MSSM extension with a mirror fourth generation, neutrino magnetic moments, and CERN LHC signatures

Tarek Ibrahim[1,2,*] and Pran Nath[2,3,+]

[1]*Department of Physics, Faculty of Science, University of Alexandria, Alexandria, Egypt*
[2]*Department of Physics, Northeastern University, Boston, Massachusetts 02115-5000, USA*
[3]*Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*

(Received 24 June 2008; published 16 October 2008)

Recent analyses have shown that a sequential fourth generation can be consistent with precision electroweak data. We consider the possibility that the new generation could be a mirror generation with $V + A$ rather than $V - A$ interactions. Specifically we consider an extension of the minimal supersymmetric standard model with a light mirror generation. Implications of this extension are explored. One consequence is an enhancement of the τ neutrino magnetic moment by several orders of magnitude consistent with the current limits on the magnetic moment of the τ. The masses of the mirror generation arise due to electroweak symmetry breaking, and if a mirror generation exists its mass spectrum must lie below a TeV, and thus should be discovered at the LHC. Mirror particles and mirror sparticles produce many characteristic signatures which should be detectable at the LHC. Heavy Higgs boson decays into mirror particles and an analysis of the forward-backward asymmetries can distinguish a mirror generation from a sequential fourth generation. The validity of the model can thus be tested at the LHC. A model of the type discussed here could arise from a more unified structure such as grand unification or strings where a mirror generation escapes the survival hypothesis, i.e., a generation and a mirror generation do not tie up to acquire a mass of size $M_{\rm GUT}$ or $M_{\rm string}$ due to a symmetry, and thus remain massless down to the electroweak scale.

DOI: 10.1103/PhysRevD.78.075013 PACS numbers: 12.60.Jv, 13.40.Em, 14.60.Pq, 14.60.St

I. INTRODUCTION

Recent investigations have shown that a fourth generation is not ruled out by the precision electroweak data if it is heavy with masses in the few hundred GeV range (or recent works see [1–7] and for early works see [8–10]). These investigations have typically assumed that the fourth generation is a sequential generation with $V - A$ type interactions. However, an intriguing possibility exists that the new generation could be a mirror generation with $V + A$ interactions. Mirror generations do arise in unified models of fundamental interactions [11–14], and thus it is natural that one consider the existence of a mirror generation. Normally one assumes the so-called survival hypothesis [12] where with n_f number of ordinary families and n_{mf} number of mirror families, only $n_f - n_{mf}$ (for $n_f > n_{mf}$) remain light, and the remainder acquire grand unified theory (GUT) or string scale size masses. However, this need not always be the case. Indeed there are many escape mechanisms where residual symmetries in breaking at the string scale or GUT scale will keep some mirror families light while others become superheavy [15,16]. Mixings between ordinary families and mirrors can arise from nonrernormalizable interactions after spontaneous breaking (see, e.g., [16,17]). Additional work on model building using mirrors can be found in [18–23] and further implications of mirrors are explored in [24–29].

In this work we make the specific assumption that there is indeed a light mirror generation with masses below the TeV scale which would be accessible at the LHC. The assumption of a full mirror generation leaves the theory anomaly free. Essentially all of the analyses valid for a sequential fourth generation regarding consistency with the precision electroweak data and other constraints should be valid for a mirror generation and we assume this to be the case. The analysis we present here differs from previous works in many respects. First we propose an extension of the minimal supersymmetric standard model with a full mirror generation which is light (mirMSSM), i.e., with masses below the TeV scale which will be accessible at the LHC. Such an extension is not considered in any of the previous works. Indeed most of the previous analyses are not in supersymmetric frameworks. Second we assume that the mixings of the mirror generation occur mostly with the third generation, and are negligible with the first two generations if they occur at all. With this assumption, the $V - A$ structure of the weak interactions for the first two generations remains intact, while the third generation can develop a small $V + A$ component. Current data on the third generation do not necessarily rule out this possibility.

If a mirror generation exists, it would be discovered at the LHC with the same amount of luminosity as for the a sequential fourth generation which is estimated to be 50 fb^{-1}. A mirror generation will lead to interesting and

*tarek@lepton.neu.edu
+nath@lepton.neu.edu

even dramatic multilepton and jet signatures which can discriminate between a mirror generation and a sequential fourth generation. Further, tests of the mirror generation can come from the decay of the heavy Higgs and via measurements of the forward-backward asymmetry. Another effect of the mixings of the mirror generation with the third generation is on magnetic moments. We analyze these in the leptonic sector in detail and show that the τ neutrino magnetic moment is enhanced by several orders of magnitude beyond what one has in the standard model. We note in passing that the term mirror has also been used in an entirely different context of mirror worlds [30,31] where one has mirror matter with their own mirror gauge group. The analysis here has no relationship with those theories.

The outline of the rest of the paper is as follows. In Sec. II we present an extension of the minimal supersymmetric standard model (MSSM) to include a fourth generation which we assume is a mirror generation and allow for a mixing of this generation with the third generation. Here the interactions in the charged and neutral current sectors are worked out including the supersymmetric interactions involving the mirrors, the chargions, and the neutralinos. Further details of mixing and interactions are given in Appendix A. An analysis of the τ neutrino magnetic moment is given in Sec. III. Here contributions arise from exchanges of the leptons from the third generation and from the mirror generation, and also from the exchanges of the sleptons and mirror sleptons. An analysis of the τ-lepton anomalous magnetic moment when mixings with the mirror family are allowed is given in Sec. IV again including exchanges from the third generation leptons and sleptons and from the mirror leptons and mirror sleptons. A discussion of the constraints on a mirror generation and a quantitative analysis of the sizes is given in Sec. V in the framework of an extended supergravity unified model [32] which includes the mirror sector. When compared with the magnetic moment analyses in MSSM with or without CP violation [33–35] one finds that the τ neutrino magnetic moment can be orders of magnitude larger than in the standard model while the magnetic moment of the τ lies within experimental bounds. A qualitative analysis of the signatures of the mirror generation at the LHC is given in Sec. VI. Here it is shown that some characteristic signatures arise, such as dominance of τs in the decay patterns of the mirror leptons which should allow one to discriminate this model from other supersymmetric models. Further, we discuss how one may distinguish a mirror generation from a sequential fourth generation. Here aside from the leptonic signatures, the decay of the heavy Higgs bosons, and the analysis of the forward-backward asymmetry would allow one to discriminate a mirror generation from a sequential fourth generation. Further details of the decay of heavy Higgs to mirror fermions are given in Appendix B. Conclusions are given in Sec. VII.

II. EXTENSION OF MSSM WITH A MIRROR GENERATION

The fourth generation which we assume to be mirror will in general mix with the other three generations. However, as is the case for the first three generations the mixings between the generations get smaller as the ratio of the masses get further apart. Thus, for example, $V_{ub} \ll V_{us}$, and we expect a similar phenomenon for mixings involving the fourth (mirror) generation, i.e., we expect $V_{uB} \ll V_{ub}$ where B is the fourth (mirror) generation bottom quark. As an example, the mixing between the first and the second can be estimated by the Gatto-Sartori-Tonin-Oakes relation [36] $V_{us} = \sqrt{m_d/m_s}$ which gives V_{us} to be about 0.2. The mixing of the first with the third can be very roughly estimated so that $V_{ub} = \sqrt{m_d/m_b}$ which gives about 0.03, i.e., a factor about 10 smaller than V_{us} [37]. If we extend this rough estimate to the fourth generation one will have mixing between the first and the fourth as $V_{uB} = \sqrt{m_d/m_B} = 0.005$ (for $m_B = 200$ GeV). Assuming similar mixings will hold in the leptonic sector one will have mixings between the first and the fourth as $\sqrt{m_e/m_E} = 0.0016$ (for $M_E = 200$ GeV) where E is the fourth (mirror) generation lepton. More detailed analyses using error bars on electroweak data show that the constraints on the enlarged Cabibbo-Kobayashi-Maskawa (CKM) matrix are more relaxed [1] (see also Sec. V). Conversely it means that with the current limits on the mixing angles the effects of the fourth generation on the analysis of the electroweak data lie well within the error bars. Here the electroweak parameters which require special attention are the S, T, U variables where larger contributions from the fourth generation are possible, but still the data can be made compatible with a fourth generation. Returning to the mixing of the fourth generation with the first two, one can easily check that small mixings of the type discussed above lead to negligible effect of the fourth generation on the phenomenology of the first two generations. For this reason we will make a simplifying assumption of neglecting the mixing effects of the fourth with the first two generations and consider below the mixing of just the third and the fourth. However, the following analysis can be straightforwardly extended to the full four generations by letting the generation index run from 1–4 keeping in mind that the fourth generation is a mirror generation. Thus under $SU(3)_C \times SU(2)_L \times U(1)_Y$ the leptons transform as follows

$$\psi_L \equiv \begin{pmatrix} \nu_L \\ \tau_L \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right), \qquad \tau_L^c \sim (1, 1, 1), \qquad \nu_L^c \sim (1, 1, 0), \tag{1}$$

where the last entry on the right-hand side of each $\sim$ is the value of the hypercharge Y defined so that $Q = T_3 + Y$. These leptons have $V - A$ interactions. Let us now consider mirror leptons which have $V + A$ interactions. Their

quantum numbers are as follows

$$\chi^c \equiv \begin{pmatrix} E^c_{\tau L} \\ N^c_L \end{pmatrix} \sim \left(1, 2, \frac{1}{2}\right), \qquad E_{\tau L} \sim (1, 1, -1), \qquad N_L \sim (1, 1, 0). \tag{2}$$

The analogous relations for the quarks are

$$q \equiv \begin{pmatrix} t_L \\ b_L \end{pmatrix} \sim \left(3, 2, \frac{1}{6}\right), \qquad t^c_L \sim \left(3^*, 1, -\frac{2}{3}\right), \qquad b^c_L \sim \left(3^*, 1, \frac{1}{3}\right), \tag{3}$$

and for the mirror quarks

$$Q^c \equiv \begin{pmatrix} B^c_L \\ T^c_L \end{pmatrix} \sim \left(3^*, 2, -\frac{1}{6}\right), \qquad T_L \sim \left(3, 1, \frac{2}{3}\right), \qquad B_L \sim \left(3^*, 1, -\frac{1}{3}\right). \tag{4}$$

For the Higgs multiplets we have the MSSM Higgs doublets which give

$$H_1 \equiv \begin{pmatrix} H^1_1 \\ H^2_1 \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}\right), \qquad H_2 \equiv \begin{pmatrix} H^1_2 \\ H^2_2 \end{pmatrix} \sim \left(1, 2, \frac{1}{2}\right). \tag{5}$$

We assume that the mirror generation escapes acquiring mass at the GUT scale and remains light down to the elctroweak scale where the superpotential of the model for the lepton part, may be written in the form

$$W = \epsilon_{ij}[f_1 \hat{H}^i_1 \hat{\psi}^j_L \hat{\tau}^c_L + f'_1 \hat{H}^j_2 \hat{\psi}^i_L \hat{\nu}^c_L + f_2 \hat{H}^i_1 \hat{\chi}^{cj} \hat{N}_L + f'_2 \hat{H}^j_2 \hat{\chi}^{ci} \hat{E}_{\tau L}] + f_3 \epsilon_{ij} \hat{\chi}^{ci} \hat{\psi}^j_L + f_4 \hat{\tau}^c_L \hat{E}_{\tau L} + f_5 \hat{\nu}^c_L \hat{N}_L. \tag{6}$$

In the above we have assumed mixings between the third generation and the mirror generation. Such mixings can arise via nonrenormalizable interactions [16]. Consider, for example, a term such as $1/M_{Pl} \nu^c_L N_L \Phi_1 \Phi_2$. If Φ_1 and Φ_2 develop vacuum expectation values of size 10^{9-10}, a mixing term of the right size can be generated.

To get the mass matrices of the leptons and the mirror leptons we replace the superfields in the superpotential by their component scalar fields. The relevant parts in the superpotential that produce the lepton and mirror lepton mass matrices are

$$W = f_1 H^1_1 \tilde{\tau}_L \tilde{\tau}^*_R + f'_1 H^2_2 \tilde{\nu}_L \tilde{\nu}^*_R + f_2 H^1_1 \tilde{N}^*_R \tilde{N}_L + f'_2 H^2_2 \tilde{E}^*_{\tau R} \tilde{E}_{\tau L} + f_3 \tilde{E}^*_{\tau R} \tilde{\tau}_L - f_3 \tilde{N}^*_R \tilde{\nu}_L + f_4 \tilde{\tau}^*_R \tilde{E}_{\tau L} + f_5 \tilde{\nu}^*_R \tilde{N}_L. \tag{7}$$

The mass terms for the lepton and their mirrors arise from the part of the Lagrangian

$$\mathcal{L} = -\frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j + \text{H.c.} \tag{8}$$

where ψ and A stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, ($\langle H^1_1 \rangle = v_1/\sqrt{2}$ and $\langle H^2_2 \rangle = v_2/\sqrt{2}$), we have the following set of mass terms written in four-spinors for the fermionic sector

$$-\mathcal{L}_m = (\bar{\tau}_R \bar{E}_{\tau R}) \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 \\ f_3 & f'_2 v_2/\sqrt{2} \end{pmatrix} \begin{pmatrix} \tau_L \\ E_{\tau L} \end{pmatrix} + (\bar{\nu}_R \bar{N}_R) \begin{pmatrix} f'_1 v_2/\sqrt{2} & f_5 \\ -f_3 & f_2 v_1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} + \text{H.c.} \tag{9}$$

Here the mass matrices are not Hermitian and one needs to use bi-unitrary transformations to diagonalize them. Thus we write the linear transformations

$$\begin{pmatrix} \tau_R \\ E_{\tau R} \end{pmatrix} = D^\tau_R \begin{pmatrix} \tau_{1_R} \\ E_{\tau 2_R} \end{pmatrix}, \qquad \begin{pmatrix} \tau_L \\ E_{\tau L} \end{pmatrix} = D^\tau_L \begin{pmatrix} \tau_{1_L} \\ E_{\tau 2_L} \end{pmatrix}, \tag{10}$$

such that

$$D^{\tau\dagger}_R \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 \\ f_3 & f'_2 v_2/\sqrt{2} \end{pmatrix} D^\tau_L = \text{diag}(m_{\tau_1}, m_{\tau_2}). \tag{11}$$

The same holds for the neutrino mass matrix

$$D^{\nu\dagger}_R \begin{pmatrix} f'_1 v_2/\sqrt{2} & f_5 \\ -f_3 & f_2 v_1/\sqrt{2} \end{pmatrix} D^\nu_L = \text{diag}(m_{\nu_1}, m_{\nu_2}). \tag{12}$$

Here τ_1, τ_2 are the mass eigenstates and we identify the τ lepton with the eigenstate 1, i.e., $\tau = \tau_1$, and identify τ_2 with a heavy mirror eigenstate with a mass in the hundreds of GeV. Similarly ν_1, ν_2 are the mass eigenstates for the neutrinos, where we identify ν_1 with the light neutrino state and ν_2 with the heavier mass eigenstate. By multiplying Eq. (11) by $D^{\tau\dagger}_L$ from the right and by D^τ_R from the left and by multiplying Eq. (12) by $D^{\nu\dagger}_L$ from the right and by D^ν_R from the left, one can equate the values of the parameter f_3 in both equations and we can get the following relation between the diagonlizing matrices D^τ and D^ν

$$m_{\tau 1} D^\tau_{R21} D^{\tau*}_{L11} + m_{\tau 2} D^\tau_{R22} D^{\tau*}_{L12} = -[m_{\nu 1} D^\nu_{R21} D^{\nu*}_{L11} + m_{\nu 2} D^\nu_{R22} D^{\nu*}_{L12}]. \tag{13}$$

Equation (13) is an important relation as it constrains the symmetry breaking parameters and this constraint must be taken into account in numerical analyses.

Let us now write the charged current interaction in the leptonic sector for the third generation and for the mirror generation with the W boson,

$$\mathcal{L}_{CC} = -\frac{g_2}{2\sqrt{2}} W^\dagger_\mu [\bar{\nu} \gamma^\mu (1 - \gamma_5) \tau + \bar{N} \gamma^\mu (1 + \gamma_5) E_\tau] + \text{H.c.} \tag{14}$$

In the mass diagonal basis the charged current interactions are given by

$$\mathcal{L}_{CC} = -\frac{g_2}{2\sqrt{2}} W^{\dagger}_{\mu} \sum_{\alpha,\beta,\gamma,\delta=1,2} \bar{\nu}_{\alpha} \gamma^{\mu} [D^{\nu\dagger}_{L\alpha\gamma} g^{L}_{\gamma\delta} D^{\tau}_{L\delta\beta} (1-\gamma_5) + D^{\nu\dagger}_{R\alpha\gamma} g^{R}_{\gamma\delta} D^{\tau}_{R\delta\beta} (1+\gamma_5)] \tau_{\beta} + \text{H.c.} \quad (15)$$

where $g^{L,R}_{\alpha\beta}$ are defined so that

$$g^{L}_{11} = 1, \qquad g^{L}_{12} = 0 = g^{L}_{21} = g^{L}_{22}, \qquad g^{R}_{11} = 0 = g^{R}_{12} = g^{R}_{21}, \qquad g^{R}_{22} = 1. \quad (16)$$

Next we consider the chargino interactions of the mirror leptons. The interaction terms in two-component notation are

$$\mathcal{L} = ig\sqrt{2} T^{a}_{ij} \lambda^{a} \psi_j A^{*}_i - \frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j + \text{H.c.} \quad (17)$$

Here $T^a = \tau^a/2$ where τ^a $(a = 1, 2, 3)$ are the Pauli matrices, and for the chargino interaction we use the generators T^1 and T^2, and W is the part of Eq. (6) given by

$$W = -f_2 H^2_1 \tilde{E}^{*}_{\tau R} \tilde{N}_L - f'_2 H^1_2 \tilde{N}^{*}_R \tilde{E}_{\tau L}. \quad (18)$$

Using the above superpotential and the fermions of the mirror generation and the supersymmetric partners of the charged Higgs for ψ and the mirror sleptons and charged Higgs for A, the interaction of the $V + A$ fourth generation with charginos in the two-component notation is given by

$$\mathcal{L} = ig[\lambda^{+} N^{c}_L \tilde{E}_{\tau R} + \lambda^{-} E^{c}_{\tau L} \tilde{N}_R] + \frac{g m_N}{\sqrt{2} M_W \cos\beta} [\tilde{N}_L \psi_{H^-_1} E^{c}_{\tau L} + \tilde{E}^{*}_{\tau R} \psi_{H^-_1} N_L] + \frac{g m_E}{\sqrt{2} M_W \sin\beta} [\tilde{N}^{*}_R \psi_{H^+_2} E_{\tau L} + \tilde{E}_L \psi_{H^+_2} N^{c}_L] + \text{H.c.}, \quad (19)$$

where $\lambda^{\pm} = \frac{\lambda^1 \mp i\lambda^2}{\sqrt{2}}$.

Now we go from two-spinor to four-spinor by defining the two four-spinors:

$$\tilde{W} = \begin{pmatrix} -i\lambda^{+} \\ i\bar{\lambda}^{-} \end{pmatrix}, \qquad \tilde{H} = \begin{pmatrix} \psi_{H^+_2} \\ \bar{\psi}_{H^-_1} \end{pmatrix}. \quad (20)$$

By using these two four-spinors, Eq. (19) for the $V + A$ generation interaction is given by

$$\mathcal{L} = -g[\bar{\tilde{W}} P_R N \tilde{E}^{*}_{\tau R} + \bar{\tilde{W}}^{c} P_R E_{\tau} \tilde{N}^{*}_R] + \frac{g m_E}{\sqrt{2} M_W \sin\beta} [\bar{\tilde{H}} P_R N \tilde{E}^{*}_{\tau L} + \bar{E}_{\tau} P_R \tilde{H}^{c} \tilde{N}_R] + \frac{g m_N}{\sqrt{2} M_W \cos\beta} [\bar{N} P_R \tilde{H} \tilde{E}_{\tau R} + \bar{\tilde{H}}^{c} P_R E_{\tau} \tilde{N}^{*}_L] + \text{H.c.} \quad (21)$$

Now we use the two-component mass eigenstates

$$\psi^{+}_1 = -i\lambda^{+}, \qquad \psi^{+}_2 = \psi_{H^+_2}, \qquad \psi^{-}_1 = -i\lambda^{-}, \qquad \psi^{-}_2 = \psi_{H^-_1}. \quad (22)$$

By defining the two-component spinors χ^{+}_i and χ^{-}_i as

$$\chi^{+}_i = V_{ij} \psi^{+}_j \qquad \chi^{-}_i = U_{ij} \psi^{-}_j \quad (23)$$

the four-component mass eigenstates are

$$\tilde{\chi}^{+}_1 = \begin{pmatrix} \chi^{+}_1 \\ \bar{\chi}^{-}_1 \end{pmatrix}, \qquad \tilde{\chi}^{+}_2 = \begin{pmatrix} \chi^{+}_2 \\ \bar{\chi}^{-}_2 \end{pmatrix}. \quad (24)$$

The matrix elements U and V that diagonalize the chargino mass matrix M_C are given by

$$U^{*} M_C V^{-1} = \text{diag}(m^{+}_{\tilde{\chi}_1}, m^{+}_{\tilde{\chi}_2}). \quad (25)$$

One can use the definitions of P_L, P_R and the above relations to get the following useful relations

$$P_L \tilde{W} = P_L \sum_{i=1}^{2} V^{*}_{i1} \tilde{\chi}^{+}_i, \qquad P_L \tilde{W}^{c} = P_L \sum_{i=1}^{2} U^{*}_{i1} \tilde{\chi}^{c}_i$$
$$P_L \tilde{H} = P_L \sum_{i=1}^{2} V^{*}_{i2} \tilde{\chi}^{+}_i, \qquad P_R \tilde{H} = P_R \sum_{i=1}^{2} U_{i2} \tilde{\chi}^{+}_i \quad (26)$$
$$P_R \tilde{H}^{c} = P_R \sum_{i=1}^{2} V_{i2} \tilde{\chi}_i{}^{c}, \qquad P_L \tilde{H}^{c} = P_L \sum_{i=1}^{2} U^{*}_{i2} \tilde{\chi}^{c}_i.$$

Using these relations and Eq. (21), the interactions of the mirror generation with chargino mass eigenstates is given by

$$-\mathcal{L}_{N-E_{\tau}-\chi^{+}} = g\bar{N}[V^{*}_{i1} P_L - \kappa_N U_{i2} P_R] \tilde{\chi}^{+}_i \tilde{E}_{\tau R} + g\bar{N}[-\kappa_{E_{\tau}} V^{*}_{i2} P_L] \tilde{\chi}^{+}_i \tilde{E}_{\tau L} + g\bar{E}_{\tau}[U^{*}_{i1} P_L - \kappa_{E_{\tau}} V_{i2} P_R] \tilde{\chi}^{c}_i \tilde{N}_R + g\bar{E}_{\tau}[-\kappa_N U^{*}_{i2} P_L] \tilde{\chi}^{c}_i \tilde{N}_L + \text{H.c.} \quad (27)$$

where $\tilde{\chi}^{c}_i$ is the charge conjugate of $\tilde{\chi}_i$ and where

$$\kappa_N = \frac{m_N}{\sqrt{2} M_W \cos\beta}, \qquad \kappa_{E_{\tau}} = \frac{m_{E_{\tau}}}{\sqrt{2} M_W \sin\beta}. \quad (28)$$

The interaction of the leptons with the chargino is given by

$$-\mathcal{L}_{\nu-\tau-\chi^{+}} = g\bar{\nu}[U_{i1} P_R - \kappa_{\nu} V^{*}_{i2} P_L] \tilde{\chi}^{+}_i \tilde{\tau}_L + g\bar{\nu}[-\kappa_{\tau} U_{i2} P_R] \tilde{\chi}^{+}_i \tilde{\tau}_R + g\bar{\tau}[V_{i1} P_R - \kappa_{\tau} U^{*}_{i2} P_L] \tilde{\chi}^{c}_i \tilde{\nu}_L + g\bar{\tau}[-\kappa_{\nu} V_{i2} P_R] \tilde{\chi}^{c}_i \tilde{\nu}_R + \text{H.c.}, \quad (29)$$

where

$$\kappa_{\tau} = \frac{m_{\tau}}{\sqrt{2} M_W \cos\beta}, \qquad \kappa_{\nu} = \frac{m_{\nu}}{\sqrt{2} M_W \sin\beta}. \quad (30)$$

A full analysis of the mirror sparticle couplings will be given elsewhere.

Next we consider the mixings of the charged sleptons and the charged mirror sleptons. The mass matrix in the basis $(\tilde{\tau}_L, \tilde{E}_L, \tilde{\tau}_R, \tilde{E}_R)$ takes the form

$$(M^2)_{\tilde{\tau}} = \begin{pmatrix} M^2_{11} M^2_{12} M^2_{13} M^2_{14} \\ M^2_{21} M^2_{22} M^2_{23} M^2_{24} \\ M^2_{31} M^2_{32} M^2_{33} M^2_{34} \\ M^2_{41} M^2_{42} M^2_{43} M^2_{44} \end{pmatrix}. \tag{31}$$

Here the terms M^2_{11}, M^2_{13}, M^2_{31}, M^2_{33} arise from soft breaking in the sector $\tilde{\tau}_L$, $\tilde{\tau}_R$. Similarly the terms M^2_{22}, M^2_{24}, M^2_{42}, M^2_{44} arise from soft breaking in the sector $\tilde{E}_L$, $\tilde{E}_R$. The terms M^2_{12}, M^2_{21}, M^2_{23}, M^2_{32}, M^2_{14}, M^2_{41}, M^2_{34}, M^2_{43}, arise from mixing between the staus and the mirrors. We assume that all the masses are of the electroweak scale so all the terms enter in the diagonalization. We diagonalize the Hermitian mass^2 matrix by the following unitary transformation

$$\tilde{D}^{\tau\dagger} M^2_{\tilde{\tau}} \tilde{D}^{\tau} = \mathrm{diag}(M^2_{\tilde{\tau}_1}, M^2_{\tilde{\tau}_2}, M^2_{\tilde{\tau}_3}, M^2_{\tilde{\tau}_4}). \tag{32}$$

A similar mass matrix exists in the sneutrino sector. In the basis $(\tilde{\nu}_L, \tilde{N}_L, \tilde{\nu}_R, \tilde{N}_R)$ it takes the form

$$(M^2)_{\tilde{\nu}} = \begin{pmatrix} m^2_{11} m^2_{12} m^2_{13} m^2_{14} \\ m^2_{21} m^2_{22} m^2_{23} m^2_{24} \\ m^2_{31} m^2_{32} m^2_{33} m^2_{34} \\ m^2_{41} m^2_{42} m^2_{43} m^2_{44} \end{pmatrix}. \tag{33}$$

As in the charged slepton sector here also the terms m^2_{11}, m^2_{13}, m^2_{31}, m^2_{33} arise from soft breaking in the sector $\tilde{\nu}_L$, $\tilde{\nu}_R$. Similarly the terms m^2_{22}, m^2_{24}, m^2_{42}, m^2_{44} arise from soft breaking in the sector $\tilde{N}_L$, $\tilde{N}_R$. The terms m^2_{12}, m^2_{21}, m^2_{23}, m^2_{32}, m^2_{14}, m^2_{41}, m^2_{34}, m^2_{43}, arise from mixing between the physical sector and the mirror sector. Again as in the charged lepton sector we assume that all the masses are of the electroweak size so all the terms enter in the diagonalization. The above matrix can be diagonalized by the following unitary transformation

$$\tilde{D}^{\nu\dagger} M^2_{\tilde{\nu}} \tilde{D}^{\nu} = \mathrm{diag}(M^2_{\tilde{\nu}_1}, M^2_{\tilde{\nu}_2}, M^2_{\tilde{\nu}_3}, M^2_{\tilde{\nu}_4}). \tag{34}$$

The physical τ and neutrino states are $\tau \equiv \tau_1$, $\nu \equiv \nu_1$, and the states τ_2, ν_2 are heavy states with mostly mirror particle content. The states $\tilde{\tau}_i$, $\tilde{\nu}_i$; $i = 1$–4 are the slepton and sneutrino states. For the case of no mixing these limit as follows

$$\begin{aligned} &\tilde{\tau}_1 \to \tilde{\tau}_L, \quad &&\tilde{\tau}_2 \to \tilde{E}_L, \quad &&\tilde{\tau}_3 \to \tilde{\tau}_R, \quad &&\tilde{\tau}_4 \to \tilde{E}_R, \\ &\tilde{\nu}_1 \to \tilde{\nu}_L, \quad &&\tilde{\nu}_2 \to \tilde{N}_L, \quad &&\tilde{\nu}_3 \to \tilde{\nu}_R, \quad &&\tilde{\nu}_4 \to \tilde{N}_R. \end{aligned} \tag{35}$$

A further discussion of the scalar mass^2 matrices is given in Appendix A.

In the mass diagonal basis the interactions of the neutrino ν and of the stau which include the mixing effects with the mirrors are given by

$$\begin{aligned} -\mathcal{L}_{\nu-\tilde{\tau}-\chi^+} = &\sum_{\alpha=1-2}\sum_{j=1-4} g\bar{\nu}_\alpha [D^{\nu\dagger}_{L\alpha 1} U_{i1} P_R - D^{\nu\dagger}_{R\alpha 1}\kappa_\nu V^*_{i2} P_L]\tilde{\chi}^+_i \tilde{D}^{\tau}_{1j}\tilde{\tau}_j + g\bar{\nu}_\alpha[-D^{\nu\dagger}_{L\alpha 1}\kappa_\tau U_{i2} P_R]\tilde{\chi}^+_i \tilde{D}^{\tau}_{3j}\tilde{\tau}_j \\ &+ g\bar{\nu}_\alpha [D^{\nu\dagger}_{R\alpha 2} V^*_{i1} P_L - D^{\nu\dagger}_{L\alpha 2}\kappa_N U_{i2} P_R]\tilde{\chi}^+_i \tilde{D}^{\tau}_{4j}\tilde{\tau}_j + g\bar{\nu}_\alpha[-D^{\nu\dagger}_{R\alpha 2}\kappa_{E_\tau} V^*_{i2} P_L]\tilde{\chi}^+_i \tilde{D}^{\tau}_{2j}\tilde{\tau}_j + \mathrm{H.c.} \end{aligned} \tag{36}$$

For $\mathcal{L}_{\tau-\tilde{\nu}-\chi^+}$ we have

$$\begin{aligned} -\mathcal{L}_{\tau-\tilde{\nu}-\chi^+} = &\sum_{\alpha=1-2}\sum_{j=1-4} g\bar{\tau}_\alpha [D^{\tau\dagger}_{L\alpha 1} V_{i1} P_R - D^{\tau\dagger}_{R\alpha 1}\kappa_\tau U^*_{i2} P_L]\tilde{\chi}^c_i \tilde{D}^{\nu}_{1j}\tilde{\nu}_j + g\bar{\tau}_\alpha[-D^{\tau\dagger}_{L\alpha 1}\kappa_\nu V_{i2} P_R]\tilde{\chi}^c_i \tilde{D}^{\nu}_{3j}\tilde{\nu}_j \\ &+ g\bar{\tau}_\alpha [D^{\tau\dagger}_{R\alpha 2} U^*_{i1} P_L - D^{\tau\dagger}_{R\alpha 2}\kappa_{E_\tau} V_{i2} P_R]\tilde{\chi}^c_i \tilde{D}^{\nu}_{4j}\tilde{\nu}_j + g\bar{\tau}_\alpha[-D^{\tau\dagger}_{R\alpha 2}\kappa_N U^*_{i2} P_L]\tilde{\chi}^c_i \tilde{D}^{\nu}_{2j}\tilde{\nu}_j + \mathrm{H.c.} \end{aligned} \tag{37}$$

Next we look at the neutral current interactions and focus on the charged leptons. Here the Z boson interactions are given by

$$\mathcal{L}_{NC} = -\frac{g}{4\cos\theta_W} Z_\mu [\bar{\tau}\gamma^\mu(4x - 1 + \gamma_5)\tau + \bar{E}_\tau\gamma^\mu(4x - 1 - \gamma_5)E_\tau], \tag{38}$$

where $x = \sin^2\theta_W$. We write the result in the mass diagonal basis and get

$$\begin{aligned} \mathcal{L}_{NC} = &-\frac{g}{2\cos\theta_W} Z_\mu \sum_{\alpha=1,2}\sum_{\beta=1,2} (\bar{\tau}_\alpha\gamma^\mu\tau_\beta)\Big(x\{D^{\tau\dagger}_{L\alpha 1} D^{\tau}_{L1\beta} + D^{\tau\dagger}_{R\alpha 1} D^{\tau}_{R1\beta} + D^{\tau\dagger}_{L\alpha 2} D^{\tau}_{L2\beta} + D^{\tau\dagger}_{R\alpha 2} D^{\tau}_{R2\beta}\} \\ &- \frac{1}{2}\{D^{\tau\dagger}_{L\alpha 1} D^{\tau}_{L1\beta} + D^{\tau\dagger}_{R\alpha 2} D^{\tau}_{R2\beta}\}\Big) + (\bar{\tau}_\alpha\gamma^\mu\gamma_5\tau_\beta)\Big(x\{-D^{\tau\dagger}_{L\alpha 1} D^{\tau}_{L1\beta} + D^{\tau\dagger}_{R\alpha 1} D^{\tau}_{R1\beta} - D^{\tau\dagger}_{L\alpha 2} D^{\tau}_{L2\beta} + D^{\tau\dagger}_{R\alpha 2} D^{\tau}_{R2\beta}\} \\ &+ \frac{1}{2}\{D^{\tau\dagger}_{L\alpha 1} D^{\tau}_{L1\beta} - D^{\tau\dagger}_{R\alpha 2} D^{\tau}_{R2\beta}\}\Big). \end{aligned} \tag{39}$$

Next we discuss the neutralino interaction. Using the parts of Eq. (17) that produce the interaction of the mirror lepton with the neutralino we have

$$\mathcal{L} = i\frac{g}{\sqrt{2}}\tau^3_{ij}\lambda^3\psi_j A^*_i + ig'\sqrt{2}Y_i\delta_{ij}\lambda'\psi_j A^*_i - \frac{1}{2}\frac{\partial^2 W}{\partial A_i \partial A_j}\psi_i\psi_j + \text{H.c.} \tag{40}$$

The part of interest in the superpotential here is

$$W = f_2 H^1_1 \tilde{N}^*_R \tilde{N}_L + f'_2 H^2_2 \tilde{E}^*_{\tau R}\tilde{E}_{\tau L}. \tag{41}$$

By using the fermions of the mirror generation and the supersymmetric partners of the neutral Higgs for ψ and the mirror sleptons and neutral Higgs for A one gets the following Lagrangian for the interactions of the mirror leptons with neutralino in the two-component notation

$$\begin{aligned}\mathcal{L} = {} & i\frac{g}{\sqrt{2}}\lambda^3[E^c_{\tau L}\tilde{E}_{\tau R} - N^c_L\tilde{N}_R] + i\frac{g'}{\sqrt{2}}\lambda'[E^c_{\tau L}\tilde{E}_{\tau R} + N^c_L\tilde{N}_R] - i\sqrt{2}g'\lambda' E_{\tau L}\tilde{E}^*_{\tau L} - \frac{g m_N}{\sqrt{2}M_W\cos\beta}[\tilde{N}_L\psi_{H^0_1}N^c_L + \tilde{N}^*_R\psi_{H^0_1}N_L] \\ & - \frac{g m_E}{\sqrt{2}M_W\sin\beta}[\tilde{E}_{\tau L}\psi_{H^0_2}E^c_{\tau L} + \tilde{E}^*_{\tau R}\psi_{H^0_2}\tilde{E}_{\tau L}] + \text{H.c.}\end{aligned} \tag{42}$$

Now we go from two-spinor to four-spinor by defining the four Majorana spinors

$$\tilde{B} = \begin{pmatrix} -i\lambda' \\ i\bar{\lambda}'\end{pmatrix}, \qquad \tilde{W}_3 = \begin{pmatrix} -i\lambda^3 \\ i\bar{\lambda}^3\end{pmatrix}, \qquad \tilde{H}_1 = \begin{pmatrix} \psi_{H^0_1} \\ \bar{\psi}_{H^0_1}\end{pmatrix}, \qquad \tilde{H}_2 = \begin{pmatrix} \psi_{H^0_2} \\ \bar{\psi}_{H^0_2}\end{pmatrix}. \tag{43}$$

The Lagrangian in terms of these fields reads

$$\begin{aligned}\mathcal{L} = {} & \frac{1}{\sqrt{2}}\tilde{N}_R[g\bar{N}P_L\tilde{W}_3 - g'\bar{N}P_L\tilde{B}] - \frac{1}{\sqrt{2}}\tilde{E}_{\tau R}[g\bar{E}_\tau P_L\tilde{W}_3 + g'\bar{E}_\tau P_L\tilde{B}] + \sqrt{2}g'\tilde{E}^*_{\tau L}\bar{\tilde{B}}P_L E_\tau \\ & - \frac{g m_N}{\sqrt{2}M_W\cos\beta}[\tilde{N}_L\bar{N}P_L\tilde{H}_1 + \tilde{N}^*_R\bar{\tilde{H}}_1 P_L N] - \frac{g m_E}{\sqrt{2}M_W\sin\beta}[\tilde{E}_{\tau L}\bar{E}_\tau P_L\tilde{H}_2 + \tilde{E}^*_{\tau R}\bar{\tilde{H}}_2 P_L E_\tau].\end{aligned} \tag{44}$$

We can write this interaction in the neutralino mass eigenstate basis $\tilde{\chi}^0_j$ where

$$X^T M_{\tilde{\chi}^0} X = \text{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}). \tag{45}$$

In writing Eq. (44) in this basis the following relations are found useful

$$\begin{aligned} P_L\tilde{W}_3 &= P_L\sum_{j=1}^4 X_{2j}\tilde{\chi}^0_j, & P_L\tilde{B} &= P_L\sum_{j=1}^4 X_{1j}\tilde{\chi}^0_j, \\ P_L\tilde{H}_1 &= P_L\sum_{j=1}^4 X_{3j}\tilde{\chi}^0_j, & P_L\tilde{H}_2 &= P_L\sum_{j=1}^4 X_{4j}\tilde{\chi}^0_j, \\ \bar{\tilde{H}}_1 P_L &= \sum_{j=1}^4 X_{3j}\bar{\tilde{\chi}}^0_j P_L, & \bar{\tilde{H}}_2 P_L &= \sum_{j=1}^4 X_{4j}\bar{\tilde{\chi}}^0_j P_L, \\ & & \bar{\tilde{B}}P_L &= \sum_{j=1}^4 X_{1j}\bar{\tilde{\chi}}^0_j P_L. \end{aligned} \tag{46}$$

Using the above, the interactions of the mirror lepton E_τ with the neutralino mass eigenstates is given by

$$\begin{aligned}-\mathcal{L}_{E_\tau-\tilde{E}_\tau-\chi^0} = {} & \frac{1}{\sqrt{2}}\sum_{j=1-4}[\bar{E}_\tau(a'_j - b'_j\gamma_5)\tilde{\chi}^0_j\tilde{E}_{\tau R} \\ & + \bar{E}_\tau(c'_j - d'_j\gamma_5)\tilde{\chi}^0_j\tilde{E}_{\tau L}] + \text{H.c.}\end{aligned} \tag{47}$$

Here

$$\begin{aligned} a'_j &= (\alpha_{E_\tau j} + \beta_{E_\tau j}), & b'_j &= (-\alpha_{E_\tau j} + \beta_{E_\tau j}), \\ c'_j &= -(\gamma_{E_\tau j} + \delta_{E_\tau j}), & d'_j &= (\gamma_{E_\tau j} - \delta_{E_\tau j}), \end{aligned} \tag{48}$$

and $\alpha_{E_{\tau j}}$, $\beta_{E_{\tau j}}$, $\gamma_{E_{\tau j}}$ and $\delta_{E_{\tau j}}$ are defined so that

$$\begin{aligned} \alpha_{E_{\tau j}} &= \frac{g m_E X^*_{4j}}{2m_W\sin\beta}, \\ \beta_{E_{\tau j}} &= eX'_{1j} + \frac{g}{\cos\theta_W}X'_{2j}\left(\frac{1}{2} - \sin^2\theta_W\right), \\ \gamma_{E_{\tau j}} &= eX'^*_{1j} - \frac{g\sin^2\theta_W}{\cos\theta_W}X^{*'}_{2j}, \\ \delta_{E_{\tau j}} &= -\frac{g m_E X_{4j}}{2m_W\sin\beta} \end{aligned} \tag{49}$$

and

$$X'_{1j} = (X_{1j}\cos\theta_W + X_{2j}\sin\theta_W),$$
$$X'_{2j} = (-X_{1j}\sin\theta_W + X_{2j}\cos\theta_W). \tag{50}$$

The above may be compared with the interactions of the τ lepton with neutralinos which are given by

$$-\mathcal{L}_{\tau-\tilde{\tau}-\chi^0} = \frac{1}{\sqrt{2}}\sum_{j=1-4}[\bar{\tau}(a_j + b_j\gamma_5)\tilde{\chi}^0_j\tilde{\tau}_L + \bar{\tau}(c_j + d_j\gamma_5)\tilde{\chi}^0_j\tilde{\tau}_R] + \text{H.c.} \tag{51}$$

Here

$$a_j = (\alpha_{\tau j} + \beta_{\tau j}), \qquad b_j = (-\alpha_{\tau j} + \beta_{\tau j}),$$
$$c_j = -(\gamma_{\tau j} + \delta_{\tau j}), \qquad d_j = (\gamma_{\tau j} - \delta_{\tau j}), \tag{52}$$

where

$$\alpha_{\tau j} = \frac{g m_\tau X_{3j}}{2 m_W \cos\beta},$$
$$\beta_{\tau j} = -eX'^*_{1j} + \frac{g}{\cos\theta_W}X'^*_{2j}\left(-\frac{1}{2} + \sin^2\theta_W\right),$$
$$\gamma_{\tau j} = -eX'_{1j} + \frac{g\sin^2\theta_W}{\cos\theta_W}X'_{2j}, \tag{53}$$
$$\delta_{\tau j} = -\frac{g m_\tau X^*_{3j}}{2 m_W \cos\beta}.$$

Rotation into the mass diagonal basis of the leptons and sleptons gives the result

$$-\mathcal{L}_{\tau-\tilde{\tau}-\chi^0} = \frac{1}{\sqrt{2}}\sum_{\alpha=1-2}\sum_{k=1-4}\sum_{j=1-4}\bar{\tau}_\alpha[(D^{\tau\dagger}_+)_{\alpha1}a_j + (D^{\tau\dagger}_-)_{\alpha1}b_j + \gamma_5((D^{\tau\dagger}_-)_{\alpha1}a_j + (D^{\tau\dagger}_+)_{\alpha1}b_j)]\tilde{\chi}^0_j(\tilde{D}^\tau)_{1k}\tilde{\tau}_k$$
$$+ \bar{\tau}_\alpha[(D^{\tau\dagger}_+)_{\alpha1}c_j + (D^{\tau\dagger}_-)_{\alpha1}d_j + \gamma_5((D^{\tau\dagger}_-)_{\alpha1}c_j + (D^{\tau\dagger}_+)_{\alpha1}d_j)]\tilde{\chi}^0_j(\tilde{D}^\tau)_{3k}\tilde{\tau}_k + \bar{\tau}_\alpha[(D^{\tau\dagger}_+)_{\alpha2}a'_j - (D^{\tau\dagger}_-)_{\alpha2}b'_j$$
$$+ \gamma_5((D^{\tau\dagger}_-)_{\alpha2}a'_j - (D^{\tau\dagger}_+)_{\alpha2}b'_j)]\tilde{\chi}^0_j(\tilde{D}^\tau)_{4k}\tilde{\tau}_k + \bar{\tau}_\alpha[(D^{\tau\dagger}_+)_{\alpha2}c'_j - (D^{\tau\dagger}_-)_{\alpha2}d'_j + \gamma_5((D^{\tau\dagger}_-)_{\alpha2}c'_j$$
$$- (D^{\tau\dagger}_+)_{\alpha2}d'_j)]\tilde{\chi}^0_j(\tilde{D}^\tau)_{2k}\tilde{\tau}_k + \text{H.c.} \tag{54}$$

where

$$D^\tau_\pm = \tfrac{1}{2}(D^\tau_L \pm D^\tau_R). \tag{55}$$

Our final result including the mixings of leptons and mirror leptons and the mixings of sleptons and of mirror sleptons are given by Eq. (15) for the W boson interactions, Eq. (36) and (37) for the chargino interactions, by Eq. (39) for the Z boson interactions, and by Eq. (54) for the neutralino interactions.

III. NEUTRINO MAGNETIC MOMENT

The discovery of neutrino masses from the solar and atmospheric data [38–43] has very significantly advanced our understanding of the basic nature of these particles. One outcome of nonvanishing neutrino masses is the possibility that they could possess nonvanishing magnetic and electric dipole moments if the neutrinos are Dirac particles while only transition magnetic moments are allowed if they are Majorana. In this analysis we assume the Dirac nature of the neutrinos. In this case the neutrinos will have nonvanishing magnetic and electric dipole moments and such moments could enter in several physical phenomena [44]. One phenomena where the moments may play a role is in the neutrino spin flip processes such as [45] $\nu_L \to \nu_R + \gamma^*$ or $\nu_L + \gamma^* \to \nu_R$. From experiment, there already exist limits on both the magnetic and the electric dipole moments of neutrinos. Our focus will be the magnetic moment of the τ neutrino which is affected by the mixing effects from the mirror leptons. (For previous work on neutrino magnetic moment with mirror effects in a different context see [20].) The current limits on the magnetic moment of the τ neutrino is [46]

$$|\mu(\nu_\tau)| \leq 1.3 \times 10^{-7}\mu_B \tag{56}$$

where $\mu_B = (e/2m_e)$ is the Bohr magneton. The magnetic moment of the neutrino arises in the standard model at one loop via the exchange of the W boson assuming one extends the standard model to include a right-handed neutrino (see Fig. 1), and in the supersymmetric models there are additional contributions arising from the chargino exchange contributions (see Fig. 2).

Neutrino masses for the first three generations are very small, i.e., from WMAP data one has $\sum_i |m_{\nu_i}| \leq$ (0.7–1) eV [47]. If the neurtinos are Dirac, one would need to explain how such tiny Dirac masses are generated which would typically require fine tunings of $O(10^{-10})$ or more. However, unlike the Majorana neutrino case for which there is a standard mechanism for the generation of small neutrino masses, i.e., seesaw, there is no standard mechanism for the generation of small Dirac neutrino masses. Indeed this topic continues to be a subject of ongoing research and several recent works can be found in [48,49]. Here, we do not go into details on this topic which would take us far afield. Thus in this work we do not make any attempt to deduce the smallness of the neutrino masses but rather assume this is the case. With this assumption we discuss below the τ neutrino magnetic moment in the extended MSSM with mirrors for the case

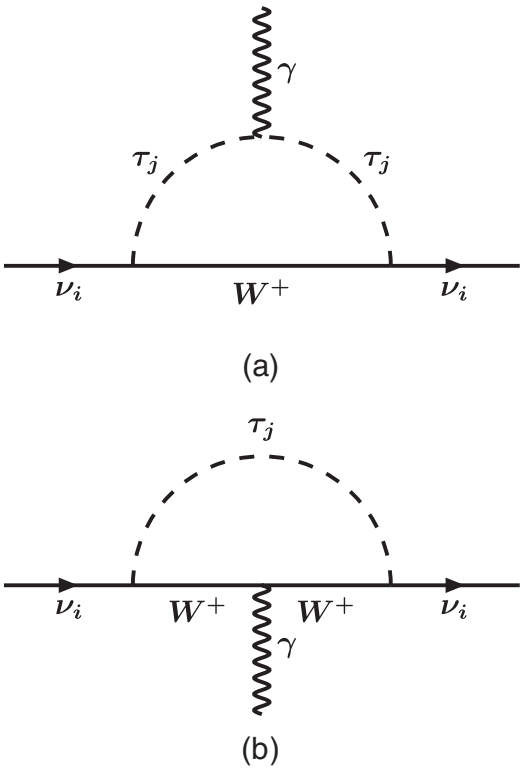

FIG. 1. The loop contributions to the magnetic dipole moment of neutrinos (ν_i) via the exchange of W^+ boson and via the exchange of leptons and mirror leptons denoted by τ_j.

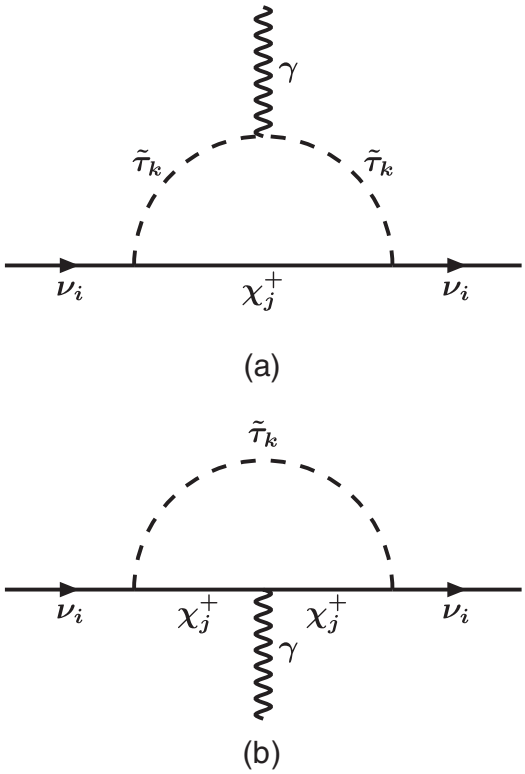

FIG. 2. The supersymmetric loop contributions to the magnetic dipole moment of neutrinos (ν_i) via the exchange of charginos (χ_j^+), sleptons, and mirror sleptons denoted by $\tilde{\tau}_k$.

when there is mixing with the mirror leptons. The contributions to be discussed arise from loops containing (1) lepton (mirror lepton) W boson and (2) scalar leptons (scalar mirrors) charginos. From Eq. (15) one can calculate the W boson, charged lepton and charged mirror lepton contributions arising from Fig. 1 to the magnetic moment of the τ neutrino in μ_B units to be

$$\mu_\nu^{(1)} = \frac{-G_F m_e}{8\pi^2\sqrt{2}} \sum_{\gamma=1}^{2}\sum_{\delta=1}^{2}\sum_{\beta=1}^{2} m_{\tau_\beta} G_1\left(\frac{m_{\tau_\beta}}{M_W}\right)(|(D_L^{\nu\dagger})_{1\gamma} g^L_{\gamma\delta}(D_L^\tau)_{\delta\beta} + (D_R^{\nu\dagger})_{1\gamma} g^R_{\gamma\delta}(D_R^\tau)_{\delta\beta}|^2 - |(D_L^{\nu\dagger})_{1\gamma} g^L_{\gamma\delta}(D_L^\tau)_{\delta\beta}$$
$$- (D_R^{\nu\dagger})_{1\gamma} g^R_{\gamma\delta}(D_R^\tau)_{\delta\beta}|^2) + \frac{3G_F m_\nu m_e}{16\pi^2\sqrt{2}} \sum_{\gamma=1}^{2}\sum_{\delta=1}^{2}\sum_{\beta=1}^{2} G_2\left(\frac{m_{\tau_\beta}}{M_W}\right)(|(D_L^{\nu\dagger})_{1\gamma} g^L_{\gamma\delta}(D_L^\tau)_{\delta\beta} + (D_R^{\nu\dagger})_{1\gamma} g^R_{\gamma\delta}(D_R^\tau)_{\delta\beta}|^2$$
$$+ |(D_L^{\nu\dagger})_{1\gamma} g^L_{\gamma\delta}(D_L^\tau)_{\delta\beta} - (D_R^{\nu\dagger})_{1\gamma} g^R_{\gamma\delta}(D_R^\tau)_{\delta\beta}|^2), \tag{57}$$

where the form factor functions $G_1(r)$ and $G_2(r)$ are given by

$$G_1(r) = \frac{4 - r^2}{1 - r^2} + \frac{3r^2}{(1 - r^2)^2}\ln(r^2),$$
$$G_2(r) = \frac{2 - 5r^2 + r^4}{(1 - r^2)^2} - \frac{2r^4}{(1 - r^2)^3}\ln(r^2). \tag{58}$$

As noted already Eq. (57) includes the contributions from the τ and from the mirror lepton. We parametrize the mixing between τ and E_τ by the angle θ, where

$$\begin{pmatrix} \tau \\ E_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}, \tag{59}$$

and the mixing between ν and N by the angle ϕ where

$$\begin{pmatrix} \nu \\ N \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \tag{60}$$

where we take $D_L^\tau = D_R^\tau$ and $D_L^\nu = D_R^\nu$ or $\theta_L = \theta_R = \theta$ and $\phi_L = \phi_R = \phi$. These are simplicity assumptions to get the size of numerical estimates and are easily improved with better understanding of mixings with mirror and ordinary leptons. We identify τ_1 with the physical τ and

τ_2 with the mirror generation lepton. When there is no risk of confusion we will set $\tau_1 = \tau$ and $\tau_2 = E$, and similarly for the ν_1 and ν_2 where we set $\nu_1 = \nu_\tau$ and $\nu_2 = N$. Now we see that the first term of Eq. (57) is proportional to the fermion mass m_{τ_β} which could be a lepton or a mirror lepton. For the lepton loop $\beta = 1$, the first term in Eq. (57) is proportional to $[\cos^2(\theta - \phi) - \cos^2(\theta + \phi)]$ and the second term is proportional to $[\cos^2(\theta - \phi) + \cos^2(\theta + \phi)]$. For the mirror lepton loop $\beta = 2$, and the first term in Eq. (57) is proportional to $[\sin^2(\theta - \phi) - \sin^2(\theta + \phi)]$ while the second term in Eq. (57) is proportional to $[\sin^2(\theta - \phi) + \sin^2(\theta + \phi)]$. Thus if the mixing between lepton and mirror leptons exist, the first term for the case of $\beta = 2$ can produce a large contribution to the neutrino magnetic moment if the mirror lepton mass is in the region of a few hundred GeV. Also if this mixing is absent, the contribution would come only from the τ-lepton loop. In this case, the first term does not contribute and the second term gives the result

$$\frac{3m_\tau m_{\nu_\tau} G_F}{4\sqrt{2}\pi^2}, \tag{61}$$

taking into account the limit $G_2(0) = 2$. Thus Eq. (57) gives for the neutrino magnetic moment the value of $3.2 \times 10^{-19}(\frac{m_\nu}{eV})\mu_B$ and agrees with the previous analyses given in the standard model [50,51]. We note that the underlying assumptions of [50,51] regarding a small Dirac mass is identical to ours except that our analysis is more general in that it includes both supersymmetry and mirror contributions.

Next we compute the supersymmetric contributions to the ν_τ magnetic moment which include the chargino, the slepton, and the mirror slepton contributions which can be calculated using Eq. (36). The result in μ_B units is

$$\begin{aligned}\mu_\nu^{(2)} = &-\frac{g^2 m_e}{16\pi^2}\sum_{k=1}^{2}\sum_{j=1}^{4}\frac{1}{m_{\chi_k^+}}\{\kappa_\nu|\tilde{D}_{1j}^{\tau}|^2\mathrm{Re}(D_{L_{11}}^{\nu\dagger}U_{k1}D_{R_{11}}^{\nu}V_{k2}) + \kappa_N|\tilde{D}_{4j}^{\tau}|^2\mathrm{Re}(D_{R_{12}}^{\nu\dagger}V_{k1}^{*}D_{L_{12}}^{\nu}U_{k2}^{*})\}G_3\left(\frac{M_{\tilde{\tau}_j}}{m_{\chi_k^+}}\right)\\ &+\frac{g^2 m_e m_{\nu_\tau}}{96\pi^2}\sum_{k=1}^{2}\sum_{j=1}^{4}\frac{1}{m_{\chi_k^+}^2}\{|\tilde{D}_{1j}^{\tau}|^2[|D_{L_{11}}^{\nu\dagger}U_{k1}|^2 + \kappa_\nu^2|D_{R_{11}}^{\nu\dagger}V_{k2}^{*}|^2] + \kappa_\tau^2|\tilde{D}_{3j}^{\tau}|^2|D_{L_{11}}^{\nu\dagger}U_{k2}|^2\\ &+|\tilde{D}_{4j}^{\tau}|^2[|D_{R_{12}}^{\nu\dagger}V_{k1}^{*}|^2 + \kappa_N^2|D_{L_{12}}^{\nu\dagger}U_{k2}|^2] + \kappa_{E_\tau}^2|\tilde{D}_{2j}^{\tau}|^2|D_{R_{12}}^{\nu\dagger}V_{k2}^{*}|^2\}G_4\left(\frac{M_{\tilde{\tau}_j}}{m_{\chi_k^+}}\right)\end{aligned} \tag{62}$$

where

$$\begin{aligned}G_3(r) &= \frac{-2}{r^2-1} + \frac{2r^2}{(r^2-1)^2}\ln(r^2),\\ G_4(r) &= \frac{3(1+r^2)}{(1-r^2)^2} + \frac{6r^2}{(1-r^2)^3}\ln(r^2).\end{aligned} \tag{63}$$

The numerical sizes of the neutrino moments $\mu_\nu^{(1)}$ and $\mu_\nu^{(2)}$ will be discussed in Sec. V.

IV. τ ANOMALOUS MAGNETIC MOMENT

An evaluation of the anomalous magnetic moment in the standard model gives $a_\tau^{\mathrm{SM}} = 117\,721(5) \times 10^{-8}$, where $a_\tau = \frac{g_\tau - 2}{2}$. The experimental limits on this parameter are [52] $-0.052 < a_\tau^{\mathrm{exp}} < 0.013$ and so the sensitivity is more than 1 order of magnitude below where one can see the effects of the τ anomalous magnetic moment. Here, we calculate the corrections to the τ anomalous magnetic moment including new physics effects from the supersymmetrized mirror sector which mixes with the τ lepton sector. Specifically we compute four different types of loop corrections to a_τ. These include the following exchanges in the loops: (1) W boson and neutral mirror leptons; (2) Z boson and charged mirror leptons; (3) chargino and scalar neutrinos-mirror scalar neutrinos, and (4) neutralino, charged scalar leptons-mirror scalar leptons. Using Eq. (15), one can write the contribution from the W boson loop so that

$$\begin{aligned}\Delta^{(1)}a_\tau = &\frac{g^2}{8}\frac{m_\tau}{16\pi^2 M_W}\sum_{\alpha,\gamma,\delta=1,2}[|(D_L^{\nu\dagger})_{\alpha\gamma}g_{\gamma\delta}^{L}(D_L^{\tau})_{\delta 1}\\ &+(D_R^{\nu\dagger})_{\alpha\gamma}g_{\gamma\delta}^{R}(D_R^{\tau})_{\delta 1}|^2 - |(D_L^{\nu\dagger})_{\alpha\gamma}g_{\gamma\delta}^{L}(D_L^{\tau})_{\delta 1}\\ &-(D_R^{\nu\dagger})_{\alpha\gamma}g_{\gamma\delta}^{R}(D_R^{\tau})_{\delta 1}|^2]h_2\left(\frac{m_{\nu_\alpha}}{M_W}\right),\end{aligned} \tag{64}$$

where

$$h_2(r) = \frac{6r^5}{(r^2-1)^3}\ln r^2 + \frac{r^5 - 11r^3 + 4r}{(r^2-1)^2}. \tag{65}$$

Using Eq. (39), one can write the contribution from the Z boson loop

$$\begin{aligned}\Delta^{(2)}a_\tau = &\frac{g^2}{4\cos^2\theta_W}\frac{m_\tau}{16\pi^2 M_Z}\sum_{j=1,2}|x[-(D_L^{\tau\dagger})_{j1}(D_L^{\tau})_{11}\\ &+(D_R^{\tau\dagger})_{j1}(D_R^{\tau})_{11} - (D_L^{\tau\dagger})_{j2}(D_L^{\tau})_{21}\\ &+(D_R^{\tau\dagger})_{j2}(D_R^{\tau})_{21}] + \frac{1}{2}[(D_L^{\tau\dagger})_{j1}(D_L^{\tau})_{11}\\ &-(D_R^{\tau\dagger})_{j2}(D_R^{\tau})_{21}]|^2 h_1\left(\frac{m_{\tau_j}}{M_Z}\right),\end{aligned} \tag{66}$$

where x is defined by Eq. (38) and

$$h_1(r) = -\frac{6r^3}{(r^2-1)^3}\ln r^2 + \frac{r^5+r^3+4r}{(r^2-1)^2}. \tag{67}$$

Next using Eq. (37), one can write the contribution from the chargino, scalar neutrino, and scalar mirror neutrino as

$$\begin{aligned}\Delta^{(3)}a_\tau = {} & \frac{g^2 m_\tau}{16\pi^2}\sum_{i=1}^{2}\sum_{j=1}^{4}\frac{1}{m_{\chi_i^+}}\{\kappa_\tau|\tilde{D}^\nu_{1j}|^2\mathrm{Re}(D^{\tau\dagger}_{L_{11}}V_{i1}D^\tau_{R_{11}}U_{i2}) + \kappa_{E_\tau}|\tilde{D}^\nu_{4j}|^2\mathrm{Re}(D^{\tau\dagger}_{R_{12}}U^*_{i1}D^\tau_{L_{12}}V^*_{i2})\}F_3\left(\frac{M^2_{\tilde{\nu}_j}}{m^2_{\chi_i^+}}\right)\\ & + \frac{g^2 m_\tau^2}{96\pi^2}\sum_{i=1}^{2}\sum_{j=1}^{4}\frac{1}{m^2_{\chi_i^+}}\{|\tilde{D}^\nu_{1j}|^2[|D^{\tau\dagger}_{L_{11}}V_{i1}|^2 + \kappa_\tau^2|D^{\tau\dagger}_{R_{11}}U^*_{i2}|^2] + \kappa_\nu^2|\tilde{D}^\nu_{3j}|^2|D^{\tau\dagger}_{L_{11}}V_{i2}|^2\\ & + |\tilde{D}^\nu_{4j}|^2[|D^{\tau\dagger}_{R_{12}}U^*_{i1}|^2 + \kappa_{E_\tau}^2|D^{\tau\dagger}_{L_{12}}V_{i2}|^2] + \kappa_N^2|\tilde{D}^\nu_{2j}|^2|D^{\tau\dagger}_{R_{12}}U^*_{i2}|^2\}F_4\left(\frac{M^2_{\tilde{\nu}_j}}{m^2_{\chi_i^+}}\right)\end{aligned} \tag{68}$$

where

$$F_3(x) = \frac{1}{(x-1)^3}(3x^2 - 4x + 1 - 2x^2\ln x), \tag{69}$$

and

$$F_4(x) = \frac{1}{(x-1)^4}(2x^3 + 3x^2 - 6x + 1 - 6x^2\ln x). \tag{70}$$

Further, using Eq. (54), one can write the contribution from the neutralino, scalar lepton, and scalar mirror lepton as

$$\begin{aligned}\Delta^{(4)}a_\tau = {} & -\frac{m_\tau}{32\pi^2}\sum_{k=1}^{4}\sum_{j=1}^{4}\frac{1}{m_{\chi_j^0}}F_1\left(\frac{M^2_{\tilde{\tau}_k}}{m^2_{\chi_j^0}}\right)|\tilde{D}^\tau_{1k}|^2\{|(D^{\tau\dagger}_+)_{11}a_j + (D^{\tau\dagger}_-)_{11}b_j|^2 - |(D^{\tau\dagger}_-)_{11}a_j + (D^{\tau\dagger}_+)_{11}b_j|^2\}\\ & + |\tilde{D}^\tau_{4k}|^2\{|(D^{\tau\dagger}_+)_{12}a'_j - (D^{\tau\dagger}_-)_{12}b'_j|^2 - |(D^{\tau\dagger}_-)_{12}a'_j - (D^{\tau\dagger}_+)_{11}b'_j|^2\} + |\tilde{D}^\tau_{3k}|^2\{|(D^{\tau\dagger}_+)_{11}c_j + (D^{\tau\dagger}_-)_{11}d_j|^2\\ & - |(D^{\tau\dagger}_-)_{11}c_j + (D^{\tau\dagger}_+)_{11}d_j|^2\} + |\tilde{D}^\tau_{2k}|^2\{|(D^{\tau\dagger}_+)_{12}c'_j - (D^{\tau\dagger}_-)_{12}d'_j|^2 - |(D^{\tau\dagger}_-)_{12}c'_j - (D^{\tau\dagger}_+)_{12}d'_j|^2\}\\ & + \frac{m_\tau^2}{96\pi^2}\sum_{k=1}^{4}\sum_{j=1}^{4}\frac{1}{m^2_{\chi_j^0}}F_2\left(\frac{M^2_{\tilde{\tau}_k}}{m^2_{\chi_j^0}}\right)|\tilde{D}^\tau_{1k}|^2\{|(D^{\tau\dagger}_+)_{11}a_j + (D^{\tau\dagger}_-)_{11}b_j|^2 + |(D^{\tau\dagger}_-)_{11}a_j + (D^{\tau\dagger}_+)_{11}b_j|^2\}\\ & + |\tilde{D}^\tau_{4k}|^2\{|(D^{\tau\dagger}_+)_{12}a'_j - (D^{\tau\dagger}_-)_{12}b'_j|^2 + |(D^{\tau\dagger}_-)_{12}a'_j - (D^{\tau\dagger}_+)_{11}b'_j|^2\} + |\tilde{D}^\tau_{3k}|^2\{|(D^{\tau\dagger}_+)_{11}c_j + (D^{\tau\dagger}_-)_{11}d_j|^2\\ & + |(D^{\tau\dagger}_-)_{11}c_j + (D^{\tau\dagger}_+)_{11}d_j|^2\} + |\tilde{D}^\tau_{2k}|^2\{|(D^{\tau\dagger}_+)_{12}c'_j - (D^{\tau\dagger}_-)_{12}d'_j|^2 + |(D^{\tau\dagger}_-)_{12}c'_j - (D^{\tau\dagger}_+)_{12}d'_j|^2\},\end{aligned} \tag{71}$$

where

$$F_1(x) = \frac{1}{(x-1)^3}(1 - x^2 + 2x\ln x), \tag{72}$$

and

$$F_2(x) = \frac{1}{(x-1)^4}(-x^3 + 6x^2 - 3x - 2 - 6x\ln x). \tag{73}$$

The numerical sizes of $\Delta^{(1)}a_\tau - \Delta^{(4)}a_\tau$ are discussed in the next section.

V. CONSTRAINTS AND SIZE ESTIMATES

There are severe phenomenological constraints on extra matter beyond the Standard Model. These constraints can be listed as follows: (1) constraints from the data on the Z width; (2) constraints from direct searches; (3) unitarity constraints on the enlarged 4×4 CKM matrix; (4) constraints from the oblique electroweak effects; and (5) constraints on Yukawas arising from keeping the theory perturbative, i.e., avoid developing a Landau pole. Many of these constraints have been investigated in the context of a sequential fourth generation [1–4,9,10,53] with the analysis of [1] being the most recent and the most detailed. We summarize the main results of these analyses below. First of all constraint (1) can be easily avoided by making the masses of the new particles greater than half the Z boson width, while (2) can be satisfied by putting lower bounds on new matter from all collider data. For example, the LEP II data puts bounds on charged leptons of about a 100 GeV, while the Tevatron puts bounds on the fourth generation quark masses so that [1] $m_{u_4} > 258$ GeV (95%

CL) and $m_{d_4} > 268$ GeV (at 95% CL). (3) Regarding the CKM unitarity constraints the enlarged CKM matrix allows a small window for mixings with the fourth generation so that [1] $|V_{14}| \leq 0.04$, $|V_{41}| \leq 0.08$, $|V_{24}| \leq 0.17$ and there are similar constraints on the other mixings which allow for non-negligible elements for mixings with the fourth generation.

Perhaps the most stringent of the constraints is (4) which comes from the oblique parameters (S, T, U) [54,55] and specifically from the oblique parameter S (for a recent review of the S, T, U fits to the electroweak data see Ref. [56,57]). Here a complete fourth generation with degenerate masses gives a contribution of about 0.2. However, this correction can be reduced when one considers splittings of the up and the down fermions in the same multiplet. Using such splittings analyses including the fourth generation allow for consistent (S, T, U) fits to the data (see, e.g., [1,3]). (5) Finally it has been shown that the Yukawa couplings can remain perturbative up to the grand unification scale for a range of fourth generation masses and Higgs boson parameters. Thus problems such as the generation of Landau pole singularities for a large fourth generation up quark mass can be avoided with appropriate parameter choices. Essentially all of the considerations valid for the sequential fourth generation are also valid for the mirror generation. Thus, for example, consider a fourth generation with up and down fermions (ψ_1, ψ_2) with hypercharge Y and masses (M_1, M_2). The transformation that takes us from fermions to mirror fermions is

$$\begin{aligned} &\text{fermions}(\psi_1, \psi_2) \leftrightarrow \text{mirror fermions}(\psi_2^c, \psi_1^c), \\ &\qquad Y \leftrightarrow -Y, \\ &\qquad M_1 \leftrightarrow M_2. \end{aligned} \tag{74}$$

Using the above, one finds that ΔS contribution from the mirror generation is the same as for the fourth sequential generation [58]. Without going into further details, we assume that fits to the electroweak data similar to those for the sequential fourth generation can be carried out for the case of the mirror generation.

Beyond the constraints on a new generation discussed above a mirror generation encounters two more issues. The first concerns avoidance of the survival hypothesis [12], i.e., a mirror generation and an ordinary generation can combine to get super heavy masses of GUT size or string scale size. However, it is well known that some of the mirror generations do escape gaining super heavy masses and remain light up to the electroweak scale [15,16]. We assume in this analysis that this indeed is the case for one mirror generation. The second issue concerns the mixing of the mirror generation with the ordinary generations. In this work we assume that the mixing primarily occurs with the third generation. In this circumstance the third generation will develop a small $V + A$ structure in addition to the expected $V - A$ structure. Indeed such a $V + A$ component for some of the third generation particles has been looked at for some time [59,60]. Here we point out that the current data regarding the third generation leaves open the possibility of new physics. For instance, the analysis of [26] finds a better fit to the precision electroweak data, and specifically a better fit to the forward-backward asymmetry A_{FB}^b of the b-quark, with additional bottom like quarks. Similarly, a model-independent measurement of the W boson helicity in the top quark decay $t \to Wb$ at D0 [61], gives for the longitudinal fraction f_0 and for the right-handed fraction (f_+) the result $f_0 = 0.425 \pm 0.166(\text{stat}) \pm 0.102(\text{syst})$ and $f_+ = 0.119 \pm 0.090(\text{stat}) \pm 0.053(\text{syst})$ while f_- is determined via the constraint $f_0 + f_+ + f_- = 1$. While the model-independent analysis above is consistent with the standard model prediction with $V - A$ structure of $f_0 = 0.697$, $f_+ = 3.6 \times 10^{-4}$, the analysis shows that a different Lorentz structure such as $V + A$ is not ruled out at the level of a few percent. A similar situation occurs in the analysis of τ lepton decays where new physics at the level of a few percent is not necessarily ruled out [62,63].

The mixing parameters and the masses of the mirror fermion sector are determined by the input parameters θ, ϕ, m_N, and m_{E_τ}, where we assume that $\theta_L = \theta_R = \theta$ and $\phi_L = \phi_R = \phi$ for the purpose of numerical investigation. However, these parameters are not independent but constrained by the symmetry breaking relation (13) which we use to determine ϕ in terms of the other parameters. The scalar sector is determined by the mixing angles $\tilde{\theta}_{1,2}$ and $\tilde{\phi}_{1,2}$ and the simplifying assumption that the scalar (mass)^2 4×4 matrix factorizes into two 2×2 block diagonal matrices. If we further assume that $M_{ij}^2 = M_{i+2j+2}^2$ we have the conditions $\tilde{\theta}_1 = \tilde{\theta}_2$ and $\tilde{\phi}_1 = \tilde{\phi}_2$. The remaining parameters are M_{11}^2 and M_{22}^2 for both the scalar τ and scalar neutrino (mass)^2 matrices. The scalar spectrum is then calculated from the formulas given in Appendix A.

The mixings between the third generation and the mirrors can affect among other things the magnetic moments. This is specifically true for the magnetic moment of the τ neutrino which we discuss next. In this case there will be two contributions, one from the non-supersymmetry sector (see Fig. 1) and the other from the supersymmetry sector (see Fig. 2). Similar contributions also arise for the anomalous magnetic moment of the τ. An analysis of these moments is given in Table I. Here we exhibit numerical sizes of the different contributions to the τ neutrino magnetic moments, i.e., $\mu_\nu^{(1)}$ and $\mu_\nu^{(2)}$ and to the anomalous magnetic moment of the τ, i.e., $\Delta^{(1)}a_\tau - \Delta^{(4)}a_\tau$. The numerical results of the table show that the contribution to the τ neutrino magnetic moment is as much as 8 orders of magnitude larger than what the model without mirror mixings will give. These results may be compared with the prediction of the standard model (extended with a right-handed neutrino) which is $\mu_\nu = O(10^{-19})(m_\nu/\text{eV})\mu_B$. The standard model value for the magnetic moment is

TABLE I. Table caption: Contributions to the magnetic moments of ν_τ and of τ including corrections from the mirror particles and mirror sparticles for a variety of mixing angles between the third generation and the mirror generation consistent with the symmetry breaking constraint of Eq. (13). The other input parameters are $\tan\beta = 20$, $m_0 = 400$, $m_{1/2} = 150$, $A_0 = 400$, $m_E = 200$, $m_N = 220$, $M_{\tilde{\tau}_{11}} = 400$, $M_{\tilde{\tau}_{22}} = 500$, $m_{\tilde{\nu}_{11}} = 420$ and $m_{\tilde{\nu}_{22}} = 520$, and $\mu > 0$. All masses are in units of GeV and all angles are in radian.

θ	$\tilde{\phi}$	$\tilde{\theta}$	$\Delta^{(1)}a_\tau \times 10^6$	$\Delta^{(2)}a_\tau \times 10^7$	$\Delta^{(3)}a_\tau \times 10^7$	$\Delta^{(4)}a_\tau \times 10^8$	$\mu_\nu^{(1)}/\mu_B \times 10^{10}$	$\mu_\nu^{(2)}/\mu_B \times 10^{10}$
0.2	0.3	0.4	5.0	18.0	2.4	−8.1	−24.0	15.0
0.15	0.35	0.45	2.8	10.0	1.4	−4.8	−14.0	8.7
0.10	0.2	0.3	1.3	4.7	0.59	−1.92	−6.2	3.8
0.09	0.0	0.2	1.06	3.8	0.47	−1.52	−4.90	3.1
0.08	0.2	0.1	0.84	3.0	0.38	−1.19	−3.95	2.4
0.07	0.1	0.0	0.65	2.30	0.29	−0.91	−3.04	1.8
0.06	0.0	0.2	0.48	1.70	0.21	−0.67	−2.23	1.4
0.05	0.2	0.1	0.33	1.18	0.15	−0.64	−1.55	0.94
0.04	0.1	0.0	0.21	0.76	0.09	−0.30	−0.99	0.60
0.03	0.0	0.2	0.12	0.43	0.05	−0.17	−0.56	0.34
0.02	0.2	0.1	0.05	0.19	0.03	−0.07	−0.25	0.15
0.01	0.1	0.0	0.013	0.048	0.006	−0.02	−0.062	0.037

too small and falls beyond any reasonable possibility of observation. In contrast the result arising from mixing with the mirror sector is only 2–3 orders of magnitude below the current limits and thus not outside the realm of observability. At the same time, we note that the contribution of the mirror sector to the anomalous magnetic moment of the τ lepton gives only a small correction to the standard model prediction.

VI. LHC SIGNATURES OF THE MIRROR SECTOR

Before discussing the LHC signatures of the mirror sector it is useful to list the new particles that arise in the model beyond those that appear in MSSM. In the fermionic sector the new particles are

$$B, T, E, N \tag{75}$$

where all fields including N are Dirac. In the bosonic sector the new particles in the mass diagonal states are

$$\tilde{B}_1, \tilde{B}_2, \tilde{T}_1, \tilde{T}_2, \tilde{E}_1, \tilde{E}_2, \tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3. \tag{76}$$

We note the appearance of three sneutrino states in Eq. (76). This is so because we started out with two extra chiral singlets, one in the MSSM sector and another in the mirror generation. Along with the two chiral neutrino states that arise from the doublets they produce four sneutrino states, one of which is in the MSSM sector and the other three are listed in Eq. (76).

In the extended MSSM with mirrors, the mirror fermions and their supersymmetric partners, the mirror sfermions, could produce interesting signatures at the LHC and at the International Linear Collider. Thus, for example, if the mirror generation mixes only with the third generation one will have decays of the following type (if $M_N > M_E + M_W$),

$$N \to E^- W^+, \qquad E^- \to \tau^- Z \to \tau^- e^+ e^-, \qquad \tau^- \mu^+ \mu^-, \qquad \tau^+ \tau^+ \tau^-. \tag{77}$$

This signal is unique in the sense that there is always at least one τ. Specifically, there is no corresponding signal where one has all three leptons of the first generation, or of the second generation or a mixture there of. These signatures are uniquely different from the leptonic signatures in MSSM, for example, from those arising from the decay of an off shell W^* [64], where $\tilde{W}^* \to \tilde{W} + \chi_2^0 \to l_1 l_2 \bar{l}_2$, i.e., with a W^* decaying into a chargino and the second lightest neutralino. Here all leptonic generations appear in all final states. Another interesting signature is the Drell-Yan process

$$pp \to Z^* \to E^+ E^- \to 2\tau 4l, 4\tau 2l, 6\tau, \tag{78}$$

where $l_1, l_2 = e, \mu$. Additionally, of course, there can be events with τs, leptons, and jets. In each case one has two opposite sign τs. Similarly one can have $pp \to Z^* \to N\bar{N}$ production. One can also have the production of mirrors via W^* exchange, i.e., via the process

$$pp \to W^* \to EN \to [\tau l_i \bar{l}_i, 3\tau, (\tau + 2\,\text{jets})] + E_T^{\text{miss}}. \tag{79}$$

Again the leptonic events always have a τ with no events of the type $l_1 l_2 \bar{l}_2$. Similarly decay chains exist with other mass hierarchies, e.g., when N is lighter than E. Additionally for the supersymmetric sector of mirMSSM one has production and decays of $\tilde{E}_{1,2}$ and $\tilde{\nu}_i$ ($i = 1, 2, 3$). For example, for the case, when $\tilde{\nu}_i$ are heavier than $\tilde{E}_k$ one has decays

$$\tilde{\nu}_i \to \tilde{E}_k^- W^+, E^- \tilde{\chi}^+ \tag{80}$$

with subsequent decays of E^-, $\tilde{E}_k^-$, etc. Thus one has processes of the type

$$pp \to \tilde{\nu}_i \tilde{\nu}_i^* \to \tilde{E}_k^+ \tilde{E}_k^- W^+ W^-, \qquad \tilde{E}_k^+ E^- W^\mp \tilde{\chi}^\pm. \quad (81)$$

Combined with the decays of the $\tilde{E}^+\tilde{E}^-$ one can get signatures with τs + leptons + jets + $E_T^{\rm miss}$ with as many as eight leptons, where all the leptons could be τs. Another important signature is the radiative decay [65] of N where

$$N \to \nu_\tau \gamma. \quad (82)$$

This decay occurs via the transition electric and magnetic moments. The lifetime for the decay is very short and once N is produced it will decay inside the detector. The signal will consist of a very energetic photon with energy in the 100 GeV range. Thus if kinematically allowed h^0, A^0 will have decays of the following types

$$(h^0, H^0, A^0) \to N\bar{N} \to 2\gamma + E_T^{\rm miss}. \quad (83)$$

Once a new generation is seen, a study of their production and decay can reveal if they are a sequential generation or a mirror generation. Let us consider the sequential fourth generation first with the superpotential

$$W_{\text{fourth-seq}} = \epsilon_{ij}[y_{4e}\hat{H}_1^i \hat{\psi}_{4L}^j \hat{e}_{4L}^c + y_{4d}\hat{H}_1^i \hat{q}_{4L}^j \hat{d}_{4L}^c + y_{4u}\hat{H}_2^j \hat{q}_{4L}^i \hat{u}_{4L}^c + y_{4\nu}\hat{H}_2^j \hat{\psi}_{4L}^i \hat{\nu}_{4L}^c] \quad (84)$$

which relate the Yukawas with the fermion masses for the fourth generation so that

$$y_{4u} = \frac{g m_{4u}}{\sqrt{2} M_W \sin\beta}, \qquad y_{4\nu} = \frac{g m_{4\nu}}{\sqrt{2} M_W \sin\beta},$$
$$y_{4e} = \frac{g m_{4e}}{\sqrt{2} M_W \cos\beta}, \qquad y_{4d} = \frac{g m_{4d}}{\sqrt{2} M_W \cos\beta}. \quad (85)$$

For the mirror generation we have

$$W_{\text{fourth}-m} = \epsilon_{ij}[f_2 \hat{H}_1^i \hat{\chi}^{cj} \hat{N}_L + f_2' \hat{H}_2^j \hat{\chi}^{ci} \hat{E}_{\tau L} + Y_B \hat{H}_2^j \hat{Q}^{ci} \hat{B}_L + Y_T \hat{H}_1^i \hat{Q}^{cj} \hat{T}_L] \quad (86)$$

and the relation among the Yukawas and the mirror fermion's masses are

$$f_2 = \frac{g M_N}{\sqrt{2} M_N \cos\beta}, \qquad Y_T = \frac{g M_T}{\sqrt{2} M_W \cos\beta},$$
$$f_2' = \frac{M_E}{\sqrt{2} M_W \sin\beta}, \qquad Y_B = \frac{g M_B}{\sqrt{2} M_W \sin\beta}. \quad (87)$$

The neutral Higgs mass eigenstates h^0, H^0, and A^0 are related to the electroweak eigenstates H_1^1 and H_2^2 by

$$H_1^1 = \frac{1}{\sqrt{2}}[v_1 + H^0 \cos\alpha - h^0 \sin\alpha + iA^0 \sin\beta]$$
$$H_2^2 = \frac{1}{\sqrt{2}}[v_2 + H^0 \sin\alpha + h^0 \cos\alpha + iA^0 \cos\beta]. \quad (88)$$

The neutral Higgs couplings of h^0, H^0 and of the CP odd Higgs boson A^0 with the sequential fourth generation in the Lagrangian take the form

$$-\mathcal{L} = \frac{g}{2M_W}\left(\frac{m_{4e}\cos\alpha}{\cos\beta}\bar{e}_4 e_4 + \frac{m_{4d}\cos\alpha}{\cos\beta}\bar{d}_4 d_4 + \frac{m_{4u}\sin\alpha}{\sin\beta}\bar{u}_4 u_4 + \frac{m_{4\nu}\sin\alpha}{\sin\beta}\bar{\nu}_4 \nu_4\right)H^0$$
$$+ \frac{g}{2M_W}\left(-\frac{m_{4e}\sin\alpha}{\cos\beta}\bar{e}_4 e_4 - \frac{m_{4d}\sin\alpha}{\cos\beta}\bar{d}_4 d_4 + \frac{m_{4u}\cos\alpha}{\sin\beta}\bar{u}_4 u_4 + \frac{m_{4\nu}\cos\alpha}{\sin\beta}\bar{\nu}_4 \nu_4\right)h^0$$
$$- \frac{ig}{2M_W}(m_{4e}\bar{e}_4\gamma_5 e_4 \tan\beta + m_{4d}\bar{d}_4\gamma_5 d_4 \tan\beta + m_{4u}\bar{u}_4\gamma_5 u_4 \cot\beta + m_{4\nu}\bar{\nu}_4\gamma_5\nu_4 \cot\beta)A^0, \quad (89)$$

while for the mirror generation it takes the form

$$-\mathcal{L} = \frac{g}{2M_W}\left(\frac{M_E\sin\alpha}{\sin\beta}\bar{E}E + \frac{M_B\sin\alpha}{\sin\beta}\bar{B}B + \frac{M_T\cos\alpha}{\cos\beta}\bar{T}T + \frac{M_N\cos\alpha}{\cos\beta}\bar{N}N\right)H^0$$
$$+ \frac{g}{2M_W}\left(\frac{M_E\cos\alpha}{\sin\beta}\bar{E}E + \frac{M_B\cos\alpha}{\sin\beta}\bar{B}B - \frac{M_T\sin\alpha}{\cos\beta}\bar{T}T - \frac{M_N\sin\alpha}{\cos\beta}\bar{N}N\right)h^0$$
$$- \frac{ig}{2M_W}(M_E\bar{E}\gamma_5 E\cot\beta + M_B\bar{B}\gamma_5 B\cot\beta + M_T\bar{T}\gamma_5 T\tan\beta + M_N\bar{N}\gamma_5 N\tan\beta)A^0. \quad (90)$$

A comparison of Eq. (89) and of Eq. (90) shows a rearrangement of α and β dependence. Thus while the down quark and the lepton vertices for a sequential generation are enhanced for large $\tan\beta$, it is the up quark vertex for a mirror generation that is enhanced. The above leads to some interesting features that distinguish a mirror generation from a sequential fourth generation.

One important consequence of the above is the following. Suppose the H^0 is heavy enough to decay into a pair of fourth generation quarks or a pair of mirror quarks

($m_{H^0} > 2m_q$, $q = u_4, d_4$). Then let us define the ratio of branching ratios $R^{H^0}_{d_4/u_4}$ as

$$R^{H^0}_{d_4/u_4} = \mathrm{BR}(H^0 \to d_4\bar{d}_4)/\mathrm{BR}(H^0 \to u_4\bar{u}_4). \quad (91)$$

Using the vertices in Eq. (89) we find

$$R^{H^0}_{d_4/u_4} = \frac{m^2_{d_4}}{m^2_{u_4}}(\cot\alpha \tan\beta)^2 P^{H^0}_{d_4/u_4}, \quad (92)$$

where $P^{H^0}_{d_4/u_4}$ is a phase space factor defined by $P^{H^0}_{d_4/u_4} = (1-4m^2_{d_4}/m^2_H)^{3/2}(1-4m^2_{u_4}/m^2_H)^{-3/2}$ (see Appendix B). Similarly if the heavy Higgs can decay into the mirror quarks ($m_{H^0} > 2m_Q$, $Q = B, T$) one has

$$R^{H^0}_{B/T} = \frac{m^2_B}{m^2_T}(\tan\alpha \cot\beta)^2 P^{H^0}_{B/T}, \quad (93)$$

where we have neglected the loop effects. Thus with a knowledge of the parameters of the Higgs sector, i.e., α and β, one has a way of differentiating a mirror generation from a sequential fourth generation. Even a more dramatic differentiation arises from the branching ratios involving the decay of the CP odd Higgs. Here one finds

$$R^{A^0}_{d_4/u_4} = \frac{m^2_{d_4}}{m^2_{u_4}}\tan^4\beta P^{A^0}_{d_4/u_4}, \quad (94)$$

where $P^{A^0}_{d_4/u_4} = (1-4m^2_{d_4}/m^2_A)^{1/2}(1-4m^2_{u_4}/m^2_A)^{-1/2}$ while a similar ratio for the decay into the mirror quarks gives (see Appendix B)

$$R^{A^0}_{B/T} = \frac{m^2_B}{m^2_T}\cot^4\beta P^{A^0}_{B/T}, \quad (95)$$

where again we have neglected possible loop effects. The above implies that for $\tan\beta \geq 2$, A^0 will dominantly decay into $d_4\bar{d}_4$ for the sequential fourth generation case, while it will decay dominantly into $T\bar{T}$ for a mirror generation. Another important way to discriminate between a sequential generation and a mirror generation is to look at the forward-backward asymmetry. Thus for the process $f\bar{f} \to f'\bar{f}'$ one may define, the forward-backward asymmetry $A_{FB} = (\int_0^1 dz(d\sigma/dz) - \int_{-1}^0 dz(d\sigma/dz))/(\int_{-1}^1 dz(d\sigma/dz))$. This asymmetry is sensitive to the $V+A$ vs $V-A$ structure of the f' fermion interaction and a measurement of it can help discriminate between a sequential generation and a mirror generation. In the above we have given a broad outline of the ways in which one might distinguish a mirror generation from a sequential fourth generation. There are many other possible chains for decay of the mirrors and mirror sparticles depending on their mass patterns. Further, more detailed analyses of signatures for the model with mirrors based on detector simulations would be useful along the line of the analysis of signatures for supergravity models [32] and for string models (for, a sample of recent works see [66–70]). Finally we comment on the flavor changing neutral current issues. It is well known that mixing with mirrors frustrates the Glashow-Iliopoulos-Maiani mechanism which suppresses flavor changing neutral current. For the current model this does not pose a problem because the mirrors do not mix with the first two generations. On the other hand one does have couplings of the Z boson which are off diagonal, $Z\bar{\tau}E$, $Z\bar{b}B$, $Z\bar{t}T$, etc. which would allow production via a Drell -Yan process of $pp \to Z^* \to \tau^+E^-$, $t\bar{T}$, $b\bar{B}$, etc., which are not allowed for a sequential generation. Of course the processes are suppressed by mixing angles.

VII. CONCLUSION

In this work we consider an extension of MSSM with an extra mirror generation which remains light down to the electroweak scale. Recent analyses indicate that an extra sequential generation is not inconsistent with the precision electroweak data, and similar considerations apply to a mirror generation. In the model we consider, we allow for mixings of the mirror generation with the third generation, and investigate some of the phenomenological implications of the model. One important effect arises on the magnetic moment of the τ neutrino, where one finds that it is enhanced by up to 8–9 orders of magnitude over what is predicted in the standard model. We also discussed the possible signatures of the mirror generation at the LHC, and find that several characteristic signatures exist which would distinguish it from a sequential generation. One such crucial test is the measurement of the forward-backward asymmetry which can discriminate between the $V-A$ vs $V+A$ interactions. It is further shown that the couplings of the mirror generation have different $\tan\beta$ dependences than those of an ordinary generation or of a sequential fourth generation.

If a mirror generation exists, it has important implications for string model building (for some recent work in D-brane and string model building see [71–76]). Typically in string model building one puts in the constraints that the difference between the number of generations n_f and the mirror generations n_{mf} (with $n_f > n_{mf}$) equals three. This assumes that the n_{mf} number of generations and mirror generations follow the survival hypothesis [12] and become superheavy. However, in unified models there are many instances where mirror generations may remain massless up to the electroweak scale. This opens a new direction for model building. Suppose, then, that one imposes only the constraint $n_f - n_{mf} = 2$ along with the condition that one mirror generation remains massless down to the electroweak scale. In this case we will have three ordinary generations and one mirror generation all light at the electroweak scale, i.e., the extended MSSM model with mirrors.

If the scenario outlined above holds, the string model building may need a revision in that the constraint of three massless generations will be relaxed. Specifically, for ex-

ample, in Kac-Moody level two heterotic string constructions one has problems getting three massless generations (see, e.g., [77]). On the other hand, if three ordinary generations and one mirror generation are massless, the rules of construction for string models change and one may need to take a fresh look at model building in string theory. Of course, the light mirror particles even if they exist need not necessarily fall into a full generation. Thus while a full generation is the simplest possibility for the cancellation of anomalies, it may happen that such cancellations may involve some exotic mirrors. This would make model building even more challenging. Many open questions remain for further study, the most important of which is a detailed dynamical model for the mixings of ordinary and mirror particles below the grand unification scale. In the analysis given in this work we assumed a phenomenological approach where we introduce mixings between the two sectors. However, a concrete mechanism is desirable to achieve a more complete understanding of the mixings of the ordinary matter and mirror matter.

ACKNOWLEDGMENTS

Interesting conversations with Emanuela Barberis, Patrick Huber, Stuart Raby and Akin Wingerter are acknowledged. This research is supported in part by NSF Grant No. PHY-0757959.

APPENDIX A: FURTHER DETAILS OF MIXINGS AND INTERACTIONS

In this section we give more explicit forms for the interactions including mixing with mirrors. We first discuss the nonsupersymmetric sector where the contributions arise from the W and Z exchanges. By parametrizing the mixing between τ and E_τ by the angle θ, and between ν and N by the angle ϕ, in the simple case where $\theta_L = \theta_R = \theta$ and $\phi_L = \phi_R = \phi$, we can write $\mathcal{L}_{CC} + \mathcal{L}_{NC}$ as

$$\begin{aligned}\mathcal{L}_{CC} + \mathcal{L}_{NC} = & -\frac{g}{2\sqrt{2}} W_\mu^\dagger \{\bar{\nu}_1 \gamma^\mu \tau_1 \cos(\theta-\phi) + \bar{\nu}_1 \gamma^\mu \tau_2 \sin(\theta-\phi) - \bar{\nu}_1 \gamma^\mu \gamma_5 \tau_1 \cos(\theta+\phi) - \bar{\nu}_1 \gamma^\mu \gamma_5 \tau_2 \sin(\theta+\phi) \\ & - \bar{\nu}_2 \gamma^\mu \tau_1 \sin(\theta-\phi) - \bar{\nu}_2 \gamma^\mu \gamma_5 \tau_1 \sin(\theta+\phi) + \bar{\nu}_2 \gamma^\mu \tau_2 \cos(\theta-\phi) + \bar{\nu}_2 \gamma^\mu \gamma_5 \tau_2 \cos(\theta+\phi)\} \\ & + \text{H.c} - \frac{g}{4\cos\theta_W} Z_\mu \{\bar{\tau}_1 \gamma^\mu (4\cos^2\theta_W - 1 + \cos 2\theta \gamma_5)\tau_1 + \bar{\tau}_2 \gamma^\mu (4\cos^2\theta_W - 1 - \cos 2\theta \gamma_5)\tau_2 \\ & + \bar{\tau}_1 \gamma^\mu \gamma_5 \sin 2\theta \tau_2 + \bar{\tau}_2 \gamma^\mu \gamma_5 \sin 2\theta \tau_1\}, \end{aligned} \tag{A1}$$

where τ_1, τ_2 are the mass eigenstates for the charged leptons, with τ_1 identified as the physical τ state, and ν_1, ν_2 are the mass eigenstates for the neutrino with ν_1 identified as the observed neutrino. We note that Eq. (A1) coincides with Eq. (1) of [21] except for the typo in the middle sign of their third line.

In the supersymmetric sector, the mass terms of the scalar leptons and scalar mirror leptons arise from the F term, the D term, and the soft supersymmetry breaking terms in the scalar potential. For example, the mixing terms between $\tilde{\tau}_L$ and $\tilde{\tau}_R$ can arise from the μ term in the superpotenital and from the trilinear coupling term of the soft breaking potential V_{soft}. This gives us the terms $M^2_{13} = M^2_{31} = m_\tau(A_\tau - \mu \tan\beta)$. The corresponding mixing terms between $\tilde{E}_{\tau L}$ and $\tilde{E}_{\tau R}$ are $M^2_{24} = M^2_{42} = m_{E_\tau}(A_{E_\tau} - \mu \cot\beta)$. We assume here that the couplings are real otherwise, we would have $M^2_{31} = m_\tau(A^*_\tau - \mu^* \tan\beta)$ and $M^2_{42} = m_{E_\tau}(A^*_{E_\tau} - \mu^* \cot\beta)$. In the general parameter space of MSSM one can fix these mixings to be zero by a proper choice of the parameters μ, A_τ, and A_{E_τ}. The other elements of the scalar mass^2 matrix can also be easily worked out. As an example, the F term produces a part of the mixing between $\tilde{\tau}_R$ and $\tilde{E}_{\tau R}$ as follows

$$V = F_i^* F_i, \; F_i = \frac{\partial W}{\partial A_i}. \tag{A2}$$

Here A_i is the scalar $\tilde{E}_{\tau L}$ and

$$\frac{\partial W}{\partial \tilde{E}_{\tau L}} = f_2' H_2^2 \tilde{E}^*_{\tau R} + f_4 \tilde{\tau}^*_R - f_2' H_2^1 \tilde{N}^*_R, \tag{A3}$$

which gives

$$\begin{aligned} V_F = & (f_2' H_2^{2*} \tilde{E}_{\tau R} + f_4 \tilde{\tau}_R - f_2' H_2^{1*} \tilde{N}_R)(f_2' H_2^2 \tilde{E}^*_{\tau R} \\ & + f_4 \tilde{\tau}^*_R - f_2' H_2^1 \tilde{N}^*_R). \end{aligned} \tag{A4}$$

After the breaking of the electroweak symmetry the V_F part of the scalar potential given above produces the following mass terms

$$\begin{aligned} -\mathcal{L}_m = & f_2'^2 \frac{v_2^2}{2} \tilde{E}_{\tau R} \tilde{E}^*_{\tau R} + f_4 f_2' \frac{v_2}{\sqrt{2}} \tilde{E}^*_{\tau R} \tilde{\tau}_R \\ & + f_4 f_2' \frac{v_2}{\sqrt{2}} \tilde{E}_{\tau R} \tilde{\tau}^*_R + f_4^2 \tilde{\tau}^*_R \tilde{\tau}_R. \end{aligned} \tag{A5}$$

Here one finds that the mixing between $\tilde{\tau}_R$ and $\tilde{E}_{\tau R}$ occurs such that the corresponding elements in the mass^2 matrix M^2_{34} and M^2_{43} are equal.

For illustrative purposes, we assume a simple mixing scenario for mixings in the scalar sector. Specifically we assume mixings among scalars and mirror scalars of the same chirality. Thus for the charged leptons we assume mixings between $\tilde{\tau}_L$ and $\tilde{E}_L$ and similarly mixings between $\tilde{\tau}_R$ and $\tilde{E}_R$, but no mixing between $\tilde{\tau}_L$, $\tilde{\tau}_R$ and between $\tilde{E}_L$ and $\tilde{E}_R$. These are obviously approximations to the more general analysis given in Sec. II. Under the above approximations the diagnolizing matrices $\tilde{D}^\tau$ and $\tilde{D}^\nu$ would have the following simple structures

$$\tilde{D}^{\tau} = \begin{pmatrix} \cos\tilde{\theta}_1 & \sin\tilde{\theta}_1 & 0 & 0 \\ -\sin\tilde{\theta}_1 & \cos\tilde{\theta}_1 & 0 & 0 \\ 0 & 0 & \cos\tilde{\theta}_2 & \sin\tilde{\theta}_2 \\ 0 & 0 & -\sin\tilde{\theta}_2 & \cos\tilde{\theta}_2 \end{pmatrix}, \quad \text{(A6)}$$

and

$$\tilde{D}^{\nu} = \begin{pmatrix} \cos\tilde{\phi}_1 & \sin\tilde{\phi}_1 & 0 & 0 \\ -\sin\tilde{\phi}_1 & \cos\tilde{\phi}_1 & 0 & 0 \\ 0 & 0 & \cos\tilde{\phi}_2 & \sin\tilde{\phi}_2 \\ 0 & 0 & -\sin\tilde{\phi}_2 & \cos\tilde{\phi}_2 \end{pmatrix}. \quad \text{(A7)}$$

In the charged leptonic sector, assuming the independent set of parameters to be $\tilde{\theta}_1$, $\tilde{\theta}_2$, M^2_{11}, M^2_{22}, M^2_{33}, and M^2_{44}, one can determine the elements $|M^2_{12}|$ and $|M^2_{34}|$ through the relations

$$\tan2\tilde{\theta}_1 = \frac{2|M^2_{12}|}{M^2_{11} - M^2_{22}}, \qquad \tan2\tilde{\theta}_2 = \frac{2|M^2_{34}|}{M^2_{33} - M^2_{44}}. \quad \text{(A8)}$$

The eigenvalues for the masses are then given by

$$\begin{aligned} M^2_{\tilde{\tau}_1} &= \frac{1}{2}(M^2_{11} + M^2_{22}) + \frac{1}{2}\sqrt{(M^2_{11} - M^2_{22})^2 + 4|M^2_{12}|^2}, \\ M^2_{\tilde{\tau}_2} &= \frac{1}{2}\left(M^2_{11} + \frac{1}{2}M^2_{22}\right) - \frac{1}{2}\sqrt{(M^2_{11} - M^2_{22})^2 + 4|M^2_{12}|^2}, \\ M^2_{\tilde{\tau}_3} &= \frac{1}{2}(M^2_{33} + M^2_{44}) + \frac{1}{2}\sqrt{(M^2_{33} - M^2_{44})^2 + 4|M^2_{34}|^2}, \\ M^2_{\tilde{\tau}_4} &= \frac{1}{2}(M^2_{33} + M^2_{44}) - \frac{1}{2}\sqrt{(M^2_{11} - M^2_{44})^2 + 4|M^2_{34}|^2}. \end{aligned} \quad \text{(A9)}$$

Similar relations hold for the scalar neutrino sector.

APPENDIX B: DECAY OF THE HEAVY HIGGS BOSONS H^0 AND A^0 INTO MIRRORS

The heavy Higgs decays into mirrors would produce some very characteristic signatures if the masses of the heavy Higgs bosons H^0 and A^0 are large enough to kinematically allow such decays. We give below the decay widths for the processes with charged mirrors

$$H^0 \to E\bar{E}, B\bar{B}, T\bar{T}, \qquad A^0 \to E\bar{E}, B\bar{B}, T\bar{T}, \quad \text{(B1)}$$

using the interactions of Eq. (90). For the decay of H^0 into charged mirrors we have

$$\begin{aligned} \Gamma(H^0 \to E\bar{E}) &= \frac{g^2 m_{H^0}}{32\pi}\left(\frac{\sin\alpha}{\sin\beta}\right)^2\left(\frac{M_E}{M_W}\right)^2\left(1 - \frac{4M^2_E}{M^{02}_H}\right)^{3/2}, \\ \Gamma(H^0 \to B\bar{B}) &= \frac{3g^2 m_{H^0}}{32\pi}\left(\frac{\sin\alpha}{\sin\beta}\right)^2\left(\frac{M_B}{M_W}\right)^2\left(1 - \frac{4M^2_B}{M^{02}_H}\right)^{3/2}, \\ \Gamma(H^0 \to T\bar{T}) &= \frac{3g^2 m_{H^0}}{32\pi}\left(\frac{\cos\alpha}{\cos\beta}\right)^2\left(\frac{M_T}{M_W}\right)^2\left(1 - \frac{4M^2_T}{M^{02}_H}\right)^{3/2}. \end{aligned} \quad \text{(B2)}$$

These may be compared with the decays of H^0 into a fourth sequential generation which are

$$\begin{aligned} \Gamma(H^0 \to e_4\bar{e}_4) &= \frac{g^2 m_{H^0}}{32\pi}\left(\frac{\cos\alpha}{\cos\beta}\right)^2\left(\frac{m_{e_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{e_4}}{M^{02}_H}\right)^{3/2}, \\ \Gamma(H^0 \to d_4\bar{d}_4) &= \frac{3g^2 m_{H^0}}{32\pi}\left(\frac{\cos\alpha}{\cos\beta}\right)^2\left(\frac{m_{d_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{d_4}}{M^{02}_H}\right)^{3/2}, \\ \Gamma(H^0 \to u_4\bar{u}_4) &= \frac{3g^2 m_{H^0}}{32\pi}\left(\frac{\sin\alpha}{\sin\beta}\right)^2\left(\frac{m_{u_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{u_4}}{M^{02}_H}\right)^{3/2}. \end{aligned} \quad \text{(B3)}$$

For the decay of A^0 into charged mirrors we have

$$\begin{aligned} \Gamma(A^0 \to E\bar{E}) &= \frac{g^2 m_{A^0}}{32\pi}\cot^2\beta\left(\frac{M_E}{M_W}\right)^2\left(1 - \frac{4M^2_E}{M^{02}_A}\right)^{1/2}, \\ \Gamma(A^0 \to B\bar{B}) &= \frac{3g^2 m_{A^0}}{32\pi}\cot^2\beta\left(\frac{M_B}{M_W}\right)^2\left(1 - \frac{4M^2_B}{M^{02}_A}\right)^{1/2}, \\ \Gamma(A^0 \to T\bar{T}) &= \frac{3g^2 m_{A^0}}{32\pi}\tan^2\beta\left(\frac{M_T}{M_W}\right)^2\left(1 - \frac{4M^2_T}{M^{02}_A}\right)^{1/2}. \end{aligned} \quad \text{(B4)}$$

These may be compared with the decays of A^0 into a fourth sequential generation which are

$$\begin{aligned} \Gamma(A^0 \to e_4\bar{e}_4) &= \frac{g^2 m_{A^0}}{32\pi}\tan^2\beta\left(\frac{m_{e_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{e_4}}{M^{02}_A}\right)^{1/2}, \\ \Gamma(A^0 \to d_4\bar{d}_4) &= \frac{3g^2 m_{A^0}}{32\pi}\tan^2\beta\left(\frac{m_{d_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{d_4}}{M^{02}_A}\right)^{1/2}, \\ \Gamma(A^0 \to u_4\bar{u}_4) &= \frac{3g^2 m_{A^0}}{32\pi}\cot^2\beta\left(\frac{m_{u_4}}{M_W}\right)^2\left(1 - \frac{4m^2_{u_4}}{M^{02}_A}\right)^{1/2}. \end{aligned} \quad \text{(B5)}$$

A study of the branching ratios will differentiate between a sequential fourth generation and a mirror fourth generation.

[1] G. D. Kribs, T. Plehn, M. Spannowsky, and T. M. P. Tait, Phys. Rev. D **76**, 075016 (2007); R. Fok and G. D. Kribs, arXiv:0803.4207.

[2] P. Q. Hung and M. Sher, Phys. Rev. D **77**, 037302 (2008).

[3] B. Holdom, J. High Energy Phys. 08 (2006) 076; 03 (2007) 063.

[4] V. A. Novikov, L. B. Okun, A. N. Rozanov, and M. I. Vysotsky, Pis'ma Zh. Eksp. Teor. Fiz. **76**, 158 (2002) [JETP Lett. **76**, 127 (2002)]; D. Fargion, M. Y. Khlopov, R. V. Konoplich, and R. Mignani, Phys. Rev. D **54**, 4684 (1996).

[5] J. E. Dubicki and C. D. Froggatt, Phys. Lett. B **567**, 46 (2003).

[6] Z. Murdock, S. Nandi, and Z. Tavartkiladze, arXiv:0806.2064.

[7] O. Cakir, H. Duran Yildiz, R. Mehdiyev, and I. Turk Cakir, arXiv:0801.0236.

[8] J. L. Hewett and T. G. Rizzo, Phys. Rev. D **35**, 2194 (1987).

[9] V. D. Barger, J. L. Hewett, and T. G. Rizzo, Mod. Phys. Lett. A **5**, 743 (1990).

[10] P. H. Frampton, P. Q. Hung, and M. Sher, Phys. Rep. **330**, 263 (2000).

[11] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Niewenhuisen and D. Z. Freedman (North-Holland, Amsterdam, 1979).

[12] H. Georgi, Nucl. Phys. **B156**, 126 (1979).

[13] F. Wilczek and A. Zee, Phys. Rev. D **25**, 553 (1982).

[14] K. S. Babu, S. M. Barr, and B. s. Kyae, Phys. Rev. D **65**, 115008 (2002).

[15] J. Bagger and S. Dimopoulos, Nucl. Phys. **B244**, 247 (1984).

[16] G. Senjanovic, F. Wilczek, and A. Zee, Phys. Lett. **141B**, 389 (1984).

[17] D. Chang, T. Hubsch, and R. N. Mohapatra, Phys. Rev. Lett. **55**, 673 (1985).

[18] I. Bars and M. Gunaydin, Phys. Rev. Lett. **45**, 859 (1980).

[19] F. del Aguila, M. Dugan, B. Grinstein, L. J. Hall, G. G. Ross, and P. C. West, Nucl. Phys. **B250**, 225 (1985).

[20] J. Maalampi and M. Roos, Phys. Rep. **186**, 53 (1990).

[21] J. Maalampi, J. T. Peltoniemi, and M. Roos, Phys. Lett. B **220**, 441 (1989).

[22] S. L. Adler, Phys. Lett. B **533**, 121 (2002).

[23] H. Chavez and J. A. Martins Simoes, Nucl. Phys. **B783**, 76 (2007); P. Csikor and Z. Fodor, Phys. Lett. B **287**, 358 (1992); V. Elias and S. Rajpoot, Phys. Lett. **134B**, 201 (1984); S. Rajpoot and J. G. Taylor, Phys. Lett. **142B**, 365 (1984).

[24] S. Nandi, A. Stern, and E. C. G. Sudarshan, Phys. Rev. D **26**, 2522 (1982).

[25] P. Langacker and D. London, Phys. Rev. D **38**, 886 (1988).

[26] D. Choudhury, T. M. P. Tait, and C. E. M. Wagner, Phys. Rev. D **65**, 053002 (2002).

[27] F. Csikor and I. Montvay, Phys. Lett. B **324**, 412 (1994).

[28] I. Montvay, arXiv:hep-ph/9708269.

[29] G. Triantaphyllou, Int. J. Mod. Phys. A **15**, 265 (2000).

[30] Yu. Kobzarev, L. B. Okun, and I. Ya. Pomeranchuk, Sov. J. Nucl. Phys. **3**, 837 (1966).

[31] R. N. Mohapatra, S. Nasri, and S. Nussinov, Phys. Lett. B **627**, 124 (2005).

[32] A. H. Chamseddine, R. Arnowitt, and P. Nath, Phys. Rev. Lett. **49**, 970 (1982); R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. **119B**, 343 (1982); L. Hall, J. Lykken, and S. Weinberg, Phys. Rev. D **27**, 2359 (1983); P. Nath, R. Arnowitt, and A. H. Chamseddine, Nucl. Phys. **B227**, 121 (1983); for a review see, P. Nath, arXiv:hep-ph/0307123.

[33] T. C. Yuan, R. Arnowitt, A. H. Chamseddine, and P. Nath, Z. Phys. C **26**, 407 (1984); D. A. Kosower, L. M. Krauss, and N. Sakai, Phys. Lett. **133B**, 305 (1983); J. L. Lopez, D. V. Nanopoulos, and X. Wang, Phys. Rev. D **49**, 366 (1994); U. Chattopadhyay and P. Nath, Phys. Rev. D **53**, 1648 (1996); T. Moroi, Phys. Rev. D **53**, 6565 (1996); **56**, 4424(E) (1997).

[34] T. Ibrahim and P. Nath, Phys. Rev. D **61**, 095008 (2000); Phys. Rev. D **62**, 015004 (2000); T. Ibrahim, U. Chattopadhyay, and P. Nath, Phys. Rev. D **64**, 016010 (2001).

[35] T. Ibrahim and P. Nath, Rev. Mod. Phys. **80**, 577 (2008).

[36] P. Ramond, R. G. Roberts, and G. G. Ross, Nucl. Phys. **B406**, 19 (1993); R. Gatto, G. Sartori, and M. Tonin, Phys. Lett. **28B**, 128 (1968); R. J. Oakes, Phys. Lett. **29B**, 683 (1969).

[37] This is actually a significant over estimate since the most recent CKM fits give a value which is even smaller, i.e., $V_{ub} = (3.93 \pm .36) \times 10^{-3}$ [78].

[38] J. N. Abdurashitov *et al.* (SAGE Collaboration), Phys. Rev. C **60**, 055801 (1999).

[39] Q. R. Ahmad *et al.* (SNO Collaboration), Phys. Rev. Lett. **89**, 011301 (2002).

[40] M. Altmann *et al.* (GNO Collaboration), Phys. Lett. B **490**, 16 (2000).

[41] M. Ambrosio *et al.* (MACRO Collaboration), Phys. Lett. B **517**, 59 (2001).

[42] S. Fukuda *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **85**, 3999 (2000).

[43] W. Hampel *et al.* (GALLEX Collaboration), Phys. Lett. B **447**, 127 (1999).

[44] J. F. Beacom and P. Vogel, Phys. Rev. Lett. **83**, 5222 (1999).

[45] A. V. Kuznetsov and N. V. Mikheev, arXiv:0712.1267.

[46] S. N. Gninenko, Phys. Lett. B **452**, 414 (1999).

[47] D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **148**, 175 (2003).

[48] P. Q. Hung, Nucl. Phys. **B805**, 326 (2008).

[49] M. Cvetic and P. Langacker, Phys. Rev. D **78**, 066012 (2008); S. Abel, A. Dedes, and K. Tamvakis, Phys. Rev. D **71**, 033003 (2005); I. Gogoladze and A. Perez-Lorenzana, Phys. Rev. D **65**, 095011 (2002); D. A. Demir, L. L. Everett, and P. Langacker, Phys. Rev. Lett. **100**, 091804 (2008); E. Ma, Nucl. Phys. **B623**, 126 (2002).

[50] L. G. Cabral-Rosetti, J. Bernabeu, J. Vidal, and A. Zepeda, Eur. Phys. J. C **12**, 633 (2000).

[51] M. Dvornikov and A. Studenikin, Phys. Rev. D **69**, 073001 (2004).

[52] S. Eidelman *et al.*, Nucl. Phys. B, Proc. Suppl. **169**, 226 (2007).

[53] M. S. Carena, H. E. Haber, and C. E. M. Wagner, Nucl. Phys. **B472**, 55 (1996).

[54] M. E. Peskin and T. Takeuchi, Phys. Rev. D **46**, 381 (1992).

[55] G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. **B369**, 3 (1992); **B376**, 444(E) (1992).
[56] J. Yao *et al.*, J. Phys. G **33**, 1 (2006).
[57] For recent fits to the S, T, U by the LEP EWWG see, http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2006/s06_stu_contours.eps.
[58] H. J. He, N. Polonsky, and S. f. Su, Phys. Rev. D **64**, 053004 (2001).
[59] M. Jezabek and J. H. Kuhn, Phys. Lett. B **329**, 317 (1994).
[60] C. A. Nelson, B. T. Kress, M. Lopes, and T. P. McCauley, Phys. Rev. D **56**, 5928 (1997).
[61] V. M. Abazov *et al.* (D0 Collaboration), Phys. Rev. Lett. **100**, 062004 (2008).
[62] M. T. Dova, J. Swain, and L. Taylor, Nucl. Phys. B, Proc. Suppl. **76**, 133 (1999).
[63] S. Singh and N. K. Sharma, Phys. Rev. D **36**, 160 (1987); **36**, 3387 (1987).
[64] P. Nath and R. Arnowitt, Mod. Phys. Lett. A **2**, 331 (1987); H. Baer, C. h. Chen, F. Paige, and X. Tata, Phys. Rev. D **50**, 4508 (1994); V. D. Barger, C. Kao, and T. j. Li, Phys. Lett. B **433**, 328 (1998). For four lepton modes see, T. Ibrahim, Phys. Rev. D **77**, 065028 (2008).
[65] A. De Rujula and S. L. Glashow, Phys. Rev. Lett. **45**, 942 (1980).
[66] D. Feldman, Z. Liu, and P. Nath, Phys. Rev. Lett. **99**, 251802 (2007); Phys. Lett. B **662**, 190 (2008); J. High Energy Phys. 04 (2008) 054.
[67] G. L. Kane, P. Kumar, and J. Shao, J. Phys. G **34**, 1993 (2007).
[68] R. Arnowitt *et al.*, Phys. Lett. B **649**, 73 (2007).
[69] P. G. Mercadante, J. K. Mizukoshi, and X. Tata, Braz. J. Phys. **37**, 549 (2007).
[70] H. Baer, H. Prosper, and H. Summy, Phys. Rev. D **77**, 055017 (2008); H. Baer, V. Barger, G. Shaughnessy, H. Summy, and L. t. Wang, Phys. Rev. D **75**, 095010 (2007).
[71] R. Blumenhagen, B. Kors, D. Lust, and T. Ott, Nucl. Phys. **B616**, 3 (2001).
[72] M. Cvetic, G. Shiu, and A. M. Uranga, Nucl. Phys. **B615**, 3 (2001).
[73] T. Kobayashi, S. Raby, and R. J. Zhang, Nucl. Phys. **B704**, 3 (2005).
[74] V. Bouchard and R. Donagi, Phys. Lett. B **633**, 783 (2006).
[75] V. Braun, Y. H. He, B. A. Ovrut, and T. Pantev, J. High Energy Phys. 05 (2006) 043.
[76] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sanchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingerter, Phys. Rev. D **77**, 046013 (2008).
[77] Z. Kakushadze and S. H. H. Tye, Phys. Rev. D **55**, 7878 (1997).
[78] C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008).