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Study of $a_0^0(980)$-$f_0(980)$ mixing from $a_0^0(980) \to f_0(980)$ transition

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Various processes have been proposed previously to study $a_0^0(980)$-$f_0(980)$ mixing through the $f_0(980) \to a_0^0(980)$ transition. Here we investigate in detail the difference between $a_0^0(980) \to f_0(980)$ and $f_0(980) \to a_0^0(980)$ transitions. It is found that the $a_0^0(980) \to f_0(980)$ transition can provide additional constraints to the parameters of $a_0^0(980)$ and $f_0(980)$ mesons. A proposal is made to study $a_0^0(980)$-$f_0(980)$ mixing from the $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980)$ reaction at the upgraded Beijing electron positron collider with the BESIII detector.

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I. INTRODUCTION

More than 30 years after their discovery, today the nature of light scalar mesons $f_0(980)$ and $a_0^0(980)$ is still in controversy. They have been described as quark-antiquark, four quarks, $K\bar{K}$ molecule, quark-antiquark-gluon hybrid, and so on. Now the study of their nature has become a central problem in the light hadron spectroscopy.

In the late 1970s, the mixing between the $a_0^0(980)$ and $f_0(980)$ resonances was first suggested theoretically in Ref. [1]. Its mixing intensity is expected to shed important light on the nature of these two resonances, and has hence been studied extensively on its different aspects and possible manifestations in various reactions [2–16]. Unfortunately, no firm experimental determination on this quantity is available yet. Obviously, more solid and precise measurements on this quantity are needed, such as by a polarized target experiment on the reaction $\pi^- p \to \eta\pi^0 n$ [4], J/ψ decays [6,15], and $dd \to \alpha\eta\pi^0$ reactions from WASA-at-COSY [12]. In Ref. [15], we pointed out that the $a_0^0(980)$-$f_0(980)$ mixing intensity can be precisely measured through the $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ reaction at the upgraded Beijing electron positron collider with the BESIII detector.

In all these previous proposals, the $f_0(980)$ is produced first, then transits to the $a_0^0(980)$ by the $a_0^0(980)$-$f_0(980)$ mixing, i.e., $f_0(980) \to a_0^0(980)$ transition. In this article, we investigate in detail the difference between $a_0^0(980) \to f_0(980)$ and $f_0(980) \to a_0^0(980)$ transitions. We define two kinds of mixing intensities ξ_{fa} and ξ_{af} for the $f_0(980) \to a_0^0(980)$ and $a_0^0(980) \to f_0(980)$ transitions, respectively. We find there are some differences between them. We find that ξ_{af} has more dependence on the parameters of $f_0(980)$, especially the $g_{fKK}/g_{f\pi\pi}$, while the ξ_{fa} has more dependence on the parameters of $a_0^0(980)$, especially $g_{aKK}/g_{a\pi\eta}$. For this reason, using the reaction of $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi\eta\pi^0$ to study only the $f_0(980) \to a_0^0(980)$ mixing is not perfect enough. For better determination of all relevant parameters for the $f_0(980)$ and $a_0^0(980)$ mesons, it would be useful to find some reaction to study $a_0^0(980) \to f_0(980)$ mixing in addition. Recently, the CLEO collaboration reported an experimental study of the $\chi_{c1} \to \pi^+\pi^-\eta$ reaction [17]. The $a_0^\pm(980)$ resonances are clearly showing up and dominant. From isospin symmetry, $\chi_{c1} \to \pi^0 a_0^0(980)$ should be produced with the same rate as $\chi_{c1} \to \pi^\pm a_0^\mp(980)$. This may provide a nice place for studying the $a_0^0(980)$-$f_0(980)$ mixing from the $a_0^0(980) \to f_0(980)$ transition by the $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0\pi\pi$ reaction. From our estimation, more than 300 events can be reconstructed by the BESIII detector in the narrow peak with a width of about 8 MeV around the mass of 990 MeV in the $\pi\pi$ invariant mass spectrum.

In the next section, we give a brief review of the theory for the $a_0^0(980)$-$f_0(980)$ mixing term. Then in the Sec. III we define two mixing intensities and tell the differences of mixing intensity. In Sec. IV we estimate the rate for the reaction of $\chi_{c1} \to \pi^0\pi\pi$. Finally we give a summary in Sec. V.

II. THE $a_0^0(980)$-$f_0(980)$ MIXING AMPLITUDE

The basic theory for the $a_0^0(980)$-$f_0(980)$ mixing was already pointed out by Achasov and collaborators [1]. For the nearly degenerate $a_0^0(980)$ (isospin 1) and $f_0(980)$ (isospin 0), both can decay into $K\bar{K}$. Because of the isospin breaking effect, the charged and neutral kaon thresholds are different by about 8 MeV. Between the charged and neutral kaon thresholds the leading term to the $a_0^0(980)$-$f_0(980)$ mixing amplitude is dominated by the unitary cuts of the intermediate two-kaon system and proportional to the difference of phase spaces for the charged and neutral kaon systems.

Considering the $a_0^0(980)$-$f_0(980)$ mixing, the propagator of $a_0^0(980)/f_0(980)$ can be expressed as [2]

$$G = \frac{1}{D_f D_a - |D_{af}|^2}\begin{pmatrix} D_a & D_{af} \\ D_{af} & D_f \end{pmatrix}, \qquad (1)$$

where D_a and D_f are the denominators for the usual propagators of $a_0^0(980)$ and $f_0(980)$, respectively:

$$D_a = m_a^2 - s - i\sqrt{s}[\Gamma^a_{\eta\pi}(s) + \Gamma^a_{K\bar{K}}(s)], \tag{2}$$

$$D_f = m_f^2 - s - i\sqrt{s}[\Gamma^f_{\pi\pi}(s) + \Gamma^f_{K\bar{K}}(s)], \tag{3}$$

$$\Gamma^a_{bc}(s) = \frac{g^2_{abc}}{16\pi\sqrt{s}}\rho_{bc}(s), \tag{4}$$

$$\rho_{bc}(s) = \sqrt{[1 - (m_b - m_c)^2/s][1 - (m_b + m_c)^2/s]}. \tag{5}$$

The D_{af} is the mixing term. From [1,4], the mixing due to $K\bar{K}$ loops gives

$$D_{af,K\bar{K}} = \frac{g_{a_0^0(980)K^+K^-}g_{f_0(980)K^+K^-}}{16\pi}\{i[\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)] - \mathcal{O}(\rho^2_{K^+K^-}(s) - \rho^2_{K^0\bar{K}^0}(s))\}. \tag{6}$$

Since the mixing is mainly coming from the $K\bar{K}$ loops, we have $D_{af} \approx D_{af,K\bar{K}}$, and this is the amplitude of $a_0^0(980)$-$f_0(980)$ mixing. From Eq. (6), D_{af} becomes large only when $\sqrt{s}$ is between $2M_{K^+}$ and $2M_{K^0}$, so it is a narrow peak with the width of about 8 MeV.

III. TWO TYPES OF REACTION AND MIXING INTENSITY

A. Two types of reaction of $a_0^0(980)$-$f_0(980)$ mixing

There are two types of reaction which can be used to study $a_0^0(980)$-$f_0(980)$ mixing: $X \to Yf_0(980) \to Ya_0^0(980) \to Y\pi^0\eta$ and $X \to Ya_0^0(980) \to Yf_0(980) \to Y\pi\pi$.

For the reaction $X \to Yf_0(980) \to Ya_0^0(980) \to Y\pi^0\eta$ as shown by the Feynman diagram in Fig. 1, the influence of various X and Y on the $a_0^0(980)$-$f_0(980)$ mixing can be removed by its comparison to the corresponding reaction $X \to Yf_0(980) \to Y\pi\pi$ as shown in Fig. 2. We define the mixing intensity ξ_{fa} for the $f_0(980) \to a_0^0(980)$ transition as the following:

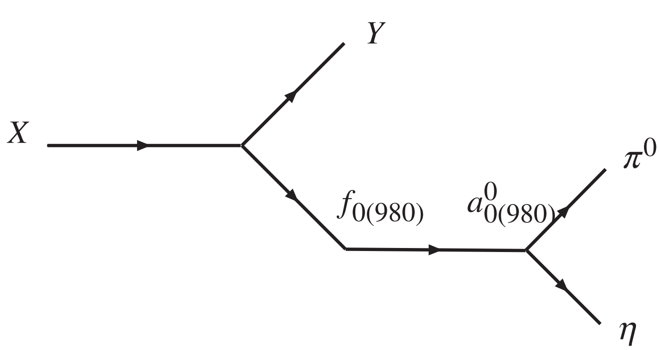

FIG. 1. The Feynman diagram of $X \to Yf_0(980) \to Ya_0^0(980) \to Y\pi^0\eta$.

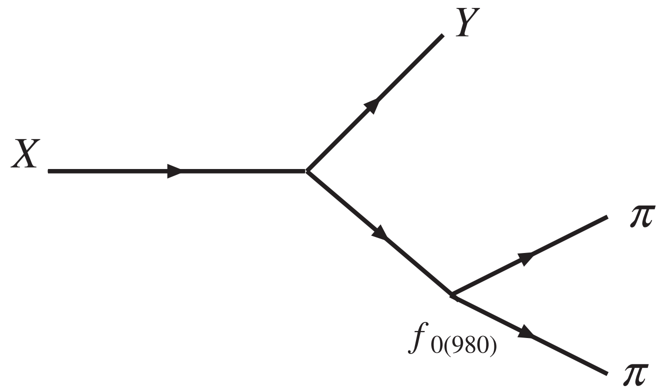

FIG. 2. The Feynman diagram of $X \to Yf_0(980) \to Y\pi\pi$.

$$\xi_{fa}(s) = \frac{d\Gamma_{X\to Yf_0(980)\to Ya_0^0(980)\to Y\pi^0\eta(s)}}{d\Gamma_{X\to Yf_0(980)\to Y\pi\pi(s)}}, \tag{7}$$

where s is the invariant mass squared of two mesons in the final state. With Eqs. (1)–(6), one can get $\xi_{fa}(s)$ as

$$\xi_{fa}(s) = \frac{|D_{af}|^2\Gamma^a_{\pi\eta}}{|D_a|^2\Gamma^f_{\pi\pi}} \tag{8}$$

$$= \left|\frac{g_{a_0^0K^+K^-}g_{f_0K^+K^-}}{g_{a_0^0\pi^0\eta}g_{f_0\pi^0\pi^0}}\right|^2\frac{|\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)|^2}{3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)} \times \frac{1}{|\frac{m_a^2-s}{\Gamma^a_{\pi\eta}\sqrt{s}} - i[|\frac{g_{a_0^0K^+K^-}}{g_{a_0^0\pi^0\eta}}|^2(\frac{\rho_{K^+K^-(s)}}{\rho_{\pi\eta(s)}} + \frac{\rho_{K^0\bar{K}^0}}{\rho_{\pi\eta}}) + 1]|^2}. \tag{9}$$

Similarly, for the reaction $X \to Ya_0^0(980) \to Yf_0(980) \to Y\pi\pi$, we define the mixing intensity ξ_{af} for the $a_0^0(980) \to f_0(980)$ transition and get its formula as the following:

$$\xi_{af}(s) = \frac{d\Gamma_{X\to Ya_0^0(980)\to Yf_0(980)\to Y\pi\pi(s)}}{d\Gamma_{X\to Ya_0^0(980)\to Y\pi^0\eta(s)}}, \tag{10}$$

$$= \left|\frac{g_{a_0^0K^+K^-}g_{f_0K^+K^-}}{g_{a_0^0\pi^0\eta}g_{f_0\pi^0\pi^0}}\right|^2\frac{|\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)|^2}{3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)} \times \frac{1}{|\frac{m_f^2-s}{\Gamma^f_{\pi\pi}\sqrt{s}} - i[|\frac{g_{f_0K^+K^-}}{g_{f_0\pi^0\pi^0}}|^2(\frac{\rho_{K^+K^-(s)}}{3\rho_{\pi\pi(s)}} + \frac{\rho_{K^0\bar{K}^0(s)}}{3\rho_{\pi\pi(s)}}) + 1]|^2}. \tag{11}$$

We can redefine that

$$R_f = |g_{f_{0(980)}K^+K^-}/g_{f_{0(980)}\pi^0\pi^0}|^2, \quad R_a = |g_{a^0_{0(980)}K^+K^-}/g_{a^0_{0(980)}\pi^0\eta}|^2, \tag{12}$$

$$A^f_{(s)} = \rho_{K^+K^-(s)}/3\rho_{\pi\pi(s)}, \qquad A^a_{(s)} = \rho_{K^+K^-(s)}/\rho_{\pi\eta(s)}, \tag{13}$$

$$B^f_{(s)} = \rho_{K^0\bar{K}^0(s)}/3\rho_{\pi\pi(s)}, \qquad B^a_{(s)} = \rho_{K^0\bar{K}^0(s)}/\rho_{\pi\eta(s)}, \tag{14}$$

$$C^f_{(s)} = (m_f^2 - s)/\Gamma^f_{\pi\pi}\sqrt{s}, \qquad C^a_{(s)} = (m_a^2 - s)/\Gamma^a_{\pi\eta}\sqrt{s}, \tag{15}$$

$$H_{(s)} = |\rho_{K^+K^-}(s) - \rho_{K^0\bar{K}^0}(s)|^2/(3\rho_{\pi\pi}(s)\rho_{\pi\eta}(s)). \tag{16}$$

The D_{af} becomes large when $\sqrt{s}$ is between the $2M_{K^+}$ and $2M_{K^0}$. In this mass range, $A^f_{(s)}$ and $A^a_{(s)}$ are real, meanwhile $B^f_{(s)}$ and $B^a_{(s)}$ are imaginary; then ξ_{af} and ξ_{fa} become

$$\xi_{af}(s) = \frac{R_aR_f \times H_{(s)}}{(C^f_{(s)} + R_f \times |B^f_{(s)}|)^2 + (1 + R_f \times A^f_{(s)})^2}, \tag{17}$$

$$\xi_{fa}(s) = \frac{R_aR_f \times H_{(s)}}{(C^a_{(s)} + R_a \times |B^a_{(s)}|)^2 + (1 + R_a \times A^a_{(s)})^2}. \tag{18}$$

B. Predictions of ξ_{af} and ξ_{fa} from various models and experiment information

From the equations given above, one can see that the mixing intensity $|\xi|$ depends on $g_{a_0^0(980)K^+K^-}$, $g_{f_0(980)K^+K^-}$, $g_{a_0^0(980)\pi^0\eta}$, $g_{f_0(980)\pi^0\pi^0}$, m_f, and m_a. Various models for the structures of $a_0^0(980)$ and $f_0(980)$ give different predictions for these coupling constants and mass [14,18–20] as listed in Table I by Nos. A–D. There have also been some experimental measurements on these coupling and mass constants [22–29] as listed by Nos. E–H. The corresponding predictions for ξ_{af} and ξ_{fa} from these various theoretical and experimental values of the coupling constants are calculated. In the calculation, the masses for K^+, K^0, π^0, and η are taken from PDG2008 [30] as $m_{K^+} = 493.7$ MeV, $m_{K^0} = 497.7$ MeV, $m_\pi = 135.0$ MeV, and $m_\eta = 547.5$ MeV, respectively. We give the value of ξ_{af} and ξ_{fa} at $\sqrt{s} = 991.4$ MeV in Table I and the dependence of $a_0^0(980)$-$f_0(980)$ mixing intensities ξ_{af} and ξ_{fa} vs two-meson invariant mass in Fig. 3. There is obviously some difference between these two mixing intensities.

C. Discussion on the difference of two mixing intensities ξ_{af} and ξ_{fa}

We know from Eqs. (17) and (18), if the $C^i_{(s)}$, $R_i \times B^i_{(s)}$, and $R_i \times A^i_{(s)}$ ($i = a$ or f) are much smaller than 1, we will have $\xi_{af}(s) \simeq \xi_{fa}(s) \simeq R_aR_f \times H_{(s)}$. Then from these mixing intensities we only get R_aR_f no matter which type of reactions one makes the measurement.

However, are these quantities really much smaller than 1? For $\sqrt{s}$ between $2M_{K^+}$ and $2M_{K^0}$, $\sqrt{s} \approx m_i$, we have

$$\begin{aligned} C^i_{(s)} &= (m_i^2 - s)/(\Gamma^i_{\pi\pi(\eta)}\sqrt{s}) \\ &= (m_i - \sqrt{s})(m_i + \sqrt{s})/(\Gamma^i_{\pi\pi(\eta)}\sqrt{s}) \\ &\approx 2(m_i - \sqrt{s})/\Gamma^i_{\pi\pi(\eta)}. \end{aligned} \tag{19}$$

Here $\Gamma^i_{\pi\pi(\eta)}$ is about 100 MeV from many experiments, $m_i \simeq 980$ MeV, then one gets $C^i_{(4M^2_{K^+})} = -0.14$ and $C^i_{(4M^2_{K^0})} = -0.30$, which are not so small.

For $R_i \times B^i_{(s)}$ and $R_i \times A^i_{(s)}$ with $A^i_{(4M^2_{K^+})} = |B^i_{(4M^2_{K^0})}| = 0$, $A^a_{(4M^2_{K^0})} \simeq |B^a_{(4M^2_{K^+})}| = 0.201$, and $A^f_{(4M^2_{K^0})} \simeq |B^f_{(4M^2_{K^+})}| = 0.044$, since from Table I the smallest R_a and R_f are 0.5 and 5.7, respectively, among various experimental determinations, the $R_i \times |B^i_{(s)}|$ and $R_i \times A^i_{(s)}$ are also larger than 0.1 and are not small enough to be neglected.

Then from Eqs. (17) and (18), one can see that besides the common numerator, ξ_{af} has additional dependence on the parameters of $f_0(980)$ while ξ_{fa} has additional dependence on the parameters of $a_0^0(980)$.

From the above analysis, we understand why ξ_{af} is different from ξ_{fa} as shown in Table I and Fig. 3. Both mixing intensities depend on six parameters: m_f, m_a, $g_{a_0^0(980)K^+K^-}$, $g_{f_0(980)K^+K^-}$, $g_{a_0^0(980)\pi^0\eta}$, $g_{f_0(980)\pi^0\pi^0}$, which are all important for understanding the nature of the $a_0^0(980)$ and $f_0(980)$ mesons, but are not well determined

TABLE I. $m_{a_0^0(980)}$ (MeV), $m_{f_0(980)}$ (MeV), and coupling constants $g_{a_0\pi\eta}$ (GeV), $g_{a_0K^+K^-}$ (GeV), $g_{f_0K^+K^-}$ (GeV), and $g_{f_0\pi^0\pi^0}$ (GeV) from various models (A–D) and experimental measurements (E–H), and calculated values of $|\xi_{af}|$ and $|\xi_{fa}|$ at $\sqrt{s} = 991.4$ MeV by Eqs. (17) and (18).

No.	Model or experiment	m_a	$g_{a_0\pi\eta}$	$g_{a_0K^+K^-}$	m_f	$g_{f_0\pi^0\pi^0}$	$g_{f_0K^+K^-}$	$\lvert\xi_{fa}\rvert$	$\lvert\xi_{af}\rvert$
A	$q\bar{q}$ model [14]	983	2.03	1.27	975	0.64	1.80	0.023	0.010
B	$q^2\bar{q}^2$ model [14]	983	4.57	5.37	975	1.90	5.37	0.068	0.062
C	$K\bar{K}$ model [18,19,21]	980	1.74	2.74	980	0.65	2.74	0.21	0.15
D	$q\bar{q}g$ model [20]	980	2.52	1.97	975	1.54	1.70	0.005	0.006
E	SND [22,23]	995	3.11	4.20	969.8	1.84	5.57	0.088	0.089
F	KLOE [24,25]	984.8	3.02	2.24	973	2.09	5.92	0.034	0.025
G	BNL [26]	1001	2.47	1.67	953.5 [27]	1.36 [27]	3.26 [27]	0.019	0.014
H	CB [28]	999	3.33	2.54	965 [29]	1.66 [29]	4.18 [29]	0.027	0.023

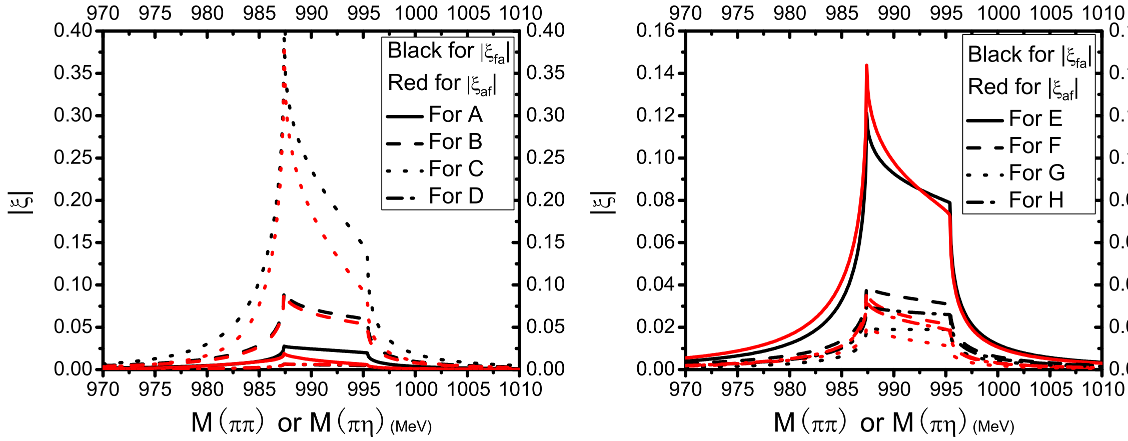

FIG. 3 (color online). Predictions for the $a_0^0(980)$-$f_0(980)$ mixing intensity ξ_{af} and ξ_{fa} vs two-meson invariant mass from various models A–-D (left) and various experimental measured parameters E–H (right).

yet. Therefore to measure ξ_{af} in addition to ξ_{fa} will be very useful for pinning down these parameters.

To further demonstrate the importance of measuring ξ_{af} in addition to ξ_{fa}, a typical example is given as follows.

For two sets of parameters given in Table II, we can see that the set No. 1 is close to the SND values in Table I. The set No. 2 changes the not well measured g_{aKK} and g_{fKK} in their experimental uncertainties. We plot the corresponding diagrams of $a_0^0(980)$-$f_0(980)$ mixing intensities ξ_{af} and ξ_{fa} vs two-meson invariant mass M_2 as shown in Fig. 4. The two sets of parameters give almost identical ξ_{fa} but very different ξ_{af}.

IV. POSSIBILITY OF MEASURING ξ_{af} FROM $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0\pi\pi$

The Feynman diagram for the reaction $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0\pi\pi$ is shown in Fig. 5.

The invariant amplitude for this reaction is

$$\mathcal{M}_{\chi_{c1}\to\pi^+\pi^-\pi^0} = g_{\chi_{c1}a_0^0(980)\pi^0}\varepsilon^{\mu}_{\chi_{c1}}(p_{\pi^0} - p_{f_0(980)})\frac{D_{af}}{D_f D_a} \times \sqrt{3}g_{f_0(980)\pi^0\pi^0}. \tag{20}$$

The coupling constant $g_{\chi_{c1}a_0^0(980)\pi^0}$ can be determined by the reaction $\chi_{c1} \to a^{\pm}_{0(980)}\pi^{\mp}$

TABLE II. Two typical parameter sets for $a_0^0(980)$ and $f_0(980)$.

No.	m_a (MeV)	$g_{a_0\pi\eta}$ (GeV)	$g_{a_0K^+K^-}$ (GeV)	m_f (MeV)	$g_{f_0\pi^0\pi^0}$ (GeV)	$g_{f_0K^+K^-}$ (GeV)
1	980	3.2	4.2	980	1.5	4.0
2	980	3.2	3.0	980	1.5	5.12

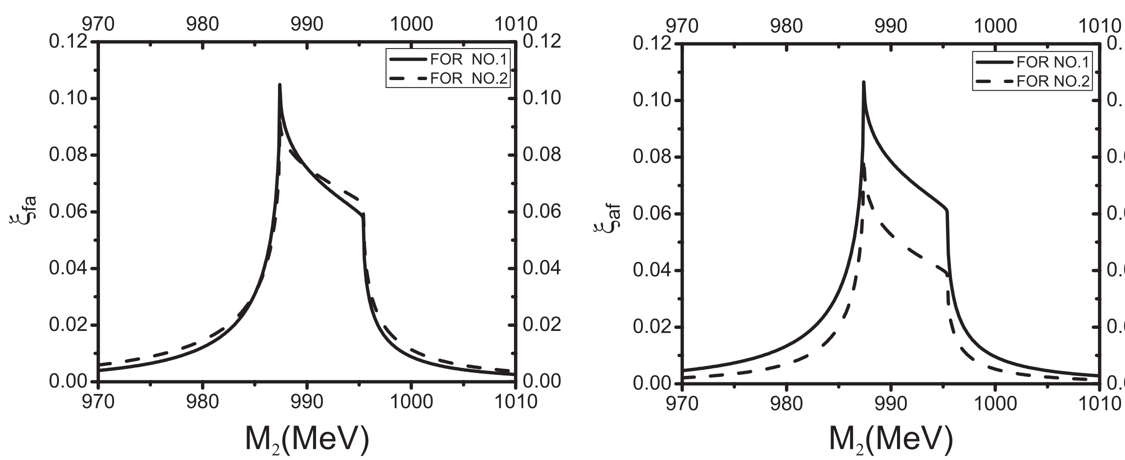

FIG. 4. Diagrams of $a_0^0(980)$-$f_0(980)$ mixing intensity ξ_{af} and ξ_{fa} vs two-meson invariant mass with parameter sets No. 1 and No. 2 of Table II.

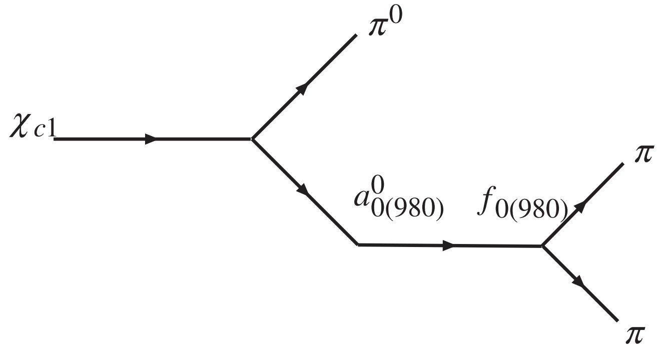

FIG. 5. Feynman diagram for $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0 \pi\pi$.

$$|g_{\chi_{c1} a_0^0(980)\pi^0}|^2 = |g_{\chi_{c1} a^+_{0(980)}\pi^-}|^2 = \frac{12\pi M^2_{\chi_{c1}} \Gamma_{\chi_{c1}} \mathrm{Br}_{(\chi_{c1}\to a^+_{0(980)}\pi^-)}}{|\vec{p}_{a^+_{0(980)}}|^3}. \tag{21}$$

According to Ref. [30], $M_{\chi_{c1}} = 3510.66$ MeV, $\Gamma_{\chi_{c1}} = 0.89$ MeV, and $\mathrm{Br}_{\chi_{c1}\to\eta\pi^+\pi^-} = (5.2 \pm 0.6) \times 10^{-3}$. The $\chi_{c1} \to a^{\pm}_{0(980)}\pi^{\mp}$ gives the dominant contribution of about 75.1% [17]. So we have $\mathrm{Br}_{(\chi_{c1}\to a^+_{0(980)}\pi^-)} \simeq 0.0052 \times 0.751/2 \simeq 2 \times 10^{-3}$. From the formulas of Eqs. (1)–(6) and parameters of setting No. H listed in Table I, we can calculate $\mathrm{Br}_{\chi_{c1}\to\pi^0 a_0^0(980)\to\pi^0 f_0(980)\to\pi^0\pi\pi}$. It is 4.6×10^{-6} and the invariant mass spectrum of $\pi^+\pi^-$ for $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$ is shown in Fig. 6.

The branching ratio of reaction $\psi_{2s} \to \gamma\chi_{c1}$ is 0.088 [30]. At the upgraded Beijing electron positron collider with BESIII detector, about 3.2×10^9 ψ_{2s} events and hence about 2.8×10^8 χ_{c1} events can be collected per year. From the branching ratio of $\chi_{c1} \to \pi^0\pi\pi = 4.6 \times 10^{-6}$, more than 1200 events are expected to be collected. Considering the reconstruction efficiency of 30%, more than 300 events can be reconstructed for this channel. Since all these events should concentrate in a narrow region of about 8 MeV around 990 MeV in the $\pi^+\pi^-$ invariant mass spectrum, the narrow peak should be easily observed; hence, ξ_{af} should be able to be measured by this reaction.

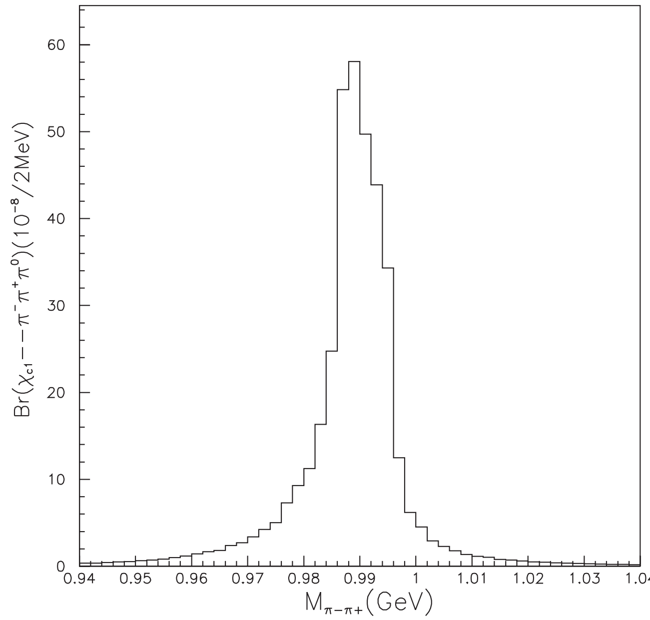

FIG. 6. $\pi^+\pi^-$ invariant mass spectrum for $\chi_{c1} \to \pi^0 a_0^0(980) \to \pi^0 f_0(980) \to \pi^0\pi^+\pi^-$.

V. SUMMARY

Various processes have been proposed previously to study $a_0^0(980)$-$f_0(980)$ mixing through the $f_0(980) \to a_0^0(980)$ transition. In this article we investigate in detail the difference between $a_0^0(980) \to f_0(980)$ and $f_0(980) \to a_0^0(980)$ transitions. Two corresponding mixing intensities ξ_{af} and ξ_{fa} are defined. It is found that, besides the common numerator, the ξ_{af} has additional dependence on the parameters of $f_0(980)$ while ξ_{fa} has additional dependence on the parameters of $a_0^0(980)$. Therefore to measure the $a_0^0(980) \to f_0(980)$ transition in addition to the $f_0(980) \to a_0^0(980)$ transition will be very useful for pinning down these parameters. We examine the possibility of measuring the $a_0^0(980)$-$f_0(980)$ mixing from $\chi_{c1} \to \pi^0\pi\pi$ for ξ_{af} at the upgraded Beijing electron positron collider with BESIII detector and find it is feasible.

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