

Signals of unparticles in low energy parity violation and the NuTeV experiment

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(Received 29 January 2008; revised manuscript received 31 August 2008; published 16 October 2008)

We have studied the possible signals of unparticle in atomic parity violation (APV) along an isotope chain and in the NuTeV experiment. The effects of unparticle physics could be observed in APV, if the uncertainty in relative neutron/proton radius shift $\delta(\Delta \frac{R_N}{R_p})$ is less than a few times 10^{-4} by measuring the parity violating electron scattering. The constraints imposed by the NuTeV experiment on unparticle physics are discussed in detail. If the NuTeV results are confirmed by future experiments, we suggest that unparticle could account for a part of the NuTeV anomaly. There exist certain regions for the unparticle parameters (Λ_U , d_U , c_{VU} , and c_{AU}), where the NuTeV discrepancy could be completely explained by unparticle effects and the strange quark asymmetry, even with or without the contributions from the isoscalarity violation, etc. It is remarkable that these parameter regions are consistent with the constraints from $b \rightarrow s\gamma$.

DOI: 10.1103/PhysRevD.78.075015

PACS numbers: 14.80.-j, 12.38.-t, 12.90.+b, 13.60.Hb

I. INTRODUCTION

Recently, H. Georgi suggests that the scale invariant stuff may exist in our world, which is made of unparticles [1,2]. The unparticle weakly couples to the standard model (SM) matter, which is suppressed by the inverse of the large energy scale from the view of the effective theory. This scenario serves as a possible component of the physics beyond the standard model above the TeV scale, which can be explored experimentally at the LHC and ILC in the future. Although conformal symmetry is widely and deeply studied in the condense matter physics (mostly in two dimension) and the superstring theory (generally in high dimension and with the supersymmetry), we know little about the conformal symmetry in four dimension and how it is manifested in particle physics experiments, if unparticle really exists.

Unparticle is very different from the ordinary matters, some exotic and remarkable properties have been demonstrated, i.e., it looks like a nonintegral number d_U of invisible massless particles with unusual phase space, where d_U is the scale dimension of the relevant unparticle operator \mathcal{O}_U . The peculiar two-point correlation function of unparticle can produce unusual interference patterns in the timelike region and so on [1,2]. The phenomenological consequences of unparticle and its possible signatures have been discussed by several authors. In a previous paper [3], we have discussed the virtual effects of unparticle in the deep inelastic scattering processes to the lowest nontrivial order of the couplings of the unparticle with the SM fields. The leading order corrections to the structure functions are the interference terms between the vector unparticle exchange amplitudes and the standard model amplitudes in

the spacelike region [3]. Unparticle production in $Z \rightarrow f\bar{f}U$ and $e^+e^- \rightarrow \gamma U$, unparticle effects in the Drell-Yan process, fermionic unparticle, its physical effects on muon anomalous magnetic moments, $B^0 - \bar{B}^0$ mixing and $D^0 - \bar{D}^0$ mixing, the lepton flavor violating processes, as well as the bounds on unparticle from the Higgs sector and the deconstruction of unparticle, etc. have also been investigated [4–20].

As we have shown in Ref. [3], the low energy effective interactions between the SM fermions and unparticle can take the following form:

$$\frac{1}{\Lambda_U^{d_U-1}} \bar{f} \gamma_\mu (c_{VU} + c_{AU} \gamma_5) f \mathcal{O}_U^\mu, \frac{1}{\Lambda_U^{d_U-1}} \bar{f} (c_{SU} + i c_{PU} \gamma_5) f \mathcal{O}_U \quad (1)$$

where d_U is the scale dimension of the relevant unparticle operator (\mathcal{O}_U^μ or \mathcal{O}_U), and \mathcal{O}_U^μ is a transverse and Hermitian operator. c_{VU} , c_{AU} , c_{SU} , and c_{PU} are dimensionless coupling constants. Those effective interactions are the leading order interactions between the SM fermions and the vector (scalar) unparticle, and universal couplings have been assumed here.

Scale invariance determines the two-point correlation function of the unparticle operator to be

$$\int d^4x e^{iP \cdot x} \langle 0 | \mathcal{O}_U(x) \mathcal{O}_U^\dagger(0) | 0 \rangle = \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{i}{(-P^2 - i\epsilon)^{2-d_U}} \quad (2)$$

$$\int d^4x e^{iP \cdot x} \langle 0 | \mathcal{O}_U^\mu(x) \mathcal{O}_U^{\nu\dagger}(0) | 0 \rangle = \frac{A_{d_U}}{2 \sin(d_U \pi)} \frac{i(-g^{\mu\nu} + P^\mu P^\nu / P^2)}{(-P^2 - i\epsilon)^{2-d_U}} \quad (3)$$

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where

$$A_{d_u} = \frac{16\pi^{5/2}}{(2\pi)^{2d_u}} \frac{\Gamma(d_u + \frac{1}{2})}{\Gamma(d_u - 1)\Gamma(2d_u)}. \quad (4)$$

In this work, we shall study the particular aspects of unparticle physics in atomic parity violation (APV) along an isotope chain and in the NuTeV experiment. Since both the APV and NuTeV experiments are important low energy probes to new physics beyond SM, which complements new physics searches at high energy colliders (such as LHC and ILC). Therefore, they should play an important role in exploring unparticle physics together with LHC and ILC in the future. The paper is organized as follows. In Sec. II, the physics effects of unparticle in APV along an isotope chain and the associated theoretical uncertainties are investigated. We concentrate on the possible signals of unparticle in the NuTeV experiment in Sec. III. Finally, we present the summary and our conclusions.

II. UNPARTICLE IN LOW ENERGY PARITY VIOLATION

Low energy parity violation observables have played an important role in uncovering and testing the structure of the electroweak sector of the standard model. They provide important information for physics beyond SM, and place stringent constraints on a variety of new physics scenarios. Low energy parity violation and collider experiments (such as LHC and ILC) provide powerfully complementary probes of new physics at the TeV scale.

The basic quantity of interest in considering weak neutral-current parity violation is the so-called weak charge Q_W . This quantity is the weak neutral-current analog of the electron-magnetic charge, which characterizes the strength of the electron axial vector times nucleus (or electron) vector weak neutral-current interaction. The most precise determination of Q_W has been obtained with the atomic parity violation in Cs (caesium) by the Boulder group [21].

There are generally large theoretical uncertainties from atomic structures for the APV observables of a single isotope. A strategy for evading these atomic structure uncertainties is to measure the ratios of parity violation observables along an isotope chain [22], and the isotopes of Ba (Barium) and Yb (Ytterbium) are currently under study at Seattle and Berkeley, respectively. Two ratios are usually considered

$$R_1 = \frac{A_{\text{PV}}^{\text{NSID}}(N') - A_{\text{PV}}^{\text{NSID}}(N)}{A_{\text{PV}}^{\text{NSID}}(N') + A_{\text{PV}}^{\text{NSID}}(N)} \quad (5)$$

$$R_2 = \frac{A_{\text{PV}}^{\text{NSID}}(N')}{A_{\text{PV}}^{\text{NSID}}(N)} \quad (6)$$

where $A_{\text{PV}}^{\text{NSID}}(N)$ is the nuclear spin-independent part of the APV observable for an atom with neutron number N , which

is given by

$$A_{\text{PV}}^{\text{NSID}}(N) = \xi Q_W \quad (7)$$

where ξ contains all the atomic structure effects for a point nucleus including the many-body corrections. For a single isotope, a significant source of theoretical uncertainties arises from the computation of the atomic structure dependent constant ξ . It was shown in Ref. [23] that taking ratios between isotopes cancels essentially all dependence on the atomic structure and the associated uncertainties. However, the nuclear structure dependence does not cancel in the ratios, especially the ratios would receive corrections from the neutron distribution. The dependence of the parity violation amplitude on the nuclear structure can be incorporated through the corrections to the weak charge, then

$$Q_W = Q_W^0 + Q_W^{\text{nucl}} + Q_W^u \quad (8)$$

where Q_W^0 is the nuclear weak charge in the standard model. Including the effects of radiative corrections, Q_W^0 becomes [24]

$$Q_W^0 = (0.9857 \pm 0.0004)(1 + 0.00782T) \times \{-N + Z[1 - (4.012 \pm 0.010)\bar{x}]\} \quad (9)$$

where $\bar{x} \equiv \sin^2\theta_w$. \bar{x} is defined at the mass scale m_Z by the modified minimal subtraction and is given by [24]

$$\bar{x} = 0.2323 + 0.00365S - 0.00261T. \quad (10)$$

In Eq. (9) and (10), S is the Peskin-Takeuchi parameter characterizing the isospin-conserving quantum loop corrections, and T characterizes the isospin-breaking corrections [25]. Q_W^u is the correction to the weak charge induced by unparticle, which can be straightforwardly calculated [26]

$$Q_W^u = \frac{24}{\Lambda_{\mathcal{U}}^2 G_F} \frac{(2\pi)^{3/2-2d_{\mathcal{U}}}\Gamma(d_{\mathcal{U}} + 1/2)}{\Gamma(d_{\mathcal{U}})(2d_{\mathcal{U}} - 1)} \times (Z + N)c_{V\mathcal{U}}c_{A\mathcal{U}}. \quad (11)$$

The scalar and pseudoscalar couplings $c_{S\mathcal{U}}$, $c_{P\mathcal{U}}$ between unparticle and SM fermions do not contribute to the weak charge of nuclei. These two couplings are forbidden by the SM symmetry if unparticle $\mathcal{O}_{\mathcal{U}}$ is a SM singlet. The nuclear structure corrections are contained in Q_W^{nucl} [22,23], which is given by

$$Q_W^{\text{nucl}} \simeq -N(q_n - 1) + Z(1 - 4\bar{x})(q_p - 1) \quad (12)$$

where $q_p = \int \rho_p(r)f(r)d^3r$, $q_n = \int \rho_n(r)f(r)d^3r$, $\rho_p(r)$, and $\rho_n(r)$ are, respectively, the proton and neutron distributions within atomic nuclei, and $f(r)$ is the spatial variation of the electron wave function inside the nucleus. In order to get an idea of the relative importance of unparticle contributions and nuclear structure effects, we follow Ref. [23] and consider a simple model in which the nucleus is treated as a sphere of uniform proton and neutron number densities out radii R_p and R_n , respectively. In this case, one obtains [23]

$$\begin{aligned} q_n &= 1 - (Z\alpha)^2 g(r_N) + \mathcal{O}((Z\alpha)^4) \\ q_p &= 1 - 0.260(Z\alpha)^2 + \mathcal{O}((Z\alpha)^4) \end{aligned} \quad (13)$$

where

$$g(r_N) = \frac{3}{10} r_N^2 - \frac{3}{70} r_N^4 + \frac{1}{450} r_N^6, \quad r_N = \frac{R_N}{R_P}. \quad (14)$$

The above estimate of q_n yields the same general trend as the detailed calculations from the Skyrme model, relativistic or nonrelativistic Hartree-Fock nuclear calculations, and the mean field methods, etc., with the absolute values differing generally by parts in a thousand or less [23]. After including both the unparticle and the nuclear structure corrections into the ratios of APV, we have

$$R_1 \simeq \frac{Q_W(N') - Q_W(N)}{Q_W(N') + Q_W(N)} = R_1^0(1 + \delta_1^{\text{nucl}} + \delta_1^{\text{u}}) \quad (15)$$

$$R_2 \simeq \frac{Q_W(N)}{Q_W(N)} = R_2^0(1 + \delta_2^{\text{nucl}} + \delta_2^{\text{u}}) \quad (16)$$

where

$$R_1^0 = \frac{Q_W^0(N') - Q_W^0(N)}{Q_W^0(N') + Q_W^0(N)} \simeq \frac{\Delta N}{N + N' - 2(1 - 4.012\bar{x})Z} \quad (17)$$

$$R_2^0 = \frac{Q_W^0(N')}{Q_W^0(N)} \simeq \frac{N' - (1 - 4.012\bar{x})Z}{N - (1 - 4.012\bar{x})Z} \quad (18)$$

which give the ratios in the standard model at the leading order. δ_1^{nucl} and δ_2^{nucl} denote the nuclear structure corrections to R_1 and R_2 , respectively, δ_1^{u} and δ_2^{u} are the corresponding corrections arising from unparticle with $N' = N + \Delta N$. Dropping the terms containing the factor $1 - 4.012\bar{x}$, which should be quite negligible due to the accidental value of $\bar{x} \approx \frac{1}{4}$, then we obtain

$$\delta_1^{\text{nucl}} = -(Z\alpha)^2 \frac{N'}{\Delta N} f'(r_N) \Delta r_N \quad (19)$$

$$\delta_1^{\text{u}} = \frac{24}{\Lambda_{\text{u}}^2 G_F} \frac{(2\pi)^{3/2-2d_{\text{u}}}\Gamma(d_{\text{u}} + 1/2)}{\Gamma(d_{\text{u}})(2d_{\text{u}} - 1)} \frac{2Z}{N + N'} c_{\text{VU}} c_{\text{AU}} \quad (20)$$

$$\delta_2^{\text{nucl}} = -(Z\alpha)^2 f'(r_N) \Delta r_N \quad (21)$$

$$\delta_2^{\text{u}} = \frac{24}{\Lambda_{\text{u}}^2 G_F} \frac{(2\pi)^{3/2-2d_{\text{u}}}\Gamma(d_{\text{u}} + 1/2)}{\Gamma(d_{\text{u}})(2d_{\text{u}} - 1)} \frac{Z}{N} \frac{\Delta N}{N'} c_{\text{VU}} c_{\text{AU}} \quad (22)$$

where $\Delta r_N = (R_{N'} - R_N)/R_P$, which denotes the shift in the neutron radius (relative to the proton radius) along the isotope chain. It is obvious that δ_2^{u} contains an explicit suppression factor $\Delta N/N'$, therefore δ_1^{u} is more sensitive to unparticle physics than δ_2^{u} for a given experimental precision. From Eqs. (19)–(22), we can see that both δ_1^{u} and δ_2^{u} depend on the product of c_{VU} and c_{AU} , and the dependence of δ_1^{nucl} on the variation in neutron radius along the isotope is enhanced by a factor $N'/\Delta N$. Thus, if one uses the ratios of APV in different isotopes to learn

about the new physics contributions from unparticle, one should have extremely precise knowledge of the shift in neutron radius.

So far the proton distribution is well-known from electric probes: electron and muon scattering, optical isotope shifts, muonic atoms, etc. Whereas there exist no reliable experimental determinations of the neutron distribution so far. Explicit studies of isotope shift Δr_N associated with $\rho_n(r)$ have been reported in Refs. [23,27], these authors showed that the spread in the predictions of Δr_N corresponds to a 100% uncertainties in the model average for Δr_N . The detailed knowledge of neutron distribution is crucial to APV. A model-independent experimental determination of the neutron distribution $\rho_n(r)$ can be achieved by parity violating electron scattering [28]. A precise determination of ρ_n using parity violating electron scattering would sufficiently constrain model calculations so as to significantly reduce the theoretical isotope shift uncertainties.

The uncertainties in the neutron distribution induce uncertainties in the ratios R_1 and R_2 , which is given by

$$\begin{aligned} \delta(\delta_1^{\text{nucl}}) &= -(Z\alpha)^2 \frac{N'}{\Delta N} [f'(r_N)\delta(\Delta r_N) + f''(r_N)\delta r_N \Delta r_N] \\ \delta(\delta_2^{\text{nucl}}) &= -(Z\alpha)^2 [f'(r_N)\delta(\Delta r_N) + f''(r_N)\delta r_N \Delta r_N] \end{aligned} \quad (23)$$

where δr_N are the uncertainties in r_N . From the view of extracting the unparticle physics limits, the impact of neutron distribution uncertainties is characterized by the ratio between $\delta(\delta_i^{\text{nucl}})$ and the unparticle physics corrections δ_i^{u} ($i = 1, 2$). The smaller the size of this ratio is, the less problematic the neutron distribution uncertainties become. Straightforwardly, we have

$$\begin{aligned} \delta(\delta_1^{\text{nucl}})/\delta_1^{\text{u}} &\simeq -(Z\alpha)^2 \frac{N'}{\Delta N} \frac{N + N'}{2Z} \frac{\Gamma(d_{\text{u}})(2d_{\text{u}} - 1)}{(2\pi)^{3/2-2d_{\text{u}}}\Gamma(d_{\text{u}} + \frac{1}{2})} \\ &\quad \times \frac{f'(r_N)\delta(\Delta r_N)\Lambda_{\text{u}}^2 G_F}{24c_{\text{VU}}c_{\text{AU}}} \\ \delta(\delta_2^{\text{nucl}})/\delta_2^{\text{u}} &\simeq -(Z\alpha)^2 \frac{N'}{\Delta N} \frac{N}{Z} \frac{\Gamma(d_{\text{u}})(2d_{\text{u}} - 1)}{(2\pi)^{3/2-2d_{\text{u}}}\Gamma(d_{\text{u}} + \frac{1}{2})} \\ &\quad \times \frac{f'(r_N)\delta(\Delta r_N)\Lambda_{\text{u}}^2 G_F}{24c_{\text{VU}}c_{\text{AU}}} \end{aligned} \quad (24)$$

where the terms containing $\delta r_N \Delta r_N$ have been dropped, since generally one has $\delta r_N \Delta r_N \ll \delta(\Delta r_N)$ [23]. It is obvious that $\delta(\delta_1^{\text{nucl}})/\delta_1^{\text{u}} \simeq \delta(\delta_2^{\text{nucl}})/\delta_2^{\text{u}}$. Although R_1 is more sensitive to unparticle physics by $\frac{N'}{\Delta N}$ as compared to R_2 , it is approximately sensitive to the neutron distribution uncertainties by the same factor as well. In order to constraint unparticle physics from APV along an isotope, the uncertainties due to neutron distribution at least should be smaller than the unparticle contributions. For demonstration, the lower bound on the absolute value of $c_{\text{VU}}c_{\text{AU}}$ as a

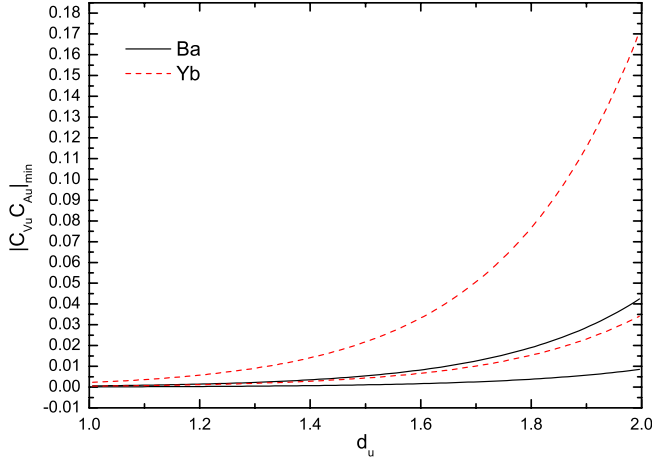


FIG. 1 (color online). The lower bound on $|c_{\nu U} c_{AU}|$ as a function of d_U imposed by extracting unparticle physics from APV along the Ba and Yb isotope. The two upper curves correspond to the uncertainty $\delta(\Delta r_N) = 1.0 \times 10^{-4}$, and the two lower curves correspond to $\delta(\Delta r_N) = 5.0 \times 10^{-4}$.

function of d_U for the Ba and Yb isotope are displayed in Fig. 1.

III. UNPARTICLE AND NUTEV ANOMALY

A few years ago, the NuTeV collaboration at Fermilab measured the value of the Weinberg angle $\sin^2 \theta_w$ in deep inelastic scattering on nuclear target with both neutrino and antineutrino beams. Having considered and examined various sources of systematic errors, the NuTeV Collaboration reported the value $\sin^2 \theta_w = 0.2277 \pm 0.0013(\text{stat}) \pm 0.0009(\text{syst})$ [29]. This result is about 3σ deviation from the world average value $\sin^2 \theta_w = 0.2227 \pm 0.00037$, which is measured in other electroweak processes [30]. A number of possible solutions to this deviation have been proposed, such as strange-antistrange asymmetry [31], tiny violation of isospin symmetry in parton distribution functions [32], nuclear physics effects [33], the effects of the next-to-leading order QCD corrections [34], and the new physics beyond the SM. For review, please see Ref. [35]. So far the NuTeV anomaly is still an open question. In the following, we will explore the possible effects of unparticle in the NuTeV experiment and the constraints on the unparticle parameters, if the NuTeV results are confirmed in future.

The NuTeV Collaboration measured the value of $\sin^2 \theta_w$ by using the ratios of the neutral-current to charged-current total cross section on iron for neutrino and antineutrino, respectively. This procedure is closely related the Paschos-Wolfenstein (PW) relation [36]

$$R^- = \frac{\sigma_{\text{NC}}^{\nu N} - \sigma_{\text{NC}}^{\bar{\nu} N}}{\sigma_{\text{CC}}^{\nu N} - \sigma_{\text{CC}}^{\bar{\nu} N}} = \frac{1}{2} - \sin^2 \theta_w \quad (25)$$

where $\sigma_{\text{CC}}^{\nu N}$ and $\sigma_{\text{NC}}^{\nu N}$ are, respectively, the charged-current

(CC) and neutral-current (NC) cross sections in neutrino-nucleon deep inelastic scattering processes, and $\sigma_{\text{CC}}^{\bar{\nu} N}$ and $\sigma_{\text{NC}}^{\bar{\nu} N}$ are corresponding antineutrino cross sections. The relationship Eq. (25) is based on the assumptions of isoscalar target and strange-antistrange symmetry of the nuclear sea. Later the corrections from unparticle physics and strange quark asymmetry would be included, nuclear physics effects and nonisoscalar corrections, etc., are discussed as well.

In Ref. [3], we have demonstrated that the cross sections for neutrino- and antineutrino-nucleon neutral-current interactions have the following form:

$$\frac{d^2 \sigma_{\text{NC}}^{\nu(\bar{\nu})A}}{dx dy} = s \frac{G_F^2}{2\pi} \left(\frac{M_Z^2}{Q^2 + M_Z^2} \right)^2 \left[xy^2 F_1^{A,\text{NC}} + (1-y) F_2^{A,\text{NC}} \right. \\ \left. \pm y \left(1 - \frac{1}{2} y \right) x F_3^{A,\text{NC}} \right] \quad (26)$$

where G_F is the Fermi constant $G_F \approx 1.166 \times 10^{-5} \text{ GeV}^2$ [30], $Q^2 = -q^2$, $x = Q^2/2p \cdot q$, $y = p \cdot q/p \cdot k$, and $s = (k+p)^2$ are the standard Bjorken deep inelastic scattering variables for the four momentum $k(p)$ of the initial state neutrino or antineutrino (nucleon), and A denotes the target. At the leading order, the structure functions $F_i^{A,\text{NC}}(x, Q^2)$ ($i = 1, 2, 3$) are expressed in terms of the quark and antiquark distributions as follows,

$$F_1^{A,\text{NC}}(x, Q^2) = \sum_q [q^A(x) + \bar{q}^A(x)] B_q^\nu(Q^2) \\ F_2^{A,\text{NC}}(x, Q^2) = \sum_q x [q^A(x) + \bar{q}^A(x)] C_q^\nu(Q^2) \\ F_3^{A,\text{NC}}(x, Q^2) = \sum_q [q^A(x) - \bar{q}^A(x)] D_q^\nu(Q^2) \quad (27)$$

with

$$B_q^\nu(Q^2) = \frac{V_q^2 + A_q^2}{2} + \frac{(c_{VU}^2 + c_{AU}^2)^2}{2G_F^2 M_Z^4 \Lambda_U^{4d_U-4}} R_{U2}^2 \\ + \frac{(V_q c_{VU} - A_q c_{AU})(c_{VU} - c_{AU})}{\sqrt{2} G_F M_Z^2 \Lambda_U^{2d_U-2}} R_{U2} \\ + \frac{(c_{SU}^2 - c_{PU}^2)^2}{4G_F^2 M_Z^4 \Lambda_U^{4d_U-4}} R_{U2}^2 \quad (28)$$

$$C_q^\nu(Q^2) = V_q^2 + A_q^2 + \frac{(c_{VU}^2 + c_{AU}^2)^2}{G_F^2 M_Z^4 \Lambda_U^{4d_U-4}} R_{U2}^2 \\ + \frac{\sqrt{2}(V_q c_{VU} - A_q c_{AU})(c_{VU} - c_{AU})}{G_F M_Z^2 \Lambda_U^{2d_U-2}} R_{U2} \quad (29)$$

$$D_q^\nu(Q^2) = 2V_q A_q + \frac{4c_{VU}^2 c_{AU}^2}{G_F^2 M_Z^4 \Lambda_U^{4d_U-4}} R_{UZ}^2 - \frac{\sqrt{2}(V_q c_{AU} - A_q c_{VU})(c_{VU} - c_{AU})}{G_F M_Z^2 \Lambda_U^{2d_U-2}} R_{UZ} \quad (30)$$

$$R_{UZ} = \frac{A_{d_U}}{2 \sin(d_U \pi)} (Q^2)^{d_U-2} (Q^2 + M_Z^2) \quad (31)$$

here $V_q = T_{3q} - 2Q_q \sin^2 \theta_w$, $A_q = T_{3q}$, Q_q , and T_{3q} are, respectively, the electric charge and the third component of the weak isospin of the quark q , and $A_{d_U} = \frac{16\pi^{5/2}}{(2\pi)^{2d_U}} \times \frac{\Gamma(d_U+1/2)}{\Gamma(d_U-1)\Gamma(2d_U)}$, which is the normalization factor of the state density for the unparticle stuff [1].

The neutrino- and antineutrino-nucleus charged-current cross sections are expressed in a similar manner,

$$\frac{d^2 \sigma_{CC}^{\nu(\bar{\nu})A}}{dx dy} = s \frac{G_F^2}{2\pi} \left(\frac{M_W^2}{Q^2 + M_W^2} \right)^2 \left[xy^2 F_1^{\nu(\bar{\nu})A,CC} + (1-y) \times F_2^{\nu(\bar{\nu})A,CC} \pm y \left(1 - \frac{1}{2}y \right) x F_3^{\nu(\bar{\nu})A,CC} \right]. \quad (32)$$

The above charge-current structure functions are given by:

$$F_1^{\nu A,CC}(x, Q^2) = \sum_{i,j} [d_j^A(x) + \bar{u}_i^A(x)] |(V_{CKM})_{ij}|^2 \quad (33)$$

$$F_2^{\nu A,CC}(x, Q^2) = 2x F_1^{\nu A,CC}(x, Q^2) \quad (34)$$

$$F_3^{\nu A,CC}(x, Q^2) = 2 \sum_{ij} [d_j^A(x) - \bar{u}_i^A(x)] |(V_{CKM})_{ij}|^2 \quad (35)$$

$$F_1^{\bar{\nu} A,CC}(x, Q^2) = \sum_{i,j} [u_i^A(x) + \bar{d}_j^A(x)] |(V_{CKM})_{ij}|^2 \quad (36)$$

$$F_2^{\bar{\nu} A,CC}(x, Q^2) = 2x F_1^{\bar{\nu} A,CC}(x, Q^2) \quad (37)$$

$$F_3^{\bar{\nu} A,CC}(x, Q^2) = 2 \sum_{ij} [u_i^A(x) - \bar{d}_j^A(x)] |(V_{CKM})_{ij}|^2 \quad (38)$$

where V_{CKM} is the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and i, j is the generation index. In the NuTeV experiment, the average value of Q^2 is about 20 GeV² ($Q^2 \ll M_Z^2, M_W^2$). Substituting Eqs. (26)–(38) into the right-hand side of Eq. (25), to leading order of the small coupling constants c_{VU} , c_{AU} , c_{SU} , and c_{PU} , we obtain the modified PW relation

$$R^- = \frac{1}{2} - \sin^2 \theta_w - \delta R^- \quad (39)$$

where the correction δR^- is

$$\delta R^- \simeq \left\{ \left(1 - \frac{7}{3} \sin^2 \theta_w \right) S_\nu + \frac{\sqrt{2} R_{UZ} (c_{VU} - c_{AU})}{G_F M_Z^2 \Lambda_U^{2d_U-2}} \times \left(-\frac{1}{3} \sin^2 \theta_w c_{AU} Q_\nu + \left[\left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w \right) c_{AU} + \frac{1}{2} c_{VU} \right] S_\nu \right) \right\} / (Q_\nu + 3S_\nu) \quad (40)$$

where $Q_\nu = \int_0^1 x [u^p(x) + d^p(x) - \bar{u}^p(x) - \bar{d}^p(x)] dx$ and $S_\nu = \int_0^1 x [s^p(x) - \bar{s}^p(x)] dx$. Since a small strange-antistrange asymmetry could be responsible for a significant fraction of the observed discrepancy, we have explicitly included this asymmetry in our analysis. From the analysis of the parton distributions [37], we choose $Q_\nu \approx 0.36$ following the authors in Ref. [35], which is better than 10% accuracy in the energy range of the NuTeV experiment. While the situation about S_ν is not so clear until now. In Ref. [38], a global fit to all available neutrino data found evidence in favor of the strange sea asymmetry. The CTEQ group [39] has performed a global QCD analysis by including the dimuon data from the CCFR and NuTeV Collaborations [40], they found that a large negative S_ν is strongly disfavored by both dimuon and other inclusive data, the strange asymmetry was in the range $-0.001 < S_\nu < 0.004$ and the most likely value was $S_\nu \sim 0.002$. Theoretically, phenomenological model calculations suggest $S_\nu \approx 0.002$ [41], which seems compatible with all the present experimental information. Since we cannot unambiguously fix the asymmetry parameter S_ν from both the global data fit and theoretical calculations, the S_ν dependence of δR^- would be illustrated later.

As is shown in Eq. (40), the correction to the PW relation induced by unparticle and the strange-antistrange asymmetry sensitively depends on the scale dimension d_U and the unparticle scale Λ_U . For illustration, we display δR^- as a function of d_U for $S_\nu = -0.001, 0.001, 0.002$, and

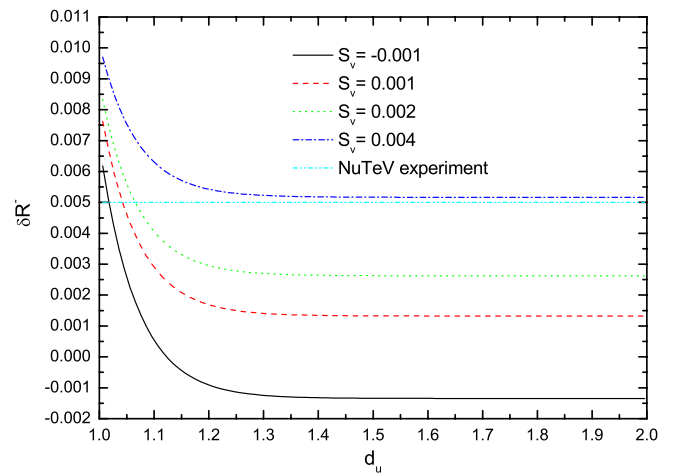


FIG. 2 (color online). The dependence of δR^- on d_U for $S_\nu = -0.001, 0.001, 0.002$, and 0.004 , respectively, with $\Lambda_U = 1$ TeV, $c_{VU} = 0.01$, and $c_{AU} = 0.002$.

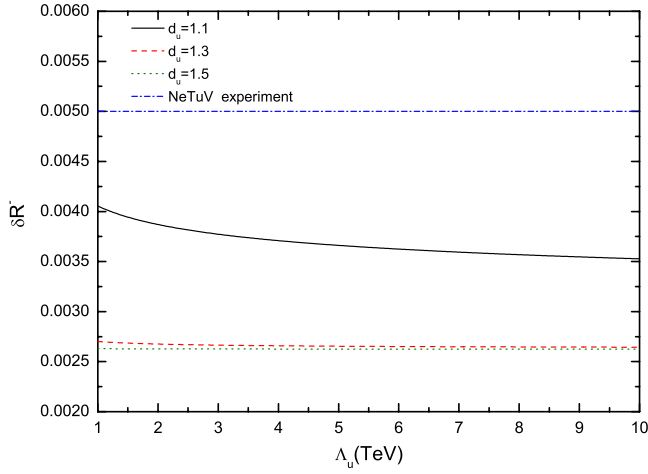


FIG. 3 (color online). The dependence of δR^- on Λ_U for $d_U = 1.1, 1.3,$ and 1.5 with $\Lambda_U = 1$ TeV, $c_{VU} = 0.01$, and $c_{AU} = 0.002$.

0.004, respectively, in Fig. 2. The variations of δR^- with respect to Λ_U are shown in Fig. 3. From Fig. 2 and 3, we see that the PW relation receives large corrections from unparticle for d_U near 1.

To get an idea of the possible role played by unparticle in dissolving NuTeV anomaly, we first discuss four special cases in the following:

- (1) No unparticle physics $c_{VU} = c_{AU} = 0$.

In this case the correction δR^- in Eq. (40) reduces to $(1 - \frac{7}{3}\sin^2\theta_w)\frac{S_v}{Q_v + 3S_v}$, which is exactly the correction to the PW relation due to the asymmetric strange sea [31].

- (2) Pure vector coupling between unparticle and SM fermion $c_{VU} \neq 0$ and $c_{AU} = 0$.

The corresponding correction to the PW relation is denoted as δR_V^- , then

$$\delta R_V^- = \left[1 - \frac{7}{3}\sin^2\theta_w + \frac{\sqrt{2}R_{UZ}c_{VU}^2}{2G_F M_Z^2 \Lambda_U^{2d_U-2}} \right] \times \frac{S_v}{Q_v + 3S_v}. \quad (41)$$

For $1 < d_U < 2$, it is obvious that R_{UZ} is negative, so that the unparticle contributions tend to cancel the contribution from the strange quark asymmetry. For the most likely value $S_v \approx 0.002$, unparticle would increase the discrepancy between the NuTeV $\sin^2\theta_w$ result and its SM value in this case.

- (3) Pure axial vector coupling between unparticle and SM fermion $c_{AU} \neq 0$ and $c_{VU} = 0$.

The correction to the PW relation is then denoted as δR_A^- , which can be straightforwardly obtained from Eq. (40)

$$\delta R_A^- = \left\{ 1 - \frac{7}{3}\sin^2\theta_w + \frac{\sqrt{2}R_{UZ}c_{AU}^2}{G_F M_Z^2 \Lambda_U^{2d_U-2}} \left[\frac{1}{2} - \frac{2}{3}\sin^2\theta_w + \frac{\sin^2\theta_w}{3} \frac{Q_v}{S_v} \right] \right\} \frac{S_v}{(Q_v + 3S_v)}. \quad (42)$$

For convenience, we define the factor $F \equiv \frac{1}{2} - \frac{2}{3}\sin^2\theta_w + \frac{\sin^2\theta_w}{3} \frac{Q_v}{S_v}$, which determines the sign of unparticle contributions relative to that of the strange quark asymmetry. If the strange asymmetry S_v is negative (i.e., $-0.001 < S_v < 0$), the factor F is negative, the contributions of unparticle increase those of asymmetric strange quark. Whereas for positive S_v (i.e., $0 < S_v < 0.004$), F is positive, consequently the signs of the contributions from unparticle and the strange quark asymmetry are opposite. For the central value $S_v \approx 0.002$, the NuTeV discrepancy would be increased by unparticle as well.

- (4) Chiral symmetric coupling limit $c_{VU} = c_{AU}$.

From Eq. (40), it is obvious that there is no correction to the PW relation from unparticle exchange, then δR^- completely comes from strange quark asymmetry.

Generally the unparticle parameters Λ_U , d_U , c_{VU} , and c_{AU} could take value beyond the four special cases considered above. We are very interested in exploring if there exists a certain parameter region, where the NuTeV discrepancy is completely accounted for by the unparticle effects and the asymmetric strange-antistrange sea. If this is true, the following relation should be satisfied

$$\begin{aligned} & \frac{\sqrt{2}R_{UZ}(c_{VU} - c_{AU})}{G_F M_Z^2 \Lambda_U^{2d_U-2}} \left\{ \frac{1}{2} S_v c_{VU} + \left[-\frac{1}{3}\sin^2\theta_w Q_v + \left(-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w \right) S_v \right] c_{AU} \right\} \\ & = 5 \times 10^{-3} (Q_v + 3S_v) - \left(1 - \frac{7}{3}\sin^2\theta_w \right) S_v. \end{aligned} \quad (43)$$

The above relation between c_{VU} and c_{AU} for $d_U = 1.1, 1.3, 1.5$ is displayed in Fig. 4, where we have chosen $\Lambda_U = 1$ TeV and $Q^2 \approx 20$ GeV². To demonstrate the S_v dependence, the parameter regions for $S_v = -0.001, 0.001, 0.002,$ and 0.004 are shown, respectively. In the case of $S_v > 0$, the contributions to δR^- from the unparticle physics and asymmetric strange quark are larger than the NuTeV discrepancy 0.005 for the parameters lying in the region between the upper curve and the lower curve of the same type (solid line, dashed line, or dotted line). Whereas the contributions are smaller than the NuTeV discrepancy for the parameters in the region above the upper curve and that below the lower curve of the same type. However, the

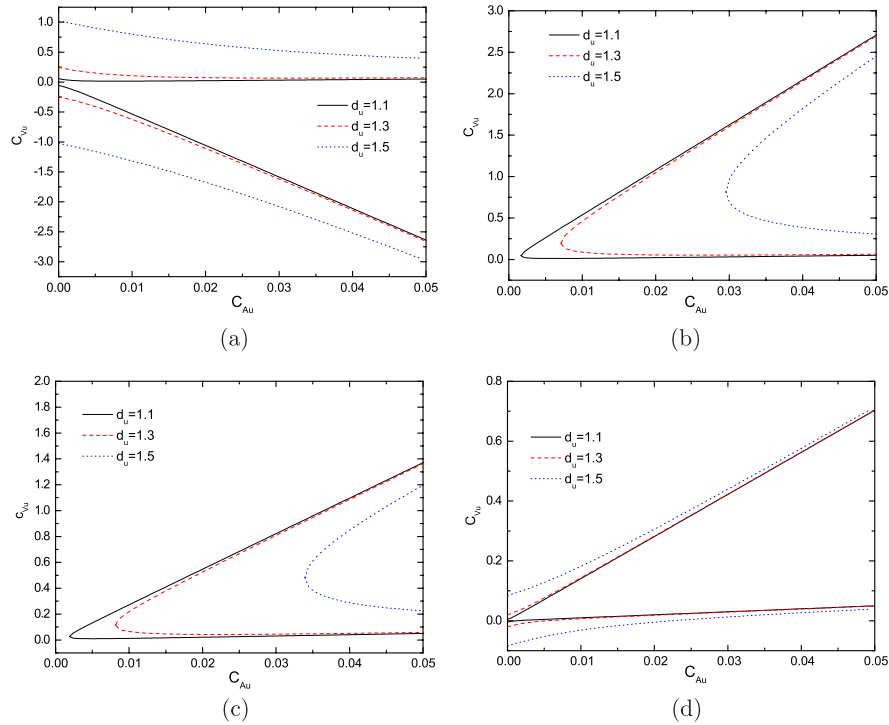


FIG. 4 (color online). The curves of c_{VU} with respect to c_{AU} for $d_U = 1.1, 1.3, 1.5$, and contributions to δR^- from isoscalar violation, etc., are assumed to be zero. Figures 4(a)–4(d) correspond to $S_v = -0.001, 0.001, 0.002$, and 0.004 , respectively.

reverse is true in the case of $S_v < 0$. Particularly we note that the NuTeV anomaly is completely dissolved by the combination of the unparticle effect and strange quark asymmetry, for the parameters on the curves.

In general, the effective unparticle couplings c_{VU} and c_{AU} are expected to be small, some literatures of unparticle physics have chosen them to be of order 10^{-2} or 10^{-3} . From Fig. 4 we see that, for the parameters on the upper curves with $S_v < 0$ and the parameters on the lower curves with $S_v > 0$, c_{VU} and c_{AU} can take values in the range from 10^{-3} to 10^{-2} , and simultaneously the NuTeV anomaly can be explained by unparticle effects and strange quark asymmetry. The corresponding parameter regions

for $d_U = 1.1, 1.3$, and 1.5 are shown in Fig. 5, where the variation of S_v in the range $-0.001 < S_v < 0.004$ is considered. Looking back at Fig. 1, we can see that for these parameter regions, the unparticle effects may be observed in APV, if the uncertainty in the isotopic relative neutron/proton radius shift $\delta(\Delta \frac{R_N}{R_p})$ is smaller than a few times 10^{-4} . Moreover, it is notable that these parameter spaces are consistent with the constraints on unparticle parameters from $b \rightarrow s\gamma$ [42].

The NuTeV experiment uses an iron target, which has an excess of neutrons over protons about 6%. This resulting isoscalar violation would introduce correction to the PW relation [43]. This violation of isoscalar is known for

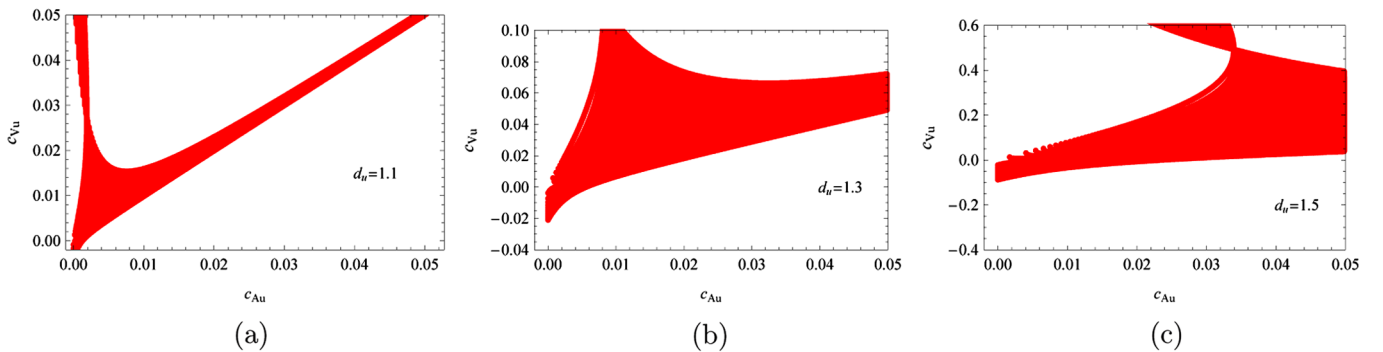


FIG. 5 (color online). The allowed parameter regions of the effective unparticle couplings c_{VU} and c_{AU} for $-0.001 < S_v < 0.004$, where the NuTeV discrepancy are completely accounted for by unparticle effect and the asymmetric strange sea. The contributions to δR^- from isoscalar violation, etc., are assumed to be zero. Figures 5(a)–5(c) correspond to $d_U = 1.1, 1.3$, and 1.5 , respectively.

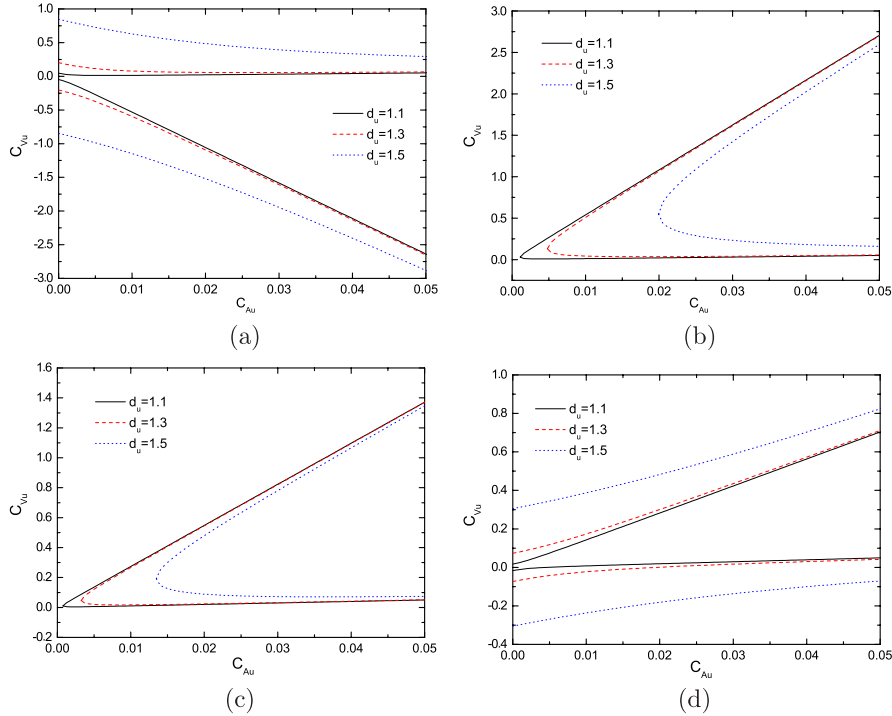


FIG. 6 (color online). The same as Fig. 4, and the contributions to δR^- from isoscalarity violation, etc., are taken to be 0.002.

good accuracy, it has been claimed to be included in the data analysis by the NuTeV Collaboration [44]. A further violation of isoscalarity could be due to the fact that isospin symmetry is violated by the parton distributions of the nucleon, i.e., $u^p \neq d^n$ and $u^n \neq d^p$, where u^p and d^p are, respectively, the up quark and down quark distributions in proton, u^n and d^n are the corresponding distributions in neutron. The contribution of this type of isospin violation (it is named as a charge symmetry violation in Ref. [32]) is expected to reduce the discrepancy of $\sin^2 \theta_W$ by about 30% [44–46].

The PW relation may receive corrections from higher order QCD contributions and nuclear physics effects (such as Fermi motion, nuclear binding, and nuclear shadowing, etc.) as well, which turn out to be small enough to be negligible [43,47]. In short, there are theoretical corrections on the NuTeV determination of $\sin^2 \theta_W$ due to QCD

effects, nuclear effects, and the violation of the assumptions on which the PW relation is based. The calculations of some corrections are model dependent. For demonstration, we assume that the contributions to δR^- from isoscalarity violation, etc., are 0.002, then the corresponding relationship between c_{Vu} and c_{Au} are shown in Fig. 6. The meaning of this figure is the same as that of Fig. 4, and the shapes of the curves in the two figures are similar to each other. Especially for the parameter values on the upper curves with $S_v < 0$ and the parameter values on the lower curves with $S_v > 0$, the unparticle couplings c_{Vu} and c_{Au} can take values of order from 10^{-3} to 10^{-2} , which are also consistent with the constraints on unparticle parameters from $b \rightarrow s\gamma$ [42]. At the same time, the residual NuTeV anomaly is completely removed by the unparticle physics and the asymmetric strange quark contribution. Explicitly, the corresponding parameter regions are shown in Fig. 7.

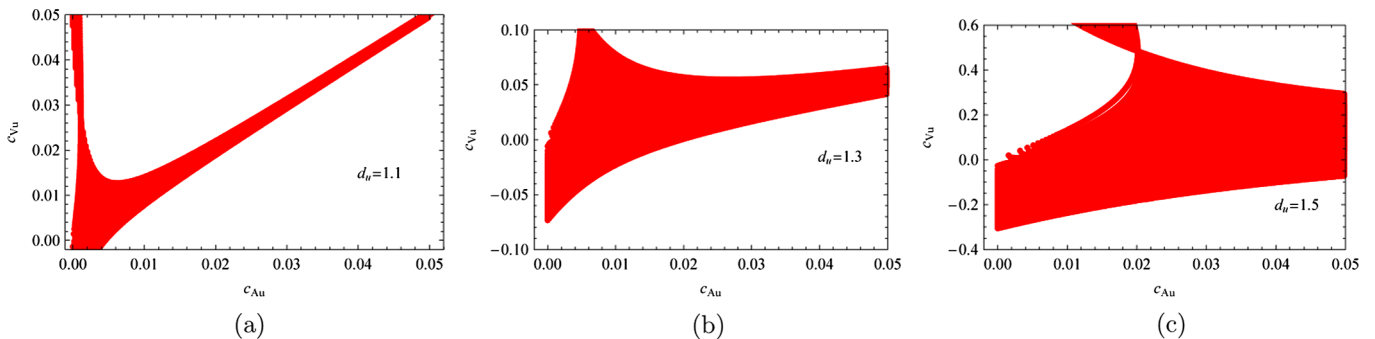


FIG. 7 (color online). The same as for Fig. 5, and the contributions to δR^- from isoscalarity violation, etc., are taken to be 0.002.

IV. SUMMARY AND CONCLUSIONS

Unparticle leads to interesting and rich phenomenology, which could be checked by experiments. In this work we have investigated the possible signals of unparticle in APV along an isotope chain and NuTeV experiment. Both the APV and NuTeV experiments would play an important role in exploring unparticle physics at lower energy, if unparticle exists in nature. Possible theoretical corrections and uncertainties within the standard model are discussed in detail. In the case of the ratios of APV observables along an isotope chain, R_1 is more sensitive to unparticle than R_2 provided the same experimental precisions. Although the theoretical uncertainties from atomic theory almost cancel, the uncertainties due to neutron distribution are crucial. The effects of unparticle could be observed, if the uncertainty in relative neutron/proton radius shift $\delta(\Delta \frac{R_N}{R_P})$ is less than a few times 10^{-4} by measuring the parity violating electron scattering.

The interpretation of the NuTeV result is still a subject of considerable debate until now. Various effects unaccounted for by the NuTeV Collaboration, have been proposed as possible remedies for the anomaly. The NuTeV results should be checked by other groups or other experiments such as the Q-Weak experiment in the future, if the NuTeV discrepancy is confirmed to be true, we suggest that unparticle could remove part of the discrepancy. The constraints imposed by the NuTeV experiment on unparticle

physics are studied in detail. We have demonstrated that there exist certain regions for the unparticle parameters (Λ_U , d_U , c_{VU} , and c_{AU}), where the unparticle physics coupled with the strange quark asymmetry could completely explain the discrepancy between the NuTeV result and the SM value with or without the contributions from the isoscalarity violation, etc. It is remarkable that these parameter regions are consistent with the constraints from $b \rightarrow s\gamma$ [42]. Meanwhile, unparticle possibly manifests itself in APV for these parameter values, if the neutron distribution uncertainty $\delta(\Delta \frac{R_N}{R_P})$ is less than a few times 10^{-4} .

Both the APV along the isotope chain and the NuTeV results put important constraints on the unparticle physics. The unparticle contributions to these observables strongly depend on the unparticle scale Λ_U and the scale dimension d_U . It is very interesting and valuable to perform a global fit to the ranges of unparticle parameters Λ_U , d_U , c_{VU} , c_{AU} allowed by the current precise electroweak data and other observables from astrophysics and cosmology, etc., which is beyond the scope of the present work.

ACKNOWLEDGMENTS

This work is partially supported by the National Natural Science Foundation of China under Grants No. 90403021 and No. KJCX2-SW-N10 of the Chinese Academy.

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