

Single-inclusive hadron production in transversely polarized pp and $\bar{p}p$ collisions with threshold resummation

Daniel de Florian,¹ Werner Vogelsang,² and Federico Wagner¹¹*Departamento de Física, FCEYN, Universidad de Buenos Aires, (1428) Pabellón 1 Ciudad Universitaria, Capital Federal, Argentina*²*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*

(Received 29 July 2008; published 24 October 2008)

We investigate the resummation of large logarithmic perturbative corrections to the partonic cross sections for single-inclusive high- p_T hadron production in collisions of transversely polarized hadrons. We perform the resummation to next-to-leading logarithmic accuracy. Phenomenological results are given for $\bar{p}p$ collisions at center-of-mass energy $\sqrt{S} = 14.5$ GeV and for pp collisions at $\sqrt{S} = 62.4$ GeV and at $\sqrt{S} = 10$ GeV, which are relevant for possible experiments at the GSI-FAIR, RHIC and J-PARC facilities, respectively. We find significant enhancements of the spin-dependent and spin-averaged cross sections, but a decrease of the double-spin asymmetry A_{TT}^π .

DOI: 10.1103/PhysRevD.78.074025

PACS numbers: 13.88.+e, 13.85.Ni

I. INTRODUCTION

In spite of extensive studies in recent years, the partonic structure of spin-1/2 nucleons is not yet completely known. Among the leading-twist collinear parton distributions, the unpolarized (f), longitudinally polarized (Δf), and transversely polarized (δf) densities, there is so far very little information about the latter. These “transversity” distributions δf are defined [1–3] as the differences of probabilities to find a parton of flavor f at scale μ and light-cone momentum fraction x with its spin aligned ($\uparrow\uparrow$) or antialigned ($\uparrow\downarrow$) with that of the transversely polarized nucleon:

$$\delta f(x, \mu) \equiv f_{\uparrow\uparrow}(x, \mu) - f_{\uparrow\downarrow}(x, \mu). \quad (1.1)$$

As is well known, there is no leading-twist gluon transversity distribution, due to the odd chirality of transversity and angular momentum conservation. The unpolarized parton distributions are recovered by taking the sum in Eq. (1.1).

Unlike the longitudinally polarized distribution functions which can be measured directly in deep-inelastic scattering (DIS), transversity is not accessible in inclusive DIS due to its chiral-odd nature [2]. Only recently has a very first “glimpse” of transversity been obtained from a combined analysis [4] of data for single-transverse spin asymmetries in semi-inclusive deep-inelastic scattering (SIDIS) [5] and e^+e^- annihilation [6]. This analysis relies on the extraction of the Collins functions [7] from e^+e^- data, which then give access to transversity in SIDIS spin asymmetries. Apart from further pursuing such studies, it will be highly desirable in the future to have more direct probes of transversity. These are available in double-transverse spin asymmetries in hadronic collisions,

$$A_{TT} \equiv \frac{\frac{1}{2}[d\sigma(\uparrow\uparrow) - d\sigma(\uparrow\downarrow)]}{\frac{1}{2}[d\sigma(\uparrow\uparrow) + d\sigma(\uparrow\downarrow)]} \equiv \frac{d\delta\sigma}{d\sigma}, \quad (1.2)$$

where the arrows denote the transverse polarization of the

scattering hadrons. A program of polarized pp collisions is now well under way at the BNL Relativistic Heavy Ion Collider (RHIC), and measurements of A_{TT} for various reactions should be feasible with sufficient beam-time for transverse polarization [8]. In the more distant future, there is also hope to have transversely polarized $\bar{p}p$ collisions at the GSI-FAIR facility, where there are plans to have an asymmetric polarized $\bar{p}p$ collider [9]. Likewise, transversely polarized pp collisions could become a possibility at the J-PARC facility [10]. In view of these opportunities, it is important to supply a theoretical framework that adequately describes the processes of interest, allowing a reliable extraction of transversity from hopefully forthcoming data.

In the present paper, we will focus on single-inclusive hadron production in transversely polarized hadronic collisions, $p^\uparrow p^\uparrow \rightarrow hX$, $\bar{p}^\uparrow p^\uparrow \rightarrow hX$, where the hadron h will for our purposes be a pion and has large transverse momentum p_T . Compared to the Drell-Yan process, which is usually considered the “golden channel” for transversity measurements in hadronic scattering [2,11–17], the spin asymmetry for hadron production is typically considerably smaller, mostly due to the large contribution from gluonic scattering in the denominator of the asymmetry [3,13,18]. On the other hand, pions are very copiously produced in hadronic scattering, resulting in much smaller statistical uncertainties, and A_{TT} in single-inclusive hadron production could thus conceivably present a viable alternative to Drell-Yan at RHIC as well as at GSI-FAIR and J-PARC [19].

Theoretical calculations of high- p_T pion production are based on the factorization theorem [20], which states that the cross section may be factorized in terms of collinear convolutions of universal parton distribution for the initial hadrons (in the spin-dependent case, transversity), a fragmentation function for the final-state pion, and short-distance parts that describe the hard interactions of the partons and are amenable to QCD perturbation theory.

The latter thus have an expansion in the strong coupling α_S , starting with a leading-order (LO) term, followed by a next-to-leading order (NLO) correction, etc. For the process we are interested in here, it was found that at RHIC energies theoretical NLO calculations are very successful in describing the experimental data [21–23]. The NLO corrections relevant for the process $pp \rightarrow \pi X$ with transversely polarized protons are available in [19].

For much lower energies, as typically available in fixed-target experiments, our previous work [24,25] has shown that the NLO framework is no longer sufficient but that all-order resummations of large logarithmic corrections are needed. Here the value of $x_T \equiv 2p_T/\sqrt{S}$, with \sqrt{S} the center-of-mass (c.m.) energy of the collision, is generally quite large, $x_T \gtrsim 0.1$. It turns out that the partonic hard-scattering cross sections relevant for $pp \rightarrow \pi X$ are then largely probed in the “threshold”-regime, where the initial partons have just enough energy to produce the high-transverse momentum parton that subsequently fragments into the hadron, and its recoiling counterpart. Relatively little phase space is then available for additional radiation of partons. In particular, gluon radiation is inhibited and mostly constrained to the emission of soft and/or collinear gluons. The cancellation of infrared singularities between real and virtual diagrams then leaves behind large double- and single-logarithmic corrections to the partonic cross sections. These logarithms appear for the first time at NLO, where they arise as terms of the form $\alpha_S \ln^2(1 - \hat{x}_T^2)$ in the rapidity-integrated cross section, where $\hat{x}_T \equiv 2\hat{p}_T/\sqrt{\hat{s}}$ with \hat{p}_T the transverse momentum of the produced parton and \hat{s} the c.m. energy of the initial partons. At yet higher (k th) order of perturbation theory, the double-logarithms are of the form $\alpha_S^k \ln^{2k}(1 - \hat{x}_T^2)$. When the threshold regime dominates, it is essential to take into account the large logarithms to all orders in the strong coupling α_S , a technique known as “threshold resummation” [26,27]. In our earlier work, we examined the effects of threshold resummation on the single-inclusive hadron cross section in [24] and on the double-longitudinal spin asymmetry A_{LL} [25]. We found very significant enhancements of the cross section by resummation, which in fact lead to a relatively good agreement between resummed theory and experimental data. We also found a moderate decrease of the resummed A_{LL} , with respect to NLO. We concluded that threshold resummation is an essential part of the theoretical description in the fixed-target kinematic regime. Its effects at higher energies (such as at RHIC) are much smaller, even though it has to be said that one is typically much further away from the threshold regime here, so that the applicability of threshold resummation is not entirely clear.

In the present paper, we apply the threshold resummation formalism to the case of partons with transverse polarization, relevant for studies of A_{TT} . While this is conceptually a relatively straightforward extension of our

previous work [24,25], it requires as a new technical ingredient the derivation of the “color-connected” Born cross sections for transverse parton polarization, which were not available so far. These are presented for the first time in one of the Appendices.

We will also use our results to obtain phenomenological predictions for cross sections and the double-transverse spin asymmetries in the kinematic regimes to be accessed by the possible GSI-FAIR, RHIC and J-PARC measurements. The remainder of this paper is organized as follows. In Sec. II, the general framework for the resummed single-inclusive hadron cross section is briefly reviewed, and the ingredients that are specific to transverse polarization are provided. Section III presents phenomenological results. We conclude in Sec. IV, and two Appendices collect some relevant expressions for each of the partonic subprocesses.

II. THE p_T DIFFERENTIAL CROSS SECTION IN PERTURBATION THEORY

We will for simplicity consider the cross section integrated over all rapidities of the produced hadron h , which turns out to simplify the analysis significantly [24,25]. The factorized spin-dependent cross section differential in the hadron’s transverse momentum p_T and its azimuthal angle ϕ with respect to the initial transverse spin directions can then be written as [24]

$$\begin{aligned} \frac{p_T^3 d\delta\sigma(x_T)}{dp_T d\phi} &= \sum_{a,b,c} \int_0^1 dx_1 \delta f_a(x_1, \mu^2) \int_0^1 dx_2 \delta f_b(x_2, \mu^2) \\ &\times \int_0^1 dz z^2 D_{h/c}(z, \mu^2) \\ &\times \int_0^1 d\hat{x}_T \delta\left(\hat{x}_T - \frac{x_T}{z\sqrt{x_1 x_2}}\right) \\ &\times \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta} \frac{\hat{x}_T^4 \hat{s}}{2} \frac{d\delta\hat{\sigma}_{ab \rightarrow cX}(\hat{x}_T^2, \hat{\eta}, \alpha_S(\mu), \mu)}{d\hat{x}_T^2 d\hat{\eta} d\phi}, \end{aligned} \quad (2.1)$$

where the $\delta f_{a,b}$ are the transversity parton distribution functions defined in Eq. (1.1), and where the $D_{h/c}$ are the parton-to-hadron fragmentation functions. Long- and short-distance contributions are separated by a factorization scale μ . We take this scale to be the same as the renormalization scale in the strong coupling constant. As before, $x_T \equiv 2p_T/\sqrt{S}$, and its partonic counterpart is \hat{x}_T .

We have $\hat{\eta}_+ = -\hat{\eta}_- = \ln[(1 + \sqrt{1 - \hat{x}_T^2})/\hat{x}_T]$. The sum in Eq. (2.1) runs over all partonic subprocesses $ab \rightarrow cX$, with partonic cross sections $d\delta\hat{\sigma}_{ab \rightarrow cX}$, defined similarly to the numerator of Eq. (1.2) for transversely polarized initial partons. As we have mentioned before, for the transversity case there are no subprocesses with initial gluons. We therefore only have four subprocesses that contribute [19]:

$$qq \rightarrow qX, \quad q\bar{q} \rightarrow qX, \quad q\bar{q} \rightarrow q'X, \quad q\bar{q} \rightarrow gX.$$

The cross sections for these have a well-known characteristic dependence on the azimuthal angle ϕ . In the c.m. frame, taking the momentum and spin directions of the initial hadrons as the z - and x -axes, respectively, this dependence is of the form $\cos(2\phi)$. We note that the expression for the spin-averaged cross section is identical to that in Eq. (2.1), with the transversity distributions and polarized partonic cross sections replaced by their standard spin-averaged counterparts. Here the sum of course also runs over partonic channels with gluons in the initial state as well.

We will only briefly describe the technical aspects of the resummation of the threshold logarithms, pointing out the specifics of the transversity case. All other details may be found in our previous papers [24,25]. The resummation of the soft-gluon contributions is achieved by taking a Mellin transform of the cross section in the scaling variable x_T^2 :

$$\frac{d\delta\sigma(N)}{d\phi} \equiv \int_0^1 dx_T^2 (x_T^2)^{N-1} \frac{p_T^3 d\delta\sigma(x_T)}{dp_T d\phi}. \quad (2.2)$$

In the same way, the partonic cross section can be expressed as

$$\begin{aligned} \frac{d\delta\hat{\sigma}_{ab\rightarrow cX}(N)}{d\phi} &\equiv \int_0^1 d\hat{x}_T^2 (\hat{x}_T^2)^{N-1} \int_{\hat{\eta}_-}^{\hat{\eta}_+} d\hat{\eta} \frac{\hat{x}_T^4 \hat{\delta}}{2} \\ &\times \frac{d\delta\hat{\sigma}_{ab\rightarrow cX}(\hat{x}_T^2, \hat{\eta})}{d\hat{x}_T^2 d\hat{\eta} d\phi}, \end{aligned} \quad (2.3)$$

where we from now on suppress the scale dependence of the cross section. In Mellin-moment space the convolutions in Eq. (2.1) become ordinary products, and threshold logarithms appear as logarithms in the moment variable N . As we discussed in [24], the resummed partonic cross section for each subprocess can be written in the rather simple form

$$\begin{aligned} \frac{d\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{res})}(N)}{d\phi} &= \delta C_{ab\rightarrow cd} \Delta_N^a \Delta_N^b \Delta_N^c J_N^d \\ &\times \left[\sum_I \delta G_{ab\rightarrow cd}^I \Delta_{IN}^{(\text{int})ab\rightarrow cd} \right] \frac{d\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{Born})}(N)}{d\phi}, \end{aligned} \quad (2.4)$$

where $\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{Born})}(N)$ denotes the LO term in the perturbative expansion of Eq. (2.3) for a given partonic process. Each of the functions J_N^d , Δ_N^i , $\Delta_{IN}^{(\text{int})ab\rightarrow cd}$ is an exponential and embodies part of the resummation. These terms all coincide with the corresponding ones for the spin-averaged

case and can be found, for example, in [24]. $\Delta_N^{a,b}$ represent the effects of soft-gluon radiation collinear to initial partons a, b , and similarly for Δ_N^c for the final-state fragmenting parton c . The function J_N^d embodies collinear, soft or hard, emission by the ‘‘nonobserved’’ recoiling parton d . Large-angle soft-gluon emission is accounted for by the factors $\Delta_{IN}^{(\text{int})ab\rightarrow cd}$, which depend on the color configuration I of the participating partons. Each of the $\Delta_{IN}^{(\text{int})ab\rightarrow cd}$ is given as

$$\ln \Delta_{IN}^{(\text{int})ab\rightarrow cd} = \int_0^1 \frac{z^{N-1} - 1}{1-z} D_{Iab\rightarrow cd}(\alpha_S((1-z)^2 p_T^2)) dz. \quad (2.5)$$

A sum over the color configurations occurs in Eq. (2.4), with $\delta G_{ab\rightarrow cd}^I$ representing a weight for each I , such that $\sum_I \delta G_{ab\rightarrow cd}^I = 1$. Finally, the coefficients $\delta C_{ab\rightarrow cd}$ contain N -independent hard contributions arising from one-loop virtual corrections. Their perturbative expansion reads:

$$\delta C_{ab\rightarrow cd} = 1 + \frac{\alpha_S}{\pi} \delta C_{ab\rightarrow cd}^{(1)} + \mathcal{O}(\alpha_S^2). \quad (2.6)$$

They can be determined for each partonic channel by expanding the resummed cross section in Eq. (2.4) to first order in α_S and comparing to the full analytic NLO calculation of Ref. [19].

The only differences between the resummation formulas for the spin-dependent and the spin-averaged cases reside in the coefficients $\delta G_{ab\rightarrow cd}^I$, $\delta C_{ab\rightarrow cd}$ and of course in the Born cross sections. These terms are all related to hard scattering, which is in general spin-dependent. We collect the moment-space expressions for the spin-dependent Born cross sections, as well as the $\delta G_{ab\rightarrow cd}^I$ and the $\delta C_{ab\rightarrow cd}$ for the various subprocess in Appendix A. The corresponding expressions for the spin-averaged case may all be found in our previous paper [24], to which we also refer the reader for further details of the calculation of the coefficients.

In order to obtain a resummed cross section in x_T^2 space, we need an inverse Mellin transform. We use the *Minimal Prescription* proposed in Ref. [28] to treat the singularity of the perturbative strong coupling constant in the resummed exponent. In order to make full use of the available fixed-order cross section, which in our case is NLO ($\mathcal{O}(\alpha_S^3)$) [19], we perform a matching to this cross section. We expand the resummed cross section to $\mathcal{O}(\alpha_S^3)$, subtract the expanded result from the resummed one, and add the full NLO cross section:

$$\begin{aligned} \frac{p_T^3 d\delta\sigma^{(\text{match})}(x_T)}{dp_T d\phi} &= \sum_{a,b,c} \int_{\text{Min.Prescr.}} \frac{dN}{2\pi i} (x_T^2)^{-N+1} \delta f_a(N, \mu^2) \delta f_b(N, \mu^2) D_{c/h}(2N+1, \mu^2) \\ &\times \left[\frac{d\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{res})}(N)}{d\phi} - \frac{d\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{res})}(N)}{d\phi} \Big|_{\mathcal{O}(\alpha_S^3)} \right] + \frac{p_T^3 d\delta\sigma^{(\text{NLO})}(x_T)}{dp_T d\phi}, \end{aligned} \quad (2.7)$$

where $\delta\hat{\sigma}_{ab\rightarrow cd}^{(\text{res})}(N)$ is the transversely polarized resummed cross section for the partonic channel $ab \rightarrow cd$ as given in Eq. (2.4), and where the Mellin integration contour is chosen according to the Minimal Prescription (see Ref. [28]). In this way, NLO is taken into account in full, and the soft-gluon contributions beyond NLO are resummed to next-to-leading logarithm (NLL). Any double-counting of perturbative orders is avoided.

III. PHENOMENOLOGICAL RESULTS

We will now apply the threshold resummation formalism outlined above to make some predictions for cross sections and spin asymmetries for single-inclusive hadron production in transversely polarized scattering. We will consider π^0 production in $\bar{p}p$ collisions at center-of-mass energy $\sqrt{S} = 14.5$ GeV, and in pp collisions at $\sqrt{S} = 62.4$ GeV and at $\sqrt{S} = 10$ GeV. These conditions correspond to the possible experiments at GSI-FAIR [9], RHIC and J-PARC [10], respectively, that we mentioned in the Introduction. In the GSI case, this energy would be achieved for an asymmetric collider with polarized anti-protons of energy $E_{\bar{p}} = 15$ GeV colliding with protons of energy $E_p = 3.5$ GeV, while for J-PARC a fixed-target setup is envisaged. We note that the current “default” energy of the RHIC polarized pp collider is $\sqrt{S} = 200$ GeV, and only a brief run at $\sqrt{S} = 62.4$ GeV has been performed. However, given that at high energies the partonic threshold regime makes a less dominant contribution to the cross sections, we refrain from presenting results for $\sqrt{S} = 200$ GeV. As we have shown in Ref. [25], at $\sqrt{S} = 62.4$ GeV the threshold approximation is still reasonably good, and in fact resummation leads to a visible improvement between the theoretical description and the RHIC data reported in [29]. In any case, it is not yet decided at which energy measurements of A_{TT} will predominantly be performed at RHIC.

We first need to choose sets of parton distribution and pion fragmentation functions. For the spin-averaged distributions we use the CTEQ6M [30] set. We follow Ref. [12] to model the essentially unknown transversity distributions by saturating the Soffer inequality [31] at a low initial scale $Q_0 \sim 0.6$ GeV for the evolution (for details see Ref. [12]). Finally, for the pion fragmentation functions we choose the “de Florian-Sassot-Stratmann” set [32]. According to Eq. (2.7), it is a great advantage to have the parton densities and fragmentation functions in moment space. Technically, since the parton distributions are typically only available in x space, we first perform a fit of a simple functional form to each distribution, of which we are then able to take Mellin moments analytically. This is done separately for each parton type and at each scale.

As we have previously mentioned, the dependence of the spin-dependent cross section on the azimuthal angle ϕ is on the form $\cos(2\phi)$. As in [19] our convention for the

GSI-FAIR and J-PARC cases is to integrate the transversely polarized cross section over the four quadrants in ϕ with alternating signs, in the form $(\int_{-\pi/4}^{\pi/4} - \int_{\pi/4}^{3\pi/4} + \int_{3\pi/4}^{5\pi/4} - \int_{5\pi/4}^{7\pi/4}) \cos(2\phi) d\phi = 4$. The unpolarized cross section is integrated over all ϕ , resulting in a factor 2π . For RHIC, we tailor our results to the PHENIX detector which covers only half of the pion’s azimuthal angle. We therefore follow [19] to integrate only over the two quadrants $-\pi/4 < \Phi < \pi/4$ and $3\pi/4 < \Phi < 5\pi/4$ here, which gives $(\int_{-\pi/4}^{\pi/4} - \int_{3\pi/4}^{5\pi/4}) \cos(2\phi) d\phi = 2$, and π for the spin-averaged cross section. It would be straightforward to adapt our calculations to the STAR detector as well. At midrapidity, we expect results very similar to the ones shown for the PHENIX case below. It will be very worthwhile to also investigate the prospects offered by STAR’s new capabilities [33] at very forward rapidities.

We first present our results for $\bar{p}p$ collisions at $\sqrt{S} = 14.5$ GeV, corresponding to the GSI-FAIR project. At this relatively modest energy one expects the perturbative threshold corrections that we resum here to be particularly important. In Fig. 1 we show the rapidity-integrated spin-averaged (left) and spin-dependent (right) cross sections at this energy. We display separately the NLL (matched) resummed cross section, the first-order ($\mathcal{O}(\alpha_s^3)$) expansion of the resummed expression, and the NLO one.

We define a “ K -factor” as the ratio of the (matched) resummed cross section over the NLO one,

$$K^{(\text{res})} = \frac{d\sigma^{(\text{match})}/dp_T d\phi}{d\sigma^{(\text{NLO})}/dp_T d\phi}. \quad (3.1)$$

This ratio is shown by the solid lines in Fig. 2. It is interesting to see that the K -factors are very large, meaning that resummation results in a large enhancement over NLO. It is worth mentioning that a previous study of the DY process at this energy also found very large K -factors for the cross sections [15].

In order to investigate how this large enhancement builds up order by order in perturbation theory, we expand the matched resummed formula beyond NLO and define the “soft-gluon K -factors” of the resummed cross section as

$$K^{(n)} \equiv \frac{d\sigma^{(\text{match})}/dp_T d\phi|_{\mathcal{O}(\alpha_s^{n+2})}}{d\sigma^{(\text{NLO})}/dp_T d\phi}. \quad (3.2)$$

For $n = 2, 3, \dots$ the $K^{(n)}$ give the additional enhancement over full NLO due to the $\mathcal{O}(\alpha_s^{2+n})$ terms in the resummed formula. Formally, $K^{(1)} = 1$ and $K^{(\infty)} = K^{(\text{res})}$ of Eq. (3.1). The results for $K^{(n)}$ are also given in Fig. 2. As can be seen in the figure, there are very large contributions even beyond NNLO, in particular, at higher p_T . It is clear that the full resummed result is required here.

The lower panels in Fig. 2 show the ratios between the $\mathcal{O}(\alpha_s^3)$ expansion of the (unmatched) resummed cross sec-

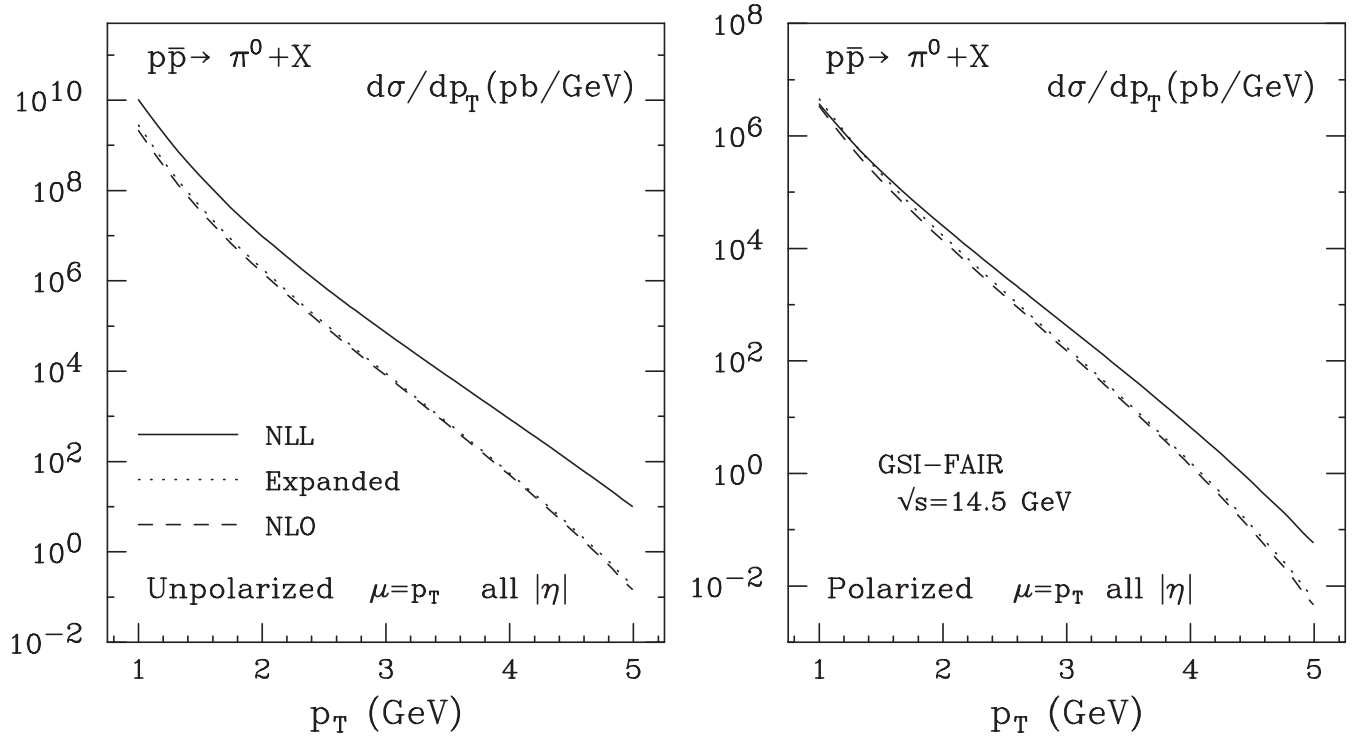


FIG. 1. Fully rapidity-integrated NLL resummed cross section, the expansion to $\mathcal{O}(\alpha_s^3)$, and the NLO cross section, for unpolarized (left) and transversely polarized (right) $\bar{p}p \rightarrow \pi^0 X$ at $\sqrt{s} = 14.5$ GeV.

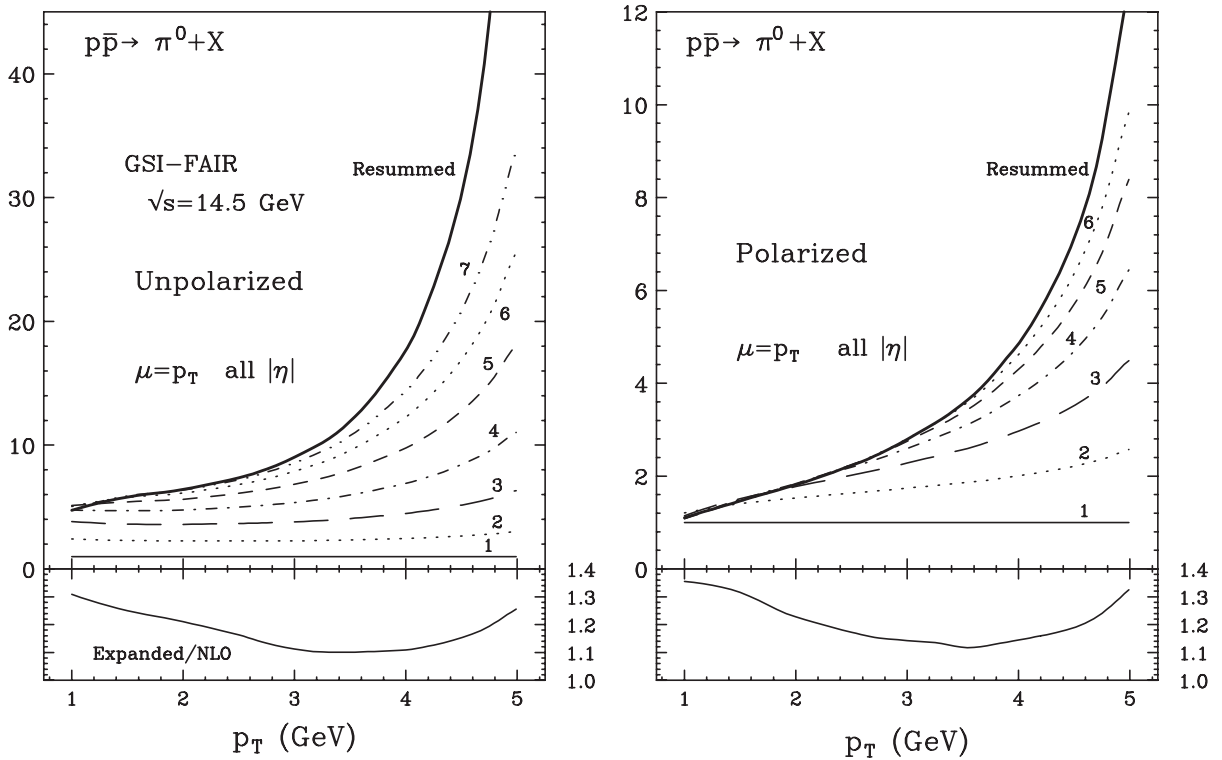


FIG. 2. Soft-gluon $K^{(n)}$ factors as defined in Eq. (3.2) for unpolarized (left) and transversely polarized (right) $\bar{p}p \rightarrow \pi^0 X$ at $\sqrt{s} = 14.5$ GeV. The numbers indicate the value of n . The lower panels show the ratios between the $\mathcal{O}(\alpha_s^3)$ expansion of the resummed cross section and the NLO cross section.

tion and the full NLO cross section. As can be observed for both the unpolarized and transversely polarized cross sections, the “soft-expansion” adequately reproduces the NLO result, implying that higher-order corrections are indeed dominated by the threshold logarithms. The large enhancements seen in the resummed K-factors are therefore not an artifact of the resummation procedure, but originate from genuine soft corrections and are expected to be reproduced in future full higher-order calculations for these observables.

To match the experimental conditions more realistically, we have to take into account the rapidity range that would be covered by the experiments. We assume this range to be $-1 < \eta_{\text{lab}} < 2.5$, where η_{lab} is the pseudorapidity of the pion in the laboratory frame. We count positive rapidity in the forward direction of the antiproton. η_{lab} is related to the c.m. pseudorapidity η_{cm} by

$$\eta_{\text{lab}} = \eta_{\text{cm}} + \frac{1}{2} \ln \frac{E_{\bar{p}}}{E_p}. \quad (3.3)$$

Therefore the rapidity interval that we use roughly corresponds to $|\eta_{\text{cm}}| \leq 1.75$ in the c.m. system. To obtain a resummed cross section for this interval, we use the approximation

$$\begin{aligned} & \frac{p_T^3 d\sigma^{(\text{match})}}{dp_T d\phi} (\eta \text{ in experimental range}) \\ &= K^{(\text{res})} \frac{p_T^3 d\sigma^{(\text{NLO})}}{dp_T d\phi} (\eta \text{ in experimental range}), \end{aligned} \quad (3.4)$$

where $K^{(\text{res})}$ is as defined in Eq. (3.1) in terms of cross sections integrated over the full region of rapidity. In other words, we “rescale” the matched resummed cross section by the ratio of NLO cross sections integrated over the experimentally relevant rapidity region or over all η , respectively. For the p_T values we are considering here, the region $-1 < \eta_{\text{lab}} < 2.5$ in fact almost coincides with the full kinematically allowed η_{cm} range. The results for the NLL resummed cross sections integrated over the range $-1 < \eta_{\text{lab}} < 2.5$ are shown in Fig. 3. We also present in the figure the uncertainties in the prediction resulting from variation of the scale μ in the range $p_T \leq \mu \leq 4p_T$. One can see that the scale uncertainty remains rather large even after resummation.

We next investigate how NLL resummation influences the double-spin asymmetry $A_{TT}^{\pi^0}$ for π^0 production. The results can be seen in Fig. 4, where we have chosen the scale $\mu = p_T$. As one can observe, there is a significant decrease of $A_{TT}^{\pi^0}$ when NLL resummation is included. Even after resummation the asymmetry appears to be large enough to be accessible experimentally.

We now turn to $p^1 p^1 \rightarrow hX$ processes and, as we have mentioned previously, we will discuss neutral pion production at $\sqrt{S} = 62.4$ GeV and at $\sqrt{S} = 10$ GeV, as relevant

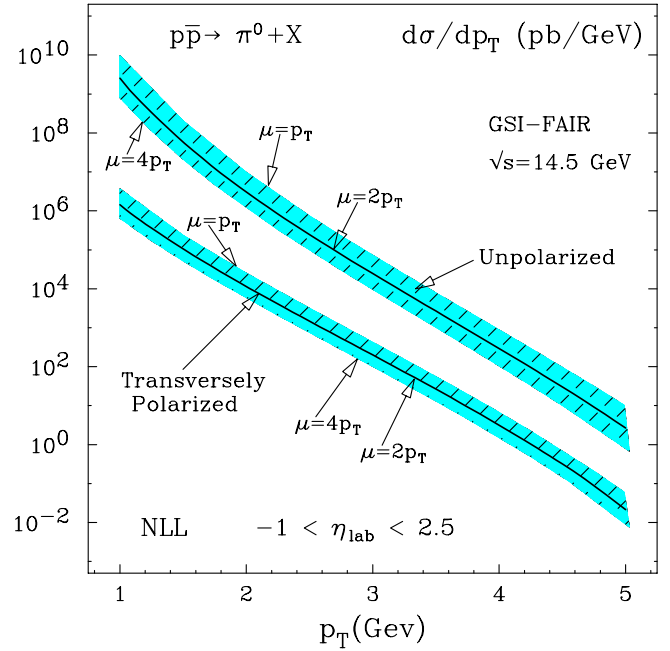


FIG. 3 (color online). NLL resummed cross section for unpolarized and transversely polarized $\bar{p}p \rightarrow \pi^0 X$ at $\sqrt{S} = 14.5$ GeV for $-1 < \eta_{\text{lab}} < 2.5$. The shaded bands represent the changes of the results if the factorization/renormalization scale is varied in the range $p_T \leq \mu \leq 4p_T$. The solid line corresponds to $\mu = 2p_T$.

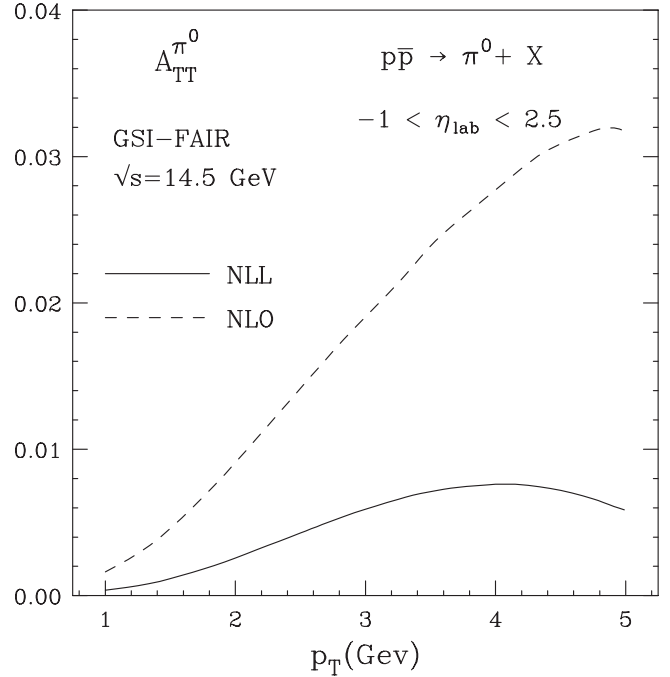


FIG. 4. NLL and NLO results for the double-spin asymmetry $A_{TT}^{\pi^0}$ in $\bar{p}p$ collisions at $\sqrt{S} = 14.5$ GeV, using “model” transversity distributions that saturate the Soffer bound [31] at a low scale.

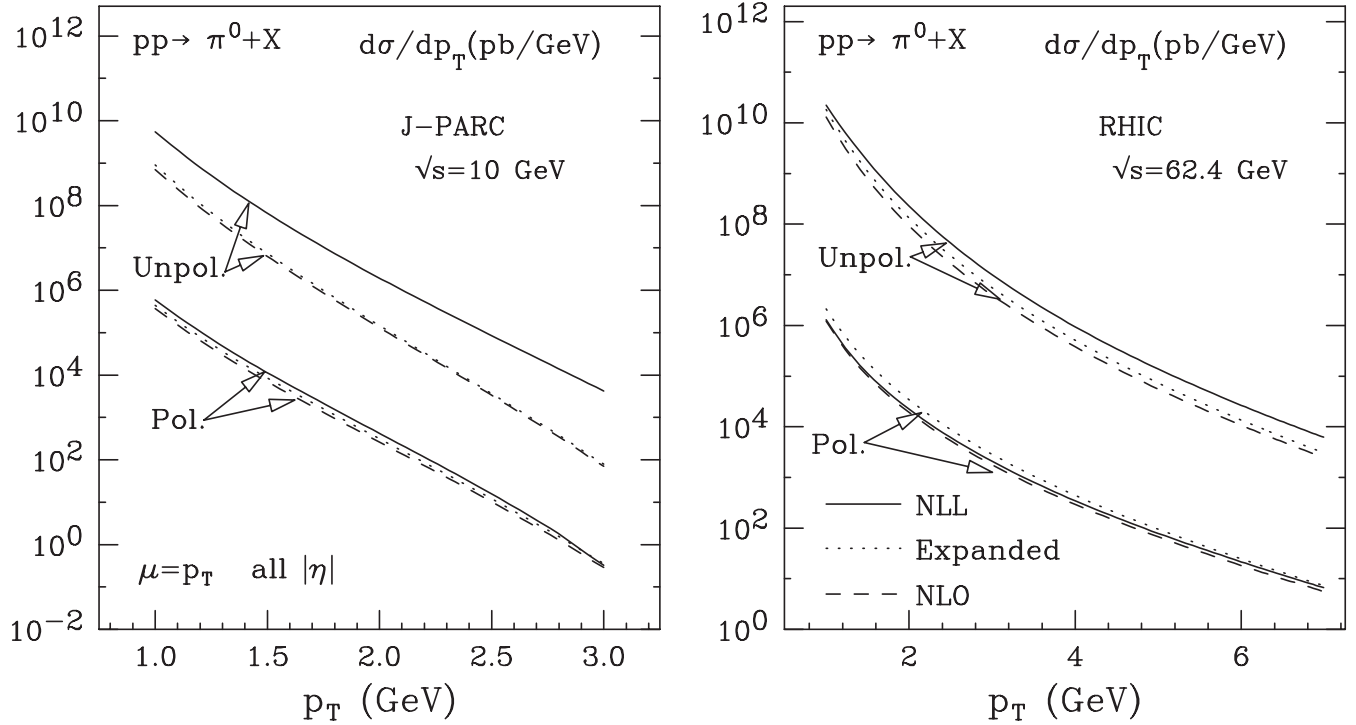


FIG. 5. Fully rapidity-integrated NLL resummed cross section, the expansion to $\mathcal{O}(\alpha_s^3)$, and the NLO cross section, for unpolarized and transversely polarized $pp \rightarrow \pi^0 X$ at $\sqrt{s} = 10$ GeV (left), and at $\sqrt{s} = 62.4$ GeV (right).

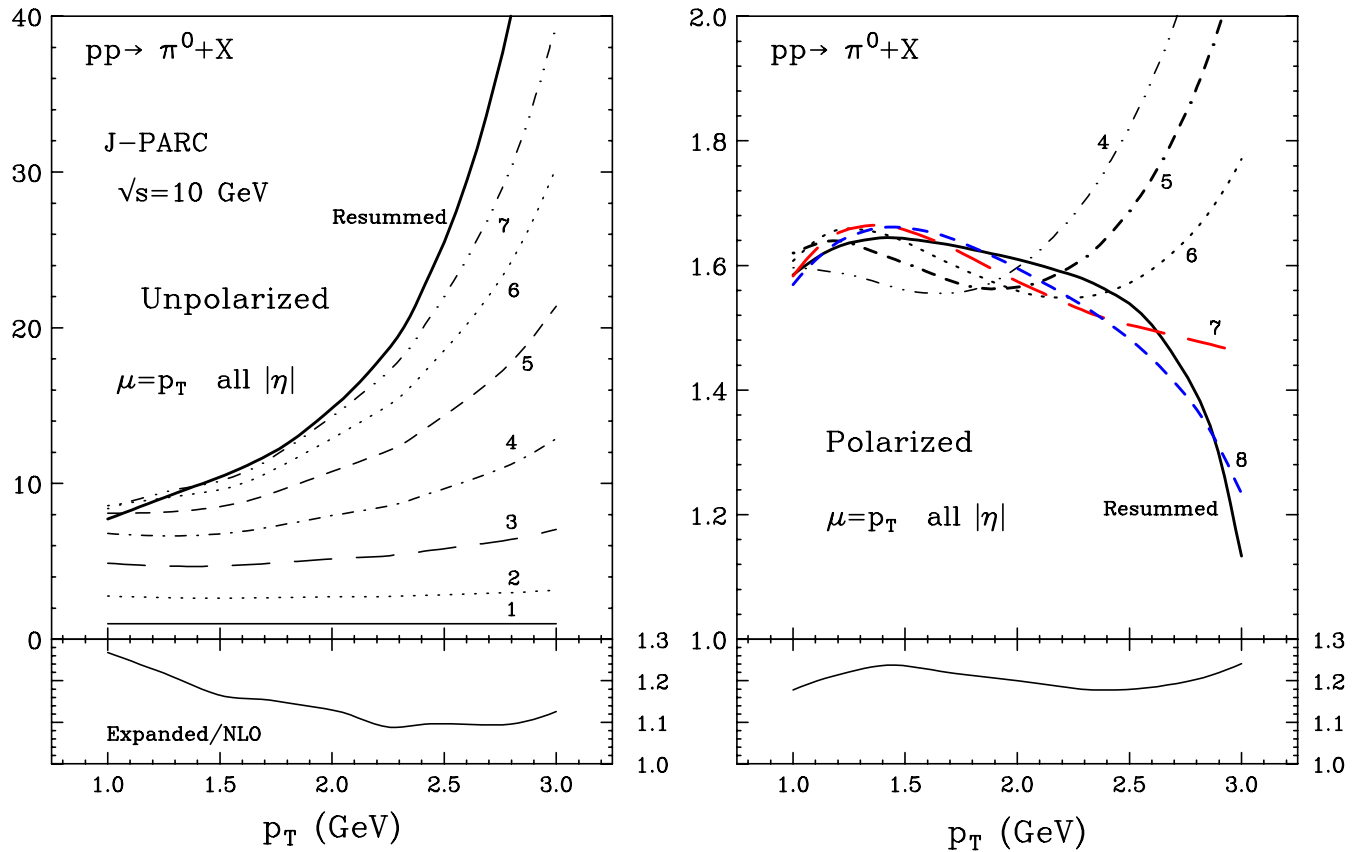


FIG. 6 (color online). Same as Fig. 2, but for pp collisions at $\sqrt{s} = 10$ GeV.

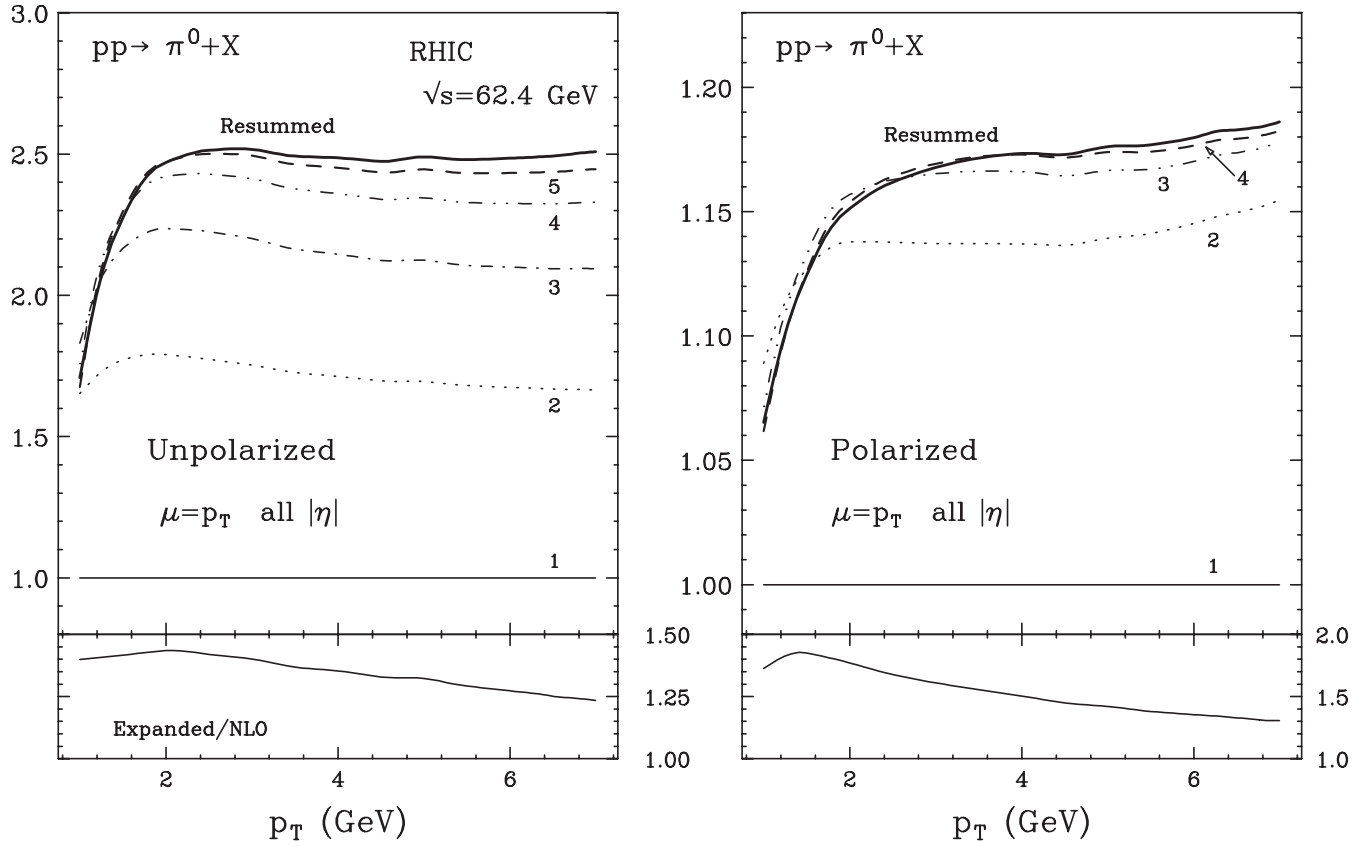


FIG. 7. Same as Fig. 6, but for collisions at $\sqrt{s} = 62.4$ GeV.

for experiments at RHIC (here, PHENIX) and J-PARC, respectively.

As before in Fig. 1 we display in Fig. 5 the rapidity-integrated spin-averaged and spin-dependent cross sections for J-PARC (left) and RHIC (right). Again, the $\mathcal{O}(\alpha_s^3)$ expansion reproduces adequately the NLO result, implying that the threshold logarithms addressed by resummation dominate the cross section in these kinematic regimes (the corresponding ratios can be seen in the lower panels of Figs. 6 and 7). This holds true, in particular, at $\sqrt{s} = 10$ GeV where one is much closer to the threshold regime. The agreement is more noticeable for the unpolarized cross section because of the large weight of the logarithmic contributions arising from the initial state gluonic subprocesses, whereas for transversely polarized scattering the expanded results slightly overestimate the NLO ones.

As can be expected, Figs. 6 and 7 show that the effects of resummation are much larger at $\sqrt{s} = 10$ GeV than at $\sqrt{s} = 62.4$ GeV. It is also interesting to notice that the K -factor is again much larger for the spin-averaged cross section than for the spin-dependent case, especially at $\sqrt{s} = 10$ GeV. This immediately implies that the spin asymmetry $A_{TT}^{\pi^0}$ will be dramatically reduced when going from NLO to the NLL resummed case.

In Figs. 6 and 7 we also show the soft-gluon $K^{(n)}$ -factors for pp collisions as we have already done in Fig. 2. As can

be observed, at $\sqrt{s} = 10$ GeV more higher-order terms are needed to reproduce the full resummed cross section than at $\sqrt{s} = 62.4$ GeV, as expected since one is much closer to the threshold regime in the first case. At $\sqrt{s} = 62.4$ GeV the $\mathcal{O}(\alpha_s^8)$ result can hardly be distinguished from the full NLL one, whereas at $\sqrt{s} = 10$ the full resummed result is required.

In order to allow comparison of our theoretical predictions with future data, we consider the regions of pseudorapidity as $\eta_{\text{cm}} > 0$ and $|\eta| < 0.38$ for J-PARC and RHIC (PHENIX), respectively, over which we again integrate. The first choice is based on the assumption of a forward spectrometer geometry with 200 mrad acceptance, similar to the one used by the COMPASS experiment at CERN, as it was also considered in Ref. [34]. The results for the two cases are presented in Fig. 8 which shows the NLL spin-averaged and spin-dependent cross sections. As before we have taken into account the theoretical uncertainties in the factorization and renormalization scales by varying $\mu = \zeta p_T$, with $\zeta = 1, 2, 4$. It is worth noticing that after resummation the scale dependence is considerably reduced in the case of RHIC.

Finally, we show in Fig. 9 our theoretical predictions for the spin asymmetry $A_{TT}^{\pi^0}$ at both J-PARC (left) and RHIC (right). As we anticipated, the NLL corrections significantly reduce the spin asymmetry with respect to the

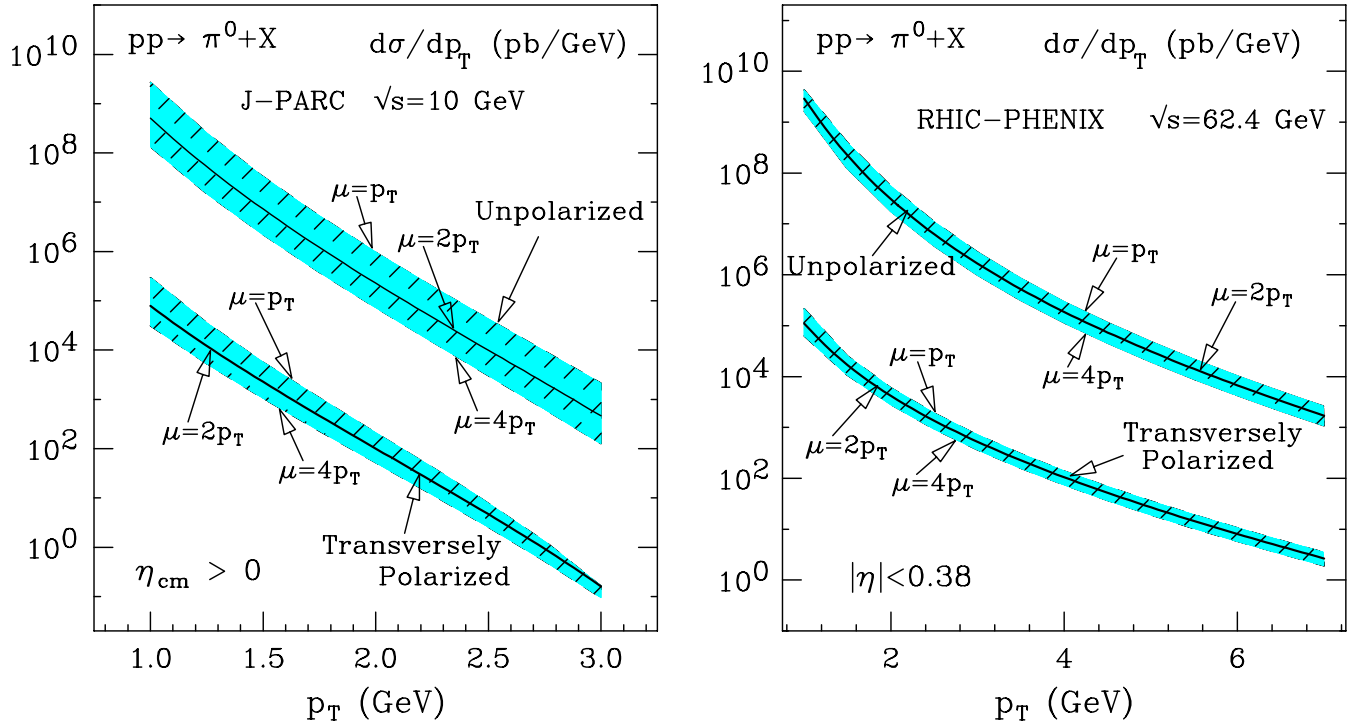


FIG. 8 (color online). NLL resummed cross section for unpolarized and transversely polarized $pp \rightarrow \pi^0 X$ at fixed-target $\sqrt{s} = 10$ GeV for $\eta_{\text{cm}} > 0$ (left) and at 62.4 GeV for $|\eta| < 0.38$ (right). The shaded bands represent the changes of the results if the factorization/renormalization scale is varied in the range $p_T \leq \mu \leq 4p_T$. The solid line corresponds to $\mu = 2p_T$.

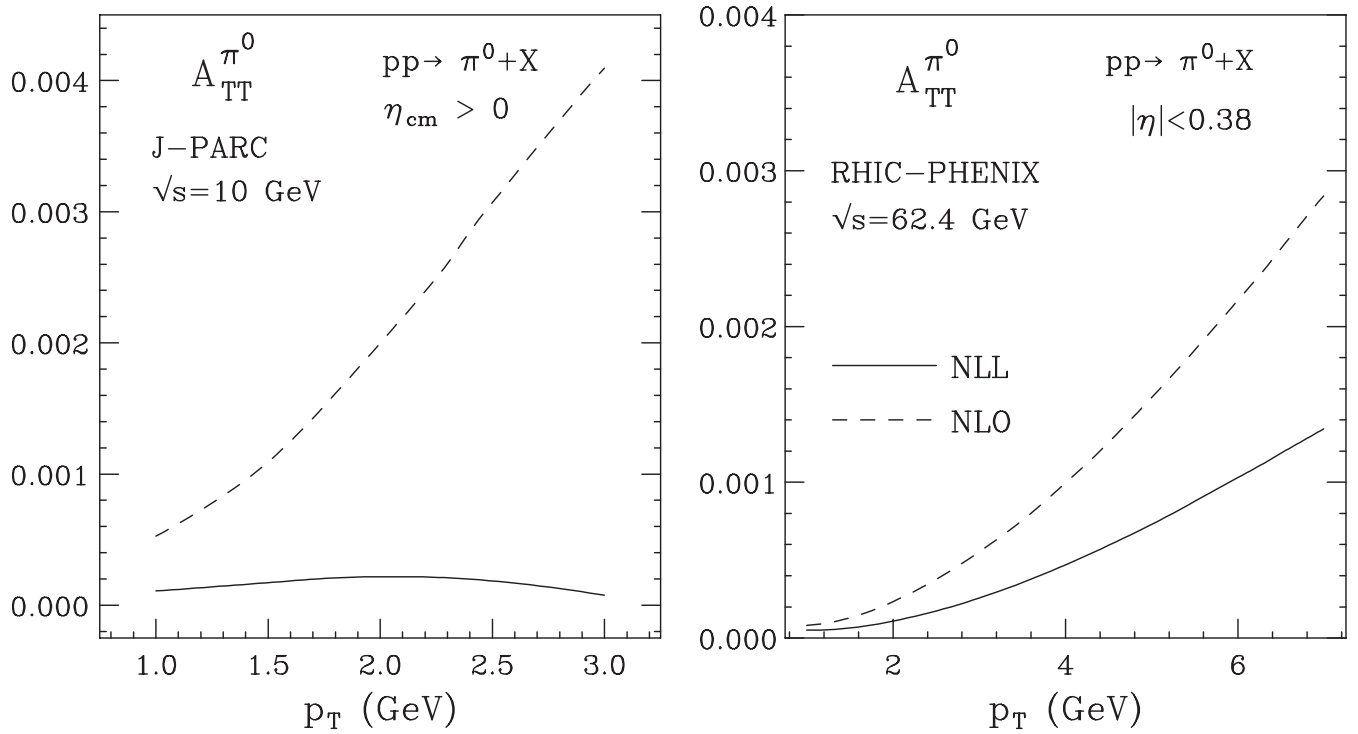


FIG. 9. Same as Fig. 4, but for pp collisions at $\sqrt{s} = 10$ GeV (left) and at $\sqrt{s} = 62.4$ GeV (right).

NLO results. This reduction is very significant for fixed-target experiments at J-PARC, whereas it is much more modest at RHIC's 62.4 GeV.

IV. CONCLUSIONS

We have studied in this paper the NLL resummation of threshold logarithms in the partonic cross sections relevant for the processes $pp \rightarrow hX$ and $\bar{p}p \rightarrow hX$ at high-transverse momentum of the hadron h , when the initial nucleons are transversely polarized. We have applied the resummation to proton-antiproton scattering at $\sqrt{S} = 14.5$ GeV, at which experiments might be carried out at GSI-FAIR, and to proton-proton scattering at $\sqrt{S} = 62.4$ GeV and at $\sqrt{S} = 10$ GeV, relevant for RHIC and J-PARC, respectively. We find that perturbative resummation produces a large enhancement of both the spin-averaged and the polarized cross sections. Its effect on the spin asymmetry is a significant reduction, especially at the two lower energies we have considered. We close by noting that for these cases power-suppressed contributions to the cross sections may be significant as well for the kinematics we have considered and will require careful theoretical study in the future.

ACKNOWLEDGMENTS

The work of D.dF. has been partially supported by Conicet, UBACyT and ANPCyT. W.V. is grateful to the

U.S. Department of Energy (Contract No. DE-AC02-98CH10886) for providing the facilities essential for the completion of his work. The work of F.W. has been supported by UBACyT.

APPENDIX A: RESULTS FOR THE VARIOUS SUBPROCESSES

In this appendix we compile the expressions for the transverse-spin-dependent Born cross sections for the various partonic subprocesses, and for the polarized process-dependent coefficients $\delta C_{ab \rightarrow cd}^{(1)}$, $\delta G_{Iab \rightarrow cd}$ that contribute to Eq. (2.4). All other ingredients of the resummation formula coincide with their expressions in the spin-averaged case and may be found in Ref. [24]. Since the $\delta C_{ab \rightarrow cd}^{(1)}$ have rather lengthy expressions, we only give their numerical values for number of active flavors $N_f = 5$, and for factorization and renormalization scales set to $\mu = Q = \sqrt{2}p_T$. In all expressions below, $C_A = 3$ and $C_F = (C_A^2 - 1)/2C_A = 4/3$. The lower index I of the coefficients $\delta G_{Iab \rightarrow cd}$ given below runs over the elements of the relevant color basis in each case. Also, $B(a, b)$ is the Beta-function.

$q\bar{q} \rightarrow q'\bar{q}'$:

$$\frac{d\delta\hat{\sigma}_{q\bar{q} \rightarrow q'\bar{q}'}^{(\text{Born})}(N)}{d\phi} = \alpha_s^2 \frac{\pi}{15} \frac{C_F}{C_A} N(N+1)(N+2) \cos(2\phi) B\left(N, \frac{7}{2}\right),$$

$$\delta G_{1q\bar{q} \rightarrow q'\bar{q}'}^{(1)} = 1, \quad \delta C_{1q\bar{q} \rightarrow q'\bar{q}'}^{(1)} = C_{1q\bar{q} \rightarrow q'\bar{q}'}^{(1)}. \quad (\text{A1})$$

$qq \rightarrow qq$:

$$\frac{d\delta\hat{\sigma}_{qq \rightarrow qq}^{(\text{Born})}(N)}{d\phi} = \alpha_s^2 \frac{\pi}{2} \frac{C_F}{C_A^2} \cos(2\phi) B\left(N+2, \frac{1}{2}\right),$$

$$\delta G_{1qq \rightarrow qq} = -1, \quad \delta G_{2qq \rightarrow qq} = 2, \quad \delta C_{1qq \rightarrow qq}^{(1)} = 21.6034(N_f = 5). \quad (\text{A2})$$

$q\bar{q} \rightarrow q\bar{q}$:

$$\frac{d\delta\hat{\sigma}_{q\bar{q} \rightarrow q\bar{q}}^{(\text{Born})}(N)}{d\phi} = \alpha_s^2 \frac{\pi}{4} \frac{C_F}{C_A} [C_A(N+2) + 2N+5] \cos(2\phi) B\left(N+2, \frac{3}{2}\right),$$

$$\delta G_{1q\bar{q} \rightarrow q\bar{q}} = 1, \quad \delta G_{2q\bar{q} \rightarrow q\bar{q}} = 0, \quad \delta C_{1q\bar{q} \rightarrow q\bar{q}}^{(1)} = 10.9783(N_f = 5). \quad (\text{A3})$$

$q\bar{q} \rightarrow gg$:

$$\frac{d\delta\hat{\sigma}_{q\bar{q}\rightarrow gg}^{(\text{Born})}(N)}{d\phi} = \alpha_s^2 \frac{\pi}{4} \frac{C_F}{C_A} [C_A^2(2N+6) - 2(2N+5)] \cos(2\phi) B\left(N+2, \frac{3}{2}\right),$$

$$\delta C_{1q\bar{q}\rightarrow gg}^{(1)} = C_{1q\bar{q}\rightarrow gg}^{(1)}, \quad \delta G_{1q\bar{q}\rightarrow gg} = \frac{5}{7}, \quad \delta G_{2q\bar{q}\rightarrow gg} = \frac{2}{7}, \quad \delta C_{1q\bar{q}\rightarrow gg}^{(1)} = C_{1q\bar{q}\rightarrow gg}^{(1)}. \quad (\text{A4})$$

APPENDIX B: SPIN-DEPENDENT COLOR-CONNECTED BORN CROSS SECTIONS

As discussed in [24,27], in order to obtain the coefficients $\delta G_{Iab\rightarrow cd}$, one needs the color-connected Born cross sections. For convenience, we list them in this Appendix for the transversely polarized case. Our choices for the color bases are the same as in Ref. [27]. We will not repeat the formulas for the soft matrices S , the anomalous dimension matrices Γ and the hard matrices H for the unpolarized case, which have been given in [24,27]. The S and Γ are spin-independent and are therefore the same for the polarized processes. Only the hard matrices δH for the polarized case are different. As in [27], we will present our results for arbitrary partonic rapidity, even though for our actual study we only need the case $\hat{\eta} = 0$. For each partonic reaction $ab \rightarrow cd$ we define the Mandelstam variables $s = (p_a + p_b)^2 = (p_c + p_d)^2$, $t = (p_a - p_c)^2 = (p_b - p_d)^2$ and $u = (p_a - p_d)^2 = (p_b - p_c)^2$. Both t and u are functions of $\hat{\eta}$. In all expressions below, $N_c = 3$ and $C_F = 4/3$.

$$q_j \bar{q}_j \rightarrow q_j \bar{q}_j$$

$$\delta H_{11}^{q_j \bar{q}_j \rightarrow q_j \bar{q}_j} = \alpha_s^2 \frac{4C_F^2}{N_c^4} \frac{tu}{s^2} \cos(2\phi),$$

$$\delta H_{12}^{q_j \bar{q}_j \rightarrow q_j \bar{q}_j} = \alpha_s^2 \frac{2C_F}{N_c^3} \left(\frac{-2tu}{N_c s^2} + \frac{u}{s} \right) \cos(2\phi) \quad (\text{B1})$$

$$= \delta H_{21}^{q_j \bar{q}_j \rightarrow q_j \bar{q}_j},$$

$$\delta H_{22}^{q_j \bar{q}_j \rightarrow q_j \bar{q}_j} = \alpha_s^2 \frac{1}{N_c^3} \left(\frac{4tu}{N_c s^2} - \frac{4u}{s} \right) \cos(2\phi).$$

$$q_j \bar{q}_j \rightarrow q_k \bar{q}_k$$

$$\delta H_{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} = \alpha_s^2 \begin{bmatrix} C_F^2 h^{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} & -C_F h^{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} \\ -C_F h^{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} & h^{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} \end{bmatrix}, \quad (\text{B2})$$

where $h^{q_j \bar{q}_j \rightarrow q_k \bar{q}_k} = 4tu \cos(2\phi)/(N_c^4 s^2)$.

$$q\bar{q} \rightarrow gg$$

$$\delta H_{11}^{q\bar{q}\rightarrow gg} = \alpha_s^2 \frac{1}{N_c^4} \cos(2\phi),$$

$$\delta H_{12}^{q\bar{q}\rightarrow gg} = N_c \delta H_{11}^{q\bar{q}\rightarrow gg} = \delta H_{21}^{q\bar{q}\rightarrow gg},$$

$$\delta H_{22}^{q\bar{q}\rightarrow gg} = N_c^2 \delta H_{11}^{q\bar{q}\rightarrow gg},$$

$$\delta H_{13}^{q\bar{q}\rightarrow gg} = \alpha_s^2 \frac{1}{N_c^3} \frac{u-t}{s} \cos(2\phi) = \delta H_{31}^{q\bar{q}\rightarrow gg}, \quad (\text{B3})$$

$$\delta H_{23}^{q\bar{q}\rightarrow gg} = N_c \delta H_{13}^{q\bar{q}\rightarrow gg} = \delta H_{32}^{q\bar{q}\rightarrow gg},$$

$$\delta H_{33}^{q\bar{q}\rightarrow gg} = \alpha_s^2 \frac{1}{N_c^2} \frac{(t-u)^2}{s^2} \cos(2\phi).$$

$$qq \rightarrow qq$$

This is the only quark-quark scattering process to consider.

$$\delta H_{11}^{qq\rightarrow qq} = \alpha_s^2 \frac{4}{N_c^3} \cos(2\phi),$$

$$\delta H_{12}^{qq\rightarrow qq} = -\alpha_s^2 \frac{2C_F}{N_c^3} \cos(2\phi) = \delta H_{21}^{qq\rightarrow qq}, \quad (\text{B4})$$

$$\delta H_{22}^{qq\rightarrow qq} = 0.$$

We finally note that the Born cross sections in Mellin-moment space given in the previous Appendix can be obtained from the above results by

$$\frac{d\delta\hat{\sigma}_{ij\rightarrow kl}^{(\text{Born})}(N)}{d\phi} = \frac{\pi}{2} \int_0^1 dv (4v(1-v))^{N+1} \text{Tr}[\delta H^{ij\rightarrow kl} S^{ij\rightarrow kl}], \quad (\text{B5})$$

where $S^{ij\rightarrow kl}$ is the soft matrix for a given partonic process, to be found in [27], and where in $\delta H^{ij\rightarrow kl}$ one has to set $u = -vs$ and $t = -(1-v)s$. The trace is in color space.

- [1] R. L. Jaffe and X. D. Ji, Phys. Rev. Lett. **67**, 552 (1991); Nucl. Phys. **B375**, 527 (1992).
[2] J. P. Ralston and D. E. Soper, Nucl. Phys. **B152**, 109 (1979).
[3] X. Artru and M. Mekhfi, Z. Phys. C **45**, 669 (1990).

- [4] M. Anselmino *et al.*, Phys. Rev. D **75**, 054032 (2007); arXiv:0807.0173.
[5] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. Lett. **94**, 012002 (2005); E. S. Ageev *et al.* (COMPASS Collaboration), Nucl. Phys. **B765**, 31 (2007). For the most

- recent experimental results, see talks by F. Bradamante (COMPASS Collaboration) and M. Diefenthaler (HERMES Collaboration) at the PKU-RBRC Workshop on Transverse Spin Physics 2008, Beijing, China.
- [6] K. Abe *et al.* (Belle Collaboration), Phys. Rev. Lett. **96**, 232002 (2006); A. Ogawa, M. Grosse-Perdekamp, R. Seidl, and K. Hasuko (Belle Collaboration), AIP Conf. Proc. **915**, 575 (2007); R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **78**, 032011 (2008).
- [7] J. C. Collins, Nucl. Phys. **B396**, 161 (1993).
- [8] G. Bunce, N. Saito, J. Soffer, and W. Vogelsang, Annu. Rev. Nucl. Part. Sci. **50**, 525 (2000); G. Bunce *et al.*, *Plans for the RHIC Spin Physics Program*, http://spin.riken.bnl.gov/rsc/report/spinplan_2008/spinplan08.pdf.
- [9] For information on the planned GSI-FAIR facility, see: <http://www.gsi.de/fair>; V. Barone *et al.* (PAX Collaboration), arXiv:hep-ex/0505054.
- [10] For information on the J-PARC facility, see: <http://j-parc.jp/index-e.html>; Y. Goto, Czech. J. Phys. **56**, F109 (2006).
- [11] J. L. Cortes, B. Pire, and J. P. Ralston, Z. Phys. C **55**, 409 (1992); A. P. Contogouris, B. Kamal, and Z. Merebashvili, Phys. Lett. B **337**, 169 (1994); V. Barone, T. Calarco, and A. Drago, Phys. Rev. D **56**, 527 (1997).
- [12] O. Martin, A. Schäfer, M. Stratmann, and W. Vogelsang, Phys. Rev. D **57**, 3084 (1998); **60**, 117502 (1999).
- [13] X. D. Ji, Phys. Lett. B **284**, 137 (1992).
- [14] W. Vogelsang and A. Weber, Phys. Rev. D **48**, 2073 (1993).
- [15] H. Shimizu, G. Sterman, W. Vogelsang, and H. Yokoya, Phys. Rev. D **71**, 114007 (2005).
- [16] M. Anselmino, V. Barone, A. Drago, and N. N. Nikolaev, Phys. Lett. B **594**, 97 (2004); A. V. Efremov, K. Goetze, and P. Schweitzer, Eur. Phys. J. C **35**, 207 (2004).
- [17] P. G. Ratcliffe, Eur. Phys. J. C **41**, 319 (2005).
- [18] R. L. Jaffe and N. Saito, Phys. Lett. B **382**, 165 (1996); J. Soffer, M. Stratmann, and W. Vogelsang, Phys. Rev. D **65**, 114024 (2002).
- [19] A. Mukherjee, M. Stratmann, and W. Vogelsang, Phys. Rev. D **72**, 034011 (2005).
- [20] S. B. Libby and G. Sterman, Phys. Rev. D **18**, 3252 (1978); R. K. Ellis, H. Georgi, M. Machacek, H. D. Politzer, and G. G. Ross, Phys. Lett. B **78**, 281 (1978); D. Amati, R. Petronzio, and G. Veneziano, Nucl. Phys. **B146**, 29 (1978); **B140**, 54 (1978); G. Curci, W. Furmanski, and R. Petronzio, Nucl. Phys. **B175**, 27 (1980); J. C. Collins, D. E. Soper, and G. Sterman, Phys. Lett. B **134**, 263 (1984); Nucl. Phys. **B261**, 104 (1985); J. C. Collins, Nucl. Phys. **B394**, 169 (1993).
- [21] A. Adare *et al.* (PHENIX Collaboration), Phys. Rev. D **76**, 051106 (2007).
- [22] J. Adams *et al.* (STAR Collaboration), Phys. Lett. B **637**, 161 (2006); Phys. Rev. Lett. **97**, 152302 (2006).
- [23] I. Arsene *et al.* (BRAHMS Collaboration), Phys. Rev. Lett. **98**, 252001 (2007).
- [24] D. de Florian and W. Vogelsang, Phys. Rev. D **71**, 114004 (2005).
- [25] D. de Florian, W. Vogelsang, and F. Wagner, Phys. Rev. D **76**, 094021 (2007).
- [26] G. Sterman, Nucl. Phys. **B281**, 310 (1987); S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989); S. Catani, M. L. Mangano, and P. Nason, J. High Energy Phys. **07** (1998) 024; E. Laenen, G. Oderda, and G. Sterman, Phys. Lett. B **438**, 173 (1998); N. Kidonakis and G. Sterman, Nucl. Phys. **B505**, 321 (1997); N. Kidonakis, G. Oderda, and G. Sterman, Nucl. Phys. **B525**, 299 (1998); **B531**, 365 (1998); R. Bonciani, S. Catani, M. L. Mangano, and P. Nason, Phys. Lett. B **575**, 268 (2003).
- [27] N. Kidonakis, G. Oderda, and G. Sterman, Nucl. Phys. **B525**, 299 (1998); **B531**, 365 (1998); N. Kidonakis and J. F. Owens, Phys. Rev. D **63**, 054019 (2001).
- [28] S. Catani, M. L. Mangano, P. Nason, and L. Trentadue, Nucl. Phys. **B478**, 273 (1996).
- [29] M. J. Tannenbaum (PHENIX Collaboration), Nucl. Phys. B, Proc. Suppl. **177**, 150 (2008).
- [30] J. Pumplin *et al.*, J. High Energy Phys. **07** (2002) 012.
- [31] J. Soffer, Phys. Rev. Lett. **74**, 1292 (1995); D. W. Sivers, Phys. Rev. D **51**, 4880 (1995).
- [32] D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D **75**, 114010 (2007); **76**, 074033 (2007).
- [33] L. C. Bland *et al.*, Eur. Phys. J. C **43**, 427 (2005).
- [34] J. Riedl, A. Schäfer, and M. Stratmann, Eur. Phys. J. C **52**, 987 (2007).