10^{-11}$.

contribution (green), which are allowed by $a_\mu^{\rm LQ} = \Delta a_\mu = (295 \pm 87.7) \times 10^{-11}$ [see Eq. (3)] within the 1σ range of data. We note that it is not possible to use the up quark loop contribution alone for the $a_\mu^{\rm LQ} = \Delta a_\mu$, since the mixing angle and couplings $h_{u\mu}$ are strongly constrained by the π leptonic decays (see Appendix B).

In order to see the impact of the mixing angle, we present in the right panel of Fig. 2 the allowed regions $a_\mu^{\rm LQ} = \Delta a_\mu$ in the $(M_{S_1} - \sin 2\theta_{\rm LQ})$ plane. We use $\alpha_{em} \leq h_{q\mu}^2 \leq 1$. The contribution dominates around $\sin 2\theta_{\rm LQ} \sim 0.7$ both for top and charm quark contributions. We see that the constraint from a_μ confines the allowed range of M_{S_1} to $M_{S_1} \lesssim 950$ GeV for top quark contribution and to $M_{S_1} \lesssim 350$ GeV for charm quark contribution at the 1σ level. The parameter space will be used for later study of LFV processes. The light leptoquark mass should be below 1 TeV, if leptoquarks with couplings of electromagnetic strength are responsible for the deviation Δa_μ. It is interesting that LHC may have a good chance to observe these particles [26].

B. Lepton flavor violating $l \to l'\gamma$ and $Z \to l\bar{l}'$ decays

In this section, we investigate the LFV decay processes generated by the same leptoquark scalar interactions. We consider only parameter space that corresponds to $a_\mu^{\rm LQ} = \Delta a_\mu$ when it is appropriate. We discuss $\mu \to e\gamma$ and $\tau \to e\gamma$, $\mu\gamma$ decays first.

In Figs. 3 and 4 we show scatter plots of the allowed parameters in $(M_{S_1}, h_{q\ell}h_{q\ell'})$ planes from bounds of $\tau \to \mu\gamma$, $\tau \to e\gamma$, and $\mu \to e\gamma$ rates. Note that in the plots we use

$$\begin{aligned} 1.5 \times 10^{-13} &\leq {\rm Br}(\mu \to e\gamma) < 1.2 \times 10^{-11}, \\ 1 \times 10^{-9} &\leq {\rm Br}(\tau \to e\gamma) < 1.1 \times 10^{-7}, \\ 1 \times 10^{-9} &\leq {\rm Br}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}, \end{aligned} \qquad (31)$$

where the upper bounds are from the current limits: Eqs. (4)–(6), while the lower bound for ${\rm Br}(\mu \to e\gamma)$ is from the expected bound in the future [28] and the lower bounds for $\tau \to l\gamma$ are for illustration. For the $\tau \to \mu\gamma$ and

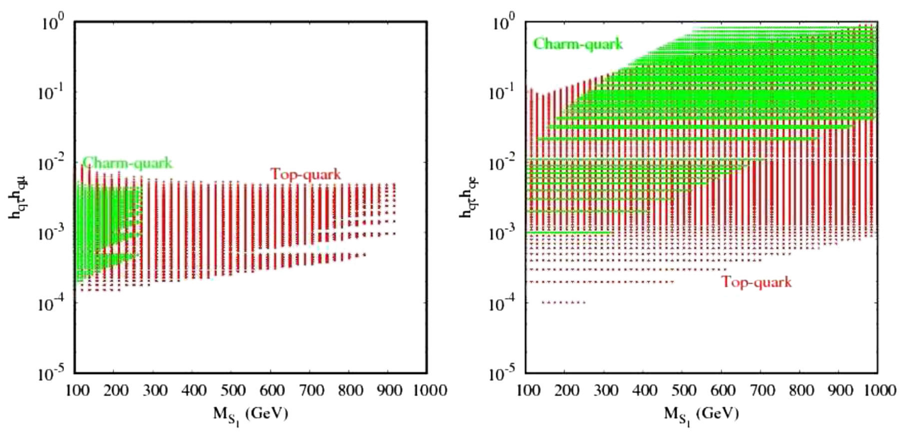

FIG. 3 (color online). Scatter plots of leptoquark parameters in $(M_{S_1}, h_{q\ell}h_{q\ell'})$ planes from $(\ell \to \ell'\gamma)$ bounds given in Eq. (31). The left (right) figure is for the $\tau \to \mu\gamma$ ($\tau \to e\gamma$) case with top and charm quark contributions.

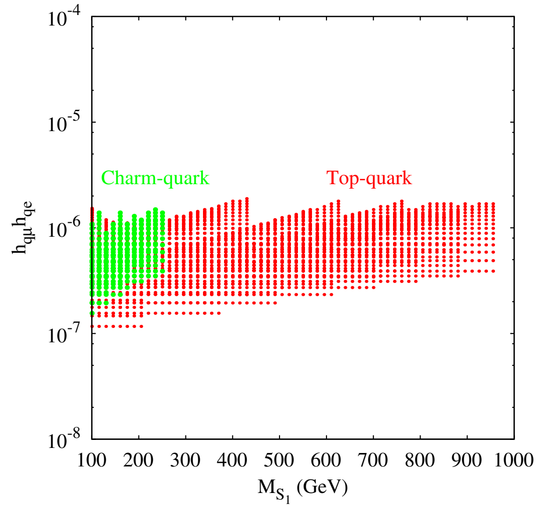

FIG. 4 (color online). Same as Fig. 3 except for the $\mu \to e\gamma$ case.

TABLE I. Constraints on the parameters $h_{q\ell}h_{q\ell'}$ ($q = t$, c) coming from radiative FCNC processes induced by the scalar leptoquark using the present experimental bounds.

Decay mode	$h_{c\ell}h_{c\ell'}$	$h_{t\ell}h_{t\ell'}$
$\tau \to \mu\gamma$	$<5.29 \times 10^{-3}$	$<9.11 \times 10^{-3}$
$\tau \to e\gamma$	<0.81	<0.82
$\mu \to e\gamma$	$<1.45 \times 10^{-6}$	$<1.92 \times 10^{-6}$

$\mu \to e\gamma$ cases the $(g-2)_\mu$ constraint is taken into account.

For different quark contribution the couplings are bounded in the following ranges: $10^{-4} \lesssim h_{q\tau}h_{q\mu} \lesssim 10^{-2}$, $10^{-3} \lesssim h_{c\tau}h_{ce} \lesssim 1$, $10^{-4} \lesssim h_{t\tau}h_{te} \lesssim 1$, and $10^{-7} \lesssim h_{q\mu}h_{qe} \lesssim 10^{-6}$. For the $\tau \to \mu\gamma$ and $\mu \to e\gamma$ cases, the allowed leptoquark masses are $m_{S_1} \lesssim$ 250–300 GeV and 1 TeV for c-quark and t-quark loop contribution, respectively. These regions are determined from the bounds and the muon $g-2$ constraint (see also

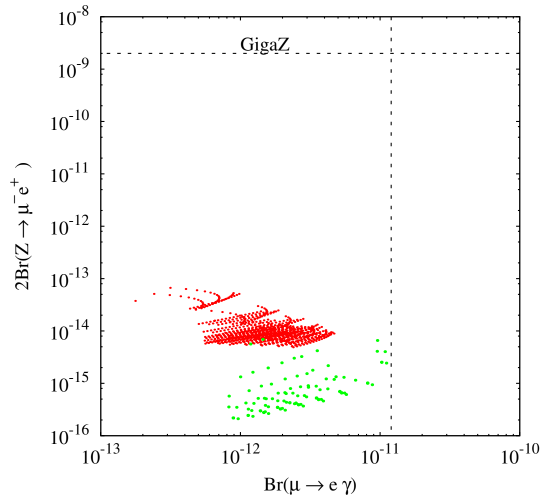

FIG. 6 (color online). The correlation between $\mathrm{Br}(\mu \to e\gamma)$ and $\mathrm{Br}(Z \to \mu e)$.

Fig. 2) at the same time. On the other hand the couplings governing $\tau \to e\gamma$ decay and those generating muon $g-2$ contribution are decoupled, the parameters corresponding to the former bounds are free from the latter constraint. The resulting allowed regions are larger in these cases. The parameters in these allowed regions will be used to predict $Z \to l\bar{l}'$ decays. To have an idea of the size the allowed couplings, we give that upper bound on $h_{q\ell}h_{q\ell'}$ obtained from the present $l \to l'\gamma$ limits in Table I. We see that the $\mu \to e\gamma$ constraint is more effective in restricting the sizes of $h_{q\ell}h_{q\ell'}$.

In Figs. 5 and 6, we give the predicted $Z \to l\bar{l}'$ rates in correlation with $\mathrm{Br}(l \to l'\gamma)$. We see that $\mathrm{Br}(Z \to \tau^\mp e^\pm)$ can reach 1.95×10^{-5}, which is comparable with the present bound, and $\mathrm{Br}(Z \to \mu^\mp \tau^\pm)$ can reach 2.34×10^{-8}, which will be accessible by future linear colliders. On the contrary, the current bound on the $\mu \to e\gamma$ decay imposes very strong constraints on the related couplings as shown in Table I. Hence the predicted $\mathrm{Br}(Z \to \mu^\mp e^\pm)$ is rather small and is too low to be observed in the near

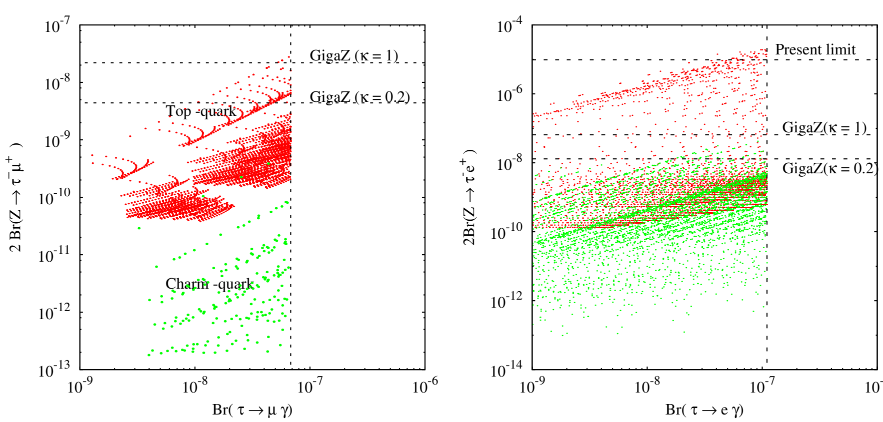

FIG. 5 (color online). The correlation between $\mathrm{Br}(\tau \to \ell'\gamma)$ and $\mathrm{Br}(Z \to \tau\ell')$ where $\tau = e$, μ.

future. In Fig. 5, we see that the $Z \to l\bar{l}'$ rates are roughly positively correlating with the $l \to l'\gamma$ rates and the top quark loop contributions are larger than the charm quark's ones. To have observable $Z \to \tau^{\mp}\mu^{\pm}$ and $Z \to \tau^{\mp}e^{\pm}$, the $\tau \to \mu\gamma$, $e\gamma$ rates are predicted to be close to the present bounds.

In this work the analysis has been performed for the scalar leptoquark case. It is possible that vector leptoquarks may also contribute to $(g-2)_\mu$ and LFV processes. As shown in Refs. [4,20], quite often $(g-2)_\mu$ and LFV processes provide more stringent constraints on vector leptoquark couplings and masses than on scalar leptoquark ones. For example, using the measured m_t and the formula given in [4], the present Δa_μ leads to a very large mass scale $\Lambda \simeq 500$ TeV in the vector leptoquark case, where Λ was defined from the relation: $4\pi/\Lambda^2 \equiv g_{\rm LQ}^2/m_{\rm LQ}^2$. The mass scale is much larger than the corresponding mass scale exhibited in Fig. 2, which is found to be $\Lambda \simeq$ few $-$ $\mathcal{O}(10)$ TeV. Similarly, in $l \to l'\gamma$ processes, the constraints on vector leptoquark parameters are usually more severe [20].

IV. CONCLUSION

Motivated by the reported discrepancy of the muon $g-2$ results, we studied the lepton flavor violating $\ell \to \ell'\gamma$ and $Z \to \ell\bar{\ell}'$ decays in the LQ model. We showed that the $g-2$ anomaly favors LQ masses in a rather low-energy regime, e.g. <1 TeV, which is within the reach of the forthcoming Large Hadron Collider.

We found that leptoquarks can generate sizable LFV $l \to l'\gamma$ decays. The present experimental limits are used to confine the leptoquark parameter space. On the other hand, it is interesting to search for these LFV effects in experiments, such as MEG, B factories, and the super-B factory.

We predict $\mathrm{Br}(Z \to \tau^{\mp}e^{\pm})$ reaching 10^{-5} and $\mathrm{Br}(Z \to \mu^{\mp}\tau^{\pm})$ reaching 2×10^{-8}, which can be accessible by present experiments and future linear colliders, such as ILC. On the contrary, the current bounds on LFV impose very strong constraints on the $\mathrm{Br}(Z \to \mu^{\mp}e^{\pm})$ and the ratio is too low to be observed in the near future. In this case, it is useful to search for the LFV effects in $\mu \to e\gamma$ decay.

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APPENDIX A: ONE-LOOP FUNCTIONS

The loop functions F_i and G_i used in Sec. II are given by

$$F_1(x) = \frac{[2 + 3x - 6x^2 + x^3 + 6x\log(x)]}{12(1-x)^4}, \tag{A1}$$

$$F_2(x) = \frac{[1 - 6x + 3x^2 + 2x^3 - 6x^2\log(x)]}{12(1-x)^4}, \tag{A2}$$

$$F_3(x) = \frac{-1}{2(1-x)^3}[3 - 4x + x^2 + 2\log(x)], \tag{A3}$$

$$F_4(x) = \frac{1}{2(1-x)^3}[1 - x^2 + 2x\log(x)], \tag{A4}$$

and

$$G_1(x) = \frac{[-2 + 9x^2 - 18x^4 + 11x^6 - 12x^6\log(x)]}{36(x^2-1)^4}, \tag{A5}$$

$$G_2(x) = \frac{1}{36(x^2-1)^4}[16 - 45x^2 + 36x^4 - 7x^6 + 12(-2 + 3x^2)\log(x)], \tag{A6}$$

$$G_3(x) = \frac{3 - 4x^2 + x^4 + 4\log(x)}{4(x^2-1)^3}, \tag{A7}$$

$$G_4(x) = \frac{2 + 9x^2 - 6x^4 + x^6 + 12x^2\log(x)}{12(x^2-1)^4}, \tag{A8}$$

$$G_5(x) = \frac{1 - 6x^2 + 3x^4 + 2x^6 - 12x^4\log(x)}{12(x^2-1)^4}, \tag{A9}$$

$$G_6(x) = \frac{1}{2(x^2-1)^3}[-1 + x^4 - 4x^2\log(x)]. \tag{A10}$$

APPENDIX B: CONSTRAINT FORM $\pi \to e\nu_e$ AND $\pi \to \mu\nu_\mu$ DECAYS

We follow [11,20] to constrain leptoquark parameters using pion decay data. Form the interactions given in Eq. (13), we obtain the effective four-Fermi interaction

$$\mathcal{L}_{\rm eff} = -\frac{h'_{ai}h'^{*}_{bj}\Gamma^{\dagger}_{R,k}\Gamma_{k,R}}{M^2_{S_k}}(\bar{e}^c_i P_L u_a)(\bar{d}_b P_R \nu^c_j) - \frac{h_{ai}h'^{*}_{bj}\Gamma^{\dagger}_{R,k}\Gamma_{k,L}}{M^2_{S_k}}(\bar{e}^c_i P_R u_a)(\bar{d}_b P_R \nu^c_j). \tag{B1}$$

By using the Fierz transformation, we can rewrite the Eq. (B1) as

$$\mathcal{L}_{\rm eff} = -\frac{1}{2M^2_{S_k}}h'_{ai}h'^{*}_{bj}\Gamma^{\dagger}_{R,k}\Gamma_{k,R}(\bar{d}_{L,b}\gamma_\mu u_{L,a})(\bar{\nu}_{L,j}\gamma^\mu e_{L,i}) + \frac{1}{2M^2_{S_k}}h_{ai}h'^{*}_{bj}\Gamma^{\dagger}_{R,k}\Gamma_{k,L}(\bar{d}_{L,b}u_{R,a})(\bar{\nu}_{L,j}e_{R,i}). \tag{B2}$$

On the other hand, the conventional interaction for the $\pi \to l\nu_l$ decay in the SM is given by

$$\mathcal{L}_{\rm eff} = -\frac{G_F V_{ud}}{\sqrt{2}}[\bar{\nu}\gamma_\mu(1-\gamma_5)l][\bar{d}\gamma^\mu(1-\gamma_5)u] + \text{H.c.}$$

Here $|V_{ud}|$ is the Cabibbo-Kobayashi-Maskawa matrix elements between the constituent of the pion meson, G_F is the Fermi couplings constant. The ratio R_{th} of the electronic and muonic decay modes is [36]

$$R_{th} = \frac{\Gamma_{\rm SM}(\pi^+ \to \bar{e}\nu_e)}{\Gamma_{\rm SM}(\pi^+ \to \bar{\mu}\nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2(1+\delta) = (1.2352 \pm 0.0001) \times 10^{-4}, \tag{B3}$$

where δ is the radiative corrections. Thus the ratio R_{th} is very sensitive to nonstandard model effects (such as multi-Higges, nonchiral leptoquarks). The experimental ratio is [18]

$$R_{\rm exp} = (1.2302 \pm 0.004) \times 10^{-4}. \tag{B4}$$

The interference between the standard model and LQ model can be expressed by

$$R_{\rm SM-LQ} = R_{th} + R_{th}\frac{m_{\pi^+}^2}{m_u + m_d}\left(\frac{1}{\sqrt{2}}\frac{\text{Re}(h_{ue}h_{ue}^{\prime *})}{G_F V_{ud} M_{S_k}^2}\frac{1}{m_e} - \frac{1}{\sqrt{2}}\frac{\text{Re}(h_{u\mu}h_{u\mu}^{\prime *})}{G_F V_{ud} M_{S_k}^2}\frac{1}{m_\mu}\right)\Gamma^\dagger_{R,k}\Gamma_{k,L}. \tag{B5}$$

At the 2σ level, we get

$$R_{\rm min} < \sum_{k=1}^{2}\left(\frac{m_\pi}{m_e}\frac{\text{Re}(h_{ue}h_{ue}^{\prime *})}{M_{S_k}^2} - \frac{m_\pi}{m_\mu}\frac{\text{Re}(h_{u\mu}h_{u\mu}^{\prime *})}{M_{S_k}^2}\right)\Gamma^\dagger_{R,k}\Gamma_{k,L} < R_{\rm max}, \tag{B6}$$

where

$$R_{\rm min} = -1.06 \times 10^{-8}\ \text{GeV}^{-2}, \tag{B7}$$

$$R_{\rm max} = 2.45 \times 10^{-9}\ \text{GeV}^{-2}. \tag{B8}$$

The total contribution to $R_{\rm SM-LQ}$ must be smaller than the differences between SM and experiment within the error limits allowed.

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