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What can we learn from $B \to a_1(1260)(b_1(1235))\pi(K)$ decays?

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We investigate the $B \to a_1(1260)(b_1(1235))\pi(K)$ decays under the factorization scheme and find many discrepancies between theoretical predictions and the experimental data. In the tree-dominated processes, large contributions from color-suppressed tree diagrams are required in order to accommodate the large decay rates of $B^- \to a_1^0\pi^-$ and $B^- \to a_1^-\pi^0$. For $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ decays which are induced by $b \to s$ transition, theoretical predictions on their decay rates are larger than the data by a factor of 2.8 and 5.5, respectively. Large electroweak penguins or some new mechanism are expected to explain the branching ratios of $B^- \to b_1^0 K^-$ and $B^- \to a_1^- \bar{K}^0$. The soft-collinear effective theory has the potential to explain large decay rates of $B^- \to a_1^0\pi^-$ and $B^- \to a_1^-\pi^0$ via a large hard-scattering form factor $\zeta_J^{B\to a_1}$. We will also show that, with proper charming penguins, predictions on the branching ratios of $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ can also be consistent with the data.

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I. INTRODUCTION

Since the first measurement on $B^0/\bar{B}^0 \to a_1^{\mp}(1260)\pi^{\pm}$ decays reported by the *BABAR* and Belle collaborations [1–3], many charmless B decays into a pseudoscalar and an axial-vector meson have been observed. Among the 18 $B \to a_1(1260)(b_1(1235)\pi(K)$[1] decay channels, 10 of them have been measured with large branching ratios. Besides decay rates, direct CP asymmetries in some $B \to (a_1, b_1)K$ channels and time-dependent CP asymmetries in $B^0/\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$ and $B^0/\bar{B}^0 \to b_1^{\pm}\pi^{\mp}$ were also studied in the two B factories [4–9]. Without any doubt, these results are helpful to investigate production mechanisms of axial-vectors in B decays, extract hadronic parameters such as strong phases in $B \to AP$ decays, and probe the structures of axial-vectors.

Charmless two-body $B \to AP$ decays have received considerable theoretical efforts [10–14]. Among these predictions, many of them are not consistent with each other: most predictions by Calderon, Munoz, and Vera [12] are larger than predictions given by Laporta, Nardulli, and Pham [11] and the QCD factorization (QCDF) approach. Predictions on $B \to a_1\pi$ by Laporta, Nardulli, and Pham (using the second sets of form factors) are very close to results in the QCDF approach. However there are large discrepancies in other predictions (see Ref. [14] for a detailed comparison between these theoretical predictions). Many results of the QCDF approach agree with the experimental data, but there still exist some deviations.

In the present paper, we intend to analyze the 18 $B \to AP$ decays with the help of experimental data. We try to check whether these problems can be removed in the perturbative QCD (PQCD) approach and the soft-collinear effective theory (SCET). Another objective is to extract the $B \to A$ form factors through $\bar{B}^0 \to a_1^+\pi^-$ and $\bar{B}^0 \to b_1^{\pm}\pi^{\mp}$ decays.

II. NAIVE FACTORIZATION APPROACH

The effective Hamiltonian describing $b \to D(D = d, s)$ transitions are given by [15]:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}}\left\{\sum_{q=u,c} V_{qb}V_{qD}^*\left[C_1 O_1^q + C_2 O_2^q + \sum_{i=3}^{10} C_i O_i\right]\right\} + \text{H.c.}, \tag{1}$$

where $V_{qb(D)}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Functions O_i are the local four-quark operators, while functions C_i are the corresponding Wilson coefficients. It is convenient to define combinations a_i of the Wilson coefficients:

$$\begin{aligned} &a_1 = C_2 + C_1/3, \qquad a_2 = C_1 + C_2/3, \\ &a_i = C_i + C_{i+1}/N_c (i = 3, 5, 7, 9), \\ &a_i = C_i + C_{i-1}/N_c (i = 4, 6, 8, 10). \end{aligned} \tag{2}$$

There exists a hierarchy for the Wilson coefficients:

$$a_1 \gg \max[a_2, a_{3-10}]. \tag{3}$$

For tree-dominated processes $B^0/\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$, the factorization formulas can be written as

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \to a_1^+\pi^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_\pi V_0^{B\to a_1}\{&V_{ub}V_{ud}^*[a_1 + a_4 + a_{10} \\ &+ r_\pi(a_6 + a_8)] + V_{cb}V_{cd}^*[a_4 + a_{10} \\ &+ r_\pi(a_6 + a_8)]\}, \end{aligned} \tag{4}$$

[1]In the following, we will use $a_1(b_1)$ to denote the $a_1(1260) \times (b_1(1235))$ meson for simplicity.

$$\mathcal{A}(\bar{B}^0 \to \pi^+ a_1^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_{a_1} f_+^{B\to\pi} \{V_{ub}V_{ud}^*[a_1 + a_4 + a_{10}] + V_{cb}V_{cd}^*[a_4 + a_{10}]\}, \quad (5)$$

where $r_\pi = 2m_0^\pi/m_B$ with m_0^π the chiral scale parameter for the pion. The CKM matrix elements for tree operators $|V_{ub}V_{ud}^*| \sim 4 \times 10^{-3}$ have the same order magnitude with those for penguin operators $|V_{cb}V_{cd}^*| \sim 8 \times 10^{-3}$. Because of the hierarchy in the Wilson coefficients, penguin contributions from the operators O_{3-10} are small compared with those from tree operators. Thus penguin contributions can be neglected in the study of branching ratios (but crucial to CP asymmetries). Combined with the $\bar{B}^0 \to \pi^+\pi^-$ data [16]

$$\mathcal{BR}(\bar{B}^0 \to \pi^+\pi^-) = (5.16 \pm 0.22) \times 10^{-6}, \quad (6)$$

we arrive at the a_1 meson decay constant and $B \to a_1$ form factor:

$$\begin{aligned} f_{a_1} &= \left[2.02 \pm 0.26 \pm 0.04 + \mathcal{O}\left(\frac{a_{3-10}}{a_1}\right)\right] f_\pi, \\ V_0^{B\to a_1} &= \left[1.55 \pm 0.28 \pm 0.03 + \mathcal{O}\left(\frac{a_{3-10}}{a_1}\right)\right] f_+^{B\to\pi}, \end{aligned} \quad (7)$$

where the uncertainties are from the experimental results for branching ratios. As a rough estimation, we take $f_\pi = 131$ MeV and $f_+^{B\to\pi} = 0.25$ which corresponds to $f_{a_1} = (265 \pm 34 \pm 6)$ MeV and $V_0^{B\to a_1} = 0.39 \pm 0.07 \pm 0.01$. These results are well consistent with predictions based on the PQCD approaches [17] and light-cone sum rules [18,19].

Now we come to the two channels $B^- \to a_1^0\pi^-$ and $B^- \to a_1^-\pi^0$ whose factorization formulas are given by

$$\begin{aligned} \sqrt{2}\mathcal{A}(B^- \to \pi^- a_1^0) = {}& \frac{G_F}{\sqrt{2}} m_B^2 f_\pi V_0^{B\to a_1} \{V_{ub}V_{ud}^*[a_1 + a_4 + a_{10} + r_\pi(a_6 + a_8)] \\ &+ V_{cb}V_{cd}^*[a_4 + a_{10} + r_\pi(a_6 + a_8)]\} \\ &+ \frac{G_F}{\sqrt{2}} m_B^2 f_{a_1} f_+^{B\to\pi} \left\{V_{ub}V_{ud}^*\left[a_2 - a_4 + \frac{1}{2}a_{10}\right] + V_{cb}V_{cd}^*\left[-a_4 + \frac{1}{2}a_{10}\right]\right\}, \end{aligned} \quad (8)$$

$$\begin{aligned} \sqrt{2}\mathcal{A}(B^- \to \pi^0 a_1^-) = {}& \frac{G_F}{\sqrt{2}} m_B^2 f_\pi V_0^{B\to a_1} \left\{V_{ub}V_{ud}^*\left[a_2 - a_4 + \frac{1}{2}a_{10} + r_\pi\left(-a_6 + \frac{1}{2}a_8\right)\right]\right. \\ &\left. + V_{cb}V_{cd}^*\left[-a_4 + \frac{1}{2}a_{10} + r_\pi\left(-a_6 + \frac{1}{2}a_8\right)\right]\right\} + \frac{G_F}{\sqrt{2}} m_B^2 f_{a_1} f_+^{B\to\pi} \{V_{ub}V_{ud}^*[a_1 + a_4 + a_{10}] \\ &+ V_{cb}V_{cd}^*[a_4 + a_{10}]\}. \end{aligned} \quad (9)$$

Because of the small values of a_{3-10}, the penguin contributions can be safely neglected:

$$\sqrt{2}\mathcal{A}(B^- \to \pi^- a_1^0) = \frac{G_F}{\sqrt{2}} m_B^2 V_{ub}V_{ud}^*[a_1 f_\pi V_0^{B\to a_1} + a_2 f_{a_1} f_+^{B\to\pi}], \quad (10)$$

$$\sqrt{2}\mathcal{A}(B^- \to \pi^0 a_1^-) = \frac{G_F}{\sqrt{2}} m_B^2 V_{ub}V_{ud}^*[a_2 f_\pi V_0^{B\to a_1} + a_1 f_{a_1} f_+^{B\to\pi}]. \quad (11)$$

Furthermore, in the hierarchy of $a_2 \ll a_1$, branching ratios are required to satisfy the following relation:

$$\begin{aligned} \mathcal{BR}(\bar{B}^0 \to a_1^+\pi^-) &= 2\mathcal{BR}(B^- \to \pi^- a_1^0), \\ \mathcal{BR}(\bar{B}^0 \to \pi^+ a_1^-) &= 2\mathcal{BR}(B^- \to a_1^-\pi^0). \end{aligned} \quad (12)$$

But the experimental data shows

$$\begin{aligned} \mathcal{BR}(B^- \to \pi^0 a_1^-) &> \mathcal{BR}(\bar{B}^0 \to a_1^-\pi^+), \\ \mathcal{BR}(B^- \to \pi^- a_1^0) &> \mathcal{BR}(\bar{B}^0 \to a_1^+\pi^-), \end{aligned} \quad (13)$$

which is dramatically different. This situation is very similar with that in $B \to \pi\pi$ decays: the branching ratio of $B^- \to \pi^0\pi^-$ is measured with almost equal magnitude with $\mathcal{BR}(\bar{B}^0 \to \pi^-\pi^+)$ but it is expected as one half of $\mathcal{BR}(\bar{B}^0 \to \pi^-\pi^+)$. To solve these problems, an efficient approach is to enhance the color-suppressed contribution which is proportional to a_2. For example, if the Wilson coefficient a_2 can be enhanced to 0.5, the branching ratios of $\mathcal{BR}(B^- \to \pi^0 a_1^-)$ and $\mathcal{BR}(B^- \to \pi^- a_1^0)$ are predicted as 20.0×10^{-6} and 16.7×10^{-6}, where we have utilized the experimental data on branching ratios of $B^0/\bar{B}^0 \to \pi^\pm a_1^\mp$. And these results are well consistent with the experimental data.

The decay constant of b_1 vanishes because of the G-parity, thus $\bar{B}^0 \to \pi^+ b_1^-$ is factorization-suppressed and only the $\bar{B}^0 \to \pi^- b_1^+$ decay survives. From the experimental results collected in Table I, we can infer that the form factors of $B \to a_1$ and $B \to b_1$ are almost equal in magnitude at maximal recoiling: $|V_0^{B\to a_1}(q^2 = 0)| \simeq |V_0^{B\to b_1}(q^2 = 0)| \simeq 0.35$. One should be careful that the two form factors have different signs, if we use the light-cone distribution amplitudes (LCDAs) of a_1 and b_1 evaluated by the QCD sum rules. The absolute value of these

TABLE I. Theoretical predictions and experimental results [1–9] on branching ratios (in unit of 10^{-6}) of $B \to a_1(b_1)\pi(K)$ decays. The QCDF predictions are quoted from Ref. [14]. In the PQCD approach, the uncertainties are from: (i) the hadronic inputs: decay constants of B meson, and shape parameters in the wave function of B meson; (ii) $\Lambda_{\rm QCD}$, the hard scale t, and the threshold resummation parameter c; (iii) the CKM matrix elements V_{ub} and γ angle. In the SCET framework, the uncertainties are from: (i) hadronic parameters: form factors and charming penguins; (ii) the CKM matrix elements. In the last row, we give the total χ^2/d.o.f. for the measured channels in the three models and see the text for the definition.

Channel	QCDF	PQCD	SCET	Exp.
$B^- \to a_1^- \pi^0$	$14.4^{+1.4+3.5+2.1}_{-1.3-3.2-1.9}$	$8.1^{+4.1+2.1+0.7}_{-2.7-1.2-0.9}$	$19.0^{+5.1+1.8}_{-4.7-1.7}$	$26.4 \pm 5.4 \pm 4.1$
$B^- \to a_1^0 \pi^-$	$7.6^{+0.3+1.7+1.4}_{-0.3-1.3-1.0}$	$6.7^{+2.9+2.8+0.5}_{-2.2-1.7-0.7}$	$17.2^{+4.7+1.7}_{-4.3-1.6}$	$20.4 \pm 4.7 \pm 3.4$
$\bar B^0 \to a_1^- \pi^+$	$23.4^{+2.3+6.2+1.9}_{-2.2-5.5-1.3}$	$15.7^{+8.3+5.9+1.2}_{-5.6-3.6-1.7}$	$17.0^{+5.8+1.6}_{-5.2-1.4}$	21.0 ± 5.4
$\bar B^0 \to a_1^+ \pi^-$	$9.1^{+0.2+2.2+1.7}_{-0.2-1.8-1.1}$	$12.7^{+5.6+6.2+0.9}_{-4.4-3.8-1.3}$	$10.7^{+2.5+1.0}_{-2.4-0.9}$	12.2 ± 4.5
$B^0/\bar B^0 \to a_1^+ \pi^-$	—	$28.1^{+13.8+12.0+2.1}_{-9.9-7.3-3.0}$	$28.2^{+6.5+2.6}_{-5.9-2.4}$	
$B^0/\bar B^0 \to a_1^- \pi^+$	—	$28.6^{+13.9+12.1+2.2}_{-10.1-7.4-3.0}$	$27.1^{+6.9+2.5}_{-6.2-2.3}$	
$\bar B^0 \to a_1^\pm \pi^\mp$	$32.5^{+2.5+8.4+3.6}_{-2.4-7.3-2.4}$	$28.3^{+13.9+12.0+2.2}_{-10.0-7.4-3.0}$	$27.7^{+6.3+2.5}_{-5.7-2.3}$	31.7 ± 3.7
$\bar B^0 \to a_1^0 \pi^0$	$0.9^{+0.1+0.3+0.7}_{-0.1-0.2-0.3}$	$0.12^{+0.07+0.02+0.02}_{-0.04-0.03-0.02}$	$5.5^{+1.7+0.6}_{-1.5-0.6}$	
$B^- \to a_1^0 K^-$	$13.9^{+0.9+9.5+12.9}_{-0.9-5.1-4.9}$	$15.4^{+7.8+10.1+2.4}_{-5.4-5.5-2.5}$	$10.5^{+3.3+1.8}_{-2.9-1.5}$	
$B^- \to a_1^- \bar K^0$	$21.6^{+1.2+16.5+23.6}_{-1.1-8.5-11.9}$	$25.5^{+12.9+18.0+3.7}_{-9.2-10.2-3.9}$	$15.5^{+5.8+2.5}_{-5.0-2.1}$	$34.9 \pm 5.0 \pm 4.4$
$\bar B^0 \to a_1^+ K^-$	$18.3^{+1.0+14.2+21.1}_{-1.0-7.2-7.5}$	$20.6^{+10.2+14.6+3.2}_{-7.3-8.5-3.3}$	$15.8^{+5.6+2.7}_{-4.9-2.3}$	$16.3 \pm 2.9 \pm 2.3$
$\bar B^0 \to a_1^0 \bar K^0$	$6.9^{+0.3+6.1+9.5}_{-0.3-2.9-3.2}$	$8.0^{+3.9+6.4+1.2}_{-2.8-3.4-1.2}$	$6.3^{+2.5+1.0}_{-2.1-0.8}$	
$B^- \to b_1^- \pi^0$	$0.4^{+0.0+0.2+0.4}_{-0.0-0.1-0.2}$	$1.0^{+0.2+0.3+0.1}_{-0.2-0.2-0.2}$	$2.0^{+0.8+0.2}_{-0.6-0.2}$	<3.3 [a]
$B^- \to b_1^0 \pi^-$	$9.6^{+0.3+1.6+2.5}_{-0.3-1.6-1.5}$	$5.1^{+3.1+3.1+0.3}_{-1.9-1.7-0.5}$	$5.0^{+1.3+0.5}_{-1.2-0.4}$	$6.7 \pm 1.7 \pm 1.0$
$\bar B^0 \to b_1^- \pi^+$	$0.3^{+0.1+0.1+0.3}_{-0.0-0.1-0.1}$	$1.4^{+0.4+0.1+0.1}_{-0.4-0.2-0.2}$	$0.6^{+0.3+0.1}_{-0.2-0.1}$	
$\bar B^0 \to b_1^+ \pi^-$	$11.2^{+0.3+2.8+2.2}_{-0.3-2.4-1.9}$	$18.7^{+9.6+8.2+1.3}_{-6.4-4.5-1.9}$	$7.7^{+2.1+0.7}_{-1.9-0.7}$	
$B^0/\bar B^0 \to b_1^+ \pi^-$	—	$14.8^{+8.5+6.6+1.3}_{-5.6-3.8-1.7}$	$5.0^{+1.8+0.6}_{-1.5-0.5}$	
$B^0/\bar B^0 \to b_1^- \pi^+$	—	$25.6^{+11.4+9.6+1.6}_{-8.3-6.0-2.6}$	$11.6^{+2.7+1.0}_{-2.5-0.9}$	
$\bar B^0 \to b_1^\pm \pi^\mp$	$11.4^{+0.4+2.9+2.5}_{-0.3-2.5-2.0}$	$20.2^{+9.9+8.1+1.4}_{-6.9-4.9-2.1}$	$8.3^{+2.1+0.7}_{-1.9-0.7}$	$10.9 \pm 1.2 \pm 0.9$
$\bar B^0 \to b_1^0 \pi^0$	$1.1^{+0.2+0.1+0.2}_{-0.2-0.1-0.2}$	$1.5^{+0.6+0.3+0.1}_{-0.5-0.3-0.2}$	$1.8^{+0.5+0.2}_{-0.4-0.1}$	<1.9 [a]
$B^- \to b_1^0 K^-$	$6.2^{+0.5+5.0+6.4}_{-0.5-2.5-5.2}$	$24.9^{+9.8+14.9+3.7}_{-7.8-9.3-3.9}$	$4.6^{+1.9+0.7}_{-1.5-0.6}$	$9.1 \pm 1.7 \pm 1.0$
$B^- \to b_1^- \bar K^0$	$14.0^{+1.3+11.5+13.9}_{-1.2-5.9-8.3}$	$55.0^{+23.6+33.5+8.0}_{-17.0-21.2-8.3}$	$8.6^{+3.8+1.4}_{-3.1-1.2}$	$9.6 \pm 1.7 \pm 0.9$ [a]
$\bar B^0 \to b_1^+ K^-$	$12.1^{+1.0+9.7+12.3}_{-0.9-4.9-30.2}$	$42.9^{+17.7+26.9+6.6}_{-13.4-16.9-6.9}$	$8.5^{+3.5+1.3}_{-2.8-1.1}$	$7.4 \pm 1.0 \pm 1.0$
$\bar B^0 \to b_1^0 \bar K^0$	$7.3^{+0.5+5.4+6.7}_{-0.5-2.8-6.5}$	$23.3^{+10.6+15.5+3.5}_{-6.8-8.8-3.6}$	$4.0^{+1.8+0.7}_{-1.4-0.6}$	<7.8 [a]
χ^2/d.o.f.	$8/\ldots$	$13/(29-5)$	$10/(29-14)$	

[a]The experimental data [9] is obtained on the assumption that the daughter decay $b_1 \to \pi\omega$ has a branching ratio $\mathcal{BR} = 1$.

form factors can be checked by the future measurements on semileptonic $B \to A$ decays such as $\bar B^0 \to (a_1^+, b_1^+) l^- \bar\nu$.

Flavor structures of $\bar B^0 \to b_1^+ K^-$ and $\bar B^0 \to a_1^+ K^-$ are the same, thus they have the same factorization formulas:

$$\begin{aligned}\mathcal{A}(\bar B^0 \to (a_1^+, b_1^+)K^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_K V_0^{B\to(a_1,b_1)}\{V_{ub}V_{us}^*[a_1 \\ + a_4 + a_{10} + r_K(a_6 + a_8)] \\ + V_{cb}V_{cs}^*[a_4 + a_{10} \\ + r_K(a_6 + a_8)]\}. \end{aligned} \tag{14}$$

The same Wilson coefficients and almost equal form factors will induce almost equal branching ratios for $\bar B^0 \to a_1^+ K^-$ and $\bar B^0 \to b_1^+ K^-$. To reduce the uncertainties, we will utilize the $\bar B^0 \to \pi^+ K^-$ decay which also has the same flavor structures with $\bar B^0 \to (a_1^+, b_1^+)K^-$. The only difference between the three modes is that the different form factors which can be extracted from tree-dominated processes $\bar B^0 \to \pi^+ \pi^-$ and $\bar B^0 \to (a_1^+, b_1^+)\pi^-$ decay. The branching ratio of $\bar B^0 \to \pi^+ K^-$ has been measured as [16]

$$\mathcal{BR}(\bar B^0 \to \pi^+ K^-) = (19.4 \pm 0.6) \times 10^{-6}, \tag{15}$$

which implies

$$\begin{aligned}\mathcal{BR}(\bar B^0 \to a_1^+ K^-) = 45.9 \times 10^{-6}, \\ \mathcal{BR}(\bar B^0 \to b_1^+ K^-) = 41.0 \times 10^{-6}. \end{aligned} \tag{16}$$

Comparing with the experimental measurements in Table I, we see that our theoretical prediction on $\mathcal{BR}(\bar B^0 \to a_1^+ K^-)$ is 2.8 times larger while the prediction on $\mathcal{BR}(\bar B^0 \to b_1^+ K^-)$ is 5.5 times larger. This discrepancy should be clarified by the theoretical studies with next-to-leading order corrections and improved experimental measurements.

Besides $\bar B^0 \to (a_1^+, b_1^+)K^-$ decays, $B^- \to a_1^- \bar K^0$ and $B^- \to b_1^0 K^-$ decays are also measured by experimentalists whose factorization formulas are

$$\mathcal{A}(B^- \to a_1^- \bar{K}^0) = \frac{G_F}{\sqrt{2}} m_B^2 f_K V_0^{B\to a_1} \bigg\{ V_{ub}V_{us}^* \bigg[a_4 - \frac{1}{2}a_{10} + r_K\bigg(a_6 - \frac{1}{2}a_8\bigg)\bigg] + V_{cb}V_{cs}^*\bigg[a_4 - \frac{1}{2}a_{10} + r_K\bigg(a_6 - \frac{1}{2}a_8\bigg)\bigg]\bigg\}, \tag{17}$$

$$\sqrt{2}\mathcal{A}(B^- \to b_1^0 K^-) = \frac{G_F}{\sqrt{2}} m_B^2 f_K V_0^{B\to b_1} \{ V_{ub}V_{us}^*[a_1 + a_4 + a_{10} + r_K(a_6 + a_8)] + V_{cb}V_{cs}^*[a_4 + a_{10} + r_K(a_6 + a_8)]\}. \tag{18}$$

In these $b \to s$ transitions, the CKM matrix elements for penguin operators are $|V_{cb}V_{cs}^*| \sim 40 \times 10^{-3}$ and those for tree operators are $|V_{ub}V_{us}^*| \sim 0.8 \times 10^{-3}$. Recalling the values for the Wilson coefficient combinations: $a_1 \sim 1$ and $a_4 \sim a_6 \sim -0.03$, we can see that contributions from tree operators with the coefficient a_1 are smaller than those from penguin operators at least by a factor of 2 in magnitude. In order to characterize the contribution from tree operators and symmetry breaking effects between B^- and $\bar{B}^0$ mesons, it is useful to define the two ratios:

$$R_1 \equiv \frac{\mathcal{BR}(B^- \to a_1^- \bar{K}^0)}{\mathcal{BR}(\bar{B}^0 \to a_1^+ K^-)} \times \frac{\tau_{\bar{B}^0}}{\tau_B^-}, \quad R_2 \equiv \frac{\mathcal{BR}(B^- \to b_1^0 K^-)}{\mathcal{BR}(\bar{B}^0 \to b_1^+ K^-)} \times \frac{\tau_{\bar{B}^0}}{\tau_B^-}, \tag{19}$$

where τ is the lifetime of the B meson. Neglecting tree operators and electroweak penguins, the ratios obey the limit:

$$R_1 = 1, \qquad R_2 = 0.5, \tag{20}$$

which are quite different from the experimental results:

$$R_1^{\mathrm{exp}} = 2.00 \pm 0.59, \qquad R_2^{\mathrm{exp}} = 1.15 \pm 0.34. \tag{21}$$

The difference between the two channels in the ratio R_1 is the tree operator and electroweak penguin operators. Since the contribution of tree operator is smaller than QCD penguins and the two kinds of amplitudes are perpendicular with each other due to the CKM angle γ close to 90°, the tree operator cannot change the branching ratio of $\bar{B}^0 \to a_1^+ K^-$ too much. Thus this does not improve theoretical predictions on R_1. Large electroweak penguins may help us to diminish the large deviation for R_1. In the $\bar{B}^0 \to b_1^+ K^-$ and $B^- \to b_1^0 K^-$ decays, the factorization formulas are exactly the same since the b_1 decay constant vanishes. Thus in order to explain the large ratio R_2, one needs some mechanism beyond factorization to enhance the ratio of R_2 by roughly 2.5.

In the above, we have analyzed the charmless nonleptonic $B \to AP$ data under the factorization approach. The decay constant of the a_1 meson and $B \to a_1$, b_1 form factors V_0 is extracted from the $\bar{B}^0 \to a_1\pi$ and $\bar{B}^0 \to b_1\pi$ decays. The form factors are consistent with the predictions evaluated in light-cone sum rules and the PQCD approach. But there exist several problems which can be summarized as the following:

(i) The Wilson coefficient combination a_2 needs to be enhanced to $a_2 = 0.5$ in order to solve the problem in $B^- \to a_1^- \pi^0$ and $B^- \to a_1^0 \pi^-$.

(ii) Since the form factors $B \to a_1$ and $B \to b_1$ are almost equal in magnitude, the $\bar{B}^0 \to a_1^+ K^-$ and $\bar{B}^0 \to b_1^+ K^-$ decays should possess similar and large branching ratios. Compared with the experimental data, theoretical predictions need to be reduced by the factors of 2.8 and 5.5, respectively.

(iii) $B^- \to a_1^- \bar{K}^0$ and $B^- \to b_1^0 K^-$ are related to $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ through relations given in Eq. (19), which also have large deviations from the data.

III. THE SOFT-COLLINEAR EFFECTIVE THEORY

The recent development of SCET places the analysis of $B \to M_1M_2$ decays on a more rigorous foundation. The SCET is a powerful method to systematically separate the dynamics at different scales: hard scale m_b (b quark mass), hard intermediate scale $\mu_{hc} = \sqrt{m_b\Lambda_{\mathrm{QCD}}}$, soft scale, and to sum large logs using the renormalization group technics. Integrating out the hard fluctuations, we arrive at the intermediate effective theory—SCET$_I$ where the factorization formulas for $B \to M_1M_2$ decays to leading power in $\lambda \equiv \sqrt{\Lambda_{\mathrm{QCD}}/m_b}$ are given by

$$\mathcal{A}(B \to M_1M_2) = \frac{G_F}{\sqrt{2}} m_B^2 \bigg\{ f_{M_1} \int du \phi_{M_1}(u) T_1(u) \zeta^{B\to M_2} + f_{M_1} \int du \phi_{M_1}(u) \int dz T_{1J}(u, z) \times \zeta_J^{B\to M_2}(z) + (1 \leftrightarrow 2) \bigg\}, \tag{22}$$

where functions ζ and ζ_J also enter into the heavy-to-light form factors. $T_1(u)$ and $T_{1J}(u, z)$ are hard kernels which can be calculated using perturbation theory. With the hard-collinear fluctuation integrated out, the final effective theory-SCET$_{II}$ is obtained where the function ζ_J can be factorized into convolutions of LCDAs with hard kernels:

$$\zeta_J^{B\to M_2}(z) = \phi_B(\omega) \otimes J(z, \omega, v) \otimes \phi_{M_2}(v). \tag{23}$$

$J(z, \omega, v)$ is the hard kernel and ϕ_B and ϕ_{M_1,M_2} are the light-cone distribution amplitudes. With our knowledge of these LCDAs, one can predict the decay amplitude by convoluting the LCDAs with the perturbatively calculated hard kernels. But there is another alternative way for phenomenological studies: one can fit experimental results, including branching ratios and CP asymmetries, to determine essential nonperturbative inputs. Note that in this way, no expansions in $\alpha_s(\sqrt{m_b \Lambda_{\rm QCD}})$ are needed and thus the exploration of the convergence is spontaneously avoided. This method is especially useful at tree level: $T_1(u)$ is a constant, and $T_{1J}(u, z)$ is a function of one argument u. It leads to a rather simple form for decay amplitudes:

$$\mathcal{A}(B \to M_1 M_2) = \frac{G_F}{\sqrt{2}} m_B^2 \bigg\{ f_{M_1} T_1 \zeta^{B\to M_2} + f_{M_1} \int du \phi_{M_1}(u) \times T_{1J}(u) \zeta_J^{B\to M_2} + (1 \leftrightarrow 2) \bigg\}, \qquad (24)$$

where the functions $\zeta^{B\to M_2}$ and

$$\zeta_J^{B\to M_2} = \int dz \zeta_J^{B\to M_2}(z) \qquad (25)$$

are treated as nonperturbative parameters to be fitted from the data. With the help of the flavor SU(3) symmetry, the $B \to AP$ decays involve only 6 parameters:

$$\begin{array}{ccc} \zeta^{B\to P}, & \zeta_J^{B\to P}, & \zeta^{B\to {}^1P_1}, \\ \zeta_J^{B\to {}^1P_1}, & \zeta^{B\to {}^3P_1}, & \zeta_J^{B\to {}^3P_1}, \end{array} \qquad (26)$$

which contribute to the $B \to P$ and $B \to A$ form factors.

Including the nonperturbative contributions from loop diagrams involving $c\bar{c}$ [20–25], the SCET can successfully explain most of $B \to PP$ and $B \to VP$ decays [21,26,27]. This phenomenological approach has many important features. In $b \to d$ transitions such as $\bar{B}^0 \to \pi^+\pi^-$, tree operators provide the dominant contributions, and contributions from charming penguins and penguin operators are subleading. From the experience in the $B \to PP$ and $B \to VP$ phenomenological study, we know that the hard-scattering form factor ζ_J is potentially large. Furthermore, as we have shown in Ref. [27], the corresponding Wilson coefficient is of order 1 which amounts to a large effective Wilson coefficient a_2. Here we take $\bar{B}^0 \to a_1^- \pi^+$ and $B^- \to a_1^- \pi^0$ as an example: if hard-scattering form factors are equal with soft form factors for the pion and a_1 meson: $\zeta = \zeta_J$, the effective Wilson coefficient equals to $a_2 \simeq \frac{\zeta_J}{\zeta + \zeta_J} = 0.5$. Thus it is easy to solve the problems in $B \to a_1 \pi$ decays under the SCET framework.

For decays induced by $b \to s$ transition, since tree operators are suppressed by the CKM matrix elements $|V_{ub}V_{us}^*/(V_{cb}V_{cs}^*)| \sim 0.02$ and penguin operators have smaller Wilson coefficients ($\max[C_{3-10}] \ll \alpha_s(2m_c)C_1$), charming penguins play a significant role. Because of the nonperturbative nature, charming penguins are totally unknown from perturbation theory and need to be extracted from data. This stuff depends on the three involved mesons: the B meson, recoiling meson and emitted meson. Thus in order to predict physical observables, too many parameters for charming penguins are required. An efficient way to reduce the independent inputs is to utilize the flavor SU(3) symmetry, and as a result only 8 parameters for charming penguins in $B \to AP$ decays are left. But even so, due to the lack of data, one can always obtain proper branching ratios of $\bar{B}^0 \to b_1^+ K^-$ and $\bar{B}^0 \to a_1^+ K^-$ by adjusting charming penguins. Despite that, there is another deficit: since the inputs, form factors, and charming penguins have been assumed to respect the SU(3) symmetry, large deviations of the ratios shown in Eqs. (20) and (21) cannot be eliminated by the SCET either.

IV. THE PERTURBATIVE QCD APPROACH

There is another commonly accepted approach to handle hadronic B decays: the perturbative QCD approach [28–30]. The basic idea of the PQCD approach is that it takes into account the transverse momentum of the valence quarks in hadrons. Decay amplitudes and form factors can be written as convolutions of wave functions with perturbatively hard kernels integrated over the longitudinal and transverse component. When considering radiative corrections, one encounters double logarithm divergences when soft and collinear momenta overlap. These large double logarithms can be resummed into the Sudakov factor. Loop corrections to the weak decay vertex also give rise to double logarithms in the threshold region. Resummation of this type of double logarithms leads to the Sudakov factor S_t. This factor decreases faster than any power of x as $x \to 0$ and changes the behavior at the endpoint region. The Sudakov factor and threshold resummation make the PQCD approach more self-consistent. This approach has successfully explained the $B \to \pi\pi$ and $B \to \pi K$ decay rates and CP asymmetries [31] together with the proper polarizations in $B \to VV$ decays [32].

In the PQCD approach, the predicted $B \to a_1$ form factor [17] is consistent with the one derived from the data, thus our PQCD prediction on $\mathcal{BR}(\bar{B}^0 \to a_1^+\pi^-)$ is in good agreement with the data. But due to the small value of a_2, the color-suppressed contribution is too small to explain the large decay rates of $B^- \to a_1^-\pi^0$ and $B^- \to a_1^0\pi^-$. The investigations of next-to-leading order corrections in Ref. [33] show that the branching ratio of $B^- \to \pi^-\pi^0$ is enhanced by the factor 4.0/3.5 while $\bar{B}^0 \to \pi^+\pi^-$ is reduced by 6.5/7.0. But even if we assume the same k factor for $B \to a_1\pi$ decays, the PQCD predictions on $B^- \to a_1^-\pi^0$ and $B^- \to a_1^0\pi^-$ are still smaller than the data. The PQCD prediction on the $B \to b_1$ form factor is large, thus the branching ratio of $\bar{B}^0 \to b_1^+\pi^-$ is 2 times larger than the experimental data and the QCDF results. From the factorization formulas of $B \to PP$ decays given in the literature [30], one can see that the contributions from hard spectator scattering diagrams are small due to the cancellation between two diagrams where a gluon is attached to either the positive quark or the antiquark in the emitted hadron. But if the emitted meson is a P-wave meson and the twist-2 LCDA is antisymmetric (like a scalar or an axial-vector meson with quantum number ${}^{2S+1}L_J = {}^1P_1$), the two diagrams give constructive contributions to make them sizable. For example, the large hard spectator scattering contributions to $B^- \to b_1^0 \pi^-$ make $\mathcal{BR}(B^- \to b_1^0\pi^-) < \frac{1}{2}\mathcal{BR}(\bar{B}^0 \to b_1^+\pi^-)$. Moreover, annihilation diagrams play an important role in the PQCD approach which often enters into decay amplitudes as imaginary. It provides the dominant strong phase which is essential to explain the large CP asymmetries. Thus unlike the situation in the QCDF approach, annihilation diagrams do not cancel with emission diagrams in $\bar{B}^0 \to b_1^+ K^-$ which results in much larger predictions on branching ratios of $\bar{B}^0 \to b_1^+ K^-$. Similar to the factorization approach, there are large differences between the PQCD approach predictions on ratios $R_{1,2}$ and those extracted from the data.

V. NUMERICAL RESULTS

In the PQCD framework and SCET framework, we calculate the decay rates, direct CP asymmetries and time-dependent CP asymmetries shown in Tables I, II, III, and IV. We have adopted the same conventions with Ref. [14] for observables in time-dependent decay widths of $B \to a_1^\pm \pi^\mp$ and $B \to b_1^\pm \pi^\mp$.[2] In the SCET calculation, we use the following values for the 14 inputs:

$$\zeta^{B\to\pi} = 0.12, \qquad \zeta_J^{B\to\pi} = 0.12, \qquad \zeta^{B\to a_1} = 0.17,$$
$$\zeta_J^{B\to a_1} = 0.17, \qquad \zeta^{B\to b_1} = -0.16, \qquad \zeta_J^{B\to b_1} = -0.16,$$
$$|A_{cc}^{3P_1P}| = 40\times 10^{-4}, \qquad \arg[A_{cc}^{3P_1P}] = 160^\circ,$$
$$|A_{cc}^{P^3P_1}| = 40\times 10^{-4}, \qquad \arg[A_{cc}^{P^3P_1}] = 145^\circ,$$
$$|A_{cc}^{1P_1P}| = 40\times 10^{-4}, \qquad \arg[A_{cc}^{1P_1P}] = 155^\circ,$$
$$|A_{cc}^{P^1P_1}| = 35\times 10^{-4}, \qquad \arg[A_{cc}^{P^1P_1}] = 100^\circ. \quad (27)$$

We should point out that this set of inputs is presented by hand instead of any reasonable way. To test the sensitivities on these parameters, we show the first uncertainty in numerical results by varying the form factors by 0.03, 20% for magnitudes of charming penguins and 20° for the phases. The second uncertainty is from CKM matrix elements. In the PQCD calculation, we have used the same inputs as those in Refs. [17,37,38]. The theoretical uncertainties are from: (i) the hadronic inputs: decay constants of the B meson, and shape parameters in the wave function of the B meson; (ii) $\Lambda_{\rm QCD}$, the hard scale t and the threshold resummation parameter c; (iii) the CKM matrix elements V_{ub} and γ angle. The factorization formula for each type of diagrams in $B \to AP$ decays is the same as those in $B \to PP$ decays which can be found in the literature.[3] Because of the same flavor structures, the hard spectator scattering diagrams often accompany with the factorizable diagrams. One only needs to consider the flavor structure for factorizable diagrams and to use meson matrices by evaluating the master equations [34]. For the CKM matrix elements, we use the updated global fit results from CKMfitter group [36]:

$$\begin{aligned} V_{ud} &= 0.97400, \qquad V_{us} = 0.22653, \\ |V_{ub}| &= (3.57^{+0.17}_{-0.17})\times 10^{-3}, \qquad V_{cd} = -0.22638, \\ V_{cs} &= 0.97316, \qquad V_{cb} = (40.5^{+3.2}_{-2.9})\times 10^{-3}, \\ \beta &= (21.7^{+0.017}_{-0.017})^\circ, \qquad \gamma = (67.6^{+2.8}_{-4.5})^\circ. \end{aligned} \quad (28)$$

Predictions in the QCDF approach are also collected in the tables to make a comparison [14] In the QCDF approach, a_2 (to be precise, α_2) are much smaller than 0.5, thus their amplitude from color-suppressed tree diagrams is not large enough to resolve the problem in $B^0/\bar{B}^0 \to a_1^\pm \pi^\mp$ and $B^0 \to (a_1^- \pi^0, a_1^0 \pi^-)$ decays. Their prediction on the branching ratio of $\bar{B}^0 \to a_1^+ K^-$ is compatible with the data. For $B \to b_1 K$, they found that decay rates are sensitive to the interference between emission diagrams and annihilation diagrams. The small decay rate of $\bar{B}^0 \to b_1^+ K^-$ arises from the destructive interference between emission diagrams and annihilations, thus the prediction on branching ratio $\bar{B}^0 \to b_1^+ K^-$ is basically consistent with the data. But their predictions on four ratios of branching fractions R_{1-4} (R_3 and R_4 are related to ratios R_1 and R_2 defined in the present paper; their ratios R_1 and R_2 characterize the magnitude of color-suppressed contributions in $B \to a_1 \pi$ decay modes) deviate from experimental data.

To be more quantitative, we also define the χ^2 for the measured observables:

$$\chi^2 = \frac{(F_{\rm the} - F_{\rm exp})^2}{F_{\rm err}^2}, \quad (29)$$

where F denotes either branching ratio or direct CP asymmetry for any specific channel. $F_{\rm the}$ denotes the theoretical prediction, $F_{\rm exp}$ denotes the central value for the observables provided by the experimentalists, and $F_{\rm err}$ denotes the experimental and theoretical errors added in quadrature. At the end of Tables I and II, we present the total χ^2/N for branching ratios and direct CP asymmetries in these three models, where N is the number of degrees of freedom. In the PQCD approach, the inputs for the light axial-vector and pseudoscalar mesons are evaluated by the QCD sum rules method. The B-meson decay constant and shape parameter, the factor in the factorization scale, $\Lambda_{\rm QCD}$, threshold resummation parameter c can be viewed as fitted parameters. But these parameters will also be constrained by the $B \to PP$, VP, VV data. In the SCET method, the 14 hadronic inputs are the fitted parameters. It should also be pointed out that this set of inputs is pre-

[2] The $\mathcal{A}_{a_1^\pm \pi^\mp}$ in the present paper correspond to $\mathcal{A}_{a_1^\mp \pi^\pm}$ defined in Refs. [16,35,36].

[3] There still exist two differences between the factorizable emission diagrams of $B \to AP$ and $B \to PP$ decays: the axial-vector meson cannot be generated by the scalar or pseudoscalar current, thus the chiraly enhanced penguins vanish; due to the vanishing decay constant, b_1 can not be factorized from the B meson and the recoiling meson.

TABLE II. Similar as Table I. but for direct CP asymmetries (in %) of $B \to a_1(b_1)\pi(K)$ decays.

Channel	QCDF	PQCD	SCET	Exp.
$B^- \to a_1^- \pi^0$	$0.5^{+0.3+0.6+12.0}_{-0.2-0.3-11.0}$	$1.6^{+0.0+0.1+0.2}_{-0.6-1.3-0.1}$	$-5.4^{+10.7+0.5}_{-10.1-0.5}$	
$B^- \to a_1^0 \pi^-$	$-4.3^{+0.3+1.4+14.1}_{-0.3-2.2-14.5}$	$-0.9^{+0.6+0.3+0.1}_{-0.3-0.3-0.1}$	$5.7^{+11.1+0.5}_{-11.3-0.5}$	
$\bar{B}^0 \to a_1^+ \pi^-$	$-3.6^{+0.1+0.3+20.8}_{-0.1-0.5-20.2}$	$12.6^{+1.8+3.5+1.0}_{-1.2-2.5-1.1}$	$21.5^{+11.3+1.6}_{-12.7-1.9}$	$7 \pm 21 \pm 15$
$\bar{B}^0 \to a_1^- \pi^+$	$-1.9 \pm 0.0 \pm 0.0^{+14.6}_{-14.3}$	$11.7^{+2.1+2.7+1.1}_{-1.9-2.0-1.1}$	$10.4^{+9.9+0.8}_{-10.6-0.9}$	$15 \pm 15 \pm 7$
$\bar{B}^0 \to a_1^0 \pi^0$	$60.1^{+4.6+6.8+37.6}_{-4.9-8.3-60.7}$	$28.9^{+7.6+42.5+2.6}_{-22.1-88.1-2.5}$	$-29.5^{+15.7+2.6}_{-13.0-2.8}$	
$B^- \to a_1^- \bar{K}^0$	$0.8^{+0.0+0.1+0.6}_{-0.0-0.1-0.0}$	$-1.0^{+0.2+0.2+0.1}_{-0.0-0.2-0.1}$	$0.3^{+0.2+0.0}_{-0.2-0.0}$	$12 \pm 11 \pm 2$
$B^- \to a_1^0 K^-$	$8.4^{+0.3+1.4+10.3}_{-0.3-1.6-12.0}$	$-6.1^{+1.1+1.3+0.6}_{-1.3-1.4-0.6}$	$-25.6^{+14.9+2.3}_{-14.8-2.4}$	
$\bar{B}^0 \to a_1^+ K^-$	$2.6^{+0.0+0.7+10.1}_{-0.1-0.7-11.0}$	$-8.9^{+1.5+2.1+0.8}_{-2.3-2.4-0.9}$	$-17.7^{+10.3+1.6}_{-10.0-1.7}$	$-16 \pm 12 \pm 1$
$\bar{B}^0 \to a_1^0 \bar{K}^0$	$-7.7^{+0.6+2.1+6.8}_{-0.6-2.2-7.0}$	$-1.8^{+0.3+0.6+0.2}_{-0.3-0.6-0.2}$	$17.9^{+10.1+1.4}_{-11.1-1.6}$	
$B^- \to b_1^- \pi^0$	$-36.5^{+4.4+18.4+82.2}_{-4.3-17.7-59.6}$	$12.5^{+16.0+23.5+1.1}_{-27.3-40.3-0.6}$	$34.1^{+27.1+3.2}_{-26.7-2.9}$	$5 \pm 16 \pm 2$
$B^- \to b_1^0 \pi^-$	$0.9^{+0.6+2.3+18.0}_{-0.4-2.7-20.5}$	$-65.8^{+11.1+13.4+4.6}_{-9.0-8.2-3.8}$	$-17.7^{+13.8+1.6}_{-14.6-1.3}$	
$\bar{B}^0 \to b_1^+ \pi^-$	$-4.0^{+0.2+0.4+26.2}_{-0.0-0.6-25.5}$	$-25.0^{+4.0+3.9+2.2}_{-4.3-4.1-1.9}$	$-42.3^{+9.3+3.6}_{-9.4-3.5}$	
$\bar{B}^0 \to b_1^- \pi^+$	$66.1^{+1.2+7.4+30.3}_{-1.4-4.8-96.6}$	$49.0^{+3.5+7.1+4.2}_{-8.0-6.0-4.0}$	0	
$\bar{B}^0 \to b_1^0 \pi^0$	$53.4^{+6.4+9.0+5.2}_{-6.3-7.3-4.7}$	$15.9^{+4.0+7.2+1.0}_{-7.8-10.7-1.4}$	$52.7^{+12.7+2.7}_{-13.9-3.9}$	
$B^- \to b_1^- \bar{K}^0$	$1.4^{+0.1+0.1+5.6}_{-0.1-0.1-0.1}$	$-0.30^{+0.02+0.00+0.03}_{-0.38-0.58-0.03}$	$-0.8^{+0.2+0.1}_{-0.2-0.1}$	$-3 \pm 15 \pm 2$
$B^- \to b_1^0 K^-$	$18.7^{+1.6+7.8+57.7}_{-1.7-6.1-44.9}$	$19.4^{+0.0+4.4+1.8}_{-0.4-4.0-1.8}$	$46.3^{+10.5+3.8}_{-8.8-3.9}$	$-46 \pm 20 \pm 2$
$\bar{B}^0 \to b_1^+ K^-$	$5.5^{+0.2+1.2+47.2}_{-0.3-1.2-30.2}$	$16.6^{+2.4+3.8+1.6}_{-2.3-3.2-1.5}$	$46.3^{+10.5+3.8}_{-8.8-3.9}$	$-7 \pm 12 \pm 2$
$\bar{B}^0 \to b_1^0 \bar{K}^0$	$-8.6^{+0.8+3.3+8.3}_{-0.8-4.2-25.4}$	$-4.3^{+1.6+1.8+0.4}_{-1.6-1.8-0.4}$	$-0.8^{+0.2+0.1}_{-0.2-0.1}$	
χ^2/d.o.f.	$4/\ldots$	$15/(29-5)$	$29/(29-14)$	

sented by hand instead of any reasonable way. With more data in the future, one can perform a comprehensive χ^2-fit.

Several remarks on the numerical results in the PQCD approach and SCET approach are in order:

(i) The predictions on $\mathcal{BR}(\bar{B}^0 \to a_1^- \pi^+)$ in both approaches are a bit smaller than experimental data, because the decay constant of $f_{a_1} = 0.238$ GeV [39] is a bit smaller than that extracted from the data.

(ii) As we expected, color-suppressed contributions to $B \to a_1 \pi$ decays are large in the SCET framework but small in the PQCD approach: SCET predictions are much larger and consistent with the present data within the uncertainties.

(iii) In the PQCD approach, $\bar{B}^0 \to b_1^- \pi^+$ occur via the so-called hard spectator scattering diagrams, despite the zero decay constant of b_1. In $B^- \to b_1^0 \pi^-$, the

TABLE III. Same as Table I but for time-dependent CP asymmetry parameters in $B^0/\bar{B}^0 \to a_1^{\pm}\pi^{\mp}$ and $B^0/\bar{B}^0 \to b_1^{\pm}\pi^{\mp}$ decays.

Observables	QCDF	PQCD	SCET	Exp.
$\mathcal{A}_{a_1\pi}$	$0.003^{+0.001+0.002+0.043}_{-0.002-0.003-0.045}$	$-0.009^{+0.002+0.002+0.001}_{-0.002-0.003-0.001}$	$0.02^{+0.08+0.00}_{-0.08-0.00}$	$-0.07 \pm 0.07 \pm 0.02$
C	$0.02^{+0.00+0.00+0.14}_{-0.00-0.00-0.14}$	$-0.12^{+0.02+0.02+0.01}_{-0.02-0.03-0.01}$	$-0.15^{+0.08+0.01}_{-0.07-0.01}$	$-0.10 \pm 0.15 \pm 0.09$
ΔC	$0.44^{+0.03+0.03+0.03}_{-0.04-0.05-0.04}$	$0.11^{+0.03+0.06+0.01}_{-0.01-0.05-0.01}$	$0.23^{+0.18+0.00}_{-0.19-0.00}$	$0.26 \pm 0.15 \pm 0.07$
S	$-0.37^{+0.01+0.05+0.09}_{-0.01-0.08-0.16}$	$-0.23^{+0.02+0.03+0.09}_{-0.01-0.03-0.14}$	$-0.45^{+0.07+0.08}_{-0.06-0.11}$	$0.37 \pm 0.21 \pm 0.07$
ΔS	$0.01^{+0.00+0.00+0.02}_{-0.00-0.00-0.02}$	$-0.03^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	$0.02^{+0.04+0.00}_{-0.05-0.00}$	$-0.14 \pm 0.21 \pm 0.06$
$\alpha_{\rm eff}^+$	$(97.2^{+0.3+1.0+4.7}_{-0.3-0.6-2.5})^\circ$	$(93.8^{+0.4+0.7+4.4}_{-0.4-0.4-2.8})^\circ$	$(103.5^{+2.4+4.0}_{-2.5-2.6})^\circ$	
$\alpha_{\rm eff}^-$	$(107.0^{+0.5+3.6+6.6}_{-0.5-2.3-3.7})^\circ$	$(99.8^{+0.5+1.5+4.2}_{-0.7-1.4-2.7})^\circ$	$(104.8^{+2.7+4.0}_{-3.2-2.6})^\circ$	
$\alpha_{\rm eff}$	$(102.0^{+0.4+2.3+5.7}_{-0.4-1.5-3.1})^\circ$	$(96.8^{+0.4+1.0+4.3}_{-0.6-0.9-2.7})^\circ$	$(104.2^{+1.8+4.0}_{-2.0-2.6})^\circ$	$(78.6 \pm 7.3)^\circ$
$\mathcal{A}_{b_1\pi}$	$-0.06^{+0.01+0.01+0.23}_{-0.01-0.01-0.23}$	$-0.27^{+0.05+0.04+0.02}_{-0.05-0.04-0.02}$	$-0.39^{+0.08+0.03}_{-0.08-0.03}$	$-0.05 \pm 0.10 \pm 0.02$
C	$-0.03^{+0.01+0.01+0.06}_{-0.02-0.02-0.01}$	$-0.03^{+0.01+0.01+0.00}_{-0.01-0.01-0.00}$	$0.07^{+0.04+0.02}_{-0.03-0.01}$	$0.22 \pm 0.23 \pm 0.05$
ΔC	$-0.96^{+0.03+0.02+0.08}_{-0.03-0.03-0.01}$	$-0.87^{+0.02+0.04+0.01}_{-0.02-0.04-0.01}$	$-0.83^{+0.08+0.03}_{-0.07-0.03}$	$-1.04 \pm 0.23 \pm 0.08$
S	$0.05^{+0.03+0.02+0.15}_{-0.03-0.02-0.26}$	$0.08^{+0.00+0.02+0.06}_{-0.01-0.02-0.04}$	$-0.46^{+0.14+0.03}_{-0.10-0.03}$	
ΔS	$0.12^{+0.04+0.04+0.08}_{-0.03-0.04-0.09}$	$-0.24^{+0.01+0.06+0.02}_{-0.02-0.07-0.01}$	$-0.17^{+0.06+0.03}_{-0.05-0.02}$	
$\alpha_{\rm eff}^+$ [a]	$(107.6^{+0.7+3.5+155.4}_{-0.2-4.9-17.8})^\circ$	$(174.1^{+0.0+2.5+5.0}_{-0.8-3.9-3.2})^\circ$	68.3°	
$\alpha_{\rm eff}^-$	$(101.3^{+0.4+2.1+4.9}_{-0.4-1.4-8.6})^\circ$	$(13.3^{+0.2+1.9+4.1}_{-0.3-1.7-2.7})^\circ$	$(3.4^{+0.2+4.6}_{-0.1-2.9})^\circ$	
$\alpha_{\rm eff}$	$(104.4^{+0.6+2.6+80.4}_{-0.3-2.1-1.6})^\circ$	$(93.7^{+0.0+0.9+4.5}_{-0.5-1.6-2.8})^\circ$	$(35.8^{+0.1+2.3}_{-0.0-1.5})^\circ$	

[a] One needs to be careful about the phase of the B-meson decay amplitudes [34]. For example, the $\bar{B}^0 \to b_1^- \pi^+$ and $B^0 \to b_1^+ \pi^-$ decay amplitudes are determined as

$$A(\bar{B}^0 \to b_1^- \pi^+) = V_{ub} V_{ud}^* T + V_{cb} V_{cd}^* P, \qquad A(B^0 \to b_1^+ \pi^-) = -[V_{ub}^* V_{ud} T + V_{cb}^* V_{cd} P]. \tag{32}$$

TABLE IV. Mixing-induced CP asymmetries in $\bar{B}^0 \to a_1^0 K_S$ and $\bar{B}^0 \to b_1^0 K_S$ decays.

Channel	PQCD	SCET
$\bar{B}^0 \to a_1^0 \pi^0$	$0.09^{+0.20+0.78+0.08}_{-0.19-0.87-0.08}$	$0.48^{+0.11+0.09}_{-0.14-0.15}$
$\bar{B}^0 \to a_1^0 K_S$	$0.71^{+0.01+0.02+0.00}_{-0.01-0.02-0.00}$	$0.85^{+0.05+0.01}_{-0.06-0.01}$
$\bar{B}^0 \to b_1^0 \pi^0$	$0.67^{+0.02+0.09+0.09}_{-0.00-0.06-0.07}$	$0.61^{+0.09+0.09}_{-0.11-0.06}$
$\bar{B}^0 \to b_1^0 K_S$	$-0.61^{+0.01+0.03+0.01}_{-0.01-0.03-0.01}$	-0.69

hard spectator scattering diagrams' contributions (tree operators), with a b_1^0 meson emitted, are sizable and cancel with the color-allowed contribution where the pion is emitted. Thus the branching ratio of $B^- \to b_1^0 \pi^-$ is smaller than one half of $\mathcal{BR}(\bar{B}^0 \to b_1^+ \pi^-)$.

(iv) In the SCET approach, $\bar{B}^0 \to b_1^- \pi^+$ only receive contributions from charming penguins and correspondingly the direct CP asymmetry in this channel is 0. The predicted branching ratio is smaller than the PQCD prediction but larger than the QCDF prediction.

(v) In the SCET, the direct CP asymmetries in $\bar{B}^0 \to a_1^+ K^-$ and $B^- \to a_1^0 K^-$ have the same sign and similar size. Moreover, their branching ratios obey the simple relation: $\mathcal{BR}(\bar{B}^0 \to a_1^+ K^-) = 2\mathcal{BR}(B^- \to a_1^0 K^-)$. It is also similar for $\bar{B}^0 \to b_1^+ K^-$ and $B^- \to b_1^0 K^-$: the direct CP asymmetries are equal with each other; the branching ratios also satisfy the relation $\mathcal{BR}(\bar{B}^0 \to b_1^+ K^-) = 2\mathcal{BR}(B^- \to b_1^0 K^-)$, where the small deviation arises from the different mass and decay width of the $\bar{B}^0$ and B^- meson.

(vi) As expected, the two ratios R_1 and R_2 are predicted with large deviations from the data:

$$R_1 = 1.16, \qquad R_2 = 0.54, \qquad \text{PQCD} \tag{30}$$

$$R_1 = 0.91, \qquad R_2 = 0.50. \qquad \text{SCET.} \tag{31}$$

(vii) Predictions on the observables in the time-dependent decay width of $B^0/\bar{B}^0 \to a_1^\pm \pi^\mp$ and $B^0/\bar{B}^0 \to b_1^\pm \pi^\mp$ are basically consistent with the experimental data except the ΔS, the $\alpha_{\rm eff}$ in $\bar{B}^0 \to a_1^\pm \pi^\mp$, and $\mathcal{A}_{b_1\pi}$. For $\bar{B}^0 \to b_1^\pm \pi^\mp$ decays, predictions on ΔC in the two approaches are close to -1 and they are consistent with the QCDF prediction [14] and the data. In the SCET framework, the angle $\alpha_{\rm eff}^+(\bar{B}^0 \to b_1^\pm \pi^\mp)$ is equal to $\frac{\pi}{2} - \beta$ which is also a consequence of the vanishing decay constant of the b_1 meson.

(viii) If only branching fractions are taken into account, the QCDF method gives the smallest χ^2: the reason is that theoretical uncertainties in this framework are large, and thus $F_{\rm err}$ is large. If only experimental errors are taken into account, the SCET method provides the smallest χ^2, while the value in the PQCD approach is extraordinarily large and the main reason is that the χ^2 for the three $B \to b_1 K$ channels is 1251. The situation can be improved by reducing the $B \to b_1$ form factors, which also make the prediction on $\bar{B}^0 \to b_1^\pm \pi^\mp$ better.

(ix) The total χ^2 for the direct CP asymmetries in the three phenomenological methods are in the same size: most of theoretical predictions on direct CP asymmetries are close to the experimental data. The largest value in the SCET for χ^2 does not mean that predictions on direct CP asymmetries in this method deviate the data a lot, as these predictions are presented only for an illustration and the total χ^2 can be reduced by tuning the magnitudes and strong phases of charming penguins with more experimental data in the future.

VI. SUMMARY

In summary, we have investigated the $B \to a_1(b_1)\pi(K)$ decays under the factorization framework and find large differences between theoretical predictions and experimental data. In tree-dominated processes $B \to a_1\pi$, large contributions from color-suppressed tree diagrams are required. In $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ decays, theoretical results are larger than data by factors of 2.8 and 5.5, respectively; meanwhile ratios R_1 and R_2 defined in Eq. (19) are much larger too. In the PQCD framework, the predicted decay rates of $B \to a_1^\pm \pi^\mp$ are consistent with data. But the other problems cannot be resolved. The SCET approach has the potential to resolve the first two problems: if large hard-scattering form factors are allowed, theoretical predictions $\mathcal{BR}(B^- \to a_1^- \pi^0)$ and $\mathcal{BR}(B^- \to a_1^0 \pi^-)$ are in good agreement with data; with the help of charming penguins, large branching ratios of $\bar{B}^0 \to (a_1^+, b_1^+)K^-$ are also pulled down to the same magnitude with the data. However, the two problems on ratios in $b \to s$ transitions remain in the present theoretical methods. These two problems may indicate some new mechanism, from the nonperturbative contributions such as final state interactions or new physics scenarios, which needs further study.

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