Vector meson ω-ϕ mixing and their form factors in the light-cone quark model

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The vector meson ω-ϕ mixing is studied in two alternative scenarios with different numbers of mixing angles, i.e., the one-mixing-angle scenario and the two-mixing-angle scenario, in both the octet-singlet mixing scheme and the quark flavor mixing scheme. Concerning the reproduction of experimental data and the Q^2 behavior of transition form factors, the one-mixing-angle scenario in the quark flavor scheme performs better than that in the octet-singlet scheme, while the two-mixing-angle scenario works well for both mixing schemes. The difference between the two mixing angles in the octet-singlet scheme is bigger than that in the quark flavor scheme.

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I. INTRODUCTION

In the investigation of the internal structure of hadrons, quarks and gluons are fundamental degrees of freedom whose behavior is controlled by quantum chromodynamics (QCD). Because of the confinement property, perturbative QCD is only applicable at large energy scales. To study hadronic properties at low energy scales, nonperturbative effects must be taken into account. Some fundamental nonperturbative QCD approaches are available, such as lattice QCD methods and QCD sum rule techniques. Different relativistic quark models also provide convenient ways to describe hadrons. The light-cone constituent quark model, which is used as an effective low energy approximation to QCD, is one of them.

The light-cone formalism [1–3] provides a convenient framework for the relativistic description of hadrons in terms of quark and gluon degrees of freedom. The hadronic wave function can be described by light-cone Fock state expansion:

$$|M\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}} + \sum |q\bar{q}g\rangle\psi_{q\bar{q}g} + \cdots, \qquad (1)$$

$$|B\rangle = \sum |qqq\rangle\psi_{qqq} + \sum |qqqg\rangle\psi_{qqqg} + \cdots. \qquad (2)$$

To simplify the problem, we take the minimal quark-antiquark Fock state description of photons and mesons to calculate their transition form factors, decay widths, and other properties.

The investigation of the electromagnetic transition processes between pseudoscalar mesons and vector mesons is helpful to understand the internal structure of mesons. The pseudoscalar transition form factors $F_{\eta\gamma}(Q^2)$ and $F_{\eta'\gamma}(Q^2)$ provide a good platform to study the η and η' mixing effects [4–6]. There are two mixing schemes when studying η-η' mixing: the octet-singlet mixing scheme and the quark flavor mixing scheme. According to other works devoted to η-η' mixing [6–9], both schemes work well when only η and η' are involved. Sometimes a second mixing angle is introduced to study η-η' mixing, especially when studying their decay constants [4,10,11]. So there are two alternative scenarios with different numbers of mixing angles: the one-mixing-angle scenario and the two-mixing-angle scenario, in both the octet-singlet mixing scheme and the quark flavor mixing scheme.

Similarly, ω-ϕ mixing can be studied through transition and decay processes. Naturally, the ω-ϕ mixing can also be studied in two mixing schemes corresponding to the η-η' mixing. Many works have been done concerning the ω-ϕ mixing [8,12–14], but only in the one-mixing-angle scenario. In this paper we extend the two-mixing-angle scenario into the study of the ω-ϕ mixing.

When studying the vector mesons, measurements of their branching fractions and transition form factors provide important tests of different models. The decays of ω, ϕ have been studied for many years [15–18]. The conversion decays $\phi \to \eta e^+e^-$ and $\omega \to \pi e^+e^-$ were collected with the CMD-2 detector in recent years [19,20], and not only their branching fractions but also related transition form factors $F_{\phi\to\eta\gamma^*}(Q^2)$, $F_{\omega\to\pi\gamma^*}(Q^2)$ in the timelike region were analyzed. Recently, there were also some new data about $\omega \to \pi\gamma$ transition form factors extracted from proton-proton collisions [21]. With the light-cone hadronic wave functions, the decay widths and transition form factors of radiative decays $V \to P\gamma$ or $P \to V\gamma$ (with $V = \omega, \phi$; $P = \pi, \eta, \eta'$) can be calculated and compared with experimental data. In this paper we try to study ω-ϕ mixing using the one-mixing-angle scenario and the two-mixing-angle scenario, respectively, with the octet-singlet and the quark flavor mixing schemes in the light-cone quark model. We give four sets of wave function parameters and vector meson mixing angles of ω-ϕ in different schemes and compare the behaviors when predicting the Q^2 evolution of the form factors.

In this paper, all the parameters of the model are redetermined by the electroweak processes according to the constraints in previous papers [9,22–24] with new experi-

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mental data from PDG (2008) [25]. In Sec. II, we give a brief review of meson light-cone wave functions and form factor calculations. In Sec. III, we exhibit the two mixing-angle scenarios in two mixing schemes in our calculation. In Sec. IV, numerical results of vector meson form factor Q^2 evolution are presented and compared with experimental data.

II. LIGHT-CONE SPIN WAVE FUNCTIONS AND TRANSITION FORM FACTORS

Based on light-cone quantization of QCD [1–3], the hadronic wave function can be expressed using the Fock state expansion:

$$|M(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{dx_i d^2\mathbf{k}_{\perp i}}{\sqrt{x_i}16\pi^3} 16\pi^3 \delta\Big(1 - \sum_{i=1}^{n} x_i\Big) \times \delta^{(2)}\Big(\sum_{i=1}^{n} \mathbf{k}_{\perp i}\Big)|n: x_i P^+, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i}, \lambda_i\rangle \psi_{n/M}(x_i, \mathbf{k}_{\perp i}, \lambda_i). \quad (3)$$

The wave function $\psi_{n/M}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$ is the amplitude for finding n constituents with momenta $(x_i P^+, \frac{m_i^2 + (x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i})^2}{x_i P^+}, x_i \mathbf{P}_\perp + \mathbf{k}_{\perp i})$, and λ_i is the helicity of the ith constituent.

For simplicity, we just take the minimal quark-antiquark Fock state description of mesons to calculate their radii, decay widths, transition form factors, and other quantities. Thus a meson Fock state ($n = 2$) is described by

$$|M(P, S_Z)\rangle = \sum_{\lambda_1,\lambda_2} \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} |x, \mathbf{k}_\perp, \lambda_1, \lambda_2\rangle \times \psi_M^{S_Z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \quad (4)$$

$$\doteq \int \frac{dx d^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3} \Psi_M^{S_Z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2). \quad (5)$$

The model wave function is given by [26–28]

$$\Psi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \varphi(x, \mathbf{k}_\perp) \chi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2). \quad (6)$$

Since there is no explicit solution of the Bethe-Salpeter equation for the mesons, the harmonic oscillator wave function in the Brodsky-Huang-Lepage (BHL) prescription [2,28] is adopted to describe the quark momentum-space wave function,

$$\varphi(x, \mathbf{k}_\perp) = \varphi_{\mathrm{BHL}}(x, \mathbf{k}_\perp) = A \exp\Big[-\frac{1}{8\beta^2}\Big(\frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x}\Big)\Big]. \quad (7)$$

$\chi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ is the spin wave function which is obtained through the Melosh-Wigner rotation or, equivalently, by proper vertices for mesons.

The instant-form state $\chi(T)$ and the front-form state $\chi(F)$ of spin-$\frac{1}{2}$ constituent quarks are related by the Melosh-Wigner rotation [27,29,30]:

$$\begin{cases} \chi_i^{\uparrow}(T) = w_i[(k_i^+ + m_i)\chi_i^{\uparrow}(F) - k_i^R \chi_i^{\downarrow}(F)] \\ \chi_i^{\downarrow}(T) = w_i[(k_i^+ + m_i)\chi_i^{\downarrow}(F) + k_i^L \chi_i^{\uparrow}(F)], \end{cases} \quad (8)$$

where $w_i = 1/\sqrt{2k_i^+(k^0 + m_i)}$, $k^{R,L} = k^1 \pm k^2$, $k^+ = k^0 + k^3 = x\mathcal{M}$; here k_i is the momentum of the quark with mass m_i, and the invariant mass of the composite system is $\mathcal{M} = \sqrt{\frac{\mathbf{k}_\perp^2 + m_1^2}{x} + \frac{\mathbf{k}_\perp^2 + m_2^2}{1-x}}$. The Melosh-Wigner rotation is essentially a relativistic effect due to the transversal motions of quarks inside the hadrons, and such an effect plays an important role in understanding the proton "spin puzzle" in the nucleon case [31,32].

In the light-cone frame, momentums of the meson and its constituents are

$$P = (P^+, P^-, \mathbf{P}_\perp) = \Big(P^+, \frac{M^2}{P^+}, \mathbf{0}_\perp\Big), \quad (9)$$

$$k_1 = \Big(xP^+, \frac{\mathbf{k}_\perp^2 + m_1^2}{xP^+}, \mathbf{k}_\perp\Big), \quad (10)$$

$$k_2 = \Big((1-x)P^+, \frac{\mathbf{k}_\perp^2 + m_2^2}{(1-x)P^+}, -\mathbf{k}_\perp\Big). \quad (11)$$

With these momentums substituted into the Melosh-Wigner rotation, we get coefficients $C_{M,S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)$ in the spin wave function

$$\chi_M^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \sum_{\lambda_1,\lambda_2} C_{M,S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \chi_1^{\lambda_1}(F) \chi_2^{\lambda_2}(F). \quad (12)$$

The same wave function can be obtained if a proper vertex is chosen for the meson [24,33], that is,

$$\bar{u}(k_1, \lambda_1)\Gamma_M v(k_2, \lambda_2), \quad (13)$$

with

$$\Gamma_P = \frac{1}{\sqrt{2}\sqrt{\mathcal{M}^2 - (m_1 - m_2)^2}} \gamma_5 \quad (14)$$

for pseudoscalar mesons, and

$$\Gamma_V = -\frac{1}{\sqrt{2}\sqrt{\mathcal{M}^2 - (m_1 - m_2)^2}} \Big(\gamma^\mu - \frac{k_1^\mu - k_2^\mu}{\mathcal{M} + m_1 + m_2}\Big) \times \epsilon_\mu(P, S_z) \quad (15)$$

for vector mesons.

The above two methods lead to the same meson light-cone spin wave function:

$$\chi_P(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \sum_{\lambda_1,\lambda_2} C_P^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) \chi_1^{\lambda_1}(F) \chi_2^{\lambda_2}(F) \quad (16)$$

for pseudoscalar mesons [27,28] (the subscription $S_Z = 0$

is omitted), where

$$\begin{cases} C_P^F(x, \mathbf{k}_\perp, \uparrow, \uparrow) & = \frac{1}{\sqrt{2}} w^{-1}(-k^L)(\mathcal{M} + m_1 + m_2) \\ C_P^F(x, \mathbf{k}_\perp, \uparrow, \downarrow) & = \frac{1}{\sqrt{2}} w^{-1}((1-x)m_1 + xm_2)(\mathcal{M} + m_1 + m_2) \\ C_P^F(x, \mathbf{k}_\perp, \downarrow, \uparrow) & = \frac{1}{\sqrt{2}} w^{-1}(-(1-x)m_1 - xm_2)(\mathcal{M} + m_1 + m_2) \\ C_P^F(x, \mathbf{k}_\perp, \downarrow, \downarrow) & = \frac{1}{\sqrt{2}} w^{-1}(-k^R)(\mathcal{M} + m_1 + m_2), \end{cases} \tag{17}$$

with $w = (\mathcal{M} + m_1 + m_2)\sqrt{x(1-x)[\mathcal{M}^2 - (m_1 - m_2)^2]}$;

$$\chi_V^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = \sum_{\lambda_1, \lambda_2} C_{V,S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)\chi_1^{\lambda_1}(F)\chi_2^{\lambda_2}(F) \tag{18}$$

for vector mesons [24], where

$$\begin{cases} C_{V,1}^F(x, \mathbf{k}_\perp, \uparrow, \uparrow) & = w^{-1}[\mathbf{k}_\perp^2 + (\mathcal{M} + m_1 + m_2)((1-x)m_1 + xm_2)] \\ C_{V,1}^F(x, \mathbf{k}_\perp, \uparrow, \downarrow) & = w^{-1}[k^R(x\mathcal{M} + m_1)] \\ C_{V,1}^F(x, \mathbf{k}_\perp, \downarrow, \uparrow) & = w^{-1}[-k^R((1-x)\mathcal{M} + m_2)] \\ C_{V,1}^F(x, \mathbf{k}_\perp, \downarrow, \downarrow) & = w^{-1}[-(k^R)^2]; \end{cases} \tag{19}$$

$$\begin{cases} C_{V,0}^F(x, \mathbf{k}_\perp, \uparrow, \uparrow) & = \frac{1}{\sqrt{2}} w^{-1}[k^L((1-2x)\mathcal{M} + (m_2 - m_1))] \\ C_{V,0}^F(x, \mathbf{k}_\perp, \uparrow, \downarrow) & = \frac{1}{\sqrt{2}} w^{-1}[2\mathbf{k}_\perp^2 + (\mathcal{M} + m_1 + m_2)((1-x)m_1 + xm_2)] \\ C_{V,0}^F(x, \mathbf{k}_\perp, \downarrow, \uparrow) & = \frac{1}{\sqrt{2}} w^{-1}[2\mathbf{k}_\perp^2 + (\mathcal{M} + m_1 + m_2)((1-x)m_1 + xm_2)] \\ C_{V,0}^F(x, \mathbf{k}_\perp, \downarrow, \downarrow) & = \frac{1}{\sqrt{2}} w^{-1}[-k^R((1-2x)\mathcal{M} + (m_2 - m_1))]; \end{cases} \tag{20}$$

$$\begin{cases} C_{V,-1}^F(x, \mathbf{k}_\perp, \uparrow, \uparrow) & = w^{-1}[-(k^L)^2] \\ C_{V,-1}^F(x, \mathbf{k}_\perp, \uparrow, \downarrow) & = w^{-1}[k^L((1-x)\mathcal{M} + m_2)] \\ C_{V,-1}^F(x, \mathbf{k}_\perp, \downarrow, \uparrow) & = w^{-1}[-k^L(x\mathcal{M} + m_1)] \\ C_{V,-1}^F(x, \mathbf{k}_\perp, \downarrow, \downarrow) & = w^{-1}[\mathbf{k}_\perp^2 + (\mathcal{M} + m_1 + m_2)((1-x)m_1 + xm_2)]. \end{cases} \tag{21}$$

These coefficients satisfy the normalization condition

$$\sum_{\lambda_1, \lambda_2} C_{M,S_z}^{F*}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) C_{M,S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = 1. \tag{22}$$

Therefore, the Fock state expansion coefficients in the light-cone wave function of the mesons are

$$\psi^{S_z}(x, \mathbf{k}_\perp, \lambda_1, \lambda_2) = C_{S_z}^F(x, \mathbf{k}_\perp, \lambda_1, \lambda_2)\varphi_{\mathrm{BHL}}(x, \mathbf{k}). \tag{23}$$

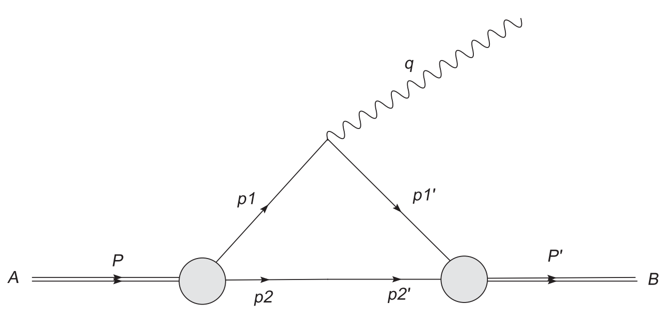

FIG. 1. The diagram for the transition form factor $F_{A\to B\gamma}$ in the Drell-Yan-West frame.

Pseudoscalar meson radii, the decay widths of pseudoscalar and vector mesons $P^\pm \to \mu^\pm \nu$, $P^0 \to \gamma\gamma$, $V \to e^+e^-$, $P \to V\gamma$, $V \to P\gamma$, and all the transition form factors of these processes can be calculated in the light-cone quark model using the above meson wave functions. Supposing that the instant-form wave functions of mesons A and B in flavor space are simply $|q_1 q_2\rangle$, the transition form factor of $A \to B\gamma^*$ is defined by [33]

$$\langle B(P')|J^\mu|A(P, \lambda)\rangle = ieF_{A\to B\gamma}(Q^2)\varepsilon^{\mu\nu\rho\sigma}\epsilon_\nu(P, \lambda)P'_\rho P_\sigma, \tag{24}$$

where $\epsilon(P, \lambda)$ is the polarization vector of the vector meson. In the Drell-Yan-West [34] frame, the kinematics are, as shown in Fig. 1,

$$\begin{cases} q & = (0, \frac{2P\cdot q}{P^+}, \mathbf{q}_\perp) \\ P & = (P^+, \frac{M_A^2}{P^+}, \mathbf{0}_\perp) \\ P' & = (P^+, \frac{M_B^2+\mathbf{q}_\perp^2}{P^+}, -\mathbf{q}_\perp) \\ p_1 & = (xP^+, \frac{m_1^2+\mathbf{k}_\perp^2}{xP^+}, \mathbf{k}_\perp) \\ p_2 & = ((1-x)P^+, \frac{m_2^2+\mathbf{k}_\perp^2}{(1-x)P^+}, -\mathbf{k}_\perp) \\ p_1' & = (xP^+, \frac{m_1^2+(\mathbf{k}_\perp-\mathbf{q}_\perp)^2}{xP^+}, x(-\mathbf{q}_\perp) + (\mathbf{k}_\perp - (1-x)\mathbf{q}_\perp)) \\ p_2' & = ((1-x)P^+, \frac{m_2^2+\mathbf{k}_\perp^2}{(1-x)P^+}, (1-x)(-\mathbf{q}_\perp) - (\mathbf{k}_\perp - (1-x)\mathbf{q}_\perp)). \end{cases} \tag{25}$$

Then, we get the transition form factor of $V \to P\gamma^*$ or $P \to V\gamma^*$ calculated by the light-cone quark model in the Drell-Yan-West frame:

$$F_{A\to B\gamma^*}(Q^2) = \frac{\langle B(P')|J^+|A(P,\lambda=+1)\rangle}{ie\varepsilon^{+\nu\rho\sigma}\epsilon_\nu(P,\lambda=+1)P'_\rho P_\sigma} = Q_{q_1} I_{VP\gamma}[m_1, m_2, A_A, \beta_A, A_B, \beta_B] - Q_{q_2} I_{VP\gamma}[m_2, m_1, A_A, \beta_A, A_B, \beta_B], \tag{26}$$

in which

$$I_{VP\gamma}[m_1, m_2, A_A, \beta_A, A_B, \beta_B] = 2\int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\frac{1}{x(1-x)}\varphi_B^*(\mathbf{k}'_\perp)\varphi_A(\mathbf{k}_\perp) \times \frac{((1-x)m_1 + xm_2)(\mathcal{M} + m_1 + m_2)(1-x) + 2(1-x)\mathbf{k}_\perp^2\sin^2(\theta-\varphi)}{(\mathcal{M} + m_1 + m_2)\sqrt{\mathcal{M}^2 - (m_1-m_2)^2}\sqrt{\mathcal{M}'^2 - (m_1-m_2)^2}}. \tag{27}$$

Here, $\varphi_{A,B}(\mathbf{k}_\perp, A_{A,B}, \beta_{A,B}) = A_{A,B}\exp[-\frac{1}{8\beta_{A,B}^2}\times(\frac{m_1^2+\mathbf{k}_\perp^2}{x} + \frac{m_2^2+\mathbf{k}_\perp^2}{1-x})]$, $\mathbf{k}'_\perp = \mathbf{k}_\perp - (1-x)\mathbf{q}_\perp$, $\mathcal{M}'^2 = \frac{m_1^2+\mathbf{k}'^2_\perp}{x} + \frac{m_2^2+\mathbf{k}'^2_\perp}{1-x}$; q is the momentum of the virtual photon, and in the Drell-Yan-West frame, $Q^2 = -q^2 = \mathbf{q}_\perp^2$; Q_{q_1} and Q_{q_2} are electric charges of q_1 and q_2. The other formulas for decay widths and form factors are presented in Appendix A.

III. TWO MIXING-ANGLE SCENARIOS IN TWO MIXING SCHEMES

There are mainly two mixing schemes concerning η-η' or ω-ϕ mixing. One is the octet-singlet mixing scheme (denoted as 08) [35,36],

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{08}^S & -\sin\theta_{08}^S \\ \sin\theta_{08}^S & \cos\theta_{08}^S \end{pmatrix}\begin{pmatrix} |\eta_8\rangle \\ |\eta_0\rangle \end{pmatrix}, \tag{28}$$

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{08}^V & -\sin\theta_{08}^V \\ \sin\theta_{08}^V & \cos\theta_{08}^V \end{pmatrix}\begin{pmatrix} |\omega_8\rangle \\ |\omega_0\rangle \end{pmatrix}, \tag{29}$$

where θ_{08}^S and θ_{08}^V are, respectively, the pseudoscalar meson mixing angle and the vector meson mixing angle in the octet-singlet mixing scheme. Here, the flavor SU(3) octet basis is $|\psi_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$ and the singlet basis is $|\psi_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$ (for $\psi = \eta$ or ω). The other is the quark-flavor basis mixing scheme (denoted as qs) [5,7]:

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{qs}^S & -\sin\theta_{qs}^S \\ \sin\theta_{qs}^S & \cos\theta_{qs}^S \end{pmatrix}\begin{pmatrix} |\eta_q\rangle \\ |\eta_s\rangle \end{pmatrix}, \tag{30}$$

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_{qs}^V & -\sin\theta_{qs}^V \\ \sin\theta_{qs}^V & \cos\theta_{qs}^V \end{pmatrix}\begin{pmatrix} |\omega_q\rangle \\ |\omega_s\rangle \end{pmatrix}, \tag{31}$$

where θ_{qs}^S and θ_{qs}^V are the pseudoscalar meson mixing angle and the vector meson mixing angle in the quark flavor mixing scheme, and the quark flavor bases are $|\psi_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $|\psi_s\rangle = s\bar{s}$ (for $\psi = \eta$ or ω).

The two schemes are equivalent to each other by $\theta_{qs} = \theta_{08} + \arctan(\sqrt{2})$ when the SU(3) symmetry is perfect. This relationship is not maintained when we take into account the $\mathrm{SU}(3)_f$ breaking by [9]:

$$|\psi_8\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d})\varphi_8^q(x, \mathbf{k}_\perp) - \frac{2}{\sqrt{6}}s\bar{s}\varphi_8^s(x, \mathbf{k}_\perp), \tag{32}$$

$$|\psi_0\rangle = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d})\varphi_0^q(x, \mathbf{k}_\perp) + \frac{1}{\sqrt{3}}s\bar{s}\varphi_0^s(x, \mathbf{k}_\perp) \tag{33}$$

for the octet-singlet scheme, in which

$$\begin{cases} \varphi_8^q(x, \mathbf{k}_\perp) & = A_8 \exp[-\frac{m_q^2+\mathbf{k}_\perp^2}{8\beta_8^2 x(1-x)}] \\ \varphi_8^s(x, \mathbf{k}_\perp) & = A_8 \exp[-\frac{m_s^2+\mathbf{k}_\perp^2}{8\beta_8^2 x(1-x)}] \\ \varphi_0^q(x, \mathbf{k}_\perp) & = A_0 \exp[-\frac{m_q^2+\mathbf{k}_\perp^2}{8\beta_0^2 x(1-x)}] \\ \varphi_0^s(x, \mathbf{k}_\perp) & = A_0 \exp[-\frac{m_s^2+\mathbf{k}_\perp^2}{8\beta_0^2 x(1-x)}], \end{cases} \quad (34)$$

and

$$|\psi_q\rangle = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi^q(x, \mathbf{k}_\perp), \quad (35)$$

$$|\psi_s\rangle = s\bar{s}\varphi^s(x, \mathbf{k}_\perp) \quad (36)$$

for the quark flavor scheme, in which

$$\begin{cases} \varphi^q(x, \mathbf{k}_\perp) & = A_q \exp[-\frac{m_q^2+\mathbf{k}_\perp^2}{8\beta_q^2 x(1-x)}] \\ \varphi^s(x, \mathbf{k}_\perp) & = A_s \exp[-\frac{m_s^2+\mathbf{k}_\perp^2}{8\beta_s^2 x(1-x)}]. \end{cases} \quad (37)$$

In the octet-singlet mixing scheme, the decay constants of the pseudoscalar mesons are given as follows,

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8 \cos\theta_{08}^S & -f_0 \sin\theta_{08}^S \\ f_8 \sin\theta_{08}^S & f_0 \cos\theta_{08}^S \end{pmatrix}. \quad (38)$$

The axial-vector anomaly and partial conservation of axial current (PCAC) lead to [13]

$$\Gamma(\eta \to \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{64\pi^3}\left(\frac{c_8}{f_{\eta_8}} \cos\theta_{08}^S - \frac{c_0}{f_{\eta_0}} \sin\theta_{08}^S\right)^2, \quad (39)$$

$$\Gamma(\eta' \to \gamma\gamma) = \frac{\alpha^2 m_\eta^3}{64\pi^3}\left(\frac{c_8}{f_{\eta_8}} \sin\theta_{08}^S + \frac{c_0}{f_{\eta_0}} \cos\theta_{08}^S\right)^2. \quad (40)$$

Combining the above with

$$F_{\eta\to\gamma\gamma^*}(Q^2) = F_{\eta_8\to\gamma\gamma^*}(Q^2)\cos\theta_{08}^S - F_{\eta_0\to\gamma\gamma^*}(Q^2)\sin\theta_{08}^S, \quad (41)$$

$$F_{\eta'\to\gamma\gamma^*}(Q^2) = F_{\eta_8\to\gamma\gamma^*}(Q^2)\sin\theta_{08}^S + F_{\eta_0\to\gamma\gamma^*}(Q^2)\cos\theta_{08}^S, \quad (42)$$

and their behavior when $Q^2 \to \infty$,

$$\lim_{Q^2\to\infty} Q^2 F_{\eta_8\to\gamma\gamma^*}(Q^2) = c_8 f_{\eta_8}, \quad (43)$$

$$\lim_{Q^2\to\infty} Q^2 F_{\eta_0\to\gamma\gamma^*}(Q^2) = c_0 f_{\eta_0}, \quad (44)$$

one can constrain the η-η' mixing angle and parameters, while the theoretical model calculation gives

$$F_{\eta_8\to\gamma\gamma^*}(Q^2) = \frac{1}{\sqrt{6}}(Q_u^2 + Q_d^2)I_{P\gamma\gamma^*}[m_u, A_{\eta_8}, \beta_{\eta_8}] - \frac{2}{\sqrt{6}}Q_s^2 I_{P\gamma\gamma^*}[m_s, A_{\eta_8}, \beta_{\eta_8}], \quad (45)$$

$$F_{\eta_0\to\gamma\gamma^*}(Q^2) = \frac{1}{\sqrt{3}}(Q_u^2 + Q_d^2)I_{P\gamma\gamma^*}[m_u, A_{\eta_0}, \beta_{\eta_0}] + \frac{1}{\sqrt{3}}Q_s^2 I_{P\gamma\gamma^*}[m_s, A_{\eta_0}, \beta_{\eta_0}]. \quad (46)$$

The decay constants and transition form factors of the vector mesons ω and ϕ are

$$\begin{pmatrix} f_\phi \\ f_\omega \end{pmatrix} = \begin{pmatrix} \cos\theta_{08}^V & -\sin\theta_{08}^V \\ \sin\theta_{08}^V & \cos\theta_{08}^V \end{pmatrix}\begin{pmatrix} f_{\omega_8} \\ f_{\omega_0} \end{pmatrix}, \quad (47)$$

$$\begin{pmatrix} F_{\phi\to\pi\gamma^*}(Q^2) \\ F_{\omega\to\pi\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_{08}^V & -\sin\theta_{08}^V \\ \sin\theta_{08}^V & \cos\theta_{08}^V \end{pmatrix}\begin{pmatrix} F_{\omega_8\to\pi\gamma^*}(Q^2) \\ F_{\omega_0\to\pi\gamma^*}(Q^2) \end{pmatrix}, \quad (48)$$

$$\begin{pmatrix} F_{\phi\to\eta\gamma^*}(Q^2) \\ F_{\phi\to\eta'\gamma^*}(Q^2) \\ F_{\omega\to\eta\gamma^*}(Q^2) \\ F_{\eta'\to\omega\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_{08}^V & -\sin\theta_{08}^V \\ \sin\theta_{08}^V & \cos\theta_{08}^V \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_{08}^S & -\sin\theta_{08}^S \\ \sin\theta_{08}^S & \cos\theta_{08}^S \end{pmatrix} \times \begin{pmatrix} F_{\omega_8\to\eta_8\gamma^*}(Q^2) \\ F_{\omega_8\to\eta_0\gamma^*}(Q^2) \\ F_{\omega_0\to\eta_8\gamma^*}(Q^2) \\ F_{\omega_0\to\eta_0\gamma^*}(Q^2) \end{pmatrix}, \quad (49)$$

in which

$$\begin{cases} F_{\omega_8\to\pi\gamma^*}(Q^2) & = \frac{1}{\sqrt{3}} I_{VP\gamma}[m_q, A_{\omega_8}, \beta_{\omega_8}, A_\pi, \beta_\pi] \\ F_{\omega_0\to\pi\gamma^*}(Q^2) & = \frac{2}{\sqrt{6}} I_{VP\gamma}[m_q, A_{\omega_8}, \beta_{\omega_8}, A_\pi, \beta_\pi] \\ F_{\omega_8\to\eta_8\gamma^*}(Q^2) & = \frac{1}{6}(\frac{2}{3} I_{VP\gamma}[m_q, A_{\omega_8}, \beta_{\omega_8}, A_{\eta_8}, \beta_{\eta_8}] - \frac{8}{3} I_{VP\gamma}[m_s, A_{\omega_8}, \beta_{\omega_8}, A_{\eta_8}, \beta_{\eta_8}]) \\ F_{\omega_8\to\eta_0\gamma^*}(Q^2) & = \frac{1}{\sqrt{18}}(\frac{2}{3} I_{VP\gamma}[m_q, A_{\omega_8}, \beta_{\omega_8}, A_{\eta_0}, \beta_{\eta_0}] + \frac{4}{3} I_{VP\gamma}[m_s, A_{\omega_8}, \beta_{\omega_8}, A_{\eta_0}, \beta_{\eta_0}]) \\ F_{\omega_0\to\eta_8\gamma^*}(Q^2) & = \frac{1}{\sqrt{18}}(\frac{2}{3} I_{VP\gamma}[m_q, A_{\omega_0}, \beta_{\omega_0}, A_{\eta_8}, \beta_{\eta_8}] + \frac{4}{3} I_{VP\gamma}[m_s, A_{\omega_0}, \beta_{\omega_0}, A_{\eta_8}, \beta_{\eta_8}]) \\ F_{\omega_0\to\eta_0\gamma^*}(Q^2) & = \frac{1}{3}(\frac{2}{3} I_{VP\gamma}[m_q, A_{\omega_0}, \beta_{\omega_0}, A_{\eta_0}, \beta_{\eta_0}] - \frac{2}{3} I_{VP\gamma}[m_s, A_{\omega_0}, \beta_{\omega_0}, A_{\eta_0}, \beta_{\eta_0}]). \end{cases} \quad (50)$$

In the quark flavor mixing scheme, the formulas are similar to those in the octet-singlet scheme as shown in Appendix B.

Up to now we just use the one-mixing-angle scenario in both the octet-singlet and the quark flavor mixing schemes. We can also introduce the two-mixing-angle scenario to do phenomenological investigation, especially when studying the decay constants of pseudoscalar mesons [4,37].

As stated in Ref. [37], the Fock state decomposition of a charge neutral meson can be generally expressed as

$$|M\rangle = C_M^8|\psi_8\rangle + C_M^0|\psi_0\rangle + C_M^g|gg\rangle + C_M^c|c\bar{c}\rangle + \cdots. \tag{51}$$

By truncating only the valence Fock states and doing phenomenological analysis, two mixing angles could be introduced for the meson state mixing. The relations are analogous to the mixing of the pseudoscalar meson decay constants [4]. To simplify the problem we just assume that the mixing angles in the valence Fock state decomposition are equal to those in the pseudoscalar meson decay constant mixing.

Take the octet-singlet mixing scheme for example,

$$\begin{pmatrix} |\eta\rangle \\ |\eta'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_8^S & -\sin\theta_0^S \\ \sin\theta_8^S & \cos\theta_0^S \end{pmatrix}\begin{pmatrix} |\eta_8\rangle \\ |\eta_0\rangle \end{pmatrix}, \tag{52}$$

where θ_8^S, θ_0^S are the two mixing angles introduced for pseudoscalar mesons η-η' in the octet-singlet mixing scheme. Then the decay constants of the pseudoscalar mesons are given by

$$\begin{pmatrix} f_\eta^8 & f_\eta^0 \\ f_{\eta'}^8 & f_{\eta'}^0 \end{pmatrix} = \begin{pmatrix} f_8\cos\theta_8^S & -f_0\sin\theta_0^S \\ f_8\sin\theta_8^S & f_0\cos\theta_0^S \end{pmatrix}. \tag{53}$$

The axial-vector anomaly and PCAC lead to

$$\Gamma(\eta \to \gamma\gamma^*) = \frac{\alpha^2 m_\eta^3}{64\pi^3}\left(\frac{\frac{c_8}{f_{\eta_8}}\cos\theta_0^S - \frac{c_0}{f_{\eta_0}}\sin\theta_8^S}{\cos(\theta_0^S - \theta_8^S)}\right)^2, \tag{54}$$

$$\Gamma(\eta' \to \gamma\gamma^*) = \frac{\alpha^2 m_\eta^3}{64\pi^3}\left(\frac{\frac{c_8}{f_{\eta_8}}\sin\theta_0^S + \frac{c_0}{f_{\eta_0}}\cos\theta_8^S}{\cos(\theta_0^S - \theta_8^S)}\right)^2. \tag{55}$$

Combined with

$$F_{\eta\to\gamma\gamma^*}(Q^2) = F_{\eta_8\to\gamma\gamma^*}(Q^2)\cos\theta_8^S - F_{\eta_0\to\gamma\gamma^*}(Q^2)\sin\theta_0^S, \tag{56}$$

$$F_{\eta'\to\gamma\gamma^*}(Q^2) = F_{\eta_8\to\gamma\gamma^*}(Q^2)\sin\theta_8^S + F_{\eta_0\to\gamma\gamma^*}(Q^2)\cos\theta_0^S, \tag{57}$$

and their $Q^2 \to \infty$ behavior in Eqs. (43) and (44), theoretical formulas Eqs. (45) and (46) can be used to constrain the $\eta\eta'$ parameters.

Similarly, ω-ϕ mixing can also be studied with the two-mixing-angle scenario:

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_0^V & -\sin\theta_8^V \\ \sin\theta_0^V & \cos\theta_8^V \end{pmatrix}\begin{pmatrix} |\omega_8\rangle \\ |\omega_0\rangle \end{pmatrix}. \tag{58}$$

In the two-mixing-angle scenario in the octet-singlet mixing scheme, the decay constants and transition form factors of the vector mesons ω and ϕ are

$$\begin{pmatrix} f_\phi \\ f_\omega \end{pmatrix} = \begin{pmatrix} \cos\theta_8^V & -\sin\theta_0^V \\ \sin\theta_8^V & \cos\theta_0^V \end{pmatrix}\begin{pmatrix} f_{\omega_8} \\ f_{\omega_0} \end{pmatrix}, \tag{59}$$

$$\begin{pmatrix} F_{\phi\to\pi\gamma^*}(Q^2) \\ F_{\omega\to\pi\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_8^V & -\sin\theta_0^V \\ \sin\theta_8^V & \cos\theta_0^V \end{pmatrix}\begin{pmatrix} F_{\omega_8\to\pi\gamma^*}(Q^2) \\ F_{\omega_0\to\pi\gamma^*}(Q^2) \end{pmatrix}, \tag{60}$$

$$\begin{pmatrix} F_{\phi\to\eta\gamma^*}(Q^2) \\ F_{\phi\to\eta'\gamma^*}(Q^2) \\ F_{\omega\to\eta\gamma^*}(Q^2) \\ F_{\eta'\to\omega\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_8^V & -\sin\theta_0^V \\ \sin\theta_8^V & \cos\theta_0^V \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_8^S & -\sin\theta_0^S \\ \sin\theta_8^S & \cos\theta_0^S \end{pmatrix} \times \begin{pmatrix} F_{\omega_8\to\eta_8\gamma^*}(Q^2) \\ F_{\omega_8\to\eta_0\gamma^*}(Q^2) \\ F_{\omega_0\to\eta_8\gamma^*}(Q^2) \\ F_{\omega_0\to\eta_0\gamma^*}(Q^2) \end{pmatrix}. \tag{61}$$

When taking $\theta_8^S = \theta_0^S = \theta_{08}^S$ and $\theta_8^V = \theta_0^V = \theta_{08}^V$, one returns to the one-mixing-angle scenario.

The two-mixing-angle scenario in the quark flavor mixing scheme is similar to that in the above octet-singlet scheme, and it can be obtained just by replacing the octet bases with the quark flavor bases as shown in Appendix B.

When the two mixing angles are not equal to each other, the mixing matrices in Eqs. (52) and (58) are not unitary. Also, due to the contributions from gluons, $c\bar{c}$, and other higher Fock states, it is possible that the left valence decompositions of the two mesons are not orthogonal to each other. This justifies the two-mixing-angle scenario as a phenomenological method to analyze the contributions from the valence part of pseudoscalar and vector mesons.

In principle, the mixing angles in the valence Fock state decomposition might not be the same as those in the pseudoscalar meson decay constant mixing. Therefore one might introduce more complicated scenarios of three mixing angles or even four mixing angles, also with different combinations. However, such procedures would be too complicated and the physical significance is also obscure; hence we do not consider these complications further in our work.

TABLE I. Decay constants, charge radii, and decay widths of pseudoscalar and vector mesons for fitting π, K, ρ parameters. The experimental data are taken from PDG (2008) [25].

	$F_{\rm exp}/f_{\rm exp}$ (GeV) (input)	$F_{\rm th}/f_{\rm th}$ (GeV) (output)
f_{π^+}	0.0922 ± 0.0001	0.0922
$\langle r_\pi^2 \rangle$ fm^2	0.45 ± 0.01	0.45
$F_{\pi^0\to\gamma\gamma^*}(0)$	0.274 ± 0.010	0.274
$f_{K^+}(K^+ \to \mu\nu)$	0.1100 ± 0.0006	0.1100
$\langle r_{K^+}^2 \rangle$ fm^2	0.31 ± 0.03	0.31
$\langle r_{K^0}^2 \rangle$ fm^2	-0.077 ± 0.010	-0.077
$f_\rho(\rho \to e^+e^-)$	0.1564 ± 0.0007	0.1564
$F_{\rho^+\to\pi^+\gamma}(0)$	0.83 ± 0.06	0.83

TABLE II. Optimized parameters that we get according to the properties of the mesons in Table I.

m_u	m_s	A_π	β_π	A_K	β_K	A_ρ	β_ρ
0.198 GeV	0.556 GeV	47.36 GeV^{-1}	0.411 GeV	68.73 GeV^{-1}	0.405 GeV	48.585 GeV^{-1}	0.373 GeV

IV. NUMERICAL RESULTS AND PREDICTIONS

A. Set π, K, ρ parameters

Following Refs. [9,22–24], we use the decay constants, radii, and decay widths to determine $m_u = m_d = m_q$ (suppose the isospin symmetry), m_s, A_M, and β_M (with $M = \pi$, K, ρ). The experimental data are updated from PDG (2008). The parameters and reproduced quantities of the mesons we obtain are listed in Tables I and II.

B. Set $\eta\eta'$, $\phi\omega$ parameters in the one-mixing-angle scenario in two mixing schemes

Take the octet-singlet scheme first. We accept the mixing angle of η-η' determined by taking into account the $Q^2 \to \infty$ behavior of the form factors of η, η' [1,5,9]; i.e., combining Eqs. (39)–(46) with the experimental pole formula, the pseudoscalar meson mixing angle can be solved:

$$\tan\theta^S = \frac{-(1+c^2)(\rho_1+\rho_2) + \sqrt{(1+c^2)^2(\rho_1+\rho_2)^2 + 4(c^2-\rho_1\rho_2)(1-c^2\rho_1\rho_2)}}{2(c^2-\rho_1\rho_2)}, \tag{62}$$

where $\rho_1 = \sqrt{\frac{\Gamma_{\eta\to\gamma\gamma^*} m_{\eta'}^3}{\Gamma_{\eta'\to\gamma\gamma^*} m_\eta^3}}$, $\rho_2 = \frac{F_{\eta\to\gamma\gamma^*}(Q^2\to\infty)}{F_{\eta'\to\gamma\gamma^*}(Q^2\to\infty)} = \frac{F_{\eta\to\gamma\gamma^*}(0)\frac{1}{1+Q^2/\Lambda_\eta^2}}{F_{\eta'\to\gamma\gamma^*}(0)\frac{1}{1+Q^2/\Lambda_{\eta'}^2}}\Big|_{Q^2\to\infty} = \rho_1\frac{\Lambda_\eta^2}{\Lambda_{\eta'}^2}$. The pole-mass parameters are taken as the CLEO Collaboration results [38]:

$$\Lambda_\eta = 774 \pm 11 \pm 16 \pm 22\ \text{MeV},\qquad \Lambda_\eta = 859 \pm 9 \pm 18 \pm 20\ \text{MeV}. \tag{63}$$

In the octet-singlet mixing scheme the constants are

$$c = \frac{c_0}{c_8}, \tag{64}$$

$$c_P = (c_\pi, c_8, c_0) = \left(1, \frac{1}{\sqrt{3}}, \frac{2\sqrt{2}}{\sqrt{3}}\right). \tag{65}$$

Thus the η-η' mixing angle in the octet-singlet scheme is $\theta_{08}^S = -16.05°$. Then we use the following constraints to set the parameters of η and η':

$$F_{\eta\to\gamma\gamma^*}(0) = \sqrt{\frac{4}{\alpha^2\pi M_\eta^3}\Gamma_{\eta\to\gamma\gamma^*}}, \tag{66}$$

$$F_{\eta'\to\gamma\gamma^*}(0) = \sqrt{\frac{4}{\alpha^2\pi M_{\eta'}^3}\Gamma_{\eta'\to\gamma\gamma^*}}, \tag{67}$$

$$F_{\rho\to\eta\gamma^*}(0) = \sqrt{\frac{3\Gamma_{\rho\to\eta\gamma}}{\alpha}\left(\frac{2m_\rho}{m_\rho^2 - m_\eta^2}\right)^3}, \tag{68}$$

$$F_{\eta'\to\rho\gamma^*}(0) = \sqrt{\frac{\Gamma_{\eta'\to\rho\gamma}}{\alpha}\left(\frac{2m_{\eta'}}{m_{\eta'}^2 - m_\rho^2}\right)^3}. \tag{69}$$

The values of these constraints coming from experimental data are displayed in the first column of Table III. With values of m_u, m_s, A_ρ, β_ρ set in Sec. IV A, we can proceed to determine the parameters of η and η' with these constraints. The reproduced decay widths given by the theoretical fit with optimized parameters are displayed in the second column of Table III.

With the parameters of η, η' set, the parameters and mixing angle of ω-ϕ are set together by the decay widths of ω, $\phi \to e^+e^-$ and the decay widths between ω, ϕ and η, η', i.e. the following constraints:

TABLE III. Experimental values [25] of the η, η' decay widths are compared with theoretical values. Parameters set in different schemes are listed below.

$F_{\rm exp}$ (GeV)		$F_{\rm th}$ (GeV) (one-angle scenario in 08 scheme)	$F_{\rm th}$ (GeV) (one-angle scenario in qs scheme)	$F_{\rm th}$ (GeV) (two-angle scenario in 08 scheme)	$F_{\rm th}$ (GeV) (two-angle scenario in qs scheme)
$F_{\eta\to\gamma\gamma^*}(0)$	0.272 ± 0.007	0.272	0.290	0.272	0.259
$F_{\eta'\to\gamma\gamma^*}(0)$	0.342 ± 0.006	0.342	0.283	0.342	0.317
$F_{\rho\to\eta\gamma^*}(0)$	1.59 ± 0.05	1.53	1.69	1.59	1.66
$F_{\eta'\to\rho\gamma^*}(0)$	1.35 ± 0.06	1.74	1.34	1.35	1.42
θ^S		$-16.05°$	$38.29°$	$\theta_8^S = -26.18°$	$\theta_q^S = 40.57°$
				$\theta_0^S = -2.85°$	$\theta_s^S = 43.89°$
Parameters		$A_{\eta 8} = 27.54\ \text{GeV}^{-1}$	$A_{\eta q} = 34.023\ \text{GeV}^{-1}$	$A_{\eta 8} = 41.65\ \text{GeV}^{-1}$	$A_{\eta q} = 34.40\ \text{GeV}^{-1}$
		$\beta_{\eta 8} = 0.505$ GeV	$\beta_{\eta q} = 0.525$ GeV	$\beta_{\eta 8} = 0.607$ GeV	$\beta_{\eta q} = 0.525$ GeV
		$A_{\eta 0} = 42.50\ \text{GeV}^{-1}$	$A_{\eta s} = 54.11\ \text{GeV}^{-1}$	$A_{\eta 0} = 32.12\ \text{GeV}^{-1}$	$A_{\eta s} = 91.39\ \text{GeV}^{-1}$
		$\beta_{\eta 0} = 0.486$ GeV	$\beta_{\eta s} = 0.525$ GeV	$\beta_{\eta 0} = 0.925$ GeV	$\beta_{\eta s} = 0.525$ GeV

$$\Gamma_{V\to e^+e^-} = \frac{4\pi\alpha^2 f_V^2}{3m_V} \qquad (V = \omega, \phi), \tag{70}$$

$$\Gamma_{V\to S\gamma} = \frac{\alpha}{3}|F_{V\to S\gamma^*}(0)|^2\left(\frac{m_V^2 - m_S^2}{2m_V}\right)^3 \quad (V = \omega, \phi; S = \pi, \eta, \eta'), \tag{71}$$

$$\Gamma_{S\to V\gamma} = \alpha|F_{S\to V\gamma^*}(0)|^2\left(\frac{m_S^2 - m_V^2}{2m_S}\right)^3 \quad (V = \omega; S = \eta'). \tag{72}$$

Combining these experimental constraints with Eqs. (47)–(50), we can get the mixing angle and parameters of ϕ and ω as listed in the second column of Table IV. With all the parameters set as shown in Tables III and IV, we can calculate the Q^2 evolving behavior of transition form factors in the spacelike region according to Eqs. (60) and (61) as shown in Figs. 2–5. Though many efforts were devoted to determining the branching ratios of the decays $V \to P\gamma$ or $P \to V\gamma$ (with $V = \omega, \phi$; $P = \pi, \eta, \eta'$), there are no experimental data about their form factors in the spacelike region. However, there are some data about these form factors in the timelike region obtained through the study of conversion decays of $V \to Pe^+e^-$ [15,16,21]. Supposing analytic continuation of the spacelike transition form factors in our model in the timelike region according to Ref. [39], we get the timelike transition form factors and compare them with the experimental data.

When changing to the quark flavor mixing scheme, we just make the replacements $c_8 \to c_q$, $c_0 \to c_s$, while

TABLE IV. Experimental data [25] for the decay constants and decay widths of ω, ϕ are compared with theoretical values. Parameters set in different schemes are listed below.

$F_{\rm exp}/f_{\rm exp}$ (GeV)		$F_{\rm th}/f_{\rm th}$ (GeV) (one-angle scenario in 08 scheme)	$F_{\rm th}/f_{\rm th}$ (GeV) (one-angle scenario in qs scheme)	$F_{\rm th}/f_{\rm th}$ (GeV) (two-angle scenario in 08 scheme)	$F_{\rm th}/f_{\rm th}$ (GeV) (two-angle scenario in qs scheme)
$f_\phi(\phi\to e^+e^-)$	0.076 ± 0.012	0.076	0.076	0.068	0.076
$f_\omega(\omega\to e^+e^-)$	0.0459 ± 0.0008	0.0458	0.0459	0.0475	0.0456
$F_{\phi\to\pi\gamma^*}(0)$	0.133 ± 0.003	0.133	0.133	0.131	0.132
$F_{\omega\to\pi\gamma^*}(0)$	2.385 ± 0.004	2.080	2.385	2.327	2.295
$F_{\phi\to\eta\gamma^*}(0)$	-0.692 ± 0.007	-0.135	-0.573	-0.581	-0.662
$F_{\phi\to\eta'\gamma^*}(0)$	0.712 ± 0.01	0.267	0.787	0.853	0.742
$F_{\omega\to\eta\gamma^*}(0)$	0.449 ± 0.02	0.477	0.572	0.453	0.457
$F_{\eta'\to\omega\gamma^*}(0)$	0.460 ± 0.03	0.482	0.383	0.450	0.470
θ^V		$42.20°$	$86.82°$	$\theta_8^V = 12.17°$	$\theta_q^V = 86.71°$
				$\theta_0^V = 77.82°$	$\theta_s^V = 93.43°$
Parameters		$A_{\omega 8} = 39.78\ \text{GeV}^{-1}$	$A_{\omega q} = 46.15\ \text{GeV}^{-1}$	$A_{\omega 8} = 215.18\ \text{GeV}^{-1}$	$A_{\omega q} = 51.58\ \text{GeV}^{-1}$
		$\beta_{\omega 8} = 0.481$ GeV	$\beta_{\omega q} = 0.374$ GeV	$\beta_{\omega 8} = 0.332$ GeV	$\beta_{\omega q} = 0.330$ GeV
		$A_{\omega 0} = 17.58\ \text{GeV}^{-1}$	$A_{\omega s} = 579.96\ \text{GeV}^{-1}$	$A_{\omega 0} = 135.52\ \text{GeV}^{-1}$	$A_{\omega s} = 52.28\ \text{GeV}^{-1}$
		$\beta_{\eta 0} = 3.726$ GeV	$\beta_{\omega s} = 0.291$ GeV	$\beta_{\eta 0} = 0.358$ GeV	$\beta_{\omega s} = 0.490$ GeV

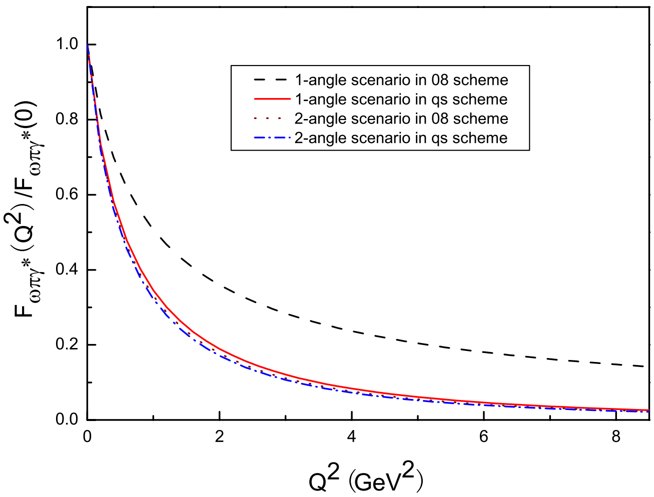

FIG. 2 (color online). Theoretical prediction of the Q^2 behavior of the normalized form factor $F_{\omega\to\pi\gamma^*}(Q^2)/F_{\omega\to\pi\gamma^*}(0)$ in the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme.

$$(c_\pi, c_q, c_s) = \left(1, \frac{5}{3}, \frac{\sqrt{2}}{3}\right). \tag{73}$$

We just suppose $\beta_{\eta_q} = \beta_{\eta_s}$ to simplify the situation. The parameters we get in the quark flavor mixing scheme are listed in the second columns of Tables III and IV.

From Tables III and IV we can see that the results in the quark flavor scheme are better than those in the octet-singlet scheme in reproducing the decay widths related to the pseudoscalar and vector meson mixing. Concerning their Q^2 behaviors after normalized by $F(Q^2)/F(0)$, the results of the two schemes can be compared with each other as shown in Figs. 2–5. Extrapolating Q^2 to the timelike region by $q_\perp \to iq_\perp$ [39], we get form factors in the timelike Q^2 region compared with the experimental data in Figs. 6 and 7. But this region is limited due to the appearance of a singularity in the numerical calculation; i.e., the form factors in a large range of the timelike region cannot be calculated simply through analytic extrapolation. In the limited timelike region our results are comparable with the experimental data. In the process $\omega \to \pi\gamma^*$, the timelike transition form factor produced by the quark flavor scheme is closer to the experimental pole formula simulation compared to the octet-singlet scheme and the vector meson

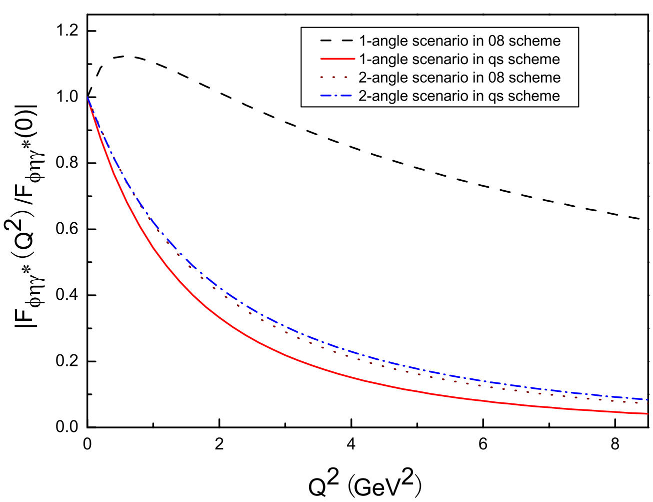

FIG. 3 (color online). Theoretical prediction of the Q^2 behavior of the normalized form factor $F_{\phi\to\eta\gamma^*}(Q^2)/F_{\phi\to\eta\gamma^*}(0)$ in the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme.

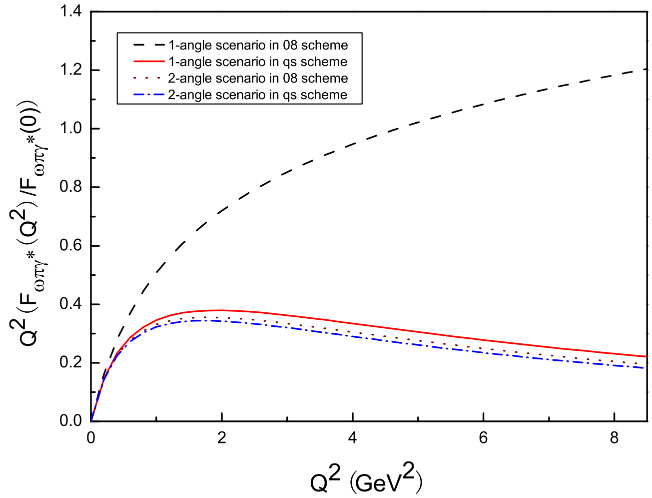

FIG. 4 (color online). Theoretical prediction of the Q^2 behavior of $Q^2 F_{\omega\to\pi\gamma^*}(Q^2)/F_{\omega\to\pi\gamma^*}(0)$ in the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme.

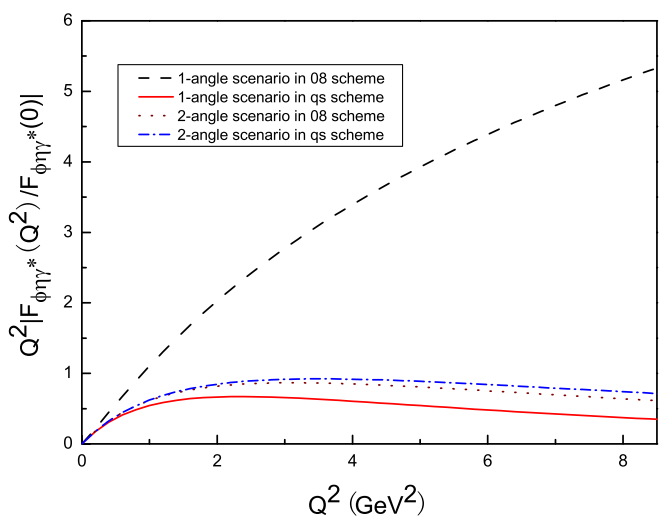

FIG. 5 (color online). Theoretical prediction of the Q^2 behavior of $Q^2 F_{\phi\to\eta\gamma^*}(Q^2)/F_{\phi\to\eta\gamma^*}(0)$ in the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme.

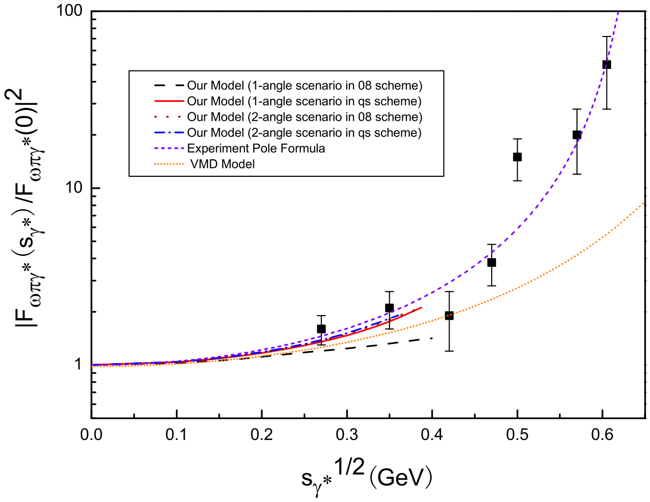

FIG. 6 (color online). The Q^2 behavior of the normalized form factor $F_{\omega\to\pi\gamma^*}(Q^2)/F_{\omega\to\pi\gamma^*}(0)$ using the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme compared with the experimental data [17,21] and the vector meson dominance model result in the timelike region.

dominance models as shown in Figs. 6 and 7. In the process $\phi \to \eta\gamma^*$, the timelike transition form factors produced by the model are comparable to the data, while there are big error bars in the experimental data. More experimental data are needed to reduce the error bars.

The mixing angles we get in the two schemes are, respectively, $\theta_{08}^S = -16.05°$, $\theta_{08}^V = 42.20°$, $\theta_{qs}^S = 38.29°$, and $\theta_{qs}^V = 86.82°$. The pseudoscalar mixing angles approximately follow the ideal SU(3) relation $\theta_{qs}^S = \theta_{08}^S + 54.7°$. They are comparable with the mixing angles determined by the mass relation in PDG (2008) and other papers [12,13]. But the vector mixing angles do not follow the relationship $\theta_{qs}^V = \theta_{08}^V + 54.7°$. If we replace $\theta_{qs}^V - 90° \to \tilde{\theta}_{qs}^V$, i.e., change to another mixing expression:

$$\begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\tilde{\theta}_{qs}^V & -\sin\tilde{\theta}_{qs}^V \\ \sin\tilde{\theta}_{qs}^V & \cos\tilde{\theta}_{qs}^V \end{pmatrix} \begin{pmatrix} -s\bar{s}\varphi^s \\ \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\varphi^q \end{pmatrix}, \quad (74)$$

it can be seen that the mixing angle from the quark flavor mixing scheme $\tilde{\theta}_{qs}^V = -3.18°$ has the opposite sign compared with the one from the octet-singlet mixing scheme $\theta_{08}^V + 54.7° - 90° = 6.9°$ [suppose they can be compared through the ideal SU(3) relationship $\theta_{qs} = \theta_{08} + 54.7°$], and their absolute values are comparable with each other and also with the vector meson mixing angle coming from the mass relation in PDG (2008). In other papers, sometimes the vector meson mixing angle is positive [13,40,41] and sometimes it is negative [14,42,43], and there is always an alternative phase convention [44]. However, their absolute values are all comparable with each other approximately. In this paper we do not use the other phase convention, but just adopt the real rotation of the octet-singlet or quark flavor bases. The results seem to prefer the quark flavor mixing scheme, which gives us a negative vector mixing angle, when introducing only one mixing angle.

The reproduction of decay widths with the one-mixing-angle scenario in the octet-singlet mixing scheme is not as good as that in the quark flavor mixing scheme. In order to improve the octet-singlet mixing scheme, it is natural to introduce the two-mixing-angle scenario.

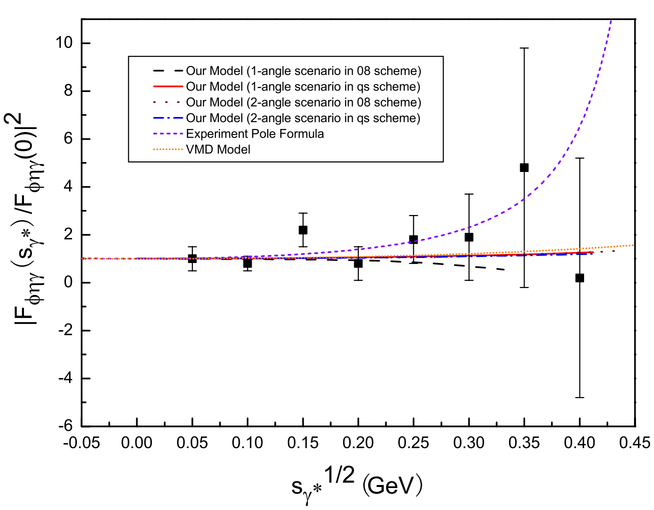

FIG. 7 (color online). The Q^2 behavior of the normalized form factor $F_{\phi\to\eta\gamma^*}(Q^2)/F_{\phi\to\eta\gamma^*}(0)$ using the one-mixing-angle scenario and the two-mixing-angle scenario in the octet-singlet mixing scheme and the quark flavor mixing scheme compared with the experimental data [19] and the vector meson dominance model result in the timelike region.

C. Set $\eta\eta'$, $\phi\omega$ parameters in the two-mixing-angle scenario in two mixing schemes

The two-mixing-angle scenario was introduced to study pseudoscalar meson η-η' mixing, especially concerning the decay constants [4,10,11]. Here we try to introduce two mixing angles to study the vector meson ω-ϕ mixing.

First we restudy η-η' mixing with the two-mixing-angle scenario in the octet-singlet mixing scheme. When introducing two mixing angles, we cannot have explicit solutions of θ_0^S, θ_8^S like in the one-mixing-angle scenario in Eq. (62). Using Eqs. (43), (44), and (54)–(57) as constraints, we can set the pseudoscalar meson mixing angles and the parameters of η, η'. The reproduction of experimental data can be improved as shown in Table III.

With the parameters of η, η' set, we can proceed to set the parameters of ω, ϕ in the two-mixing-angle scenario in the octet-singlet mixing scheme. Using the constraints Eqs. (70)–(72) combined with Eqs. (47)–(50), we get the parameters and reproduction of the decay widths listed in the fourth column in Table IV. Obviously the experimental data are better reproduced in the two-mixing-angle scenario than in the one-mixing-angle scenario.

Though the two mixing angles we get deviate a lot from each other ($\Delta\theta_{08}^S = \theta_0^S - \theta_8^S = 23.33°$, $\Delta\theta_{08}^V = \theta_0^V - \theta_8^V = 65.65°$), the average value of the two mixing angles is comparable with the one-mixing-angle result: $\bar{\theta}_{08}^S = \frac{\theta_0^S+\theta_8^S}{2} = -14.52° \sim \theta_{08}^S = -16.05°$, $\bar{\theta}_{08}^V = \frac{\theta_0^V+\theta_8^V}{2} = 45.00° \sim \theta_{08}^V = 42.20°$.

As reviewed in Ref. [8], both the octet-singlet mixing scheme and the quark flavor mixing scheme can be introduced with two mixing angles, while the results from the η-η' study show that the difference of the two mixing angles in the quark flavor scheme is much smaller than that in the octet-singlet scheme. This suggests that the one-mixing-angle approximation is more reasonable in the quark flavor scheme. So we introduce the two-mixing-angle scenario in the quark flavor mixing scheme to study not only η-η' mixing but also ω-ϕ mixing. The steps are similar to those in the octet-singlet mixing scheme, just by changing the octet and singlet bases to the quark flavor bases and replacing the constants c_8, c_0 by c_q, c_s. The results we get are listed in the final columns of Tables III and IV.

We can see that the reproduction of the experimental data in the two-mixing-angle scenario is also improved compared with the one-mixing-angle scenario in the quark flavor scheme. The differences between the two mixing angles in the quark flavor scheme are much smaller than those in the octet-singlet scheme: $\Delta\theta_{qs}^S = \theta_s^S - \theta_q^S = 3.32° \ll \Delta\theta_{08}^S$, $\Delta\theta_{qs}^V = \theta_s^V - \theta_q^V = 6.72° \ll \Delta\theta_{08}^V$. And their average values are also close to the one-mixing-angle scenario results: $\bar{\theta}_{qs}^S = \frac{\theta_s^S+\theta_q^S}{2} = 42.23° \sim \theta_{qs}^S = 38.29°$, $\bar{\theta}_{qs}^V = \frac{\theta_s^V+\theta_q^V}{2} = 90.07° \sim \theta_{qs}^V = 86.82°$. These results also explain why the one-mixing-angle approximation in the quark flavor scheme is more reasonable when studying η-η' and ω-ϕ mixing.

Now we have four sets of parameters which can be used to reproduce the decay widths of the mesons and calculate the transition form factors of the mesons. The reproduction of the decay widths is improved by introducing two mixing angles in both schemes. The Q^2 evolving behavior of the transition form factors is shown and compared in Figs. 2–7. It is interesting to notice that the three curves we get from the one-mixing-angle scenario in the quark flavor scheme, and the two-mixing-angle scenario in the quark flavor scheme and the octet-singlet scheme are close to each other; however, the curve produced by the one-mixing-angle scenario in the octet-singlet scheme deviates from the other three. Concerning the Q^2 behavior of the meson form factors, the mixing-angle results, and the decay widths fit by the model, the best choices is the two-mixing-angle scenario in the quark flavor mixing scheme and in the octet-singlet mixing scheme. The one-mixing-angle scenario in the quark flavor scheme is also acceptable, while the one-angle-mixing scenario in the octet-singlet mixing scheme deviates a lot from the other three and may be the worst one of the four choices.

V. CONCLUSION

The light-cone quark model is a useful approach to study hadronic properties in the low energy region which is related to nonperturbative QCD. With the decay widths, form factors, and radii of the mesons as constraints, we set the mixing angles and wave function parameters of the pseudoscalar mesons η, η' and the vector mesons ω, ϕ with two mixing-angle scenarios in two mixing schemes. Comparing theoretical results with experimental data, we find that the results from the quark flavor mixing scheme are better than those from the octet-singlet mixing scheme, and the results of the two-mixing-angle scenario are better than those of the one-mixing-angle scenario. We calculate the transition form factors in the spacelike region using the two mixing-angle scenarios in two mixing schemes, respectively, and compare their behavior. By extrapolating the form factors to the limited timelike region, our results are comparable with the experimental data. The absolute values of the vector meson mixing angles we get in the two mixing schemes are comparable with each other. If one only introduces one mixing angle to study processes related to pseudoscalar and vector meson mixing, the quark flavor mixing scheme is more reliable than the octet-singlet mixing scheme. When introducing two mixing angles, both schemes work well.

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APPENDIX A

After getting the wave functions of the mesons through the Melosh-Wigner rotation or vertices in Eq. (13) equivalently, we can calculate the decay constant f_P of a charged pseudoscalar meson P:

$$f_P = I_{P\mu\nu}[m_{q_1}, m_{q_2}, A_P, \beta_P], \tag{A1}$$

in which

$$I_{P\mu\nu}[m_{q_1}, m_{q_2}, A_P, \beta_P] = 2\sqrt{3}\int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3}\varphi_P(\mathbf{k}_\perp)\frac{m_{q_1}(1-x)+m_{q_2}x}{\sqrt{\mathbf{k}_\perp^2+(m_{q_1}(1-x)+m_{q_2}x)^2}}. \tag{A2}$$

The form factor of a pseudoscalar meson P is

$$F_P(Q^2) = Q_{q_1} I_{PP}[m_{q_1}, m_{q_2}, A_P, \beta_P] + Q_{q_2} I_{PP}[m_{q_2}, m_{q_1}, A_P, \beta_P], \tag{A3}$$

in which

$$I_{PP}[m_{q_1}, m_{q_2}, A_P, \beta_P] = \int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \varphi_P^*(x, \mathbf{k}'_\perp)\varphi_P(x, \mathbf{k}_\perp) \frac{(m_{q_1}(1-x) + m_{q_2}x)^2 + \mathbf{k}_\perp \cdot \mathbf{k}'_\perp}{\sqrt{(m_{q_1}(1-x) + m_{q_2}x)^2 + \mathbf{k}_\perp^2}\sqrt{(m_{q_1}(1-x) + m_{q_2}x)^2 + \mathbf{k}'^2_\perp}}. \tag{A4}$$

The transition form factor of a pseudoscalar meson $F_{P\to\gamma\gamma^*}(Q^2)$ is

$$F_{P\to\gamma\gamma^*}(Q^2) = Q_q^2 I_{P\gamma\gamma^*}[m_q, A_P, \beta_P], \tag{A5}$$

in which

$$I_{P\gamma\gamma^*}[m_q, A_P, \beta_P] = 4\sqrt{6}\int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \varphi_P(x, \mathbf{k}_\perp) \times \frac{m_q}{x\sqrt{\mathbf{k}_\perp^2 + m^2}} \frac{x(1-x)}{m_q^2 + \mathbf{k}'^2_\perp}. \tag{A6}$$

The decay constant of a neutral vector meson V is

$$f_V = 2\sqrt{3}\int \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \frac{1}{\sqrt{x(1-x)}} \varphi_V(x, \mathbf{k}_\perp) \times \frac{2\mathbf{k}_\perp^2 + m_q(\mathcal{M} + 2m_q)}{\sqrt{\mathbf{k}_\perp^2 + m_q^2}(\mathcal{M} + 2m_q)}. \tag{A7}$$

APPENDIX B

In the two-mixing-angle scenario in the quark flavor mixing scheme, the mixing of the vector mesons is defined by

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_q^V & -\sin\theta_s^V \\ \sin\theta_q^V & \cos\theta_s^V \end{pmatrix} \begin{pmatrix} |\omega_q\rangle \\ |\omega_s\rangle \end{pmatrix}; \tag{B1}$$

the decay constants and transition form factors of the vector mesons are

$$\begin{pmatrix} f_\phi \\ f_\omega \end{pmatrix} = \begin{pmatrix} \cos\theta_q^V & -\sin\theta_s^V \\ \sin\theta_q^V & \cos\theta_s^V \end{pmatrix} \begin{pmatrix} f_{\omega_q} \\ f_{\omega_s} \end{pmatrix}, \tag{B2}$$

$$\begin{pmatrix} F_{\phi\to\pi\gamma^*}(Q^2) \\ F_{\omega\to\pi\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_q^V & -\sin\theta_s^V \\ \sin\theta_q^V & \cos\theta_s^V \end{pmatrix} \begin{pmatrix} F_{\omega_q\to\pi\gamma^*}(Q^2) \\ 0 \end{pmatrix}, \tag{B3}$$

$$\begin{pmatrix} F_{\phi\to\eta\gamma^*}(Q^2) \\ F_{\phi\to\eta'\gamma^*}(Q^2) \\ F_{\omega\to\eta\gamma^*}(Q^2) \\ F_{\eta'\to\omega\gamma^*}(Q^2) \end{pmatrix} = \begin{pmatrix} \cos\theta_q^V & -\sin\theta_s^V \\ \sin\theta_q^V & \cos\theta_s^V \end{pmatrix} \otimes \begin{pmatrix} \cos\theta_q^S & -\sin\theta_s^S \\ \sin\theta_q^S & \cos\theta_s^S \end{pmatrix} \times \begin{pmatrix} F_{\omega_q\to\eta_q\gamma^*}(Q^2) \\ 0 \\ 0 \\ F_{\omega_s\to\eta_s\gamma^*}(Q^2) \end{pmatrix}, \tag{B4}$$

in which

$$\begin{cases} F_{\omega_q\to\pi\gamma^*}(Q^2) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(2Q_u I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_\pi, \beta_\pi] - 2Q_d I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_\pi, \beta_\pi]) = I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_\pi, \beta_\pi] \\ F_{\omega_q\to\eta_q\gamma^*}(Q^2) = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}(2Q_u I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_{\eta_q}, \beta_{\eta_q}] + 2Q_d I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_{\eta_q}, \beta_{\eta_q}]) \\ \qquad = \frac{1}{3} I_{VP\gamma}[m_q, A_{\omega_q}, \beta_{\omega_q}, A_{\eta_q}, \beta_{\eta_q}] \\ F_{\omega_s\to\eta_s\gamma^*}(Q^2) = 2Q_S I_{VP\gamma}[m_S, A_{\omega_s}, \beta_{\omega_s}, A_{\eta_s}, \beta_{\eta_s}]. \end{cases} \tag{B5}$$

In the two-mixing-angle scenario, θ_q^S, θ_s^S, θ_q^V, and θ_s^V are fit separately. When setting $\theta_q^S = \theta_s^S = \theta_{qs}^S$, $\theta_q^V = \theta_s^V = \theta_{qs}^V$, one returns back to the one-mixing-angle scenario.

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