Four generations, the electroweak phase transition, and supersymmetry

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(Received 24 April 2008; published 23 October 2008)

We calculate the strength of the electroweak phase transition in a supersymmetric model with four chiral generations. The additional chiral fermions (and scalar partners) lower the critical temperature and thus strengthen the first-order phase transition. The scalar partners stabilize the potential, leading to an effective theory that is bounded from below. We identify the ensemble of parameters where $\phi_c/T_c \gtrsim 1$ simultaneous with obtaining a large enough Higgs mass. Our calculations focus on a subset of the full four generational supersymmetric parameter space: We take the pseudoscalar heavy, $\tan\beta = 1$, and neglect all subleading contributions to the effective potential. We find that the region of parameter space with a strong first-order phase transition requires $m_{\tilde{q}'}/m_{q'} \lesssim 1.1$ while the constraint on the lightest Higgs mass requires $m_{\tilde{q}'}/m_{q'} \gtrsim 1$ with $m_{q'} \gtrsim 300$ GeV. We are led to an intriguing prediction of quarks and squarks just beyond the current Tevatron direct search limits that are poised to be discovered quickly at the LHC.

I. INTRODUCTION

The origin of the matter asymmetry is a deep mystery that remains unsolved. Conditions that can lead to a dynamical asymmetry between baryons and antibaryons were articulated years ago by Sakharov [1]: baryon number violation, C and CP violation, and a departure from thermal equilibrium. All three conditions are satisfied by the standard model as it passes through the electroweak phase transition. But, the CP violation is too small [2], and the phase transition is not strongly first order (e.g., [3–7]), given the direct search bounds on the Higgs boson from LEP. New physics with large CP violation is trivial to introduce into the model; weak scale supersymmetry is an obvious example (care needs only to be taken to ensure that induced electric dipole moments are within the experimental bounds). Even with a new source of CP violation, if the phase transition is not strong enough first order, any generated baryon asymmetry will be washed out [8]. New physics that enhances the first order phase transition, however, is generally much more tricky to achieve.

In the early 1990s it was realized that the electroweak phase transition could be enhanced by modifying the cubic coupling of the finite-temperature effective potential [3]. Nontrivial modifications of the cubic coupling could arise from additional scalars with order one couplings to the Higgs. In the minimal supersymmetric standard model (MSSM), the scalar superpartners to the top quarks (stops) can play precisely this role [9–12]. It has long been advocated that the region of MSSM parameter space with a light stop (and a light Higgs) can yield a strong enough phase transition. Unfortunately, the combination of direct searches for the lightest Higgs boson and direct searches for stops have virtually ruled out this possibility. The remaining parameter space [13] requires a large hierarchy between the left-handed and right-handed stops to ensure the Higgs satisfies the LEP bound.

Methods to strengthen the first-order phase transition beyond the SM and MSSM are now widely discussed [14–25]. Several of these ideas add a singlet field, such as in the NMSSM or nMSSM. Another related idea is to simply cut off the SM at a low scale, adding the effects of higher dimensional operators [26,27] (which can be UV completed by integrating out a singlet).

Yet another interesting possibility, and the one we will focus on in this paper, is to add more particles with modestly strong couplings to the Higgs. This was proposed in [18]; the additional particles have quantum numbers such that they mix with the MSSM charginos and neutralinos. Heavy particles that receive their mass entirely or dominantly from electroweak symmetry breaking can have a substantial impact on the electroweak phase transition. In this paper we consider a modification to the MSSM similar in spirit to [18] to enhance the phase transition. Namely, we add a fourth generation of particles (and sparticles) to the MSSM. Larger couplings to the Higgs are automatic simply due to the direct search bounds from LEP and Tevatron on fourth generation fermions.

That the electroweak phase transition could be enhanced in a four generation supersymmetric model was considered before in [28]. They performed an interesting numerical study that also found that the electroweak phase transition can be enhanced when $m_{\tilde{q}'}/m_{q'}$ is not much larger than 1. However, the present limits on fourth generation quark masses rule out their parameter space, and moreover, they allowed $\tan\beta$ to far exceed 1, implying that the b' Yukawa coupling was nonperturbative. In our analysis, we first systematically analyze the origin of the contributions that allow the phase transition to become first order. This allows us to make a clear distinction between supersymmetric and nonsupersymmetric theories with heavy chiral fermions. We then identify the viable region of parameter space where the Yukawa couplings are under control ($\tan\beta \simeq 1$) and all other bounds are satisfied.

A fourth generation has historically been thought to be strongly disfavored by the absence of flavor mixing, the $Z \to \nu \bar{\nu}$ constraint, and electroweak precision data. (Some early reviews can be found in [29–31].) All of these objections can be straightforwardly overcome, as was recently emphasized in [32]. Below we summarize these results in the form of the parameter space that is allowed. It is interesting that the constraints from electroweak precision data can be overcome without or with an electroweak scale Majorana mass for the fourth generation right-handed neutrinos. If a Majorana mass does indeed exist, lepton number is violated at the electroweak scale, and thus scenarios of baryogenesis that rely on an earlier generation of $B - L$ number (such as leptogenesis) do not work here [33,34]. Electroweak baryogenesis is one of the few mechanisms not sensitive to this source of lepton number violation and thus becomes even more interesting to study.

II. SETUP

We consider a low energy supersymmetric theory with a fourth chiral generation of matter, the "4MSSM" (for an early discussion, see [35]). A fourth chiral generation of matter does affect electroweak precision observables. One of the main results of [32] was to show that it can be made completely consistent with electroweak precision data so long as there are modest mass splittings between the isospin partners in the quark and lepton doublets. This splitting causes a modest reduction in the positive contribution to S, simultaneous with a positive contribution to T, allowing ensembles of parameters that are within the 68% CL ellipse of the LEP Electroweak Working Group. For example, the mass spectrum $m_{\nu'} = 100$ GeV, $m_{\ell'} = 155$ GeV, $m_{t'} = 310$ GeV, $m_{b'} = 260$ GeV, and $m_h = 115$ GeV is perfectly acceptable. There is strong sensitivity to the mass *differences* with only mild sensitivity to the overall scale of the particles. We will present results for both the electroweak preferred ratio $m_{t'}/m_{b'} \sim 1.2$ as well as $m_{t'} = m_{b'}$ for comparison.

With supersymmetry, there are additional contributions to electroweak precision observables from superpartners (e.g., [36]). We have not included these contributions to optimize the parameter set to match electroweak data, simply because many more parameters enter the fit that can be freely adjusted without affecting our results for the electroweak phase transition. We therefore take fourth generation Yukawa couplings consistent with [32] and take the scalar partner masses to be degenerate, eliminating this potential additional contribution to isospin violation.

We neglect all subleading contributions to the zero-temperature and finite-temperature effective potential. Subleading here refers to couplings smaller than about 1. We retain the contributions from gauge bosons, but we neglect light fermions ($u, d, c, s, b, e, \mu, \tau$), Higgs bosons (the quartic is small), and all superpartners other than $\tilde{t}'_{1,2}$, $\tilde{b}'_{1,2}$, $\tilde{t}_{1,2}$. We also neglect contributions from fourth gen-

eration leptons because the number of degrees of freedom per particle is only $1/3$ that of quarks and the bounds on the mass from the nonobservation in experiment are much weaker than for quarks.

III. SUPERSYMMETRY WITH $\tan\beta = 1$

In the limit $\tan\beta \to 1$, several aspects of supersymmetry drastically simplify. From the definition of $\tan\beta \equiv \langle H_u \rangle / \langle H_d \rangle$, we see the vacuum expectation values (vevs) are equal, $v_u = v_d = v/\sqrt{2}$, where $v = 246$ GeV, and the Yukawa couplings

$$y_f = \frac{2m_f}{v} \tag{1}$$

are the same for the up-type and down-type fermions. The fourth generation quarks t' and b' have large Yukawa couplings,

$$y_{t',b'} = 2.1\left(\frac{m_{t',b'}}{260 \text{ GeV}}\right), \tag{2}$$

where 260 GeV is shown for convenience in comparison to the (approximate) present direct search bounds from the Tevatron [37,38]. Note that these Yukawa couplings are a factor of $\sqrt{2}$ larger than in a nonsupersymmetric model, since the t' and b' acquire their mass only through couplings to the up-type and down-type Higgs, respectively.

If $\tan\beta \neq 1$, either the up-type or down-type Higgs vev is reduced, and thus to hold the masses of the fermions fixed, either $y_{t'}$ or $y_{b'}$ must increase. Setting $\tan\beta = 1$ allows the largest possible physical fourth generation fermion masses with the smallest Yukawa couplings. Since y_f cannot be arbitrarily large for perturbative calculations at the weak scale to be valid, the parameter choice $\tan\beta = 1$ really just maximizes the cutoff scale of the model. Even with this adjustment, the cutoff scale is low. This can be estimated by running the one-loop renormalization group equations for the Yukawa couplings up to $\sim 4\pi$. We show the scale of the Landau pole as a function of fermion mass in Fig. 1. Note that requiring $y_f^2/(4\pi) \lesssim 1$, which is about as large as can be considered for one-loop calculations, implies $y_f \lesssim 3.5$, corresponding to $m_f \lesssim 450$ GeV; we will not consider fermion masses that much exceed this value.

In the limit $\tan\beta \to 1$ the Higgs sector also drastically simplifies. The general tree-level potential for the neutral components of the Higgs in the MSSM is [39]

$$V = (m_{H_u}^2 + \mu^2)|H_u^0|^2 + (m_{H_d}^2 + \mu^2)|H_d^0|^2$$
$$- (bH_u^0 H_d^0 + \text{c.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \tag{3}$$

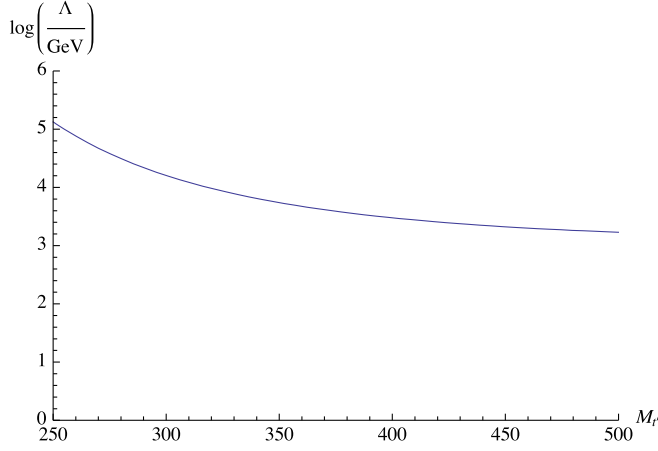Expanding the neutral components in the limit $\tan\beta = 1$,

FIG. 1 (color online). An estimate of the cutoff scale of the 4MSSM as a function of the fourth generation fermion mass by running the one-loop renormalization group equations of the quark Yukawas up to where they encounter a Landau pole ($y_{f'} \sim 4\pi$).

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ v \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} h \\ H \end{pmatrix}, \quad (4)$$

where the α rotation matrix diagonalizes the Higgs fluctuations (h, H) into mass eigenstates. In the limit that the second Higgs doublet "decouples" ($m_{A^0, H, H^{\pm}} \gg m_h$), the mixing angle $\alpha \rightarrow -\beta$, and thus the lightest Higgs is simply $h = (H_u^0 + H_d^0 - \sqrt{2}v)$. In this limit the tree-level Higgs potential vanishes, since h corresponds to the excitation of a D-flat direction.

Since electroweak precision data prefers $m_{t'}/m_{b'} \simeq 1.2$, this could be arranged either by adjusting just these two Yukawa couplings $y_{t'}/y_{b'} = 1.2$ or instead adjusting $\tan\beta = 1.2$. These two scenarios are nearly equivalent for our purposes, and so we choose to set $\tan\beta = 1$. The alternative, $\tan\beta = 1.2$, would give a tree-level contribution to the Higgs potential. The contribution to the (mass)^2 is, however, just $0.03M_Z^2$. As we will see, the one-loop radiative corrections from quarks and squarks will be far larger than this, so it is safe to completely neglect tree-level contributions even if $\tan\beta$ were allowed to vary slightly from 1.

In addition to taking $\tan\beta = 1$, we also choose supersymmetric parameters such that the mass eigenstates of $\tilde{t}'_{1,2}$ and $\tilde{b}'_{1,2}$ correspond to the gauge eigenstates $\tilde{t}'_{L,R}$ and $\tilde{b}'_{L,R}$. This is done purely to simplify our calculation. It is a rather conservative approximation, since it is well known that increasing the off-diagonal contribution to the squark mass matrix leads to an enhancement in the one-loop contribution to the Higgs mass (e.g., see [40]). We expect that the parameter space with a strong first-order phase transition will enlarge as this restriction is relaxed. Note that since the off-diagonal left-right contribution to the up-type and down-type squark mass matrix is equal to $m_f(A_f - \mu)$

(where again, $\tan\beta = 1$), this simplification corresponds to the specific parameter choice $A_f = \mu$.

Finally, as we discussed above, the Higgs potential simplifies in the limit m_{A^0}, $m_{H^{\pm}}$, $m_{H^0} \gg m_h$. This is a common assumption in the electroweak phase transition literature: The operational advantage is that the low energy theory is effectively a one-Higgs-doublet model that is drastically simpler to analyze at finite temperature.

IV. ONE-LOOP EFFECTIVE POTENTIAL

In the 4MSSM with $\tan\beta = 1$, loop corrections entirely determine the Higgs potential. We are interested in the loop corrections to just the scalar fluctuation $\phi = (h + v)/\sqrt{2}$. At one loop the effective potential for the Higgs is determined from the Coleman-Weinberg potential

$$V_1 = \sum_i \frac{n_i}{64\pi^2} M_i(\phi)^4 \left(\ln\frac{M_i(\phi)^2}{\mu^2} - c_i\right), \quad (5)$$

where $M_i(\phi)$ are the field-dependent masses and μ is the renormalization scale ($\overline{\mathrm{MS}}$ scheme). We generally use M_i to refer to ϕ-dependent (and temperature-dependent) masses and m_i to refer to $M_i(v)$ at zero temperature. The c_i's are constants corresponding to $5/6$ for gauge bosons and $3/2$ for fermions and scalars. The degeneracies per particle are $n_q = -12$ (for each $q = t, t', b'$), $n_{\tilde{q}_L} = n_{\tilde{q}_R} = 6$, $n_{W_T} = 4$, $n_{Z_T} = 2$, $n_{W_L} = 2$, $n_{Z_L} = 1$.

Expanding the effective potential as given above, evidently the minimum is not necessarily located at the proper electroweak breaking scale $v = 246$ GeV. This is easily remedied by imposing a renormalization condition on the mass parameter such that the minimum is enforced to be at v. This amounts to adding the v-dependent contribution to the effective potential,

$$\begin{aligned} \Delta V &= -\frac{dV_1(\phi = v)}{dv^2}\phi^2 \\ &= -\sum_i \frac{n_i}{32\pi^2} M_i^2(v)\frac{dM_i^2(v)}{dv^2}\left(\ln\frac{M_i^2(v)}{\mu^2} + \frac{1}{2} - c_i\right)\phi^2. \end{aligned}$$

$$(6)$$

The masses used in the effective potential are $\overline{\mathrm{MS}}$ masses that differ from the physical (pole) masses through finite and log-dependent corrections. The running fermion masses are given at one loop by

$$m_f|_{\mathrm{pole}} = m_f(\mu)\left[1 + \frac{\alpha_s}{\pi}\left(\frac{4}{3} + \ln\frac{\mu^2}{m_f^2}\right)\right]. \quad (7)$$

Since the fourth generation fermions overwhelmingly dominate the contributions to the (zero- and finite-temperature) effective potential, we take $\mu = \sqrt{m_{t'}m_{b'}}$, i.e., the scale of the largest electroweak breaking masses in the problem. This tends to minimize the higher-order corrections to the potential, though our calculations are not particularly sensitive to the precise choice of renormaliza-

tion scale. We note also that by fixing $\tan\beta = 1$ at $\mu = \sqrt{m_{t'}m_{b'}}$, we have neglected higher-order electroweak running of $\tan\beta$ between μ and υ.

The running scalar masses also differ from their physical pole masses through one-loop corrections depending on not only the gluon but also gluino diagrams [36]. This correction is generally numerically smaller than the correction to the fermion mass, typically less than a few %. Moreover, since the correction is gluino mass dependent, relating the pole mass to the running mass requires specifying an otherwise unfixed parameter in our model. We choose instead to simply take $m_{\tilde{f},\text{pole}} = m_{\tilde{f}}(m_{\tilde{f}})$, thus neglecting the difference between the pole and running mass for the squarks.

V. FINITE-TEMPERATURE ONE-LOOP EFFECTIVE POTENTIAL

The finite-temperature contributions to the effective potential are [3–7]

$$V_T = V_{T1} + V_{\text{ring}},\tag{8}$$

where

$$V_{T1} = \sum_i \frac{n_i}{2\pi^2} J_i\left(\frac{M_i^2}{T^2}\right)T^4,\tag{9}$$

$$V_{\text{ring}} = -\frac{T}{12\pi}\sum_{k=W_L,Z_L} n_k(\bar{M}_k^3 - M_k^3),\tag{10}$$

and

$$\bar{M}_k^2 = M_k^2 + \Pi_k.\tag{11}$$

The field-dependent fermion and scalar masses are

$$M_f(\phi)^2 = 2y_f^2\phi^2,\tag{12}$$

$$M_{\tilde{f}}(\phi)^2 = m_{\text{soft}}^2 + M_f(\phi)^2.\tag{13}$$

Explicit expressions for the thermal masses of the SM gauge bosons can be found in, e.g., Ref. [41]. The finite-temperature contributions depend on the thermal functions

$$J_{B,F}(y^2) = \int_0^\infty dx\,x^2 \ln\left[1 \mp \exp\left(-\sqrt{x^2 + y^2}\right)\right].\tag{14}$$

Often a high-temperature approximation is employed to estimate these integrals. In our case, due to the large Yukawa couplings, this approximation is generally not appropriate. Consequently, all computations given below evaluate the thermal functions $J_{B,F}$ numerically. The ring contribution (10) is only relevant for the longitudinal components of the W and Z. No contributions from scalars are included here since the squarks receive a contribution from soft supersymmetry breaking, and thus they remain heavy in the $\phi \to 0$ limit.

We have self-consistently included contributions in the finite-temperature effective potential the same as those included in the zero-temperature effective potential. Namely, we include contributions from t', b', t, their superpartners $\tilde{t}'_{L,R}$, $\tilde{b}'_{L,R}$, $\tilde{t}_{L,R}$, as well as the transverse and longitudinal components of W, Z.

In the standard model, the electroweak phase transition becomes second order when the Higgs mass approaches 70 GeV [42]. Qualitatively, this is because the transverse modes of W and Z, which would drive a first-order phase transition in the standard model with a lighter Higgs mass, develop a thermal mass from nonperturbative effects. If the transverse thermal masses are large, they effectively remove the cubic term from the finite-temperature potential when the effective potential is reset to zero at $\phi = 0$. In our model, the first-order phase transition is mostly driven by squarks. In fact, when the W and Z are neglected in our model, ϕ_c and T_c do not change significantly and the phase transition remains first order. Therefore, we do not expect nonperturbative effects encountered in the standard model at larger Higgs masses to significantly affect our calculations of the strength of the phase transition in the 4MSSM.

VI. EFFECTS OF NEW HEAVY PARTICLES

The effects of heavy particles (that receive their mass dominantly from electroweak symmetry breaking) on the electroweak phase transition can be broadly characterized as follows. Consider the effective potential at T_c, where there are two degenerate minima $V_{\text{eff}}(0, T_c) = V_{\text{eff}}(\phi_c, T_c)$ located at $\phi = 0$ and $\phi = \phi_c$. Now add to this a new particle that satisfies $M(\phi_c)/T_c \gg 1$. The phase transition strength is modified in two ways from the contributions of the new particle. One is through corrections to the finite-temperature contribution; the other is through the zero-temperature Coleman-Weinberg potential.

A. Finite-temperature effects

The contributions from bosons and fermions with masses larger than the critical temperature, $m \gg T_c$, can be characterized by how they contribute at large field values $\phi \gg T$ and small field values $\phi \ll T$. At large field values, we can take a *low temperature* approximation to the finite-temperature effective potential. In this limit, the contribution from fermions or bosons becomes

$$V_{T1}|_{T \ll m} = -|n|\left(\frac{M(\phi)}{2\pi T}\right)^{3/2}T^4\exp\left[-\frac{M(\phi)}{T}\right],\tag{15}$$

where n counts the number of degrees of freedom per boson or fermion with field-dependent mass $M(\phi)$. Clearly, when $M(\phi) \gg T$, which is equivalent to $\phi \gg T$ (with order one or larger Yukawa couplings), the contribution to the effective potential from fermions or bosons is exponentially suppressed.

At small field values, we can take a high-temperature approximation to the thermal contribution to the effective potential. The leading order contribution is the field-independent constant

$$V_{T1}|_{T\gg m} = -|n|c_{B,F}\frac{\pi^2}{90}T^4, \qquad (16)$$

where $c_{B,F} = (1, 7/8)$ for a boson or fermion contribution.

The combination of (15) and (16) implies that the introduction of a heavy fermion or boson causes a substantial negative shift in the potential at $\phi = 0$ while causing a negligible shift in the potential at $\phi = \phi_c$. As an illustration, we show in Fig. 2(a) the effect of adding one additional heavy fermionic degree of freedom ($n_f = -1$, for illustration) that obtains a mass of 400 GeV entirely from electroweak symmetry breaking. Readjusting the minimum $V_{\text{eff}}(\phi = 0) = 0$ shifts the potential up for all field values, thereby removing the second minimum at $\phi = \phi_c$, and thus restoring electroweak symmetry. We must *lower* the temperature further in order to have the second minimum reappear in the effective potential with the new heavy fermion or boson.

B. Zero-temperature effects

The second effect of heavy bosons and fermions is that they also modify the zero-temperature effective potential. Here, however, the effect of fermions and bosons is different. There are two contributions whose origin is ultimately the Coleman-Weinberg potential. One contribution is to the quartic coupling (5), while the second contribution is the quadratic term (6). For smaller field values, i.e., $\phi \lesssim \mu$, the dominant contribution is from the quadratic term. Since we choose $\mu \simeq m_{q'}$, the log term drops out of (6), giving an
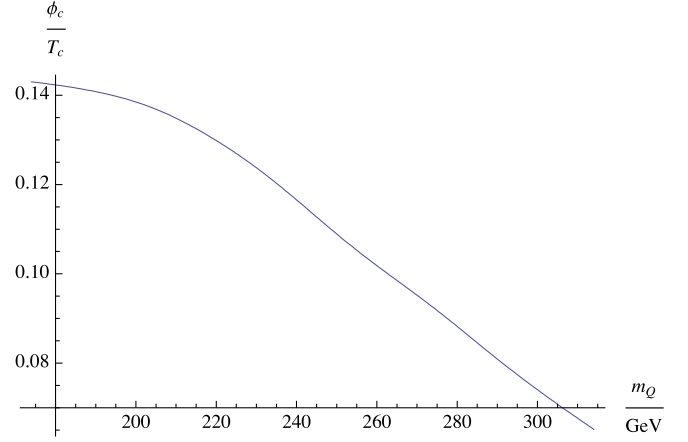
FIG. 3 (color online). This plot shows that the electroweak phase transition order parameter decreases if an additional heavy chiral quark is added to the theory without its corresponding superpartner.

overall *negative* (*positive*) contribution to the effective potential from fermions (bosons).

The negative contribution from fermions at modest field values actually overpowers the effect from the finite-temperature contributions discussed above. This is illustrated in Fig. 2(b). The net result is that introducing a new heavy chiral fermion causes a *decrease* in ϕ_c/T_c as the mass of the fermion is increased. The resulting decrease in the strength of the electroweak phase transition with one additional chiral fermion is shown in Fig. 3.

Adding bosonic contributions cancels the contribution from fermions in the Coleman-Weinberg potential. This cancellation is one-loop exact in the limit $m_{\tilde{f}'} = m_{f'}$, i.e., no SUSY breaking contribution to the scalar mass. This makes it clear that we need both heavy fermions *and*
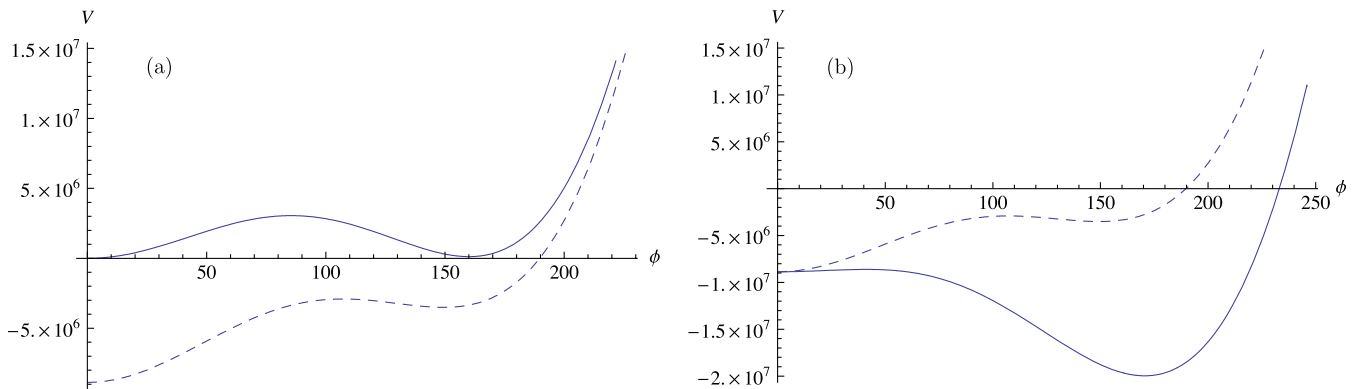
FIG. 2 (color online). An illustration of the effect of adding one additional heavy fermion that obtains a mass of 400 GeV entirely from electroweak symmetry breaking. The figure (a) on the left shows the effective potential at the critical temperature T_c (solid line) in the 4MSSM with $m_{t'} = m_{b'} = 400$ GeV and $m_{\tilde{q}'}/m_{q'} = 1.05$. Adding an additional heavy fermion (with $n = -1$) to the effective potential is shown (dashed line), except that *only* the finite-temperature contribution, V_{T1}, is included in (a). Figure (b) on the right shows the effect of including just the finite-temperature contribution (dashed line), identical to Figure (a), and then the effect of including both the finite temperature as well as the zero-temperature contribution V_1 (solid line). The net effect shown in Figure (b) solid line is that the global minimum at $\phi \simeq \phi_c$ decreases and thus T_c increases.
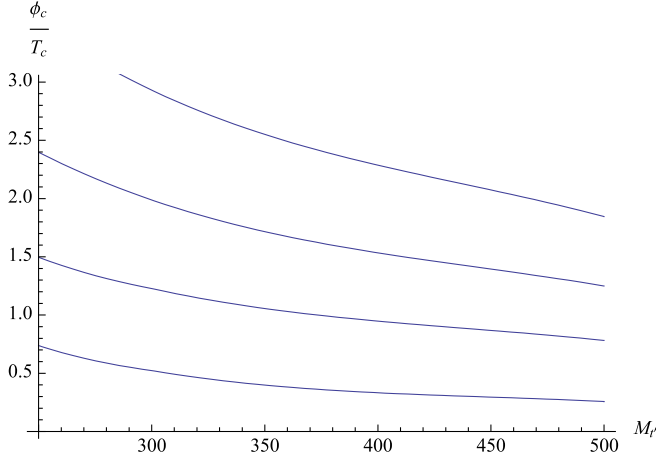
FIG. 4 (color online). Contour plot with the ratio $m_{\tilde{q}'}/m_{q'}$ fixed, from top to bottom, as 1.05, 1.1, 1.15, 1.2. The masses of the fourth generation quarks are taken to be equal.
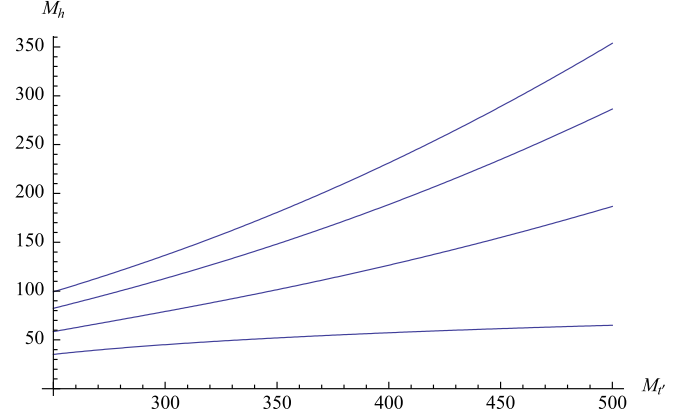

FIG. 5 (color online). Contour plot of the lightest Higgs mass as a function of the fourth generation quark mass. Each contour corresponds to a fixed ratio $m_{\tilde{q}'}/m_{q'}$, from top to bottom, 1.2, 1.1, 1.0, 0.95 with $m_{t'} = m_{b'}$ and $m_{\tilde{t}} = m_{\tilde{q}'}$.

scalars with equal numbers of degrees of freedom and similar masses to utilize the mechanism of Ref. [18] to lower ϕ_c/T_c.

Reference [18] estimated that order ten or more degrees of freedom is needed to sufficiently enhance the phase transition to achieve $\phi_c/T_c \gtrsim 1$. A fourth generation quark corresponds to adding 24 degrees of freedom. (We could have equivalently added degrees of freedom in other ways, such as several pairs of vectorlike supersymmetric lepton doublets that only get mass through the Higgs mechanism. This is another interesting possibility that we will not explore here [43].) We have calculated the strength of the phase transition for a range of quark and squark masses. The results are shown in Fig. 4.

VII. LIGHTEST HIGGS MASS IN THE 4MSSM

Given the parameter choice $\tan\beta = 1$, the tree-level Higgs potential vanishes, and thus the lightest Higgs mass also vanishes at leading order. It is well known that loop corrections from splitting the masses of the top quark from the stops in the Coleman-Weinberg potential provide large corrections to the tree-level value. In the 4MSSM, we can split not only the top and stops, but also split the fourth generation quarks from squarks. Since the one-loop contribution to the Higgs quartic coupling is proportional to y_f^4, even a small splitting between f and $\tilde{f}$ has a very important effect on the Higgs mass. A one-loop estimate of the lightest Higgs mass in the 4MSSM can be obtained by taking $d^2(V_0 + V_1)/d\phi^2$ at $\phi = v$. This gives our rough estimate for the Higgs mass

$$m_h^2 = \sum_{f=t,t',b'} \frac{3}{2\pi^2} \frac{m_f^4}{v^2} \ln\frac{m_{\tilde{f}}^2}{m_f^2}, \qquad (17)$$

where again $v = 246$ GeV and $m_{A^0,H,H^\pm} \gg m_h$. In Fig. 5 we show Higgs mass plotted against different $m_{t'} = m_{b'}$

masses, where all squark masses were taken to be degenerate $m_{\tilde{t}'} = m_{\tilde{b}'} = m_{\tilde{t}}$. Each contour has the fourth generation squark-to-quark mass ratio, $m_{\tilde{f}'}/m_{f'}$, fixed to the values shown. Clearly, when the splitting between the fourth generation squark and quark ($\overline{\text{MS}}$) masses vanishes, there is an insufficient one-loop contribution from top/stop loops to raise the Higgs mass much above about 60 GeV. Nevertheless, for even a small splitting between fourth generation squarks and quarks, one can easily obtain a one-loop contribution to the Higgs mass that far exceeds the LEP bound so long as $m_f \gtrsim 300$ GeV.

VIII. RESULTS

Combining our calculation of the phase transition with our calculation of the Higgs mass, we can find the allowed region in parameter space where the first order phase transition is strong $\phi_c/T_c \gtrsim 1$ while the Higgs mass satisfies the LEP bound $m_h > 115$ GeV. We have computed this for the mass ratio $m_{t'}/m_{b'} = 1$ in Fig. 6 and $m_{t'}/m_{b'} = 1.2$ in Fig. 7. In Fig. 6(a) we show contours of increasing ϕ_c/T_c, illustrating that it is straightforward to obtain the strength of the phase transition to significantly exceed $\phi_c/T_c = 1$. In Fig. 6(b) we show contours of increasing Higgs mass, illustrating that it is also straightforward to obtain a lightest Higgs mass that significantly exceeds $m_h = 115$ GeV.

Note that our plots are with respect to the pole masses of quarks and squarks (as well as the ratio $m_{\tilde{q}'}/m_{q'}$). The quantities that enter the effective potential are $\overline{\text{MS}}$-renormalized masses, which differ (as we discussed above) for fermion masses. Since the fermion pole mass is larger than its $\overline{\text{MS}}$-renormalized counterpart by about 5%, the ratio of pole masses can be as small as 0.95 while the ratio of $\overline{\text{MS}}$ masses is still larger than 1. This is why the fourth generation contributions to the Higgs mass (17)
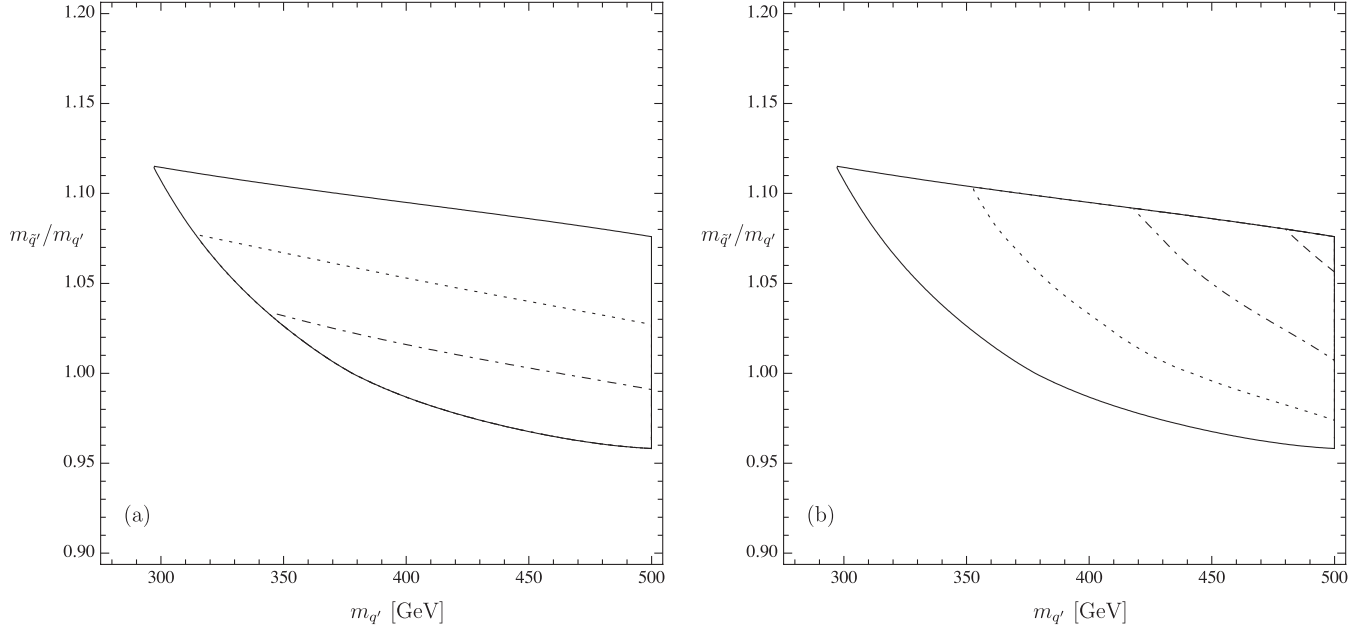
FIG. 6. The region in the quark/squark mass plane where the electroweak phase transition is strongly first order is shown. The regions shown in the left- and right-hand side figures (a) and (b) are identical: the upper boundary (the solid line) is determined by $\phi_c/T_c = 1$ while the lower boundary is determined by $m_h = 115$ GeV. The dotted and dashed contours on the left-hand side figure (a) correspond to $\phi_c/T_c = 1.5, 2.0$, respectively. The dotted, dot-dashed, and dotted contours on the right-hand side figure (b) correspond to the Higgs masses $m_h = 150, 200, 250$ GeV, respectively.

remain positive even when the pole mass ratio $m_{\tilde{q}'}/m_{q'}$ is smaller than 1.

These results suggest that even though only one-loop approximations for the effective potential and the Higgs mass calculation were employed, we are not near any

critical boundary, and so a more refined calculation is expected to only modestly adjust the parameter regions we have shown. For instance, there are several effects that can increase the Higgs mass to values higher than our calculation. One is moving away from the parameter
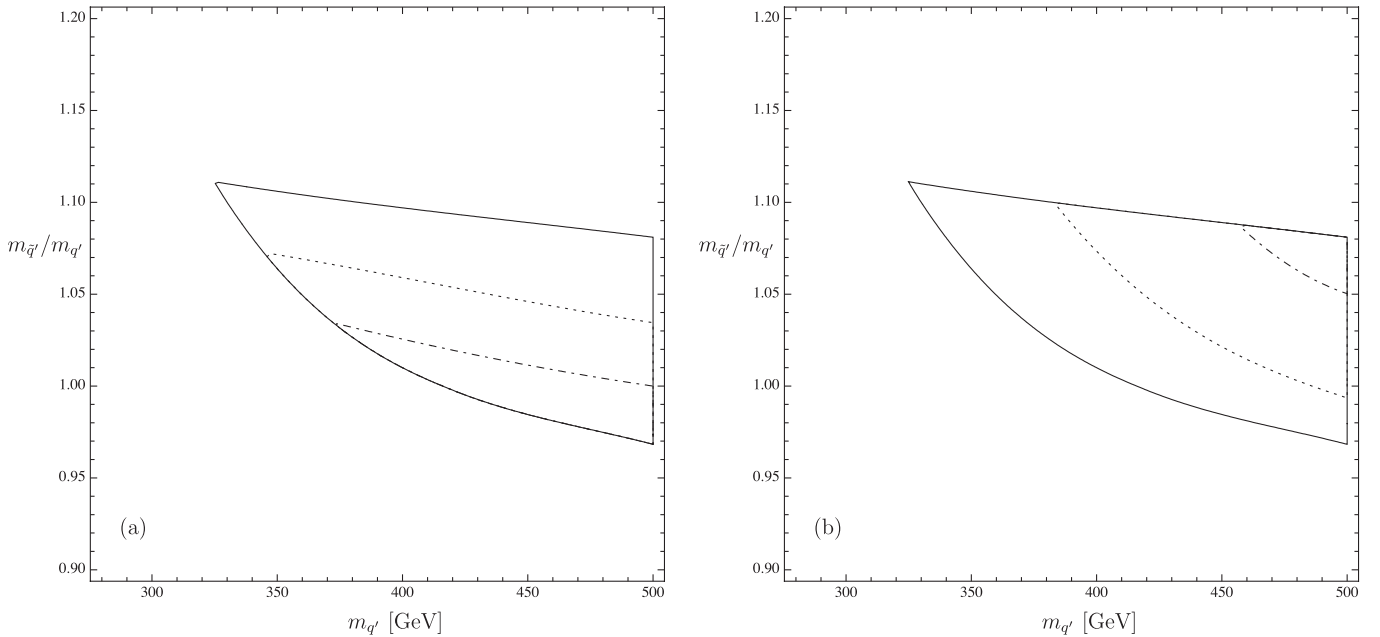

FIG. 7. The same as Fig. 6 but we take $m_{t'}/m_{b'} = 1.2$, as favored by electroweak precision data. The basic shape and size of the region is the same, illustrating that our results are not particularly sensitive to the heavy fourth generation quark mass ratios.

choice $A_f = \mu$, where the off-diagonal contribution from squarks provides a modest increase. Another is separating the third generation squark masses from the fourth generation squark masses. The latter effect is also modest: in Fig. 5, the contour 0.95 corresponds to just the contribution from the third generation, and one can see that there is a small increase as the squark mass (equal to the fourth generation quark mass) increases. Similar statements also hold for the finite-temperature effective potential, since again our results show that there are model parameters where the 4MSSM model has an electroweak phase transition with ϕ_c/T_c that is well above the critical first order boundary $\simeq 1$.

IX. CONCLUSIONS

We have calculated the strength of the electroweak phase transition in a supersymmetric model with four chiral generations. We find there is an intriguing region of parameter space, with fourth generation quarks heavier than about 300 GeV and the squark-to-quark mass ratio $1 \lesssim m_{\tilde{q}'}/m_{q'} \lesssim 1.1$, where $\phi_c/T_c > 1$. Within this region of parameter space we showed the Higgs can be easily heavier than the LEP bound of 115 GeV.

This suggests that a viable model of electroweak baryogenesis could indeed be a low energy supersymmetric model with a fourth generation of chiral fermions. The main drawback to this approach is the appearance of a Landau pole in the Yukawa couplings below about 10 TeV, requiring a UV completion. What we have shown is the strength of the first-order phase transition can be large enough to prevent the washout of a baryon asymmetry. This model also has several new sources of CP violation,

ubiquitous in low energy supersymmetry, that could be used to satisfy Sakharov's CP violation criteria. Examples of sufficient CP violation that have been employed in other supersymmetric electroweak baryogenesis scenarios [9–11] include the phase of the Higgsino mass parameter μ as well as the gaugino mass parameters $M_{1,2}$.

It is *coincidental* that the region of parameter space where the first order transition is strong enough combined with obtaining a large enough Higgs mass (taking $\tan\beta = 1$) happens to be just beyond the current Tevatron direct search bounds [37,38]. If we are lucky, the Tevatron could begin to see evidence for new physics in the form of both an extra chiral generation as well as superpartners in the very near future. The LHC, however, can easily cover this parameter space. Indeed, the mechanism to enhance the first order phase transition described here is expected to be found or ruled out with only a modest amount of data from the LHC.

Note added:—As this paper was being completed, Ref. [44] appeared, speculating that the electroweak phase transition could be enhanced with a fourth generation *without* supersymmetry. Unfortunately, this does not work, as we show in Fig. 3 where just adding fermions actually *decreases* ϕ_c/T_c because of the effects of the fermions on the zero-temperature effective potential.

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